

$$\begin{aligned}
 f'(x) &= a f(x-2h) + b f(x-h) + c f(x+h) + d f(x+2h) \\
 &= a \left(f(x) - 2h f'(x) + 2h^2 f''(x) - \frac{8h^3}{6} f'''(x) \right) + \\
 &\quad b \left(f(x) - h f'(x) + h^2 f''(x) - \frac{h^3}{6} f'''(x) \right) + \\
 &\quad c \left(f(x) + h f'(x) + h^2 f''(x) + \frac{h^3}{6} f'''(x) \right) + \\
 &\quad d \left(f(x) + 2h f'(x) + 2h^2 f''(x) + \frac{8h^3}{6} f'''(x) \right) + O(h^4)
 \end{aligned}$$

$$\begin{aligned}
 &= (a+b+c+d)f(x) + h(-2a-b+c+2d)f'(x) + \\
 &\quad h^2(2a+b+c+2d)f''(x) + \frac{h^3}{6}(-8a-b+c+8d)f'''(x) \\
 &\quad + O(h^4)
 \end{aligned}$$

$$a+b+c+d = 0 \quad \text{--- (A)}$$

$$h(-2a-b+c+2d) = 1 \quad \text{--- (B)}$$

$$a+4b+4c+d = 0 \quad \text{--- (C)}$$

$$-8a-b+c+8d = 0 \quad \text{--- (D)}$$

$$(C) - (A) \rightarrow b = -c$$

$\therefore a = -d$ using (A) again

Using above & (D), $c = -8d$

$$\therefore b = 8d$$

$$\text{Using (B), } d = -\frac{1}{12h}$$

$$\therefore a = \frac{1}{12h}, b = -\frac{8}{12h}, c = \frac{8}{12h}$$