Optimal Strain Sensor Placement Strategy for Deepwater Risers

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April 4, 2018

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Outline

I. Background

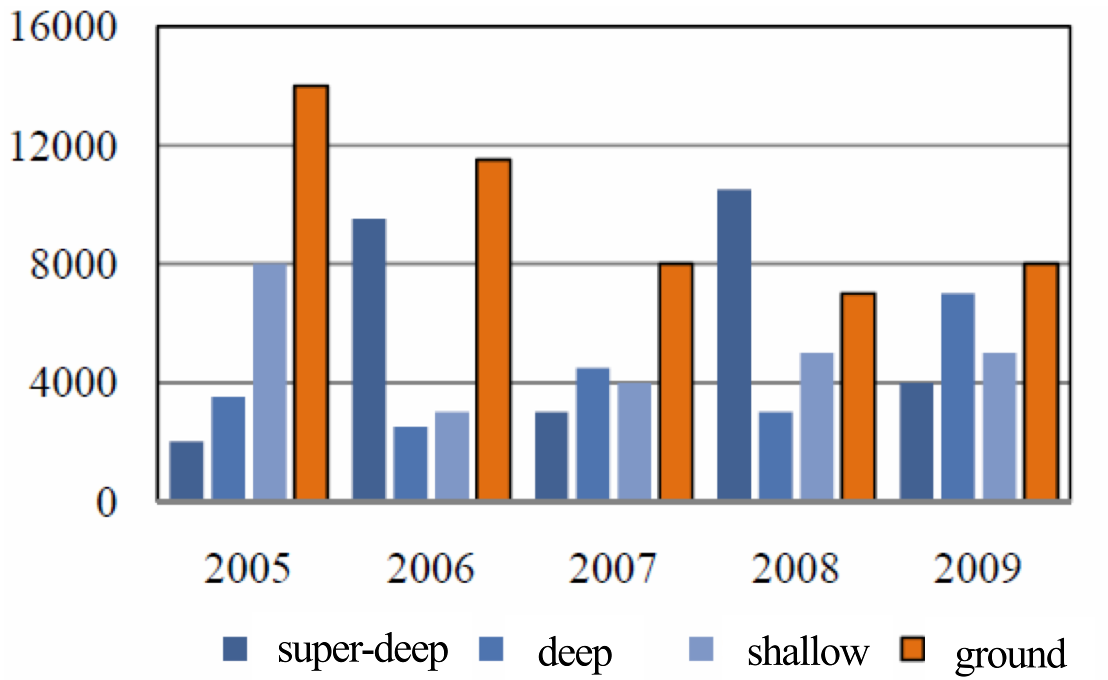
