Time-Varying Policy Rules and Monetary Policy (Mis)perceptions*

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Abstract

This paper estimates a time-varying random-coefficient forward-looking Taylor rule for the United States using a novel kernel-weighted continuously-updating time-varying generalised methods of moments estimator. Given time-varying coefficients of the policy rule, we derive an explicit time series for the implied natural rate of interest ($r_t^{\#}$), which we argue is a strong proxy for the perceptions of monetary policymakers. We evaluate this against the actual natural rate of interest (r_t^{*}) to measure (mis)perceptions of the long-run equilibrium interest rate. This paper documents the FED's conduct of monetary policy and explains key periods of instability during which policymakers underestimate or overestimate the natural rate of interest.

Keywords: Interest rates, Natural rate of interest, Kalman filter, Instrument variables, Generalised methods of moments, Time-varying parameters, Taylor rule, Monetary Policy

JEL classification: C14, C36, C32, E43, E52, E58

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1 Introduction

The natural rate of interest (r_t^* or r-star) has played an increasingly important role in the conduct of monetary policy. In their attempt to stabilise output and inflation, policymakers regularly communicate their estimation of the equilibrium interest rate and how it has influenced their setting of policy rates. Perceptions of r-star are therefore a crucial determinant of priorities between inflation and output stability. The Taylor (1993) rule has long been a conventional tool to model this stance. It suggests that policymakers ought to set policy rates in response to deviations in inflation from its target and output from its potential. In addition, the equilibrating intercept in Taylor-type rules provides an estimate of the natural rate of interest that achieves an optimal balance between these policy objectives. The estimation of such monetary policy rules are well documented within the existing literature and relate to wider discussions including, but not limited to, whether monetary policy satisfies the Taylor principle (e.g., Taylor, 1999; Clarida, Galí and Gertler, 2000; Orphanides, 2004), optimal monetary policy design (Woodford, 2001) and interest rate persistence (e.g., Rudebusch, 2002; Coibion and Gorodnichenko, 2012).

This paper is less concerned with such thoroughly treated aspects of the existing literature and instead, focuses on the perceptions of policymakers concerning the state of the economy, which we argue is well proxied for by the time-varying natural rate implicit in the conduct of monetary policy. We derive explicit estimates of these implicit beliefs by relaxing the assumption of time-invariance underlying standard empirical approaches to monetary policy. In particular, classical formulations of the Taylor rule assume a long-run equilibrium interest rate that is fixed over time in addition to static weights on deviations in inflation from its target and output from its potential. There are strong reasons to suggest neither of these assumptions are true.

As for the former, the assumption that the equilibrium interest rate is constant has been widely challenged. However, in doing so, many studies depart from the standard Taylor rule specification. This is perhaps expected given the intercept within prototypical policy rules may be interpreted as the perceived natural rate assumed by the central bank in its determination of nominal policy rates (Taylor, 1993; 1999). The estimation of the objective or actual natural rate is therefore independent of a reaction function and contingent on more structural parameters. For instance, Laubach and Williams (2003) and Holston, Laubach and Williams (2017) implement a semi-structural approach to estimate the natural rate of interest, which is carved primarily from variation in growth. Their novel application of the Kalman filter in a multi-stage maximum likelihood procedure reveals a secular decline in the long-run equilibrium interest rate across the advanced world. These general findings have been shown to be robust to a number of empirical extensions and variations (see for instance Clark and Kozicki, 2005; Mésonnier and Renne, 2007; Berger and Kempa, 2014; Lewis and Vazquez-Grande, 2018; Krustev, 2019).

Whilst many similar attempts have been made to measure r-star in the literature (e.g., Barsky et al., 2014; Hamilton et al., 2016; Johannsen and Mertens, 2016; Kiley, 2015; Lubik and Matthes, 2015; Pescatori and Turunen, 2016), the perceived rate assumed by central banks in their conduct of monetary policy has received very little attention. This theoretical rate is particularly important, as it reveals the perception of policymakers concerning the true long-run equilibrium interest rate, the implications of which are extensive. For instance, Ajello et al. (2020; 2021) capture mistakes in these perceptions using scenario-based analysis and find that the welfare cost of overestimating the natural rate of interest is greater than the cost of underestimating it. Whilst highly informative, the authors do not propose a formal methodology to quantify these policy misperceptions. In fact, to the best of our knowledge, no attempts have yet been made to estimate the implied time-varying natural rate of interest arising from the Taylor rule using ex-post data.

In a similar vein, some studies have attempted to measure market perceptions of the monetary policy rule rather than the perceptions of policymakers concerning the state of the macroeconomy. For instance, Hamilton et al. (2011) directly estimate a market-perceived policy rule using macroeconomic news and find evidence of variation in the associated response parameters. Bauer et al. (2022) attempt to capture these perceptions in a time-varying context by estimating a market-perceived policy rule from forecasts using panel techniques and a state-space model (see Carvalho and Nechio (2014) for a similar exercise in a time-invariant setting using household expectations and professional forecasts). Whilst market perceptions are useful in determining policy effectiveness, policymaker perceptions are perhaps of even greater significance to the conduct of monetary policy and are therefore the central focus of this paper.

As for the latter assumption concerning static reaction coefficients in Taylor-type rules, relative weights associated with deviations in output and inflation are likely inconstant due to changing objectives and preferences of monetary policymakers, both of which are regularly influenced by new information learnt by the central bank over time (see for instance Favero and Rovelli, 2003). This time-variation is empirically consistent with a number of studies that find breaks in the reaction coefficients of the Taylor rule (e.g., Clarida et al., 2000; Smets and Wouters, 2007; Coibion and Gorodnichenko, 2011; Sims and Zha, 2006).

Despite these significant findings, few have approached the Taylor rule from a time-varying perspective. Amongst those studies that have, the Kalman (1960) filter is frequently implemented within a maximum likelihood framework. For instance, Boivin (2006) sources real-time ex-ante data, which assumes a constant short-term nominal rate over a given forecast horizon to mitigate endogeneity in the estimation of a forward-looking monetary policy rule with drifting coefficients. Perhaps more closely related to our work, Kim and Nelson (2006) also estimate a forward-looking Taylor rule that allows for time-varying parameters using ex-post data. Issues of potential endogeneity are resolved by way of a Heckman-type bias-correcting two-step procedure that accounts for both nonlinearity and heteroskedasticity.

Alternatively, Cogley and Sargent (2001; 2005) implement the Kalman filter in a Vector Auto Regression (VAR) framework to estimate the time-varying parameters of the policy rule (refer to Canova and Gambetti (2004) for a similar exercise within a Structural VAR (SVAR) framework). Aside from the Kalman filter, Orphanides and Williams (2005) also allow for time-varying parameters using VAR and OLS techniques to investigate structural changes from the policymakers view in the implementation of monetary policy. Similarly, Sims and Zha (2006) estimate a SVAR model in which they allow for time-variation in the reaction coefficients of the Taylor rule to investigate regime switching. Finally, Owyang and Ramey (2004) use a markov-switching model to measure these shifts in the parameters of the Federal Reserve's policy rule.

Although differing in methodology, each of these studies detect some degree of time-variation in the reaction coefficients of the policy rule, which serves as strong motivation for our research, particularly as time-varying priorities of monetary policymakers imply time-varying perceptions of r-star. However, many of these studies do not convincingly resolve issues of endogeneity and impose numerous restrictions on the estimation process that involve unnecessary complexity when gauging these priorities. In fact, most of these attempts fail to directly estimate a forward-looking policy rule with time-varying random-coefficients, nor exploit these estimated time-varying parameters to reveal the implied time-varying inflation target or implied long-run equilibrium interest rate, which we believe, above all else, is the main novelty in our research.

This is perhaps because efficient procedures to mitigate endogeneity associated with expected variables present within the policy rule have not yet been readily applicable to time-varying random-coefficient models. We implement a methodology that achieves precisely that. This paper extends the framework of Clarida et al. (2000), hearafter CGG, in which ex-post data is employed to estimate a static forward-looking Taylor rule using instrument variables (IV) and generalised methods of moments (GMM). We outline a methods of moments extension to the kernel-weighted time-varying estimation approach inspired by Ciu et al. (2023) and developed in a series of published work by Giraitis et al. (2014; 2018; 2021), to estimate a time-varying random-coefficient forward-looking policy rule using ex-post data. Given estimated nonconstant coefficients, we analyse the stance of monetary policy on inflation and output stability across our sample.

Our contribution is therefore twofold; not only do we estimate a time-varying forward-looking Taylor rule, but in doing so yield estimated time-varying coefficients that allow us to derive the implied natural rate of interest, hereafter $r_t^{\#}$ or r-hash. We argue that this is a strong proxy for the beliefs and perceptions of monetary policymakers on the state of the economy and define monetary policy misperceptions as the absolute deviation in r-hash from r-star. In doing so, we construct a framework to quantify these misperceptions, so as to evaluate the accuracy of monetary policy. If r-hash is exceedingly high, policymakers overestimate r-star, monetary conditions tighten and lead to higher unemployment and lower inflation. Conversely, if r-hash is lower than r-star, conditions loosen and lead to lower unemployment and higher inflation.

Our results find substantial time variation in the coefficients of the Taylor rule. In particular, we document varying policy responses to the inflation and output gap, in addition to a high degree of policy inertia. We find that during the pre-Volcker era, in which the Taylor principle is violated, policymakers persistently misperceive the neutral rate. In contrast, policymakers become increasingly accurate in their perceptions over the subsequent post-Volcker period, in which the Taylor principle is satisfied, with minimal deviations in r-hash from r-star. This paper also finds evidence to suggest that the Federal Reserve actively sought to correct its underestimation of r-star over the pre-Volcker period, at times overshooting its judgment in the early part of the post-Volcker period. In addition, we find evidence that the implied and actual natural rate of interest are cointegrated, suggesting movements in r-star are ultimately reflected in the beliefs of policymakers.

The remainder of this paper is structured as follows. Section 2 outlines the theoretical framework. Section 3 discusses the data, instruments, kernel functions and bandwidth parameters used in our estimation. Section 4 presents and discusses the main results. Section 5 concludes.

2 Theoretical Framework

2.1 Time-Varying Taylor Rules

To begin, we introduce the specification of the static forward-looking Taylor rule with interest rate smoothing as in CGG (2000). The central bank's target policy rate may be defined as follows:

$$i_t^* = i^\# + \delta_\pi(E[\pi_t|\Omega_t] - \pi^*) + \delta_{\tilde{y}}E[\tilde{y}_t|\Omega_t] \tag{1}$$

where $i^\#$ is the nominal equilibrating interest rate, π^* is the inflation target, π_t is the inflation rate and $\tilde{y}_t$ is the output gap, whose expectation is conditional on the central bank's information set Ω_t in period t. We define a constant parameter $\delta_0 = i^\# - \delta_\pi\pi^*$, to yield the following equation:

$$i_t^* = \delta_0 + \delta_\pi E[\pi_t|\Omega_t] + \delta_{\tilde{y}}E[\tilde{y}_t|\Omega_t] \tag{2}$$

As central banks tend to adjust short-term interest rates in sequential incremental steps, we introduce a smoothing mechanism describing how actual policy rates adjust to the target policy rate:

$$i_t = [1-\rho]i_t^* + \rho(L)i_t + u_t \tag{3}$$

where ρ is the smoothing parameter. To derive a policy rule for the actual interest rate, we substitute the target policy rate (1) into the partial adjustment mechanism (3) to yield the following:

$$i_t = (1-\rho)(\delta_0 + \delta_\pi E[\pi_t|\Omega_t] + \delta_{\tilde{y}}E[\tilde{y}_t|\Omega_t]) + \rho(L)i_t + u_t \tag{4}$$

Under rational expectations, we may derive the following policy reaction function:[1]

$$i_t = (1-\rho)(\delta_0 + \delta_\pi\pi_t + \delta_{\tilde{y}}\tilde{y}_t) + \rho(L)i_t + \varepsilon_t \tag{5}$$

Note that equation (5) may be expressed as a linear function of auxiliary coefficients:[2]

$$i_t = \psi_0 + \psi_\pi\pi_t + \psi_{\tilde{y}}\tilde{y}_t + \psi_\rho(L)i_t + \varepsilon_t \tag{6}$$

Given a set of predetermined instruments $\mathbf{x}_t$ within the information set Ω_t that are assumed uncorrelated with the error term u_t, we may write the following set of orthogonality conditions:

$$E[i_t - (1-\rho)(\delta_0 + \delta_\pi\pi_t + \delta_{\tilde{y}}\tilde{y}_t) - \rho(L)i_t|\mathbf{x}_t] = 0 \tag{7}$$

GMM is used to address potential endogeneity in (5) given stochastic variation in the model coefficients. To contextualise this approach, we first outline the static GMM framework in the notation of Hayashi (2000), which we also estimate in Section 4 before moving to the time-varying case.

[1] Note that the error term $\varepsilon_t = (1-\rho)\Big(\psi_\pi(\pi_t - E[\pi_t|\Omega_t]) + \delta_{\tilde{y}}(\tilde{y}_t - E[\tilde{y}_t|\Omega_t])\Big) + v_t$ is a linear combination of the forecast error of inflation and output and the exogenous error u_t; orthogonal to variables in the information set.

[2] Where $\psi_0 = (1-\rho)\delta_0$, $\psi_\pi = (1-\rho)\delta_\pi$, $\psi_{\tilde{y}} = (1-\rho)\delta_{\tilde{y}}$ and $\psi_\rho = \rho$. We may then estimate a linear regression and extract coefficients from the auxiliary vector $\boldsymbol{\psi}$. Alternatively we may simply estimate the nonlinear representation.

Consider the following model for the federal funds rate:

$$i_t = \mathbf{z}_t'\boldsymbol{\delta} + \varepsilon_t \qquad (t = 1, 2, ..., T) \tag{8}$$

where $\mathbf{z}_t$ is a $L \times 1$ vector of regressors, $\boldsymbol{\delta}$ is a $L \times 1$ coefficient vector, and ε_t is the error term. Given $\mathbf{x}_t$ is a $K \times 1$ vector of predetermined instruments, the exogeneity conditions are given by:

$$E[\mathbf{x}_t(i_t - \mathbf{z}_t'\boldsymbol{\delta})] = \mathbf{0} \quad \text{or} \quad E(\mathbf{g}) = E[\mathbf{g}(\mathbf{w}_t; \boldsymbol{\delta})] = \mathbf{0} \tag{9}$$

where but for parsimony we define $\mathbf{g} = \mathbf{g}(\mathbf{w}_t; \boldsymbol{\delta}) \equiv \mathbf{x}_t(i_t - \mathbf{z}_t'\boldsymbol{\delta})$. Let $\boldsymbol{\delta}^0$ be the $L \times 1$ parameter vector of interest and consider the system of K simultaneous equations in L unknowns such that:

$$E[\mathbf{g}(\mathbf{w}_t; \boldsymbol{\delta}^0)] = \mathbf{0} \tag{10}$$

Orthogonality in (10) implies that $\boldsymbol{\delta}^0$ is one solution to this system of simultaneous equations. The sample analogue of the population moments $E[\mathbf{g}(\mathbf{w}_t; \boldsymbol{\delta}^0)]$ is the sample mean of the function $\mathbf{g}$ evaluated at some hypothetical parameter vector $\boldsymbol{\delta}$, which may be expressed as follows:

$$\begin{aligned} \mathbf{g}_T(\boldsymbol{\delta}) &\equiv \frac{1}{T}\sum_{t=1}^{T} \mathbf{g}(\mathbf{w}_t; \boldsymbol{\delta}) = \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_t \cdot (i_t - \mathbf{z}_t'\boldsymbol{\delta}) \\ &= \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_t \cdot i_t - \Big(\frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_t\mathbf{z}_t'\Big)\boldsymbol{\delta} \equiv \mathbf{s}_{\mathbf{xi}} - \mathbf{S}_{\mathbf{xz}}\boldsymbol{\delta} \end{aligned} \tag{11}$$

where $\mathbf{s}_{\mathbf{xi}}$ and $\mathbf{S}_{\mathbf{xz}}$ are the sample moments of $\boldsymbol{\sigma}_{\mathbf{xi}}$ and $\boldsymbol{\Sigma}_{\mathbf{xz}}$. Given the sample analogue $\mathbf{g}_T(\boldsymbol{\delta}) = \mathbf{0}$, we have that $\mathbf{S}_{\mathbf{xz}}\boldsymbol{\delta} = \mathbf{s}_{\mathbf{xi}}$. If however $K > L$, the model is over-identified, $\mathbf{S}_{\mathbf{xz}}$ is no longer invertible and we choose $\boldsymbol{\delta}$ optimally by minimising the distance.

The distance between any two K-dimensional vectors $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ is given by the quadratic $(\boldsymbol{\xi} - \boldsymbol{\eta})'\mathbf{W}(\boldsymbol{\xi} - \boldsymbol{\eta})'$, where $\mathbf{W}$ is a symmetric $K \times K$ positive definite weighting matrix. The GMM estimator is therefore a solution to the following minimisation problem:

$$\hat{\boldsymbol{\delta}}(\mathbf{W}) \equiv \text{argmin}_{\boldsymbol{\delta}} J(\boldsymbol{\delta}, \mathbf{W}) \equiv T\mathbf{g}_T(\boldsymbol{\delta})'\mathbf{W}\mathbf{g}_T(\boldsymbol{\delta}) \tag{12}$$

Without loss of generality, if $\mathbf{g}_T(\boldsymbol{\delta})$ is linear in $\boldsymbol{\delta}$, the objective function is quadratic in $\boldsymbol{\delta}$, therefore $J(\boldsymbol{\delta}, \mathbf{W}) = T(\mathbf{s}_{\mathbf{xi}} - \mathbf{S}_{\mathbf{xz}}\boldsymbol{\delta})'\mathbf{W}(\mathbf{s}_{\mathbf{xi}} - \mathbf{S}_{\mathbf{xz}}\boldsymbol{\delta})$. Given ergodicity, stationarity and the rank condition for identification hold, the minimiser of equation (12) is the celebrated GMM estimator:

$$\hat{\boldsymbol{\delta}}(\mathbf{W}) = (\mathbf{S}_{\mathbf{xz}}'\mathbf{W}\mathbf{S}_{\mathbf{xz}})^{-1}\mathbf{S}_{\mathbf{xz}}'\mathbf{W}\mathbf{s}_{\mathbf{xi}} \tag{13}$$

Choosing weighting matrix $\mathbf{W}$ optimally, minimises the asymptotic variance. Thus, if $\mathbf{W} = \widehat{\mathbf{S}}^{-1}$ such that $\widehat{\mathbf{S}} \rightarrow_p \mathbf{S}$, where $\mathbf{S}$ is the optimal weighting matrix of interest, the estimator satisfying this efficiency condition is the optimal GMM estimator that is a solution to the following problem:

$$\hat{\boldsymbol{\delta}}(\widehat{\mathbf{S}}^{-1}) \equiv \text{argmin}_{\boldsymbol{\delta}} J(\boldsymbol{\delta}, \widehat{\mathbf{S}}^{-1}) \equiv T\mathbf{g}_T(\boldsymbol{\delta})'\widehat{\mathbf{S}}^{-1}\mathbf{g}_T(\boldsymbol{\delta}) \tag{14}$$

What remains is to consistently estimate this optimal weighting matrix. We consider the Continuously Updating (CU) GMM estimator, which estimates matrix $\mathbf{S}$ simultaneously as a function of $\boldsymbol{\delta}$ and is defined as a solution to the following minimisation problem within the static framework:

$$\hat{\boldsymbol{\delta}}\big(\widehat{\mathbf{S}}^{-1}(\boldsymbol{\delta})\big) \equiv \operatorname{argmin}_{\boldsymbol{\delta}} J\big(\boldsymbol{\delta}, \widehat{\mathbf{S}}^{-1}(\boldsymbol{\delta})\big) \equiv T\mathbf{g}_T(\boldsymbol{\delta})'\widehat{\mathbf{S}}^{-1}(\boldsymbol{\delta})\mathbf{g}_T(\boldsymbol{\delta}) \tag{15}$$

Given conditionally heteroskedastic errors, $\widehat{\mathbf{S}}(\boldsymbol{\delta})$ may be written explicitly as:[3]

$$\widehat{\mathbf{S}}(\boldsymbol{\delta}) = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t\mathbf{x}_t'\hat{\varepsilon}_t^2 = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t\mathbf{x}_t'(i_t - \mathbf{z}_t'\boldsymbol{\delta})^2 \tag{16}$$

Assuming an optimally chosen weighting matrix $\mathbf{S}$, the minimised distance asymptotically follows a chi-squared distribution. Hansen's (1982) test of over-identifying restrictions may be written as:

$$J = J\big(\hat{\boldsymbol{\delta}}(\widehat{\mathbf{S}}^{-1}), \widehat{\mathbf{S}}^{-1}\big) = T\mathbf{g}_T\big(\hat{\boldsymbol{\delta}}(\widehat{\mathbf{S}}^{-1})\big)'\widehat{\mathbf{S}}^{-1}\mathbf{g}_T\big(\hat{\boldsymbol{\delta}}(\widehat{\mathbf{S}}^{-1})\big) \to_d \chi^2_{K-L} \tag{17}$$

Consider now a time-varying random-coefficient model for the federal funds rate:

$$\begin{gathered} i_t = \mathbf{z}_t'\boldsymbol{\delta}_t + \varepsilon_t \qquad (t = 1, 2, ..., T) \\ \mathbf{z}_t = \boldsymbol{\Phi}_t'\mathbf{x}_t + \mathbf{u}_t \end{gathered} \tag{18}$$

where $\mathbf{z}_t$ is a $k \times 1$ vector of regressors, $\boldsymbol{\delta}_t$ is a $k \times 1$ coefficient vector and ε_t is a scalar error term. $\mathbf{x}_t$ is a $l \times 1$ instrument vector, $\boldsymbol{\Phi}_t'$ is a $k \times l$ coefficient matrix and $\mathbf{u}_t$ is a $l \times 1$ error vector.[4]

We assume the standard exogeneity conditions also hold in the time-varying case:

$$E[\mathbf{x}_t\varepsilon_t] = 0 \qquad E[\mathbf{x}_t\mathbf{u}_t'] = 0 \tag{19}$$

This paper uses a kernel-weighted approach inspired by Giraitis et al. (2021) and Ciu et al. (2023) to estimate the time-path of our coefficient vector $\boldsymbol{\delta}_t$. In this paper, we adapt the GMM framework in which the IV estimator is a special case by constructing the kernel-weighted sample moments:

$$\mathbf{g}_t(\boldsymbol{\delta}_t) = K_t^{-1}\sum_{i=1}^{T}k_{it}\mathbf{x}_t(i_t - \mathbf{z}_t'\boldsymbol{\delta}_t) \quad \text{where} \quad K_t = \sum_{i=1}^{T}k_{it} \tag{20}$$

Sample moments of $\boldsymbol{\sigma_{xy}}$ and $\boldsymbol{\Sigma_{xz}}$ are weighted analogously:

$$\mathbf{s}_{t,\mathbf{xi}} = K_t^{-1}\sum_{i=1}^{T}k_{it}\mathbf{x}_t i_t \quad \text{and} \quad \mathbf{S}_{t,\mathbf{xz}} = K_t^{-1}\sum_{i=1}^{T}k_{it}\mathbf{x}_t\mathbf{z}_t' \tag{21}$$

We define the kernel $k_{it} = K\big(\frac{|i-t|}{H}\big)$, wherein $H = T^h$ is the bandwidth. $K(x)$ is a non-negative continuous bounded kernel function with a bounded first order derivative such that $\int K(x)\partial x = 1$.

