Longer-Run Equilibrium Interest Rates: Evidence From The United Kingdom*

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Abstract

The natural rate of interest has recently been in decline, yet little is known about its historical variation. This paper estimates r-star across two and a half centuries for the United Kingdom between 1700-1950. Results suggest that the long-run equilibrium interest rate rose persistently during much of the eighteenth and nineteenth century, and only began its decline around the turn of the twentieth. Historical decompositions suggest structural changes in productivity, demography and risk are largely responsible for this reduction. I argue secular stagnation is unique to contemporary history, insofar as r-star was ascending across the Late Modern Era.

Keywords: Natural rate of interest, Secular stagnation, Kalman filter, Vector Autoregression

JEL classification: C32, E43, E52, N13, N33, N43

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1 Introduction

The natural rate of interest (r*) has received significant attention over recent years. As the long-run equilibrium interest rate consistent with output and price stability, it has understandably played an increasingly important role in the conduct of monetary policy since its introduction. R-star is now a key criterion in the determination of policy rates, such that they are neither overly contractionary nor expansionary. Given its latent nature, the theoretical natural rate of interest must be estimated rather than directly observed. Secular decline in these estimates have been well documented across the advanced world, significantly limiting the scope of monetary policy.

However, much of the existing literature has only focused on trends across contemporary history; passing next to no judgment on the evolution of long-run equilibrium real interest rates during the Late Modern period. This era is particularly important, as appreciating historical trends might better inform us of the wider context in which recent declines in the natural real rate of interest are taking place. In addition, the validity and extent of established theories that offer an explanation to trends in r-star in terms of long-run shifts in the supply and demand for savings such as the secular stagnation hypothesis, would certainly benefit from historical scrutiny.

This paper seeks to answer a straightforward, yet highly relevant question; what are the longer-run trends in the long-run equilibrium interest rate and to what extent are they explained by core drivers posited by existing research? I adapt a benchmark semi-structural New Keynesian model to estimate the 'longer-run' equilibrium interest rate using a broad series of annual macroeconomic data for the United Kingdom between 1700-1950. Given historical estimates, this paper attempts to explain long-run variation in r-star using historical decompositions arising from a recursively identified Structural Vector Autoregression (SVAR) model that is a function of well documented drivers for which data is available such as productivity, demography, debt and risk.

This empirical investigation yields a number of novel results. Firstly, r-star rose persistently for the better part of the eighteenth and nineteenth century before falling throughout the twentieth. This decline was further exacerbated by World War I, World War II, and the Great Depression. Secondly, shocks to productivity, demography and risk can explain a large fraction of the variation in r-star over this secular stagnation. Historical trends in the United Kingdom suggest that a slowdown in total factor productivity, an increasing old-age dependency ratio and a rising term spread around the twentieth century are core suspects behind the trend reversal. Furthermore, had it not been for sharp increases in government debt around the First and Second World War, the natural rate of interest may have fallen even further. Thirdly, given r-star has converged to the zero lower neutral bound over recent years, this paper argues that secular stagnation is a phenomenon relatively unique to contemporary history, and that the natural rate is simply returning to the prevailing average it maintained at the start of the Late Modern Era.

The remainder of this paper is structured as follows. Section 2 reviews the core literature concerning the estimation and drivers of r-star. Section 3 outlines the framework used to estimate the long-run equilibrium interest rate. Section 4 outlines the historical data used in the estimation process. Section 5 discusses particulars concerning the empirical implementation. Section 6 presents and discusses the main findings of the paper. Finally, Section 7 concludes.

2 Literature

This study is concerned with the estimation of the natural rate of interest and its long-run drivers. As for the former, research by Laubach and Williams (2003) and Holston, Laubach and Williams (2017), hereafter HLW, is perhaps of most significance. The authors implement a Kalman filter to estimate r-star in a now benchmark maximum likelihood procedure. This semi-structural approach focuses on medium-long run fluctuations in the equilibrium real interest rate that are influenced by low-frequency nonstationary processes. Their results highlight a secular decline in the neutral rate across the advanced world since the 1960s. In addition, the authors find substantial comovement between natural rates of interest, indicating global factors driving their variation.

These empirical findings are robust to a number of extensions (e.g. Clark and Kozicki, 2005; Mésonnier and Renne, 2007; Trehan and Wu, 2007; Berger and Kempa, 2014; Lewis and Vazquez-Grande, 2018). However, a consistent theme across research inspired by this particular methodology are highly imprecise estimates of r-star, owing to filter and parameter uncertainty. Despite a number of alternative models and estimation techniques, including more structural approaches that focus instead on the higher-frequency components of r-star (e.g. Barsky, Justiniano and Melosi, 2014; Kiley, 2015; Lubik and Matthes, 2015; Hamilton et al., 2016; Johannsen and Mertens, 2016; Pescatori and Turunen, 2016; Christensen and Rudebusch, 2019), no other dominant alternative presents itself in the existing literature, and as such, the model has become a key point of reference in estimating the long-run equilibrium interest rate across the advanced world.

Given its centrality, I maintain and adapt the HLW (2017) framework to estimate the longer-run equilibrium interest rate using a broad series of macroeconomic data for the United Kingdom. In particular, this paper recasts the state-space system into a form more suitable for annual data whilst preserving the fundamental relationship between state variables of the model. These modifications relate entirely to the lag structure found in the dynamic Investment-Savings (IS) curve and Phillips Curve (PC), which is assumed to be adaptive as opposed to a linear combination of multiple lagged quarters. Using this adjusted empirical specification, I estimate the natural rate of interest across a rich sample that ranges two-and-a-half centuries in the United Kingdom.

As for the causes of such decline in the natural rate of interest, existing research has emphasised the importance of certain macroeconomic trends that have influenced the long-run supply and demand for savings (see for instance Carvalho, Ferrero and Nechio, 2016; Gagnon et al., 2016; Gordon, 2016; Rachel and Smith, 2017; Eggertsson et al., 2019; Rachel and Summers, 2019). These structural trends include but are not restricted to; adverse changes in demography, slowing technological progress and innovation, falling (relative) prices of investment goods, an increasing scarcity of safe assets, stronger risk aversion and greater capital and income inequality.

This paper explains the historical variation in r-star in terms of factors for which data is available in the United Kingdom. Incidentally, these variables are also at the centre of recent literature concerning the primary drivers of the natural rate such as demography, productivity, debt, and risk. As for the first, higher rates of average life expectancy and declining rates of fertility are thought to have increased the supply and decreased the demand for savings respectively. As for the second, declining productivity growth, and by extension, investment opportunities, has also given rise to downward pressures on the demand for capital and savings. As for the third, enlarged government deficits and public debt are thought to have placed upward pressures on r-star due to crowding out. As for the fourth, greater risk aversion in terms of higher risk premiums and increased demand for safe assets is also believed to have driven down the natural rate of interest in recent years. Many of these trends are often cited when explaining secular decline in r-star across the advanced world and the extent of their historical relevance is further investigated in this paper.

It is clear from the existing literature that studies estimating the natural rate rarely do so prior to the twentieth century. Among the few that have, Jordà et al. (2022) seek to investigate the long-run equilibrium real interest rate across Europe over a sample stretching back as far as the fourteenth century. The crux of their empirical analysis is on the consequences of pandemics, which, unlike wars, depress the natural rate for decades. Despite it not being the primary focus of their research, r-star is modelled as a simple random walk, argued to be flexible enough to capture secular trends. Whilst this is tractable over numerous centuries, time variation in the medium-long run frequency component of the equilibrium interest rate may require a more intricate approach. Hamilton et al. (2016) also conduct an extensive study of real interest rates in the United States, albeit this does not extend beyond the 1800s, nor does it seek to filter out the latent natural rate from the historical data. Instead, the authors use moving averages of ex-post rates to pass judgment on r-star, whilst acknowledging this to be a noisy measure of the theoretical rate consistent with stability.

In a related strand of literature, a number of studies have attempted to measure the real interest rate over an even longer horizon using similar techniques. Perhaps of most relevance to this paper is the extensive work of Schmelzing (2020), in which moving averages of real rates are shown to exhibit 'suprasecular' decline since the fourteenth century, maintaining what is argued to be their true historical trend and calling into question the narrative that secular stagnation is a uniquely recent phenomenon affecting the advanced world. Whilst informative, there are important caveats to this methodology. Moving averages of real rates used to infer upon the natural rate of interest by allowing for persistent change are often characterised by large shifts that last for decades. In other words, this approach struggles to separate long-term from short-term variation in the natural rate of interest during periods of harsh volatility. In particular, movements in inflation and output are not explicitly accounted for by reduced-form univariate approaches and are therefore not entirely consistent with the Wicksellian interest rate that prevails during medium-long run pressures, which are particularly important during various periods of the historical sample.

In this regard, there are strong reasons to use multivariate filtering approaches to estimate the long-run equilibrium real interest rate over a longer-run horizon. Trends identified in the literature suggest that the natural rate of interest has been in decline over much of the last century, reaching a new normal close to the zero lower neutral bound - but has this trend always been the case? What behaviour does the equilibrium interest rate exhibit prior to this period, and how does this relate to trends in its established drivers? This paper seeks to answer such questions. I find that the natural rate has not always been in secular decline and is instead returning to an old normal from which it in fact increased until the turn of the twentieth century, reversing its trend due to structural changes in its central drivers. I argue secular stagnation is a phenomenon relatively unique to contemporary history, insofar as it contrasts itself to the Late Modern Era in terms of the evolution of r-star.

3 Specification

This paper uses the HLW (2017) system to estimate the natural rate of interest (r*) over a longer-run horizon. In particular, the canonical small open economy New Keynesian model of Galí and Monacelli (2005) is relaxed by specifying reduced equations of the Investment-Savings (IS) curve and Phillips Curve (PC) that permit shocks to the output gap $\tilde{y}_t$ and inflation π_t as follows:

$$\tilde{y}_t = \phi_y(L)\tilde{y}_t + \phi_r(L)\tilde{r}_t + \epsilon_{\tilde{y},t} \tag{1}$$

$$\pi_t = \varphi_\pi(L)\pi_t + \varphi_y(L)\tilde{y}_t + \epsilon_{\pi,t} \tag{2}$$

where $\tilde{r}_t$ is the real interest rate gap given by deviations in the real interest rate from the natural rate of interest $r_t - r_t^*$, and the error terms $\epsilon_{\tilde{y},t}$ and ϵ_{π_t} capture any transitory shocks to output and inflation respectively. The law of motion for natural rate of interest is given by the following:

$$r_t^* = g_t + z_t \tag{3}$$

where g_t is the trend growth rate of the natural rate of output y_t^* and z_t is the error term capturing unobserved factors driving the real equilibrium interest rate. This relation follows directly from the textbook Ramsey optimal growth model.[1] The transition equations of the system are as follows:

$$y_t^* = (L)y_t^* + (L)g_t + \epsilon_{y*,t} \tag{4}$$

$$g_t = (L)g_t + \epsilon_{g,t} \tag{5}$$

$$z_t = (L)z_t + \epsilon_{z,t} \tag{6}$$

where equation (4) defines potential output as a random walk with drift g_t, which itself is defined as a random walk in equation (5), as is the unobserved component of the natural rate of interest in equation (6). It is assumed that the error terms in transition equations (4) to (6) are contemporaneously uncorrelated and normally distributed, with the following variance-covariance matrix:

$$\begin{bmatrix} \sigma_{\tilde{y}}^2 & 0 & \cdots & \cdots & 0 \\ 0 & \sigma_\pi^2 & \ddots & & \vdots \\ \vdots & \ddots & \sigma_{y*}^2 & \ddots & \vdots \\ \vdots & & \ddots & \sigma_g^2 & 0 \\ 0 & \cdots & \cdots & 0 & \sigma_z^2 \end{bmatrix} \tag{7}$$

The general structure is therefore similar to the HLW (2017) system except for the order of the lag polynomials within the state equations given the use of lower frequency data. These adjustments to the lag structure of the dynamic IS and PC equations are further detailed in Section 5.