II. FEM Modelling

III. Optimal Sensor Placement Strategy

IV. Discussion

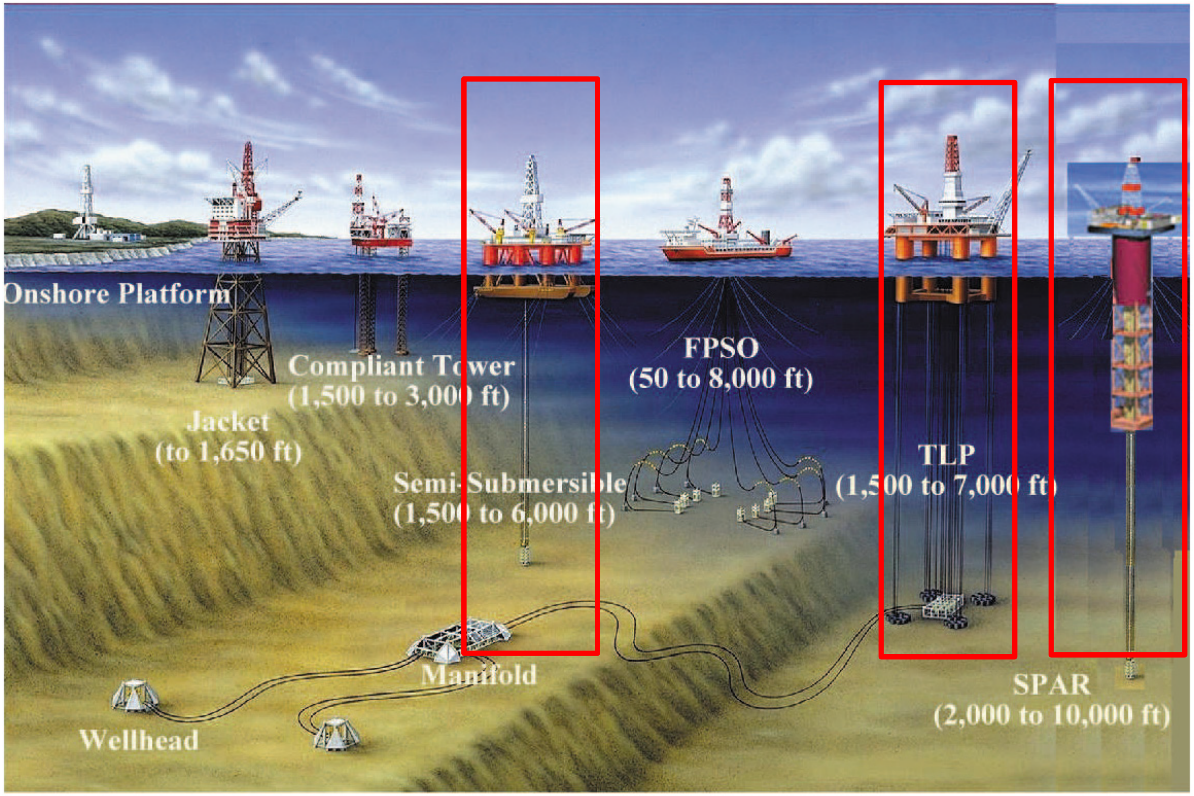
I.1 Research Need

Ocean contains **70%** of the total oil and natural-gas on earth, and in 2015, **12%** are drilled from deep water (>= 900m).

