

### Question 1.

- a) The sum of the arrow securities is \$0.98. Therefore, the interest rate per period is  $1 / 0.98 - 1 = 2.04\%$  to 2 d.p.

This can be shown by arbitrage arguments. If the interest rate is less than 2.04% then you can borrow to buy all 5 Arrow securities for \$0.98. In the next period, one of the Arrow securities will pay out \$1 which is more than the amount you have to repay including interest which results in a riskless profit. Correspondingly, if the interest rate is higher than 2.04% then you can short all 5 Arrow securities to receive \$0.98 which will be deposited at the interest rate. In the next period, you will only have a liability of \$1 from one of the Arrow securities but the proceeds from your deposit will be higher than \$1 resulting in a riskless profit.

- b) Since the current price of the financial asset is \$100, using Arrow securities to price this asset must arrive at the same result to prevent arbitrage.

$$\text{So } 90 \times 0.15 + 95 \times 0.16 + 100 \times 0.25 + 105 \times 0.22 + X \times 0.20 = 100$$

$$X = \$116$$

- c) If X were \$110, then the current price of the financial asset priced by the Arrow securities would be \$98.80

State	Price of asset in next period	Price of Arrow security paying \$1 in current period	Columns multiplied
1	90.00	0.15	13.50
2	95.00	0.16	15.20
3	100.00	0.25	25.00
4	105.00	0.22	23.10
5	110.00	0.20	22.00
Sum			98.80

If you short the financial asset for \$100 and buy the number of Arrow securities required to cover the liability for each state e.g. for state 1, you buy 90 Arrow securities which cost you \$13.50 and this ensures that you have the \$90 to cover your short position in that state in the next period. This will cost a total of \$98.80 based on the table above which means you have a surplus of \$1.20 with all liabilities covered in the next period i.e. arbitrage.

- d) First we need to know the payout of the option in each of the state then we can use the same technique as above to price the option. Payout will be  $\max(0, \text{price of asset} - \text{strike})$ . The result is tabulated below and arbitrage free price for option is \$5.64.

State	Payout of Option	Price of Arrow security paying \$1 in current period	Columns multiplied
1	0.00	0.15	0.00
2	0.00	0.16	0.00
3	2.00	0.25	0.50
4	7.00	0.22	1.54
5	18.00	0.20	3.60
		<b>Sum</b>	<b>5.64</b>