[1] This result is contingent on the assumption of a representative household with a CES utility function and constant relative risk aversion, which is the inverse of the intertemporal elasticity of substitution in consumption. In steady state, optimisation yields r* as a function of the growth rate of per capita consumption and the rate of time preference.

4 Data

Given data used in this paper spans over two centuries, it is useful to categorise the sample into periods of significance that facilitate our analysis. For that purpose, the years prior to 1750 shall be referred to as the Early Modern Era and the years after shall be referred to as the Late Modern Era. Within the Late Modern Era, the First Industrial Revolution is dated between 1760 to 1840 and the Second Industrial Revolution is dated between 1870 to 1914. As is known, World War I takes place between 1914 to 1918, the Great Depression occurs between 1929 to 1939, and World War II between 1939 to 1945. The Contemporary Era begins at the end of our sample, from 1945, and is a period in which the natural rate of interest has been well documented by the existing literature (see HLW, 2017). For this reason, my investigation focuses on trends strictly prior to this period, albeit using contemporary estimates of r-star as a key point of reference.

This paper uses a broad series of annual macroeconomic data published by the Bank of England to estimate the natural rate of interest between 1700 to 1950. Nominal short-term interest rates are sourced from the Official Bank Rate series published by the Bank of England. CPI is from the Shumpeter-Gilboy Index between 1700-1750 (see Mitchell, 1988), from Crafts and Mills (1994) between 1750-1770, Feinstein (1991; 1998) between 1770-1914 and the ONS between 1914-1950 (see O'Donoghue et al., 2004). Finally, Real GDP is sourced from the geographically-consistent real headline series; a chained volume measure constructed from an array of sources (in particular, refer to Feinstein, 1972; Solomou and Weale, 1991; Sefton and Weale, 1995; Broadberry et al., 2015). All data is collated contiguously in Thomas and Dimsdale (2017).[2]

Figure 1 plots the historical series for real GDP, CPI inflation and ex-post real interest rates in addition to ex-ante real interest rates calculated using inflation expectations also sourced from the Bank of England dataset (refer to Thomas and Dimsdale, 2017). Each series clearly exhibits a substantial degree of volatility coinciding with periods of macroeconomic expansion and contraction. WWI and WWII separate themselves as events associated with strong volatility, albeit given the extensive sample size under scrutiny, they are certainly not unique. For instance, inflation at the start of the 1800s was clearly unprecedented in the history of the United Kingdom, which is often attributed to the strain on aggregate supply due to the Napoleonic Wars and upward pressure on aggregate demand due to the Industrial Revolution (Gilboy, 1936).

In the post-war period, real rates seem to begin trending down to very low and even negative levels. Given the natural rate of interest has been in stagnation over much of the last half-century, this suggests a subsequent rise and secular decline until the present (Jordà et al., 2019). However, focusing our discussion on such a narrow period can be potentially misleading. Contrary to what most studies have concluded over the Contemporary Era, it is unclear what trend real interest rates follow across the wider Late Modern Era in the United Kingdom. Given the presence of transitory shocks across this period, there is strong motivation to estimate the long run equilibrium real rate of interest associated with inflation and output stability.

[2] Refer to the '*Millennium of Macroeconomic Data for the United Kingdom*' OBRA dataset published by the Bank of England (Thomas and Dimsdale, 2017). All data used in this paper to estimate the longer-run equilibrium interest rate are found within this historical dataset. See Appendix A.3 for further commentary on data construction and sources.

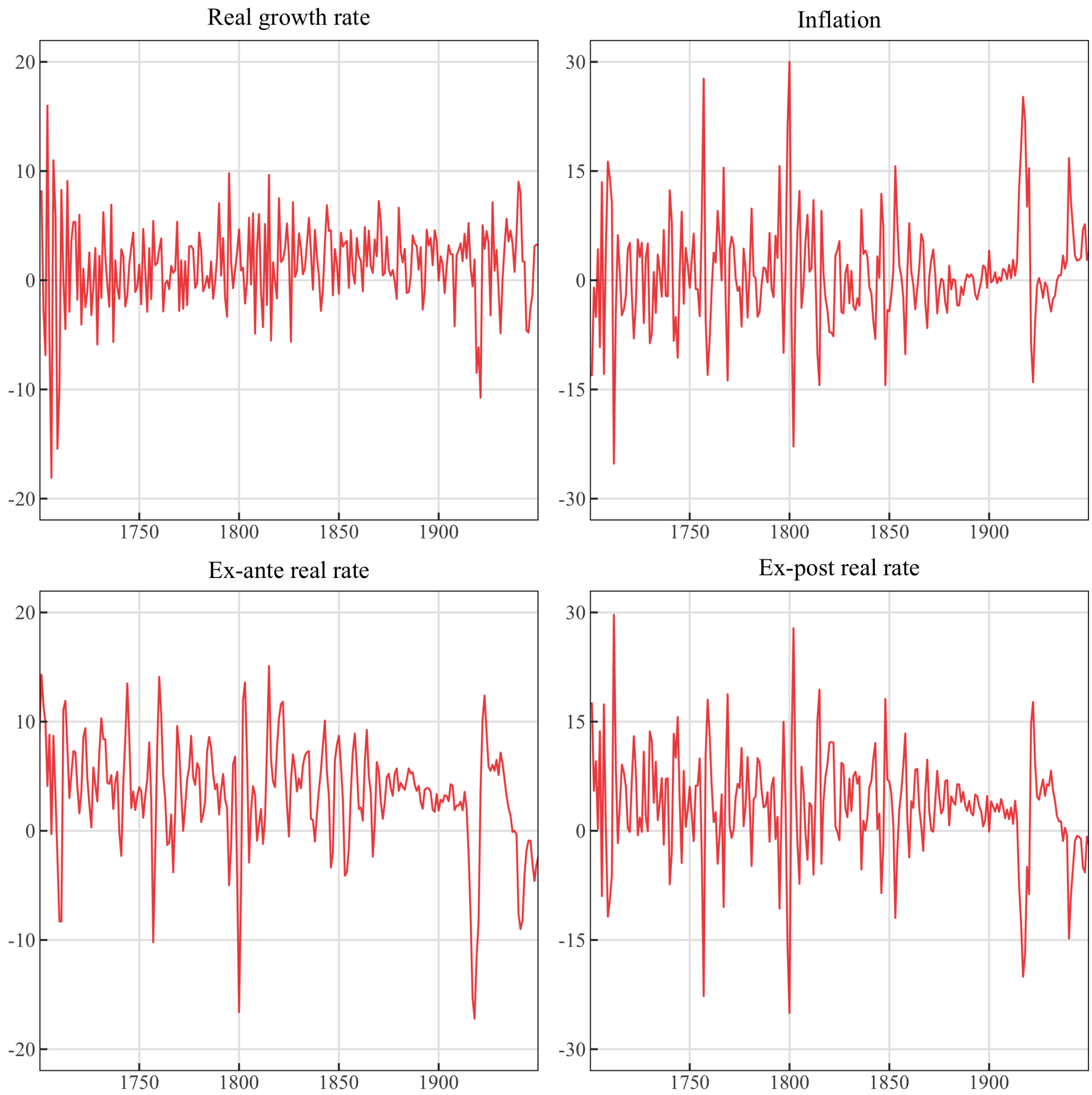

Figure 1. Historical growth, inflation and interest rates

Note: Annual data for the United Kingdom between 1700-1950 in red. The real growth rate is the rate of change in real GDP. Inflation is the rate of change in CPI. Ex-post real rates are determined by the Fisher relation. Ex-ante real rates are calculated using inflation expectations from Thomas and Dimsdale (2017).

5 Estimation

To motivate a multivariate model, it is useful to first consider univariate approaches to estimate the natural rate of interest. In the absence of variation, sample means of ex post real interest rates may be sufficient to pass judgment on r-star. However, factors influencing the supply and demand for savings are seldom constant, shifting the equilibrium interest rate across time. If these movements are sufficiently large and persistent, sample averages would be a poor estimate of r-star. One way to allow for persistent changes is by computing moving averages of historical real interest rates.

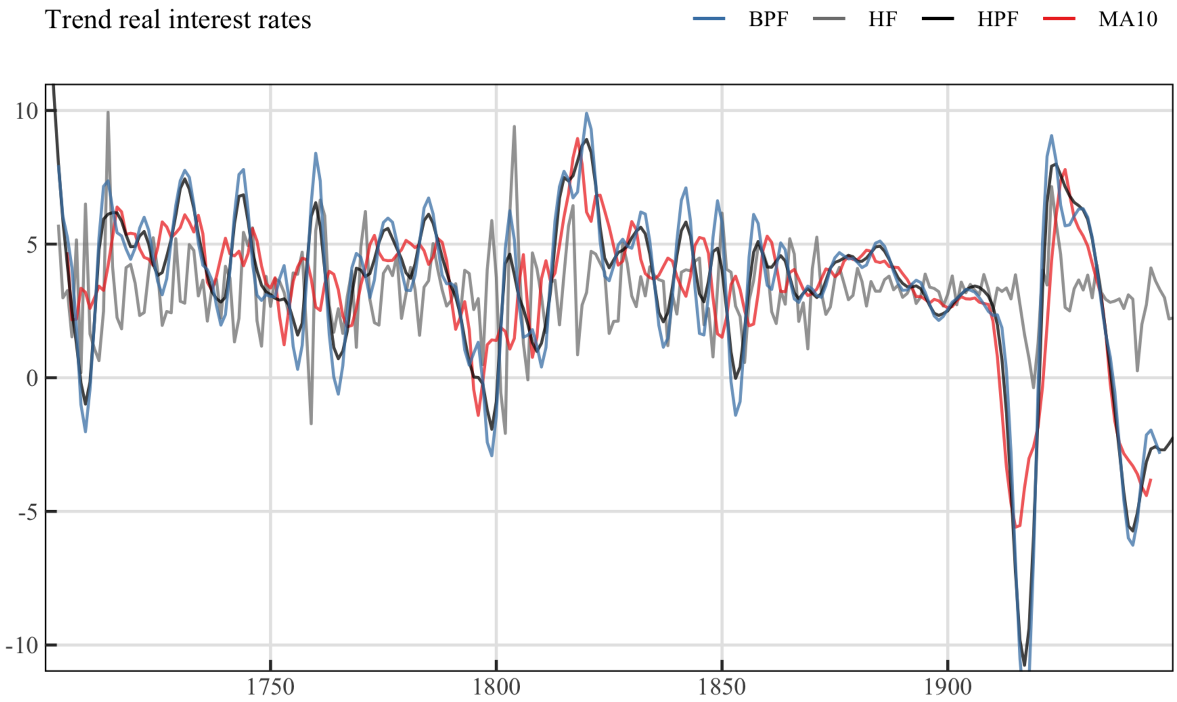

Figure 2. Trend real interest rates

Note: Univariate estimates between 1700-1950. Band-pass (Baxter and King, 1999) filter (BPF) estimates are presented in blue. Hamilton (2018) filter (HF) estimates are presented in grey, with the recommended look-ahead period of h = 2. Hodrick-Prescott filter (HPF) estimates are presented in black, with λ = 6.25 as in Ravn and Uhlig (2002) given annual data. Ten year moving averages (MA10) are presented in red.

In addition to ten-year moving averages, Figure 2 presents other examples of common statistical approaches that separate medium or long-term trends from short-term variations such as the band-pass filter, the Hamilton (2018) filter, and the Hodrick-Prescott filter. In principle, these techniques would be sufficient to extract the natural rate of interest when both inflation and output are stable. However, given the sample length and our knowledge of periods during which this is certainly not the case, univariate approaches may yield unreliable estimates by ascribing movements in the real rate to its underlying trend (Hamilton et al., 2016). One potential solution is to use forward rates on longer-term securities that capture market perceptions of the natural rate of interest. However, term premia may be a harsh source of variation in forward rates, also rendering them an unreliable proxy for future long-run equilibrium real interest rates.