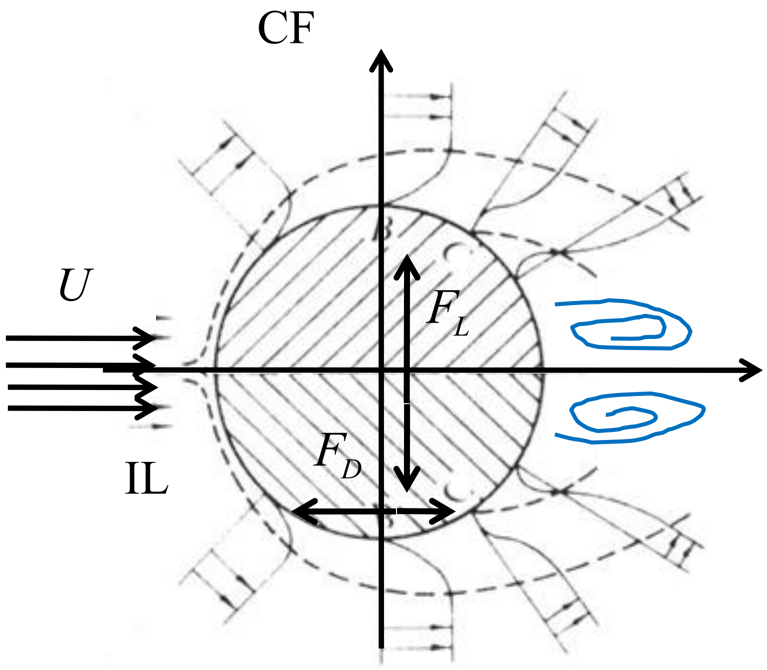


I.2 Types of Platform

Semi-submersible, tension-leg (TLP), SPAR, FPSO, etc



I.3 Vortex-induced Vibration: Mechanism



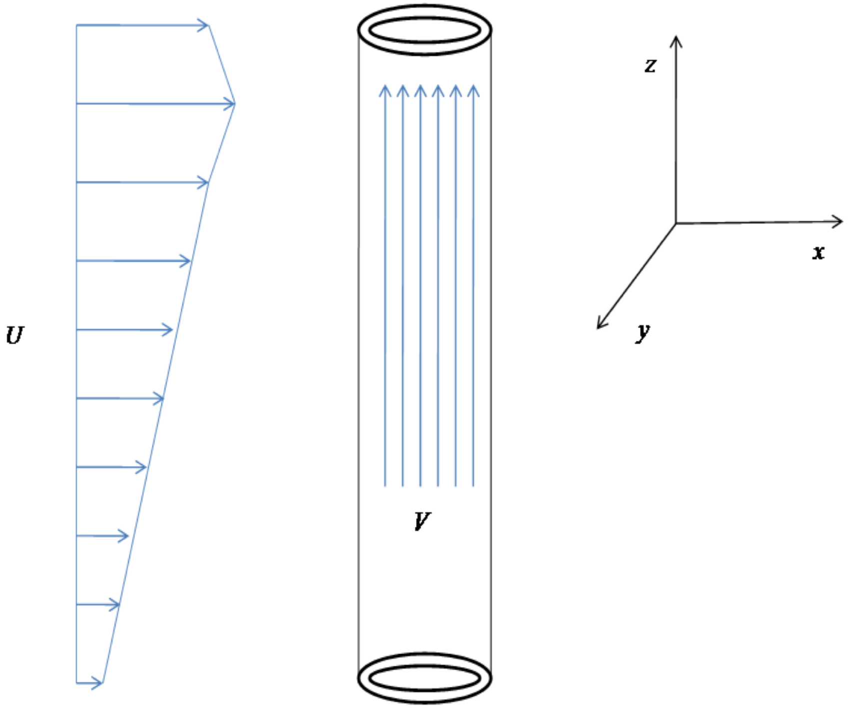
$$F_L = \frac{1}{2} C_L \left(\frac{A}{D}\right) \rho_w D \left(U_z - \frac{\partial x}{\partial t}\right)^2 \cos \omega_s t$$

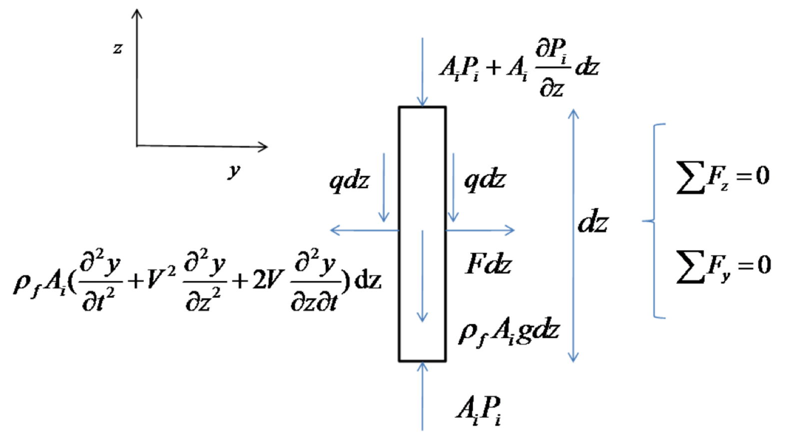
$$F_D^{'} = \frac{1}{2} C_D^{'} \rho_w D U_z^{\,2} \cos \omega_s^{'} t$$

