Time-Varying Implicit Inflation Targets*

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Abstract

Empirical approaches to monetary policy rules typically assume static preferences between inflation and output stability. I relax this assumption by estimating a forward-looking Taylor rule with interest rate smoothing using a time-varying kernel-weighted estimation approach. This paper employs two data-driven optimal bandwidth selection procedures to support its empirical results. Given kernel estimated time-varying reaction coefficients of the Taylor rule, this paper documents the historical variation in the objectives of policymakers and derives time-varying inflation targets implicit in the Federal Reserve's conduct of monetary policy in the United States.

Keywords: Taylor rule, Monetary Policy, Time-varying parameters

JEL classification: C26, E43, E52, E58

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1 Introduction

The Taylor (1993) rule suggests that policymakers should adjust short-term nominal policy rates in reaction to deviations in inflation from its target and output from its potential. The estimation of such rules are well documented in the literature, albeit their stability has received less attention. With nominal policy rates poised to rise across the advanced world, interest in such rules is once again being renewed. This paper seeks to better understand the objectives of policymakers and their evolution across time by implementing a simple, yet novel estimation approach that attempts to capture structural changes in the reaction coefficients of the Taylor rule.

There are strong empirical reasons to suggest that parameters of monetary policy rules are inconstant, due to the changing perceptions and preferences of policymakers between output and price stability. This has been supported by numerous studies that find breaks in the coefficients of the Taylor rule (e.g., Judd and Rudebusch, 1998, Taylor, 1999; Clarida, Galí and Gertler, 2000; Orphanides, 2004; Coibion and Gorodnichenko, 2011).

Despite these empirical findings, few studies have approached monetary policy rules from a time-varying framework. Amongst those that have, the Kalman (1960) filter is frequently employed in a maximum likelihood framework. For instance, Boivin (2006), Kim and Nelson (2006) and Trehan and Wu (2007) estimate forward and backward-looking specifications by maximum likelihood. Alternatively, Cogley and Sargent (2001) employ a Vector Auto Rregression (VAR) methodology whereas Sims and Zha (2006) construct a Structural VAR (SVAR) model to estimate the time-varying parameters. Although differing in approach, each confirm time-variation in the coefficients of monetary policy rules that correspond to periods of macroeconomic instability.

However, studies using such techniques to model time-variation tend to abstract from classical formulations of the Taylor rule by imposing restrictive assumptions on the reaction coefficients. In addition, few methodologies simultaneously address issues of potential endogeneity in the estimation process, which is a well known problem in the existing literature. More importantly, existing work has yet to exploit estimated time-varying reaction coefficients so as to reveal the implied time-varying beliefs of policymakers concerning the inflation target. This implied rate of inflation derived from the policy rule is particularly useful, as it allows us to evaluate the extent to which monetary policymakers have implicitly targeted the objective rate of inflation set out in their explicit mandate (see Judd and Rudebusch, 1998; Clarida, Galí and Gertler, 2000).

To resolve these issues, this paper estimates a forward-looking time-varying Taylor rule with interest rate smoothing using non-parametric kernel-weighted estimators. This approach has the advantage of imposing relatively minimal restrictions on the coefficients of the monetary policy rule. Such estimation, however, is not without its challenges. Ordinary Least Squares (OLS) estimation may yield inconsistent estimates of the Taylor coefficients due to issues of potential endogeneity in the monetary policy reaction function. Instrument Variables (IV) may resolve such endogeneity, however, the selection of instruments is itself a precarious endeavour. Whilst problematic, Carvalho, Nechio and Tristo (2021) show that the amount of endogeneity is in fact minimal in empirically relevant sample sizes and OLS may even outperform IV in some instances. As the choice of econometric technique is therefore nuanced, I estimate the Taylor rule using both time-varying OLS and time-varying IV estimators. To check the robustness of these results, I use two optimal bandwidth selection procedures and corroborate the range of parameters used in the initial kernel-weighted estimation of the Taylor rule.

As the inflation target and natural rate of interest cannot be identified simultaneously, I estimate the latter using the Kalman filter in a multi-step maximum-likelihood procedure designed by Holston, Laubach and Williams (2017). Given kernel estimated coefficients of the forward-looking Taylor rule, I derive the implicit beliefs about the inflation target and evaluate the stance of policy using the objective rate set out in the Federal Reserve's mandate since its inception in the mid 1990's. Periods when the implicit inflation target is below the objective target rate are interpreted as periods of under-targeting, during which monetary policy is tighter than optimal. In contrast, periods when the implicit inflation target is above the objective target rate are interpreted as periods of over-targeting, during which monetary policy is looser than optimal.

A number of important results emerge from this dynamic time-series analysis. Firstly, there is substantial time variation in the coefficients of the Taylor rule across all monetary policy regimes. Secondly, the Taylor principle holds on average up until the turn of the century, after which the coefficient associated with deviations in inflation from its target falls below unity. Thirdly, the priority of output stability has generally been rising over the last half-century, captured by an increasing coefficient associated with deviations in output from its potential. Fourthly, there has been a high and marginally increasing degree of inertia in monetary policy across all regimes reflected in the smoothing parameter of the Taylor rule. Finally, monetary policymakers have indeed implicitly pursued an average 2% inflation target, deviating from it frequently during periods of implicit targeting and sharply during crises at the turn of the century. In addition, policymakers have generally been inclined to under-target inflation over the last few decades.

The remainder of this paper is structured as follows. Section 2 outlines the theoretical framework concerning time-varying kernel-weighted OLS and IV estimators employed in this paper. Section 3 discusses particulars concerning the selection of data, instruments, kernels and bandwidths. Section 4 presents and discusses the main empirical results of the paper. Finally, Section 5 concludes.

2 Theoretical Framework

The forward-looking Taylor rule with interest rate smoothing may be written as in Clarida, Galí and Gertler, (2000). The central bank's target policy rate is defined as follows:

$$i_t^* = i^* + \beta_\pi(E[\pi_{t,k}|\Omega_t] - \pi^*) + \beta_{\tilde{y}} E[\tilde{y}_{t,q}|\Omega_t] \tag{1}$$

where i^* is the nominal equilibrium interest rate, π^* is the actual inflation target, $\pi_{t,k}$ is the inflation rate in period $t+k$ and $\tilde{y}_{t,q}$ is the output gap in period $t+q$, whose expectation is conditional on information set Ω_t. We may define a constant $\beta_0 = i^* - \beta_\pi \pi^*$, to yield:

$$i_t^* = \beta_0 + \beta_\pi E[\pi_{t,k}|\Omega_t] + \beta_{\tilde{y}} E[\tilde{y}_{t,q}|\Omega_t] \tag{2}$$

The smoothing mechanism describing how actual policy rates adjust to the target rate is:

$$i_t = [1-\rho] i_t^* + \rho(L) i_t + u_t \tag{3}$$

where ρ is the smoothing parameter. The policy rule for the actual interest rate is derived by substituting the target rate (1) into the partial adjustment mechanism (3) to yield:

$$i_t = (1-\rho)(\beta_0 + \beta_\pi E[\pi_{t,k}|\Omega_t] + \beta_{\tilde{y}} E[\tilde{y}_{t,q}|\Omega_t]) + \rho(L) i_t + u_t \tag{4}$$

Under rational expectations, the policy reaction function may be specified as:[1]

$$i_t = (1-\rho)(\beta_0 + \beta_\pi \pi_t + \beta_{\tilde{y}} \tilde{y}_t) + \rho(L) i_t + \varepsilon_t \tag{5}$$

Given time-variation in the parameter vector, the Taylor rule (5) is estimated by the kernel-weighted Time-Varying OLS (TV-OLS) and Time-Varying IV (TV-IV) estimator analysed in Giraitis et al. (2014; 2018; 2021). To outline this approach, consider the following:

$$y_t = \mathbf{x}_t' \boldsymbol{\beta}_t + u_t \quad t = 1, ..., T \tag{6}$$

where $\mathbf{x}_t$ is a $k \times 1$ vector of regressors, $\boldsymbol{\beta}_t$ is a $k \times 1$ coefficient vector and u_t is a scalar error term. The coefficient vector $\boldsymbol{\beta}_t$ is given by the following kernel TV-OLS estimator:

$$\hat{\boldsymbol{\beta}}_t = \Big(\sum_{t=1}^{T} k_{ti} \mathbf{x}_t \mathbf{x}_t'\Big)^{-1} \Big(\sum_{t=1}^{T} k_{ti} \mathbf{x}_t y_t'\Big) \tag{7}$$

[1] Note that (5) may be expressed as a linear function of auxiliary coefficients: $i_t = \phi_0 + \phi_\pi \pi_t + \phi_{\tilde{y}} \tilde{y}_t + \phi_\rho(L) i_t + \varepsilon_t$, where $\phi_0 = (1-\rho)\beta_0$, $\phi_\pi = (1-\rho)\beta_\pi$, $\phi_{\tilde{y}} = (1-\rho)\beta_{\tilde{y}}$ and $\phi_\rho = \rho$. This allows us to estimate a linear regression and extract coefficients from the auxiliary vector $\boldsymbol{\phi}$.