Despite these issues, we still observe that most estimates suggest a downward trend beginning around the end of the nineteenth century, followed by large movements in real rates across the first half of the twentieth century. However, long-run trends in the real rate are unclear prior to this. In this regard, equations (1) to (6) may be cast into a state-space system to estimate the parameters by the maximisation of a likelihood function provided by the Kalman filter (refer to Appendix A for the full representation of the model). Given observation and transition equations, this recursive algorithm for updating a projection for a dynamic system yields linear unbiased estimates of the state variables and provides a way to compute their uncertainty. This is not without calibration. In particular, initialisation of the state vector and estimation of the error variance are central issues.

As for the former, it is relatively straightforward to derive the unconditional mean and autocovariance of the autoregressive processes as a function of model parameters. However, this task is made difficult for the output gap due to unspecified dynamics of the nominal policy rate. The typical solution that is implemented in this paper is to apply the Hodrick-Prescott filter to initialise the state vector. As for the latter, given real growth rates and interest rates are often influenced by highly persistent shifts, maximum likelihood estimation would likely return estimates of the standard deviations of innovations σ_g and σ_z that are biased towards zero. To fix this 'pile-up' problem (Stock, 1994), the median unbiased estimator is used to derive signal-to-noise ratios $\lambda_g = \sigma_g/\sigma_{y*}$ and $\lambda_z = \phi_r\sigma_z/\sigma_{\tilde{y}}$ that are imposed as restrictions on the remaining model parameters (Stock and Watson, 1998). This procedure follows directly from HLW (2017). In particular, the natural rate of output is first estimated barring the real rate gap and assuming constant trend growth. The median unbiased estimate for the ratio between the standard deviations of innovations for the natural rate of output and its trend growth rate is then derived and subsequently imposed as a restriction in the second stage of the estimation that includes the real rate gap. Finally, the ratio for the component unrelated to the trend growth rate is derived in a similar manner and imposed in the third stage to estimate the remaining parameters of the model.[3]

It is worth noting that the implementation of the Kalman filter in a maximum likelihood framework to estimate r-star over such an extensive horizon poses a number of unique challenges. Perhaps of most significance is the effect of extreme-tailed phenomena that undermine the assumption that all stochastic innovations are Gaussian in distribution. One potential solution to this particular issue is the inclusion of omitted factors in the measurement equations that control for the presence and extent of these harsh shocks to the output gap. This could accommodate sharp and persistent changes in output and thus allow the model to parse other influences into transitory and permanent components. However, given the extensive size of our sample, it would be difficult to identify, let alone control for shocks strong enough to potentially violate this assumption. Whilst it is probable that certain historical events such as both World Wars and the Great Depression distort estimates of r-star, this methodology is still able to reveal general trends in the equilibrium interest rate during periods of such volatility. The Great Recession is analogous in this regard over which much of the existing literature continues to estimate the natural rate of interest, despite the fact that it likely exhibited similar pressures on the assumptions of the Kalman filter.

Another unique challenge that transpires from the use of historical data is that of identification due to potentially flat IS and Phillips curves. In particular, the low frequency of annual data may lack the information needed to estimate the relationship between the output gap, real rate gap and inflation. In addition, the longer horizon may involve structural breaks in the relationship between these variables. In such instances, one would also expect more uncertainty around estimates of the natural rate of interest due to a lack of observability (Fiorentini et al., 2018). Few studies related to r-star have focused their analysis on the United Kingdom to gauge the true extent of these issues. The one notable exception is HLW (2017), whose estimates suggest that whilst the Phillips curve is well identified ($\varphi_y = 0.49$), the elasticity of the current output gap to past real rate gaps is close

[3] Confidence intervals for these estimates and their corresponding standard errors are calculated using a constraint Monte Carlo procedure, which accounts for filter and parameter uncertainty (see Hamilton (1986) for further details).

to zero ($\phi_r = -0.01$). However, this result relies on higher frequency data over a narrow horizon and it is unclear whether it is true in lower frequencies over the historical sample. With this caveat in mind, I proceed to estimate the natural rate of interest and return to the issue of identification in the following section given estimates of the model parameters.

Given quarterly data is unavailable for the historical period in question, the model is specified to reflect annual time series data. The main alteration is in the order of the lag polynomials within the state equations, where the output gap is related to one autoregressive lag in addition to the real rate gap, and inflation is related to one autoregressive lag and the output gap.[4] The three transition equations are otherwise identical to those specified in HLW (2017). The final system of equations to be estimated by maximum likelihood are expressed concisely as follows:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1-\phi_y & 1-\phi_r & -\phi_r \\ -\varphi_y & 0 & 0 \end{bmatrix} \begin{bmatrix} y^*_{t-1} \\ g_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \phi_y & 0 & \phi_r \\ \varphi_y & \varphi_\pi & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{\tilde{y},t} + \epsilon_{y*,t} \\ \epsilon_{\pi,t} \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} y^*_t \\ g_t \\ z_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y^*_{t-1} \\ g_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{y*,t} \\ \epsilon_{g,t} \\ \epsilon_{z,t} \end{bmatrix} \tag{9}$$

Where (8) is the measurement equation involving the dynamic IS and Phillips curve in addition to the law of motion for the natural rate of interest, and (9) is the corresponding transition equation.

6 Results

6.1 Parameter Estimates

Parameter estimates are reported in Table 1. Estimates of the ratios λ_g and λ_z indicate substantial time variation in the trend growth rate and natural rate of interest. These estimates are relatively higher than those in HLW (2017), likely due to greater volatility over the historical sample. Slope coefficients of the measurement equations (ϕ_r and φ_y) suggest that whilst the output gap is well identified, the real rate gap is not. However, it is worth noting that the slope of the IS curve is marginally higher than what is typically estimated using quarterly data over a much shorter horizon. Finally, inflation reacts strongly to transitory variation in the output gap given by φ_y, which itself is fairly persistent as seen by ϕ_y. In respect to the standard deviations, clearly inflation is poorly explained by its own lag and the lag of the output gap, as indicated by a high σ_π. Variation in the unobservable component and the natural rate of interest are also substantially higher compared to parallel estimates of the same parameters using quarterly data (HLW, 2017). Once again, these are perhaps symptoms of using annual data over a much longer horizon in which a number of shocks influence r-star. This is also captured by relatively higher average standard errors for the natural rate of interest, natural rate of output and the trend growth rate of output.

[4] Lag selection in the state equations is justified given the use of lower frequency data in which it is found that most of the feedback in inflation and the output gap is accounted for within the year and that the interest rate affects output in one period, which subsequently affects inflation in the next (see Bank of England (1999) for a further discussion).

Table 1. Parameter estimates

Parameter		Standard error	
λ_g	0.042	r^*_{avg}	3.495
λ_z	0.023	y^*_{avg}	1.654
ϕ_y	0.717	g_{avg}	0.723
ϕ_r	-0.033		
φ_y	0.574		
$\sigma_{\tilde{y}}$	0.759	r^*_{fin}	3.842
σ_π	3.727	y^*_{fin}	1.421
σ_{y*}	0.965	g_{fin}	0.683
σ_g	0.041		
σ_z	0.529		
σ_{r*}	0.531	T	250

Note: Estimated parameters of the Holston, Laubach and Williams (2017) model for the United Kingdom between 1700 to 1950. Average and final standard errors of the natural rate of interest, the natural rate of output and its trend growth are reported in the last column. σ_{r*} is calculated as the square root of $\sigma_g^2 + \sigma_z^2$.

Such wide confidence bands are a reflection of the uncertainty around estimates of the slope of the dynamic IS curve. However, it is worth pointing out that the magnitude of error does not entirely invalidate these results. Imprecision arising from parameter and filter uncertainty is a well known issue that even plagues quarterly estimates of r-star over more narrow samples. Notwithstanding, much can still be gained from analysing general trends in the natural rate of interest and for this reason I proceed to present and analyse the core empirical results of the estimated model.

6.2 Longer-Run Equilibrium Interest Rates

Given the size of the historical sample, I separate this analysis into parts by isolating three phases in the evolution of r-star. The aggregate series is presented in Appendix B. Much of the sample is characterised by wars in which Britain played a significant role. The number of business cycles, their duration, and the volatility in estimates of the natural rate of interest and trend growth rate are a clear reflection of the shocks that took place over this period. This is particularly relevant, as large sums of long-term debt resulting from wartime finance places upward pressure on rates due to crowding out, in addition to greater risk-premiums and potential capital scarcity.

With this in mind, the first phase is presented in Figure 3 between 1700 to 1750. My estimates reveal substantial variation in r-star during the initial part of the half-century, which coincides with the War of the Spanish Succession (1701-1714). By the end of this period the natural rate of interest declines sharply by approximately 2.5-3% before recovering over much of the subsequent decade. Britain was also straddled with unprecedented long-term public debt burdens as a result of the war (Carlos et al., 2013), which may have contributed to the upward pressure on r-star. Despite a number of recessions that ensued in Britain, the natural rate of interest remains relatively stable over the rest of this period, averaging roughly 1.5-2% across significant events such as the Great Northern War (1717-1720) and the war of the Austrian Succession (1740-1748).

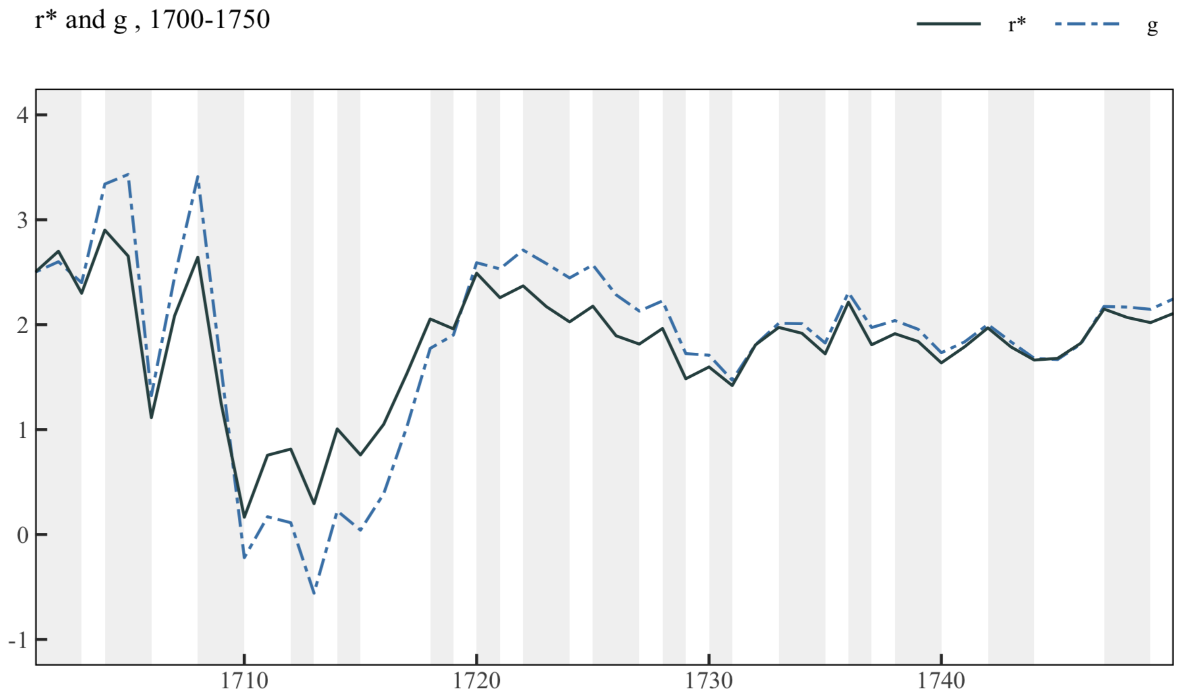

Figure 3. Prelude: 1700-1750

Note: Estimated series for the natural rate of interest (r) in black and trend growth rate (g) in blue between 1700-1750 in the United Kingdom. Recession bands in grey are constructed from Broadberry et al. (2022).*

The second phase is presented in Figure 4 between 1750 to 1875. In the latter half of the eighteenth century, the natural rate and trend growth rate begin to rise. This was the case despite major events such as the Seven Years' War (1756-1763), the American Revolutionary War (1775-1783), the French Revolutionary War (1792-1802) and the Napoleonic War (1802-1813). Estimates suggest that this ascent persists right until the last quarter of the nineteenth century, pushing r-star and trend growth to unprecedentedly high levels. It is clear that across much of the nineteenth century, the number of cycles and their duration diminish substantially. However this was certainly not an era free from war, rather Britain was largely unstrained by most of the conflicts she was involved in and bathed in the rewards of many of her victories in Asia, the Pacific, and Africa.

