ESD 40.317 Financial Systems Design
Homework 7

Due: 19 Aug before Midnight

We will be continuing with the Constant Elasticity of Variance (CEV) model for risk-neutral stock price evolution with β = 0.75. Recall the SDE:

$$dX = rXdt + \sigma X^{\beta} dW$$

We will be computing the price of the same European up-and-in call option but with Monte Carlo simulations. The terms are: expiration in 0.5 years, strike price of $100 and EKI barrier at $115 which is only observed at maturity. The current stock price x_0 is $110 with a volatility of 25%. The interest rate is constant at 2%.

Use the Milstein scheme to simulate the price path of the underlying asset. For the CEV process, the scheme will be

$$X_{t+\Delta t} = X_t + rX_t\Delta t + \sigma X_t^{\beta}\sqrt{\Delta t}Z + \frac{1}{2}\beta\sigma^2 X_t^{2\beta-1}\Delta t(Z^2-1) \text{ where } Z \sim N(0,1)$$

Use parallel processing with ipyparallel in parts (a) to (c).

(a) Use the Standard Monte Carlo method to price the option. Recall that the price of the options is ~4.35% of X_0 based on the previous homework assignment. Therefore, try experimenting with the lowest number of simulations and number of steps to get close to this value. Report your results.

(b) Use the asset price, X_T , as a control variable to price (a) again. Implement the algorithm where each parallel process returns the covariance matrix of the partial sample then calculate the covariance matrix of the combined sample using the piecewise formula given in the lecture slides to fully take advantage of parallel processing.

(c) Instead of the barrier being observed at maturity only, we will try and price the same option but with the barrier observed continuously which is called American Knock-In (AKI) in the industry. This means the option will knock in if the asset trades above the barrier at any point during the lifetime of the option. The price is ~6.5% of X_0. Therefore, try experimenting with the lowest number of simulations and number of steps to get close to this value. How does the value of N or number of steps required to get to the appropriate accuracy compare to the EKI version of the option? (Hint: there are many possible ways to check the barrier condition but one potential way which is more memory efficient is to keep track of the maximum of the asset price in each simulation path).