The kernel function is defined as $k_{ti} = K\left(\frac{|t-i|}{H}\right)$, where $H = T^h$ is the bandwidth. $K(x)$ is a non-negative continuous bounded kernel function with a bounded first derivative such that $\int K(x)\partial x = 1$. Examples of such kernels include but are not limited to:

$$K(x) = (3/4)(1 - x^2)I(|x| \leqslant 1) \qquad \text{Epanechnikov kernel}$$

For kernels with infinite support assume additionally that $K(x) \leqslant Ce(-cx^2), |\dot{K}(x)| \leqslant C(1 + x^2)^{-1}, x \geqslant 0$, for some $C > 0$ and $c > 0$. Examples include but are not limited to:

$$K(x) = (1/\sqrt{2\pi})e^{-x^2/2} \qquad \text{Gaussian kernel}$$

Alternatively, consider in addition to (6) the following reduced equation:

$$\mathbf{x}_t = \boldsymbol{\psi}_t'\mathbf{z}_t + \boldsymbol{v}_t \tag{8}$$

where $\mathbf{z}_t$ is a $l \times 1$ instrument vector, $\boldsymbol{\psi}_t'$ is a $k \times l$ coefficient matrix and $\boldsymbol{v}_t$ is a $l \times 1$ error vector. Assume the following exogeneity conditions hold for the error term and vector:

$$E[\mathbf{z}_t u_t] = 0 \quad \text{and} \quad E[\mathbf{z}_t \boldsymbol{v}_t'] = 0 \tag{9}$$

The kernel TV-IV estimator for $\boldsymbol{\beta}_t$ is given by:

$$\tilde{\boldsymbol{\beta}}_t = \left(\sum_{t=1}^{T} k_{ti}\widehat{\boldsymbol{\psi}}_t'\mathbf{z}_t\mathbf{x}_t'\right)^{-1}\left(\sum_{t=1}^{T} k_{ti}\widehat{\boldsymbol{\psi}}_t'\mathbf{z}_t y_t'\right) \tag{10}$$

where $\widehat{\boldsymbol{\psi}}_t$ denotes the time-varying first-stage OLS estimate of $\boldsymbol{\psi}_t$.

Giraitis et al. (2021) also propose a time-varying Hausman (H) test statistic:

$$H_t = K_t^2 K_{2,t}^{-1} V_{T,t}' \hat{\Sigma}_{\hat{v}\hat{v},t}^{-1} V_{T,t} \hat{\sigma}_{\hat{u},t}^{-2} \rightarrow_d \chi_r^2 \tag{11}$$

where the following variables are defined:

$$\begin{aligned}
K_t &= \Sigma_{t=1}^{T} k_{ti}; & K_{2,t} &= \Sigma_{t=1}^{T} k_{ti}^2;\\
\hat{\Sigma}_{\hat{\boldsymbol{v}}\hat{\boldsymbol{v}},t} &= K_t^{-1}\Sigma_{t=1}^{T} \boldsymbol{v}_t\boldsymbol{v}_t'; & \hat{\sigma}_{\hat{u},t}^{-2} &= K_t^{-1}\Sigma_{t=1}^{T} k_{ti}\hat{u}_t;\\
V_{T,t} &= \left(\Sigma_{t=1}^{T} k_{ti}\hat{\mathbf{x}}_t\hat{\mathbf{x}}_t'\right)^{1/2}\left(\Sigma_{t=1}^{T} k_{ti}\mathbf{x}_t\mathbf{x}_t'\right)^{1/2}\left(\tilde{\boldsymbol{\beta}}_t - \hat{\boldsymbol{\beta}}_t\right)
\end{aligned}$$

where $\hat{\boldsymbol{v}}_t$ and $\hat{u}_t$ are first and second-stage residuals and $\hat{\mathbf{x}}_t$ are first-stage fitted values.

Additionally, the Hansen (J) test statistic is given by:

$$J_t = K_t K_{2,t}^{-1} \hat{\sigma}_t^{-2} \Big(\sum_{t=1}^{T} k_{ti} \mathbf{z}_t' \hat{u}_t \Big) \Big(\sum_{t=1}^{T} k_{ti} \mathbf{z}_t \hat{z}_t' \Big)^{-1} \Big(\sum_{t=1}^{T} k_{ti} \mathbf{z}_t \hat{u}_t \Big) \rightarrow_d \chi_r^2 \tag{12}$$

This paper proceeds to estimate a time-varying specification of a forward-looking Taylor rule with interest rate smoothing by TV-OLS and TV-IV. The following section discusses particulars concerning the implementation before presenting the empirical results.

3 Estimation

3.1 Data and Instruments

The Taylor rule is estimated using quarterly data from 1960:1 to 2019:4. The nominal interest rate is the federal funds rate taken from the Board of Governors. Inflation is calculated as the percentage change in core Personal Consumption Expenditure (PCE) sourced from the Bureau of Economic Analysis (BEA). The output gap is computed using the Congressional Budget Office (CBO) estimate of potential GDP. I rely on the instrument selection in Clarida, Galí and Gertler, (2000), wherein each variable and four of their lags are included in the matrix $\mathbf{z}_t$. Money growth is the percentage change in M2 published by the Board of Governors. Price inflation is calculated as the percentage change in a composite index of goods published by the Bureau of Labor Statistics (BLS), and the interest rate spread between long and short-term bonds is the difference between the 10-year and 3-year Treasury bill rate, published by the Federal Reserve.

3.2 Kernel and Bandwidth Selection

As is necessary in kernel-weighted estimation, the selection of kernel and bandwidth requires attention. In this paper, the baseline kernel is Gaussian with infinite support due to its desirable properties. For robustness, I select the Epanechnikov kernel with finite support. In the case of the former, all observations contribute to the estimation, whereas only local observations are used in the finite case. Whilst such kernels are widely employed in the time-series literature, any kernel function may be used in practice.

The choice of bandwidth is a delicate matter that has non-trivial implications in kernel-weighted estimation. As there is no clear consensus concerning the selection of bandwidth parameter across the existing literature, I choose an array for sufficient number of observations. As this regulates smoothness, I check the robustness of this range with two popular data-driven optimal bandwidth selection procedures in Section 4.2.

4 Results and Discussion

4.1 Time-varying Taylor rules

To motivate this analysis, the Taylor rule is first estimated by standard OLS and IV. I examine the stability of equation (5) across subsamples of particular relevance to the historical conduct of monetary policy in the United States; the (1) pre-Volcker (1960:1-1979:2), (2) Volcker-Greenspan (1979:3-2005:4), (3) Greenspan-Bernanke (1987:3-2013:4), and (4) Bernanke-Yellen (2006:1-2018:1) era. Static results are reported in Tables 1-2.

Differences between OLS and IV estimates are clearly negligible, however standard errors associated with the latter are marginally less than those associated with the former. Assuming a 2% inflation target, for the Taylor principle to hold, we expect that $\beta_\pi > 1$ and $\beta_y > 0$ (Woodford, 2001). Both OLS and IV estimates of β_π are below unity during the pre-Volcker period, albeit greater than unity thereafter. The response to deviations in inflation from its target seems to be increasing between the pre-Volcker and Volcker-Greenspan era before declining across subsequent monetary policy regimes.

OLS and IV estimates of $\beta_{\tilde{y}}$ are above zero across policy regimes. In particular, the response to the output gap is increasing from the start of the sample until the Greenspan-Bernanke era, whereafter it is in decline. Judgment is generally suspended on the reaction coefficient for inflation and output during this period due to insignificant results.

Estimates of the smoothing parameter ρ are high and increasing across policy regimes, implying a rising degree of inertia in monetary policy. These empirical results corroborate the existing literature and indicate considerable variation in the coefficients of the Taylor rule across monetary policy regimes. What is clearly limiting in this cursory analysis is the discrete nature of our interpretation of the reaction coefficients associated with deviations in inflation from its target and output from its potential.

To investigate such time-variation in the coefficients of the Taylor rule, the following time-varying specification of equation (5) is estimated:

$$i_t = (1 - \rho_t)(\beta_{0,t} + \beta_{\pi.t}\pi_t + \beta_{\tilde{y},t}\tilde{y}_t) + \rho_t(L)i_t + \varepsilon_t \tag{13}$$

where the time-varying vector of interest $\boldsymbol{\beta}_t = [\beta_{0,t}, \beta_{\pi,t}, \beta_{\tilde{y},t}, \rho_t]$ is estimated by TV-OLS and TV-IV. Dynamic results including robustness checks are presented in Figures 1-2.

Differences between TV-OLS and TV-IV remain negligible. Time-varying Hansen (J) and Hausman (H) test statistics are presented in Appendix A.1-2. I note that estimates of the reaction coefficient to the inflation gap seem to diverge post-1990, during which TV-OLS is systematically larger than TV-IV. This is also the case for the constant parameter, which has implications for our estimates of the implicit inflation target.

Table 1. OLS Estimates

	Pre-Volcker 1960:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2018:1
β_π	0.93***	2.21***	1.82***	1.65
	(0.39)	(0.55)	(0.52)	(1.14)
$\beta_{\tilde{y}}$	0.87***	1.21***	1.25***	0.54
	(0.30)	(0.48)	(0.32)	(0.38)
ρ	0.77***	0.83***	0.87***	0.88***
	(0.07)	(0.07)	(0.02)	(0.05)
R^2	0.90	0.88	0.96	0.86

*Note: Significance at the 90/95/99% confidence level are denoted by */**/*** respectively. Robust standard errors are reported in parentheses. Uncertainty is propagated by the delta method.*

Table 2. IV Estimates

	Pre-Volcker 1960:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2017:4
β_π	0.97***	2.20***	1.99***	1.66
	(0.30)	(0.52)	(0.59)	(1.05)
$\beta_{\tilde{y}}$	0.84***	1.09***	1.16***	0.60*
	(0.26)	(0.41)	(0.30)	(0.33)
ρ	0.73***	0.81***	0.84***	0.85***
	(0.06)	(0.07)	(0.03)	(0.06)
J	0.82	0.34	0.38	0.21
H	0.34	0.77	0.01	0.09
R^2	0.90	0.89	0.97	0.87

*Note: Instruments detailed in Section 3. Significance at the 90/95/99% confidence level is denoted by */**/*** respectively. Robust standard errors reported in parentheses. P-values reported for Hansen (J) and Hausman (H) test statistics. Uncertainty is propagated using the delta method.*