This long stretch of 'peace' that persisted through to the early twentieth century is often known as *Pax Britannica* (1815-1914) and may explain why the natural rate saw a relatively unperturbed rise. The First Industrial Revolution also played an important role in Britain closer to the end of this period and its contribution to the ascension in r-star shall be discussed in relation to changes in total factor productivity growth. By the end of this particular phase, the natural rate of interest increases by approximately 7 percentage points. This upward trend in the equilibrium real interest rate over such a vast horizon serves as clear evidence of long-run drivers that transcend transitory shocks to the British economy. In this regard, there is strong motivation to investigate the potential drivers that would have contributed to this sustained rise. Finally, the trend growth rate of output follows a similar trajectory to the natural rate of interest during this period, suggesting an era of macroeconomic expansion in Britain that generally persisted for over a century.

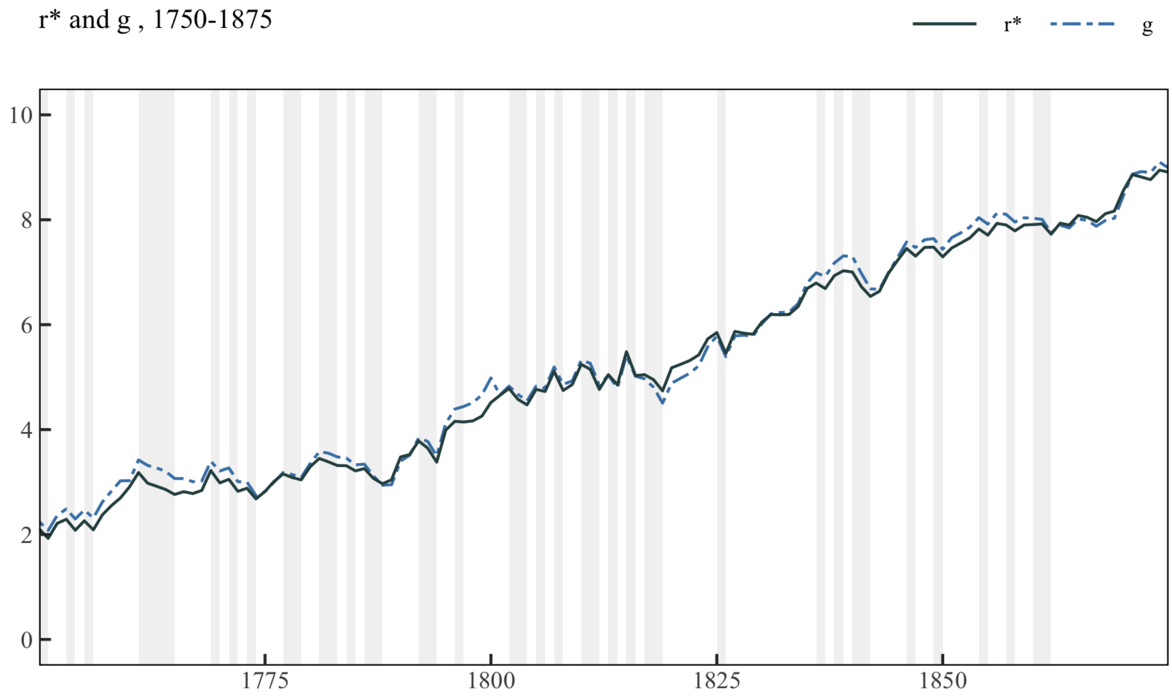

Figure 4. Ascension: 1750-1875

Note: Estimated series for the natural rate of interest (r) in black and trend growth rate (g) in blue between 1750-1875 in the United Kingdom. Recession bands in grey are constructed from Broadberry et al. (2022).*

The third phase is presented in Figure 5 between 1875-1950. In the last quarter of the nineteenth century, the natural rate of interest and trend growth rate exhibit signs of secular decline. This trend seems to stabilise around the turn of the century until the First World War. My estimates suggest that the implications of WWI on the natural rate of interest and trend growth rate were extensive. In particular, the post-war recession coincides with a reduction of approximately 2-2.5%. In context, this is more than twice the estimated reduction in r-star that the United Kingdom had experienced during the Great Recession. The equilibrium interest rate remains permanently depressed until the start of the Great Depression, which coincides with a further reduction of roughly 0.5-1%, albeit soon after which the equilibrium real interest rate recovers up until the Second World War.

The post-WWII recession also has a negative impact on the natural rate of interest and trend growth rate, albeit not to the extent of WWI. In particular, r-star falls by approximately 0.5-1% and remains depressed thereafter until the close of the historical sample. Given the existing literature has established a sustained decline in the natural rate of interest over the last century, my estimates suggest that this secular decline actually began just before the turn of the twentieth century and was heavily exacerbated by the First World War. The Great Depression and the Second World War also play a key role in depressing the natural rate of interest and trend growth rate to unprecedented lows. Whilst these events are highly significant, it is clear that the downward trend in r-star begins well before they transpire. By the end of this phase, it declines by approximately 3-3.5%. If recent estimates of r-star in the existing literature are accurate, this series precedes further reductions in the natural rate of interest through to the present, particularly exacerbated by the Great Recession.

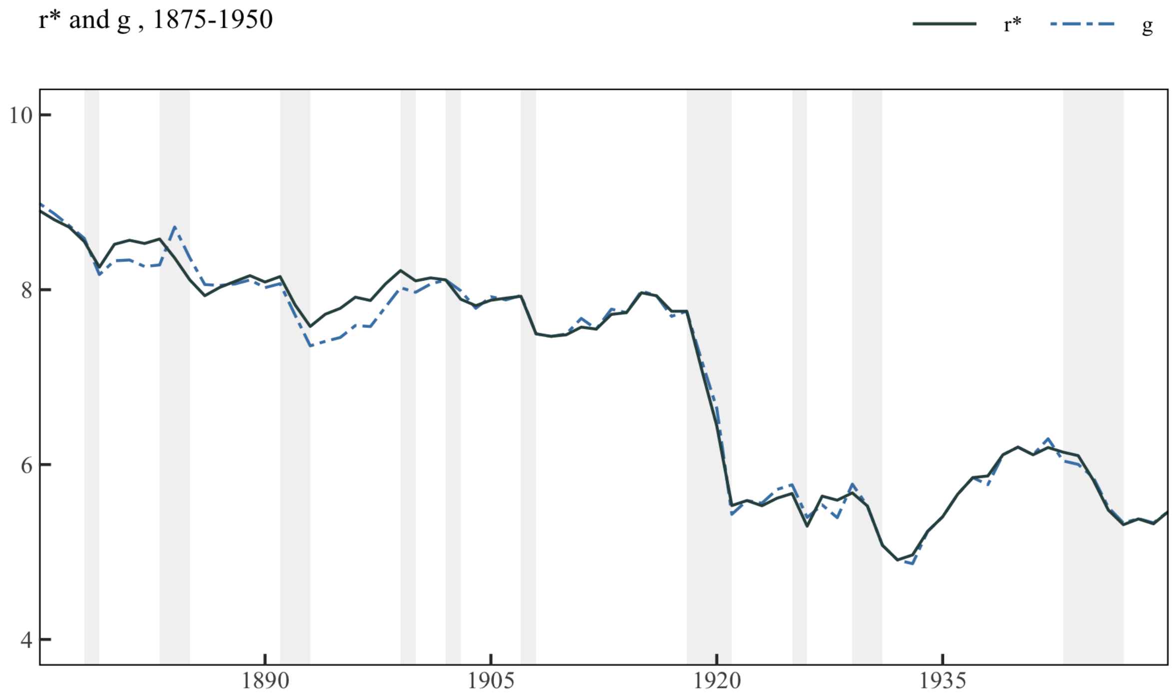

Figure 5. Decline: 1875-1950

Note: Estimated series for the natural rate of interest (r) in black and trend growth rate (g) in blue between 1875-1950 in the United Kingdom. Recession bands in grey are constructed from Broadberry et al. (2022).*

6.3 Even Longer-Run Interest Rates

Whilst my estimates of the natural rate of interest reveal periods of long run ascension and stagnation, they capture a relatively short part of an even longer history of real interest rates. Shmelzing (2020) offers perhaps the most authoritative documentation of this history and shows, using seven year moving averages, that ex-post rates locally (in the United Kingdom) and globally have been in decline over the last eight centuries (see Figure 6). Although useful over longer horizons given time series which exhibit clear trends with minimal noise, moving averages often struggle to parse short-term and long-term variation, particularly during times of volatility. In this regard, the work of Shmelzing (2020) does well to explain the general trend of real rates, but the historical evolution of the equilibrium rate consistent with output and inflation stability may perhaps differ.

Estimating r-star over such a horizon using more advanced multivariate filtering techniques is a major challenge for economic historians. Barring data and measurement issues, the imposition of structure becomes increasingly untenable as the sample extends further into history. These periods predate a plethora of market mechanisms and institutions that facilitate the linkage between macro-fundamentals necessary for the estimation of r-star. Furthermore, the frequency and magnitude of crises across this broad history threaten core assumptions underlying multivariate filters. Reduced-form time-series models may well be more appropriate in this regard. My choice of sample horizon (coinciding roughly with the inception of the Bank of England) is therefore no coincidence and I leave the challenge of estimating the even longer-run equilibrium interest rate to future research.