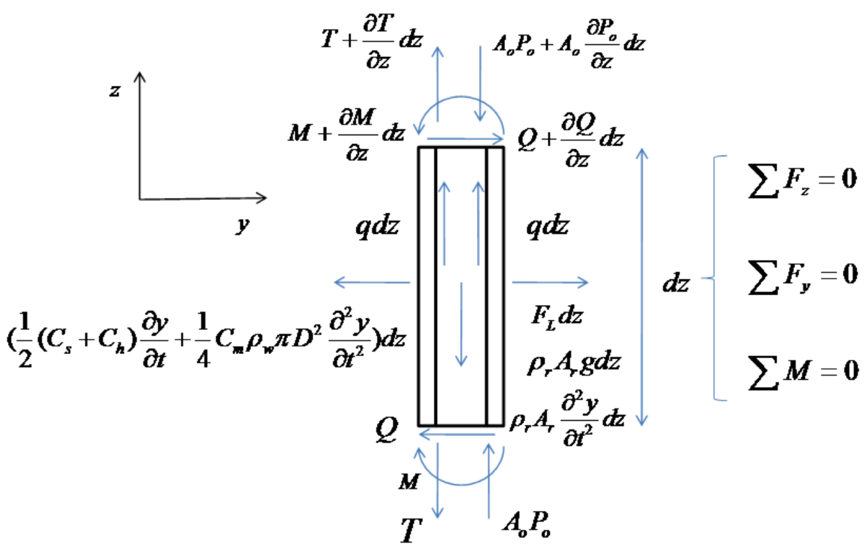
I.4 Vortex-induced Vibration: Damage on Risers

Time	Cause	Description	Leakage (Barrel)
2002	**VIV**	Connection Failure from platform	267
2003	Storm	Breakage	1421
2003	Ocean current	Breakage due to fatigue	74
2003	**VIV**	Breakage due to fatigue	137
2004	Wave load	Connection Failure from platform	48
2004	Storm	Connection Failure from platform	1034
2005	Wave load	Connection Failure from platform	426

II. Modelling: FEM







II. Modelling: FEM

- **Dynamic Equation**
  - Cross-flow

$$\bar{m}\frac{\partial^2 y}{\partial t^2}+(C_s+C_h)\frac{\partial y}{\partial t}+\frac{\partial}{\partial z}\left\{[m_f V^2-T_{top}+(m_r+m_f-\rho_w A_o)g(L-z)]\frac{\partial y}{\partial z}\right\}+EI\frac{\partial^4 y}{\partial z^4}=\frac{1}{2}C_L(\frac{A}{D})\rho_w D(U-\frac{\partial x}{\partial t})^2 cos\omega_s t$$

  - In-flow

$$\bar{m}\frac{\partial^2 x}{\partial t^2}+(C_s+C_h)\frac{\partial x}{\partial t}+\frac{\partial}{\partial z}\left\{[m_f V^2-T_{top}+(m_r+m_f-\rho_w A_o)g(L-z)]\frac{\partial x}{\partial z}\right\}+EI\frac{\partial^4 x}{\partial z^4}=\frac{1}{2}C_D^{'}\rho_w DU^2 cos\omega_s^{'} t$$

II. Modelling: Modal Analysis

- **Modal Analysis**

$$[M][\ddot{u}]+[C][\dot{u}]+[K][u]=[F]$$

$$[u]=\sum_{n}\phi_n q_n(t)$$

$$M_r\ddot{q}_r(t)+C_r\dot{q}_r(t)+K_r q_r(t)=F_r(t)$$

$$M_r=\phi_r^T[M]\phi_r \quad C_r=\phi_r^T[C]\phi_r \quad K_r=\phi_r^T[K]\phi_r \quad F_r(t)=\phi_r^T[F]$$

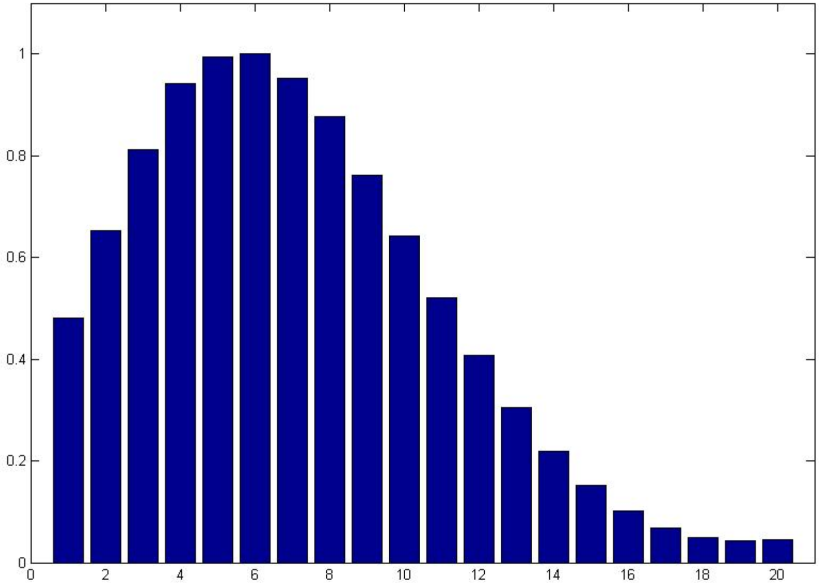
- **Reduction Criterion (Modal Selection)**

