ESD 40.317 Financial Systems Design
Homework 4

Due: 13 July before midnight

All homework must be completed in your groups.

1) Imagine a financial asset current priced at $100. There are only 5 possible states for this asset price in the next period. They are $90, $95, $100, $105, $X. There are Arrow securities trading with a payout corresponding to each of these states (recall that an Arrow security pays out $1 in a specific state and $0 for all other states).

State	Price of asset in State at next period	Price of Arrow security with payout corresponding to state in current period
1	$90	$0.15
2	$95	$0.16
3	$100	$0.25
4	$105	$0.22
5	$X	$0.20

Assume that you can buy or short both the asset and the Arrow securities in any amount for the questions below.

a) What is the implied interest rate per period based on the prices of the Arrow securities if there is no arbitrage in the system (assume it is the same for both borrowing and deposits)?
b) What should X be to prevent arbitrage in the system?
c) What if X were $110? How do you construct an arbitrage opportunity?
d) We now add a European call option on the financial asset with a strike of $98 to this 2 period finite state world. How much should this option cost based on the arbitrage free prices that you have found from b)?

2) Derive the following central difference equation using the method of undetermined coefficients:

$$f'(x) \approx (1/(12h))(f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h))$$

Next, determine its order of accuracy by finding the errors for the function $(x) = \exp(x)\sin(\pi x)$ at x = 0.5 for h = 0.1, 0.01, and 0.001, and then finding the relationship between these errors. You will need to submit a Jupyter Notebook file.