[3] If the population moment conditions are ergodic-stationary but serially correlated, the time-varying conditionally heteroskedastic and autocorrelated consistent estimate may be written generally as: $\widehat{\mathbf{S}}_{\text{HAC}} = T^{-1}\Sigma_{i=1}^{t-1}w_{it}(\tilde{\boldsymbol{\Gamma}}_i(\boldsymbol{\delta}) + \tilde{\boldsymbol{\Gamma}}_i'(\boldsymbol{\delta}))$ where w_{ti} are kernel weights given a bandwidth parameter and $\tilde{\boldsymbol{\Gamma}}_i(\boldsymbol{\delta}) = T^{-1}\Sigma_{s=i+1}^{t}\mathbf{g}_t(\tilde{\boldsymbol{\delta}})\mathbf{g}_{s-i}(\boldsymbol{\delta})'$.

[4] Refer to Giraitis et al. (2021) and Ciu et al. (2023) for more details concerning the conditions for $\boldsymbol{\delta}_t$ and $\boldsymbol{\Phi}_t$.

Examples of such kernels with finite support include, but are not limited to:

$$
\begin{aligned}
K(x) &= (1/2)I(|x| \leqslant 1) && \text{Uniform kernel} \\
K(x) &= (3/4)(1-x^2)I(|x| \leqslant 1) && \text{Epanechnikov kernel}
\end{aligned}
$$

If $K(x)$ has infinite support, we assume in addition that $K(x) \leqslant Ce(-cx^2), |\dot{K}(x)| \leqslant C(1+x^2)^{-1}, x \geqslant 0$, for some $C > 0$ and $c > 0$. Examples of such kernels with infinite support include:

$$
K(x) = (1/\sqrt{2\pi})e^{-x^2/2} \qquad \text{Gaussian kernel}
$$

The Time-Varying GMM (TV-GMM) estimator is therefore the minimiser of the time-varying objective function $J_t(\boldsymbol{\delta}_t, \mathbf{W}_t) \equiv \mathbf{g}_t(\boldsymbol{\delta}_t)'\mathbf{W}_t\mathbf{g}_t(\boldsymbol{\delta}_t)$ given a kernel function and bandwidth parameter:

$$
\hat{\boldsymbol{\delta}}_t(\mathbf{W}_t) = (\mathbf{S}'_{t,\mathbf{xz}}\mathbf{W}_t\mathbf{S}_{t,\mathbf{xz}})^{-1}\mathbf{S}'_{t,\mathbf{xz}}\mathbf{W}_t\mathbf{s}_{t,\mathbf{xi}} \tag{22}
$$

For $\mathbf{W}_t = \widehat{\mathbf{S}}_t^{-1}$, such that $\widehat{\mathbf{S}}_t \rightarrow_p \mathbf{S}_t$, the optimal TV-GMM estimator is given by:

$$
\hat{\boldsymbol{\delta}}_t\big(\widehat{\mathbf{S}}_t^{-1}\big) \equiv \text{argmin}_{\boldsymbol{\delta}_t} J_t\big(\boldsymbol{\delta}_t, \widehat{\mathbf{S}}_t^{-1}\big) \equiv \mathbf{g}_t(\boldsymbol{\delta}_t)'\widehat{\mathbf{S}}_t^{-1}\mathbf{g}_t(\boldsymbol{\delta}_t) \tag{23}
$$

whose sample analogue may be written explicitly as follows:

$$
\begin{aligned}
\hat{\boldsymbol{\delta}}_t = &\Big(\big(\sum_{i=1}^{T} k_{it}\mathbf{z}_t\mathbf{x}_t'\big)\big(\sum_{i=1}^{T} k_{it}\mathbf{x}_t\mathbf{x}_t'\hat{\varepsilon}_t^2\big)^{-1}\big(\sum_{i=1}^{T} k_{it}\mathbf{x}_t\mathbf{z}_t'\big)\Big)^{-1} \\
&\times\big(\sum_{i=1}^{T} k_{it}\mathbf{z}_i\mathbf{x}_t'\big)\big(\sum_{i=1}^{T} k_{it}\mathbf{x}_t\mathbf{x}_t'\hat{\varepsilon}_t^2\big)^{-1}\big(\sum_{i=1}^{T} k_{it}\mathbf{x}_t i_t\big)
\end{aligned} \tag{24}
$$

The TV CU-GMM estimator may be defined as the solution to the problem:

$$
\hat{\boldsymbol{\delta}}_t\big(\widehat{\mathbf{S}}_t^{-1}(\boldsymbol{\delta}_t)\big) \equiv \text{argmin}_{\boldsymbol{\delta}_t} J_t\big(\boldsymbol{\delta}_t, \widehat{\mathbf{S}}_t^{-1}(\boldsymbol{\delta}_t)\big) \equiv \mathbf{g}_t(\boldsymbol{\delta}_t)'\widehat{\mathbf{S}}_t^{-1}(\boldsymbol{\delta}_t)\mathbf{g}_t(\boldsymbol{\delta}_t) \tag{25}
$$

The time-varying optimal weighting matrix given conditional heteroskedasticity is:[5]

$$
\widehat{\mathbf{S}}_t(\boldsymbol{\delta}_t) = K_t^{-1}\sum_{i=1}^{T} k_{it}\mathbf{x}_t\mathbf{x}_t'\hat{\varepsilon}_t^2 = K_t^{-1}\sum_{i=1}^{T} k_{it}\mathbf{x}_t\mathbf{x}_t'(i_t - \mathbf{z}_t'\boldsymbol{\delta}_t)^2 \tag{26}
$$

Finally, we outline the general time-varying specification of Hansen's J-test:

$$
\begin{gathered}
J_T = \Big(\widehat{\boldsymbol{\Omega}}_2^{-\frac{1}{2}}\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\boldsymbol{V}_{T,t}\Big)'\Big(\widehat{\boldsymbol{\Omega}}_2^{-\frac{1}{2}}\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\boldsymbol{V}_{T,t}\Big) \rightarrow_d \chi^2_{K-L} \\
\text{where} \quad \boldsymbol{\Omega}_2 = \text{plim}_{T\to\infty}\text{Var}\big(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\boldsymbol{V}_{T,t}\big) \quad \text{and} \quad \boldsymbol{V}_{T,t} = \tilde{\mathbf{S}}_t^{-\frac{1}{2}}K_{2,t}^{-\frac{1}{2}}\sum_{i=1}^{T} k_{it}\mathbf{g}_t(\hat{\boldsymbol{\delta}}_t)
\end{gathered} \tag{27}
$$

[5] Similarly, if moments are serially correlated, the time-varying heteroskedastic and autocorrelated consistent estimate $\widehat{\mathbf{S}}_t(\boldsymbol{\delta}_t)$ is: $\widehat{\mathbf{S}}_t(\boldsymbol{\delta}_t) = \boldsymbol{\Sigma}_{s=-T}^{T} w_{it}\hat{\Gamma}(s)$, where $\hat{\boldsymbol{\Gamma}}(s) = K_{2,t}^{-1}\Sigma_{i=1}^{T-s}k_{it}k_{i+s,t}\big(\mathbf{g}_s(\boldsymbol{\delta}_t) - T^{-1}\Sigma_{s=1}^{T}\mathbf{g}_s(\boldsymbol{\delta}_t)\big)\big(\mathbf{g}'_{i+s}(\boldsymbol{\delta}_t) - T^{-1}\Sigma_{s=1}^{T}\mathbf{g}_s(\boldsymbol{\delta}_t)\big)$, $K_{2,t} = \Sigma_{i=1}^{T}k_{it}^2$ and $\hat{\boldsymbol{\Gamma}}(s) = \hat{\boldsymbol{\Gamma}}(-s)$ for $s < 0$ (Giraitis et al., 2014; 2018; 2021; Ciu et al., 2023).

This result follows straightforwardly from Giraitis et al. (2021) where similar tests are discussed.

2.2 Implied Natural Rates of Interest

Consider now stochastic time-variation in random coefficients of the forward-looking Taylor rule:

$$i_t = (1-\rho_t)(\delta_{0,t} + \delta_{\pi,t}\pi_t + \delta_{\tilde{y},t}\tilde{y}_t) + \rho_t(L)i_t + u_t \tag{28}$$

The time-varying coefficient vector $\boldsymbol{\delta}_t = [\delta_{0,t}, \delta_{\pi,t}, \delta_{\tilde{y},t}, \rho_t]$ is estimated using TV CU-GMM.[6] Given that $\delta_{0,t} = i_t^{\#} - \delta_{\pi,t}\pi^*$ and $i_t^{\#} = r_t^{\#} + \pi^*$, the implied time-varying long run equilibrium interest rate may be expressed as a function of the following time-varying parameters $\delta_{0,t}$ and $\delta_{\pi,t}$:

$$r_t^{\#} = \delta_{0,t} - \pi^*(1-\delta_{\pi,t}) \tag{29}$$

We note here that as r-hash is a function of the estimated coefficients $\delta_{0,t}$ and $\delta_{\pi,t}$, the uncertainty must be propagated. Given the implied rate is linear in these coefficients, we express its variance-covariance as a function of the variance-covariance matrix of ($\boldsymbol{\delta}_t$) and its coefficient vector ($\boldsymbol{\Lambda}_t$):

$$\begin{gathered} r_t^{\#} = \sum_i^n \Lambda_{i,t}\delta_{i,t} = \boldsymbol{\Lambda}_t\boldsymbol{\delta}_t, \quad \text{where} \quad \Sigma_t^{r^{\#}} = \sum_i^n\sum_j^n \Lambda_{i,t}\Sigma_{ij,t}^{\delta}\Lambda_{j,t} = \boldsymbol{\Lambda}_t\boldsymbol{\Sigma}_t^{\boldsymbol{\delta}}\boldsymbol{\Lambda}_t' \\ \text{and} \quad \boldsymbol{\Sigma}_t^{\boldsymbol{\delta}} = E[(\boldsymbol{\delta}_t - \boldsymbol{\mu}_t) \otimes (\boldsymbol{\delta}_t - \boldsymbol{\mu}_t)] \end{gathered} \tag{30}$$

We approach the implied natural rate of interest in a similar manner to Woodford (2001), in which it is shown that formulations of the Taylor rule that do not incorporate real disturbances and that only account for contemporaneous feedback from the policy objectives are suboptimal due to fixed intercepts that fail to stabilise inflation and the output gap period-by-period. Given standard determinacy conditions for the reaction coefficients hold, our time-varying framework ensures that both objectives are stabilised at all times periods by adjustments in the time-varying intercept term ($\delta_{0,t}$) that tracks variation in the natural rate of interest.[7]

As discussed in Woodford (2001), this is in keeping with Taylor's original prescription, which describes the intercept as capturing "the central bank's estimate of the equilibrium real rate of interest" (Taylor, 1999, p.325). We argue that these estimates are subject to error, which is a function of the difference between the perception of policymakers about r-star and the actual natural rate of interest. This framing of the equilibrium real interest rate within a forward-looking Taylor rule; as the perceived rather than actual natural rate of interest, is also consistent with Judd and Rudebusch (1998), in which the natural rate of interest within a forward-looking Taylor rule with smoothing is clearly interpreted as the 'belief' of policymakers about the equilibrium real interest rate (see in particular Judd and Rudebusch, 1998, pp.10-11).

[6] Our estimation relies on the assumption that both short term rates and inflation are stationary. Augmented Dickey-Fuller tests confirm this hypothesis, albeit with less evidence against the null for the former in smaller sample sizes.

[7] Policy rules such as those presented in this paper are consistent with optimal equilibrium if $\delta_y > 0$ and $\delta_\pi > 1$. If the reaction coefficient associated with deviations in inflation from its target falls below unity, the target real rate adjusts to accommodate changes in inflation rather than stabilise it with output.

We therefore conclude, that given ex-post policy rates, $r_t^{\#}$ is a good proxy for the perceptions of policymakers concerning the state of the macroeconomy. To evaluate these (mis)perceptions, we use semi-structural estimates of the actual natural rate of interest (r_t^*) as a comparator; derived from structural equations that capture both demand and supply side pressures, rather than from reaction coefficients associated with priorities and preferences of the central bank. In this regard, we consider r-star an objective benchmark to evaluate the subjective beliefs of policymakers implicit in their determination of policy rates. In particular, we define misperception as any absolute deviation in the implied natural rate from the actual natural rate of interest.[8]

What remains in the computation of r-hash is to assume a value for the inflation target. As the implied natural rate of interest and inflation target cannot be identified simultaneously, one may only be derived under explicit assumptions about the other. Given our focus is on the perceptions of r-star, we assume a value for the latter. Whilst studies have estimated an array of inflation targets pursued by the Federal Reserve, we refer to the prescriptions of Taylor (1993) and fix this to 2%. In doing so, we assume that the Federal Reserve obeys this target in its historical attempt to set policy rates to achieve inflation and output stability. Whilst this is a relatively strong assumption, we note that this has been the implicit and explicit target of the Federal Reserve for the last two decades and continues to be the desired rate of inflation to date.

It is, however, unclear whether the inflation target prior to this period is itself inconstant and therefore time-varying. Notwithstanding this, it is unlikely that policymakers targeted excessively high rates of inflation in the historical conduct of monetary policy nor rates of deflation for that matter. We leave the empirical estimation of such potential variation to future research and assume that historical target rates do not significantly deviate from a 2% target.

2.3 Actual Natural Rates of Interest

To estimate r-star we use the benchmark semi-structural Holston, Laubach and Williams (2017), hereafter HLW, model. In particular, w e relax the open economy framework of Galí and Monacelli (2005) by specifying equations of the dynamic Investment Savings (IS) curve and New Keynesian Phillips Curve (NKPC) that permit shocks to the output gap $\tilde{y}_t$ and inflation π_t respectively:

$$\tilde{y}_t = \boldsymbol{\phi}_y(L)\tilde{y}_t + \boldsymbol{\phi}_r(L)\tilde{r}_t + \varepsilon_{\tilde{y},t} \tag{31}$$

$$\pi_t = \boldsymbol{\varphi}_\pi(L)\pi_t + \boldsymbol{\varphi}_y(L)\tilde{y}_t + \varepsilon_{\pi,t} \tag{32}$$

where $\varepsilon_{\tilde{y},t}$ and $\varepsilon_{\pi,t}$ denote transitory shocks to output and inflation, $\tilde{r}_t$ is the real interest rate gap given by the deviation in real rates from the natural rate of interest $r_t - r_t^*$, wherein r_t^* captures persistent shocks to the relationship between r_t and $\tilde{y}_t$. The law of motion for the natural rate is:

$$r_t^* = g_t + z_t \tag{33}$$

[8] If $r^{\#} > r^*$, policymakers overestimate the natural rate and policy becomes overly contractionary. In contrast, if $r^{\#} < r^*$, policymakers underestimate the natural rate and policy becomes overly expansionary. If however $r^{\#} = r^*$, policymakers perfectly perceive the natural rate and policy is consistent with the optimal equilibrium.

where g_t is the trend growth rate of the natural rate of output and z_t is the error term capturing other factors driving r-star. The transition equations of the state-space system may be written as:

$$y_t^* = (L)y_t^* + (L)g_t + \varepsilon_{y*,t} \tag{34}$$

$$g_t = (L)g_t + \varepsilon_{g,t} \tag{35}$$

$$z_t = (L)z_t + \varepsilon_{z,t} \tag{36}$$

where (34) defines log potential output y_t^* as a random walk with stochastic drift g that also follows a random walk process (35). Finally, (36) captures the unobserved component of the natural rate of interest, which itself is a random walk. We assume shocks are contemporaneously uncorrelated and normally distributed, whose variance-covariance matrix $\mathbf{\Sigma}_\varepsilon$ is given by:

$$\begin{bmatrix} \sigma_{\tilde{y}}^2 & 0 & \cdots & \cdots & 0 \\ 0 & \sigma_\pi^2 & \ddots & & \vdots \\ \vdots & \ddots & \sigma_{y*}^2 & \ddots & \vdots \\ \vdots & & \ddots & \sigma_g^2 & 0 \\ 0 & \cdots & \cdots & 0 & \sigma_z^2 \end{bmatrix} \tag{37}$$

Model linearity in the unobserved state vector allows us to estimate the natural rate of interest, the natural rate of output and its trend growth using the Kalman filter. However, as real growth rates and interest rates are often influenced by highly persistent shocks, MLE is likely to return estimates of the standard deviations of innovations σ_g and σ_z that are biased towards zero. To resolve this 'pile-up' problem (Stock, 1994), we follow HLW by using the median unbiased estimator to derive estimates of ratios $\lambda_g = \sigma_g/\sigma_{y*}$ and $\lambda_z = \phi_r\sigma_z/\sigma_{\tilde{y}}$ that are imposed on the remaining parameters (Stock and Watson, 1998). What follows is a brief outline of the specification and implementation.

3 Estimation

3.1 Specification and Implementation

Given our interest in using estimates of the actual natural rate (r_t^*) as a comparator, our calibrations to the HLW model are minimal. The observation equations (31)-(32) are thus specified as follows:

$$\tilde{y}_t = \sum_{i=1}^{2} \phi_{y,i}\tilde{y}_{t-i} + \frac{\phi_r}{2}\sum_{i=1}^{2} \tilde{r}_{t-i} + \varepsilon_{\tilde{y},t} \tag{38}$$

$$\pi_t = \varphi_\pi \pi_{t-1} + \frac{1-\varphi_\pi}{3}\sum_{i=2}^{4} \pi_{t-i} + \varphi_y \tilde{y}_{t-1} + \varepsilon_{\pi,t} \tag{39}$$

where we impose a similar lag structure in both the dynamic IS and Phillips curve, such that two lags of the output gap and the average of two lags of the real rate gap enter the former and one lag of the output gap in addition to the first and average of the second to fourth lag of inflation enter the latter. The measurement equations (34)-(36) are first order random walks specified as follows:

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_{y^*,t} \tag{40}$$

$$g_t = g_{t-1} + \varepsilon_{g,t} \tag{41}$$

$$z_t = z_{t-1} + \varepsilon_{z,t} \tag{42}$$

The final set of measurement and transition equations to be estimated are summarised as follows (see Appendix A for the complete state-space representation and maximum likelihood procedure):

$$\begin{aligned} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} &= \begin{bmatrix} 1-\phi_{y,1} & -\phi_{y,2} & 1-\phi_r & -\phi_r \\ -\varphi_y & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1}^* \\ y_{t-2}^* \\ g_{t-1} \\ z_{t-1} \end{bmatrix} + \\ &+ \begin{bmatrix} \phi_{y,1} & \phi_{y,2} & 0 & 0 & \phi_r \\ \varphi_y & 0 & \varphi_\pi & 1-\varphi_\pi & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ \pi_{t-1} \\ \pi_{t-2|4} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\tilde{y},t} + \varepsilon_{y^*,t} \\ \varepsilon_{\pi,t} \end{bmatrix} \end{aligned} \tag{43}$$

where the system (43) captures the measurement equations, which also includes the law of motion for the natural rate of interest and the system (44) captures the transition equations of the model:

$$\begin{bmatrix} y_t^* \\ y_{t-1}^* \\ g_t \\ z_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{t-1}^* \\ y_{t-2}^* \\ g_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{y^*,t} \\ 0 \\ \varepsilon_{g,t} \\ \varepsilon_{z,t} \end{bmatrix} \tag{44}$$

We cast these equations into a state-space system and estimate the parameters by maximising the likelihood function computed by the Kalman filter. We initialise the state vector by calculating the conditional expectation and covariance matrix using the Hodrick-Prescott filter. Given estimates of the standard deviations of innovations are likely biased, we impose median unbiased estimates of λ_g and λ_z in the chronology of our estimation á la Hoslton, Laubach and Williams (2017).[9]

In particular, we first estimate the natural rate of output barring the real rate gap and assuming constant trend growth. We subsequently derive the median unbiased estimate for the ratio between the standard deviations of innovations for the natural rate of output and its trend growth rate, which

[9] Following directly from the HLW (2017) model, our specification of the vector of unobserved states also contains three lags of potential output y^* in the first stage, one lag of the trend growth rate g in the second and a second lag of the trend growth rate in the third in addition to two lags of the unobservable component z.

we impose as a restriction in the second stage of our estimation that includes the real rate gap. We derive a ratio for the component unrelated to trend growth in a similar manner, which we impose in the third stage to estimate the remaining model parameters by maximum likelihood.[10]

3.2 Data and Instrument Selection

To estimate the implied natural rate ($r_t^{\#}$) we rely on the instrument selection in CGG by including up to four lags of each variable in our specification of the matrix $\mathbf{x}_t$. The nominal interest rate is the federal funds rate. Inflation is the percentage change in core personal consumption expenditure (PCE). The output gap is derived using Congressional Budget Office (CBO) estimates of potential GDP. Money growth is the percentage change in the M2 stock published by the Board of Governors of the Federal Reserve. Commodity inflation is computed as the percentage change in a composite index of goods (including energy prices) published by the Bureau of Labor Statistics (BLS). The interest rate spread between long-term and short-term bonds is the difference between the 10-year and 3-year treasury bill rate also published by the Board of Governors.