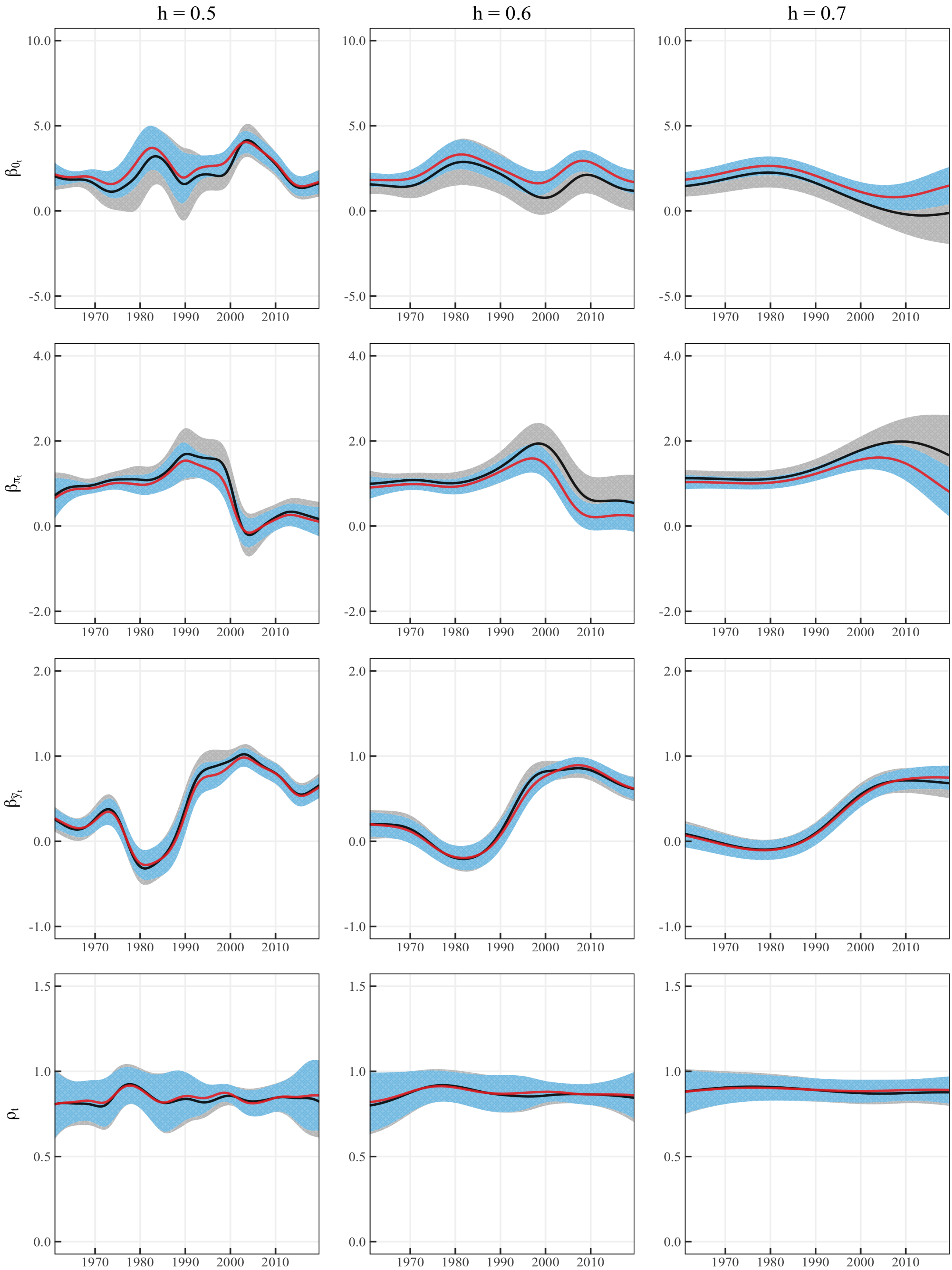

Figure 1. Time-Varying Taylor Rule Estimates (K(x): Gaussian)

Note: Time-varying estimates of equation (13) by TV-OLS (red) and TV-IV (black) with bandwidth parameters h = {0.5,0.6,0.7}. Kernel function is fixed as Gaussian. Instrument selection is detailed in Section 3. Shaded regions in blue (OLS) and grey (IV) correspond to the 90% confidence band.

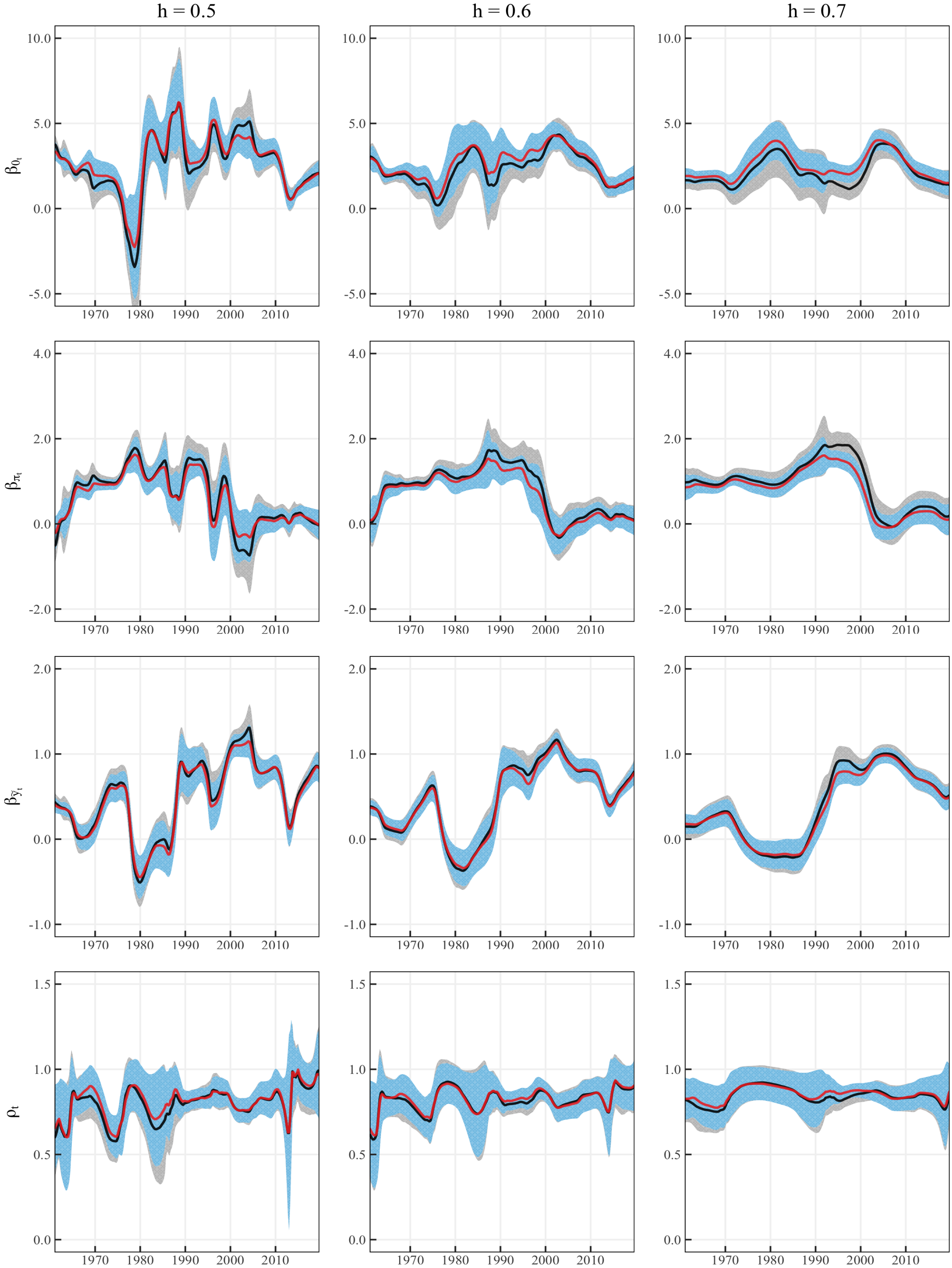

Figure 2. Time-Varying Taylor Rule Estimates (K(x): Epanechnikov)

Note: Time-varying estimates of equation (13) by TV-OLS (red) and TV-IV (black) with bandwidth parameters h = {0.5,0.6,0.7}. Kernel function is fixed as Epanechnikov. Instruments are detailed in Section 3. Shaded regions in blue (OLS) and grey (IV) correspond to the 90% confidence band.

Baseline results using a Gaussian kernel function corroborate the rise in $\beta_{t,\pi}$ during the initial part of the post-Volcker period, before declining substantially at the turn of the century. The priorities on inflation stability seem to stabilise well below unity during the Bernanke-Yellen era. Baseline estimates of $\beta_{t,\tilde{y}}$ reveal a decline in sensitivity to the output gap during the pre-Volcker period, rising substantially thereafter. The priorities on output stability seem to decline once again during the Bernanke-Yellen period. Finally, estimates of the smoothing parameter ρ_t confirm a high degree of inertia that is rising marginally consistently across monetary policy regimes in the United States.

As in Clarida, Gali and Gertler (2000), the Taylor principle is violated during the pre-Volcker period, wherein $\beta_{t,\pi}$ lies just below unity. It holds on average thereafter until the Bernanke-Yellen era, wherein $\beta_{t,\pi}$ falls below unity once again. The Taylor principle is also violated when $\beta_{t,\tilde{y}}$ briefly falls below zero during the initial part of the post-Volcker period. It is worth noting that the turn of the century seems to be a key point of change in the priorities of output and price stability, which is perhaps attributable to the ensuing recessions in the United States. What is most clear is that all the coefficients of the Taylor rule are subject to significant time-variation. These findings therefore encourage the use of techniques that capture structural changes in the parameters of the monetary policy rule so as to track the dynamic preferences of policymakers at all periods.

These results are generally robust to alternative specifications of the bandwidth parameter h as evidenced in Figure 1. As expected, the smoothness is sensitive to bandwidth selection. Notwithstanding, general trends in the estimated coefficients of the Taylor rule prevail. Empirical results seem to also be robust to alternative kernels as evidenced in Figure 2. Whilst the finitely supported Epanechnikov kernel function yields estimates that are evidently less smooth than the infinitely supported Gaussian kernel, results corroborate baseline trends that are themselves robust to alternative specifications of h. Despite this, suboptimal selection of kernel and bandwidth clearly risks under or over-smoothing.

4.2 Optimal bandwidth selection

Whilst there is no strict rule concerning the selection of either bandwidth or kernel function, this research would benefit from the imposition of some optimal criterion. In this paper I follow much of the kernel-based time-series literature and focus primarily on optimal bandwidth selection due to its significance in the estimation process. To achieve this, two such criteria are imposed; the minimisation of (1) an adjusted Akaike Information Criterion (AIC) as in Cai (2007) and (2) a Cross-Validation (CV) function as in Racine and Li (2004). This section briefly outlines each approach before re-estimating the forward-looking Taylor rule with optimal bandwidth parameters. Further details concerning both selection procedures can be found in Appendix B.