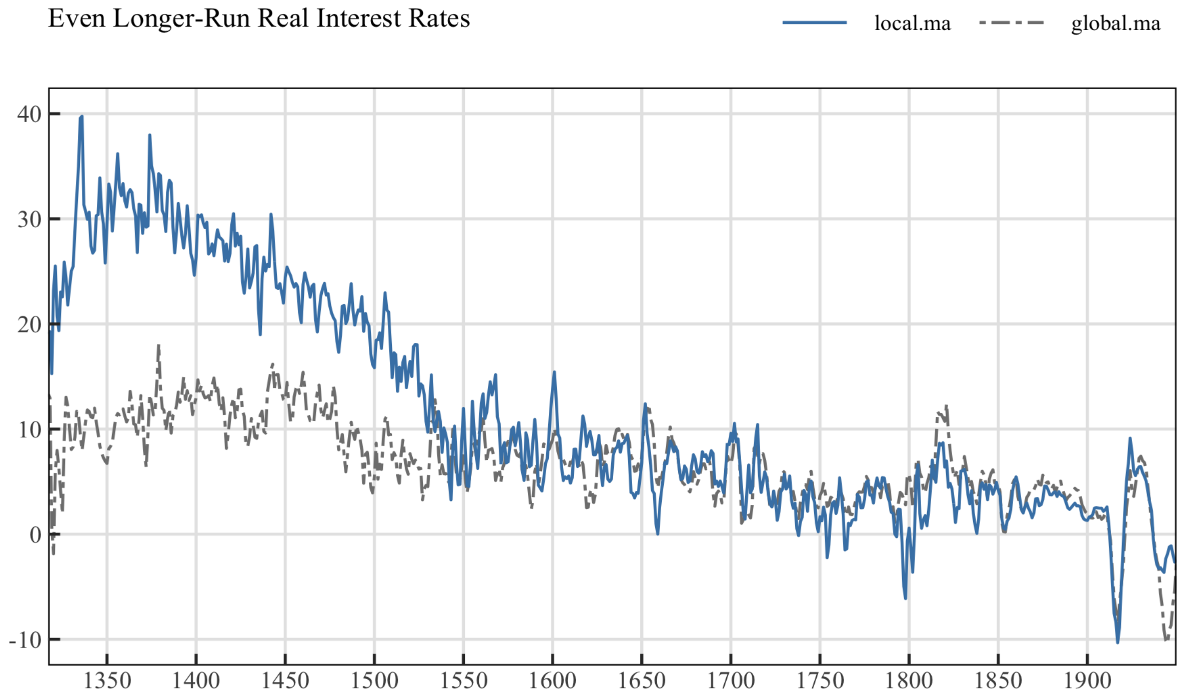

Figure 6. Even longer-run real interest rates

Note: Seven year moving averages of (local) ex-post real rates for the United Kingdom in blue and (global) ex-post rates in grey between 1317-1950. Global ex-post rates are calculated using weighted averages of real rates across advanced economies. Data used to compute this series is taken from Schmelzing (2020).

Given this investigation does not attempt to pass judgment on r-star prior to the eighteenth century, Figure 7 presents aggregate estimates of the natural rate of interest alongside seven year moving averages of local and global ex-post rates computed in Schmelzing (2020) for the duration of our sample. Although not directly comparable, this period of ascension estimated in the natural rate of interest beginning around the mid-eighteenth century can also be observed in averages of the local ex-post real rate. Whilst this does not persist as long as in r-star, the decline does coincide with the end of the nineteenth century and persists until the end of the sample. It is worth emphasising that moving averages exhibit large shifts that in many instances persist for decades, which is a known symptom of its difficulty in parsing underlying variation in the real interest rate.

Why is secular stagnation widely considered a surprising phenomenon, given our knowledge of the historical trend in real rates of interest? This paper offers a simple explanation; our ignorance of the historical trend in the natural rate of interest. Recent developments in r-star relative to its longer-run trend might be an explanation as to why the secular stagnation hypothesis has once again resurfaced (Summers, 2015). As one-sided Kalman filter estimates clearly demonstrate, the decline in r-star is an unprecedented phenomenon in the Late Modern and Contemporary Era. Yet our understanding of neutral rates consistent with inflation and output stability prior is still unclear, rendering recent developments in r-star even more startling. In this regard, if moving averages of real rates are any indication of long-run trends in the natural rate of interest over such an extensive horizon (as in Schmelzing, 2020), its behaviour across recent history is indeed far from unique.

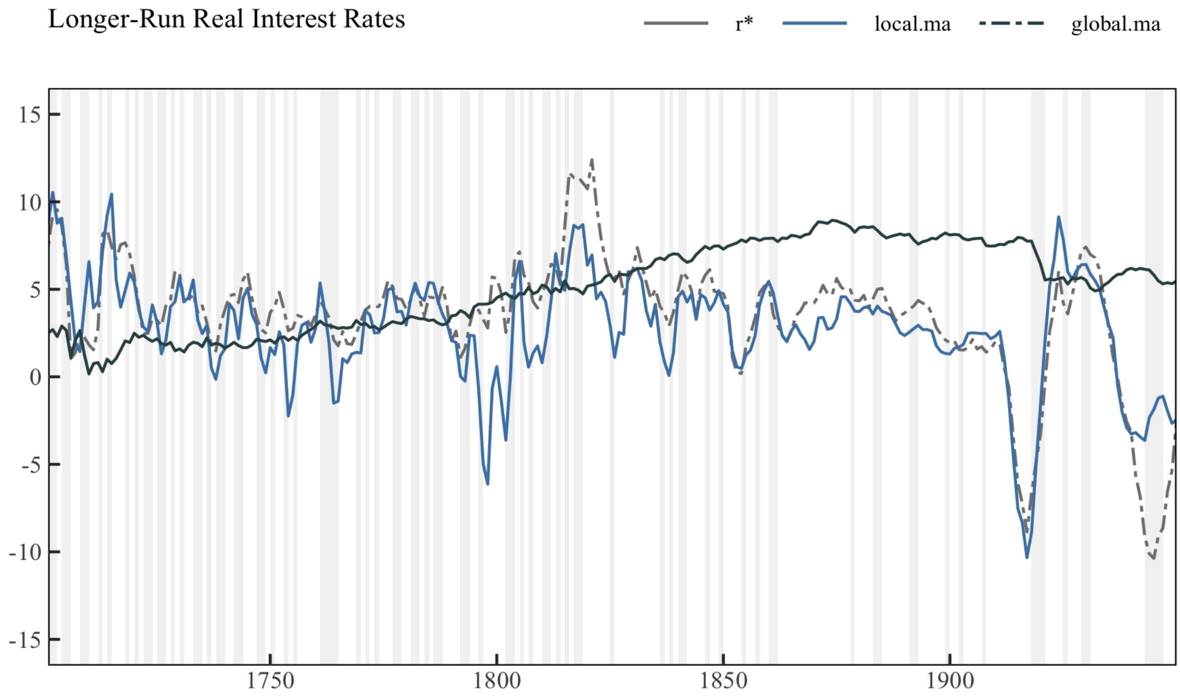

Figure 7. Longer-run real interest rates

Note: Estimated series for the natural rate of interest (r) are represented in black and seven year moving averages of local ex-post real rates are represented in blue between 1700-1950. Global seven year moving averages in grey are calculated using weighted averages of real rates across a number of advanced economies (see Schmelzing, 2020). Recession bands in grey are constructed from Broadberry et al. (2022).*

Notwithstanding, my estimates indicate a clear shift in the long-run trend of r-star by the end of the nineteenth century. What are the reasons for this reversal and to what extent can they be explained by changes in long-run drivers proposed by existing research? What follows is an explicit attempt to understand these shifts in the long-run supply and demand for savings beyond the last century.

6.4 Historical Drivers of the Natural Rate

This section investigates core drivers of the natural rate of interest established in the existing literature and for which historical data is available. In particular, I investigate the long-run relationship between r-star, productivity, demography, public debt, and private risk. The decline in growth rates of total factor productivity and their contributions to reductions in r-star is widely documented in the existing literature (refer to Rachel and Smith, 2017; Rachel and Summers, 2019). In the Ramsey model, lower productivity growth reduces expected future income, and greater savings needed to sustain future consumption results in lower rates of capital accumulation, which subsequently implies a lower capital-to-output ratio and a higher marginal product of capital. As the real interest rate equates to the marginal product of capital, reductions in productivity growth create downward pressures on the natural rate of interest. However, total factor productivity growth, whilst slower than expected (Crafts and Harley, 1992), grew steadily during the Industrial Revolution. Given the textbook Ramsey model, we would expect this to sustain a ceteris paribus increase in the natural rate of interest. In reality, it was only until the 1870s that Britain experienced reductions in labour and total factor productivity, which persisted through to the '80s before further, albeit more modest reductions in the decade prior to the First World War (Crafts and Mills, 2020).

The historical series for total factor productivity growth sourced from Thomas and Dimsdale (2017) is presented in Figure 8 between 1761 to 1950.[5] Moving averages (see Appendix D) reveal a gradual rise in the growth rate of productivity from the start of the sample, peaking in the 1870s before strong reductions that persist for decades. Total factor productivity struggles to recover and declines further in the 1900s. These trends confirm much of the literature in regard to the history of productivity in the United Kingdom around the end and beginning of the nineteenth and twentieth century respectively, suggesting a decline in r-star during this period.

Demographic changes in the advanced world have also driven recent trends in r-star. Declining rates of fertility and rising rates of average life expectancy are thought to be important contributors (see for instance Carvalho et al., 2016; Gagnon et al., 2016). According to this particular strand of the literature, shifts in the age distribution across the advanced world due to the mid-twentieth century baby boom has created asymmetries in the distribution of savings behaviour. In the standard overlapping-generations framework, subsequent bulges in the middle-aged demographic relative to the rest of the population have meant greater savings and lower expected returns (see Eggertsson et al., 2019). The United Kingdom experienced a sharper boom in fertility rates post-WWI, yet it was short-lived relative to the sustained rise post-WWII.

However, these demographic trends are recent phenomena, and in contrast to the wider historical narrative. In particular, it was only until the early twentieth century, that the United Kingdom experienced structural shifts in old-age demography. To demonstrate this, I temporally interpolate quinquennial demographic data from Wrigley and Schofield (1981) and derive an annual series for the old-age dependency ratio in the United Kingdom between 1761-1841, which I then extend to 1950 using data from the Human Mortality Database (HMD).[6] The historical series is presented in Figure 8. The data suggests that the ratio of old-age dependents to the working age population had actually been in gradual decline since the beginning of the sample until the early 1900s, rising sharply thereafter. These trends in demography suggest upward pressures on r-star followed by an accelerated reduction around the turn of the twentieth century.

Shifts in private risk are also thought to have contributed to long-run variation in the natural rate of interest. Risk premium shocks, which capture precautionary savings due to greater uncertainty, in addition to scarcities in the supply of safe assets, have driven down r-star in the advanced economies (see for instance Caballero and Farhi, 2017; Del Negro et al., 2017; Borio et al., 2019). These pressures on the equilibrium real interest rate follow directly from shifts in the supply and demand for aggregate savings. However, long-run trends in private risk during late modern history suggest an alternative narrative. To highlight this point, I determine the spread between long-term (10 month) and short-term (3 month) bonds to proxy for the term premium between 1761 to 1950

[5] To the best of my knowledge, data prior to this period is scarce for the United Kingdom. Given this is the shortest series among the selected historical drivers of the natural rate of interest, I restrict the following analysis to this horizon.

[6] Time variation in the old-age dependency ratio provided by Wrigley and Schofield (1981) is relatively negligible during this period, rendering cubic spline interpolation uncontroversial. Ratios calculated using series sourced from the HMD define old-age dependents as being above 60 to remain consistent with data from Wrigley and Schofield (1981).

using data from Thomas and Dimsdale (2017) and present this series in Figure 8. Moving averages (see Appendix D) reveal a relatively stable, if not declining term spread until the twentieth century, rising thereafter. Providing this is a good proxy for risk preferences, we expect marginal contributions to the rise in r-star followed by downward pressure over its decline.

Finally, recent strands within the existing literature suggest that public policy may have worked to prevent sharper reductions in the natural rate of interest in the advanced world (see Rachel and Summers, 2019). In particular rising public debt, increasing old-age payments, and greater public provision of credit have likely increased the natural rate of interest over recent years. The reasons for this arise from a number of models. The flow-based IS-LM framework suggests that permanent increases in government deficits lead to higher equilibrium real interest rates in the long-run. Following similar logic, temporary shifts to the deficit that reverse any shift in the IS schedule would therefore have only temporary consequences on the real rate. In the neoclassical model assuming complete markets and infinitely-lived agents, the Ricardian Equivalence suggests that representative households offset any changes to fiscal policy by consumption smoothing with private savings. Government borrowing therefore has no impact on the equilibrium interest rate. However, the neoclassical framework does allow a channel for government policy to influence interest rates through its pressure on the private capital stock. Greater expenditure tightens resource constraints and expands labour supply; as the marginal product of capital rises, investment is more profitable. Given the capital-to-labour ratio is fixed, interest rates are therefore only higher transitionally.

