100,00

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Middle East Technical University
Department of Computer Engineering

CENG 477

Fall '18

Midterm Exam #1

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- **Duration:** **120** minutes.
- **Grading:**
  - Each of the **15** TRUE-FALSE questions is worth **2** points.
  - Each of the **10** Multiple-choice questions is worth **5** points.
  - Each of the **2** Classical-type questions is worth **10** points.
  - For TRUE-FALSE and multiple-choice questions **4** wrong points cancel out **1** correct point.

26: 10/10
27: 10

- **Asking questions:** <u>is not allowed.</u> If you decide that a question is wrong:
  - DO NOT ask the proctor about a clarification.
  - Indicate clearly your objection and your proposed answer on the **first page of the question booklet.**
- **Mark** your group ID (as A or B) on your answer sheet.
- **Turn in** your question booklet (this booklet) together with the answer sheet. Otherwise your answer sheet will not be evaluated, and you will receive a zero from this exam.
- GOOD LUCK !

→ I believe there is a problem with question 27. The question states a 400x300 image, but illustrates a 401x301 image due to zero-based indexing, showing the top-left pixel as (0,0) and bottom-right pixel as (400,300); while it should be (399,299). OK. You are right. This also means that the middle point is not in the middle of a pixel as in the illustration, but between four pixels => OK.

While solving the question, I disregarded the illustration and took the bottom-right pixel as (399,299) and the dimensions as 400x300 ✓

→ Also, for question 15, I assume that rectangular images can be scaled. ✓ OK.

All that aside, cool exam :)

We start with TRUE-FALSE questions, mark $\boxed{A}$ box for TRUE, $\boxed{B}$ box for FALSE on your <u>answer sheet</u>

F $\boxed{1}$ In Blinn-Phong shading of a shiny sphere, when we increase the specular exponent (shininess), the specular highlight on the sphere will get larger.

F $\boxed{2}$ The specular (Blinn-Phong shading) component of the ray tracing illumination model depends on the viewer's position and the light position, but it does not depend on the normal vector of the surface.

F $\boxed{3}$ In bilinear interpolation for texture mapping, the nearest two pixels' colors are interpolated to find the final color.

T $\boxed{4}$ In ray tracing, with everything else being constant, the image size of the objects become smaller if the image plane is brought closer to the camera.

T $\boxed{5}$ The surface color obtained by texture mapping can be used as an object's reflectance coefficient in ray tracing computations.

F $\boxed{6}$ In a k-D tree used to partition a 3D scene, each interior node has $k = 3$ children.

F $\boxed{7}$ No objects will be in shadow if there are three or more light sources that are separated by 120° in a scene. seperated by 120° on an arbitrary plane, I assume...

T $\boxed{8}$ The surface of a unit sphere can be modeled by a parametric equation with two parameters.

T $\boxed{9}$ In ray tracing, the color of a pixel is independent from the colors of neighboring pixels.

F $\boxed{10}$ Vectors remain unchanged by all modeling transformations.

F $\boxed{11}$ The dot product of any two vectors gives the cosine of the angle between them.

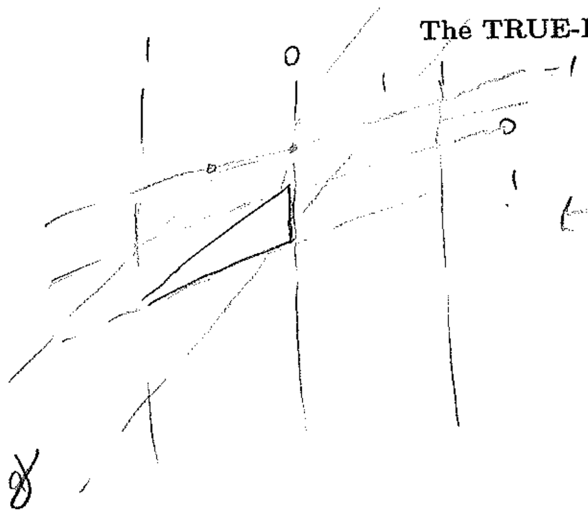
F $\boxed{12}$ The barycentric coordinates of a point inside a triangle will add up to 1 even if some of them may be negative. ?

