Equilibrium Interest Rates and Monetary Policy (Mis)perceptions*

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Abstract

This paper estimates a time-varying random-coefficient forward-looking Taylor rule for the United States using a novel non-parametric time-varying continuously-updating generalised methods of moments estimator. Given time-varying parameters of the policy rule, we derive an explicit time series for the implied natural rate of interest ($r^{\#}$), which we argue is a proxy for the perceptions of monetary policymakers. We evaluate this against the actual natural rate of interest (r^{*}) to measure misperceptions of the long-run equilibrium interest rate. This paper documents the Federal Reserve's historical conduct of monetary policy and explains periods of instability wherein policymakers underestimate or overestimate the natural rate of interest.

Keywords: Equilibrium interest rate, Natural rate of interest, Kalman filter, Generalised methods of moments, Time-varying parameter models, Policy rules, Monetary policy perceptions

JEL classification: C14, C36, C32, E43, E52, E58

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1 Introduction

The natural rate of interest (r^* or r-star) has become increasingly relevant to monetary policy over recent years. In their ambition to stabilise output and inflation, policymakers regularly communicate their assessment of the equilibrium interest rate, and how it has influenced their determination of interest rates. Perceptions of r-star are therefore a key determinant of priorities between inflation and output stability. The Taylor (1993) rule has long been a conventional tool to model this stance. It suggests that policymakers ought to determine policy rates in response to deviations in inflation from its target and output from its potential. In addition, the equilibrating intercept in Taylor-type policy rules yields an implicit estimate of the natural rate of interest that is associated with optimal inflation and output stability. The empirical estimation of reaction functions are well documented in the existing literature and relate to wider discussions including, but not limited to; satisfying the Taylor principle (e.g., Taylor, 1999; Clarida, Galí and Gertler, 2000; Orphanides, 2004), optimal monetary policy design (Woodford, 2001), and interest rate persistence or monetary policy inertia (e.g., Rudebusch, 2002; Coibion and Gorodnichenko, 2012).

This research is less concerned with such thoroughly treated aspects of the existing literature and instead, focuses on the perceptions of policymakers concerning the state of the macroeconomy, which we argue is well proxied for by the time-varying natural rate that is implicit in the conduct of monetary policy. We derive explicit estimates of these implicit beliefs by relaxing the assumptions of time-invariance underlying standard estimation approaches to such rules. In particular, classical formulations of the Taylor rule assume a long-run equilibrium interest rate that is fixed over time in addition to static weights on deviations in inflation from its target and output from its potential. There are strong reasons to suggest neither of these assumptions are true.

As for the former, the assumption that the equilibrium interest rate is constant has been widely challenged. However, in doing so, many studies depart from the standard Taylor rule specification. This is perhaps expected, given the intercept within prototypical policy rules may be interpreted as the perceived natural rate assumed by the central bank in its determination of nominal policy rates (Taylor, 1993; 1999). The estimation of the objective or actual natural rate is therefore independent of a reaction function and based on theoretically justified relationships. For instance, Laubach and Williams (2003) and Holston, Laubach and Williams (2017) implement a semi-structural approach to measure this natural rate of interest, which is derived primarily from variation in growth. Their novel application of the Kalman filter within a multi-stage maximum likelihood procedure reveals a secular decline in the long-run equilibrium interest rate across the advanced world. These general findings have been shown to be robust to a number of empirical extensions and variations (see for instance Clark and Kozicki, 2005; Mésonnier and Renne, 2007; Berger and Kempa, 2014; Lewis and Vazquez-Grande, 2018; Krustev, 2019).

Whilst many similar attempts have been made to measure r-star in the literature (e.g., Barsky et al., 2014; Hamilton et al., 2016; Johannsen and Mertens, 2016; Kiley, 2015; Lubik and Matthes, 2015; Pescatori and Turunen, 2016), the perceived rate assumed by central banks in their conduct of monetary policy has received very little attention. This theoretical rate is particularly important, as it reveals the perception of policymakers concerning the true long-run equilibrium interest rate, the implications of which are extensive. For instance, Ajello et al. (2020; 2021) capture mistakes in these perceptions using scenario-based analysis and find that the welfare cost of overestimating the natural rate of interest is greater than the cost of underestimating it. Whilst highly informative, the authors do not propose a formal methodology to quantify these policy misperceptions. In fact, to the best of our knowledge, no attempts have yet been made to estimate the implied time-varying natural rate of interest arising from the Taylor rule using ex-post data.

In a similar vein, some studies have attempted to measure market perceptions of the monetary policy rule rather than the perceptions of policymakers concerning the state of the macroeconomy. For instance, Hamilton et al. (2011) estimate a market-perceived monetary policy rule using macroeconomic news and find evidence of time-variation in the associated response parameters. Bauer et al. (2022) attempt to capture these perceptions in a time-varying context by estimating a market-perceived policy rule from forecasts using panel techniques and a state-space model (see Carvalho and Nechio (2014) for a similar exercise in a time-invariant setting using household expectations and professional forecasts). Whilst market perceptions are useful in determining policy effectiveness, policymaker perceptions are perhaps of even greater significance to the conduct of monetary policy and are therefore the central focus of this paper.

As for the latter assumption concerning constant reaction coefficients in monetary policy rules, relative weights associated with deviations in output and inflation are likely inconstant due to changing policy objectives and preferences of policymakers, both of which are regularly influenced by new information learnt by the central bank over time (see for instance Favero and Rovelli, 2003). This time-variation is consistent with a number of empirical studies that reveal breaks in the reaction coefficients of the monetary policy rule (e.g., Clarida et al., 2000; Smets and Wouters, 2007; Coibion and Gorodnichenko, 2011; Sims and Zha, 2006).

Despite this significant empirical evidence, few have approached the Taylor rule from a time-varying perspective. Among those that have, the Kalman (1960) filter is often implemented within a maximum likelihood setting. For instance, Boivin (2006) sources real-time ex-ante data, which assumes a constant short-term nominal rate over a given forecast horizon to mitigate endogeneity in the estimation of a forward-looking policy rule with drifting coefficients. Perhaps more closely related to our work, Kim and Nelson (2006) also estimate a forward-looking Taylor rule that allows for time-varying parameters using ex-post data. Issues of potential endogeneity in the estimation are resolved by way of a Heckman-type bias-correcting two-step procedure that accounts for both nonlinearity and heteroskedasticity.

Alternatively, Cogley and Sargent (2001; 2005) implement the Kalman filter in a Vector Auto Regression (VAR) framework to estimate the time-varying parameters of the policy rule (refer to Canova and Gambetti (2004) for a similar exercise within a Structural VAR (SVAR) framework). Aside from the Kalman filter, Orphanides and Williams (2005) also permit time-varying parameters using VAR and OLS techniques to investigate structural changes from the policymakers view in the implementation of monetary policy. Similarly, Sims and Zha (2006) estimate a SVAR model in which they allow for time-variation in the reaction coefficients of the Taylor rule to investigate regime switching. Finally, Owyang and Ramey (2004) use a markov-switching model to measure these shifts in the parameters of the Federal Reserve's policy rule.

Although differing in methodology, each of these studies detect some degree of time-variation in the reaction coefficients of the policy rule, which serves as strong motivation for our research, particularly as time-varying priorities of monetary policymakers imply time-varying perceptions of r-star. However, many of these studies do not convincingly resolve problems of endogeneity and impose numerous restrictions on the estimation process that involve unnecessary complexity when gauging these priorities. In fact, most of these attempts fail to directly estimate a forward-looking policy rule with time-varying random-coefficients, nor exploit these estimated time-varying parameters to reveal the implied time-varying inflation target or implied long-run equilibrium interest rate, which we believe, above all else, is the main novelty in our research.

This is possibly because efficient procedures to mitigate the endogeneity associated with expected variables present within the policy rule have not yet been readily applicable to time-varying random-coefficient models. We implement a methodology that achieves precisely that. This paper extends the framework of Clarida et al. (2000), hearafter CGG, in which ex-post data is employed to estimate a constant forward-looking Taylor rule using instrument variables (IV) and generalised methods of moments (GMM). We outline a methods of moments extension to the kernel-weighted time-varying estimation approach, inspired by Ciu et al. (2023), and developed in a series of published research by Giraitis et al. (2014; 2018; 2021), to estimate a time-varying random-coefficient forward-looking policy rule using ex-post data. Given estimated nonconstant coefficients, we analyse the stance of monetary policy on inflation and output stability across our sample.

Our contribution is therefore twofold; not only do we estimate a time-varying forward-looking Taylor rule, we use these estimated time-varying parameters to derive estimates of the implied natural rate of interest, hereafter $r^{\#}$ or r-hash. We argue that this variable is a strong proxy for the beliefs and perceptions of monetary policymakers on the state of the economy and define monetary policy misperceptions as the absolute deviation in r-hash from r-star. In doing so, we construct a framework to quantify these misperceptions, so as to evaluate the accuracy of monetary policy. If r-hash is exceedingly high, policymakers overestimate r-star, monetary conditions tighten and lead to higher unemployment and lower inflation. Conversely, if r-hash is lower than r-star, conditions loosen and lead to lower unemployment and higher inflation.

Our results reveal substantial time variation in the coefficients of the Taylor rule. In particular, we document varying policy responses to the inflation and output gap, in addition to a high degree of policy inertia. We find that during the pre-Volcker era, in which the Taylor principle is violated, policymakers persistently misperceive the natural rate. In contrast, policymakers become increasingly accurate in their perceptions over the subsequent post-Volcker era, during which the Taylor principle is satisfied, with minimal deviations in r-hash from r-star. This paper also finds evidence to suggest that the Federal Reserve actively sought to correct its underestimation of r-star over the pre-Volcker period, at times overshooting its judgment in the early part of the post-Volcker period. In addition, we find evidence that the implied and actual natural rate of interest are cointegrated, suggesting movements in r-star are ultimately reflected in the beliefs of policymakers.

The remainder of this paper is structured as follows. Section 2 outlines the theoretical framework. Section 3 discusses the data, instruments, kernel functions and bandwidth parameters used in our estimation. Section 4 presents and discusses the main results. Section 5 concludes.

2 Theoretical Framework

2.1 Time-Varying Taylor Rules

To begin, we introduce the specification of the static forward-looking Taylor rule with interest rate smoothing as in CGG (2000). The central bank's target policy rate may be defined as follows:

$$i_t^* = i^\# + \delta_\pi (E[\pi_t|\Omega_t] - \pi^*) + \delta_{\tilde{y}} E[\tilde{y}_t|\Omega_t] \tag{1}$$

where $i^\#$ is the nominal equilibrating interest rate, π^* is the inflation target, π_t is the inflation rate and $\tilde{y}_t$ is the output gap, whose expectation is conditional on the central bank's information set Ω_t in period t. We define a constant parameter $\delta_0 = i^\# - \delta_\pi \pi^*$, to yield the following equation:

$$i_t^* = \delta_0 + \delta_\pi E[\pi_t|\Omega_t] + \delta_{\tilde{y}} E[\tilde{y}_t|\Omega_t] \tag{2}$$

As central banks tend to adjust short-term interest rates in sequential incremental steps, we introduce a smoothing mechanism describing how actual policy rates adjust to the target policy rate:

$$i_t = (1-\rho) i_t^* + \rho(L) i_t + u_t \tag{3}$$

where ρ is the smoothing parameter. To derive a policy rule for the actual interest rate, we substitute the target policy rate (1) into the partial adjustment mechanism (3) to yield the following:

$$i_t = (1-\rho)(\delta_0 + \delta_\pi E[\pi_t|\Omega_t] + \delta_{\tilde{y}} E[\tilde{y}_t|\Omega_t]) + \rho(L) i_t + u_t \tag{4}$$

Under rational expectations, we may derive the following policy reaction function:[1]

$$i_t = (1-\rho)(\delta_0 + \delta_\pi \pi_t + \delta_{\tilde{y}} \tilde{y}_t) + \rho(L) i_t + \varepsilon_t \tag{5}$$

Note that equation (5) may be expressed as a linear function of auxiliary coefficients:[2]

$$i_t = \psi_0 + \psi_\pi \pi_t + \psi_{\tilde{y}} \tilde{y}_t + \psi_\rho(L) i_t + \varepsilon_t \tag{6}$$

Given a set of predetermined instruments $\mathbf{x}_t$ within the information set Ω_t that are assumed uncorrelated with the error term u_t, we may write the following set of orthogonality conditions:

$$E[i_t - (1-\rho)(\delta_0 + \delta_\pi \pi_t + \delta_{\tilde{y}} \tilde{y}_t) - \rho(L) i_t | \mathbf{x}_t] = 0 \tag{7}$$

GMM is used to address potential endogeneity in (5) given stochastic variation in the model coefficients. To contextualise this approach, we first outline the static GMM framework in the notation of Hayashi (2000), which we also estimate in Section 4 before moving to the time-varying case.