As for (1), the optimal bandwidth for a nonparametric kernel regression may be determined by selecting the parameter h that minimises a modified AIC(h) function:

$$\text{AIC}(h) = \log(\hat{\sigma}^2) + \frac{2(T_h + 1)}{T - T_h - 2} \tag{14}$$

where $\hat{\sigma}^2 = T^{-1}\Sigma_{t=1}^{T}(y_t - \hat{y}_t)^2$ and T_h is the trace of the smoothing matrix $\mathbf{H}_h$ (refer to Appendix B.1 for further details). Thus, the optimal bandwidth parameter is a solution to:

$$h^*_{\text{AIC}} = \text{argmin}_{h>0}\text{AIC}(h) \tag{15}$$

As for (2), the optimal bandwidth for a nonparametric kernel regression can also be determined by selecting the parameter h that minimises the following CV(h) function:

$$\text{CV}(h) = \frac{1}{T}\sum_{t=1}^{T}\big(y_t - \hat{g}_{-t}(x_t)\big)^2 I(x_t) \tag{16}$$

where $\hat{g}_{-t}(x_t)$ is the 'leave-one-out' estimator expressed as:

$$\hat{g}_{-t}(x_t) = K_{h,t}^{-1}\sum_{i\neq t}^{T} k_{h,ti}y_i \quad \text{where} \quad K_{h,t} = \sum_{i\neq t}^{T} k_{h,ti} \tag{17}$$

Thus, the optimal bandwidth parameter is a solution to:

$$h^*_{\text{CV}} = \text{argmin}_{h>0}\text{CV}(h) \tag{18}$$

Given numerical optimisation of (16) tends to yields multiple local minima, I evaluate the CV(h) function for a grid of bandwidths and perform a search for the global minimum.

Re-estimation results given optimal bandwidths are presented in Figures 3-4. TV-J and TV-H statistics are reported in Appendix A.3-4. Between the AIC and CV approach, optimal bandwidth parameters for the Gaussian and Epanechnikov kernel seem to average around 0.5 and 0.6 respectively. These results clearly support the range of parameters h selected in this study and corroborate trends identified earlier in the paper given an array of bandwidths. Interestingly, these specifications yield similar results between kernel functions. Additionally, h^* is almost identical in TV-OLS and TV-IV, wherein the latter, optimal bandwidths are calculated at both stages of the estimation. To facilitate the analysis between monetary policy regimes, subsample averages are presented in Appendix C. Whilst this paper adopts two approaches to select h optimally, the optimal choice of bandwidth parameter and kernel function still remains an open problem within the literature, whose solution is left for future research.

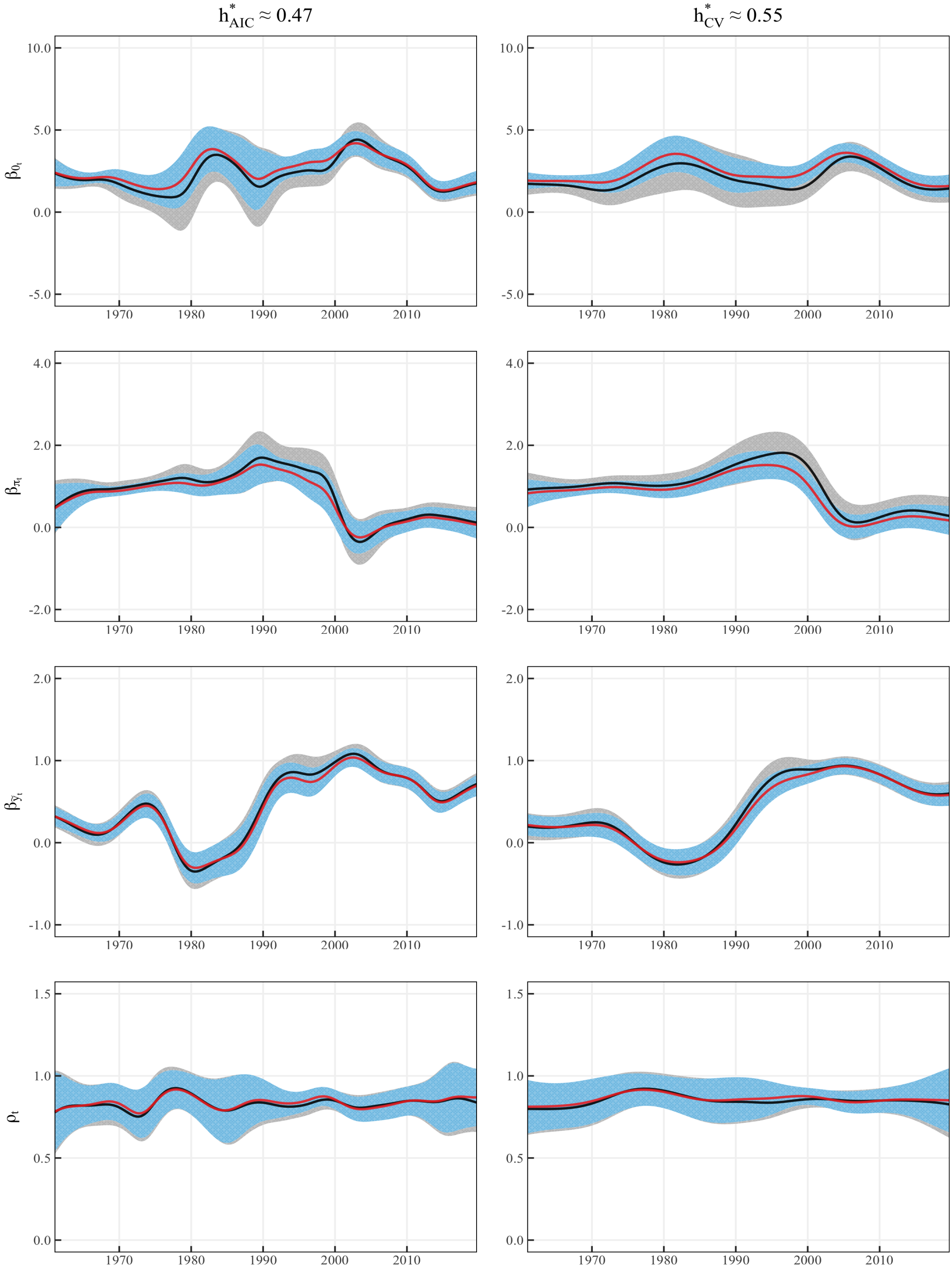

Figure 3. Time-Varying Estimates with Optimal Bandwidths (K(x): Gaussian)

Note: Time-varying estimates of (13) by TV-OLS (red) and TV-IV (black) given $h^*_{AIC} = 0.47$ *and* $h^*_{CV} = 0.55$*. Kernel function is fixed as Gaussian. Instruments are detailed in Section 3.1. Shaded regions in blue (TV-OLS) and grey (TV-IV) correspond to the 90% confidence band.*

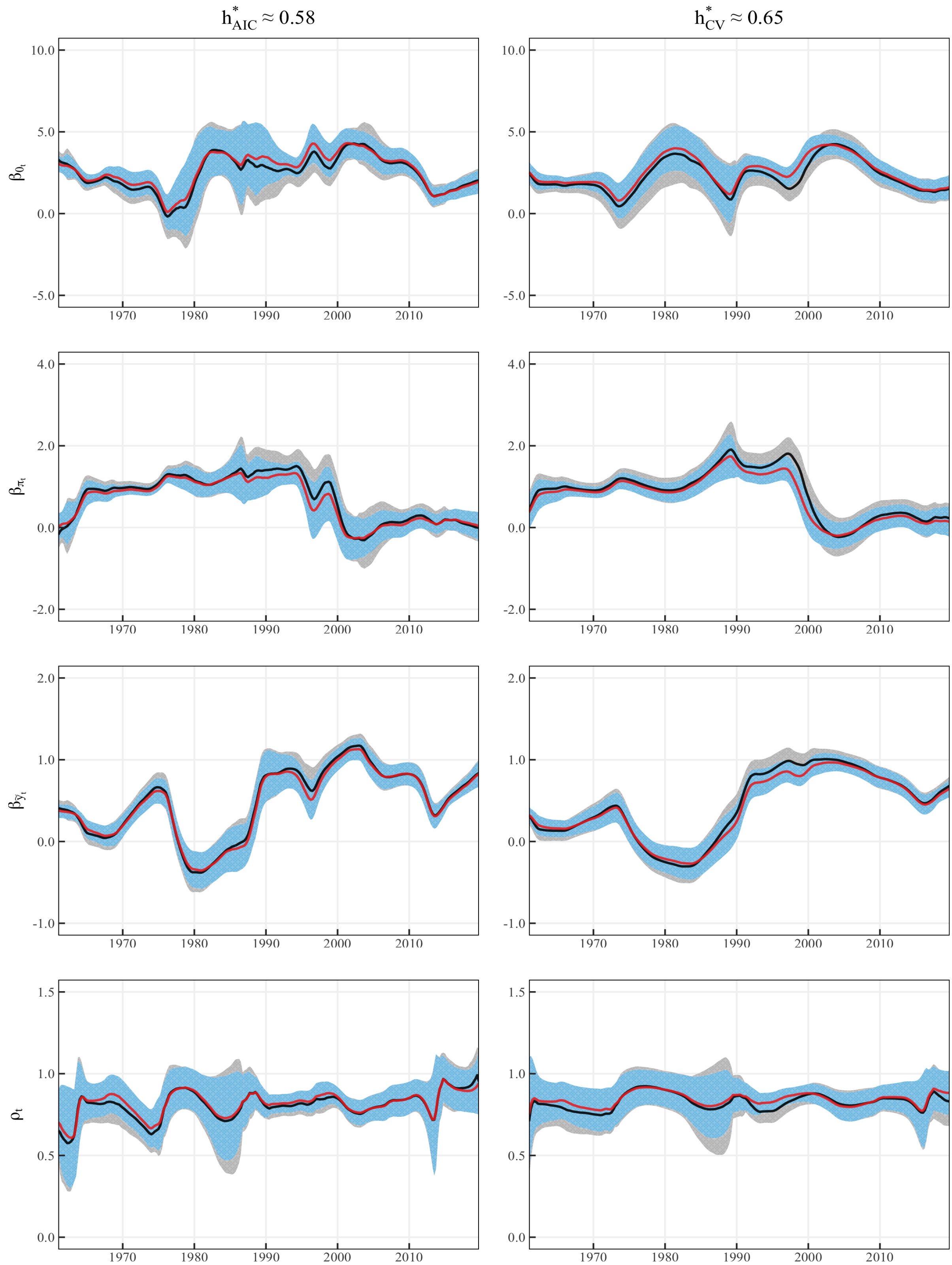

Figure 4. Time-Varying Estimates with Optimal Bandwidths (K(x): Epanechnikov)