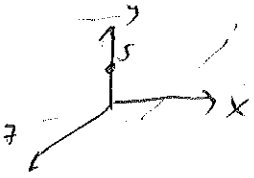
F $\boxed{13}$ The running-time complexity of ray tracing grows quadratically with the number of pixels.

T $\boxed{14}$ Diffuse shading components of a surface point are the same for cameras located at different points.

✓F $\boxed{15}$ A rectangular image cannot be used for texture mapping of a triangle.

The TRUE-FALSE questions END here.





16 Which of the following matrices can be used to draw the reflection of an object from a mirror with plane equation y=5?

A)
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

B)
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

C)
$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & -1 & 0 & -5 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

D)
$$\begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

E)
$$\begin{bmatrix} -1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

slow check: ✓

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

17 Assume that a 3 × 3 modeling transformation matrix is defined as follows:

$$m = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (M^{-1})^T \qquad m^{-1} = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What is the correct transformation matrix to transform the normals?

A)
$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

B)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

C)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

D)
$$\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

E)
$$\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

18 Assume that we want to store 8 different vertices and a closed-mesh made up of 12 triangles to represent a shape. Each vertex is made up of 3 floating point numbers (assume one float occupies 4 bytes). Each index is represented using an integer value (assume one integer also occupies 4 bytes). If the only extra information that the file contains is the number of vertices (one integer) and number of triangular faces (another integer), how many total bytes will be used to represent this mesh using an Indexed-Face-Set representation that only supports triangles?

A) 216
B) 232
C) 248
D) 256
E) 282

Faces → 12 bytes × 12 = 144
Vertices → 12 bytes × 8 = 96
extra: 8 bytes

248

Assuming we want linear output. Otherwise, a gamma of 2.2 is good for the human visual system.

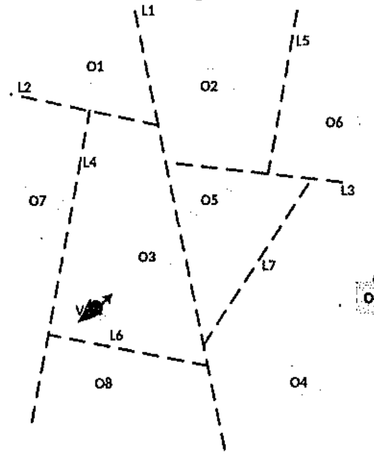
19 Assume that a display device has a gamma (γ) value of 2.5. What is the most appropriate gamma-correction value that you should use to prepare an image for this display device?

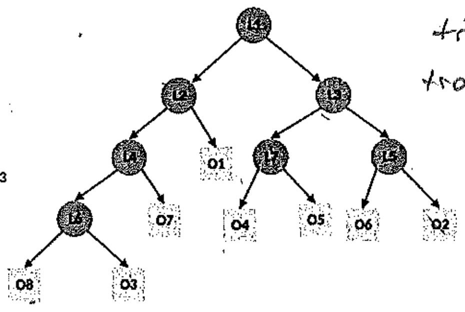
A) 0.2
B) 0.3
C) 0.4
D) 1.5
E) 2.5

$\frac{1}{2.5} = 0.4$

Then, we would use, 0.88.

20 Consider the BSP Tree given below for the 8 objects, O1, ...,O8. traversal dips: traverse other side, traverse camera side





not sure!

O6, O2, O4, O5, O1, O7, O8, O3

If the camera is in the same sub-space as in O3 as shown in the figure above, which of the following correctly list the leaf nodes in a back-to-front ordering with respect to the camera?

A) O8, O7, O3, O1, O4, O5, O6, O2
B) O6, O2, O4, O5, O1, O7, O8, O3
C) O4, O5, O6, O2, O8, O7, O1, O3
D) O6, O2, O5, O4, O7, O1, O8, O3
E) O2, O6, O5, O4, O7, O1, O3, O8

$\vec{w}_r$ $\vec{n}$ $\vec{w}_i$

in unit

21 Find the reflection of the vector $\vec{w}_i = [1/\sqrt{14} \quad 2/\sqrt{14} \quad 3/\sqrt{14}]$ along the surface normal $\vec{n} = [0 \quad 1/\sqrt{2} \quad 1/\sqrt{2}]$. - unit