[1] Note that the error term $\varepsilon_t = (1-\rho)\Big(\psi_\pi(\pi_t - E[\pi_t|\Omega_t]) + \delta_{\tilde{y}}(\tilde{y}_t - E[\tilde{y}_t|\Omega_t])\Big) + v_t$ is a linear combination of the forecast error of inflation and output and the exogenous error u_t; orthogonal to variables in the information set.

[2] Where $\psi_0 = (1-\rho)\delta_0$, $\psi_\pi = (1-\rho)\delta_\pi$, $\psi_{\tilde{y}} = (1-\rho)\delta_{\tilde{y}}$ and $\psi_\rho = \rho$. We may then estimate a linear regression and extract coefficients from the auxiliary vector $\boldsymbol{\psi}$. Alternatively we may simply estimate the nonlinear representation.

Consider the following model for the federal funds rate:

$$i_t = \mathbf{z}_t'\boldsymbol{\delta} + \varepsilon_t \qquad (t = 1, 2, ..., T) \tag{8}$$

where $\mathbf{z}_t$ is a $L \times 1$ vector of regressors, $\boldsymbol{\delta}$ is a $L \times 1$ coefficient vector, and ε_t is the error term. Given $\mathbf{x}_t$ is a $K \times 1$ vector of predetermined instruments, the exogeneity conditions are given by:

$$E[\mathbf{x}_t(i_t - \mathbf{z}_t'\boldsymbol{\delta})] = \mathbf{0} \quad \text{ or } \quad E(\mathbf{g}) = E[\mathbf{g}(\mathbf{w}_t; \boldsymbol{\delta})] = \mathbf{0} \tag{9}$$

where but for parsimony we define $\mathbf{g} = \mathbf{g}(\mathbf{w}_t; \boldsymbol{\delta}) \equiv \mathbf{x}_t(i_t - \mathbf{z}_t'\boldsymbol{\delta})$. Let $\boldsymbol{\delta}^0$ be the $L \times 1$ parameter vector of interest and consider the system of K simultaneous equations in L unknowns such that:

$$E[\mathbf{g}(\mathbf{w}_t; \boldsymbol{\delta}^0)] = \mathbf{0} \tag{10}$$

Orthogonality in (10) implies that $\boldsymbol{\delta}^0$ is one solution to this system of simultaneous equations. The sample analogue of the population moments $E[\mathbf{g}(\mathbf{w}_t; \boldsymbol{\delta}^0)]$ is the sample mean of the function $\mathbf{g}$ evaluated at some hypothetical parameter vector $\boldsymbol{\delta}$, which may be expressed as follows:

$$\begin{aligned} \mathbf{g}_T(\boldsymbol{\delta}) &\equiv \frac{1}{T}\sum_{t=1}^{T}\mathbf{g}(\mathbf{w}_t; \boldsymbol{\delta}) = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t \cdot (i_t - \mathbf{z}_t'\boldsymbol{\delta}) \\ &= \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t \cdot i_t - \Big(\frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t\mathbf{z}_t'\Big)\boldsymbol{\delta} \equiv \mathbf{s}_{\mathbf{xi}} - \mathbf{S}_{\mathbf{xz}}\boldsymbol{\delta} \end{aligned} \tag{11}$$

where $\mathbf{s}_{\mathbf{xi}}$ and $\mathbf{S}_{\mathbf{xz}}$ are the sample moments of $\boldsymbol{\sigma}_{\mathbf{xi}}$ and $\boldsymbol{\Sigma}_{\mathbf{xz}}$. Given the sample analogue $\mathbf{g}_T(\boldsymbol{\delta}) = \mathbf{0}$, we have that $\mathbf{S}_{\mathbf{xz}}\boldsymbol{\delta} = \mathbf{s}_{\mathbf{xi}}$. If however $K > L$, the model is over-identified, $\mathbf{S}_{\mathbf{xz}}$ is no longer invertible, and we choose $\boldsymbol{\delta}$ optimally by minimising the distance.

The distance between any two K-dimensional vectors $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ is given by the quadratic $(\boldsymbol{\xi} - \boldsymbol{\eta})'\mathbf{W}(\boldsymbol{\xi} - \boldsymbol{\eta})'$, where $\mathbf{W}$ is a symmetric $K \times K$ positive definite weighting matrix. The GMM estimator is therefore a solution to the following minimisation problem:

$$\hat{\boldsymbol{\delta}}(\mathbf{W}) \equiv \text{argmin}_{\boldsymbol{\delta}} J(\boldsymbol{\delta}, \mathbf{W}) \equiv T\mathbf{g}_T(\boldsymbol{\delta})'\mathbf{W}\mathbf{g}_T(\boldsymbol{\delta}) \tag{12}$$

Without loss of generality, if $\mathbf{g}_T(\boldsymbol{\delta})$ is linear in $\boldsymbol{\delta}$, the objective function is quadratic in $\boldsymbol{\delta}$, therefore $J(\boldsymbol{\delta}, \mathbf{W}) = T(\mathbf{s}_{\mathbf{xi}} - \mathbf{S}_{\mathbf{xz}}\boldsymbol{\delta})'\mathbf{W}(\mathbf{s}_{\mathbf{xi}} - \mathbf{S}_{\mathbf{xz}}\boldsymbol{\delta})$. Given ergodicity, stationarity, and the rank condition for identification hold, the minimiser of equation (12) is the celebrated GMM estimator:

$$\hat{\boldsymbol{\delta}}(\mathbf{W}) = (\mathbf{S}_{\mathbf{xz}}'\mathbf{W}\mathbf{S}_{\mathbf{xz}})^{-1}\mathbf{S}_{\mathbf{xz}}'\mathbf{W}\mathbf{s}_{\mathbf{xi}} \tag{13}$$

Choosing weighting matrix $\mathbf{W}$ optimally, minimises the asymptotic variance. Thus, if $\mathbf{W} = \widehat{\mathbf{S}}^{-1}$ such that $\widehat{\mathbf{S}} \rightarrow_p \mathbf{S}$, where $\mathbf{S}$ is the optimal weighting matrix of interest, the estimator satisfying this efficiency condition is the optimal GMM estimator that is a solution to the following problem:

$$\hat{\boldsymbol{\delta}}(\widehat{\mathbf{S}}^{-1}) \equiv \text{argmin}_{\boldsymbol{\delta}} J(\boldsymbol{\delta}, \widehat{\mathbf{S}}^{-1}) \equiv T\mathbf{g}_T(\boldsymbol{\delta})'\widehat{\mathbf{S}}^{-1}\mathbf{g}_T(\boldsymbol{\delta}) \tag{14}$$

What remains is to consistently estimate this optimal weighting matrix. We consider the Continuously Updating (CU) GMM estimator, which estimates matrix $\mathbf{S}$ simultaneously as a function of $\boldsymbol{\delta}$ and is defined as a solution to the following minimisation problem within the static framework:

$$\hat{\boldsymbol{\delta}}\big(\widehat{\mathbf{S}}^{-1}(\boldsymbol{\delta})\big) \equiv \text{argmin}_{\boldsymbol{\delta}} J\big(\boldsymbol{\delta}, \widehat{\mathbf{S}}^{-1}(\boldsymbol{\delta})\big) \equiv T\mathbf{g}_T(\boldsymbol{\delta})'\widehat{\mathbf{S}}^{-1}(\boldsymbol{\delta})\mathbf{g}_T(\boldsymbol{\delta}) \tag{15}$$

Given conditionally heteroskedastic errors, $\widehat{\mathbf{S}}(\boldsymbol{\delta})$ may be written explicitly as:[3]

$$\widehat{\mathbf{S}}(\boldsymbol{\delta}) = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t\mathbf{x}_t'\hat{\varepsilon}_t^2 = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t\mathbf{x}_t'(i_t - \mathbf{z}_t'\boldsymbol{\delta})^2 \tag{16}$$

Assuming an optimally chosen weighting matrix $\mathbf{S}$, the minimised distance asymptotically follows a chi-squared distribution. Hansen's (1982) test of over-identifying restrictions may be written as:

$$J = J\big(\hat{\boldsymbol{\delta}}(\widehat{\mathbf{S}}^{-1}), \widehat{\mathbf{S}}^{-1}\big) = T\mathbf{g}_T\big(\hat{\boldsymbol{\delta}}(\widehat{\mathbf{S}}^{-1})\big)'\widehat{\mathbf{S}}^{-1}\mathbf{g}_T\big(\hat{\boldsymbol{\delta}}(\widehat{\mathbf{S}}^{-1})\big) \to_d \chi^2_{K-L} \tag{17}$$

Consider now a time-varying random-coefficient model for the federal funds rate:

$$\begin{aligned} i_t &= \mathbf{z}_t'\boldsymbol{\delta}_t + \varepsilon_t \qquad (t = 1, 2, ..., T) \\ \mathbf{z}_t &= \boldsymbol{\Phi}_t'\mathbf{x}_t + \mathbf{u}_t \end{aligned} \tag{18}$$

where $\mathbf{z}_t$ is a $k \times 1$ vector of regressors, $\boldsymbol{\delta}_t$ is a $k \times 1$ coefficient vector and ε_t is a scalar error term. $\mathbf{x}_t$ is a $l \times 1$ instrument vector, $\boldsymbol{\Phi}_t'$ is a $k \times l$ coefficient matrix, and $\mathbf{u}_t$ is a $l \times 1$ error vector.[4]

We assume the standard exogeneity conditions also hold in the time-varying case:

$$E[\mathbf{x}_t\varepsilon_t] = 0 \qquad E[\mathbf{x}_t\mathbf{u}_t'] = 0 \tag{19}$$

This paper uses a kernel-weighted approach inspired by Giraitis et al. (2021) and Ciu et al. (2023) to estimate the time-path of our coefficient vector $\boldsymbol{\delta}_t$. In this paper, we adapt the GMM framework, in which the IV estimator is a special case, by constructing the kernel-weighted sample moments:

$$\mathbf{g}_t(\boldsymbol{\delta}_t) = K_t^{-1}\sum_{i=1}^{T}k_{it}\mathbf{x}_t(i_t - \mathbf{z}_t'\boldsymbol{\delta}_t) \quad \text{where} \quad K_t = \sum_{i=1}^{T}k_{it} \tag{20}$$

Sample moments of $\boldsymbol{\sigma_{xy}}$ and $\boldsymbol{\Sigma_{xz}}$ are weighted analogously:

$$\mathbf{s}_{t,\mathbf{xi}} = K_t^{-1}\sum_{i=1}^{T}k_{it}\mathbf{x}_t i_t \quad \text{and} \quad \mathbf{S}_{t,\mathbf{xz}} = K_t^{-1}\sum_{i=1}^{T}k_{it}\mathbf{x}_t\mathbf{z}_t' \tag{21}$$

We define the kernel $k_{it} = K\big(\frac{|i-t|}{H}\big)$, wherein $H = T^h$ is the bandwidth. $K(x)$ is a non-negative continuous bounded kernel function with a bounded first order derivative such that $\int K(x)\partial x = 1$.

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[3] If the population moment conditions are ergodic-stationary but serially correlated, the time-varying conditionally heteroskedastic and autocorrelated consistent estimate may be written generally as: $\widehat{\mathbf{S}}_{\text{HAC}} = T^{-1}\Sigma_{i=1}^{t-1}w_{it}(\tilde{\boldsymbol{\Gamma}}_i(\boldsymbol{\delta}) + \tilde{\boldsymbol{\Gamma}}_i'(\boldsymbol{\delta}))$ where w_{ti} are kernel weights given a bandwidth parameter and $\tilde{\boldsymbol{\Gamma}}_i(\boldsymbol{\delta}) = T^{-1}\Sigma_{s=i+1}^{t}\mathbf{g}_t(\tilde{\boldsymbol{\delta}})\mathbf{g}_{s-i}(\boldsymbol{\delta})'$.

[4] Refer to Giraitis et al. (2021) and Ciu et al. (2023) for more details concerning the conditions on $\boldsymbol{\delta}_t$ and $\boldsymbol{\Phi}_t$.