Additionally, modern micro-founded macroeconomic models in which the Ricardian equivalence does not hold, also suggest a link between government policy and the natural rate of interest. Given a finite planning horizon, agents adjust their consumption and savings in response to transfers due to their knowledge that future generations have to service what they do not. Furthermore, heterogeneity in the Marginal Propensity to Consume (MPC) across agents imply that any transfers away from low-MPC towards high-MPC agents will increase the aggregate propensity to consume and decrease the aggregate desire to save, thereby increasing real rates. Finally, public policy that reduces risk by increasing the provision of safe assets also pushes real rates higher. This particular channel of precautionary saving has been well documented in the literature and is the main driver considered in this paper (see Caballero et al., 2016; Caballero and Farhi, 2018).

In this regard, were it not for the positive influence of fiscal policy in recent years, equilibrium interest rates may well have been far lower. However, such trends in public policy are also a recent phenomenon that do not necessarily inform us of the wider historical narrative. I therefore collect data from Thomas and Dimsdale (2017) on government debt between 1761 to 1950 as arguably the most significant indicator of public policy. Data in Figure 8 reveals a sustained rise in government debt at the start of the sample followed by a prolonged decline over much of the nineteenth century. The start of the twentieth century is met with unprecedented increases in public debt coinciding with the First and Second World War. These trends suggest that shifts in public policy in terms of debt likely exerted downward pressure on the natural rate of interest over much of its ascension and upward pressure on r-star during much of its decline. What follows in the remainder of this paper is an empirical attempt to explain longer-run trends in the natural rate of interest in terms of these core drivers established by the existing literature.

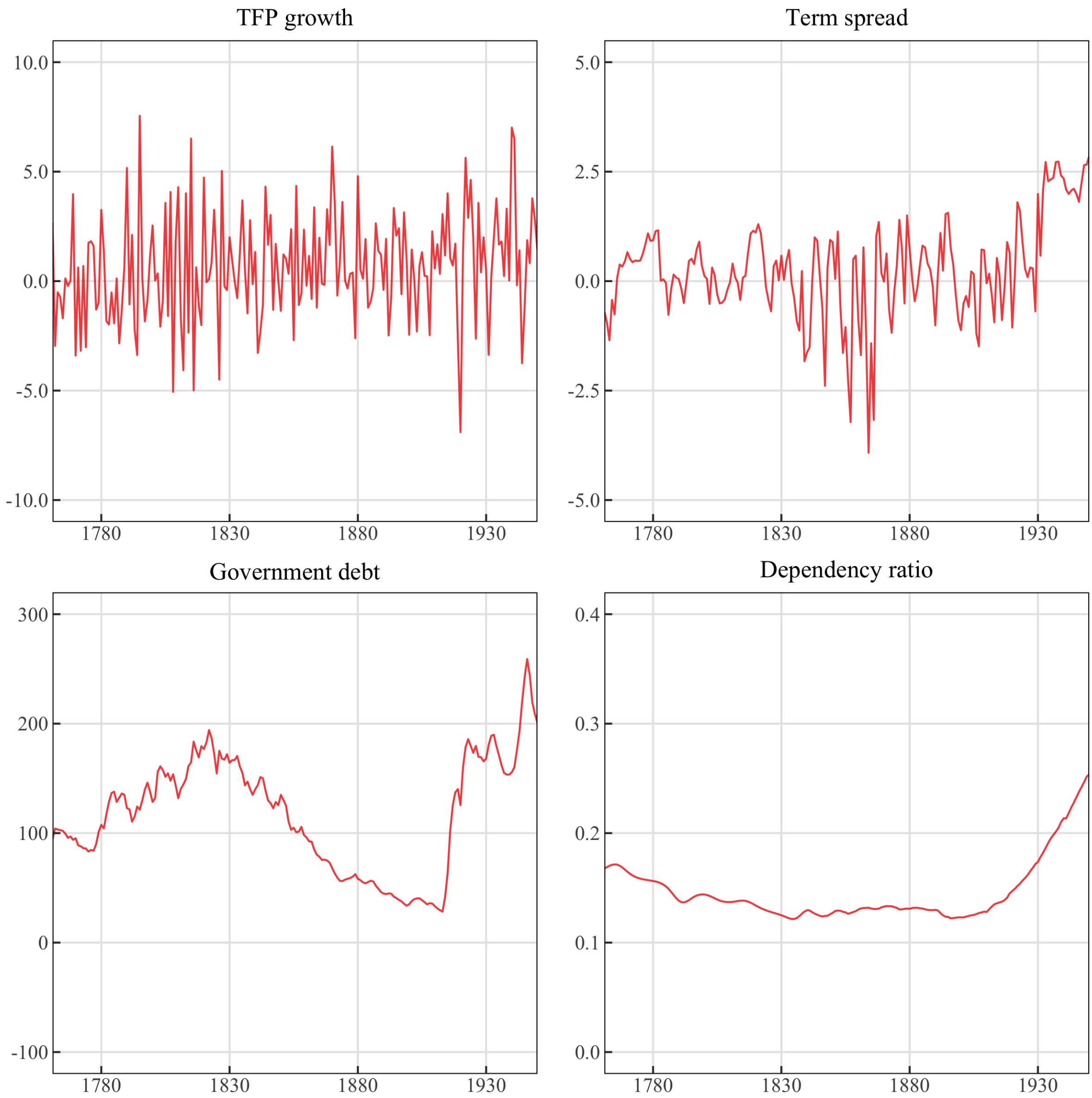

Figure 8. Historical productivity, risk, debt and demography

Note: Annual time series for the United Kingdom between 1761 to 1950 in red. The term spread measures the difference between interest rates on long-term (10 month) and short-term (3 month) bonds as a proxy for risk. Government or public sector debt is measured as a percentage of nominal GDP. The demographic old-age dependency ratio is calculated as the ratio of old dependents (60+) to the working age population.

Augmented Dickey Fuller tests reveal non-stationarity in all factors barring TFP growth at the 5% confidence level. I find evidence of at least two cointegrating vectors via the Johansen test and estimate the following Vector Error Correction Model (VECM) to characterise the joint relationship:

$$\Delta r_t^* = \alpha(r_{t-1}^* - \beta' X_{t-1}) + \gamma' \Delta X_t + \epsilon_t \tag{10}$$

where X_t is the vector of drivers of the natural rate of interest. Estimation results are presented in Table 2 for a VECM with various cointegrating vectors in addition to tests for Granger causality.

Table 2. Cointegration

	r*	TFP	Spread	Debt	Ratio
ADF (p)	0.78	0.01	0.25	0.83	0.89
$r = 1, k = 2$					
Cointegrating vector (S.E.)	1 (-)	0.55 (0.22)	-0.52 (0.18)	0.43 (0.33)	-0.37 (0.25)
Error correction coefficient (S.E.)	-0.03 (0.01)	-0.05 (0.02)	-0.01 (0.02)	-0.02 (0.01)	0.01 (0.05)
Granger test (p)	0.35	0.01	<0.01	0.03	0.06
$r = 2, k = 2$					
Cointegrating vector (S.E.)	1 (-)	0 (-)	-0.40 (0.09)	0.14 (0.07)	-0.22 (0.14)
	0 (-)	1 (-)	-0.38 (0.22)	0.27 (0.11)	-0.33 (0.08)
Error correction coefficient (S.E.)	-0.02 (0.01)	-0.08 (0.04)	-0.04 (0.01)	-0.03 (0.02)	0.07 (0.08)
Granger test (p)	0.33	< 0.01	0.02	0.04	0.08

Note: Estimates of a vector error correction model with one and two cointegrating vectors. r is chosen by the Johansen test. k is chosen by the Akaike Information Criterion. Standard errors reported in parentheses.

It is worth noting that the natural rate of interest is generated from an estimated model with its own uncertainty. In this regard, caution must be exercised when interpreting results from any estimated model in which it features; the objective here is therefore to simply characterise its comovement. With this in mind, estimates of the cointegrating vector suggest a long-run relationship between the equilibrium interest rate and its core drivers that conform to theory. In particular, there exists a positive relationship with total factor productivity growth and public debt. In addition, there exists a negative relationship with the term spread and old-age dependency ratio. The loading coefficients associated with all drivers barring demography are also negative and withing range, suggesting a slow correction back to equilibrium. To summarise these relationships further, Figure 9 presents the forecast error variance decompositions for the natural rate of interest in terms of shocks to its drivers. Whilst the Cholesky decomposition is sensitive to ordering, the overall empirical findings of a long-run relationship between the natural rate of interest and its drivers is robust.

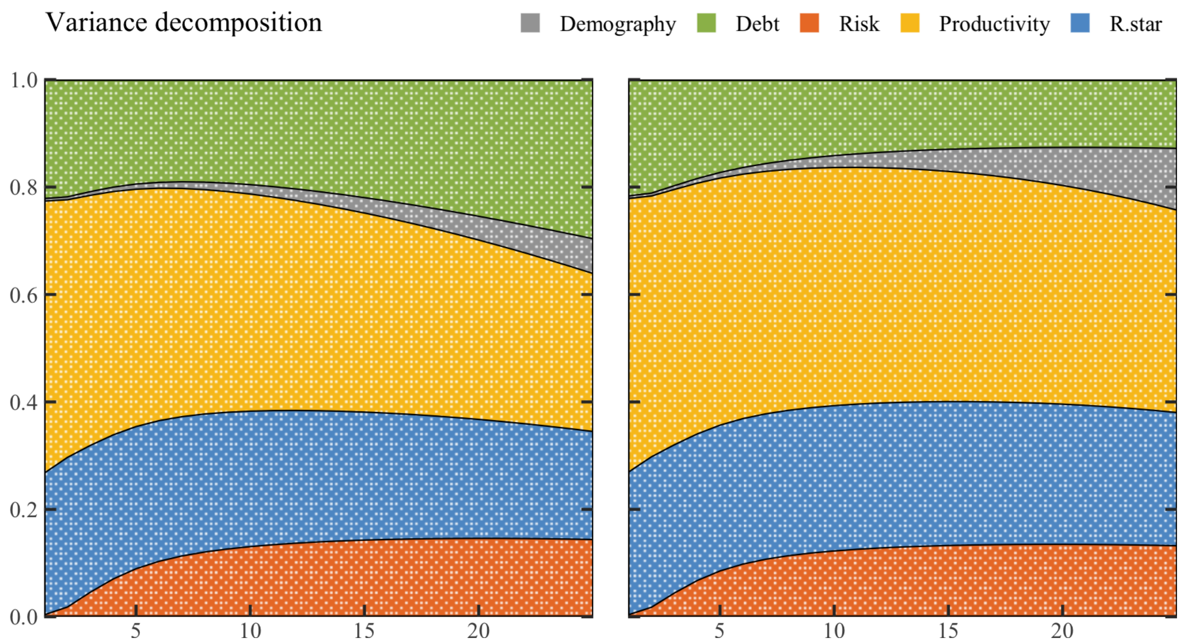

Figure 9. Variance decomposition of r-star

Note: Variance decomposition of the natural rate of interest (r) over 25 periods. Areas represent individual contributions of exogenous shocks to the error variance of r-star. Left: VECM (r=1). Right: VECM (r=2).*