Note: Time-varying estimates of (13) by TV-OLS (red) and TV-IV (black) given $h^*_{AIC} = 0.58$ *and* $h^*_{CV} = 0.65$*. Kernel function is fixed as Epanechnikov. Instruments are detailed in Section 3.1. Shaded regions in blue (TV-OLS) and grey (TV-IV) correspond to the 90% confidence band.*

4.3 Implicit inflation targets

Using the theoretical framework in Section 2, we may solve for the time-varying implicit inflation target π_t^* as a function of the time-varying model parameters (refer to Clarida, Gali and Gertler (2000) for a similar exercise in the time-invariant framework):[2]

$$\pi_t^* = \frac{r_t^* - \beta_{0,t}}{\beta_{\pi,t} - 1} \tag{19}$$

The inflation target and natural rate of interest cannot be identified simultaneously. In this regard, I use the benchmark semi-structural Holston, Laubach and Williams (2017) model to estimate the natural rate of interest (r_t^*).

In particular, I specify the following reduced equations of the dynamic IS and Phillips Curve that permit shocks to the output gap $\tilde{y}_t$ and inflation π_t respectively:

$$\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \tfrac{a_r}{2}\textstyle\sum_{j=1}^{2}(r_{t-j} - r_{t-j}^*) + \mu_{\tilde{y},t} \tag{20}$$

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi)\pi_{t-2,4} + b_y \tilde{y}_{t-1} + \mu_{\pi,t} \tag{21}$$

where $\pi_{t-2,4}$ represents the average of the second to fourth lag of inflation, $\mu_{\tilde{y},t}$ and $\mu_{\pi,t}$ denote transitory shocks, and r_t^* captures persistent shocks to the relationship between r_t and $\tilde{y}_t$. The law of motion for the natural rate of interest is defined as follows:

$$r_t^* = g_t + z_t \tag{22}$$

where g_t is the trend growth rate of potential output and z_t is the error term capturing other factors driving the natural rate of interest. In addition, I specify the following transition equations of the state-space model as detailed in Holston, Laubach and Williams (2017):

$$y_t^* = y_{t-1}^* + g_{t-1} + \mu_{y*,t} \tag{23}$$

$$g_t = g_{t-1} + \mu_{g,t} \tag{24}$$

$$z_t = z_{t-1} + \mu_{z,t} \tag{25}$$

where (23) defines log potential output y_t^* as a random walk with drift g that also follows a random walk process specified in equation (24). Finally, (25) captures the unobserved component of the natural rate, which itself is also a random walk.[3]

[2] Note that π_t^* is nonlinear in these coefficients and thus the error must be propagated accordingly. I use the delta method to achieve this and use the associated standard errors to construct confidence bands.

[3] I assume here that the error terms in each transition equation are contemporaneously uncorrelated and normally distributed with variance parameters; $\mu_{y*,t} \sim (N, \sigma_{y*}^2)$, $\mu_{g,t} \sim (N, \sigma_g^2)$ and $\mu_{z,t} \sim (N, \sigma_z^2)$.

The resulting state-space system is estimated using a maximum likelihood procedure that imposes restrictions estimated by the median unbiased estimator (Stock and Watson, 1998) at each stage of the estimation process (see Appendix D for further details).[4]

One-sided Kalman filter estimates of the long-run equilibrium interest rate are presented in Figure 5. As is well documented in the literature, estimates of r-star exhibit secular declination towards the zero-lower neutral bound. As these estimates are merely an intermediate input, this paper does not attempt to explain such decline. Instead, I simply note that consistent with the literature, standard errors associated with estimates of r-star are large and attributable to filter and parameter uncertainty in estimating the latent variables of the state-space model (see Appendix D for complete parameter estimates).

Estimates of the implicit inflation target computed using r-star are presented in Figure 6 for all specifications of TV-IV between 1990:1-2019:4. Estimates prior to this period are characterised by sharp volatility for reasons that likely pertain to when the FED began targeting inflation, which only began implicitly in the mid 1990's and explicitly in 2012 (Famiglietti and Garriga, 2021). To evaluate the stance of policymakers, this paper concerns itself with the last two and a half decades in which there exists some benchmark target for inflation. Estimates over this horizon allow us to determine explicitly the extent to which the Federal Reserve has implicitly deviated from the 2% target inflation rate set out in both its implicit and explicit monetary policy mandate.

These results confirm that monetary policymakers have indeed been targeting an inflation rate in the region of 2% since the 1990's. Across most combinations of kernel and bandwidth, we observe significant volatility in the targeting of inflation circa the crisis of 2001. Barring this exception, the Federal Reserve seems to deviate little on average from its mandated target. In fact, volatility in the implied inflation target seems to be limited to periods prior to the adoption of an explicit target, implying some discretion in monetary policy during the era of an implicit mandate. Finally, I note that in most instances policymakers seems to generally favour marginally under-targeting inflation, suggesting policy has been slightly tighter than optimal over the last few decades.

Large confidence bands associated with estimates of the inflation target are clearly due to the fact that they are a function of r-star. As I use the Kalman filter to compute the natural rate of interest, much of the propagated uncertainty is from filter and parameter uncertainty. Whilst alternative measures may resolve this (such as moving averages of the real federal funds rate), persistent shocks that are not resolved in shorter frequencies are clearly problematic in a time-varying setting. In this regard, Kalman filter estimates of r-star are perhaps more suitable for computing the time paths of implicit inflation targets.

[4]Confidence intervals for these estimates and their standard errors are calculated using a Monte Carlo procedure under constraints, which accounts for filter and parameter uncertainty (see Hamilton, 1986).

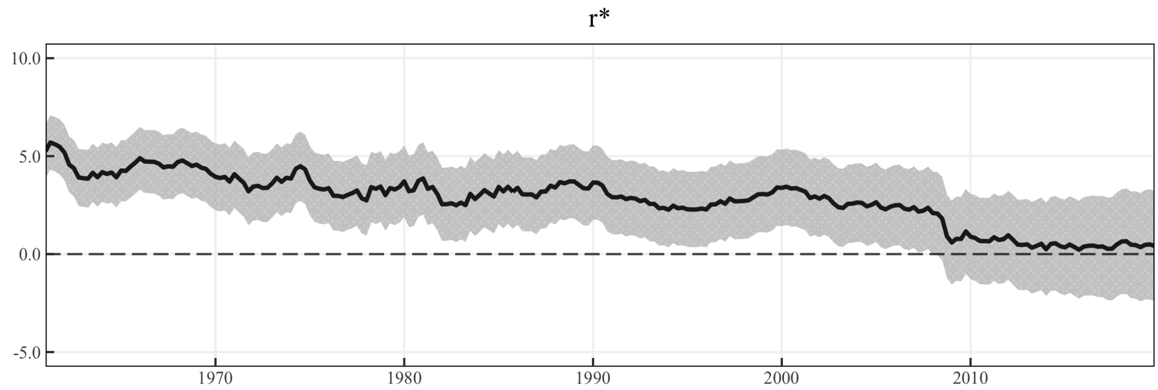

Figure 5. Natural Rate of Interest

Note: HLW (2017) model estimates of the natural rate of interest (r-star). Shaded regions in grey correspond to the 90% confidence band calculated using Monte Carlo (refer to Hamilton, 1986).