Examples of such kernels with finite support include, but are not limited to:

$$K(x) = (3/4)(1 - x^2)I(|x| \leqslant 1) \qquad \text{Epanechnikov kernel}$$

If $K(x)$ has infinite support, we assume in addition that $K(x) \leqslant Ce(-cx^2), |\dot{K}(x)| \leqslant C(1 + x^2)^{-1}, x \geqslant 0$, for some $C > 0$ and $c > 0$. Examples of such kernels with infinite support include:

$$K(x) = (1/\sqrt{2\pi})e^{-x^2/2} \qquad \text{Gaussian kernel}$$

The Time-Varying GMM (TV-GMM) estimator is therefore the minimiser of the time-varying objective function $J_t(\boldsymbol{\delta}_t, \mathbf{W}_t) \equiv \mathbf{g}_t(\boldsymbol{\delta}_t)'\mathbf{W}_t\mathbf{g}_t(\boldsymbol{\delta}_t)$ given a kernel function and bandwidth parameter:

$$\hat{\boldsymbol{\delta}}_t(\mathbf{W}_t) = (\mathbf{S}'_{t,\mathbf{xz}}\mathbf{W}_t\mathbf{S}_{t,\mathbf{xz}})^{-1}\mathbf{S}'_{t,\mathbf{xz}}\mathbf{W}_t\mathbf{s}_{t,\mathbf{xi}} \tag{22}$$

For $\mathbf{W}_t = \widehat{\mathbf{S}}_t^{-1}$, such that $\widehat{\mathbf{S}}_t \rightarrow_p \mathbf{S}_t$, the optimal TV-GMM estimator is given by:

$$\hat{\boldsymbol{\delta}}_t\big(\widehat{\mathbf{S}}_t^{-1}\big) \equiv \text{argmin}_{\boldsymbol{\delta}_t} J_t\big(\boldsymbol{\delta}_t, \widehat{\mathbf{S}}_t^{-1}\big) \equiv \mathbf{g}_t(\boldsymbol{\delta}_t)'\widehat{\mathbf{S}}_t^{-1}\mathbf{g}_t(\boldsymbol{\delta}_t) \tag{23}$$

whose sample analogue may be written explicitly as follows:

$$\begin{aligned} \hat{\boldsymbol{\delta}}_t = &\Big(\big(\sum_{i=1}^{T} k_{it}\mathbf{z}_t\mathbf{x}_t'\big)\big(\sum_{i=1}^{T} k_{it}\mathbf{x}_t\mathbf{x}_t'\hat{\varepsilon}_t^2\big)^{-1}\big(\sum_{i=1}^{T} k_{it}\mathbf{x}_t\mathbf{z}_t'\big)\Big)^{-1} \\ &\times\big(\sum_{i=1}^{T} k_{it}\mathbf{z}_i\mathbf{x}_t'\big)\big(\sum_{i=1}^{T} k_{it}\mathbf{x}_t\mathbf{x}_t'\hat{\varepsilon}_t^2\big)^{-1}\big(\sum_{i=1}^{T} k_{it}\mathbf{x}_t i_t\big) \end{aligned} \tag{24}$$

The TV CU-GMM estimator may be defined as the solution to the problem:

$$\hat{\boldsymbol{\delta}}_t\big(\widehat{\mathbf{S}}_t^{-1}(\boldsymbol{\delta}_t)\big) \equiv \text{argmin}_{\boldsymbol{\delta}_t} J_t\big(\boldsymbol{\delta}_t, \widehat{\mathbf{S}}_t^{-1}(\boldsymbol{\delta}_t)\big) \equiv \mathbf{g}_t(\boldsymbol{\delta}_t)'\widehat{\mathbf{S}}_t^{-1}(\boldsymbol{\delta}_t)\mathbf{g}_t(\boldsymbol{\delta}_t) \tag{25}$$

The time-varying optimal weighting matrix given conditional heteroskedasticity is:[5]

$$\widehat{\mathbf{S}}_t(\boldsymbol{\delta}_t) = K_t^{-1}\sum_{i=1}^{T} k_{it}\mathbf{x}_t\mathbf{x}_t'\hat{\varepsilon}_t^2 = K_t^{-1}\sum_{i=1}^{T} k_{it}\mathbf{x}_t\mathbf{x}_t'(i_t - \mathbf{z}_t'\boldsymbol{\delta}_t)^2 \tag{26}$$

Finally, we outline the general time-varying specification of Hansen's J-test:

$$\begin{aligned} &J_T = \Big(\widehat{\boldsymbol{\Omega}}_2^{-\frac{1}{2}}\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\boldsymbol{V}_{T,t}\Big)'\Big(\widehat{\boldsymbol{\Omega}}_2^{-\frac{1}{2}}\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\boldsymbol{V}_{T,t}\Big) \rightarrow_d \chi^2_{K-L} \\ &\text{where} \quad \boldsymbol{\Omega}_2 = \text{plim}_{T\rightarrow\infty}\text{Var}\big(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\boldsymbol{V}_{T,t}\big) \quad \text{and} \quad \boldsymbol{V}_{T,t} = \tilde{\mathbf{S}}_t^{-\frac{1}{2}}K_{2,t}^{-\frac{1}{2}}\sum_{i=1}^{T}k_{it}\mathbf{g}_t(\hat{\boldsymbol{\delta}}_t) \end{aligned} \tag{27}$$

[5] Similarly, if moments are serially correlated, the time-varying heteroskedastic and autocorrelated consistent estimate $\widehat{\mathbf{S}}_t(\boldsymbol{\delta}_t)$ is: $\widehat{\mathbf{S}}_t(\boldsymbol{\delta}_t) = \boldsymbol{\Sigma}_{s=-T}^{T} w_{it}\hat{\Gamma}(s)$, where $\hat{\boldsymbol{\Gamma}}(s) = K_{2,t}^{-1}\Sigma_{i=1}^{T-s}k_{it}k_{i+s,t}\big(\mathbf{g}_s(\boldsymbol{\delta}_t) - T^{-1}\Sigma_{s=1}^{T}\mathbf{g}_s(\boldsymbol{\delta}_t)\big)\big(\mathbf{g}_{i+s}'(\boldsymbol{\delta}_t) - T^{-1}\Sigma_{s=1}^{T}\mathbf{g}_s(\boldsymbol{\delta}_t)\big)$, $K_{2,t} = \Sigma_{i=1}^{T}k_{it}^2$ and $\hat{\boldsymbol{\Gamma}}(s) = \hat{\boldsymbol{\Gamma}}(-s)$ for $s < 0$ (Giraitis et al., 2014; 2018; 2021; Ciu et al., 2023).

This result follows straightforwardly from Giraitis et al. (2021) where similar tests are discussed.

2.2 Implied Natural Rate of Interest

Consider now stochastic time-variation in random coefficients of the forward-looking Taylor rule:

$$i_t = (1 - \rho_t)(\delta_{0,t} + \delta_{\pi,t}\pi_t + \delta_{\tilde{y},t}\tilde{y}_t) + \rho_t(L)i_t + u_t \tag{28}$$

The time-varying coefficient vector $\boldsymbol{\delta}_t = [\delta_{0,t}, \delta_{\pi,t}, \delta_{\tilde{y},t}, \rho_t]$ is estimated using TV CU-GMM.[6] Given that $\delta_{0,t} = i_t^{\#} - \delta_{\pi,t}\pi^*$ and $i_t^{\#} = r_t^{\#} + \pi^*$, the implied time-varying long run equilibrium interest rate may be expressed as a function of the following time-varying parameters $\delta_{0,t}$ and $\delta_{\pi,t}$:

$$r_t^{\#} = \delta_{0,t} - \pi^*(1 - \delta_{\pi,t}) \tag{29}$$

We note here that as r-hash is a linear function of estimated coefficients $\delta_{0,t}$ and $\delta_{\pi,t}$, uncertainty must be propagated. In general, the variance-covariance of a series of t functions may be expressed as a function of the variance-covariance matrix of parameters ($\boldsymbol{\delta}_t$) and their coefficient vector ($\boldsymbol{\Lambda}_t$):

$$\begin{gathered} r_t^{\#} = \sum_i^n \Lambda_{i,t}\delta_{i,t} = \boldsymbol{\Lambda}_t\boldsymbol{\delta}_t, \quad \text{where} \quad \Sigma_t^{r^{\#}} = \sum_i^n\sum_j^n \Lambda_{i,t}\Sigma_{ij,t}^{\delta}\Lambda_{j,t} = \boldsymbol{\Lambda}_t\boldsymbol{\Sigma}_t^{\boldsymbol{\delta}}\boldsymbol{\Lambda}_t' \\ \text{and} \quad \boldsymbol{\Sigma}_t^{\boldsymbol{\delta}} = E[(\boldsymbol{\delta}_t - \boldsymbol{\mu}_t) \otimes (\boldsymbol{\delta}_t - \boldsymbol{\mu}_t)] \end{gathered} \tag{30}$$

We approach the implied natural rate of interest in a similar manner to Woodford (2001), in which it is shown that formulations of the Taylor rule that fail to incorporate real disturbances, and that only account for contemporaneous feedback from the policy objectives are suboptimal, because of fixed intercepts that fail to stabilise inflation and the output gap period-by-period. Given standard determinacy conditions for the reaction coefficients hold, our time-varying framework ensures that both objectives are stabilised at all time periods by adjustments in the time-varying intercept term ($\delta_{0,t}$) that tracks variation in the natural rate of interest.[7]

As discussed in Woodford (2001), this is in keeping with Taylor's original prescription, which describes the intercept as capturing "the central bank's estimate of the equilibrium real rate of interest" (Taylor, 1999, p.325). We argue that these estimates are subject to error, which is a function of the difference between the perception of policymakers about r-star and the actual natural rate of interest. This framing of the equilibrium real interest rate within a forward-looking Taylor rule; as the perceived rather than actual natural rate of interest, is also consistent with Judd and Rudebusch (1998), in which the natural rate of interest within a forward-looking Taylor rule with smoothing is clearly interpreted as the "belief" of policymakers about the equilibrium real interest rate (see in particular Judd and Rudebusch, 1998, pp.10-11).

[6] Our estimation relies on the assumption that both short term rates and inflation are stationary. Augmented Dickey-Fuller tests confirm this hypothesis, albeit with less evidence against the null for the former in smaller sample sizes.

[7] Policy rules such as those presented in this paper are consistent with optimal equilibrium if $\delta_{\tilde{y}} > 0$ and $\delta_{\pi} > 1$. If the reaction coefficient associated with deviations in inflation from its target falls below unity, the target real rate adjusts to accommodate changes in inflation rather than stabilise it with output.

We therefore argue, that given ex-post policy rates, $r^{\#}$ is a reasonable proxy for the perceptions of monetary policymakers regarding the state of the macroeconomy. To evaluate these perceptions, we use semi-structural estimates of the actual natural rate of interest (r^*) as a comparator; derived from theoretical relationships that capture demand and supply side pressures, rather than reaction coefficients associated with relative priorities and preferences of the central bank. In this regard, we consider r-star an objective benchmark to evaluate the subjective beliefs of policymakers implicit in their determination of policy rates. In particular, we define misperception as any absolute deviation in the implied natural rate from the actual natural rate of interest.[8]

What remains in the computation of r-hash is to assume a value for the inflation target. As the implied equilibrium interest rate and inflation target cannot be identified simultaneously, one may only be derived under explicit assumptions about the other. Given our focus is on the perceptions of r-star, we assume a value for the latter. Whilst many studies have estimated an array of historical inflation targets pursued by the Federal Reserve, our baseline model defers to the prescriptions of Taylor (1993) and fix this to 2%. Despite it being the implicit and explicit target over the last few decades (Shapiro and Wilson, 2019), we recognise that this is a relatively strong assumption to maintain over the early years of our sample. In fact, it is uncertain whether the inflation target is itself constant and therefore time-varying prior to it being set explicitly. To this end, we examine the sensitivity of our results to a time-varying inflation target as derived in Ireland (2007) within the standard New Keynesian framework and also compare our findings to the natural rate of interest.

2.3 Actual Natural Rate of Interest

To measure the natural rate of interest, we employ the benchmark semi-structural Holston, Laubach and Williams (2017), hereafter HLW, model. In particular, we relax the canonical New Keynesian model by specifying reduced equations of the Investment Savings (IS) and Phillips Curve (PC):

$$\tilde{y}_t = \boldsymbol{\phi}_y(L)\tilde{y}_t + \boldsymbol{\phi}_r(L)\tilde{r}_t + \varepsilon_{\tilde{y},t} \tag{31}$$

$$\pi_t = \boldsymbol{\varphi}_\pi(L)\pi_t + \boldsymbol{\varphi}_y(L)\tilde{y}_t + \varepsilon_{\pi,t} \tag{32}$$

where $\varepsilon_{\tilde{y},t}$ and $\varepsilon_{\pi,t}$ denote transitory shocks to output and inflation, $\tilde{r}_t$ is the real interest rate gap given by the deviation in real rates from the natural rate of interest $r_t - r_t^*$, wherein r_t^* captures persistent shocks to the relationship between r_t and $\tilde{y}_t$. The law of motion for the natural rate is:[9]

$$r_t^* = g_t + z_t \tag{33}$$

where g_t is the trend growth rate of the natural rate of output and z_t is the error term that captures other factors driving r-star. The transition equations of the state-space model may be expressed as:

[8] If $r^{\#} > r^*$, policymakers overestimate the natural rate and policy becomes overly contractionary. In contrast, if $r^{\#} < r^*$, policymakers underestimate the natural rate and policy becomes overly expansionary. If however $r^{\#} = r^*$, monetary policymakers perfectly perceive the natural rate and policy is consistent with the optimal equilibrium.