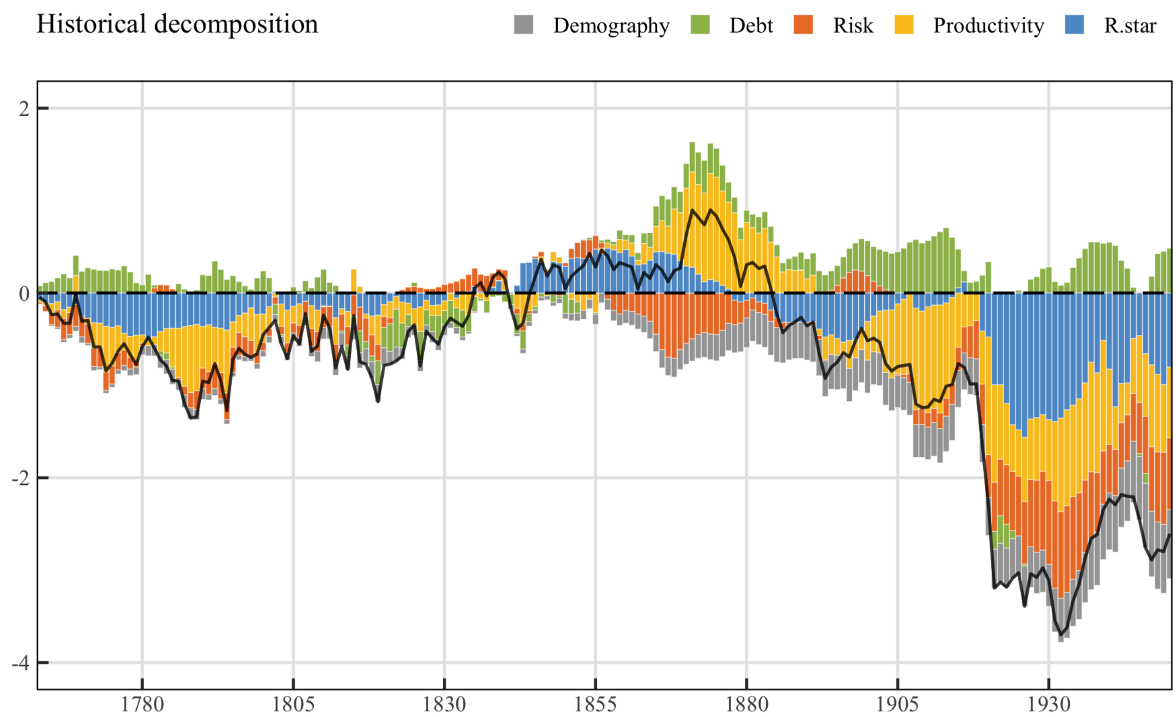

Figure 10. Historical decomposition of r-star

Note: Historical VAR decomposition of the natural rate of interest (r) between 1761-1950. The stochastic component of the long-run equilibrium interest rate to which all shocks aggregate is represented in black.*

Variance decompositions of the natural rate of interest suggest that some core drivers posited by the literature explain more of the variation in r-star than others. In particular, shocks to total factor productivity growth seem to have the largest influence, explaining approximately 50% of the total variation over the historical sample. This contribution marginally diminishes in the long-run, albeit clearly remains the most dominant relative to other drivers. Shocks to government debt also play a sizeable role, explaining roughly 20% of this variation. In contrast, demography and private risk play a relatively smaller role over shorter horizons, albeit grow over time to explain approximately 5-10% and 15-20% of the variation in r-star over longer horizons respectively.

These results suggest productivity growth and public debt play a greater role than documented in the existing literature. Additionally, demography and private risk are thought to be key contributors, yet estimates suggest they play a lesser role in explaining variation in r-star. This is perhaps explained by the fact that these variance decompositions capture variation over a historical sample that is characterised by unique pressures to the United Kingdom economy. These pressures, arising from phenomena such as multiple wars, the industrial revolution and various economic crises, may have affected the contributions of each driver in non-standard ways relative to recent history.

To parse these pressures on the equilibrium interest rate, further analysis is therefore required. Whilst heterogeneous overlapping-generation (OLG) approaches seem tempting, strong structural restrictions on the large historical sample may be suboptimal, particularly due to unique challenges in calibration and underlying volatility in each of the drivers. In this regard, I let the data do most of the talking and derive the historical decomposition from a recursively identified structural VAR model with imposed zero contemporaneous restrictions. Results are presented in Figure 10.

Historical decompositions suggest that selected core drivers explain far more variation in r-star over its stagnation relative to its ascension. In particular, public debt, private risk, and demography explain little variation in r-star during the latter eighteenth and early nineteenth century. Shocks to productivity play a slightly larger, albeit negligible role in driving r-star during this period. These results suggest that much of the rise and fall in r-star over the latter part of the nineteenth century were due to significant pressures from TFP growth. This is somewhat intuitive, given the peak and subsequent decline in productivity around the 1870s that persisted towards WWI.

Shocks to demography explain a negligible fraction of variation in r-star during its ascension. However, it becomes increasingly relevant after the mid-nineteenth century, which coincides with a stagnant and subsequently rising old-age dependency ratio towards the turn of the century. Similarly, shocks to the term spread initially explain only a small fraction of the variation in r-star prior to the mid-nineteenth century, after which contributions becomes substantial around the point of the trend reversal in r-star and during the early half of the twentieth century. These pressures also coincide with well documented shifts in the term spread over the same period.

Similar to shocks in productivity, shifts in private risk and demography collectively account for a large amount of the long-run stagnation in the natural rate of interest within the United Kingdom, particularly after the First World War. These findings corroborate a large body of empirical research that suggests such factors are of particular relevance in driving down the long-run equilibrium interest rate over the last half-century across the advanced world (see for instance Del Negro et al., 2019; Rachel and Summers, 2019; Kiley, 2020; Cesa-Bianchi et al., 2022).

Finally, this decomposition suggests that were it not for changes in public debt, the natural rate of interest would have declined further over the latter half of the sample. This is clearly captured by positive pressures during periods that coincide with sharp increases in government debt around the start of the twentieth century. Such movements in public policy also coincide with both World Wars, which would have exerted upward pressures on the natural rate of interest, insulating it from harsher downward adjustments due to variation in other drivers. These findings are also consistent with recent studies on the role of public policy in determining r-star (Rachel and Summers, 2019).

7 Conclusion

This paper estimates the natural rate of interest in the United Kingdom between 1700-1950. Findings reveal a sustained upward trend in r-star across much of the Late Modern Era before the start of secular stagnation around the turn of the twentieth century. This sustained decline in the natural rate of interest is further exacerbated by both World Wars and the Great depression, pressuring r-star to unprecedented lows. These novel results preface the widely documented downward trend in r-star over the last half-century. Together, they suggest that the long-run equilibrium interest rate in the United Kingdom is now returning back to the zero lower neutral bound. In other words, whilst secular stagnation is relatively unique to contemporary history, the present era of low natural rates of interest is not an entirely unfamiliar one to monetary policymakers.

This paper estimates the cointegrating relationship between natural rates of interest and theoretical drivers established in the existing literature. In particular, I investigate the link between the equilibrium interest rate, productivity, demography, private risk and public debt. Empirical results suggest a long-run relationship between r-star and its drivers that conforms to theory. To determine their individual contributions, I estimate a structural vector autoregression model that provides the historical error variance decomposition of the long-run equilibrium interest rate in terms of its drivers. Findings reveal strong pressures from productivity in addition to moderate pressures from demography and private risk around the trend reversal in r-star and over much of its subsequent decline. In addition, had it not been for the rise in public debt around the World Wars, the natural rate of interest may well have declined even further than estimated.

Although they are able to explain much of its stagnation, traditional drivers posited by existing research struggle to explain the estimated ascension in r-star, which persists for more than century. Further analysis is therefore required around shifts in the long-run supply and demand for savings across much of the Late Modern period. This sustained ascension may be explained by potentially omitted drivers such as inequality, as well as alternative dimensions to the factors investigated in this paper for which data was scarce over the historical sample. In addition, time series techniques such as those imposed in this paper may lack the theoretical rigour found in more structural models when teasing out the extent and nature of these core drivers. Given this was a period characterised by numerous wars and crises, volatility may also be undermining the long-run relationship between r-star and drivers established in the existing literature. I leave the inclusion of potentially omitted drivers, structural approaches to the historical equilibrium interest rate, and the use of econometric techniques to control for macroeconomic instability, to future extensions of this work.

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APPENDIX

Longer-Run Equilibrium Interest Rates:
Evidence From The United Kingdom

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This document provides all additional material for the paper "*Longer-Run Equilibrium Interest Rates: Evidence From The United Kingdom*." Appendix A outlines the adjusted multi-stage Holston, Laubach and Williams (2017) maximum-likelihood procedure used to estimate the annual natural rate of interest (r-star). Appendix B presents aggregate estimates of the long-run equilibrium interest rate and trend growth rate in the United Kingdom between 1700-1950. Appendix C outlines historical annual data used to estimate the natural rate of interest and its sources in addition to data used to investigate its historical drivers. Finally, Appendix D presents five-year and ten-year moving averages of the historical series used to estimate the natural rate of interest, in addition to moving averages of its historical drivers.

A Adapted HLW System

State-space representation:

$$\mathbf{y}_t = \mathbf{A}'\mathbf{x}_t + \mathbf{H}'\xi_t + \mathbf{w}_t$$

$$\xi_t = \mathbf{F}\xi_{t-1} + \mathrm{v}_t$$

A.1 First Stage Specification:

$$\mathbf{y}_t = \begin{bmatrix} y_t, \pi_t \end{bmatrix}' \qquad \mathbf{x}_t = \begin{bmatrix} y_{t-1}, \pi_{t-1} \end{bmatrix}' \qquad \xi_t = \begin{bmatrix} y_t^*, y_{t-1}^* \end{bmatrix}'$$

$$\mathbf{A}' = \begin{bmatrix} \phi_y & 0 \\ \varphi_y & \varphi_\pi \end{bmatrix} \qquad \mathbf{H}' = \begin{bmatrix} 1 & -\phi_y \\ 0 & -\varphi_y \end{bmatrix} \qquad \mathbf{F} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \qquad \mathbf{Q} = \begin{bmatrix} \sigma_{y*}^2 & 0 \\ 0 & 0 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_1 = \begin{bmatrix} \phi_y, \varphi_\pi, \varphi_y, g, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*} \end{bmatrix}$$

A.2 Second Stage Specification:

$$\mathbf{y}_t = \begin{bmatrix} y_t, \pi_t \end{bmatrix}' \qquad \mathbf{x}_t = \begin{bmatrix} y_{t-1}, r_{t-1}, \pi_{t-1}, 1 \end{bmatrix}' \qquad \xi_t = \begin{bmatrix} y_t^*, y_{t-1}^*, g_{t-1} \end{bmatrix}'$$

$$\mathbf{A}' = \begin{bmatrix} \phi_y & \phi_r & 0 & \phi_0 \\ \varphi_y & 0 & \varphi_\pi & 0 \end{bmatrix} \qquad \mathbf{H}' = \begin{bmatrix} 1 & -\phi_y & \phi_g \\ 0 & -\varphi_y & 0 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{Q} = \begin{bmatrix} \sigma_{y*}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\lambda_g \sigma_{y*})^2 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_1 = \begin{bmatrix} \phi_y, \phi_r, \phi_0, \phi_g, \varphi_\pi, \varphi_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*} \end{bmatrix}$$

A.3 Third Stage Specification

$$\mathbf{y}_t = \left[y_t, \pi_t\right]' \qquad \mathbf{x}_t = \left[y_{t-1}, r_{t-1}, \pi_{t-1}\right]' \qquad \xi_t = \left[y_t^*, y_{t-1}^*, g_{t-1}, z_{t-1}\right]'$$

$$\mathbf{A}' = \begin{bmatrix} \phi_y & \phi_r & 0 \\ \varphi_y & 0 & \varphi_\pi \end{bmatrix} \qquad \mathbf{H}' = \begin{bmatrix} 1 & -\phi_y & -\phi_r & -\phi_r \\ 0 & -\varphi_y & 0 & 0 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{Q} = \begin{bmatrix} (1+\lambda_g^2)\sigma_{y*}^2 & 0 & (\lambda_g \sigma_{y*})^2 & 0 \\ 0 & 0 & 0 & 0 \\ (\lambda_g \sigma_{y*})^2 & 0 & (\lambda_g \sigma_{y*})^2 & 0 \\ 0 & 0 & 0 & \left(\frac{\lambda_z \sigma_{\tilde{y}}}{\phi_r}\right)^2 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_1 = \left[\phi_y, \phi_r, \varphi_\pi, \varphi_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*}\right]$$

B One-Sided Kalman Filter Estimates

Figure B.1. Complete one-sided estimates for r* and g

C Business Cycle Dates

Table A.1. Turning Points: 1700-1950

Peak	Trough	Peak	Trough
1701	1703	1792	1794
1704	1706	1796	1797
1708	1710	1802	1804
1712	1713	1805	1806
1714	1715	1807	1808
1718	1719	1810	1812
1720	1721	1813	1814
1722	1724	1815	1816
1725	1727	1817	1819
1728	1729	1825	1826
1730	1731	1836	1837
1733	1735	1838	1839
1736	1737	1840	1842
1738	1740	1846	1847
1742	1744	1849	1850
1747	1749	1854	1855
1750	1751	1857	1858
1753	1754	1860	1862
1755	1756	1878	1879
1761	1765	1883	1885
1769	1770	1891	1893
1771	1772	1899	1900
1773	1774	1902	1903
1777	1779	1907	1908
1781	1783	1918	1921
1784	1785	1925	1926
1786	1788	1929	1931
1792	1794	1943	1947

Note: Business cycles (annual turning points) reported in Broadberry et al. (2023) between 1700-1950.