A) $[2/\sqrt{56} \quad -6/\sqrt{56} \quad 4/\sqrt{56}]$
B) $[2/\sqrt{56} \quad 4/\sqrt{56} \quad -6/\sqrt{56}]$
C) $[-2/\sqrt{56} \quad 4/\sqrt{56} \quad 6/\sqrt{56}]$
D) $[-2/\sqrt{56} \quad 6/\sqrt{56} \quad 4/\sqrt{56}]$
E) $[2/\sqrt{56} \quad 6/\sqrt{56} \quad -4/\sqrt{56}]$

$\vec{w}_r = 2(\vec{n}\cdot\vec{w}_i)\vec{n} - \vec{w}_i$

$= (0, \frac{10}{\sqrt{56}}, \frac{10}{\sqrt{56}}) - \vec{w}_i = \frac{1}{\sqrt{56}}(-2,6,4)$

22 Find the intersection point of a ray with origin $o = [3 \quad 2 \quad 1]$ and direction $\vec{d} = [5 \quad 2 \quad 8]$ with a plane with surface normal $\vec{n} = [1/\sqrt{2} \quad 0 \quad -1/\sqrt{2}]$ and passing through the point $a = [6 \quad 4 \quad 7]$.

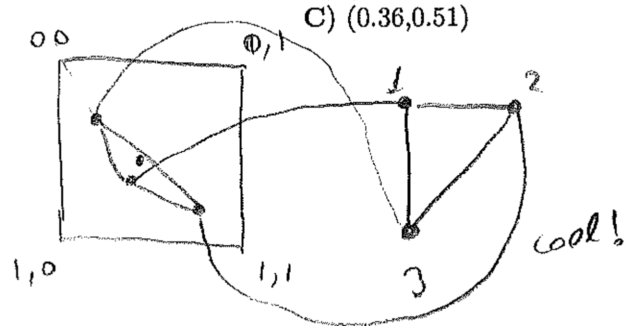
A) $[6 \quad 7 \quad 2]$
B) $[8 \quad 4 \quad 9]$ - cross-checked ✓
C) $[6 \quad 2 \quad 8]$
D) $[1/\sqrt{2} \quad 2 \quad -1/\sqrt{2}]$
E) $[-1/\sqrt{2} \quad 3 \quad -2/\sqrt{2}]$

$[8, 4, 9]$

$\vec{n}\cdot(a-(o+t\vec{d})) = 0 \qquad \vec{n}\cdot(3-5t, ?, 6-8t) \Rightarrow 3-5t+8t-6=0, \ t=1$

23 Consider a triangle with the following texture coordinates at its vertices: $(u_1, v_1) = (0.4, 0.6)$, $(u_2, v_2) = (0.8, 0.8)$, and $(u_3, v_3) = (0.2, 0.3)$. What will be the texture coordinates of the point on the triangle identified by the following Barycentric coordinates: $\alpha = 0.2$, $\beta = 0.3$, associated with vertices 1 and 2 respectively.

A) (0.36,0.36)
B) (0.42,0.36)
C) (0.36,0.51)
D) (0.42,0.51)
E) none of the above



$0.2\times0.4 + 0.3\times0.8 + 0.5\times0.2 = 0.42$

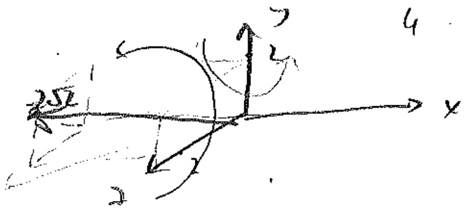
$0.2\times0.6 + 0.3\times0.8 + 0.5\times0.3 = 0.51$

$\boxed{24}$ Given the following transformation definitions:

- $R_x(\theta)$: Rotate a point around the x axis counter-clockwise by θ degrees
- $R_y(\theta)$: Rotate a point around the y axis counter-clockwise by θ degrees
- $R_z(\theta)$: Rotate a point around the z axis counter-clockwise by θ degrees

Which of the following transformations will rotate any given point along the line passing through $P_1 = (0, 0, 0)$ and $P_2 = (-2\sqrt{2}, 2, 2)$ by an angle of θ degrees counter-clockwise?