[9] This arises from the textbook Ramsey model assuming representative household with a CES utility function and constant relative risk aversion, which is the inverse of the intertemporal elasticity of substitution. Optimisation yields an equilibrium interest rate that is a function of the growth rate of per capita consumption and the rate of time preference.

$$y_t^* = (L)y_t^* + (L)g_t + \varepsilon_{y*,t} \tag{34}$$

$$g_t = (L)g_t + \varepsilon_{g,t} \tag{35}$$

$$z_t = (L)z_t + \varepsilon_{z,t} \tag{36}$$

where (34) defines log potential output y_t^* as a random walk with stochastic drift g that also follows a random walk process (35). Finally, (36) captures the unobserved component of the natural rate of interest, which itself is a random walk. We assume shocks are contemporaneously uncorrelated and normally distributed. Refer to Appendix A for the full implementation of the system.

Model linearity in the unobserved state vector allows us to estimate the natural rate of interest, the natural rate of output and its trend growth using the Kalman filter. However, as real growth rates and interest rates are often influenced by highly persistent shocks, MLE is likely to return estimates of the standard deviations of innovations σ_g and σ_z that are biased towards zero. To resolve this 'pile-up' problem (Stock, 1994), we follow HLW by using the median unbiased estimator to derive estimates of ratios $\lambda_g = \sigma_g/\sigma_{y*}$ and $\lambda_z = \phi_r\sigma_z/\sigma_{\tilde{y}}$ that are imposed on the remaining parameters (Stock and Watson, 1998). What follows is a brief outline of the specification and implementation.

3 Estimation

3.1 Specification and Implementation

Given our interest in using estimates of the actual natural rate (r_t^*) as a comparator, our calibrations to the HLW model are minimal. The observation equations (31)-(32) are thus specified as follows:

$$\tilde{y}_t = \sum_{i=1}^{2} \phi_{y,i}\tilde{y}_{t-i} + \frac{\phi_r}{2}\sum_{i=1}^{2} \tilde{r}_{t-i} + \varepsilon_{\tilde{y},t} \tag{37}$$

$$\pi_t = \varphi_\pi \pi_{t-1} + \frac{1-\varphi_\pi}{3}\sum_{i=2}^{4} \pi_{t-i} + \varphi_y \tilde{y}_{t-1} + \varepsilon_{\pi,t} \tag{38}$$

where we impose a similar lag structure in both the dynamic IS and Phillips curve, such that two lags of the output gap and the average of two lags of the real rate gap enter the former and one lag of the output gap in addition to the first and average of the second to fourth lag of inflation enter the latter. The measurement equations (34)-(36) are first order random walks specified as follows:

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_{y*,t} \tag{39}$$

$$g_t = g_{t-1} + \varepsilon_{g,t} \tag{40}$$

$$z_t = z_{t-1} + \varepsilon_{z,t} \tag{41}$$

where the error terms are distributed as follows: $\varepsilon_{y*,t} \sim (N, \sigma_{y*}^2)$, $\varepsilon_{g,t} \sim (N, \sigma_g^2)$, $\varepsilon_{z,t} \sim (N, \sigma_z^2)$

The final set of measurement and transition equations to be estimated are summarised as follows (see Appendix A for the complete state-space representation and maximum likelihood procedure):

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1-\phi_{y,1} & -\phi_{y,2} & 1-\phi_r & -\phi_r \\ -\varphi_y & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y^*_{t-1} \\ y^*_{t-2} \\ g_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \phi_{y,1} & \phi_{y,2} & 0 & 0 & \phi_r \\ \varphi_y & 0 & \varphi_\pi & 1-\varphi_\pi & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ \pi_{t-1} \\ \pi_{t-2|4} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\tilde{y},t} + \varepsilon_{y*,t} \\ \varepsilon_{\pi,t} \end{bmatrix} \tag{42}$$

where the system (42) captures the measurement equations, which also includes the law of motion for the natural rate of interest, and the system (43) captures the transition equations of the model:

$$\begin{bmatrix} y^*_t \\ y^*_{t-1} \\ g_t \\ z_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y^*_{t-1} \\ y^*_{t-2} \\ g_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{y*,t} \\ 0 \\ \varepsilon_{g,t} \\ \varepsilon_{z,t} \end{bmatrix} \tag{43}$$

We cast these equations into a state-space system and estimate the parameters by maximising the likelihood function computed by the Kalman filter. We initialise the state vector by calculating the conditional expectation and covariance matrix using the Hodrick-Prescott filter. Given estimates of the standard deviations of innovations are likely biased, we impose median unbiased estimates of λ_g and λ_z in the chronology of our estimation á la Hoslton, Laubach and Williams (2017).[10]

In particular, we first estimate the natural rate of output barring the real rate gap and assuming constant trend growth. We subsequently derive the median unbiased estimate for the ratio between the standard deviations of innovations for the natural rate of output and its trend growth rate, which we impose as a restriction in the second stage of our estimation that includes the real rate gap. We derive a ratio for the component unrelated to trend growth in a similar manner, which we impose in the third stage to estimate the remaining model parameters by maximum likelihood.[11]

We replicate this methodology rather than sourcing two-sided estimates directly from the authors simply due to the construction of our baseline sample that excludes the pandemic for which particular adjustments are made to the measurement equations of the state-space model.

[10] Following directly from the HLW (2017) model, our specification of the vector of unobserved states also contains three lags of potential output y^* in the first stage, one lag of the trend growth rate g in the second and a second lag of the trend growth rate in the third in addition to two lags of the unobservable component z.

[11] Confidence intervals for these estimates and their corresponding standard errors are calculated using a constraint Monte Carlo procedure, which accounts for filter and parameter uncertainty (see Hamilton (1986) for further details).

3.2 Data and Instrument Selection

To estimate the implied natural rate ($r^{\#}$) we rely on the instrument selection in CGG by including up to four lags of each regressor in our specification of the matrix $\mathbf{x}_t$. The nominal interest rate is the federal funds rate. Inflation is the percentage change in core personal consumption expenditure (PCE). The output gap is derived using Congressional Budget Office (CBO) estimates of potential GDP. Money growth is the percentage change in the M2 stock published by the Board of Governors of the Federal Reserve. Commodity inflation is computed as the percentage change in a composite index of goods (including energy prices) published by the Bureau of Labor Statistics (BLS). The interest rate spread between long-term and short-term bonds is the difference between the 10-year and 3-month treasury bill rate also published by the Federal Reserve.

To estimate the actual natural rate of interest (r^*), we source data for real GDP and core PCE from the Bureau of Economic Analysis (BEA). As before, inflation is computed as the percentage change in core PCE. Inflation expectations used to derive ex-ante real interest rates are calculated as the four-quarter average of the lagged inflation rate. As before, the quarterly short-term interest rate is the average of the monthly federal funds rate published by the Board of Governors of the Federal Reserve. All data is sourced from the FED Economic Database (FRED).

Our quarterly data ranges from 1961:1 to 2019:4. Given our focus is on the historical conduct of monetary policy in the United States, we classify our sample into five key periods of interest: the pre-Volcker period from 1961:1 to 1979:2, the Volcker-Greenspan era from 1979:3 to 2005:4, the Greenspan-Bernanke era from 1987:3 to 2013:4, the Bernanke-Yellen era from 2006:1 to 2017:4, and the Yellen-Powell era from 2014:1 to 2019:4.[12] Whilst data was available after this period, we conclude our baseline sample just prior to the coronavirus pandemic, despite methods to account for it as outlined in Holston, Laubach and Williams (2020; 2023). The authors rightly argue that such extreme-tailed events violate the Kalman filter, in which innovations are assumed Gaussian in distribution. Furthermore, transitory shocks in the Phillips curve are assumed serially uncorrelated, which is largely inconsistent with responses to the pandemic. Thus, estimates of r-star during this period could potentially be distorted, and as such, may not warrant inclusion. Notwithstanding, we do not ignore this period, but instead use the adjusted specification of the measurement equations in the state-space model to test the sensitivity of our results to the inclusion of the pandemic.

3.3 Kernel and Bandwidth Selection

Our preferred baseline kernel is Gaussian, owing to its desirable properties as discussed in Giraitis et al. (2014). Unbounded support is perhaps a suitable benchmark, particularly given limited prior knowledge of the distribution of the underlying data process. We test the sensitivity of our results to the choice of kernel function by estimating the policy rule using an Epanechnikov kernel with finite support, which is also popular in its class due to its desirable properties. In the case of such functions, only local observations in the sample contribute to the estimation of $\boldsymbol{\delta}_t$. In contrast, all sample observations are weighted in the case of infinite support such as the Gaussian kernel.

[12] This particular division of the sample follows from the literature (refer to Clarida et al. (2000) and Carvalho et al. (2021) for a similar breakdown). We additionally investigate the terms of individual chairs across the historical sample.

We select a bandwidth $h = 0.6$ for our baseline case. As this parameter controls the smoothness and roughness of our results, caution must be invoked given higher bandwidths lead to oversmoothing whereas lower bandwidths lead to undersmoothing. Whilst there is no universal rule for optimal bandwidth selection, we choose a bandwidth that allows for a sufficient number of observations given our sample size.[13] We test the sensitivity of our results using bandwidths marginally below and above our baseline ($h = 0.5$ and $h = 0.7$). As we later discuss, the choice of bandwidth and kernel have non-trivial implications for our estimation of the time-varying parameters.

4 Results

To motivate our time-varying estimation approach, we first estimate the monetary policy rule using CU GMM. We examine the stability of the time-invariant parameters across subsamples and report these results in Table 1, in addition to point estimates of the implied natural rate of interest ($r^{\#}$).[14] For the Taylor principle to hold, that is for monetary policy to be stabilising, we expect $\delta_\pi > 1$ and $\delta_{\tilde{y}} > 0$. Estimates of δ_π are significant and below unity during the pre-Volcker period, albeit much greater than unity during the Volcker-Greenspan era. We find that the response to the inflation gap declines thereafter across the remaining future periods. Estimates of $\delta_{\tilde{y}}$ are statistically significant and above zero in all periods. We find that responses to the output gap have also declined in recent years between the Bernanke-Yellen and Yellen-Powell era.

Estimates of the smoothing parameter ρ are significantly high and increasing marginally across all periods of our sample, indicating considerable inertia in monetary policy. Finally, estimates of r-hash indicate a rise and secular fall in the perceptions of the natural rate. Assuming this is a good proxy for the perceptions of the Federal Reserve, this suggests long-run pessimism about r-star. In addition, our estimates show that the Great Recession had a sizeable influence on r-hash, forcing perceptions closer to the lower bound. In particular, the implied natural rate declines by more than 1.5 percentage points between the Greenspan-Bernanke and Bernanke-Yellen era. These findings suggest substantial time variation in the coefficients associated with deviations in inflation from its target and output from its potential, motivating our estimation of a time-varying policy rule.

4.1 Parameter Estimates

We begin this empirical investigation by first estimating the baseline case given a Gaussian kernel function and bandwidth parameter of $h = 0.6$. We therefore proceed to specify the time-varying kernel-weighted sample moment function outlined in equation (20) accordingly:

[13] We employ a data-driven bandwidth selector to loosely calibrate this parameter. In particular, we implement leave-one-out cross-validation techniques to triangulate h^*, which is an orthodox approach to optimal bandwidth selection. Optimisation of the cross validation function suggests a parameter in the range of 0.582-0.633 between kernels, which we subsequently approximate to 0.6. Refer to Appendix B for further details concerning these particular methods.

[14] Our estimates in the first and second column are close in proximity to Clarida, Galí and Gertler (2000). However, they are nonidentical for three reasons; (1) we use core PCE to derive our time series for inflation, whereas the authors use the GDP deflator and CPI, (2) our data for the Volcker-Greenspan era extends right until the beginning of Bernanke's term rather than ceasing it at 1996:1, which was the latest data point available to the authors at the time, and (3) we use the CU-GMM estimator, which estimates the optimal weighting matrix simultaneously rather than iteratively.

$$\mathbf{g}_t(\boldsymbol{\delta}_t) = K_t^{-1}\sum_{i=1}^{T}(1/\sqrt{2\pi})e^{-\frac{1}{2}\left(\frac{i-t}{H}\right)^2}\Big(\mathbf{x}_i\big(i_i - (1-\rho_t)(\delta_{0,t} + \delta_{\pi,t}\pi_i + \delta_{\tilde{y},t}\tilde{y}_i) - \rho_t(L)i_i\big)\Big) \quad (44)$$

Results are presented in Figure 1.[15] Time-varying estimates of the Taylor rule coefficients reveal substantial structural change. Despite apparent differences in magnitude, our results corroborate some general trends identified by the invariant estimation. To demonstrate this, we take subsample averages of these time-varying estimates and present them in Table 2. These results confirm a rise in δ_π over the post-Volcker period, albeit to a lesser extent than in the time-invariant case. Similar to our previous results, our estimates reveal a sharp decline in the average response to deviations in inflation from its target between the Greenspan-Bernanke and Bernanke-Yellen era.