To estimate the actual natural rate of interest (r_t^*), we source data for real GDP and core PCE from the Bureau of Economic Analysis (BEA). As before, inflation is computed as the percentage change in core PCE. Inflation expectations used to derive ex-ante real interest rates are calculated as the four-quarter average of lagged inflation. As before, the quarterly short-term interest rate is the average of the monthly federal funds rate published by the Board of Governors of the Federal Reserve. All data is sourced from the FED Economic Database (FRED).

Our quarterly data ranges from 1961:1 to 2019:4. Given our focus is on the historical conduct of monetary policy within the United States, we identify five periods of interest to our analysis: (1) the pre-Volcker period from 1961:1-1979:2, (2) the Volcker-Greenspan era from 1979:3-2005:4, (3) the Greenspan-Bernanke era from 1987:3-2013:4, (4) the Bernanke-Yellen era from 2006:1-2017:4 and (5) the Yellen-Powell era from 2014:1-2019:4. We refer to 1979:3-2019:4 as the post-Volcker period.[11] Whilst data exists after this period, we restrict our analysis prior to the COVID-19 pandemic despite methods to account for it developed in Holston, Laubach and Williams (2020; 2023). The authors rightly argue that such extreme-tailed events violate the Kalman filter, in which innovations are assumed Gaussian. In addition, transitory shocks in the Phillips curve are assumed serially uncorrelated; largely inconsistent with responses to the pandemic. Thus, estimates of r-star over this period may likely be distorted, and as such does not warrant inclusion in our sample.

3.3 Kernel and Bandwidth Selection

Our preferred baseline kernel is Gaussian, owing to its desirable properties as discussed in Giraitis et al. (2014). Unbounded support is perhaps an ideal benchmark, particularly given limited prior knowledge of the distribution of the underlying data process. We test the sensitivity of our results to the choice of kernel by estimating the policy rule using two alternatives with finite support; the

[10] Confidence intervals for these estimates and their corresponding standard errors are calculated using a constraint Monte Carlo procedure, which accounts for filter and parameter uncertainty (see Hamilton (1986) for further details).

[11] This particular categorisation follows from the existing literature (refer to Clarida et al. (2000) and Carvalho et al. (2021) for a similar breakdown). We additionally investigate the terms of all chairs of the FED across our sample.

Epanechnikov (parabolic) and uniform (rectangular) kernel. In the case of such kernels, only local observations in the sample contribute to the estimation of $\boldsymbol{\delta}_t$. In contrast, all sample observations are weighted in the case of infinite support (such as the Gaussian kernel).

We select a bandwidth $h = 0.6$ for our baseline case. As this parameter controls the smoothness and roughness of our results, caution must be invoked given higher bandwidths lead to oversmoothing and lower bandwidths lead to undersmoothing. Whilst there is no universal rule for optimal bandwidth selection, we choose a bandwidth that allows for a sufficient number of observations given our sample size. We test the sensitivity of our results using bandwidths marginally below and above our baseline; $h = 0.5$ and $h = 0.7$. As we later discuss, the choice of bandwidth and kernel have non-trivial implications for our estimation of the time-varying parameters.

4 Results

To motivate our time-varying estimation approach, we first regress the Taylor rule as in equation (5) using GMM. We examine the stability of the time-invariant rule across subsamples outlined in Section 3.2. Table 1 reports these results in addition to the implied natural rate of interest ($r^{\#}$).

Table 1. CU-GMM Estimates

	Pre-Volcker 1961:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2017:4	Yellen-Powell 2014:1-2019:4
δ_π	0.96*** (0.35)	2.29*** (0.71)	1.94*** (0.54)	1.70** (0.80)	1.23 (0.99)
$\delta_{\tilde{y}}$	0.86*** (0.24)	1.01*** (0.36)	1.10*** (0.28)	0.55** (0.25)	0.33 (0.27)
ρ	0.73*** (0.07)	0.80*** (0.05)	0.83*** (0.03)	0.84*** (0.05)	0.93*** (0.09)
$r^{\#}$	1.38*** (0.25)	2.45*** (0.20)	2.22*** (0.11)	0.68*** (0.23)	0.32 (0.27)
J	0.72	0.33	0.45	0.67	0.52

Note: Estimates of equation (28) by GMM.[12] *Instruments include four lags of inflation, the output gap, the federal funds rate, money growth, commodity price inflation and the long-short spread. Significance at the 90/95/99% confidence level are denoted by */**/*** respectively. Robust standard errors are reported in parentheses. P-values are reported for the associated Hansen J-test statistic at the 5% significance level.*

[12] Our estimates in the first and second column are close in proximity to Clarida, Galí and Gertler (2000). However, they are nonidentical for three reasons; (1) we use core PCE to derive our time series for inflation, whereas the authors use the GDP deflator and CPI, (2) our data for the Volcker-Greenspan era extends right until the beginning of Bernanke's term rather than ceasing it at 1996:1, which was the latest data point available to the authors at the time, and (3) we use the CU-GMM estimator, which estimates the optimal weighting matrix simultaneously rather than iteratively.

For the Taylor principle to hold, that is for monetary policy to be stabilising, we expect $\delta_\pi > 1$ and $\delta_y > 0$. Estimates of δ_π are significant and below unity during the pre-Volcker period, albeit much greater than unity during the Volcker-Greenspan era. We find that the response to the inflation gap declines thereafter across the remaining regimes. Estimates of $\delta_{\tilde{y}}$ are statistically significant and above zero across all periods. We find that responses to the output gap have also declined in recent years between the Bernanke-Yellen and Yellen-Powell era.

Estimates of the smoothing parameter ρ are significantly high and increasing marginally across all periods of our sample, indicating considerable inertia in monetary policy. Finally, estimates of r-hash indicate a rise and secular fall in the perceptions of the natural rate. Assuming this is a good proxy for the ex-post beliefs of the central bank, this suggests long-run pessimism about r-star. In addition, our estimates show that the Great Recession had a sizeable influence on r-hash, forcing perceptions closer to the lower bound. In particular, the implied natural rate declines by more than 1.5 percentage points between the Greenspan-Bernanke and Bernanke-Yellen era.

It is clear from these results that there is substantial time variation in the coefficients associated with both deviations in inflation from its target and the output from its potential, in addition to the coefficient of inertia; naturally motivating our estimation of a time-varying policy rule.

4.1 Estimating a Time-Varying Taylor Rule

We begin by estimating the baseline case. Given a Gaussian kernel function and bandwidth parameter of $h = 0.6$, we specify the kernel-weighted average of the sample moment function (20):

$$\mathbf{g}_t(\boldsymbol{\delta}_t) = K_t^{-1} \sum_{i=1}^{T} (1/\sqrt{2\pi}) e^{-\frac{1}{2}\left(\frac{i-t}{H}\right)^2} \Big(\mathbf{x}_i \big(i_i - (1-\rho_t)(\delta_{0,t} + \delta_{\pi,t}\pi_i + \delta_{\tilde{y},t}\tilde{y}_i) - \rho_t(L) i_i \big) \Big) \tag{45}$$

Results are presented in Figure 1.[13] Time-varying estimates of the Taylor coefficients reveal substantial structural change. Despite apparent differences in magnitude, our results corroborate some general trends identified by the GMM estimation of (5). To demonstrate this, we take subsample averages of these time-varying estimates and present them in Table 2. These results confirm a rise in δ_π over the post-Volcker period, albeit to a lesser extent than in the time-invariant case. Similar to our previous results, our estimates reveal a sharp decline in the average response to deviations in inflation from its target between the Greenspan-Bernanke and Bernanke-Yellen era.

Averages of $\delta_{\tilde{y}}$ are generally lower than in the time-invariant case, yet our estimates corroborate a sustained increase in the average sensitivity to the output gap. Estimates of the smoothing parameter ρ are significantly large, albeit stable on average across all our subsamples, confirming a high degree of inertia in the conduct of monetary policy. Finally, average estimates of r-dagger also indicate a rise and fall in the perceptions of the actual natural rate, albeit to higher extent on average than in the invariant case. We find that the Taylor principle is initially violated during the pre-Volcker period but holds on average thereafter. As shown in Figure 1, it is undermined during the Bernanke-Yellen-Powell era when the average response to the inflation gap falls below unity.

[13] Time-varying Sargan-Hansen J-test statistics for overidentification are reported in Appendix C.

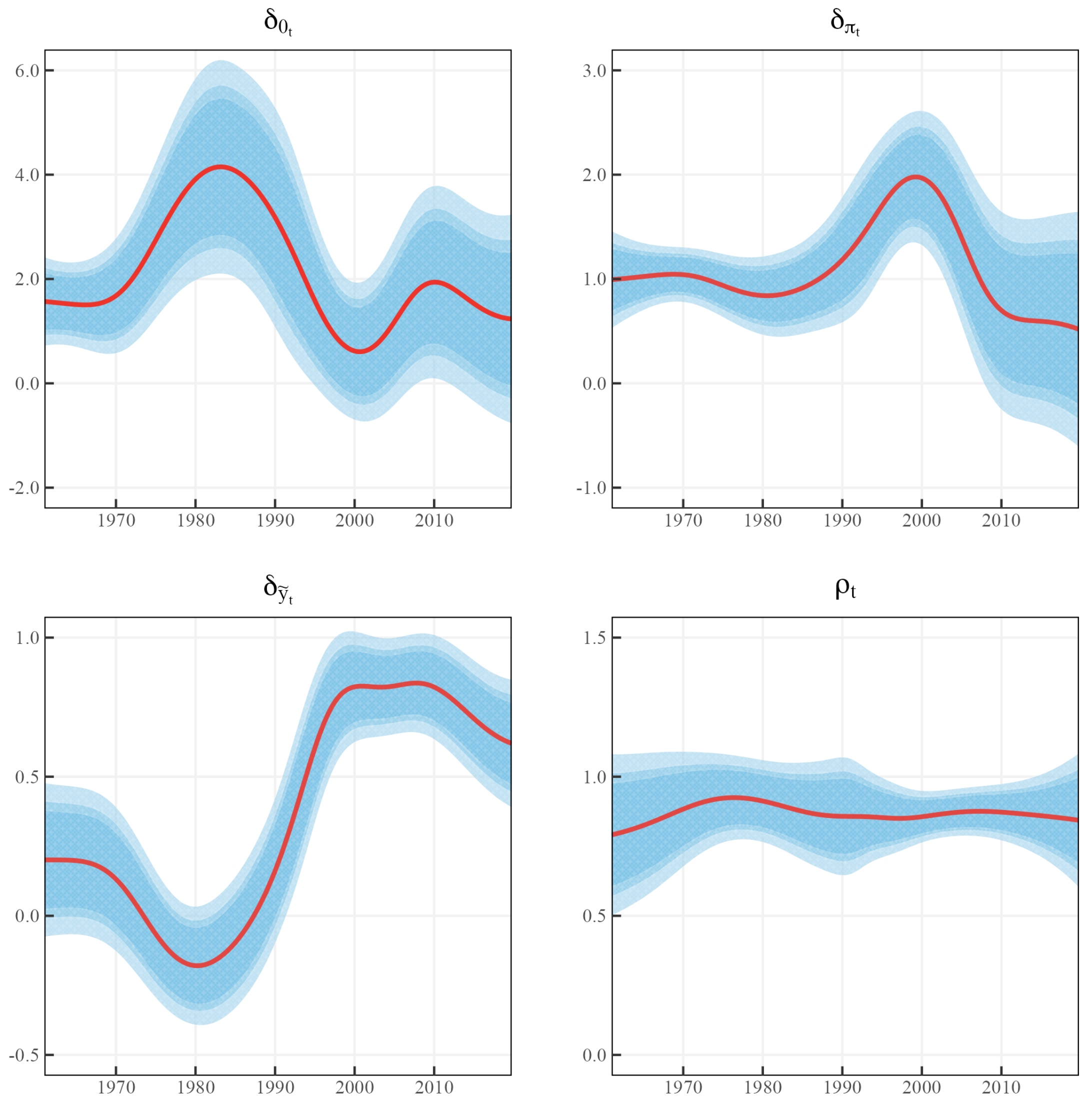

Figure 1. Baseline Time-Varying Estimates

Note: Time-varying policy rule parameter estimates by TV CU-GMM. K(x): Gaussian. h = 0.6. Instrument selection is detailed in Section 3.2. Shaded regions in blue correspond to the 90/95/99% confidence bands.

To check the robustness of our findings, we relax our baseline specification along two dimensions. First, we investigate the sensitivity to bandwidth selection whilst maintaining a Gaussian kernel. As the bandwidth parameter controls the smoothness (roughness) of our estimates, we are cautious to not choose a bandwidth that oversmooths or undersmooths. In this regard, we deviate marginally by choosing a bandwidth just below ($h = 0.5$) and above ($h = 0.7$) our baseline case.

Second, we investigate the sensitivity to kernel selection whilst maintaining the baseline bandwidth parameter ($h = 0.6$). As the Gaussian kernel is perhaps the most widely used within its class of infinitely supported kernel functions, we select two widely used kernel functions with finite support, namely the Epanechnikov (parabolic) and uniform (rectangular) kernel, where the latter is equivalent to a rolling window estimator. In this regard, our analysis tests the robustness to support rather than kernels within the same class that likely yield similar results.

Table 2. Subsample Averages

	Pre-Volcker 1961:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2017:4	Yellen-Powell 2014:1-2019:4
$\bar{\delta}_\pi$	0.98***	1.38***	1.34***	0.72**	0.57
	(0.12)	(0.23)	(0.29)	(0.35)	(0.40)
$\bar{\delta}_{\tilde{y}}$	0.07**	0.37***	0.65***	0.77***	0.67***
	(0.04)	(0.09)	(0.08)	(0.08)	(0.08)
$\bar{\rho}$	0.87***	0.87***	0.86***	0.87***	0.85***
	(0.08)	(0.06)	(0.05)	(0.05)	(0.07)
$\bar{r}^\#$	2.09***	3.15***	2.41***	1.11**	0.53
	(0.30)	(0.32)	(0.22)	(0.54)	(1.10)
$\bar{J}$	0.37	0.33	0.34	0.29	0.27

*Note: Averages of time-varying policy rule parameters estimated by TV CU-GMM given a Gaussian kernel function and bandwidth parameter h = 0.6. Instrument selection is detailed in Section 3.2. Significance at the 90/95/99% confidence level are denoted by */**/*** respectively. Average standard errors are reported in parentheses. Average p-values are reported for the time-varying Hansen J-test statistic.*

Results for alternative bandwidth selection are presented in Figure 2. Our estimates are generally robust to alternative specifications of h.[14] As for $h = 0.5$, we find that our estimates are characterised by greater volatility and uncertainty, yet they corroborate trends in the Taylor coefficients identified by the baseline case. In particular, the estimated coefficient associated with deviations in inflation from its target is stable and increasing until 2000:1, whereafter it exhibits an even sharper decline before recovering in the latter periods of our sample. Estimates of $\delta_{\tilde{y}}$ further corroborate a sustained rise in the sensitivity to the output gap before declining slightly post 2003:1. Finally, estimates of the smoothing parameter ρ are more volatile yet vary around the same averages identified in our baseline case. The Taylor principle generally holds until the Great Recession, after which the estimated inflation reaction coefficient falls below unity.

As for $h = 0.7$, we find that our estimates are characterised by less volatility and uncertainty, yet they also corroborate similar trends in the Taylor coefficients identified by the baseline case. The notable exception is in the coefficient δ_π, which continues to increase at a marginally diminishing rate during the latter of our sample, albeit with a higher degree of uncertainty. In particular, it averages 2.06 over the Bernanke-Yellen era. Estimates of $\delta_{\tilde{y}}$ and ρ exhibit a similar behaviour to the baseline case. The Taylor principle holds firmly across the post-Volcker period. We note that whilst higher bandwidths account for a larger set of observations, time-varying estimates become overly smooth, and thus caution ought to be taken when interpreting these results. The converse is also true for lower bandwidths with the risk of undersmoothing.

[14] Subsample averages for all specifications are found in Appendix D.

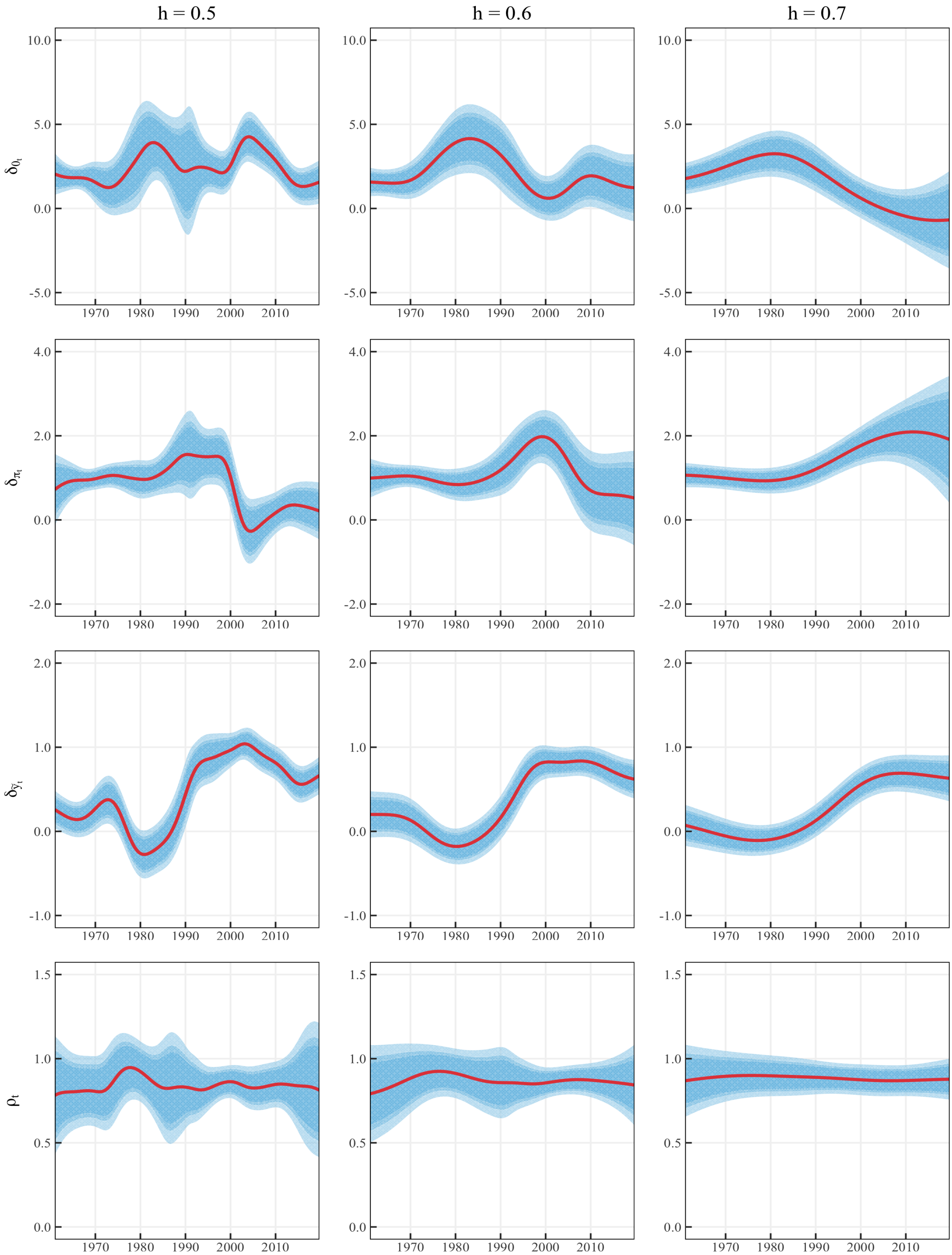

Figure 2. Alternative Bandwidth Parameters

Note: Time-varying parameters of the Taylor rule estimated by TV CU-GMM. K(x): Gaussian. h = 0.5, 0.6, 0.7. Instrument selection is detailed in Section 3.2. Shaded regions in blue correspond to the 90/95/99% confidence bands. Refer to Appendix B for estimates using all combinations of kernel and bandwidth.