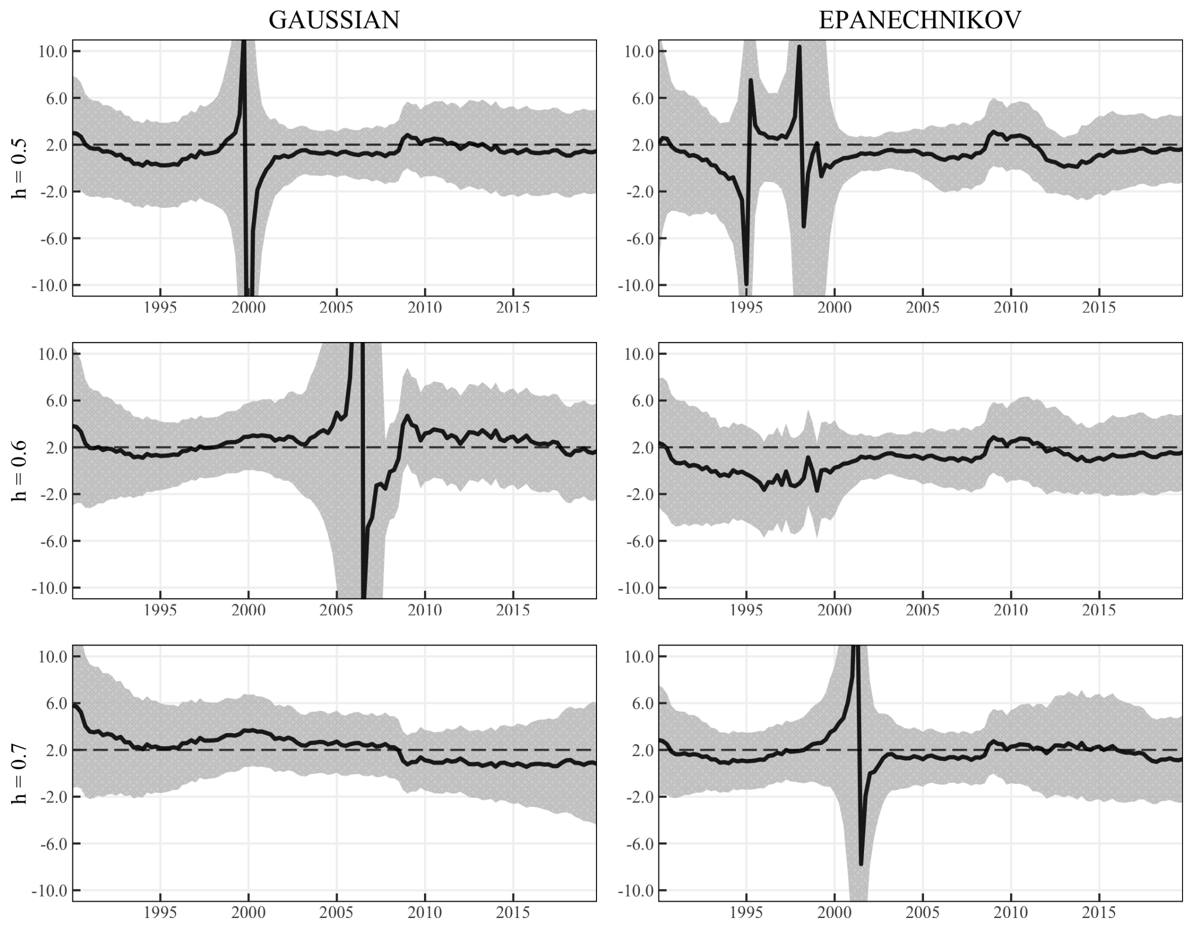

Figure 6. Implicit Time-Varying Inflation Targets

Note: Implied inflation targets derived from estimated coefficients of the Taylor rule by TV-IV given all combinations of kernel function and bandwidth parameter. Shaded regions in grey correspond to the 90% confidence band. Uncertainty is propagated using the delta method.

5 Conclusion

This paper estimates a time-varying random-coefficient specification of a forward-looking Taylor rule for the United States using kernel-weighted time-varying estimators between 1960:1-2019:4. Empirical findings reveal substantial variation in the coefficients of the reaction function that is robust to kernel and bandwidth selection. In particular, the reaction to deviations in inflation from its target has risen since the pre-Volcker period until the latter part of the Volcker-Greenspan era. At the turn of the century, priorities on inflation stability decline substantially and stabilise below unity during the Bernanke-Yellen era. In contrast, priorities on output stability increase at the start of the Volcker-Greenspan period until the turn of the century, declining thereafter during the Bernanke-Yellen era. Furthermore, estimates of the interest rate smoothing parameter are large and marginally increasing across regimes since the pre-Volcker period, indicating substantial inertia in the Federal Resreve's historical conduct of monetary policy.

This paper uses TV-OLS and TV-IV techniques to estimate the monetary policy rule. Whilst the latter attempts to deal with potential issues of endogeneity, results differ minimally between estimators. This support the conclusions of Carvalho, Nechio and Tristo (2021), whom find similar results between estimators in a time-invariant context and argue that as monetary policy shocks explain a small fraction of the variance in typical Taylor rule regressors, bias due to endogeneity is minimal. In addition, two data-driven optimal bandwidth selection procedures are imposed to corroborate the range of parameters used in the kernel-weighted estimation of the Taylor rule. Using bandwidth parameters selected by a modified Akaike Information Criterion and Cross-Validation, this paper consolidates trends identified in the coefficients of the monetary policy rule.

Finally, this paper estimates the long-run equilibrium interest rate (r-star) to compute the time-varying implied inflation target from the coefficients of the Taylor rule. Implicit beliefs about the inflation target in the conduct of monetary policy suggest that the Federal Reserve has indeed pursued an average 2% target over the last few decades. Much of the volatility in this implied rate derived from the Taylor rule is concentrated in the era of an implicit inflation targeting regime and during recessions circa the turn of the century, after which policymakers deviate little from the inflation target set out in the Federal Reserve's mandate. This suggests some discretion in the determination of policy rates prior to the introduction of an explicitly binding rate. Finally, monetary policymakers generally seem to systematically, albeit marginally, under-target inflation, implicit in their determination of policy rates since the inception of an implicit and explicit inflation target.

References

Boivin, J. (2006) 'Has U.S. Monetary Policy Changed? Evidence from Drifting Coefficients and Real-Time Data,' *Journal of Money, Credit and Banking*, 38(5), 1149-1173.

Cai, Z. (2007) 'Trending time-varying coefficient time series models with serially correlated errors,' *Journal of Econometrics*, 136(1), 163-188.

Carlos, C., Nechio, F. and Tiago, T. (2021) 'Taylor rule estimation by OLS,' *Journal of Monetary Economics*, 124, 140-154.

Clarida, R., Galí, J. and Gertler, M. (2000) 'Monetary policy rules and macroeconomic stability: evidence and some theory,' *Quaterly Journal of Economics*, 115(1), 147-180.

Cogley, T. and Sargent, T. (2001) 'Evolving Post-World War II U.S. Inflation Dynamics,' *National Bureau of Economic Research, Macroeconomics Annual*, 16(1), 331-373.

Coibion, O. and Gorodnichenko, Y. (2011) 'Monetary Policy, Trend Inflation, and the Great Moderation: An Alternative Interpretation,' *American Economic Review*, 101(1), 341-70.

Giraitis, L., Kapetanios, G. and Yates, T. (2014) 'Inference on stochastic time-varying coefficient models,' *Journal of Econometrics*, 179(1), 46-65.

Giraitis, L., Kapetanios, G. and Yates, T. (2018) 'Inference on multivariate heteroscedastic time varying random coefficient models,' *Journal of Time Series Analysis*, 39(2), 129-149.

Giraitis, L., Kapetanios, G. and Marcellino, M. (2021) 'Time-varying instrumental variable estimation,' *Journal of Econometrics*, 224(2), 394-415.

Hamilton, J.D. (1986) 'A Standard Error for the Estimated State Vector of a State Space Model', *Journal of Econometrics*, 33, pp.387-397.

Holston, K., Laubach, T. and Williams, J. (2017) 'Measuring the natural rate of interest: International trends and determinants', *Journal of International Economics*, 108(1), pp.S59-S75.

Hurvich, C.M., Simonoff, J.S. and Tsai, C. (1998) 'Smoothing Parameter Selection in Nonparametric Regression Using an Improved Akaike Information Criterion,' *Journal of the Royal Statistical Society. Series B (Statistical Methodology)*, 60(2), 271-293.

Judd, J.P. and Rudebusch, G. (1998) 'Taylor's rule and the Fed, 1970-1997,' *Economic Review*, Federal Reserve Bank of San Francisco, 3-16.

Kalman, R. (1960) 'A New Approach to Linear Filtering and Prediction Problems,' *American Society of Mechanical Engineers Journal of Basic Engineering*, 82, 35-45.

Kim, C-J. and Nelson, C. (2006) 'Estimation of a Forward-Looking Monetary Policy Rule: A Time-Varying Parameter Model Using Ex-Post Data,' *Journal of Monetary Economics*, 53(8), 1949-1966.

Orphanides, A. (2004) 'Monetary policy rules, macroeconomic stability, and inflation: A view from the trenches,' *Journal of Money, Credit and Banking*, 36(2), 151-175.

Racine, J. and Qi, L. (2004) 'Nonparametric estimation of regression functions with both categorical and continuous data,' *Journal of Econometrics*, 119(1), 99-130.

Sims, C. and Zha, T. (2006) 'Were There Regime Switches in U.S. Monetary Policy?' *American Economic Review*, 96(1), 54-81.

Stock, J. and Watson, M. (1998) 'Median unbiased estimation of coefficient variance in a time-varying parameter model', *American Statistical Association*, 93(441), pp.349-358.

Taylor, J. (1993) 'Discretion versus policy rules in practice,' *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214.

Taylor, J. (1999) 'A Historical Analysis of Monetary Policy Rules,' NBER Chapters, in: Monetary Policy Rules. 319-348. National Bureau of Economic Research, Inc.

Trehan, B. and Wu, T. (2007) 'Time-varying equilibrium real rates and monetary policy analysis,' *Journal of Economic Dynamics and Control*, 31(5), 1584-1609.

Woodford, M. (2001) 'The Taylor Rule and Optimal Monetary Policy.' *American Economic Review*, 91(2), pp.232-237.

Appendix

A. Test Statistics

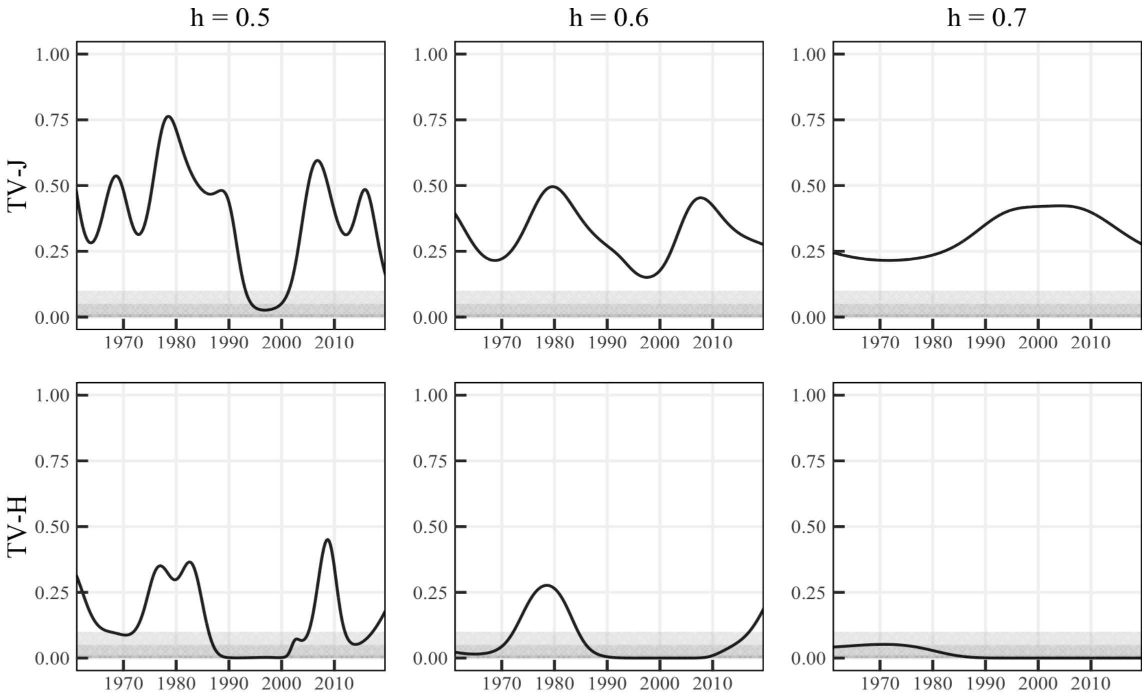

Figure A.1. Time-varying J and H statistics (p-values) (K(x): Gaussian)