Averages of $\delta_{\tilde{y}}$ are generally lower than in the time-invariant case, yet our estimates corroborate a sustained increase in the average sensitivity to the output gap. Estimates of the smoothing parameter ρ are significantly large, albeit stable on average across all our subsamples, confirming a large degree of inertia in the conduct of monetary policy. Finally, average estimates of r-hash also indicate a rise and fall in perceptions of the actual natural rate, albeit to a greater extent on average than in the invariant case. We find that the Taylor principle is initially violated during the pre-Volcker period but holds on average thereafter. As shown in Figure 1, it is undermined during the Bernanke-Yellen-Powell era when the response to the inflation gap falls below unity.

To check the robustness of our findings, we relax our baseline specification along two dimensions. First, we investigate the sensitivity to bandwidth selection whilst maintaining the Gaussian kernel. As the bandwidth controls the smoothness of our estimates, we are cautious to not choose a parameter that oversmooths or undersmooths. In this regard, we deviate marginally by selecting a value just below ($h = 0.5$) and above ($h = 0.7$) our baseline case. Second, we check the sensitivity to kernel selection whilst maintaining the baseline bandwidth parameter. Given the Gaussian kernel is perhaps the most widely used within its class of infinitely supported kernel functions, we select the Epanechnikov (parabolic) kernel with finite support as an alternative, which is also widely celebrated within the literature. In this regard, our analysis tests the robustness to support rather than kernels within the same class that likely yield similar results.

Results for alternative bandwidths are presented in Figure 2. Our time-varying estimates are generally robust to alternative specifications of the parameter h. As for $h = 0.5$, we find that results are characterised by sharper volatility and uncertainty, yet they generally corroborate trends in the Taylor coefficients identified in the baseline case. In particular, the estimated coefficient associated with deviations in inflation from its target is stable and increasing until 2000:1, whereafter it exhibits an even sharper decline before recovering in the latter periods of our sample. Estimates of $\delta_{\tilde{y}}$ further corroborate a sustained rise in the sensitivity to the output gap before declining marginally after 2003:1. Time-varying estimates of the smoothing parameter ρ are also more volatile yet vary around the same averages identified in our baseline case. Finally, it is worth noting that the Taylor principle generally prevails until the Great Recession, after which the estimated inflation coefficient falls below unity wherein it remains until the conclusion of our sample.

[15] Time-varying p-values associated with the J-test are reported in Appendix C for all combinations of kernel function and bandwidth parameter. For much of the sample we fail to reject the null, suggesting some validity in the instruments.

As for $h = 0.7$, we find that our results are characterised by lower volatility and uncertainty, yet also corroborate a number of trends in the parameters of the Taylor rule that are identified in the baseline case. The notable exception is in the coefficient δ_π, which continues to increase at a marginally diminishing rate during the latter of our sample, albeit with a higher degree of uncertainty. In particular, it averages 2.06 over the Bernanke-Yellen era. Estimates of $\delta_{\tilde{y}}$ and ρ exhibit a similar behaviour to the baseline results. The Taylor principle holds firmly across the post-Volcker period. We note that, whilst higher bandwidths account for a larger number of observations, time-varying estimates are overly smooth; thus, caution must be taken when interpreting these empirical results. The converse is also true for lower bandwidths due to the risk of undersmoothing.

Results for alternative kernel functions are presented in Figure 3. Our time-varying estimates are also generally robust to alternative specifications of the kernel function. Whilst unsurprisingly more volatile than the Gaussian kernel with infinite support, estimates from the finitely supported Epanechnikov kernel are able to corroborate a number of estimated trends in the Taylor coefficients identified within the benchmark specification. In particular, δ_π is also close to unity during the pre-Volcker period, before rising gradually up until the pre-crisis period. It declines sharply thereafter, violating the Taylor principle and pushing the coefficient δ_π well into negative territory. Our time-varying estimates of the sensitivity of policy to the output gap $\delta_{\tilde{y}}$ convey a comparatively similar, albeit far more volatile trend than the baseline case across alternative bandwidths. Estimates of the smoothing parameter exhibit a clear but volatile upward trend, confirming the high and increasing inertia in monetary policy as estimated in the time-invariant case.

In general, the Taylor principle seems to consistently prevail up until 2000:1, prior to which δ_π averages above unity across most specifications. Time-varying estimates of the inflation parameter decline sharply thereafter, recovering only to remain below unity by the close of our sample. The Taylor rule therefore calls into question the stability of monetary policy during this period. Finally, what is most apparent across all specifications, is the substantial time-variation in the parameters of the Taylor rule, justifying techniques such as those used within this paper to capture structural changes in the priorities of the Federal Reserve between output and inflation stability. We conclude here that our results are robust to alternative specifications and proceed to use these time-varying parameters to compute the implied natural rate of interest.

4.2 Between Chairs

To further investigate the stability of monetary policy and the implied natural rate of interest given time-varying estimates of the policy parameters, we take subsample averages by chairs of the FED. These results are reported in Table 3. We find the average response to inflation is weakening across the tenure of chairs in the pre-Volcker era, rising immediately prior to, and well after the Volcker term. In fact, these priorities peak during Greenspan's term at 1.58 percentage points and decline towards the end of our sample, albeit with reduced statistical significance.

The response to the output gap follows a similar trend. In particular, we find that priorities on output stability rise substantially over the post-Volcker period, peaking during the Bernanke term and declining thereafter. Given the Taylor principle holds for responses to the inflation gap above unity and responses to the output gap above zero, our results suggest that monetary policy during the Martin and Greenspan term was stabilising. We also note minimal time-variation in the inertial coefficient across chairs of the Federal Reserve, which remains high and close to unity across our sample, suggesting sluggish monetary policy rate adjustments.

Finally, estimates of the implied equilibrium interest rate suggest a sharp rise and protracted decline in perceptions of the actual rate. In particular, r-hash rises across the pre-Volcker period and peaks during the Volcker term at around 3.81 percentage points. It then subsequently declines gradually across the post-Volcker period, reaching a level close to zero by the end of our sample. These empirical results strongly motivate our following analysis concerning the time-varying implicit natural rate that captures the perceptions of monetary policymakers concerning the objective equilibrium interest rate. We therefore proceed to use these parameter estimates to derive r-hash.

4.3 Implied Equilibrium Rates

Estimates of the implied natural rate of interest are presented in Figure 4 across all combinations of kernel and bandwidth. Empirical results for the baseline specification demonstrate a clear rise and protracted fall in estimates of r-hash. Such secular pessimism follows similar trends in r-star identified in the literature. Given this series reflects the ex-post beliefs of monetary policymakers concerning the natural rate of interest, our findings suggest that the perceptions of the central bank regarding r-star have declined since the 1980s by around 3.5-4 percentage points. This stagnation persists right until the very end of our sample, where r-hash is seen to converge towards zero. This is a striking result that indicates sustained negative pressures on the Federal Reserve's historical estimation of the equilibrating interest rate in the United States.

This rise and fall in the time-varying implied natural rate is robust to alternative specifications of kernel and bandwidth. As expected, estimates of r-hash are smoother and rougher across higher and lower bandwidths respectively. We find that volatility rises substantially when estimating the Taylor rule with finitely supported kernels. In addition, estimates of the implied natural rate given such kernels fall and rise sharply just before and after the 1980s. Finally, it is worth noting that for finite kernel functions and lower bandwidth parameters, there is a sharp reduction in the perception of the actual natural rate immediately after the Great Recession, forcing r-hash in some instances into negative territory, indicating harsh revisions by the Federal Reserve in their estimation of the actual natural rate given the severity of the financial crisis.

4.4 Zero Lower Bound

Given conventional monetary policy rules, such as those considered in this paper, are less effective at stabilising the economy at the zero lower bound, we decide to estimate the time-varying Tayor rule using shadow rates during which it is a binding constraint. We do this primarily to characterise the stance of monetary policy across this period and to check the robustness of our estimates of the priorities between inflation and output stability. Given time-varying parameter estimates of the forward looking Taylor rule with smoothing, we are also able to derive the implicit series for the implied equilibrium interest rate that accounts for the lower bound environment.

Whilst a range of shadow rates are available to us within the literature, we choose the Wu-Xia (2016) rate released and maintained by the Federal Reserve Bank of Atlanta. The authors compute these estimates from a shadow rate term structure model (SRTSM) that is able to capture the stance of unconventional monetary policy.[16] Our implementation is given by the piecewise function:

$$i_t = \left\{ \begin{array}{ll} f_t, & \text{if } f_t \geqslant \underline{i} \\ s_t, & \text{if } s_t < \underline{i} \end{array} \right\} \tag{45}$$

Such that the interest rate is equal to the federal funds rate f_t if it is above the lower bound $\underline{i}$, and equal to the shadow rate s_t only if it falls below it. As discussed in Wu and Xia (2016), we fix this lower bound to $\underline{i} = 0.25\%$, given the FED has paid this rate on reserves since 2008. We therefore impose this in the estimation process and present our baseline findings in Figure 5.

Our findings suggest relatively minimal deviations in the priorities of policy makers over the period in which the lower bound is binding. In particular, we note marginally higher priorities on inflation and output stability, with negligible differences in inertia. Perhaps most noticeable is the greater uncertainty that surrounds estimates of the policy parameters arising after the crisis. Whilst changes to the time-varying intercept may suppress significant changes to the implied natural rate of interest, larger standard errors are likely to exacerbate propagated uncertainty in r-hash.

Estimates of r-hash presented in Figure 6 derived from the time-varying coefficients also reveal minimal deviations to the baseline case over the binding lower bound constraint. In particular, perceptions of r-star are only marginally lower on average across all specifications. As an exception, we detect a much sharper decline and recovery in r-hash given lower bandwidths and finite kernels (particularly for $h = 0.5$). What is clear across specifications is the rise in propagated uncertainty around the implicit natural rate circa the zero lower bound period. Notwithstanding, we argue that our results are largely robust to the use of shadow rates, which seek to capture periods of unconventional monetary policy. In this regard, we proceed to estimate and implement the actual natural rate as an objective comparator to gauge monetary policy misperceptions.

4.5 Actual Equilibrium Rates

We proceed to estimate the actual natural rate of interest using the HLW (2017) model detailed in Section 2.3. We present the estimated parameters of the model in Table 4. Time variation in trend growth and the natural rate of interest for the United States is substantial as indicated by relative median-unbiased estimates of the innovations to σ_g and σ_z, in ratios λ_g and λ_z respectively. Slope parameters ϕ_r and φ_y associated with the interest rate gap and output gap are relatively large and statistically significant, suggesting that they are reasonably identified. In accordance with existing studies, two-sided Kalman filter estimates of the natural rate of interest are imprecise with a sample average standard error of approximately 1.2 percentage points.

[16] Refer to Black (1995) for the orginial proposition of this framework. In addition, see for instance Bullard (2012) and Krippner (2013) for alternative proxies of near-zero policy rates that also reflect unconventional monetary policy accommodations. We also note wider considerations around the sensitivity of these shadow rates to model specification and data selection, as discussed in Christensen and Rudebusch (2015) and Bauer and Rudebusch (2016).

We focus on results from the Kalman smoother for the simple reason that our estimates of the implied natural rate are also computed using what are effectively two-sided estimators, naturally rendering smoothed estimates a more appropriate comparison. In this regard, our analysis focuses on the medium-long run rather than short-run frequency of policy. Estimates of the actual equilibrium interest rate arising from the Holston, Laubach and Williams (2017) model are presented in Figure 7. As documented across the existing literature, r^* exhibits clear trends of secular decline since the start of our sample. This was exacerbated by the Great Recession, after which estimates of the equilibrium real interest rate fall to unprecedented lows. The natural rate of interest remains depressed for many years thereafter before recovering marginally.

Reasons for this long-run decline in r-star are well documented in the literature and generally attributed to long-term trends in demography, productivity, risk preferences, public policy, and inequality (see for instance Carvalho, Ferrero and Nechio, 2016; Gagnon et al., 2016; Gordon, 2016; Rachel and Smith, 2017; Eggertsson et al., 2019; Rachel and Summers, 2019). Large imprecision in our estimates are often attributed to parameter and filter uncertainty arising from estimating the state variables. In addition, the peak in variance of a shock to the latent state variable arising from simultaneously estimating the model by maximum likelihood is also of some concern. Whilst this issue is addressed with the medium unbiased estimator, uncertainty around r-star prevails. Finally, the estimation of r-star during periods of harsh volatility is a precarious endeavour; such extreme tailed events may violate the Kalman filter, in which stochastic innovations are assumed Gaussian. In this regard, caution ought to be taken in interpreting these empirical results.

Whilst we do not attempt to offer an explicit solution to this particular issue, we note that our derived standard errors for the implied natural rate are substantially lower (approximately 0.78 on average in the baseline case). In particular, our estimates neither suffer from filter uncertainty nor issues of pile-up in the estimation process. This is primarily due to the use of generalised methods of moments, which is a clear advantage over alternative modelling approaches to time-variation.