$$\prod^r=\frac{|F_r|^2}{2\omega_r C_r}$$

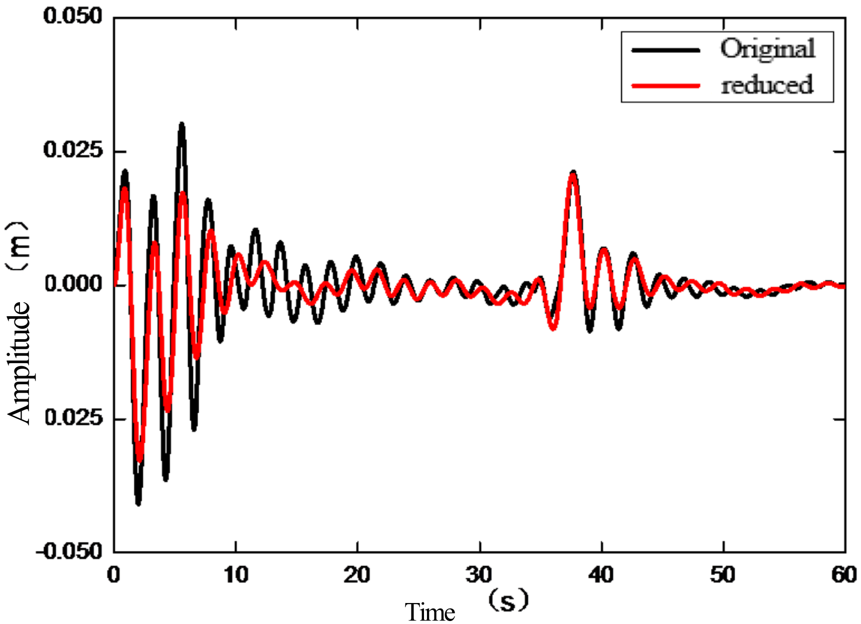
$$\beta_r=\frac{\prod^r}{\prod^{\max}}$$

II. Modelling: Reduction

- **Reduction**



The relative motion energy of the first 20 mode for riser



The comparison between “reduced” and true model at 24m

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III. Optimal Strain Sensor Placement

- **Criterion: based on H_2 norm (modal sensitivity)**
  - Function reformulation

$$M_m\ddot{q}_m + D_m\dot{q}_m + K_m q_m = \phi^T B_o u$$

$$y = C_o\phi q_m + C_o^{'}\phi\dot{q}_m$$

$$M_m = \phi^T M\phi = I$$

$$D_m = \phi^T D\phi = diag[2\zeta_1\omega_1, 2\zeta_2\omega_2, ...2\zeta_n\omega_n]$$

$$K_m = \phi^T K\phi = diag[\omega_1^2, \omega_2^2, ...\omega_n^2]$$

$$\dot{x} = A_m x + B_m u$$

$$y = C_m x$$

$$x_i = \begin{bmatrix} q_{mi} \\ \dot{q}_{mi} \end{bmatrix}$$

III. Optimal Strain Sensor Placement

- **Criterion: based on H_2 norm (modal sensitivity)**
  - Transfer Function (Laplace)

$$G(\omega) = \sum_{i=1}^{k} G_{mi}(\omega) = \sum_{i=1}^{k} \frac{(C_{moi} + j\omega C'_{moi})b_{mi}}{\omega_i^2 - \omega^2 + 2j\zeta_i\omega_i\omega}$$

$$G(\omega_i) \cong G_{mi}(\omega_i) = \frac{(-jC_{moi} + \omega_i C'_{moi})b_{mi}}{2\zeta_i\omega_i^2}$$