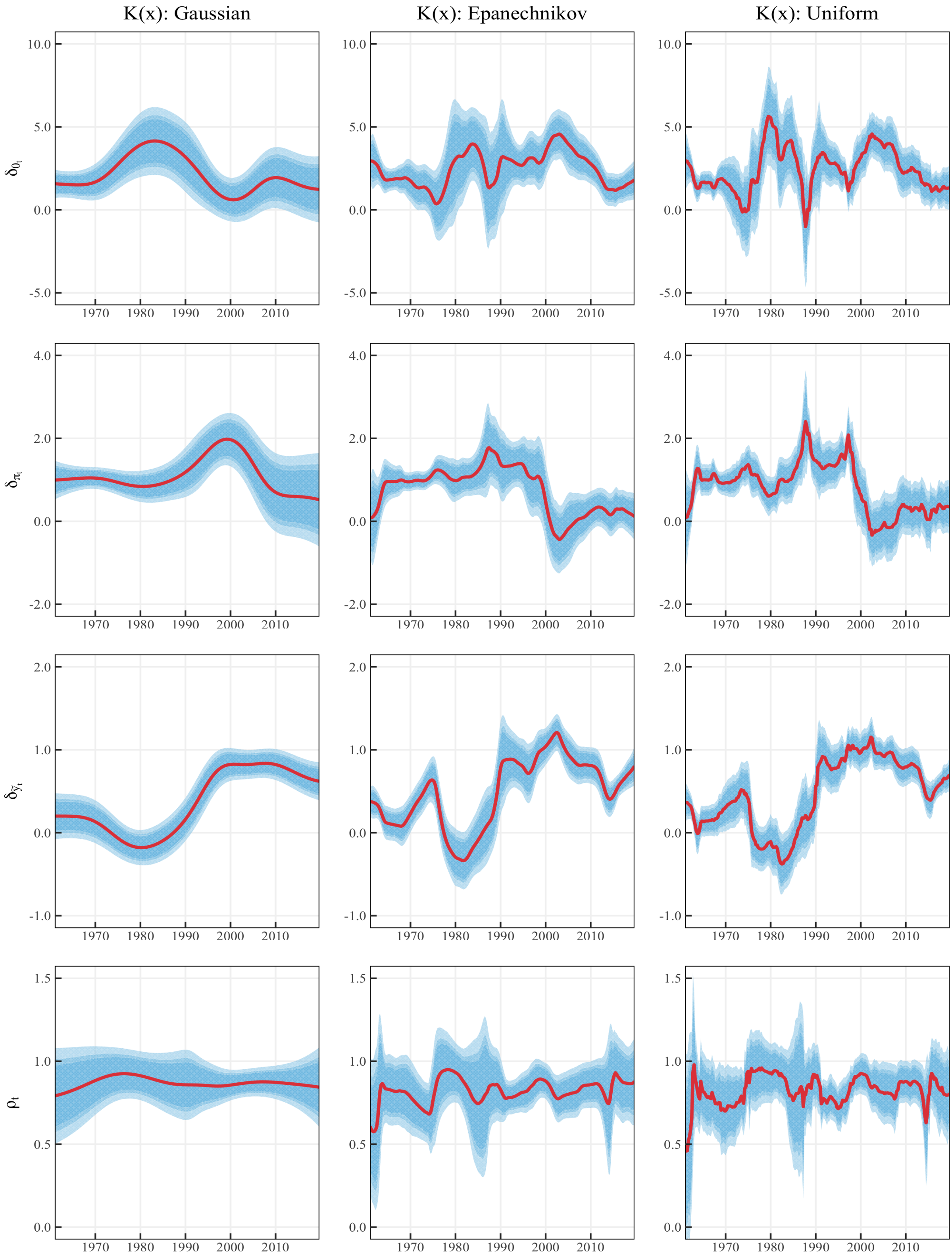

Figure 3. Alternative Kernel Functions

Note: Time-varying parameters of the Taylor rule estimated by TV CU-GMM. K(x): Gaussian, Epanechnikov, Uniform. h = 0.6. Instrument selection is detailed in Section 3.2. Shaded regions in blue correspond to the 90/95/99% confidence bands. Refer to Appendix B for estimates using all kernels and bandwidths.

Results for alternative kernel functions are presented in Figure 3. Both the Epanechnikov and uniform kernels yield unsurprisingly similar results given they each maintain finite support. Estimates derived from the latter are demonstrably more volatile than the former given the nature of rectangular density. Both are more volatile than the Gaussian case with infinite support, yet corroborate estimated trends in the Taylor coefficients identified in the baseline case. In particular, δ_π is also close to unity during the pre-Volcker period before gradually rising until the pre-crisis period. It declines sharply thereafter, violating the Taylor principle and pushing δ_π well into negative territory. Estimates of the sensitivity to the output gap $\delta_{\tilde{y}}$ communicate a comparatively similar, albeit more volatile trend to the baseline Gaussian case and its alternative bandwidths. Estimates of the smoothing parameter exhibit a clear but volatile upward trend, confirming the high and increasing inertia in monetary policy as estimated in the time-invariant case presented in Table 1.

We complete our estimation of the time-varying Taylor rule in (28) with remaining combinations of kernel function and bandwidth selection. These results are all presented in Appendix B and C. The Taylor principle seems to consistently prevail until 2000:1, prior to which, δ_π averages above unity across all specifications. Estimates of δ_π decline sharply thereafter, recovering only to remain below unity by the end of our sample. The Taylor rule thus calls into question the stability of monetary policy during this period. Finally, what is most apparent across all specifications, is the substantial time variation in the parameters of the Taylor rule, justifying techniques such as those used in this paper to capture structural changes in priorities of the Federal Reserve between output and inflation stability. We conclude here that our results are robust to alternative specifications and proceed to use these time-varying estimates of the Taylor coefficients to derive the time-varying implied natural rate of interest (r-hash).

4.2 Policy Parameters by FED Chairs

To further investigate the stability of monetary policy and the implied natural rate of interest given time-varying estimates of the policy parameters, we take subsample averages by chairs of the FED. Our results are presented in Table 3. We find the average response to inflation is weakening across the tenure of chairs in the pre-Volcker era, rising immediately prior to, and well after the Volcker term. In fact, these priorities peak during Greenspan's term at 1.58 percentage points and decline towards the end of our sample, albeit with reduced statistical significance.

The response to the output gap follows a similar trend. In particular, we find that priorities on output stability rise substantially over the post-Volcker period, peaking during the Bernanke term and declining thereafter. Given the Taylor principle holds for responses to the inflation gap above unity and responses to the output gap above zero, our results suggest that monetary policy during the Martin and Greenspan term was stabilising. We also note minimal time-variation in the inertial coefficient across chairs of the Federal Reserve, which remains high and close to unity across our sample, suggesting sluggish policy rate adjustments.

Finally, estimates of the implied natural rate of interest (r-hash) suggest a sharp rise and protracted decline in perceptions of the natural rate. In particular r-hash rises across the pre-Volcker period and peaks during the Volcker term at 3.81 percentage points. It subsequently declines over the post-Volcker period and reaches the zero lower neutral bound by the conclusion of our sample. These empirical results strongly motivate our following analysis concerning the time-varying implicit natural rate that captures the perceptions of monetary policymakers concerning the actual equilibrium interest rate. We therefore proceed to use these parameter estimates to derive r-hash.

Table 3. Subsample Averages by Chair

	Martin 1961:1-1970:1	Burns 1970:1-1978:1	Miller 1978:1-1979:3	Volcker 1979:3-1987:3
$\bar{\delta}_\pi$	1.02***	0.96***	0.85***	0.89**
	(0.13)	(0.11)	(0.14)	(0.16)
$\bar{\delta}_{\tilde{y}}$	0.19**	-0.02	-0.17***	-0.12***
	(0.10)	(0.09)	(0.08)	(0.08)
$\bar{\rho}$	0.83***	0.91***	0.92***	0.89***
	(0.10)	(0.07)	(0.06)	(0.06)
$\bar{r}^{\#}$	1.59***	2.44***	3.39***	3.81***
	(0.43)	(0.60)	(0.76)	(0.85)
$\bar{J}$	0.41	0.32	0.34	0.29

	Greenspan 1987:3-2006:1	Bernanke 2006:1-2014:1	Yellen 2014:1-2018:1	Powell 2018:1-2019:4
$\bar{\delta}_\pi$	1.58***	0.77***	0.58	0.54
	(0.26)	(0.36)	(0.39)	(0.43)
$\bar{\delta}_{\tilde{y}}$	0.58***	0.81***	0.69***	0.63***
	(0.09)	(0.07)	(0.08)	(0.09)
$\bar{\rho}$	0.86***	0.87***	0.86***	0.85***
	(0.05)	(0.04)	(0.06)	(0.09)
$\bar{r}^{\#}$	2.85***	1.34	0.61	0.34
	(0.83)	(1.01)	(1.08)	(1.14)
$\bar{J}$	0.36	0.31	0.24	0.35

*Note: Subsample averages by chairs of the FED. K(x): Gaussian. h = 0.6. Significance at the 90/95/99% confidence level are denoted by */**/*** respectively. Average standard errors are reported in parentheses. Average p-values are reported for the time-varying J-test. See Appendix D for results using all K(x) and h.*

4.3 Estimating the Implied Natural Rate of Interest

Estimates of the implied natural rate of interest are presented in Figure 4 across all combinations of kernel and bandwidth parameter. Empirical results for the baseline specification demonstrate a rise and protracted fall in r-hash. This secular 'pessimism' follows long-run trends in r-star identified within the literature. Given r-hash captures the beliefs of the monetary policymaker at the time of setting policy rates, we conclude that the perceptions of the central bank about the natural rate of interest have declined since the 1980's by approximately 3.5-4 percentage points. This declination has persisted right until the end of our sample, where r-hash converges to the zero lower bound.

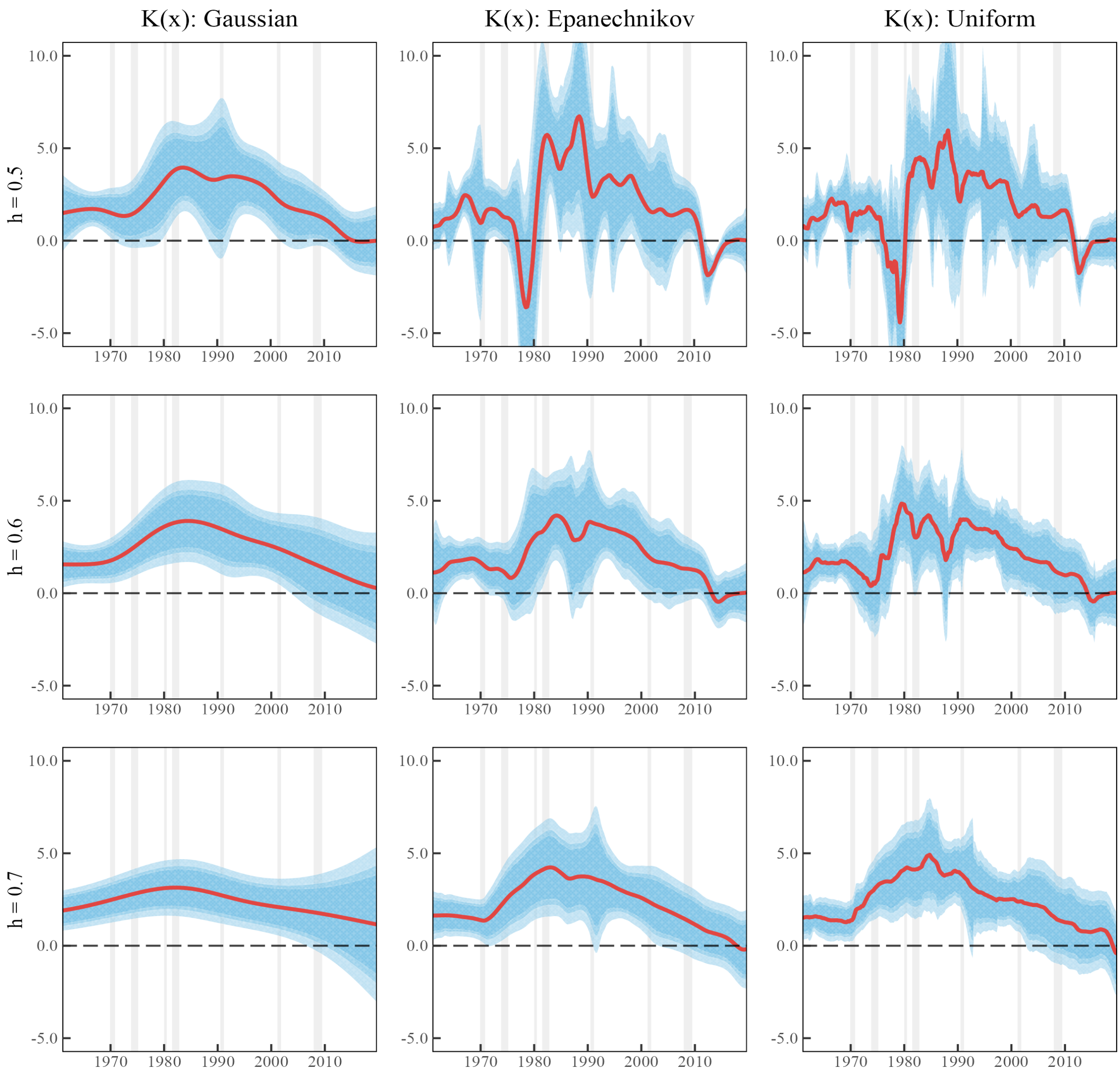

Figure 4. Implied Natural Rate of Interest

Note: Implied natural rates of interest derived from estimated time-varying coefficients of the Taylor rule. Shaded regions in blue correspond to the 90/95/99% confidence bands. Shaded bars in grey indicate recessionary periods in the United States dated by the National Bureau of Economic Research (NBER).

This rise and fall in the time-varying implied natural rate of interest is robust to alternative specifications of both kernel function and bandwidth parameter. As expected, estimates of $r^{\#}$ become smoother and rougher with higher and lower bandwidths respectively. We find that volatility rises substantially when estimating the Taylor rule with finitely supported kernels. In addition, estimates of the implied natural rate given such kernels fall and rise sharply just before and after the 1980's. Finally, it is worth noting that for finite kernel functions and lower bandwidth parameters, there is a sharp reduction in the perception of the actual natural rate immediately after the Great Recession, forcing r-hash in some instances into negative territory, indicating harsh revisions by the Federal Reserve in their understanding of the actual natural rate given the severity of the financial crisis.

4.4 Robustness to the Zero Lower Bound

Given conventional monetary policy rules at the effective lower bound may be ineffective at stabilising the economy, we estimate a Taylor rule given shadow rates during which the zero lower bound (ZLB) is a binding constraint. We do this primarily to characterise the stance of monetary policy during this period and to check the robustness of our estimates of the priorities between inflation and output stability. Given robust time-varying parameter estimates of the forward looking Taylor rule with smoothing, we are also able to derive the implicit series for r-hash that accounts for the ZLB environment. In doing so, we are able to check the sensitivity of perceptions about r-star to periods during which conventional monetary policy is less effective.

Whilst a range of shadow rates are available to us within the literature, we choose the Wu-Xia (2016) rate released and maintained by the Federal Reserve Bank of Atlanta. The authors derive these estimates from a shadow rate term structure model (SRTSM) that is able to capture the stance of unconventional monetary policy.[15] Our implementation is given by the piecewise function:

$$i_t = \left\{ \begin{array}{ll} f_t, & \text{if } f_t \geqslant \underline{i} \\ s_t, & \text{if } s_t < \underline{i} \end{array} \right\} \tag{46}$$

Such that the interest rate is equal to the federal funds rate f_t if it is above the lower bound $\underline{i}$, and equal to the shadow rate s_t only if it falls below it. As discussed in Wu and Xia (2016), we fix this lower bound to $\underline{i} = 0.25\%$, given the FED has paid this rate on reserves since 2008. We therefore impose this in our time-varying GMM estimation and present baseline results in Figure 5.

Our findings suggest relatively minimal deviations in the priorities of policy makers over the period in which the lower bound is binding. In particular, we note marginally higher priorities on inflation and output stability, with negligible differences in inertia. Perhaps most noticeable is the greater uncertainty surrounding estimates of the policy parameters arising after the crisis. Whilst changes to the time-varying intercept may suppress significant changes to the implied natural rate of interest, larger standard errors are likely to exacerbate propagated uncertainty in r-hash.

[15] Refer to Black (1995) for the orginial proposition of this framework. In addition, see for instance Bullard (2012) and Krippner (2013) for alternative proxies of near-zero policy rates that also reflect unconventional monetary policy accommodations. We also note wider considerations around the sensitivity of these shadow rates to model specification and data selection, as discussed in Christensen and Rudebusch (2015) and Bauer and Rudebusch (2016).

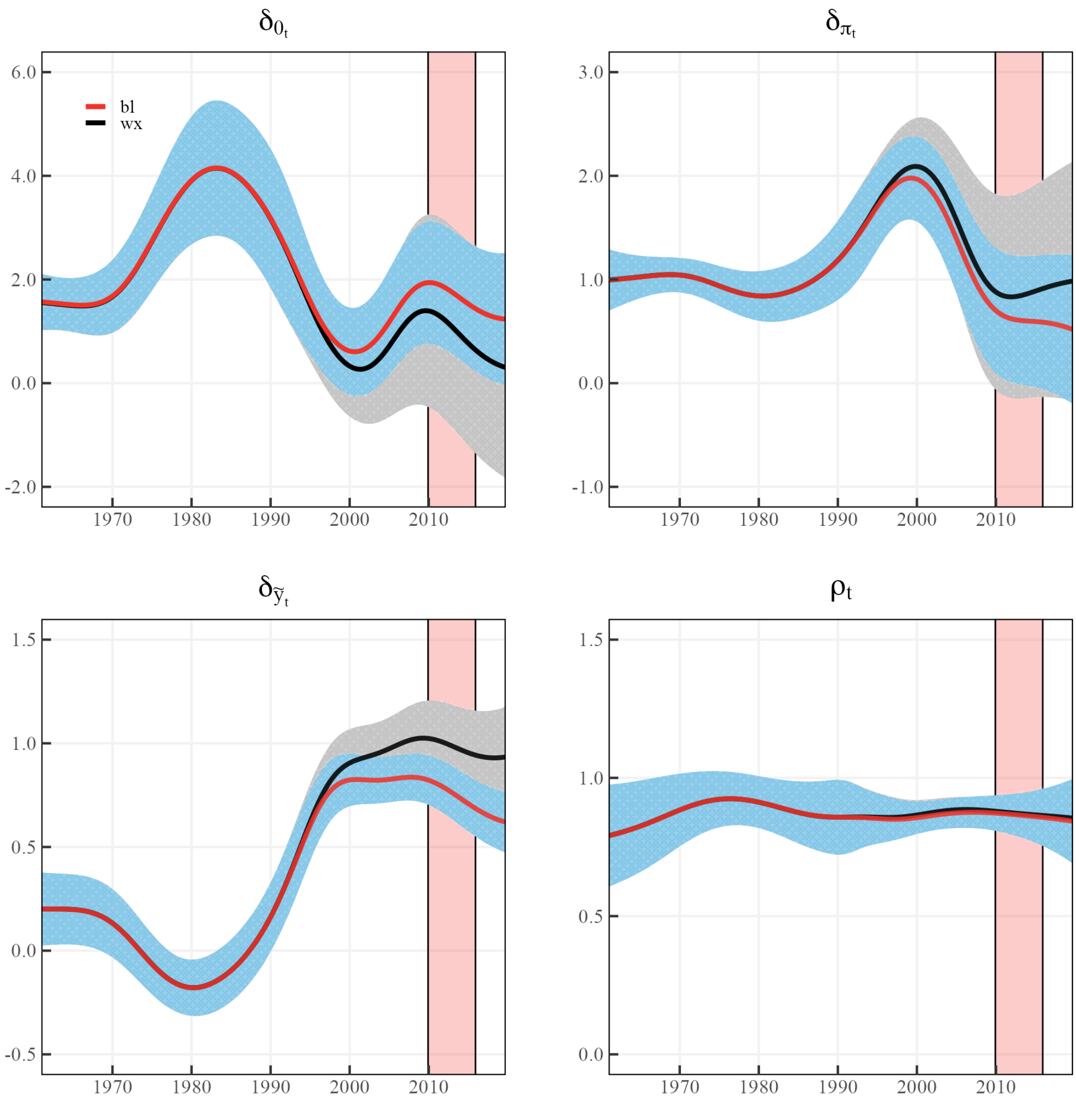

Figure 5. ZLB Robust Parameter Estimates

Note: Time-varying policy rule parameter estimates by TV CU-GMM. K(x): Gaussian. h = 0.6. Baseline estimates (bl) are presented in red. Estimates using Wu-Xia (2016) shadow rates (wx) are presented in black. Shaded regions in red correspond to periods in which the lower bound is binding between 2008:4-2015:4. Shaded regions in blue (baseline) and grey (shadow rates) correspond to the 90% confidence band.