4.6 Monetary Policy Perceptions

Given these estimates of the implied and actual natural rate of interest, what follows is an explicit framework for measuring (mis)perceptions in monetary policy. We present these estimated natural rates jointly in Figure 8 and proceed to identify several periods of divergence between the two time series, during which monetary policymakers either underestimate or overestimate the natural rate of interest. To facilitate this analysis, we also plot their area difference in Figure 9. These results document a rich history of monetary policy conduct within the United States in respect to policy perceptions of the equilibrium real interest rate. In particular, we identify roughly four significant phases in the setting of nominal policy rates by the Federal Reserve across our sample.

Firstly, policymakers during the pre-Volcker period suffer major inaccuracies in their targeting of r-star. This era is characterised by substantial underestimation of the equilibrium rate, implying that monetary policy was likely overly expansionary, leading to high inflationary pressure. Indeed, this corroborates events during the pre-Volcker period, in which the Great Inflation afflicted the United States between 1965-1982. Secondly, policymakers during the Volcker and early Greenspan term tend to overestimate the equilibrium interest rate. This implies that monetary policy was likely overly contractionary during this period, resulting in disinflationary pressure. These trends coincide with significant phenomena during this period, such as the Volcker Disinflation between 1980-1983 and the Great Moderation that followed thereafter from 1984.

Thirdly, monetary policymakers during the latter Greenspan and early Bernanke term seem to underestimate r-star once more. This period coincides roughly with the decade immediately prior to the Great Recession, in which monetary policy was overly expansionary, leading to low levels of unemployment and rising inflation. In the latter part of Bernanke's term following the financial crisis, policymakers appear to briefly overestimate the natural rate of interest. Finally, perceptions of r-star are generally in decline across the much of the Yellen-Powell era suggesting policymakers have underestimated the equilibrium real interest rate in recent years. Given that the Powell period consists of only a few observations, we look at sample extensions in the next section.

Our results indicate that during the pre-Volcker period, in which the Taylor principle was putatively violated, policymakers were historically inaccurate in their perceptions of r-star. In contrast, these perceptions improve during the post-Volcker period, with comparatively smaller deviations from the actual equilibrium interest rate. In particular, we estimate that the absolute deviation in r-hash from r-star was on average, 1.8 percentage points during the pre-Volcker period, and roughly 0.1 percentage points over the remaining sample for the baseline case. These findings demonstrate that monetary policymakers have become more effective in tracking the equilibrium interest rate over time. Finally, we reiterate here that these empirical results are generally robust to alternative specifications of both kernel and bandwidth, with only a few, relatively minor exceptions.

4.7 Sample Extensions

Responses to the coronavirus pandemic and subsequent supply and demand-side pressures created unprecedented changes in economic activity and inflation within the United States. Such volatility challenges key assumptions underlying the estimation of the natural rate. In particular, the Kalman filter assumes all stochastic innovations are Gaussian in distribution, which is clearly violated by the extremities of the pandemic. In addition, transitory shocks to inflation are assumed free from serial correlation, which is also inconsistent with the sequence of lockdowns imposed in response to the spread of coronavirus. To extend our analysis, we source estimates of r-star from a modified HLW (2023) model that accounts for shifts directly related to the restrictions imposed during the pandemic, in addition to time-varying volatility in shocks to the measurement equations.

The model is adjusted along two important dimensions. To resolve the issue of extreme outliers associated with the pandemic, the authors follow Lenza and Primiceri (2022) by introducing time-varying volatility in the stochastic innovations to the output gap and inflation equations over this period. This effectively suppresses outliers within the maximum likelihood estimation procedure to yield measures of the natural rate of interest that would otherwise be distorted due to the harsh effects of the pandemic. To resolve issues of potential serial correlation arising from sequences of lockdowns, the authors incorporate a persistent, albeit transitory supply shock, in order to capture the impact of restrictions on economic activity within the output gap during this period.

We re-estimate the forward looking Taylor rule over this extended sample from 1961:1-2024:2 and extract estimates of the implied equilibrium interest rate. To investigate the accuracy of monetary policy over this period, we superimpose these implied natural rates on estimates of the actual equilibrium interest rate that control for the pandemic to gauge the deviation between series. These results are presented in Figure 10 for all combinations of kernel and bandwidth.

Our estimates of the implied natural rate of interest are generally robust to the inclusion of the coronavirus pandemic. Compared with estimates of the actual natural rate of interest, we observe more sizeable periods of overestimation during the Volcker-Greenspan years. More interestingly, perceptions of the equilibrium rate clearly reach unprecedented lows circa the pandemic and begin to subsequently rise during the inflation surge observed across much of the advanced world. Two-sided filter estimates of the natural rate of interest are relatively stable if not marginally increasing during this period, suggesting monetary policymakers underestimated the equilibrium interest rate before correcting their estimation of r-star towards the latter part of our sample.

These findings are generally consistent with the experience of the pandemic, in which the Federal Reserve slashed interest rates to stimulate spending, but suggest that monetary policymakers may have been overly pessimistic about the extent to which the natural rate of interest had truly declined. The subsequent rate hikes in response to a range of supply and demand pressures within the United States to curb inflation is also consistent with the rise in r-hash, which has now converged to the natural rate of interest. These estimates corroborate the argument that policymakers have become more accurate in their measurement of the real equilibrium interest rate over time, as indicated by smaller deviations from r-star relative to the former part of our sample.

4.8 Inflation Targets

Time-varying estimates of the implied natural rate of interest depend upon the identification of the inflation target, which we fix at 2% across all our kernel weighted specifications of the Taylor rule. The Federal Reserve explicitly defined this target in 2012, prior to which it implicitly pursued a target within the same neighbourhood since the mid-1990s (Shapiro and Wilson, 2019; Wells, 2024). However, the assumption that monetary policymakers pursued the same target across former parts of our sample requires greater scrutiny, particularly given our knowledge of substantial inflationary pressures present in the United States during the pre-Volcker period.

To gauge the impact of alternative inflation targets on our estimates of the implied equilibrium rate, we defer to measures derived from the existing literature. In particular, we employ estimates of a time-varying inflation target from a canonical New Keynesian framework outlined in Ireland (2007). The model shares many features of Clarida, Gali and Gertler (1999) and Woodford (2003) but generalises the Taylor (1993) rule, thereby allowing the inflation target to adjust to other shocks in the economy. The time series for this target is presented in Figure 11 between 1961:1-2004:2.[17] These results indicate that the inflation target of the Federal Reserve experienced sharp volatility across the former parts of our sample. In particular, π^* rose persistently until the mid-1970s only to

[17] We are grateful to Peter Ireland for providing this time series. Given our knowledge of the inflation target implicitly pursued by the Federal Reserve in its historical conduct of monetary policy since the mid-1990s, we extend these estimates by assuming it is fixed at 2% until 2012:1, after which it remains fixed at this rate with certainty.

decline persistently across the Volcker and post-Volcker period. It reaches the 2% level implicitly pursued by the Federal Reserve around the mid-1990s and seems to be relatively stable thereafter. Given this variation in the inflation target, it is therefore necessary to consider the impact on our estimates of the implied equilibrium interest rate derived from the Taylor rule.

We recompute measures of r-hash with a time-varying inflation target and present these results in Figure 12. It is worth noting that estimates of the implied equilibrium interest rate are computed from estimates of the reaction coefficient to the inflation gap within the Taylor rule in addition to the inflation target. As the former is close in proximity to unity during the initial part of the sample, time-variation in the inflation target has a negligible impact on our estimates of r-hash computed in equation (29), which are approximately equal to the intercept of the policy rule. As the inflation target begins to converge to the 2% level towards the latter part of the sample, our estimates of the implied natural rate of interest given a time-varying inflation target yield marginal differences to the baseline specifications with a time-invariant target rate.

In particular, our estimates of monetary policy perceptions are generally robust to the incorporation of a time-varying target. In the baseline specification, we observe marginally longer periods of underestimation that extend from the pre-Volcker to the early Volcker period, in addition to marginally higher periods of overestimation during the early Greenspan years. Given our estimates of the implied equilibrium interest rates inherits its smoothness from two-sided estimates of the reaction coefficient to the inflation gap, the observed increase in transitory volatility is predominantly due to higher frequency structural measures of the inflation target, albeit this variation is relatively negligible and does not alter the underlying long-run trends in the perceptions of r-star.

4.9 Cointegration

Our results suggest long-run comovement between the perceived and actual natural rate of interest. We briefly investigate such a relationship using a standard error-correction model (ECM). Standard Augmented Dickey-Fuller (ADF) tests confirm r-star is a nonstationary process. This is expected, given the natural rate of interest is modelled as such in the HLW (2017) system. In addition, ADF tests suggest that implied natural rates of interest are nonstationary across all kernel functions and bandwidth parameters. Standard Johansen tests for cointegration detect one cointegrating relation that links the perceived natural rate of interest and the actual natural rate of interest. As both series are found to be integrated of the first order, we therefore estimate the following ECM specification:

$$\Delta r_t^{\#} = \alpha(r_{t-1}^{\#} - \beta r_{t-1}^{*}) + \gamma \Delta r_t^{*} + \varepsilon_t \tag{46}$$

We note here that given the actual and implied equilibrium interest rates are themselves the product of estimated models with sizeable uncertainty, caution ought to be invoked when interpreting any results from an estimated model in which they feature; our objective here is therefore to primarily characterise their comovement and to simply establish if these series are related over the long-run. In light of this caveat, we present our results in Table 5 given estimates derived from the baseline Gaussian kernel. For robustness, we estimate the model across all bandwidth parameters (refer to Appendix F for estimated models given alternative kernel functions and bandwidth parameters).

Estimated short-run and long-run coefficients clearly indicate a significant positive relationship between actual and perceived natural rates of interest. This is somewhat intuitive, as we expect for movements in r-star to be ultimately reflected in the beliefs and perceptions of the monetary policymaker, whose objectives are output and inflation stability. Finally, error-correction coefficients of the model are negative and within the range required for convergence across all specifications of the bandiwdth parameter, indicating a gradual adjustment to equilibria.

We further examine this long-run relationship in a time-varying error correction model, which we estimate using methods outlined in Giraitis et al. (2018). In particular, minimising the kernel-weighted residual sum of squares $\sum_{t=1}^{T} k_{it} u_t^2$ yields the time-varying least squares estimator:

$$\hat{\beta}_t = \Big(\sum_{t=1}^{T} k_{it} x_t x_t'\Big)^{-1} \Big(\sum_{t=1}^{T} k_{it} x_t y_t\Big) \tag{47}$$

We use this technology to estimate an error correction model, wherein the impact multiplier, long-run multiplier, and error correction coefficient are assumed to be stochastic time-varying parameters. The estimated time-varying specification of the model may therefore be written as follows:

$$\Delta r_t^{\#} = \alpha_t (r_{t-1}^{\#} - \beta_t r_{t-1}^{*}) + \gamma_t \Delta r_t^{*} + \varepsilon_t \tag{48}$$

We present results for our baseline Gaussian kernel and bandwidth parameter in Figure 13 (refer to Appendix F for estimated time-varying models using alternative kernel functions and bandwidth parameters). The long-run cointegrating relationship has risen substantially over the post-Volcker period towards unity, which suggests that latent variation in the natural rate of interest is having an increasingly positive and proportional impact on the perceptions of policymakers. In addition, our results suggest that the error correction coefficient has remained negative for a sizeable part of our sample with a similarly gradual speed of adjustment. These empirical findings largely corroborate our initial point estimates of the fixed coefficient error correction model summarised in Table 5.

4.10 Policy Shocks

Policy misperceptions have significant implications for macroeconomic fluctuations. To highlight this, Figure 8 plots dates identified across Romer and Romer (1989; 1994; 2023) marking when the Federal Open Market Committee (FOMC) announced their decision "to exert a contractionary influence on the economy in order to reduce inflation", that is, when policy rates became explicitly disinflationary.[18] These dates are detailed in Appendix G. Our findings show that instances when the FOMC take decisions to increase policy rates are largely clustered around the pre-Volcker and Volcker period, each corresponding to upward pressures on the deviation between perceptions of the natural rate and the actual natural interest rate. Given r-star is declining moderately during this period, much of these pressures are driven by changes in r-hash.

[18] Romer and Romer (2023) have since updated these dates to include expansionary monetary policy shocks. Given only one such date arises, we focus our analysis mainly on negative policy shocks. To our knowledge, no other attempts have been made to chart these dates using a narrative approach. We are therefore restricted to those identified in Romer and Romer (1989; 1994; 2023) and thank David Romer for his comments and suggestions pertaining to these dates.