  - H_2 norm

$$\| G \|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} tr(G^*(\omega)G(\omega))d\omega$$

  - For the i^{th} modal

$$\| G_i \|_2^2 = \sum_{j=1}^{r} \| G_{ij} \|_2^2$$

$$\| G_{ij} \|_2 = \frac{\| B_{mi} \|_2 \| C_{mji} \|_2}{2\sqrt{\zeta_i\omega_i}}$$

III. Optimal Strain Sensor Placement

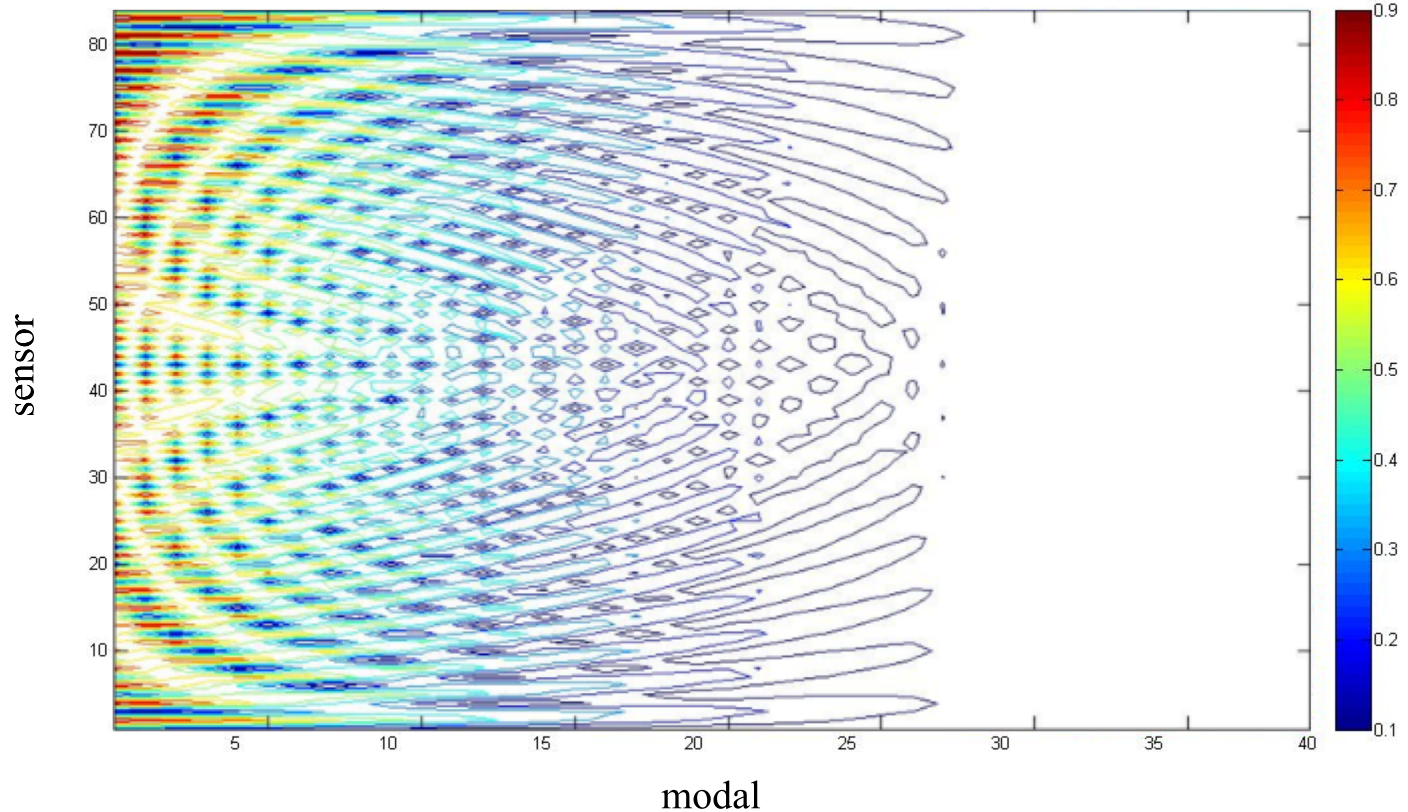
- **Criterion: based on H_2 norm (modal sensitivity)**
  - Optimal Selection Matrix