Estimates of r-hash presented in Figure 6 derived from time-varying coefficients also reveal minimal deviations to the baseline case during the binding lower bound period. In particular, perceptions of r-star are only marginally lower on average across all specifications. As an exception, we detect a much sharper decline and recovery in r-hash given lower bandwidths and finite kernels (particularly for $h = 0.5$). What is clear across specifications is the rise in propagated uncertainty around the implicit natural rate circa the zero lower bound period. Notwithstanding, we argue that our results are largely robust to the use of shadow rates, which seek to capture periods of unconventional monetary policy. In this regard, we proceed to estimate and implement the actual natural rate as an objective comparator to gauge policy misperceptions.

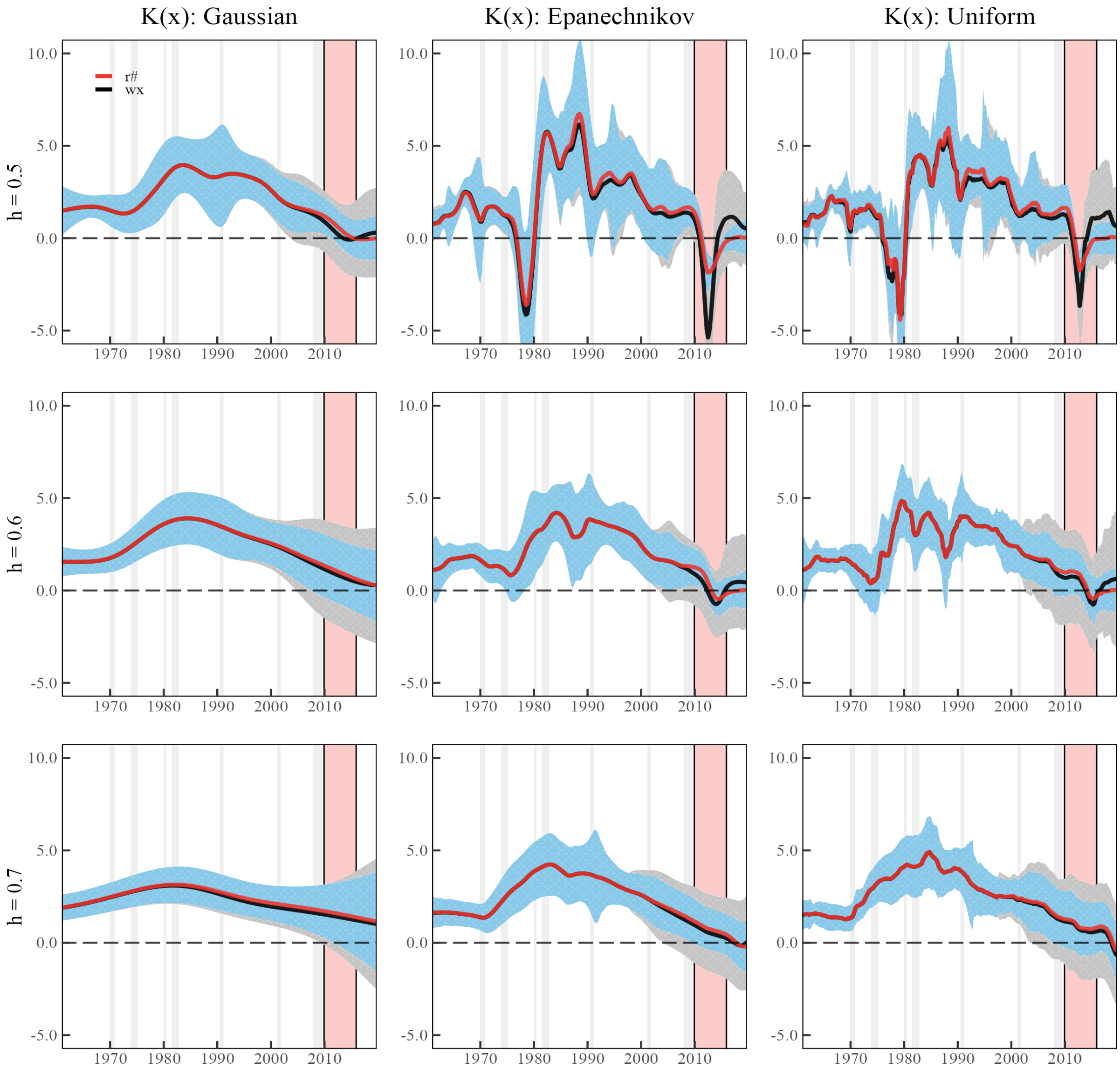

Figure 6. ZLB Robust Implied Natural Rates

Note: Baseline estimates (r-hash) are presented in red. Estimates using shadow rates (wx) are presented in black. Shaded regions in red correspond to periods in which the lower bound is binding between 2008:4-2015:4. Shaded regions in blue (baseline) and grey (shadow rates) correspond to the 90% confidence band.

4.5 Estimating the Actual Natural Rate of Interest

We proceed to estimate the actual natural rate of interest using the HLW (2017) model detailed in Section 2.3. We present the estimated parameters of the model in Table 4. Time variation in trend growth and the natural rate of interest for the United States is substantial as indicated by relative median-unbiased estimates of the innovations to σ_g and σ_z, in ratios λ_g and λ_z respectively. Slope parameters ϕ_r and φ_y associated with the interest rate and output gap are relatively large and statistically significant suggesting that they are well identified. In accordance with much of the literature, one-sided Kalman filter estimates for the natural rate of interest in the United States are imprecise with an average standard error of roughly 1.2 percentage points.

Table 4. Parameter Estimates

Parameter		Standard error	
λ_g	0.053	r^*_{avg}	1.180
λ_z	0.036	y^*_{avg}	1.526
$\Sigma\phi_y$	0.943	g_{avg}	0.400
ϕ_r	-0.068		
φ_y	0.078		
$\sigma_{\tilde{y}}$	0.339	r^*_{fin}	1.712
σ_π	0.789	y^*_{fin}	2.028
σ_{y*}	0.573	g_{fin}	0.545
σ_g	0.121		
σ_z	0.178		
σ_{r*}	0.215	T	1961:1-2019:4

Note: Estimated parameters of the Holston, Laubach and Williams (2017) model. Average and final standard errors of the natural rate of interest, the natural rate of output and its trend growth are reported in the very last column. Standard errors are calculated as in Hamilton (1986). σ_g is expressed as an annual rate.

One-sided estimates of the actual natural rate of interest are presented in Figure 7. For completeness, we also include estimates of the trend growth rate, output gap and real rate gap. Consistent with much of the literature, the natural rate of interest exhibits a secular decline. This was exacerbated by the Great Recession, during which the natural rate of interest declined by approximately 1.5-2 percentage points. R* has since persisted at the zero lower neutral bound for over a decade. Estimates of the trend growth rate exhibit a similar decline, albeit diverge from the natural rate of interest after the crisis by approximately 1.5-2 percentage points at the end of our sample. Finally, we also note that the output gap and real rate gap coincide with recessionary periods and exhibit an inverse relationship, which is consistent with the structure of the model.

Reasons for this secular decline in r-star are well documented in the literature and generally attributed to long-term trends in demography, productivity, risk preferences, public policy and inequality (see for instance Carvalho, Ferrero and Nechio, 2016; Gagnon et al., 2016; Gordon, 2016; Rachel and Smith, 2017; Eggertsson et al., 2019; Rachel and Summers, 2019). Large imprecision in our estimates are often attributed to parameter and filter uncertainty arising from estimating the state variables. In addition, the peak in variance of a shock to the latent state variable arising from simultaneously estimating the model by maximum likelihood is also of some concern. Whilst this issue is addressed with the medium unbiased estimator, uncertainty around r-star prevails. Finally, the estimation of r-star during periods of high volatility is a precarious endeavour; such extreme tailed events may violate the Kalman filter, in which stochastic innovations are assumed Gaussian. In this regard, caution ought to be taken in interpreting these results.

Whilst we do not attempt to offer an explicit solution to this particular issue, we note that our derived standard errors for the implied natural rate are substantially lower (approximately 0.78 on average in the baseline case). In particular, our estimates neither suffer from filter uncertainty nor issues of pile-up in the estimation process. This is primarily due to the use of generalised methods of moments, which is a clear advantage over alternative approaches to time-variation.

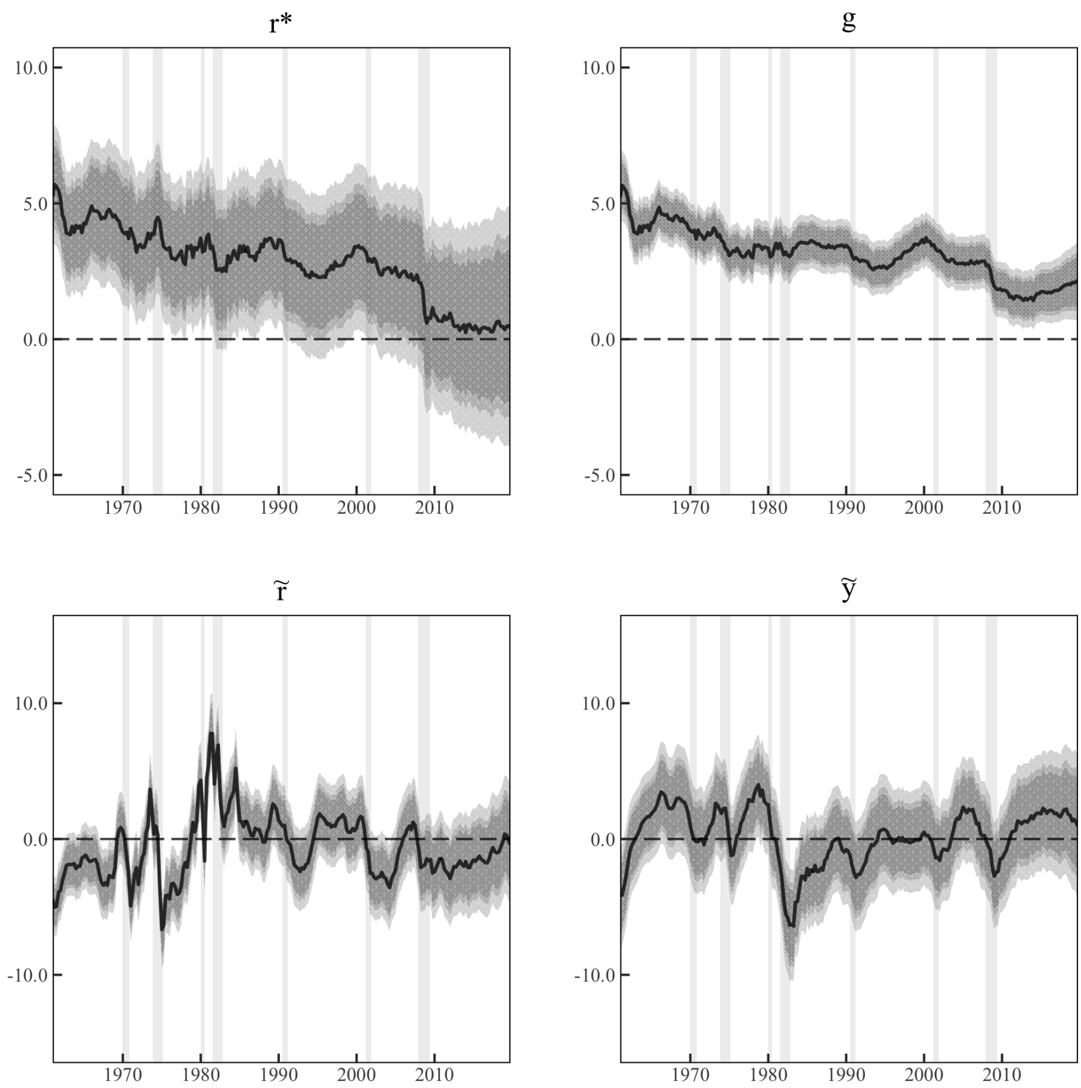

Figure 7. HLW Model Estimates

Note: One-sided estimates of the natural rate of interest, trend growth rate, output gap and real rate gap. Shaded regions in grey correspond to the 90/95/99% confidence bands. Standard errors are computed using Hamilton's (1986) Monte Carlo procedure that accounts for both filter and parameter uncertainty.

4.6 Monetary Policy (Mis)perceptions

Given our estimates of the implied and actual natural rate of interest, what follows is an explicit framework for measuring (mis)perceptions in monetary policy. We present these estimated natural rates jointly in Figure 8 and proceed to identify several periods of divergence between the two time series, during which monetary policymakers either underestimate or overestimate the natural rate of interest. To facilitate this analysis, we also plot their area difference in Figure 9. These results document a rich history of monetary policy conduct within the United States in respect to policy perceptions of the equilibrium real interest rate. In particular, we identify roughly four significant phases in the setting of policy rates by the Federal Reserve across our sample.

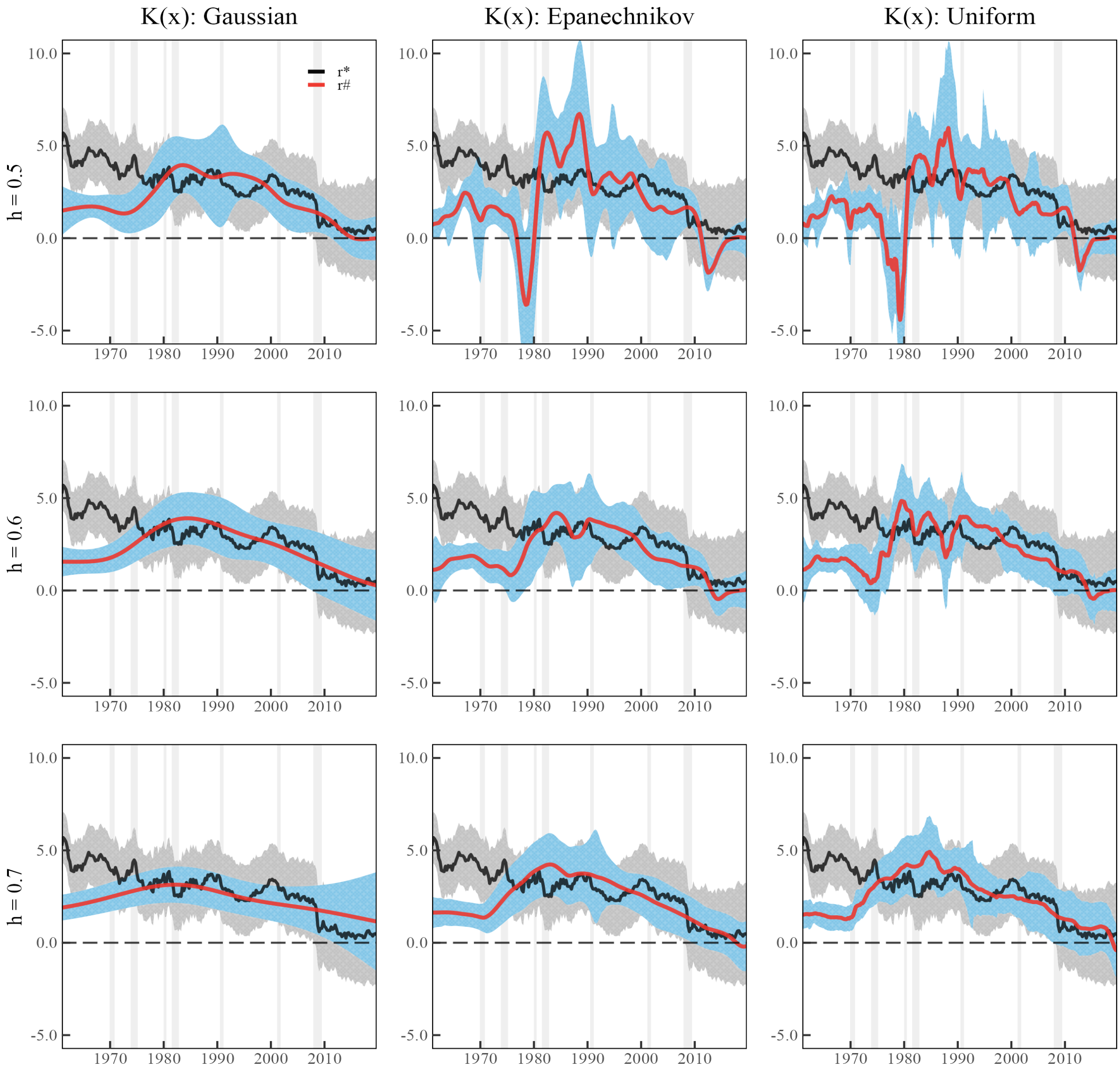

Figure 8. Natural Rates of Interest

Note: Time-varying estimates of r-star (black) and r-hash (red) for all combinations of kernel function and bandwidth parameter. Shaded regions in grey (r-star) and blue (r-hash) represent the 90% confidence band.

Firstly, policymakers during the pre-Volcker period suffer major inaccuracies in their targeting of r-star. This period is characterised by substantial underestimation of the actual natural rate, implying monetary policy was likely overly expansionary, leading to strong inflationary pressure. Indeed, this corroborates events during the pre-Volcker period, in which the Great Inflation afflicted the US economy between 1965-1982. Secondly, policymakers during much of the Volcker-Greenspan era tend to overestimate the natural rate of interest up until the turn of the century. This implies monetary policy was likely overly contractionary during this period, leading to deflationary pressure. These trends also coincides with significant macroeconomic phenomena that take place during the initial part of the post-Volcker period such as the Volcker Disinflation between 1980-1983 and the Great Moderation between 1984-2007.

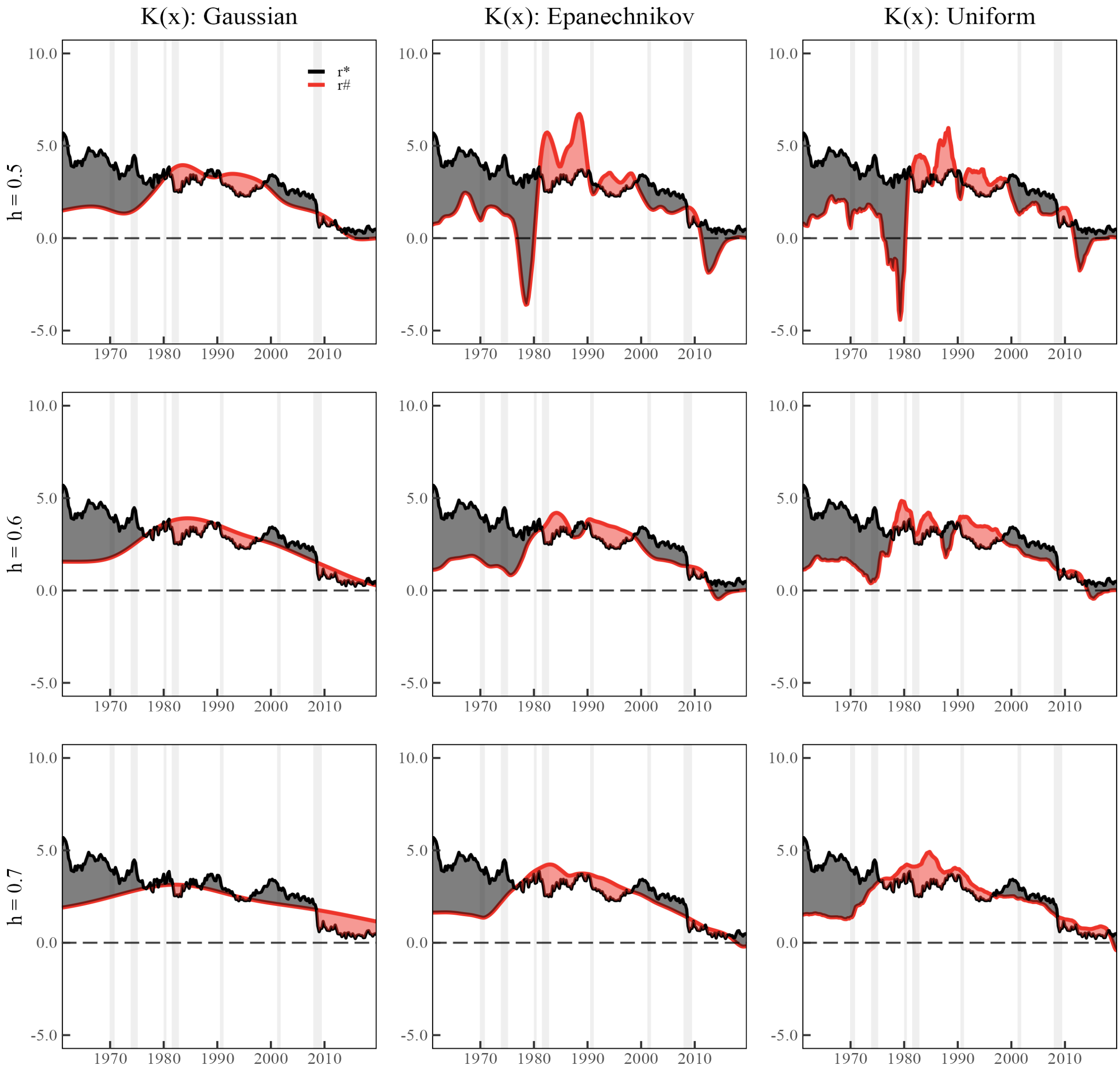

Figure 9. Policy (Mis)perceptions: Area Difference

Note: Area of the deviations in r-hash (red) from r-star (black). Shaded regions in grey are associated with periods of underestimation (overly expansionary policy), where r-hash < r-star. Shaded regions in red are associated with periods of overestimation (overly contractionary policy), where r-hash > r-star.