This is somewhat intuitive given rising anti-inflationary sentiment prior to and during the Volcker regime. Our findings suggest that policymakers consistently revised their perceptions of r-star upwards during this period, reflecting the growing attempts to tighten monetary policy at the time. In fact, the cluster of positive revisions in nominal policy rates during the latter 1970s and the early 1980s is followed by almost a decade of overestimation of the natural rate of interest. This occurs once again in the latter 1980s, suggesting that monetary policymakers, in reaching for r-star, have historically overshot their implicit beliefs about the equilibrium interest rate after decisions to disinflate the economy, insofar as the natural rate was subsequently well below what was implied by the Federal Reserve's conduct of monetary policy at the time.

It is worth mentioning here that the narrative approach from which these dates are determined is not without critique. As Nakamura and Steinsson (2018) have aptly pointed out, the selection of these dates do not follow a formal methodology and is therefore difficult to replicate. In addition, such a small sample size (of $n = 10$ shocks) may not be sufficiently large to average out potential random correlation with omitted factors. Notwithstanding these issues, the narrative approach still remains a useful way to capture exogenous monetary policy shocks within the literature.

5 Conclusion

This paper estimates a time-varying random-coefficient forward-looking Taylor rule with smoothing for the United States using a kernel-weighted time-varying continuously-updating generalised methods of moments estimator. Our findings reveal substantial time variation in all the coefficients of the monetary policy reaction function that is generally robust to kernel function and bandwidth parameter selection. Estimates suggest that the Taylor principle prevailed for much of our sample, only to be violated around the Great Recession. In addition, we find that historical monetary policy in the United States is characterised by substantial and marginally rising inertia.

Given time-varying estimates of the policy parameters, we derive a series for the time-varying implied natural rate of interest (r-hash). This is the natural rate of interest perceived by the central bank in its determination of policy rates given its priorities between the inflation and output gap. Our empirical results show a rise and protracted fall in the perceptions of policymakers concerning r-star. In particular, these perceptions have been in decline since the 1980s, converging to zero by the end of our historical sample. The inception of such secular pessimism coincides with the Great Moderation and has been exacerbated by the Great Recession. Using time-varying shadow rates, we show that our results are robust to a binding lower bound constraint.

We estimate the actual (semi-structural) natural rate of interest (r-star) and use this as an objective comparator to explicitly measure and analyse policy (mis)perceptions. Given our time-varying results, we are able to identify multiple periods of inaccuracy in the estimation of r-star that corroborate key periods of macroeconomic fluctuation across chairs and regimes of the Federal Reserve. More importantly, our empirical analysis suggests that monetary policymakers in the post-Volcker period have substantially improved their perceptions of the long-run equilibrium interest rate relative to the pre-Volcker period, during which the average absolute deviation between r-hash and r-star was significantly higher in comparison to the rest of the sample.

Our analysis is generally robust to extensions of the sample, towards the latter of which we observe harsh swings in output and inflation due to a range of supply and demand-side pressures. Most interestingly, we identify a fall and rise in the perceptions of the natural rate of interest, coinciding with the pandemic and subsequent inflation surges experienced across the advanced world. In addition, we relax the assumption of a constant inflation target by incorporating New Keynesian measures of a time-varying inflation target during the former part of our sample. Our results reveal marginal differences between these cases, which lends robust support to the underestimation and overestimation identified in the pre-Volcker and immediate post-Volcker periods.

Using narrative-based dates for exogenous monetary policy shocks, this paper finds evidence to suggest that the Federal Reserve has historically overshot its position on r-star during the early post-Volcker period in its endeavour to correct its underestimation, insofar as r-star was below what was implied by monetary policy. In addition, we find evidence of a positive long-run relationship between the implied and actual equilibrium interest rate, suggesting that beliefs of monetary policymakers are ultimately aligned with movements in the natural rate of interest.

The results in this paper rest on kernel-weighted estimators, which are sensitive to kernel and bandwidth selection. Whilst there is no particular rule of thumb concerning either, our estimation may be improved with various optimal bandwidth selection criteria. We leave such an endeavour to future extensions. Notwithstanding, our TV CU-GMM estimates of r-hash are characterised by less uncertainty than Kalman filter estimates of r-star, yet exhibit similar trends of secular decline. These findings are of significance in the analysis of monetary policy, and our investigation provides an explicit framework to quantify not only the priorities between inflation and output stability but also the ex-post precision of policymakers in evaluating the state of the macroeconomy.

Table 1. Point Estimates

	Pre-Volcker 1961:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2017:4	Yellen-Powell 2014:1-2019:4
δ_π	0.96	2.29	1.94	1.70	1.23
	(0.35)	(0.71)	(0.54)	(0.80)	(0.99)
$\delta_{\tilde{y}}$	0.86	1.01	1.10	0.55	0.33
	(0.24)	(0.36)	(0.28)	(0.25)	(0.27)
ρ	0.73	0.80	0.83	0.84	0.93
	(0.07)	(0.05)	(0.03)	(0.05)	(0.09)
r$^{\#}$	1.38	2.45	2.22	0.68	0.32
	(0.25)	(0.20)	(0.11)	(0.23)	(0.27)
J	0.72	0.33	0.45	0.67	0.52

Note: Parameter estimates of the Taylor rule by CU-GMM. Robust standard errors are reported in parentheses. P-values are reported for the associated Hansen J-test statistic at the $\alpha = 5\%$ *significance level.*

Table 2. Subsample Averages

	Pre-Volcker 1961:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2017:4	Yellen-Powell 2014:1-2019:4
$\bar{\delta}_\pi$	0.98	1.38	1.34	0.72	0.57
	(0.12)	(0.23)	(0.29)	(0.37)	(0.40)
$\bar{\delta}_{\tilde{y}}$	0.07	0.37	0.65	0.77	0.67
	(0.04)	(0.09)	(0.08)	(0.08)	(0.08)
$\bar{\rho}$	0.87	0.87	0.86	0.87	0.85
	(0.08)	(0.06)	(0.05)	(0.05)	(0.07)
$\bar{r}^{\#}$	2.09	3.15	2.41	1.11	0.53
	(0.30)	(0.32)	(0.22)	(0.54)	(1.10)
$\bar{J}$	0.37	0.33	0.34	0.29	0.27

Note: Average parameter estimates of the Taylor rule by TV CU-GMM. K(x): Gaussian. h = 0.6. Average standard errors are reported in parentheses. Average p-values are reported for the associated TV-J statistic.

Table 3. Chair Averages

	Martin 1961:1-1970:1	Burns 1970:1-1978:1	Miller 1978:1-1979:3	Volcker 1979:3-1987:3
$\bar{\delta}_{\pi}$	1.02 (0.13)	0.96 (0.11)	0.85 (0.14)	0.89 (0.16)
$\bar{\delta}_{\tilde{y}}$	0.19 (0.10)	-0.02 (0.09)	-0.17 (0.08)	-0.12 (0.08)
$\bar{\rho}$	0.83 (0.10)	0.91 (0.07)	0.92 (0.06)	0.89 (0.06)
$\bar{r}^{\#}$	1.59 (0.43)	2.44 (0.60)	3.39 (0.76)	3.81 (0.85)
$\bar{J}$	0.41	0.32	0.34	0.29

	Greenspan 1987:3-2006:1	Bernanke 2006:1-2014:1	Yellen 2014:1-2018:1	Powell 2018:1-2019:4
$\bar{\delta}_{\pi}$	1.58 (0.26)	0.77 (0.36)	0.58 (0.39)	0.54 (0.43)
$\bar{\delta}_{\tilde{y}}$	0.58 (0.09)	0.81 (0.07)	0.69 (0.08)	0.63 (0.09)
$\bar{\rho}$	0.86 (0.05)	0.87 (0.04)	0.86 (0.06)	0.85 (0.09)
$\bar{r}^{\#}$	2.85 (0.83)	1.34 (1.01)	0.61 (1.08)	0.34 (1.14)
$\bar{J}$	0.36	0.31	0.24	0.35

Note: Average parameter estimates of the Taylor rule by time-varying GMM between chairs of the Federal Reserve. K(x): Gaussian. h = 0.6. Average standard errors are reported in parentheses. Average p-values are reported for the associated TV-J statistic. See Appendix E for results given combinations of K(x) and h.

Table 4. Parameter Estimates

Parameter		Standard error	
λ_g	0.053	r^*_{avg}	1.180
λ_z	0.036	y^*_{avg}	1.526
$\Sigma\phi_y$	0.943	g_{avg}	0.400
ϕ_r	-0.068		
φ_y	0.078		
$\sigma_{\tilde{y}}$	0.339	r^*_{fin}	1.712
σ_π	0.789	y^*_{fin}	2.028
σ_{y*}	0.573	g_{fin}	0.545
σ_g	0.121		
σ_z	0.178		
σ_{r*}	0.215	T	1961:1-2019:4

Note: Estimated parameters of the Holston, Laubach and Williams (2017) model. Average and final standard errors of the natural rate of interest, the natural rate of output and its trend growth are reported in the very last column. Standard errors are calculated as in Hamilton (1986). σ_g is expressed as an annual rate.

Table 5. ECM Estimates

Bandwidth (h)	0.5	0.6	0.7
Cointegrating vector (S.E.)	0.46 (0.06)	0.41 (0.06)	0.30 (0.03)
Error correction coefficient (S.E.)	-0.02 (0.01)	-0.04 (0.02)	-0.01 (0.01)
Granger test (p)	0.03	< 0.01	0.05
ADF test (p)	0.58	0.41	0.10

Note: Results for bivariate error-correction models in r-star and r-hash. K(x): Gaussian. h = 0.5, 0.6, 0.7. Standard errors are reported in parentheses. See Appendix F for results given combinations of K(x) and h.

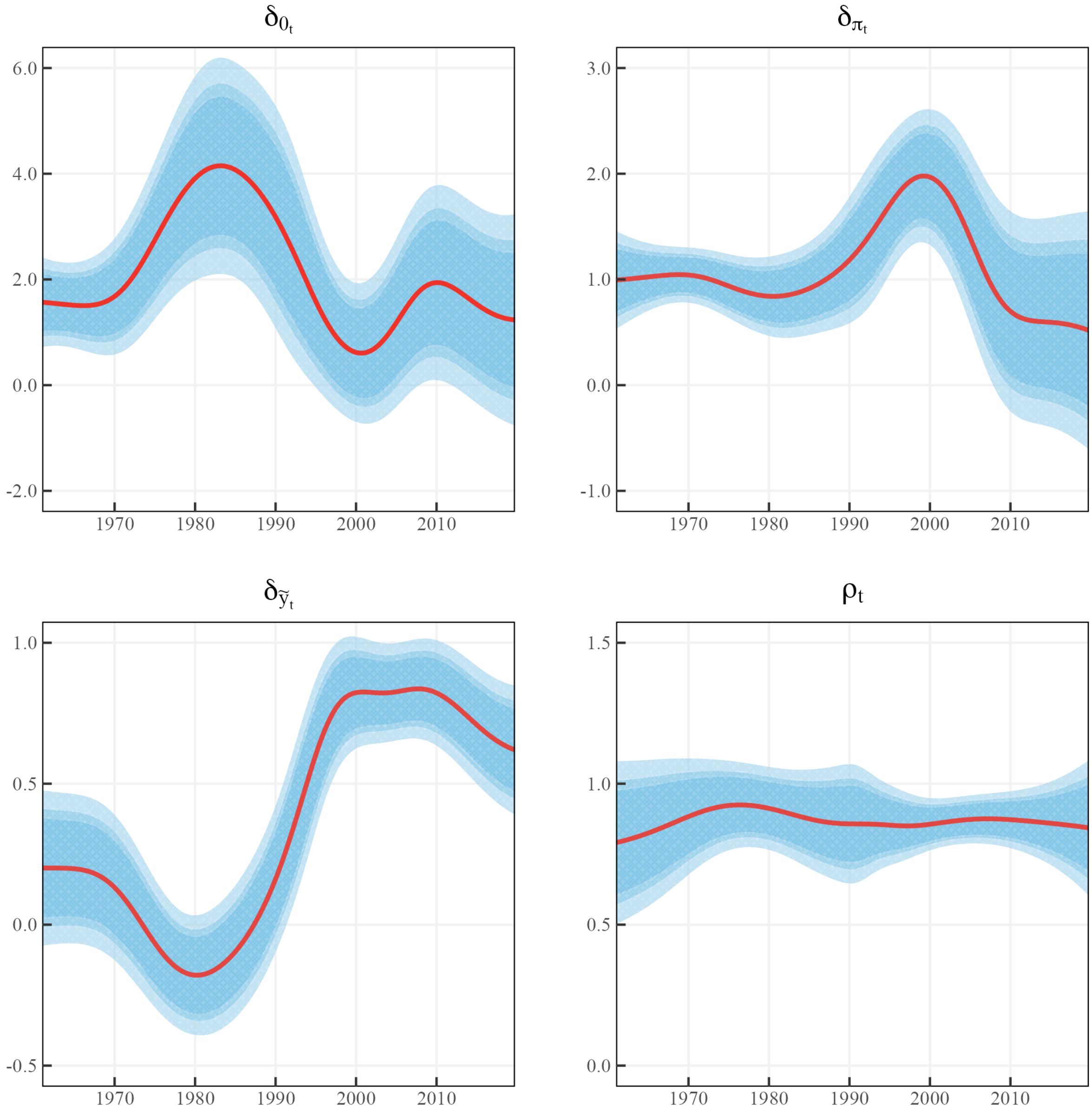

Figure 1. Baseline Dynamic Estimates

Note: Time-varying parameter estimates of a forward looking Taylor rule with smoothing by TV CU-GMM. K(x): Gaussian. h = 0.6. Shaded regions in blue correspond to the respective 90/95/99% confidence bands.