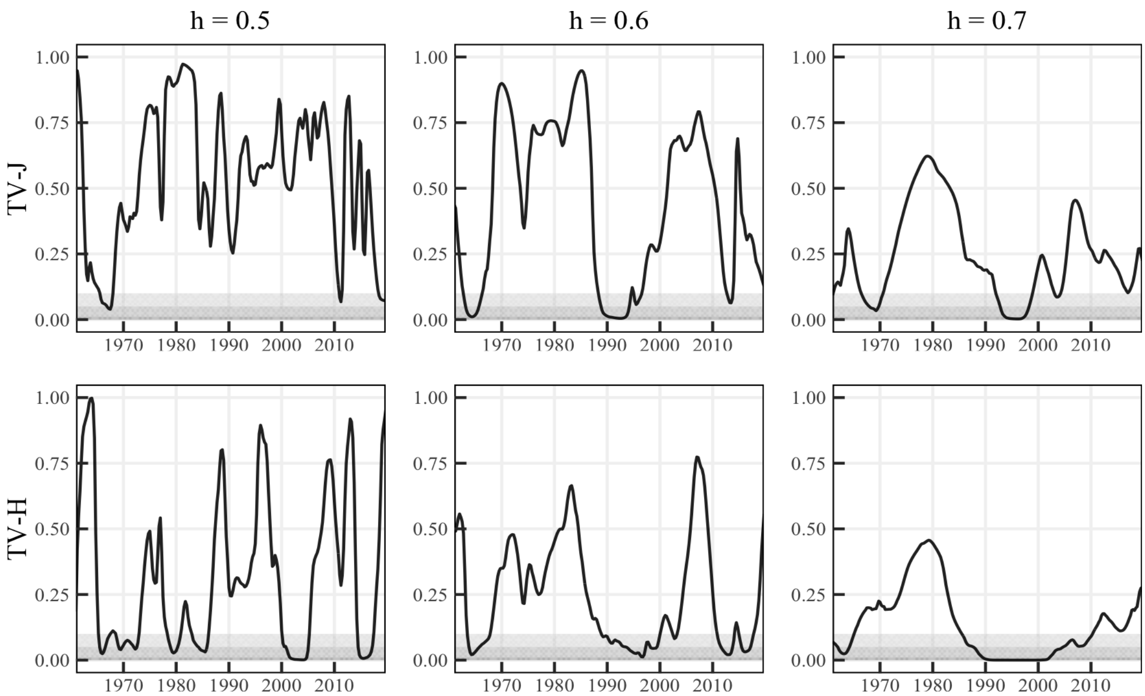

Figure A.2. Time-varying J and H statistics (p-values) (K(x): Epanechnikov)

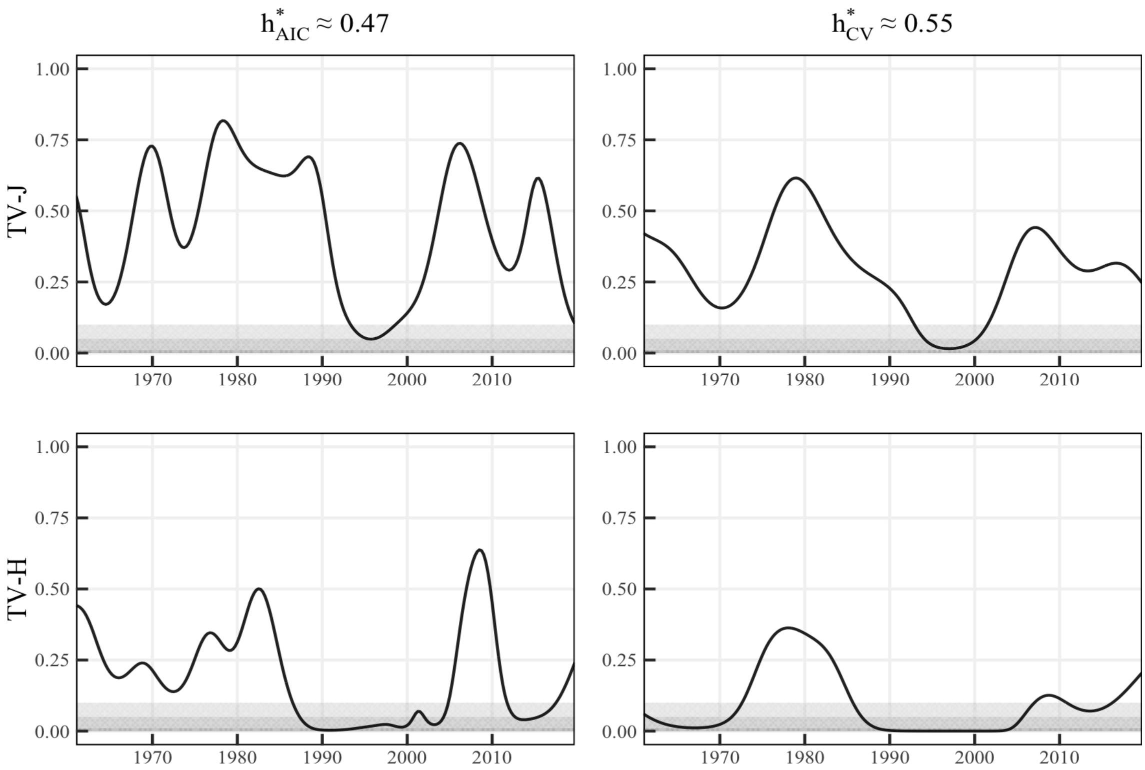

Figure A.3. Time-varying J and H statistics (p-values) (K(x): Gaussian)

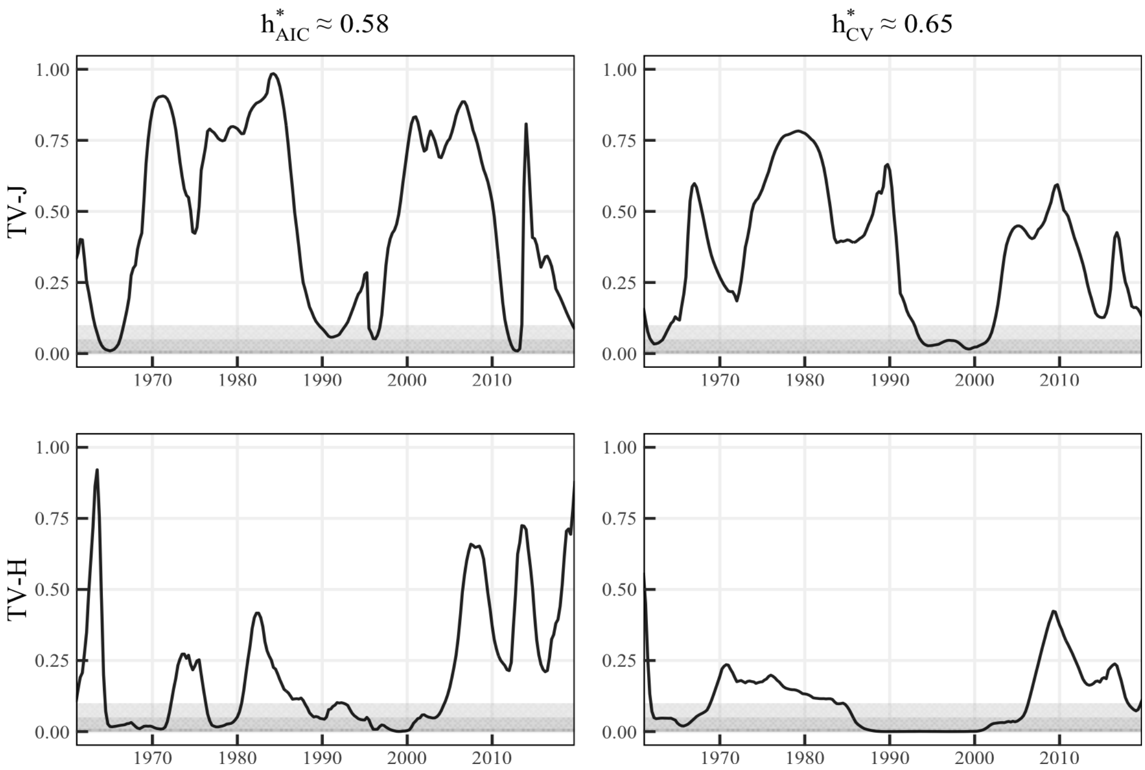

Figure A.4. Time-varying J and H statistics (p-values) (K(x): Epanechnikov)

B. Optimal Bandwidths

B.1 Modified Akaike Information Criterion

To define $\mathbf{H}_h$, first note estimators (7) and (10) may be written as:

$$\bar{\boldsymbol{\beta}}_t = \left(\mathbf{w}_t'\mathbf{x}\right)^{-1}\mathbf{w}_t'\mathbf{y} \tag{B.1}$$

where $\mathbf{x}$ and $\mathbf{w}_t$ are $T \times k$ matrices such that:

$$\mathbf{w}_t^{\text{OLS}} = \mathbf{D}_t\mathbf{x} \quad \text{and} \quad \mathbf{w}_t^{\text{IV}} = \mathbf{D}_t\hat{\mathbf{x}} \tag{B.2}$$

where $\mathbf{D}_t$ is a $T \times T$ diagonal matrix such that $(\mathbf{D}_t)_{i,i} = k_{ti}$.