Thirdly, monetary policymakers during the latter of the Greenspan-Bernanke era seem to underestimate r-star once more. This coincides with the decade prior to the Great Recession, in which monetary policy was overly expansionary, leading to low unemployment and rising inflation. Finally, policy perceptions generally fluctuate around the natural rate of interest during the Bernanke-Yellen-Powell era with minimal deviation. Both rates converge to the zero lower neutral bound by the end of our sample. These general findings are also robust to alternative kernel and bandwidth selection with some minor exceptions. To demonstrate the magnitude of these policy misperceptions, we derive an explicit series for the deviation between r-hash and r-star, which we present in Figure 10 for all combinations of kernel functions and bandwidth parameters.

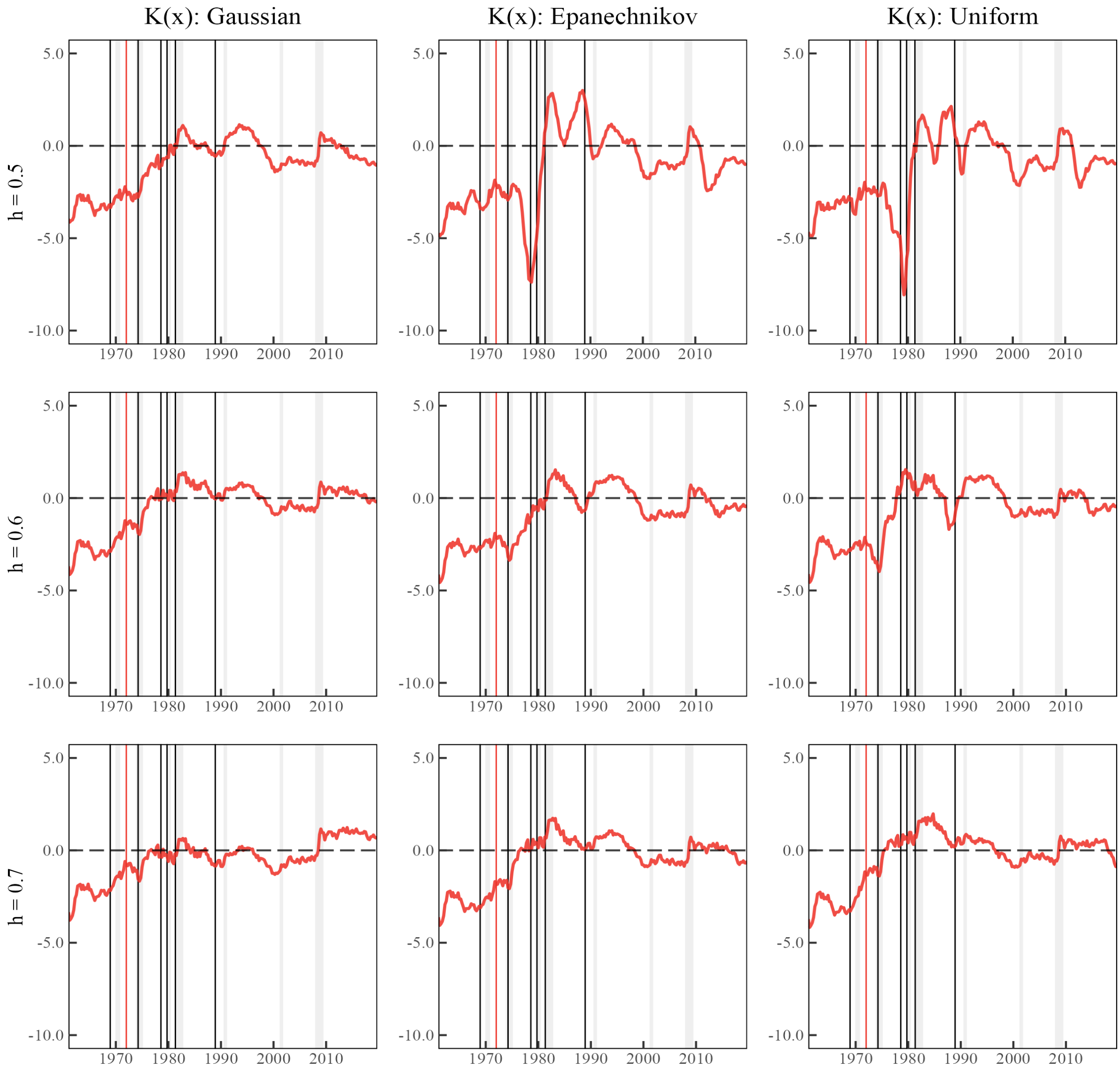

Figure 10. Policy (Mis)perceptions: Differenced Series

Note: Time series of the difference between r-hash and r-star. Negative values are associated with periods of underestimation (overly expansionary policy). Positive values are associated with periods of overestimation (overly contractionary policy). Black and red vertical lines correspond to negative and positive exogenous monetary policy shocks respectively outlined across Romer and Romer (1989; 1994; 2023).

Our results show that during the pre-Volcker period, in which the Taylor principle was putatively violated, policymakers were generally inaccurate in their perceptions of r-star. In contrast, these perceptions improve over the post-Volcker period with comparatively smaller deviations from the actual natural rate of interest. In particular, we estimate that the absolute deviation in r-hash from r-star was on average 2 percentage points during the pre-Volcker period and only 0.15 percentage points during the post-Volcker period in our baseline case. These findings suggests that monetary policymakers have become more effective in tracking the natural rate of interest over time. Finally, we reiterate here that these empirical results are largely robust to alternative specifications of both kernel functions and bandwidth parameters.

4.7 Cointegration

Our results suggest long-run comovement between the perceived and actual natural rate of interest. We briefly investigate such a relationship using a standard error-correction model (ECM). Standard Augmented Dickey-Fuller (ADF) tests confirm r-star is a nonstationary process. This is expected given the natural rate of interest is modelled as such in the HLW (2017) system. In addition, ADF tests suggest that the implied natural rates are also nonstationary across all kernel functions and bandwidth parameters. Standard Johansen tests for cointegration find one cointegrating relation that links the perceived natural rate of interest and the actual natural rate of interest. As actual and implied natural rates are I(1), we therefore choose to estimate the following ECM:

$$\Delta r_t^{\#} = \alpha(r_{t-1}^{\#} - \beta r_{t-1}^{*}) + \gamma \Delta r_t^{*} + \varepsilon_t \tag{47}$$

We note here that given r-star and r-hash are themselves a product of estimated models, caution must be invoked when interpreting any results from an estimated model in which they feature; our objective here is therefore to simply characterise their long-run comovement. Given this caveat, we report the results in Table 5 for the baseline Gaussian kernel (refer to Appendix E for estimated models given alternative kernel functions and bandwidth parameters).

Estimated short-run and long-run coefficients clearly indicate a significant positive relationship between actual and perceived natural rates of interest. This is somewhat intuitive as we expect movements in r-star to be ultimately reflected in the beliefs and perceptions of the monetary policymaker whose objectives are output and inflation stability. Finally, error-correction coefficients of the model are negative and within the range required for convergence across all specifications of the bandiwdth parameter. These estimates indicate a gradual adjustment to equilibria.

Table 5. ECM Estimates

1961:1-2019:4	$h = 0.5$	$h = 0.6$	$h = 0.7$
Short-run est. coefficient (SE)	0.05 (0.01)	0.07 (0.02)	0.02 (0.01)
Long-run est. coefficient (SE)	0.40 (0.05)	0.35 (0.05)	0.26 (0.02)
Error-correction coefficient (SE)	-0.02 (0.01)	-0.04 (0.02)	-0.01 (0.01)
ADF test (p)	0.58	0.41	0.10

Note: Results are reported for three bivariate error-correction models in which r-hash varies according to the bandwidth parameter h = 0.5, 0.6, 0.7. The kernel function is fixed as Gaussian. All estimates using alternative kernel functions are presented in Appendix E. Standard errors are reported in parentheses.

In the spirit of time-varying estimation, we briefly examine the long-run relationship established in equation (47) using a kernel-weighted cointegration approach as in Kapetanios et al. (2020) based on estimators outlined in Giraitis et al. (2018). In particular, minimisation of the kernel-weighted residual sum of squares $\sum_{t=1}^{T} k_{it} u_t^2$ yields the time-varying ordinary least squares estimator:

$$\hat{\beta}_t = \left(\sum_{t=1}^{T} k_{it} x_t x_t'\right)^{-1} \left(\sum_{t=1}^{T} k_{it} x_t y_t\right) \tag{48}$$

We use this technology to estimate an error correction model in which the impact multiplier, long-run multiplier and error correction coefficient are assumed to be stochastic time-varying parameters. The estimated time-varying specification may therefore be expressed as follows:

$$\Delta r_t^{\#} = \alpha_t (r_{t-1}^{\#} - \beta_t r_{t-1}^{*}) + \gamma_t \Delta r_t^{*} + \varepsilon_t \tag{49}$$

We present results for our baseline Gaussian kernel and varying bandwidths in Figure 11 (refer to Appendix E for estimated time-varying models given alternative kernel functions and bandwidth parameters). The long-run cointegrating relationship has risen substantially over the post-Volcker period towards unity, which suggests that latent variation in the natural rate of interest is having an increasingly positive and proportional impact on the perceptions of policymakers. In addition, our results suggest that the error correction coefficient has remained negative for a sizeable part of our sample with a similarly gradual speed of adjustment. These empirical findings largely corroborate our initial estimates of the fixed coefficient error correction model (47) summarised in Table 5.

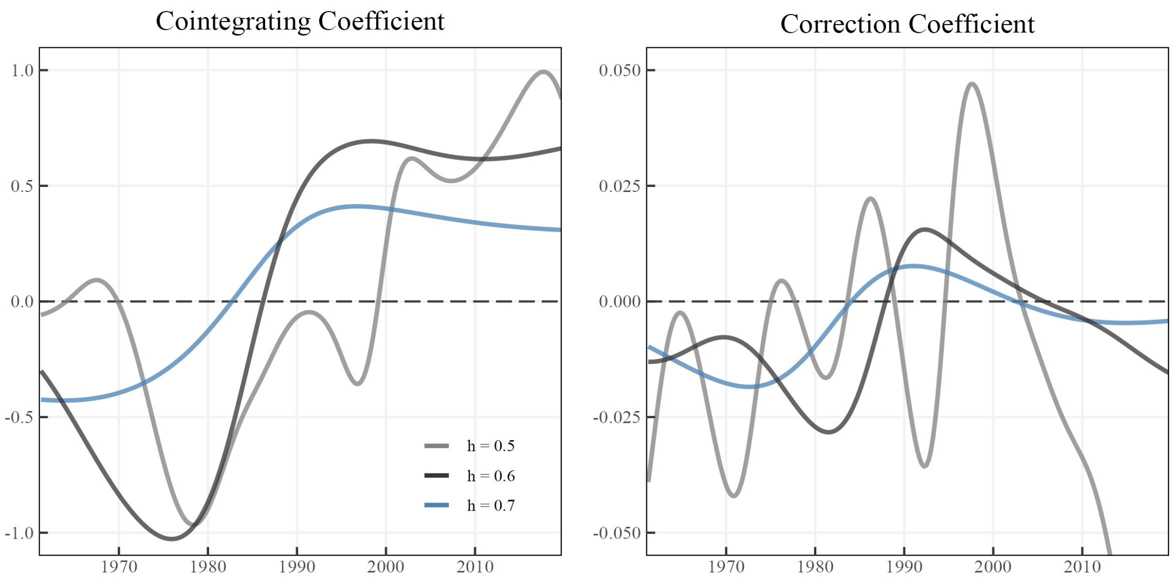

Figure 11. Time-varying ECM Estimates

Note: Time-varying estimates of the (long-run) cointegrating relationship and error correction coefficient (speed of adjustment) as in (49). R-hash varies according to the bandwidth parameter h = 0.5, 0.6, 0.7. The kernel function is fixed as Gaussian. All estimates using alternative kernels are found in Appendix E.

4.8 Exogenous Policy Shocks

Policy misperceptions have significant implications for macroeconomic fluctuations. To highlight this, Figure 10 plots dates identified across Romer and Romer (1989; 1994; 2023) marking when the Federal Open Market Committee (FOMC) announced their decision "to exert a contractionary influence on the economy in order to reduce inflation," that is, when policy rates became explicitly disinflationary.[16] These dates are detailed in Appendix F. Our results show that instances when the FED takes the decision to hike policy rates are largely clustered around the pre-Volcker and Volcker period, each corresponding to upward pressures on the deviation between central bank perceptions of the natural rate and the equilibrium interest rate. Given r-star is declining moderately over this period, much of these pressures are driven by changes in r-hash.

This is somewhat intuitive given rising anti-inflationary sentiment prior to and during the Volcker regime. Our findings show that policymakers consistently revised their perceptions of r-star upwards during this period. In fact, the cluster of upward revisions in policy rates during the latter 1970's and early 1980's is followed by almost a decade of overestimation. This occurs once more in the latter 1980's and early 1990's, suggesting that monetary policymakers, in reaching for r-star, have historically overshot their implicit beliefs about the equilibrium interest rate after decisions to disinflate the economy, insofar as the actual natural rate of interest was subsequently well below what was implied by the conduct of monetary policy.

It is worth mentioning here that the narrative approach from which these dates are derived is not without critique. As Nakamura and Steinsson (2018) have rightly pointed out, the selection of these dates do not follow a formal methodology and is therefore difficult to replicate. In addition, such a small sample size (of $n = 10$ shocks) may not be sufficiently large to average out potential random correlation with omitted factors. Notwithstanding these issues, the narrative approach still remains a useful way to capture exogenous monetary policy shocks within the literature.

4.8.1 Alternative Benchmarks

Given the natural rate of interest is associated with significant uncertainty, its use as a benchmark rate warrants further scrutiny. Multivariate Kalman filter approaches to r-star are often considered preferable to alternative univariate methods. Whilst these techniques may also be able to parse the long-term trends in real interest rates from its short-term variation, they struggle to do so at times of inflation and output instability. During such volatility, univariate approaches tend to attribute variation in real interest rates to their trend (see Hamilton et al. 2016). Notwithstanding, Kalman filter estimates used in this paper are clearly not immune from issues of uncertainty.

To check the robustness of our analysis concerning policy perceptions, we implement a range of alternative univariate and multivariate methods to measure the natural rate of interest. In regards to the former, we use the band-pass filter (Baxter and King, 1999), the Hamilton (2018) filter and the Stock and Watson (2007) Unobserved Components with Stochastic Volatility (UCSV) model

[16] Romer and Romer (2023) have since updated these dates to include expansionary monetary policy shocks. Given only one such date arises, we focus our analysis mainly on negative policy shocks. To our knowledge, no other attempts have been made to chart these dates using a narrative approach. We are therefore restricted to those identified in Romer and Romer (1989; 1994; 2023) and thank David Romer for his comments and suggestions pertaining to these dates.

to parse variation in real interest rates. As for the latter, a number of more advanced multivariate approaches to measure the natural rate of interest are available to us within the existing literature. However, for ease of our analysis we restrict this investigation to those released and maintained by various branches of the Federal Reserve. In addition to estimates from the Holston, Laubach and Williams (2017) model, we also source estimates of the natural rate of interest from the Lubik and Matthes (2015) time-varying parameter vector autoregression (TVP-VAR) model and Del Negro et al. (2017) dynamic stochastic general equilibrium model.

Given this paper employs a range of alternative measures, we average across estimates of r-star to yield a series that may be used for comparative analysis. These results are presented in Figure 12 for the baseline Gaussian kernel (refer to Appendix G for results using all combinations of kernels and bandwidths). Our findings clearly demonstrate substantial differences between univariate and multivariate estimates for reasons discussed. Notwithstanding, the former exhibit similar long-run trends to that of r-hash, in which we observe a rise and secular decline. In this regard, we proceed to use these alternative, less restrictive measures of the equilibrium real interest rate to investigate the robustness of our analysis on policy perceptions.

In a number of cases, these results corroborate trends identified using one-sided Kalman filter estimates of r-star. This is particularly true given finite kernel functions at lower bandwidth parameters, in addition to results from the Hamilton filter. For example, policymakers are still shown to have underestimated r-star in a number of instances during the pre-Volcker period, albeit the magnitude and frequency of these misperceptions are more varied than previously identified. In addition, policymakers continue to overestimate and subsequently underestimate r-star during the Greenspan term. Periods of overestimation during the Bernanke-Yellen era identified in a number of specifications using multivariate filter estimates are also prevalent, albeit results derived from univariate measures suggest this is to a much larger extent than identified.

Despite robust evidence in various parts of the sample, we also find periods where univariate estimates clearly yield differing results. In particular, we detect periods of strong overestimation and underestimation during the latter part of the pre-Volcker period and the initial part of the post-Volcker period respectively. In addition, policymakers substantially overestimate r-star during the end of the Greenspan term. Such results seem at odds with our earlier estimates but for reasons we think are entirely due to the fact that univariate measures struggle to parse variation in real rates from their trend and that shocks to real rates persist for years, in some instances decades. Although they impose less restrictions, this is an expected issue associated with univariate methods that must be handled with great caution in comparative historical analysis.

As regards multivariate estimates, results presented in Figure 12 largely corroborate our earlier analysis mainly due to the inclusion of HLW estimates derived earlier in our analysis. Whilst TVP-VAR model estimates yield systematically lower measures of r-star across our sample, they still identify extensive periods of overestimation across much of the Volcker-Greenspan era. Although we plot estimates from the DSGE framework implemented in Del Negro et al. (2017), we choose to omit them from multivariate averages despite their wide usage. This is primarily because such methods return a natural rate consistent with equilibrium over the business-cycle that is not directly comparable to estimates reflecting the medium to long run frequency.

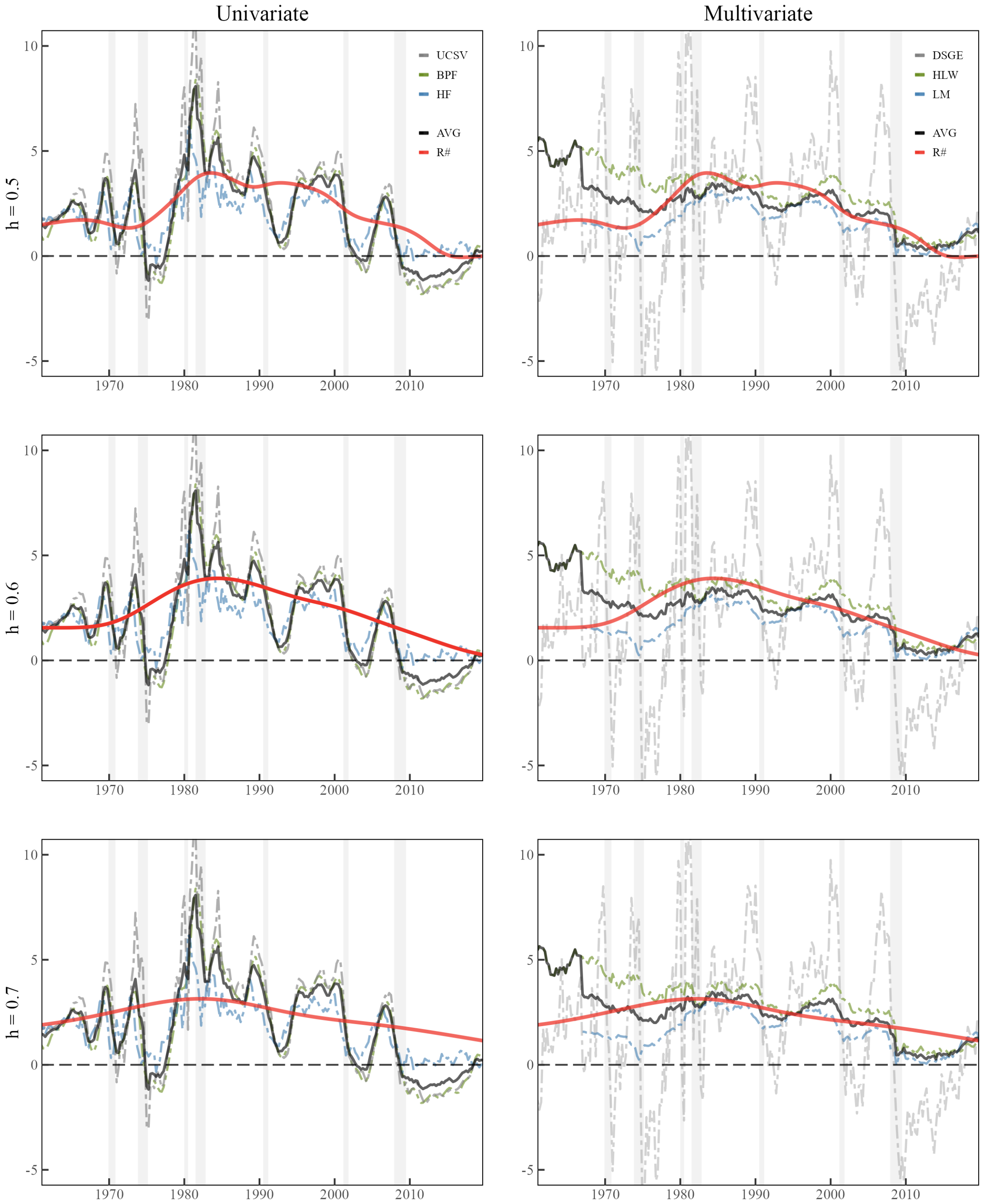

Figure 12. Alternative Measures

Note: Left column: Unobserved components with stochastic volatility (UCSV) model estimates are presented in grey, band-pass filter (BPF) estimates are presented in green and Hamilton filter (HF) estimates are presented in blue, given h = 8, p = 4. Right column: DSGE (2017) estimates are presented in grey, HLW (2017) estimates are presented in green, and LM (2015) estimates are presented in blue. Averages of estimates are presented in black across both univariate and multivariate columns. Policy perceptions (r-hash) are presented in red, and vary by bandwidth parameter h = 0.5, 0.6, 0.7. The kernel is fixed as Gaussian.