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \phantom{\sigma_{211}} & \phantom{\sigma_{212}} & & j^{th}\text{ sensor} & & \phantom{\sigma_{21r}} \end{array} \\
 & \begin{bmatrix}
\sigma_{211} & \sigma_{212} & \cdots & \sigma_{21j} & \cdots & \sigma_{21r} \\
\sigma_{221} & \sigma_{222} & \cdots & \sigma_{22j} & \cdots & \sigma_{22r} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\sigma_{2i1} & \sigma_{2i2} & \cdots & \sigma_{2ij} & \cdots & \sigma_{2ir} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\sigma_{2k1} & \sigma_{2k2} & \cdots & \sigma_{2kj} & \cdots & \sigma_{2kr}
\end{bmatrix} \\
i^{th}\text{ modal (row } i\text{)} &
\end{array}
$$

$$
\sigma_{2ij} = \frac{\| G_{ij} \|_2}{\| G \|_2} \qquad \| G \|_2 = \sum_{i=1}^{k} \| G_i \|_2^2 = \sum_{i=1}^{k} \sum_{j=1}^{r} \| G_{ij} \|_2^2
$$

III. Optimal Strain Sensor Placement

- **Criterion: based on H_2 norm**
  - Optimal Selection Matrix



III. Optimal Strain Sensor Placement

- **Criterion: based on H_2 norm**
  - Reduction based on sensor sensitive similarity

$$g_i = \begin{bmatrix} \| G_{1i} \|_2^2 \\ \| G_{2i} \|_2^2 \\ \vdots \\ \| G_{ki} \|_2^2 \end{bmatrix}$$

$$r_{ij} = \frac{g_i^T g_j}{\| g_i \|_2 \| g_j \|_2}$$

III. Optimal Strain Sensor Placement

- **Procedure**
  - 1. Define the set of candidate locations, determine the number of sensors r based on the total number of excited modes k being considered ($r >= k$).
  - 2. Reduce the candidate set based on modal sensitivity $\boldsymbol{H_2}$ (exhaustive search).
  - 3. Make further reduction based on sensor sensitive similarity.

III. Optimal Strain Sensor Placement

- **Evaluation + Kalman Filter**

$$x_k = Fx_{k-1} + Gu_{k-1} + \omega_{k-1}$$
$$y_k = \boxed{Cx_k + \upsilon_k}$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_k(y_k - C\hat{x}_k^-)$$

