Introduction to Machine Learning (CSCI-UA.473): Homework 1

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Submission Instructions

You must typeset the answers using LATEX and compile them into a single PDF file. Name the pdf file as ⟨Your-NetID⟩_hw0.pdf and the notebook containing the coding portion as ⟨Your-NetID⟩_hw0.ipynb. The PDF file should contain solutions to both the theory portion and the coding portion. Submit the files through the following Google Form - https://forms.gle/Vqj9ry6o3mqim6Hm6 The due date is **September 20, 2022, 11:59 PM**. You may discuss the questions with each other but each student must provide their own answer to each question.

Questions

Probability and Calculus

Question 1 (10 points)

Two players take turns trying to kick a ball into the net in soccer. Player 1 succeeds with probability 1/5 and Player 2 succeeds with the probability 1/4. Whoever succeeds first wins the game and the game is over. Assuming that Player 1 takes the first shot, what is the probability that Player 1 wins the game? Please derive your answer.

Question 2 (10 points)

You know that 1% of the population have COVID. You also know that 90% of the people who have COVID get a positive test result and 10% of people who do not have COVID also test positive. What is the probability that you have COVID given that you tested positive?

Question 3 (10 points)

Let the function $f(x)$ be defined as:

$$f(x) = \begin{cases} 0 & for\ x < 0 \\ \frac{1}{(1+x)} & otherwise. \end{cases} \tag{1}$$

Is $f(x)$ a PDF? If yes, then prove that it is a PDF. If no, then prove that it is not a PDF.

Question 4 (10 points)

Assume that X and Y are two independent random variables and both have the same density function:

$$f(x) = \begin{cases} 2x & if\qquad\quad 0 \leq x \leq 1 \\ 0 & otherwise \end{cases} \tag{2}$$

What is the value of $\mathbb{P}(X + Y \leq 1)$?

Question 5 (10 points)

Let X be a random variable which belongs to a Uniform distribution between 0 and 1: $X \sim Unif(0,1)$. Let $Y = g(X) = e^X$. What is the value of $\mathbb{E}(Y)$?

Question 6 (10 points)

Suppose that the number of errors per computer program has a Poisson distribution with mean 5. We have 125 program submissions. Let $X_1, X_2, \ldots, X_{125}$ denote the number of errors in the programs. What is the value of $\mathbb{P}(\bar{X}_n < 5.5)$?

Question 7 (10 points)

Let $X_n = f(W_n, X_{n-1})$ for $n = 1, \ldots, P$, for some function $f()$. Let us define the value of variable E as

$$E = ||C - X_P||^2, \tag{3}$$

for some constant C. What is the value of the gradient $\frac{\partial E}{\partial X_0}$?

Linear Algebra

Question 8 (10 points)

Let A be the matrix $\begin{bmatrix} 2 & 6 & 7 \\ 3 & 1 & 2 \\ 5 & 3 & 4 \end{bmatrix}$ and let x be the column vector $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$. Let A^T and x^T denote the transpose of A and x respectively. Compute Ax, A^T and $x^T A$.

Question 9 (10 points)

Find out if the following matrices are invertible. If yes, find the inverse of the matrix.

(a)

$$\begin{bmatrix} 6 & 2 & 3 \\ 3 & 1 & 1 \\ 10 & 3 & 4 \end{bmatrix} \tag{4}$$

(b)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 1 & 4 & 5 \end{bmatrix} \tag{5}$$

Question 10 (10 points)

What is an Eigen Value of a matrix? What is an Eigen Vector of a matrix? Describe one method (any method) you would use to compute both of them. Use the above described method to compute the Eigen Values of the matrix:

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ -2 & 2 & 1 \end{bmatrix} \tag{6}$$