Parallel Algorithms
TD-Communication Networks

Oguz Kaya

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1 Pipelined point-to-point communication

Given a communication network, let $d(i,j)$ be the minimum distance between the processors P_i and P_j. The networks consists in identical links in terms of bandwidth B and latency L. We are interested in finding the point-to-point communication cost for a message of size m between these two processors. In what is called "store-and-forward" routing protocal, to transmit a message, we have to perform an send/receive for each traversed link in the network, which yields the total communication cost

$$c_{i,j}(m) = d(i,j)\Big(L + \frac{m}{B}\Big) = d(i,j)L + d(i,j)mb$$

Indeed, this is not an efficient communication since during each send/receive on a link, the rest of the network is completely unused. In order to attenuate this limitation, we can employ the idea of *pipelining*; we can divide the message into r pieces of size m/r (assuming that r divides m), then transmit these pieces one after the other so that each processor in the routing path between P_i and P_j sends the message piece of size m/r that it received in the previous step, while it receives another piece of size m/r simultaneously.

Question 1

a) Find the complexity $c_{i,j}(m,r)$ for the transmission of a message of size m in a pipelined manner using r pieces, in terms of r, $d(i,j)$, L, and b.

b) What is the best value of r that minimizes this communication cost?

2 Broadcast on a unidirectional ring

A Broadcast can be performed on a unidirectional ring in $(p-1)(L+mb)$ time with a simple algorithm where each processor receives the entire message from the processor to the left, then transmits it to the processor to the right. However, this yields an inefficient use of the network since only one link is used at a time in the network while other $(p-1)$ links are unused.

The classical technique for preventing such inefficiencies is to "pipeline" the operations, which is heavily used in modern processors to to cut each complex instruction into smaller pieces, and execute these in different units of the processors in parallel by pipelining such pieces of multiple instructions that are independent.

Here, we will employ the same idea to utilize the entire network by communicating in parallel between pairs of processors and reducing the overall communication cost. Instead of sending the entire message of size m, we will divide the message into r pieces of size m/r (assuming that r divides m), then communicate these smaller messages one after the other to the next processor in the ring.

Question 2

a) Design an algorithm Broadcast$(k, addr, m)$ that sends the message at the address $addr$ of size m in the processor P_k to all other processors in a pipelined manner using r pieces/stages. Assume that r divides m.

b) Find the total communication cost in terms of p, L, m, b, and r.

c) What is the value of r that minimizes this communication cost?

3 Scatter on a hypercube

Question 3

a) For a hypercube network topology, write an algorithm SCATTER($k, msg[m], addr[P][m]$) that sends a message of size m in the array $addr[i]$ of the processor P_k into the array msg in the processor P_i for all $1 \leq i \leq p$.

b) Find the total communication cost.

4 Transposition on a unidirectional ring

On a unidirectional ring with p processors, we suppose that each processor has a message $sendMsg[i]$ of size N to each other processor P_i on the ring for i $(0 \leq i < p)$ (including the processor itself).

Question 4

a) Develop an algorithm TRANSPOSITION($sendMsg[p][N], recvMsg[p][N]$) on a unidirectional ring in which each processor P_i sends its local array $sendMsg[j]$ to the processor P_j and receives the array send to it by P_j in its $recvMsg[j]$, for all $0 \leq j < N$.

b) Find the communication cost in terms of L, b, N, and P.

5 Grids, hypercubes, and binary trees

Question 5

a) What is the diameter of a complete binary tree topology with the height h?

b) What is the number of unique routing paths of minimal length between two processors with coordinates (x_1, y_1) and (x_2, y_2) on a grid with dimensions $n \times n$?

c) Prove that the bisection width of a d-cube (having 2^d processors) is 2^{d-1}.