A) $R_x(-45^o)R_y(-45^o)R_z(\theta)R_y(45^o)R_x(45^o)$
B) $R_x(-45^o)R_y(-45^o)R_y(\theta)R_y(45^o)R_x(45^o)$
C) $R_x(-45^o)R_y(-45^o)R_x(\theta)R_y(45^o)R_x(45^o)$
D) All of the above
E) None of the above

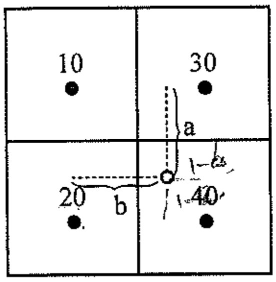


$\boxed{25}$ Assume that the uv coordinates of a texture point is indicated by the empty circle in the diagram below. Its distance to the top pixel is given by $a = 0.75$ and the left pixel by $b = 0.60$. The numbers above filled circles indicate the intensities of different pixels. Compute the final color that should be used for this texture point assuming <u>bilinear interpolation</u>.

A) 28.5
B) 29.0
C) 29.5
D) 28.0
E) 30.0

$\frac{3}{5} \cdot \frac{3}{4} = 0.45$

0.25×0.4 2



1 → $40ab + 20a(1-b)$
$+ 30(1-a)b + 10(1-a)(1-b)$
$= 40ab + 20a - 20ab$
$+ 30b - 30ab + 10 + 10ab$
$- 10a - 10b$

$6 + 4.5 + 1 + 18.$
$= 29.5$

The Multiple-choice questions END here.

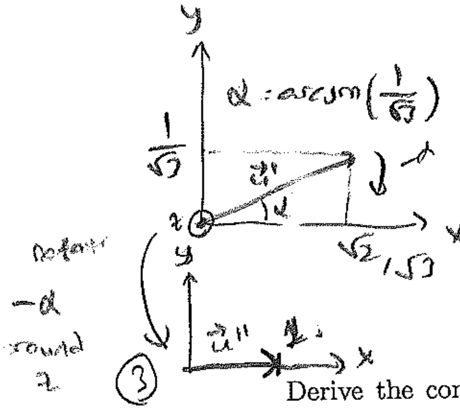
$0ab + 10a + 20b + 10$

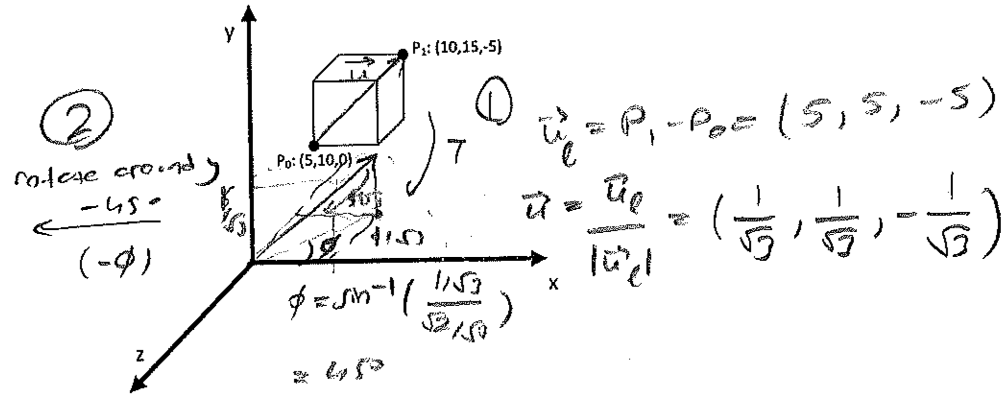
$7.5 + 12 + 10 = 29.5$

Classical questions BEGIN here. You must show your work with a clear writing. You can use the back of the page if needed.

Assuming + is CCW and − is CW for rotations.

26 Consider the 3D cube shown below.





Derive the composite transformation matrix as a multiplication of basic transformation matrices to rotate the cube θ degrees around the axis defined **from vertex** P_0 **to vertex** P_1. **The direction of the rotation axis is from** P_0 to P_1. Write your solution as a sequence of basic transformations. In other words, you do **not** need to write or multiply any 4×4 matrices. You may indicate basic transformations as $R_*(\alpha)$: rotate around *-axis (* is either x, y, or z) α degrees and $T(tx, ty, tz)$: translate tx, ty, and tz units. You may use the *arccos*, i.e., cos^{-1} and the *arcsin*, i.e., sin^{-1} functions to indicate angles in your rotations.

Honestly, I prefer the alternative orthonormal basis method. But let's go!