5 Conclusion

In this paper, we estimate a forward-looking time-varying Taylor rule with interest rate smoothing for the United States using a kernel-weighted time-varying continuously-updating generalised methods of moments estimator. Our findings reveal substantial time variation in the coefficients of the Federal Reserve's reaction function that is generally robust to kernel function and bandwidth parameter selection. Estimates suggest that the Taylor principle prevailed for much of our sample but is violated around the Great Recession. In addition, the historical conduct of monetary policy in the United States is characterised by substantial and marginally rising inertia.

Given time-varying estimates of the policy parameters, we derive a series for the time-varying implied natural rate of interest (r-hash). This is the natural rate of interest perceived by the central bank in its determination of policy rates given its priorities between the inflation and output gap. Our empirical results show a rise and protracted fall in the perceptions of policymakers concerning r-star. In particular, these perceptions have been in decline since the 1980's, converging to the zero lower bound by the end of our sample. The inception of such secular pessimism coincides with the Great Moderation and has been exacerbated by the Great Recession. Using time-varying shadow rates, we show that our results are robust to a binding zero lower bound.

We estimate the actual (semi-structural) natural rate of interest (r-star) and use this as an objective comparator to explicitly measure and analyse policy (mis)perceptions. Given our time-varying results, we are able to identify multiple periods of inaccuracy in the estimation of r-star that corroborate key periods of macroeconomic volatility across chairs and regimes of the Federal Reserve. More positively, our empirical analysis suggests that monetary policymakers in the post-Volcker period have substantially improved their perceptions of the long-run equilibrium interest rate relative to the pre-Volcker period, during which the average absolute deviation between r-hash and r-star was significantly higher in comparison to the rest of the sample.

Using narrative-based dates for exogenous monetary policy shocks, this paper finds evidence to suggest that the Federal Reserve has historically overshot its position on r-star during the early post-Volcker period in its attempt to correct its underestimation, insofar as r-star was below what was implied by monetary policy. In addition, we find evidence of a positive long-run relationship between the implied and actual natural rate of interest, suggesting that the beliefs of the Federal Reserve are ultimately aligned with movements in r-star. With some exceptions, a number of our findings concerning monetary policy misperceptions are largely robust to less restrictive univariate and alternative multivariate measures of the equilibrium interest rate.

This paper rests on the use of a kernel-weighted estimator, which depends on a choice of kernel and bandwidth. Whilst there is no strict rule of thumb concerning either, our estimation may well be improved with optimal selection criteria such as cross validation. We leave such an endeavour to future research. Despite this, our estimates of r-hash are characterised by less uncertainty than one-sided Kalman filter estimates of r-star, yet exhibits similar long-run trends of secular decline. These findings are of significance in the analysis of monetary policy, and our analysis provides an explicit framework to quantify not only the priorities between inflation and output stability but also the precision of policymakers in evaluating the state of the macroeconomy.

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APPENDIX

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This document provides all additional material for the paper "*Time-Varying Taylor Rules and Monetary Policy (Mis)perceptions.*" Appendix A outlines the three stages of the HLW (2017) maximum likelihood system used to estimate the natural rate of interest (r-star). Appendix B presents final TV CU-GMM estimates of the policy rule using all combinations of kernel function and bandwidth parameter. Appendix C presents time-varying J-test statistics across all specifications. Appendix D presents subsample averages to facilitate analysis across policy regimes. Appendix E presents fixed and time-varying ECM results for the relationship between r-hash and r-star. Appendix F presents business cycle and exogenous policy shock dates. Appendix G presents estimates of the natural rate of interest derived from a range of univariate and multivariate techniques given all combinations of kernel and bandwidth.

A Natural Rate of Interest

Stage I Specification:

$$\mathbf{y}_t = \Big[y_t, \pi_t\Big]' \qquad \mathbf{x}_t = \Big[y_{t-1}, y_{t-2}, \pi_{t-1}, \pi_{t-2,4}\Big]' \qquad \xi_t = \Big[y_t^*, y_{t-1}^*, y_{t-2}^*\Big]'$$

$$\mathbf{H}' = \begin{bmatrix} 1 & -\phi_{y,1} & -\phi_{y,2} \\ 0 & -\varphi_y & 0 \end{bmatrix} \quad \mathbf{A}' = \begin{bmatrix} \phi_{y,1} & \phi_{y,2} & 0 & 0 \\ \varphi_y & 0 & \varphi_\pi & 1-\varphi_\pi \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad \mathbf{Q} = \begin{bmatrix} \sigma_{y*}^2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_1 = \Big[\phi_{y,1}, \phi_{y,2}, \varphi_\pi, \varphi_y, g, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*}\Big]$$

Stage II Specification:

$$\mathbf{y}_t = \Big[y_t, \pi_t\Big]' \qquad \mathbf{x}_t = \Big[y_{t-1}, y_{t-2}, r_{t-1}, r_{t-2}, \pi_{t-1}, \pi_{t-2,4}, 1\Big]' \qquad \xi_t = \Big[y_t^*, y_{t-1}^*, y_{t-2}^*\Big]'$$

$$\mathbf{H}' = \begin{bmatrix} 1 & -\phi_{y,1} & -\phi_{y,2} & \phi_g \\ 0 & -\varphi_y & 0 & 0 \end{bmatrix} \quad \mathbf{A}' = \begin{bmatrix} \phi_{y,1} & \phi_{y,2} & \frac{\phi_r}{2} & \frac{\phi_r}{2} & 0 & 0 & \phi_0 \\ \varphi_y & 0 & 0 & 0 & \varphi_\pi & 1-\varphi_\pi & 0 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{Q} = \begin{bmatrix} \sigma_{y*}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (\lambda_g \sigma_{y*})^2 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_2 = \Big[\phi_{y,1}, \phi_{y,2}, \phi_r, \phi_0, \phi_g, \varphi_\pi, \varphi_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*}\Big]$$

Stage III Specification:

$$\mathbf{y}_t = \Big[y_t, \pi_t\Big]' \qquad \mathbf{x}_t = \Big[y_{t-1}, y_{t-2}, r_{t-1}, r_{t-2}, \pi_{t-1}, \pi_{t-2,4}\Big]' \qquad \xi_t = \Big[y_t^*, y_{t-1}^*, y_{t-2}^*, g_{t-1}, g_{t-2}, z_{t-1}, z_{t-2}\Big]'$$

$$\mathbf{H}' = \begin{bmatrix} 1 & -\phi_{y,1} & -\phi_{y,2} & \frac{-\phi_r}{2} & \frac{-\phi_r}{2} & \frac{-\phi_r}{2} & \frac{-\phi_r}{2} \\ 0 & -\varphi_y & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{A}' = \begin{bmatrix} \phi_{y,1} & \phi_{y,2} & \frac{\phi_r}{2} & \frac{\phi_r}{2} & 0 & 0 \\ \varphi_y & 0 & 0 & 0 & \varphi_\pi & 1-\varphi_\pi \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} (1+\lambda_g^2)\sigma_{y*}^2 & 0 & 0 & (\lambda_g \sigma_{y*})^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (\lambda_g \sigma_{y*})^2 & 0 & 0 & (\lambda_g \sigma_{y*})^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (\frac{\lambda_z \sigma_{\tilde{y}}}{\phi_r})^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_3 = \Big[\phi_{y,1}, \phi_{y,2}, \phi_r, \varphi_\pi, \varphi_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*}\Big]$$

B Time-Varying CU-GMM Estimates

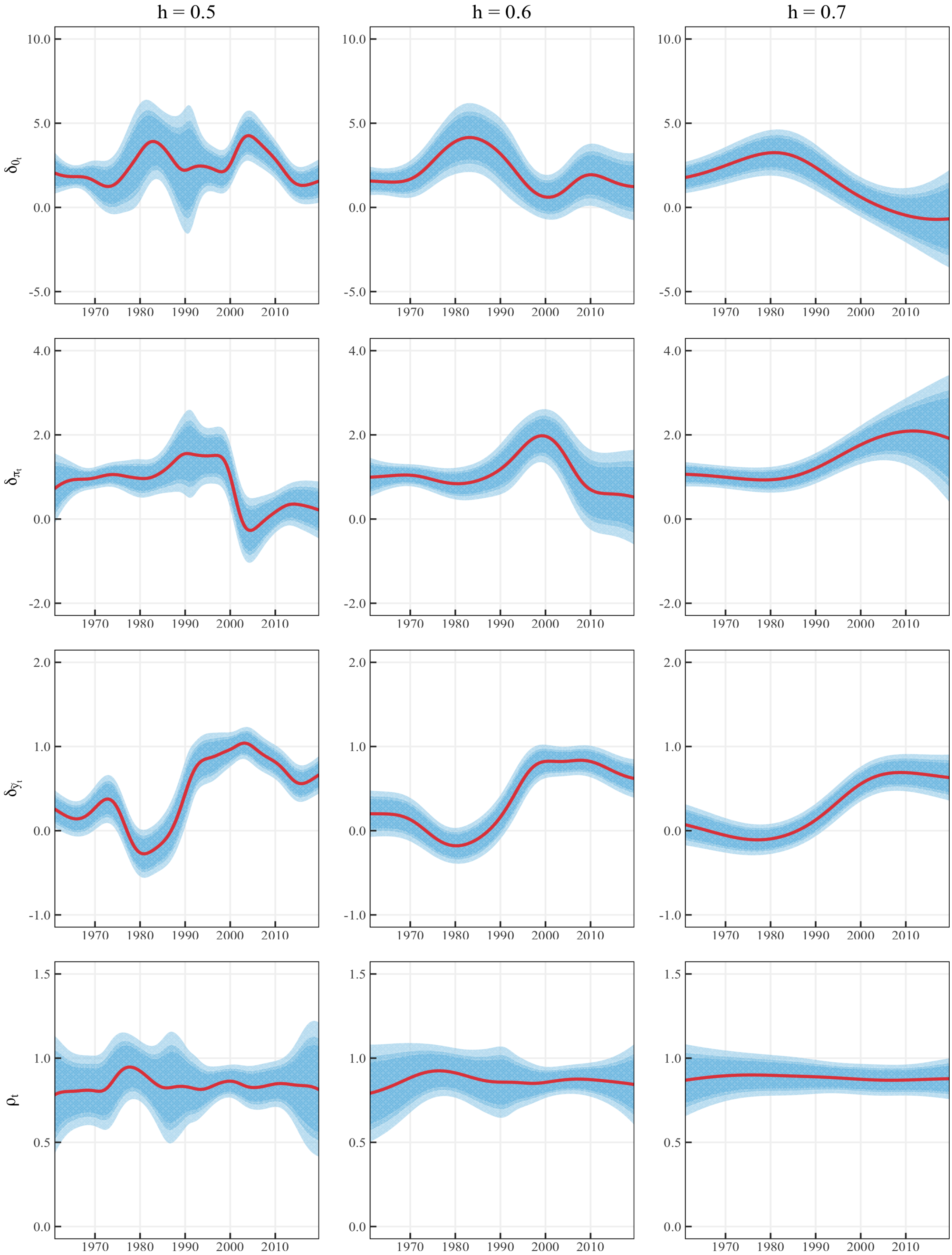

Figure B.1. Time-Varying Taylor Rule Estimates, $K(x)$: Gaussian

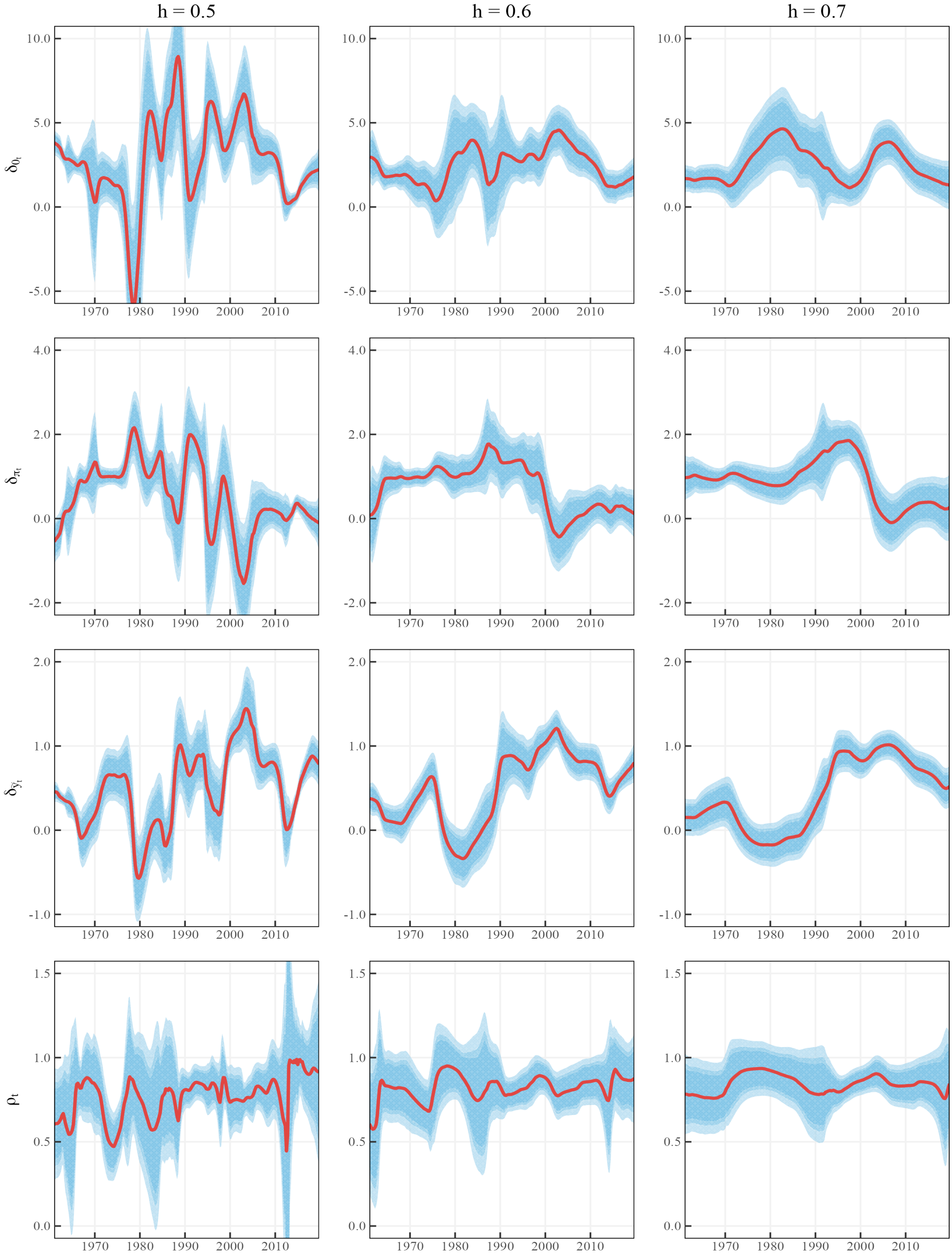

Figure B.2. Time-Varying Taylor Rule Estimates, $K(x)$: Epanechnikov

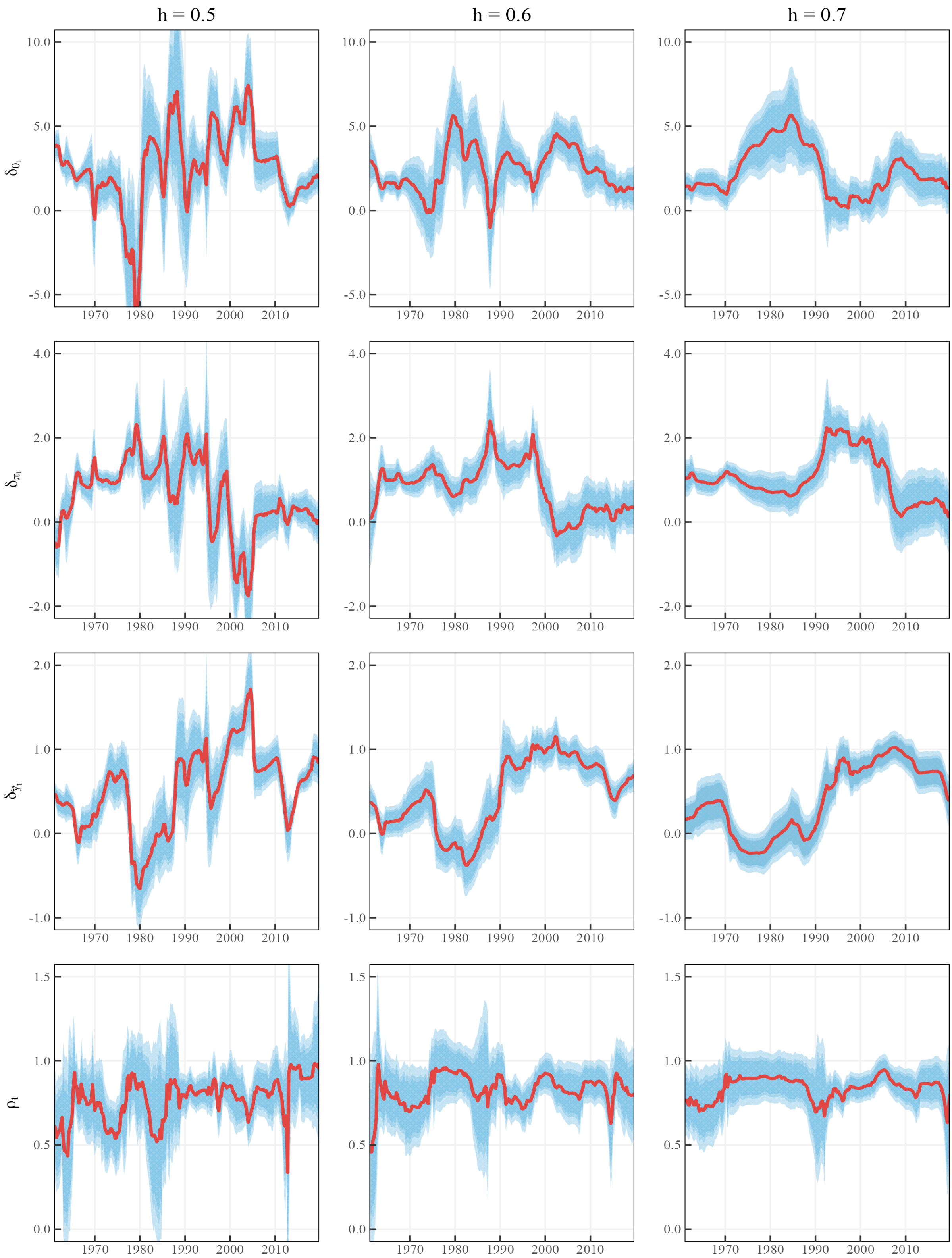

Figure B.3. Time-Varying Taylor Rule Estimates, $K(x)$: Uniform

C Time-Varying J-Test Statistics

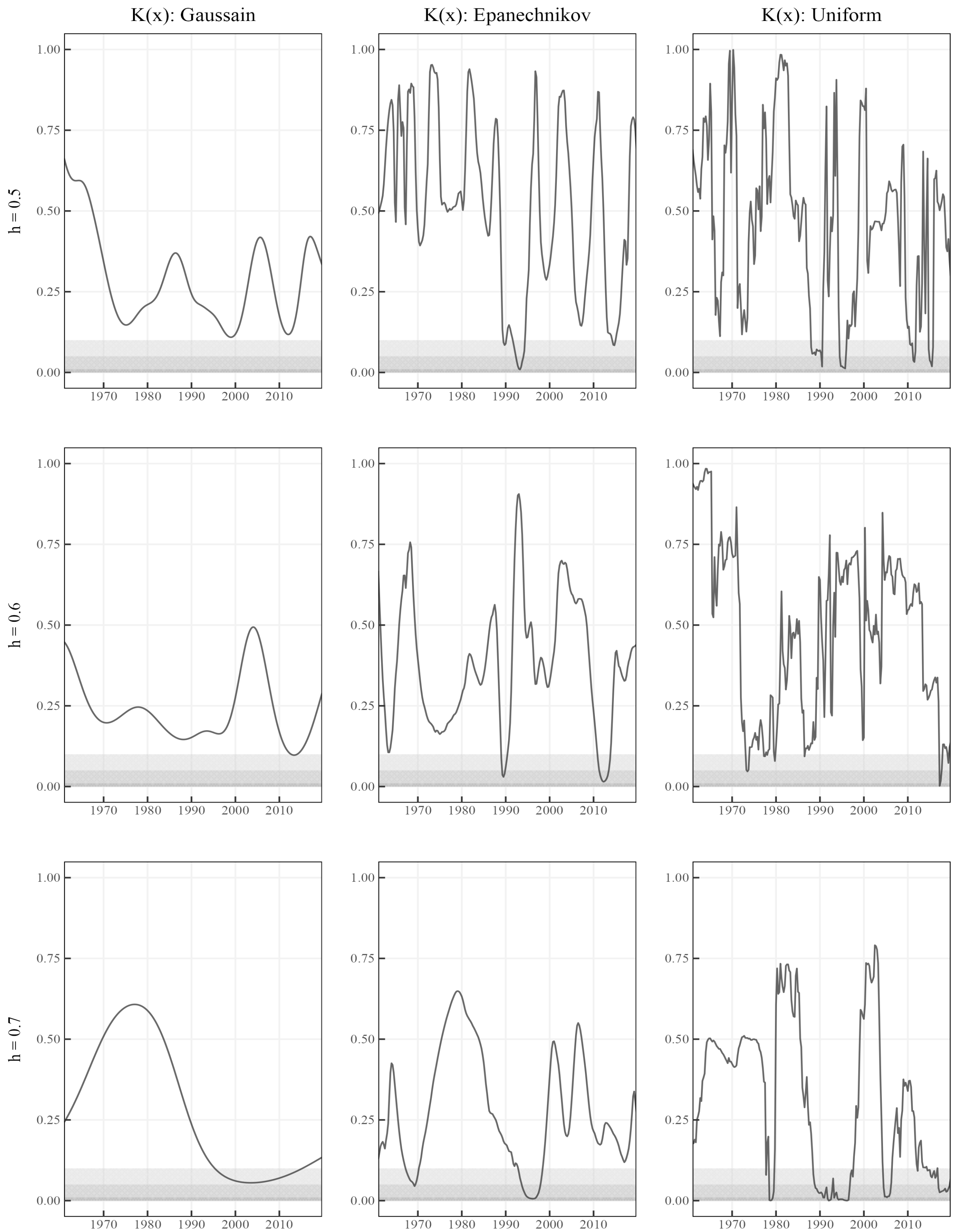

Figure C.1. Sargan-Hansen J-Test Statistics (p-value)