D Data

Real Gross Domestic Product

GDP is sourced from Thomas and Dimsdale (2017) published by the Bank of England. I employ the geographically-consistent real headline series, which is a chained volumetric composite time-series from multiple sources (Feinstein, 1972; Solomou and Weale, 1991; Broadberry et al. 2015).

Consumer Price Inflation

CPI inflation is sourced from Thomas and Dimsdale (2017) published by the Bank of England. This is an composite series, sourced from the Shumpeter-Gilboy Index between 1700-1750 (refer to Mitchell, 1988), from Crafts and Mills (1994) between 1750-1770, from Feinstein (1991; 1998) between 1770-1914 and from the ONS between 1914-1950 (refer to O'Donoghue et al. 2004).

Real Interest Rates

Nominal interest rates are sourced directly from the Official Bank Rate data published by the Bank of England. Inflation expectations and consumer price index inflation used to derive ex-ante and ex-post real interest rates respectively are sourced from Thomas and Dimsdale (2017) published by the Bank of England. The Fisher equation is used to compute the ex-ante and ex-post real rate.

Total Factor Productivity

Total Factor Productivity (TFP) growth is sourced from Thomas and Dimsdale (2017) published by the Bank of England. It is the ratio of real output to combined units of labor and capital input.

Term Spread

The term spread is defined as the difference between short-term (3 month) bonds and long-term (10 month) bonds. To calculate this proxy for the risk premium, I source annual data on long-term and short-term securities from Thomas and Dimsdale (2017) published by the Bank of England.

Government Debt

Government debt or net public sector debt is sourced from Thomas and Dimsdale (2017) published by the Bank of England. It is measured as a percentage of nominal gross domestic product (GDP).

Old-Age Dependency Ratio

The old-age dependency ratio is a composite series constructed from Wrigley and Schofield (1981) between 1761-1841. I interpolate this quinquennial data using cubic spline interpolation. To compute the dependency ratio, I take the ratio of old dependents (60+) to the working age population. This series is finally extended to 1950 using data from the Human Mortality Database (HMD).

E Moving Averages

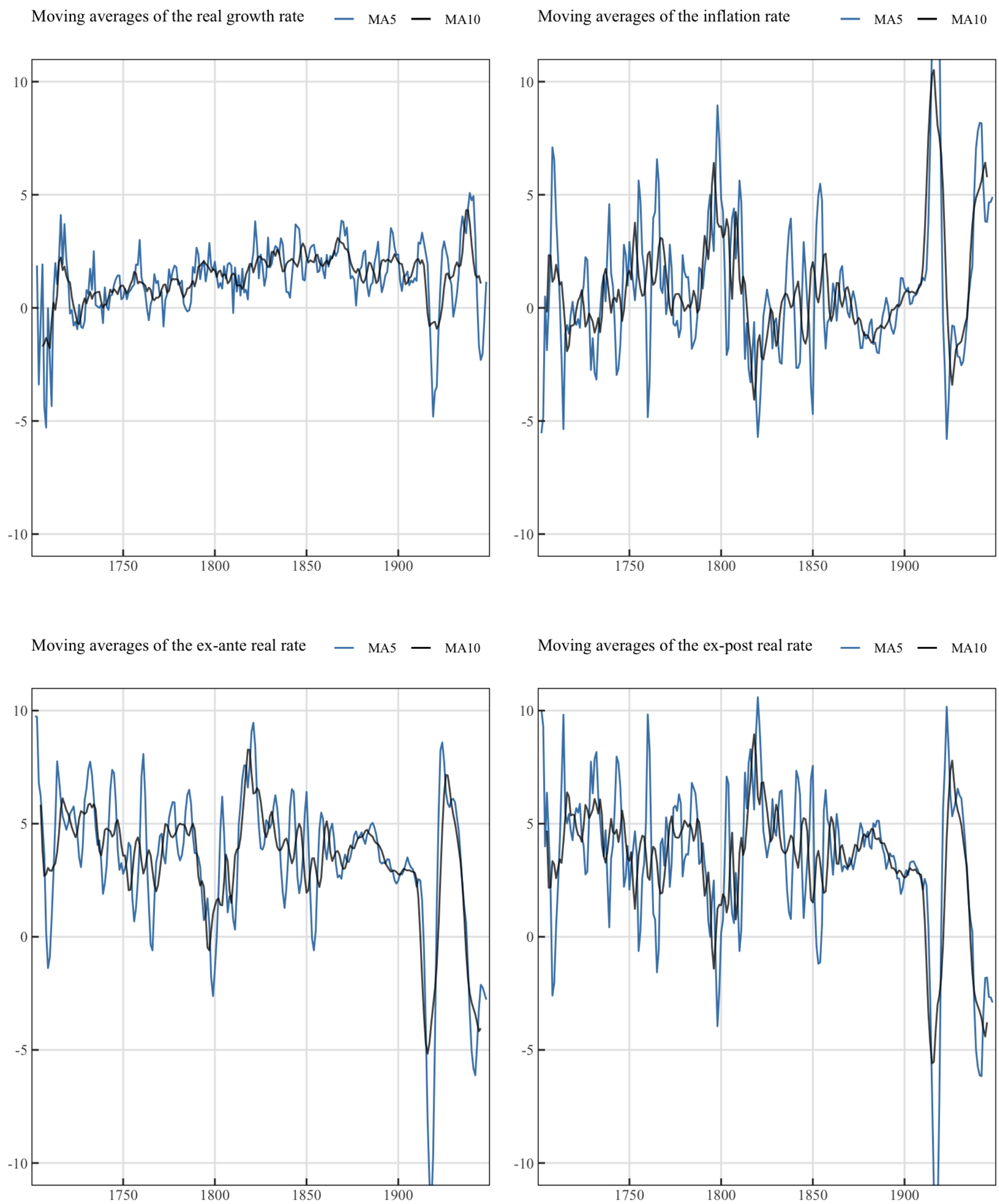

Figure C.1. Moving averages of growth, inflation and interest rates

Note: Annual time series between 1700-1950. Real growth rates are rates of change in real GDP. Inflation is the rate of change in CPI. Ex-post real rates are determined by the Fisher relation. Ex-ante real rates are calculated using inflation expectations by Thomas and Dimsdale (2017). Five year moving averages (MA5) are represented in blue. Ten year moving averages (MA10) are represented in black.

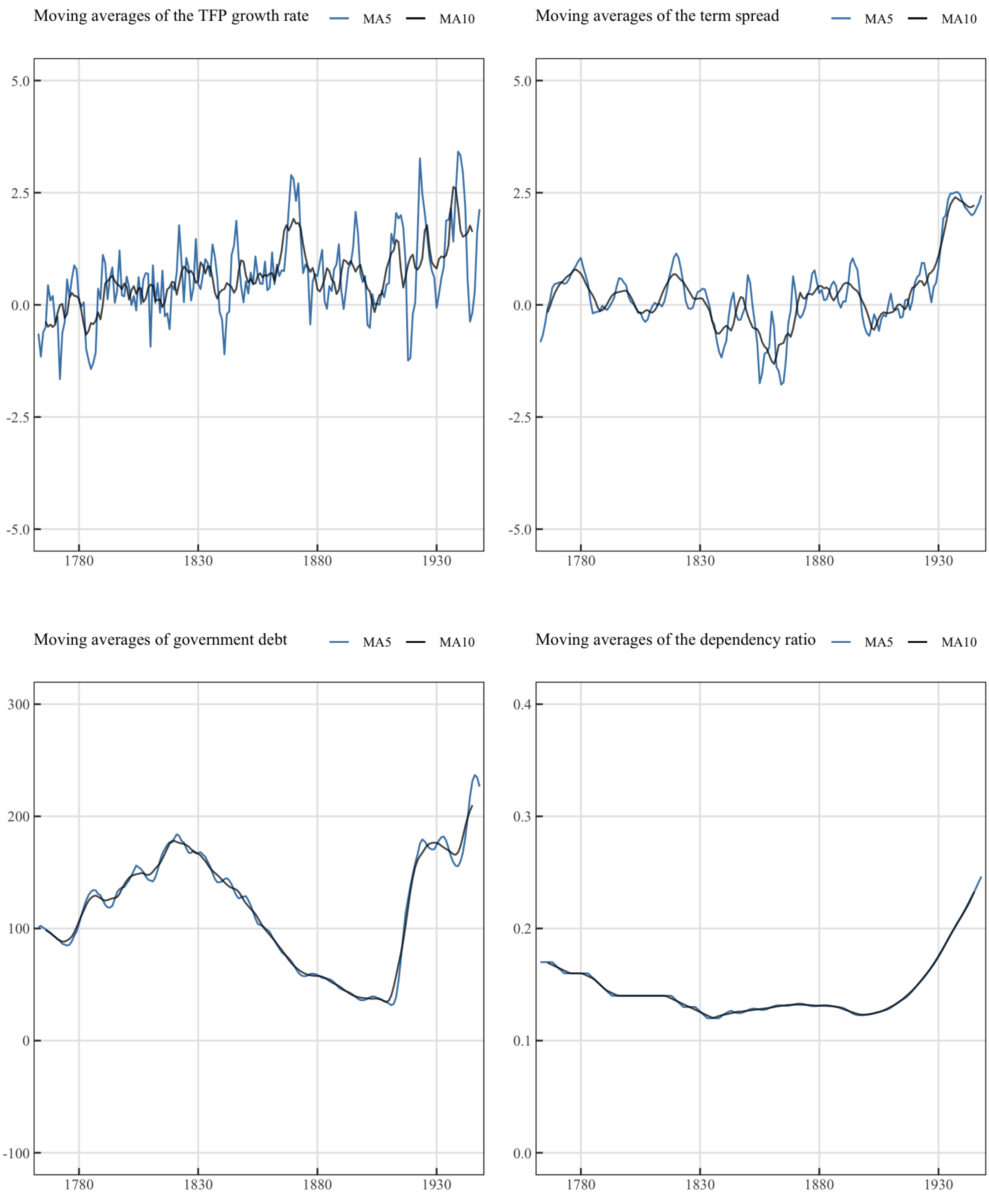

Figure C.2. Moving averages of productivity, risk, debt and demography

Note: Annual time series between 1760-1950. The term spread measures the difference between interest rates on long-term (10 month) and short-term (3 month) bonds as a proxy for risk. Government or public sector debt is measured as a percentage of nominal GDP. The demographic old-age dependency ratio is calculated as the ratio of old dependents (60+) to the working age population. Five year moving averages (MA5) are represented in blue. Ten year moving averages (MA10) are represented in black.