We may rewrite the model specified in equations (6) and (8) as:

$$\hat{\mathbf{y}} = \mathbf{H}_h\mathbf{y} \tag{B.3}$$

where $\mathbf{H}_h$ is a $T \times T$ smoothing matrix defined as:

$$\mathbf{H}_h = \mathbf{D}_t\mathbf{G}_t \quad \text{where} \quad \mathbf{G}_t = \left(\mathbf{w}_t'\mathbf{x}\right)^{-1}\mathbf{w}_t' \tag{B.4}$$

B.2 Least Squares Cross-Validation

Consider the following error criterion for the estimator $\hat{g}(\cdot)$:

$$\text{ISE}[\hat{g}(\cdot)] = \int (\hat{g}(x) - g(x))^2 f(x)dx \tag{B.5}$$

where the objective function to be minimised is defined as:

$$\text{MISE}[\hat{g}(\cdot)|X_t] = \mathbb{E}[\text{ISE}[\hat{g}(\cdot)]|X_t] = \int \mathbb{E}[(\hat{g}(x) - g(x))^2|X_t]f(x)dx \tag{B.6}$$

The idea here is to use the sample twice in a cross-validatory way; to construct the estimator and to evaluate its performance, such that data used for the former is not used for the latter. The simplest approach is to compare Y_t with the *leave-one-out* estimate of g, that is computed barring the t-th datum (X_t, T_t), yielding the cross-validation error function:

$$\text{CV}(h) = \frac{1}{T}\sum_{t=1}^{T}\left(Y_t - \hat{g}_{-t}(X_t)\right)^2 \tag{B.7}$$

Each time one observation is omitted, the remaining data points are used to fit the data and predict the omitted value. Cross-validation is therefore a method to estimate the prediction error, which is an approximation of the error criterion. Given minimisation may yield several local minima, the CV loss curve is graphed to identify the global solution.

C. Subsample Averages

Table C.1. Subsample averages of TV-OLS Estimates (K(x): Gaussian)

	Pre-Volcker 1960:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2018:1
$\bar{\beta}_{\pi,t}$	0.91***	0.97***	0.68***	0.16
	(0.12)	(0.21)	(0.21)	(0.17)
$\bar{\beta}_{\tilde{y},t}$	0.18***	0.43***	0.73***	0.70***
	(0.05)	(0.10)	(0.09)	(0.07)
$\bar{\rho}_t$	0.84***	0.85***	0.84***	0.84***
	(0.08)	(0.06)	(0.05)	(0.07)

*Note: Significance at the 90/95/99% confidence level are denoted by */**/*** respectively. Robust errors reported in parentheses. Kernel is Gaussian. Bandwidth is the average of the two optimal criteria* $\bar{h}^* \approx 0.5$.

Table C.2. Subsample averages of TV-IV Estimates (K(x): Gaussian)

	Pre-Volcker 1960:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2017:4
$\bar{\beta}_{\pi,t}$	0.98***	1.12***	0.82***	0.21
	(0.15)	(0.28)	(0.28)	(0.21)
$\bar{\beta}_{\tilde{y},t}$	0.18***	0.47***	0.77***	0.71***
	(0.04)	(0.11)	(0.10)	(0.09)
$\bar{\rho}_t$	0.84***	0.84***	0.84***	0.84***
	(0.08)	(0.07)	(0.05)	(0.07)
$\bar{J}_t$	0.46	0.32	0.28	0.44
$\bar{H}_t$	0.19	0.10	0.10	0.19

*Note: Instruments; four lags of inflation, output gap, federal funds rate, money growth, commodity price inflation and long-short spread. Significance at the 90/95/99% confidence level are denoted by */**/*** respectively. Average robust standard errors are reported in parentheses. Average p-values are reported for test statistics. Kernel is Gaussian. Bandwidth is the average of the two optimal criteria* $\bar{h}^* \approx 0.5$.

Table C.3. Subsample averages of TV-OLS Estimates (K(x): Epanechnikov)

	Pre-Volcker 1960:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2018:1
$\bar{\beta}_{\pi,t}$	0.85***	0.85***	0.55***	0.14
	(0.14)	(0.25)	(0.19)	(0.17)
$\bar{\beta}_{\tilde{y},t}$	0.25***	0.49***	0.78***	0.66***
	(0.08)	(0.12)	(0.11)	(0.08)
$\bar{\rho}_t$	0.80***	0.82***	0.83***	0.85***
	(0.09)	(0.07)	(0.05)	(0.08)

*Note: Significance at the 90/95/99% confidence level are denoted by */**/*** respectively. Robust errors reported in parentheses. Kernel is parabolic. Bandwidth is the average of the two optimal criteria* $\bar{h}^* \approx 0.6$.

Table C.4. Subsample averages of TV-IV Estimates (K(x): Epanechnikov)

	Pre-Volcker 1960:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2017:4
$\bar{\beta}_{\pi,t}$	0.90***	1.00***	0.69***	0.21
	(0.15)	(0.29)	(0.23)	(0.20)
$\bar{\beta}_{\tilde{y},t}$	0.24***	0.52***	0.81***	0.67***
	(0.08)	(0.13)	(0.11)	(0.08)
$\bar{\rho}_t$	0.79***	0.82***	0.82***	0.85***
	(0.09)	(0.07)	(0.05)	(0.08)
$\bar{J}_t$	0.48	0.42	0.33	0.45
$\bar{H}_t$	0.28	0.21	0.18	0.26

*Note: Instruments; four lags of inflation, output gap, federal funds rate, money growth, commodity price inflation and long-short spread. Significance at the 90/95/99% confidence level are denoted by */**/*** respectively. Average robust standard errors are reported in parentheses. Average p-values are reported for test statistics. Kernel is parablic. Bandwidth is the average of the two optimal criteria* $\bar{h}^* \approx 0.6$.

D. Natural Rate of Interest

Stage I Specification:

$$\mathbf{y}_t = \Big[y_t, \pi_t\Big]' \quad \mathbf{x}_t = \Big[y_{t-1}, y_{t-2}, \pi_{t-1}, \pi_{t-2,4}\Big]' \quad \xi_t = \Big[y_t^*, y_{t-1}^*, y_{t-2}^*\Big]'$$

$$\mathbf{H}' = \begin{bmatrix} 1 & -a_{y,1} & -a_{y,2} \\ 0 & -b_y & 0 \end{bmatrix} \quad \mathbf{A}' = \begin{bmatrix} a_{y,1} & a_{y,2} & 0 & 0 \\ b_y & 0 & b_\pi & 1-b_\pi \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} \sigma_{y*}^2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_1 = \Big[a_{y,1}, a_{y,2}, b_\pi, b_y, g, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*}\Big]$$

Stage II Specification:

$$\mathbf{y}_t = \Big[y_t, \pi_t\Big]' \quad \mathbf{x}_t = \Big[y_{t-1}, y_{t-2}, r_{t-1}, r_{t-2}, \pi_{t-1}, \pi_{t-2,4}, 1\Big]' \quad \xi_t = \Big[y_t^*, y_{t-1}^*, y_{t-2}^*\Big]'$$

$$\mathbf{H}' = \begin{bmatrix} 1 & -a_{y,1} & -a_{y,2} & a_g \\ 0 & -b_y & 0 & 0 \end{bmatrix} \quad \mathbf{A}' = \begin{bmatrix} a_{y,1} & a_{y,2} & \frac{a_r}{2} & \frac{a_r}{2} & 0 & 0 & a_0 \\ b_y & 0 & 0 & 0 & b_\pi & 1-b_\pi & 0 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} \sigma_{y*}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (\lambda_g \sigma_{y*})^2 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_2 = \Big[a_{y,1}, a_{y,2}, a_r, a_0, a_g, b_\pi, b_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*}\Big]$$

Stage III Specification:

$$\mathbf{y}_t = \Big[y_t, \pi_t\Big]' \quad \mathbf{x}_t = \Big[y_{t-1}, y_{t-2}, r_{t-1}, r_{t-2}, \pi_{t-1}, \pi_{t-2,4}\Big]' \quad \xi_t = \Big[y_t^*, y_{t-1}^*, y_{t-2}^*, g_{t-1}, g_{t-2}, z_{t-1}, z_{t-2}\Big]'$$

$$\mathbf{H}' = \begin{bmatrix} 1 & -a_{y,1} & -a_{y,2} & \frac{-a_r}{2} & \frac{-a_r}{2} & \frac{-a_r}{2} & \frac{-a_r}{2} \\ 0 & -b_y & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{A}' = \begin{bmatrix} a_{y,1} & a_{y,2} & \frac{a_r}{2} & \frac{a_r}{2} & 0 & 0 \\ b_y & 0 & 0 & 0 & b_\pi & 1-b_\pi \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} (1+\lambda_g^2)\sigma_{y*}^2 & 0 & 0 & (\lambda_g \sigma_{y*})^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (\lambda_g \sigma_{y*})^2 & 0 & 0 & (\lambda_g \sigma_{y*})^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (\frac{\lambda_z \sigma_{\tilde{y}}}{a_r})^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_3 = \Big[a_{y,1}, a_{y,2}, a_r, b_\pi, b_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*}\Big]$$

Table D.1. Parameter Estimates

Parameter		Standard error	
λ_g	0.053	r^*_{avg}	1.180
λ_z	0.036	y^*_{avg}	1.526
Σa_y	0.943	g_{avg}	0.400
a_r	-0.068		
b_y	0.078		
$\sigma_{\tilde{y}}$	0.339	r^*_{fin}	1.712
σ_π	0.789	y^*_{fin}	2.028
σ_{y*}	0.573	g_{fin}	0.545
σ_g	0.121		
σ_z	0.178		
σ_{r*}	0.215	T	1960:1-2019:4

Note: Estimated parameters of the Holston, Laubach and Williams (2017) model. Average and final standard errors of the natural rate of interest, the natural rate of output and its trend growth are reported in the very last column. Standard errors are calculated as in Hamilton (1986). σ_g is expressed as an annual rate.