Ein paar zahlentheoretischer Beweise

Gleichung (1)

$$\forall n : (n \in (\mathbb{N}/\mathbb{P}) \land n \neq 4) \implies n \mid (n-1)! \tag{1}$$

Beweis von (1)

$$n = ab \land a \neq b \implies a < b < n \implies (ab-a) \mid (ab-1)! \land (ab-b)|(ab-1)! \tag{2}$$

$$(2) \implies \mid ab(a-1)(b-1) \mid (ab-1)! \implies ab \mid (ab-1)! \implies n \mid (n-1)! \tag{3}$$

$$n = a^2 \implies (a^2 - a) \mid (ab-1)! \land (ab-2a)|(ab-1)! \implies 1 < a^2 - 2a \tag{4}$$

$$(4) \implies 2 < a^2 - 2a + 1 \implies 2 < (a-1)^2 \implies \sqrt{2} + 1 < a \implies 2 < a \implies n \neq 4 \tag{5}$$

$$(2) \land (5) \implies \forall n : (n \in (\mathbb{N}/\mathbb{P}) \land n \neq 4) \implies n \mid (n-1)! \tag{6}$$

$$\blacksquare \tag{7}$$

Gleichung (7)

$$\forall((2^n - 1) \in \mathbb{P})) \implies n \in \mathbb{P} \tag{8}$$

Beweis von Gleichung (7)

$$(a^n - 1 \in \mathbb{P} \land a \in (\mathbb{N}/\{1\})) \implies (a-1)\sum_{k=0}^{n-1} a^k = a^n - 1 \implies (a-1) = 1 \lor \sum_{k=0}^{n-1} a^k = 1 \tag{9}$$

$$\sum_{k=0}^{n-1} a^k = 1 \implies 1 = \frac{1-a^n}{1-a} \implies a = a^n \implies (a = 1 \implies 1^n - 1 \in \mathbb{P}) \lor ((n = 1) \in \mathbb{P}) \tag{10}$$

$$(8) \wedge (9) \implies a = 2 \implies 2^n - 1 \in \mathbb{P} \tag{11}$$

$$n \notin \mathbb{P} : n = ab \wedge (a, b) \in (\mathbb{N}/\{1\}) \implies (2^a)^b - 1 \in \mathbb{P} \tag{12}$$

$$(8) \implies (2^a - 1) = 1 \vee \frac{1 - 2^{ab}}{1 - 2^a} = 1 \tag{13}$$

$$((2^a - 1 = 1) \implies a = 1) \vee ((\frac{1 - 2^{ab}}{1 - 2^a} = 1) \implies (2^{ab} = 2^a) \implies b = 1) \tag{14}$$

$$(13) \implies ((a = 1) \vee (b = 1)) \implies n \in \mathbb{P} \tag{15}$$

$$(10) \wedge (15) \implies \forall((2^n - 1) \in \mathbb{P})) \implies n \in \mathbb{P} \tag{16}$$

$$\blacksquare \tag{17}$$

Gleichung (18)

$$\forall n \in \mathbb{N} : (n \neq 2(2k + 1) \wedge (k \in \mathbb{N}) \implies n = b^2 - c^2 \wedge ((b, c) \in \mathbb{N}^2) \tag{18}$$

Beweis von Gleichung (18)

$$b > c \implies b = (a + c) \implies n = (a + c)^2 - c^2 \implies n = a(2c + a) \tag{19}$$

$$\neg(2 \perp n) \implies a = 1 \implies n = 2(b - 1) + 1 \tag{20}$$

$$2 \perp n \implies (((2 \perp b) \wedge (2 \perp c)) \vee \neg(2 \perp b) \wedge (2 \perp c)) \implies (2 \perp a) \wedge ((2 \perp c) \vee \neg(2 \perp c)) \tag{21}$$

$$(2 \perp a) \implies (4 \perp n) \implies n \neq 2(2k + 1) \tag{22}$$

$$\blacksquare \tag{23}$$