* Let the transformation matrix be M. First, we have to translate all vertices so that P_0 is at the origin: $\boxed{M = T(5,10,0)\, M_0\, T(-5,-10,0)}$ (see ①)
* Then, we can bring the $\vec{u}$ vector to the xy plane by rotating it $-45°$ degrees around the y-axis: $\boxed{M_0 = R_y(+45°)\, M_1\, R_y(-45°)}$ (see ②)
* Then, we can bring the new $\vec{u}'$ vector to the x axis by rotating it $-\alpha$ degrees around z, where $\alpha = \arcsin(1/\sqrt{3})$: $\boxed{M_1 = R_z(\alpha)\, M_2\, R_z(-\alpha)}$ (see ③)
* Finally, now that our vector ($\vec{u}''$) is aligned with x, we can perform the rotation around the x axis: $\boxed{M_2 = R_x(\theta)}$
* Since we have embedded the inverse operations as well, we can easily write the full composite matrix:

final result

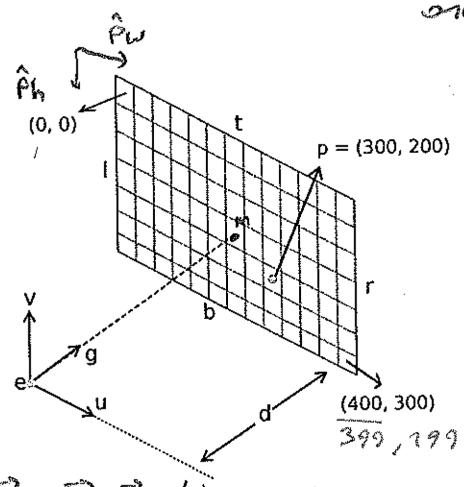
$$M = T(5,10,0)\, R_y(+45°)\, R_z\left(\arcsin\left(\tfrac{1}{\sqrt{3}}\right)\right) R_x(\theta)\, R_z\left(-\arcsin\left(\tfrac{1}{\sqrt{3}}\right)\right) R_y(-45°)\, T(-5,-10,0)$$

* The question states that the image is 400x300, but the illustrated image plane is 401x301 (zero-indexed, last is 400,300 instead of 399,299). I will assume the illustration is wrong (m passes between pixels, not from the middle of one)

27 Assume that you are given the following configuration for ray tracing defined in a right-handed world coordinate system:

10

Eye position (e):	[3, 4, 5]
View direction (g):	[0, 0, −1]
Up vector (v):	[0, 1, 0]
Near plane (NP) distance (d):	10
NP left coordinate (l):	−10
NP right coordinate (r):	10
NP bottom coordinate (b):	−7.5
NP top coordinate (t):	7.5
Image width (pixels):	400
Image height (pixels):	300
Image origin:	top-left



$\vec{u} = \vec{g} \times \vec{v} = (1, 0, 0)$

Given the configuration above, find the world coordinate of pixel $p(x = 300, y = 200)$.

Let m be the midpoint of the image plane, $\hat{p}_h$ a vector advancing one pixel by height and $\hat{p}_w$ a vector advancing one pixel by width

m corresponds to the middle, which is between four pixels, at $(199.5, 149.5)$ assuming zero-based indexing

Then, we can formulate $p = m + 100.5\,\hat{p}_w + 50.5\,\hat{p}_h$ (1)

$m = e + d\vec{g} = (3, 4, -5)$, $\hat{p}_w = \frac{r-l}{n_w}\vec{u} = \frac{1}{20}\vec{u} = (0.05, 0, 0)$, $\hat{p}_h = \frac{t-b}{n_h}\vec{v} = -\frac{1}{20}\vec{v}$
$= (0, -0.05, 0)$

$p = (3, 4, -5) + 100.5(0.05, 0, 0) + 50.5(0, -0.05, 0)$, using (1)
$= (8.025, 1.475, -5)$ //

Compute the world coordinate of the primary ray passing through the same pixel p at ray parameter $t = 2$. Assume that at $t = 1$ the ray will be on the image plane.

$r(t) = e + \underbrace{(p - e)}_{\vec{d}}\,t \Rightarrow r(1) = p$, on the image plane ✓

⇓

$r(2) = e + 2(p - e) = 2p - e = (16.05, 2.95, -10) - (3, 4, 5)$

$= (13.05, -1.05, -15)$

Excellent ✓