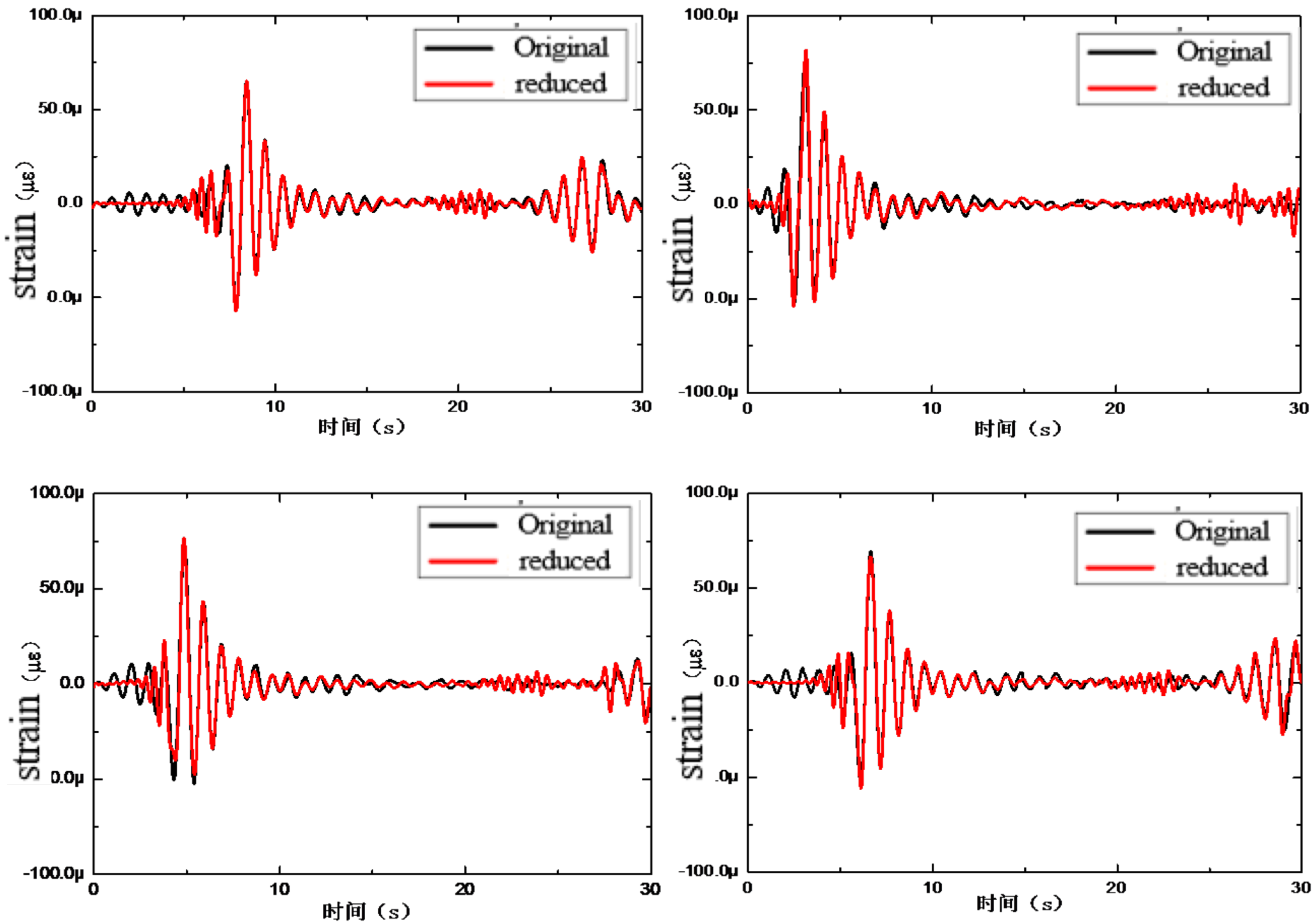
$$K_k = P_k^- C^{\mathrm{T}} (\mathrm{C}\, P_k^- C^{\mathrm{T}} + R_k)^{-1}$$

- **Criterion**

$$\beta = \frac{\sum_i^n \| (\overline{\varepsilon}_i - \varepsilon_i) \|}{\sum_i^n \| (\varepsilon_i) \|}$$

III. Optimal Strain Sensor Placement

- **Result**
  - 30 strain sensors: [4,7,8,11,12,14,15,16,19,22,23,27,28,29,30,57,60,61,62,63,68,69,70,71,73,74,75,76,79,80]



III. Optimal Strain Sensor Placement

- **Result**
  - Comparison

Comparison of accuracy between different sensor arrangement

	Optimal	Case 1	Case 2	Case 3
β	0.336	0.344	0.348	0.349

Comparison of accuracy between different number of optimal sensor arrangement

	30	32	34	36	38
β	0.336	0.332	0.334	0.326	0.326

IV. Discussion

- **Sensor optimal placement + Kalman Filter**

- **References**

1. Li, X., Ren, P., Zhou, Z. and Ou, J. An optimized FBG-based fatigue monitoring strategy on deepwater risers. In *Optical Metrology and Inspection for Industrial Applications III*.
2. Bai, S., Li, X., Xie, Z., Zhou, Z. and Ou, J., 2014. A wireless fatigue monitoring system utilizing a bio-inspired tree ring data tracking technique. *Sensors*.