D Subsample Averages

Table D.1. Subsample Averages, $K(x)$: Gaussian

	Pre-Volcker 1960:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2017:4
		$h = 0.5$		
$\bar{\delta}_\pi$	0.96***	1.05***	0.76***	0.20
	(0.15)	(0.27)	(0.28)	(0.21)
$\bar{\delta}_{\tilde{y}}$	0.18**	0.50***	0.80***	0.73***
	(0.10)	(0.11)	(0.10)	(0.08)
$\bar{\rho}$	0.84***	0.84***	0.84***	0.84***
	(0.09)	(0.07)	(0.06)	(0.08)
$\bar{J}$	0.37	0.24	0.22	0.27
		$h = 0.6$		
$\bar{\delta}_\pi$	0.98***	1.38***	1.34***	0.72**
	(0.12)	(0.23)	(0.29)	(0.37)
$\bar{\delta}_{\tilde{y}}$	0.07**	0.37***	0.65***	0.77***
	(0.04)	(0.09)	(0.08)	(0.08)
$\bar{\rho}$	0.87***	0.87***	0.86***	0.87***
	(0.08)	(0.06)	(0.05)	(0.05)
$\bar{J}$	0.37	0.33	0.34	0.27
		$h = 0.7$		
$\bar{\delta}_\pi$	0.99***	1.39***	1.72***	2.06**
	(0.11)	(0.15)	(0.21)	(0.36)
$\bar{\delta}_{\tilde{y}}$	0.04	0.26***	0.48***	0.68***
	(0.08)	(0.08)	(0.08)	(0.09)
$\bar{\rho}$	0.89***	0.88***	0.88***	0.87***
	(0.06)	(0.04)	(0.04)	(0.04)
$\bar{J}$	0.48	0.24	0.11	0.08

Table D.2. Subsample Averages, $K(x)$: Epanechnikov

	Pre-Volcker 1960:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2017:4
		$h = 0.5$		
$\bar{\delta}_\pi$	0.89***	0.62*	0.29	0.10
	(0.21)	(0.43)	(0.38)	(0.16)
$\bar{\delta}_{\tilde{y}}$	0.33***	0.57***	0.77***	0.55***
	(0.13)	(0.19)	(0.17)	(0.11)
$\bar{\rho}$	0.71***	0.77***	0.80***	0.85***
	(0.13)	(0.09)	(0.11)	(0.19)
$\bar{J}$	0.66	0.49	0.42	0.34
		$h = 0.6$		
$\bar{\delta}_\pi$	0.91***	0.93***	0.62***	0.20
	(0.16)	(0.29)	(0.28)	(0.20)
$\bar{\delta}_{\tilde{y}}$	0.24***	0.54***	0.83***	0.68***
	(0.09)	(0.13)	(0.12)	(0.08)
$\bar{\rho}$	0.79***	0.83***	0.83***	0.85***
	(0.10)	(0.07)	(0.06)	(0.08)
$\bar{J}$	0.33	0.44	0.42	0.30
		$h = 0.7$		
$\bar{\delta}_\pi$	0.96***	1.17***	0.94***	0.22
	(0.14)	(0.24)	(0.26)	(0.25)
$\bar{\delta}_{\tilde{y}}$	0.12	0.46***	0.76***	0.79***
	(0.10)	(0.10)	(0.09)	(0.08)
$\bar{\rho}$	0.84***	0.85***	0.84***	0.84***
	(0.08)	(0.06)	(0.06)	(0.06)
$\bar{J}$	0.30	0.28	0.23	0.26

Table D.3. Subsample Averages, $K(x)$: Uniform

	Pre-Volcker 1960:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2017:4
		$h = 0.5$		
$\bar{\delta}_\pi$	0.91***	0.76**	0.42	0.15
	(0.18)	(0.41)	(0.39)	(0.20)
$\bar{\delta}_{\tilde{y}}$	0.31***	0.60***	0.84***	0.62***
	(0.12)	(0.18)	(0.16)	(0.10)
$\bar{\rho}$	0.71***	0.77***	0.80***	0.85***
	(0.12)	(0.09)	(0.08)	(0.13)
$\bar{J}$	0.54	0.46	0.36	0.36
		$h = 0.6$		
$\bar{\delta}_\pi$	0.96***	1.02***	0.77***	0.21
	(0.17)	(0.26)	(0.28)	(0.26)
$\bar{\delta}_{\tilde{y}}$	0.19**	0.53***	0.83***	0.71***
	(0.10)	(0.12)	(0.10)	(0.08)
$\bar{\rho}$	0.80***	0.84***	0.83***	0.84***
	(0.10)	(0.08)	(0.06)	(0.07)
$\bar{J}$	0.54	0.46	0.54	0.48
		$h = 0.7$		
$\bar{\delta}_\pi$	0.97***	1.41***	1.32***	0.44*
	(0.13)	(0.23)	(0.28)	(0.33)
$\bar{\delta}_{\tilde{y}}$	0.08	0.43***	0.70***	0.85***
	(0.11)	(0.11)	(0.10)	(0.09)
$\bar{\rho}$	0.82***	0.84***	0.83***	0.85***
	(0.08)	(0.06)	(0.06)	(0.05)
$\bar{J}$	0.40	0.32	0.22	0.17

E Cointegration

Table E.1. ECM Estimates, $K(x)$: Gaussian

1961:1-2019:4	$h = 0.5$	$h = 0.6$	$h = 0.7$
Short-run est.	0.05	0.07	0.02
coefficient (SE)	(0.01)	(0.02)	(0.01)
Long-run est.	0.40	0.35	0.26
coefficient (SE)	(0.05)	(0.05)	(0.02)
Error-correction	-0.02	-0.04	-0.01
coefficient (SE)	(0.01)	(0.02)	(0.01)
ADF test (p)	0.58	0.41	0.01

Table E.2. ECM Estimates, $K(x)$: Epanechnikov

1961:1-2019:4	$h = 0.5$	$h = 0.6$	$h = 0.7$
Short-run est.	0.04	0.05	0.03
coefficient (SE)	(0.09)	(0.02)	(0.01)
Long-run est.	0.51	0.40	0.41
coefficient (SE)	(0.10)	(0.06)	(0.05)
Error-correction	0.01	-0.04	-0.05
coefficient (SE)	(0.01)	(0.01)	(0.02)
ADF test (p)	0.48	0.84	0.75

Table E.3. ECM Estimates, $K(x)$: Uniform

1961:1-2019:4	$h = 0.5$	$h = 0.6$	$h = 0.7$
Short-run est.	0.02	0.04	0.03
coefficient (SE)	(0.01)	(0.02)	(0.04)
Long-run est.	0.40	0.38	0.37
coefficient (SE)	(0.08)	(0.06)	(0.06)
Error-correction	-0.02	-0.03	0.01
coefficient (SE)	(0.01)	(0.02)	(0.01)
ADF test (p)	0.53	0.63	0.95

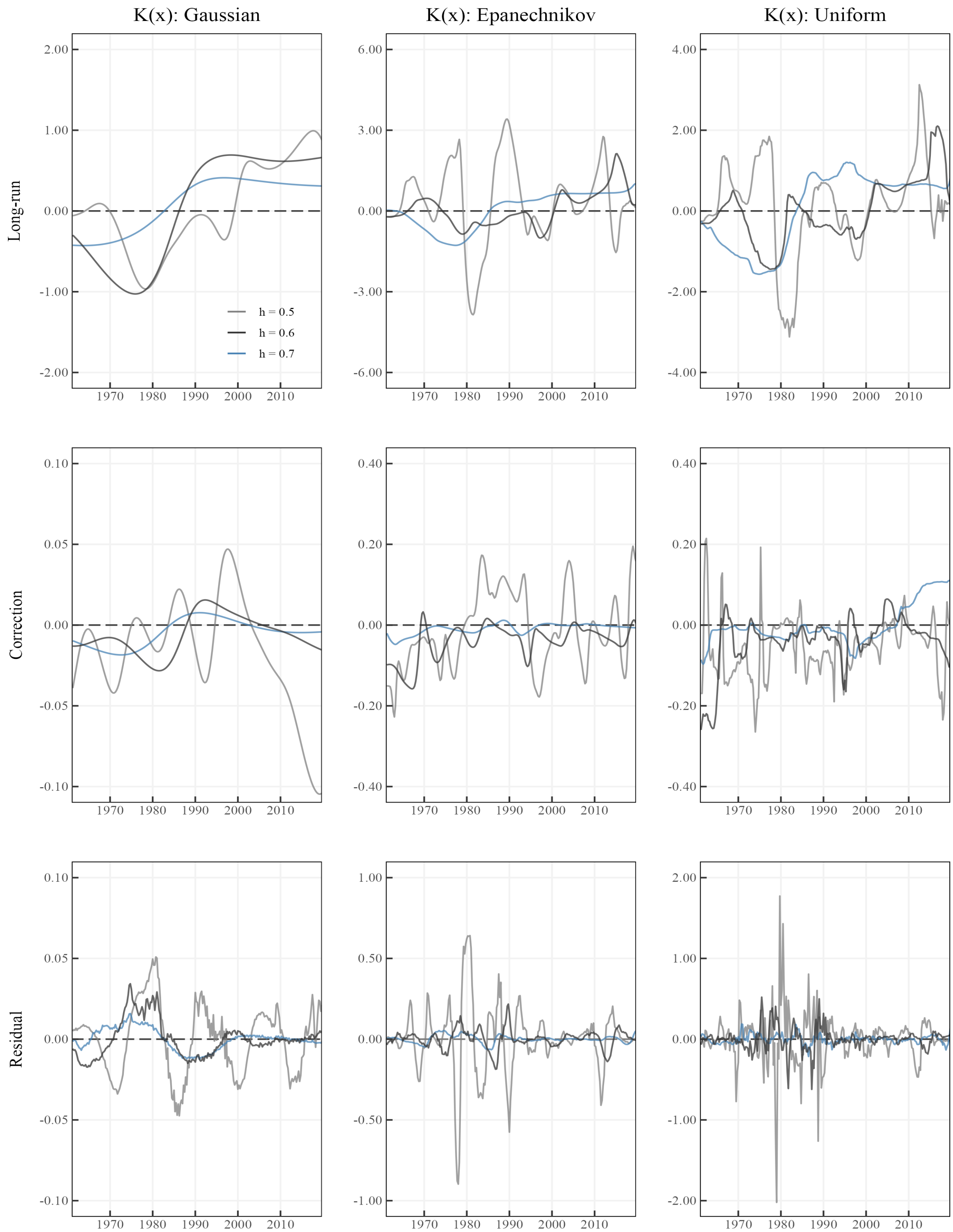

Figure E.1. TV-ECM Estimates

F Critical Dates

Table F.1. NBER Dates

April 1960 - February 1961
December 1969 - November 1970
November 1973 - March 1975
January 1980 - July 1980
July 1981 - November 1982
July 1990 - March 1991
March 2001 - November 2001
December 2007 - June 2009

Table F.2. RR Dates

October 1947 -
August 1995 -
September 1958 -
December 1968 -
January 1972 +
April 1974 -
August 1978 -
October 1979 -
May 1981 -
December 1988 -

G Alternative Measures

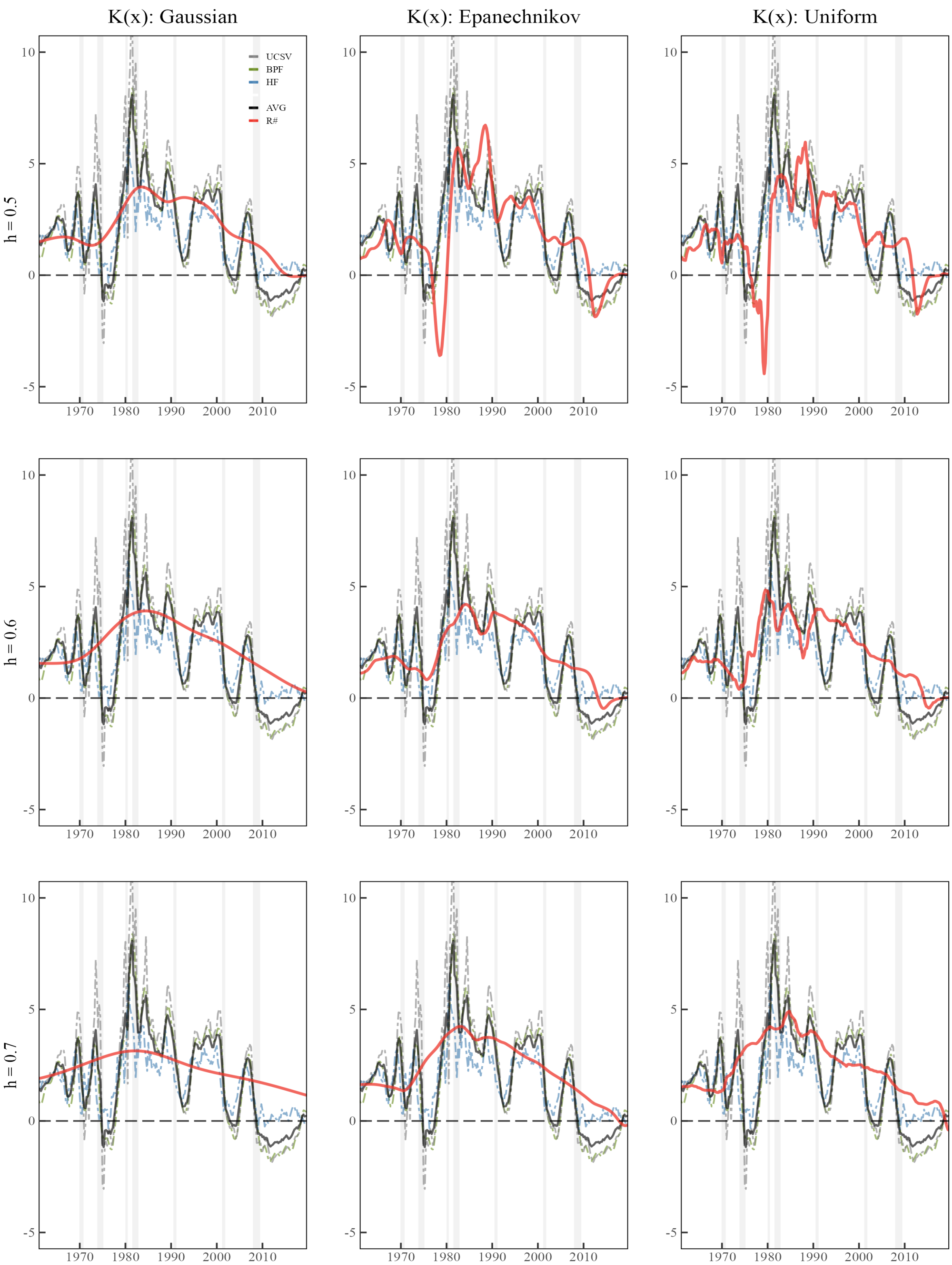

Figure G.1. Alternative Univariate Measures

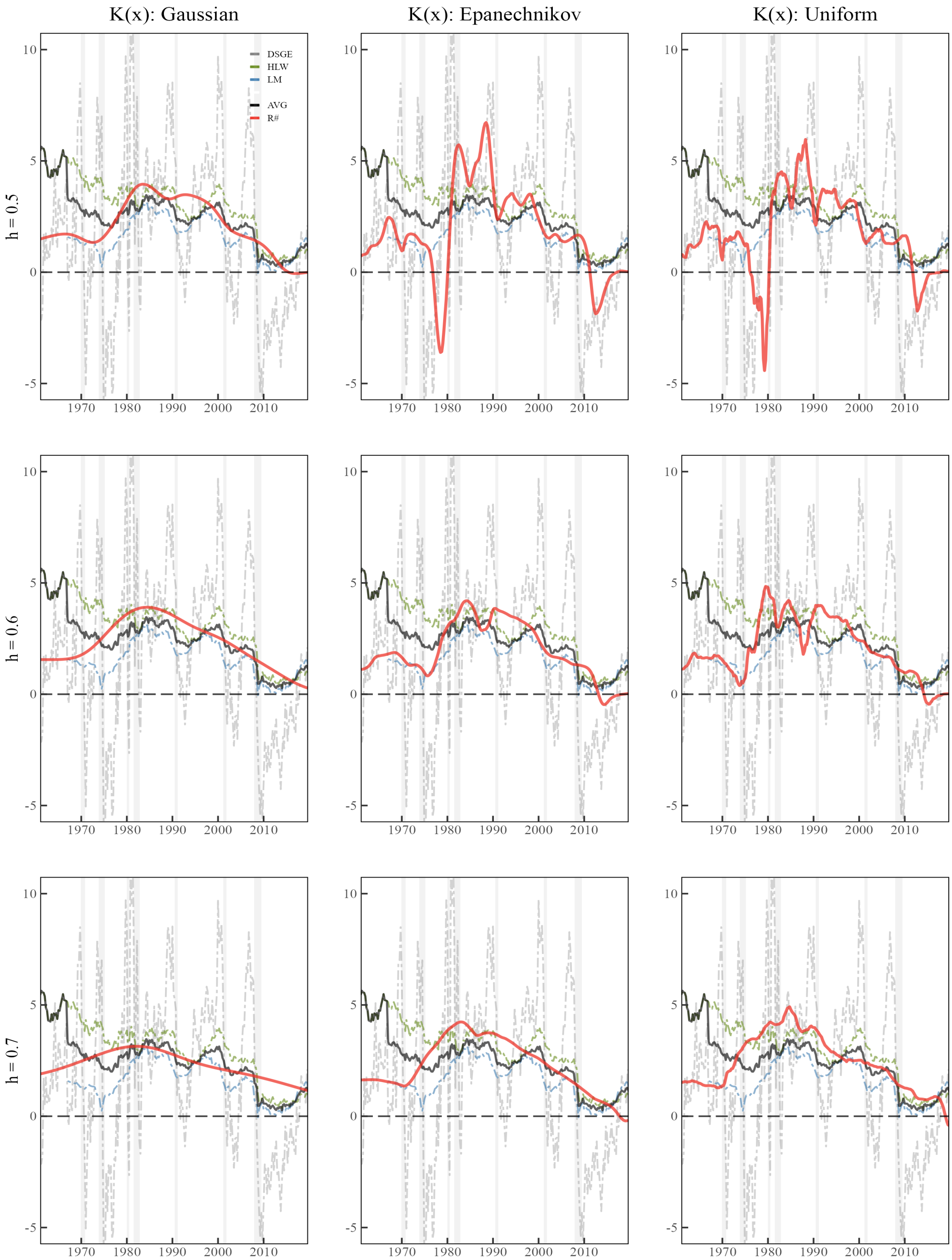

Figure G.2. Alternative Multivariate Measures