ESD 40.317 Financial Systems Design
Homework 5

Due: 27 July before midnight

All homework must be completed in your groups.

1) Consider the ODE $y'(t) = f(t, y(t)) = -10y$ with initial condition $y(0) = 1$.

Use Euler's method to solve this ODE from $t = 0$ to $t = 3$. You will find that smaller values of h (i.e. step size) will work properly, but larger values of h will not work properly. Experiment until you find the smallest value of h which works properly.

For any ODE of the form $y'(t) = -ay$ where a is positive with $y(0) = 1$, try to derive an inequality between a and h which must hold in order for Euler's method to be stable, i.e. to work properly. Hint: $y(t) = e^{-at}$ and note that the Euler algorithm for this form of ODE can be simplified to $\hat{y}_{i+1} = (1 - ah)\hat{y}_i$

Instead of using Euler's method for this ODE, i.e.,

$$\hat{y}_{i+1} = \hat{y}_i + h(-10\hat{y}_i)$$

try using

$$\hat{y}_{i+1} = \hat{y}_i + h(-10\hat{y}_{i+1})$$

Rearrange this second equation to express $\hat{y}_{i+1}$ as a function of $\hat{y}_i$. Then use the rearranged equation to solve the ODE from t =0 to t =3. Now which values of h are stable or unstable?

2) Similar to the In Class Exercise, use LU decomposition to evaluate the function y(x) for x in the interval (0,1) given the following linear 2nd order ODE boundary value problem. Use a step size of 0.001

$$y'' - y' = xe^x - 2x + 2$$

$$y(0) = 1$$

$$y'(1) = \frac{1}{2}e + 2$$

Using the solution $y = \left(1 + \frac{1}{2}x^2 - x\right)e^x + x^2$, plot the approximated curve for y with the actual values for y. Plot the error of the approximation using the actual values as well.