Math 147 Topology

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Topological Subspaces:

Example: Consider the topological space  2 with the dictionary order.

Q: What are the open sets in the subspace x?

A: All sets are open. We can draw a small open interval about any point, so the set of any one point is open, and any set is the union of such sets. Because of all sets are open, we can consider this topology the discrete topology on x.

Example: Consider x as a subspace of  2, but this time with the usual topology. Again, all sets are open, as we can construct an open ball of radius ½ about any point that doesn’t intersect any others the same way we can construct an open interval.

Small Facts about Subspaces:

Let (X,FX) and (Y,FY) be topological spaces with (S,FS) a subspace of X and (T,FT) a subspace of Y. Then…

1)

2)

3) Suppose is continuous. Then f restricted to S is continuous.

4) Suppose is continuous and . Let be such that

. Then g is continuous.

Proofs:

1. Let C be closed. , so such that . X-U is closed in X since U is open in X. We claim the closed set we desire is X-U.

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Suppose and A is closed. Then , so . But, is open in S, so C is closed in S.

1. Let . f is continuous, so . , so is the intersection of an open set with S and is open. Therefore, f|s is continuous.
2. Let . Then, such that . is continuous so . Since ,

Therefore, is open in X. But , so is open and g is continuous.

In metric spaces, the idea of an open ball is very useful. So, we would like to extend the idea of an open ball to more general topological spaces.

Definition: Let (X,F) be a topological space and such that , U is a union of elements in . Then, we say is a **basis** for F.

Note: this basis is not necessarily minimal, like the basis of a vector space, but it is certainly more useful the more specific we get.

Example: with the half-open interval topology was *defined* by the basis .

Example:  2 with the dictionary topology was similarly defined by a basis of open intervals.

Theorem: Let X be a set and be a collection of subsets such that:

1. , and

Let F=. Then F is a topology on X with basis

Proof: yet to be shown