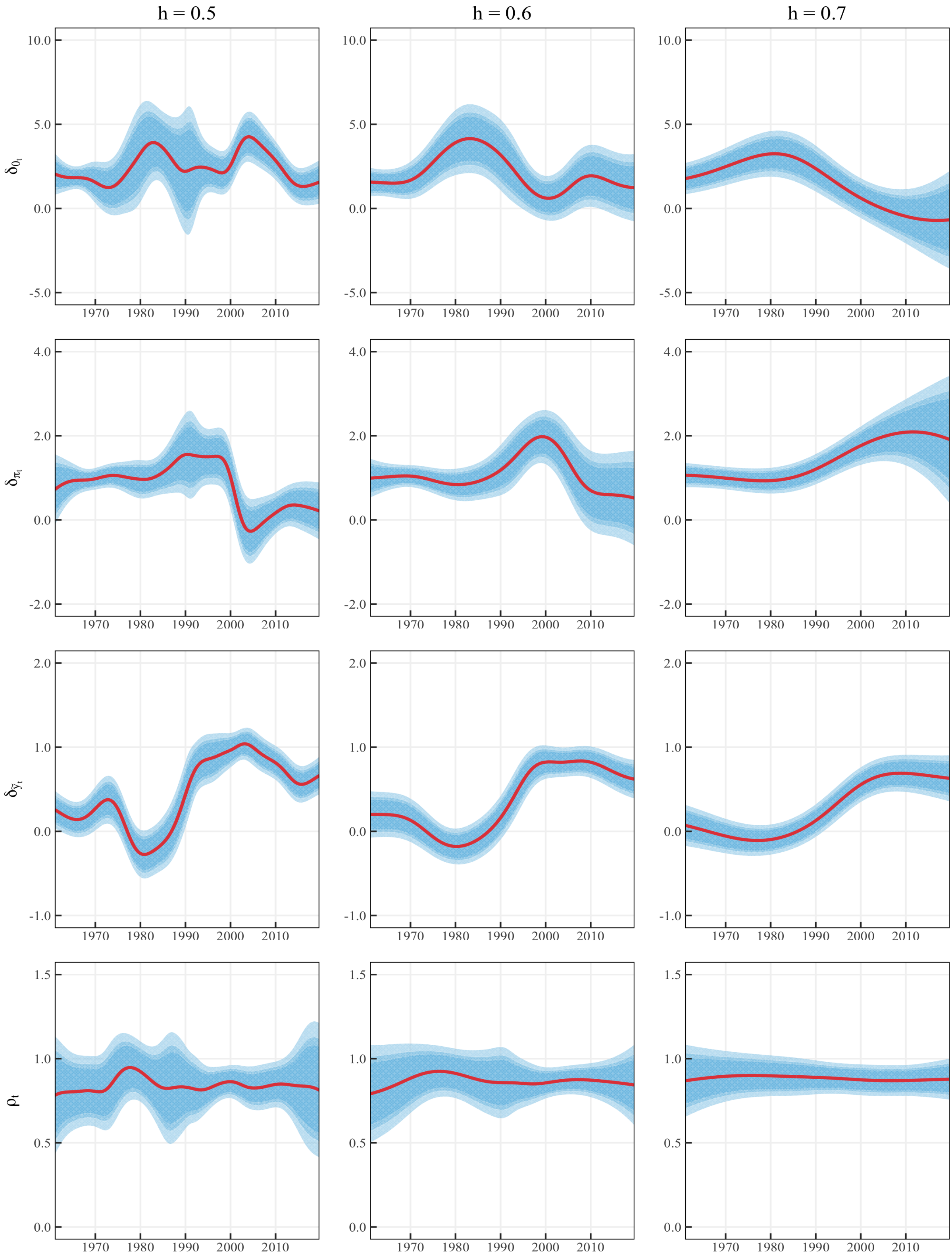

Figure 2. Alternative Bandwidth Parameters

Note: Time-varying parameter estimates of a forward looking Taylor rule with smoothing by TV CU-GMM. K(x): Gaussian. h = 0.5, 0.6, 0.7. Shaded regions in blue correspond to the 90/95/99% confidence bands. See Appendix C for estimates using all combinations of kernel functions K(x) and bandwidth parameters h.

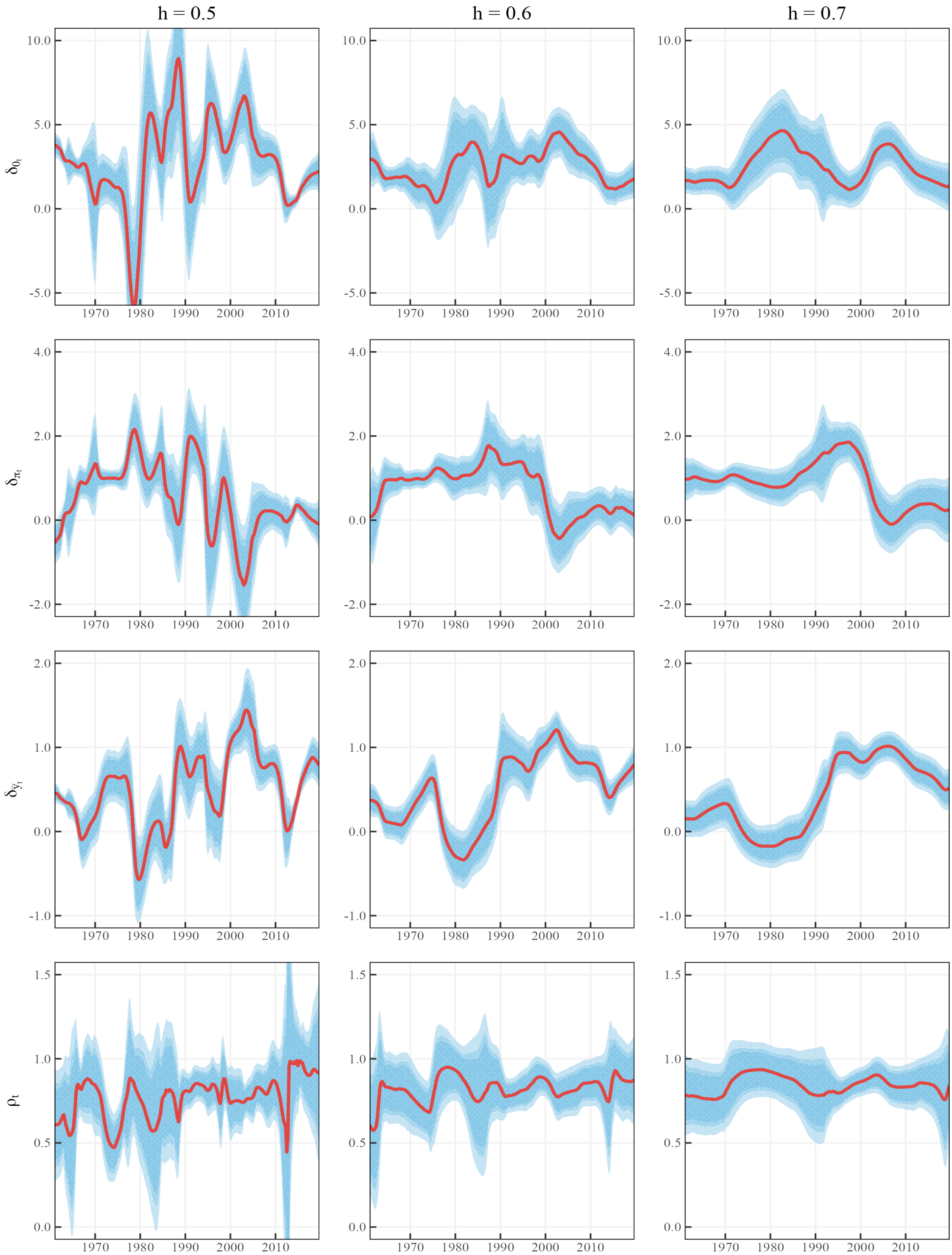

Figure 3. Alternative Kernel Functions

Note: Time-varying parameter estimates of a forward looking Taylor rule with smoothing by TV CU-GMM. K(x): Epanechnikov. h = 0.5, 0.6, 0.7. Shaded regions in blue represent the 90/95/99% confidence bands. See Appendix C for estimates using all combinations of kernel functions K(x) and bandwidth parameters h.

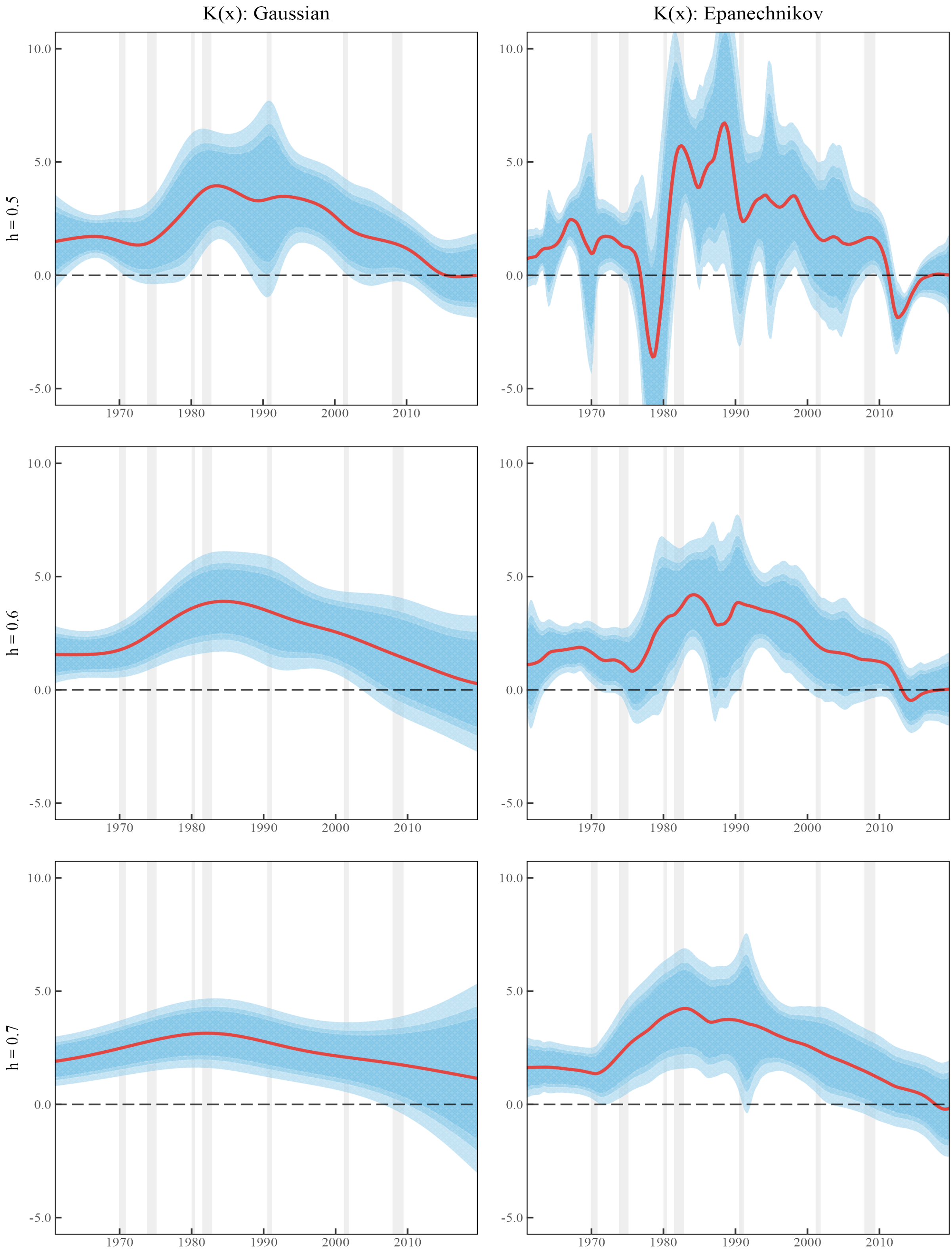

Figure 4. Implied Natural Rate of Interest

Note: Implied natural rates of interest derived from estimated time-varying coefficients of the Taylor rule. Shaded regions in blue correspond to the 90/95/99% confidence bands. Shaded vertical bars in grey indicate recessionary periods within the United States, dated by the National Bureau of Economic Research (NBER).

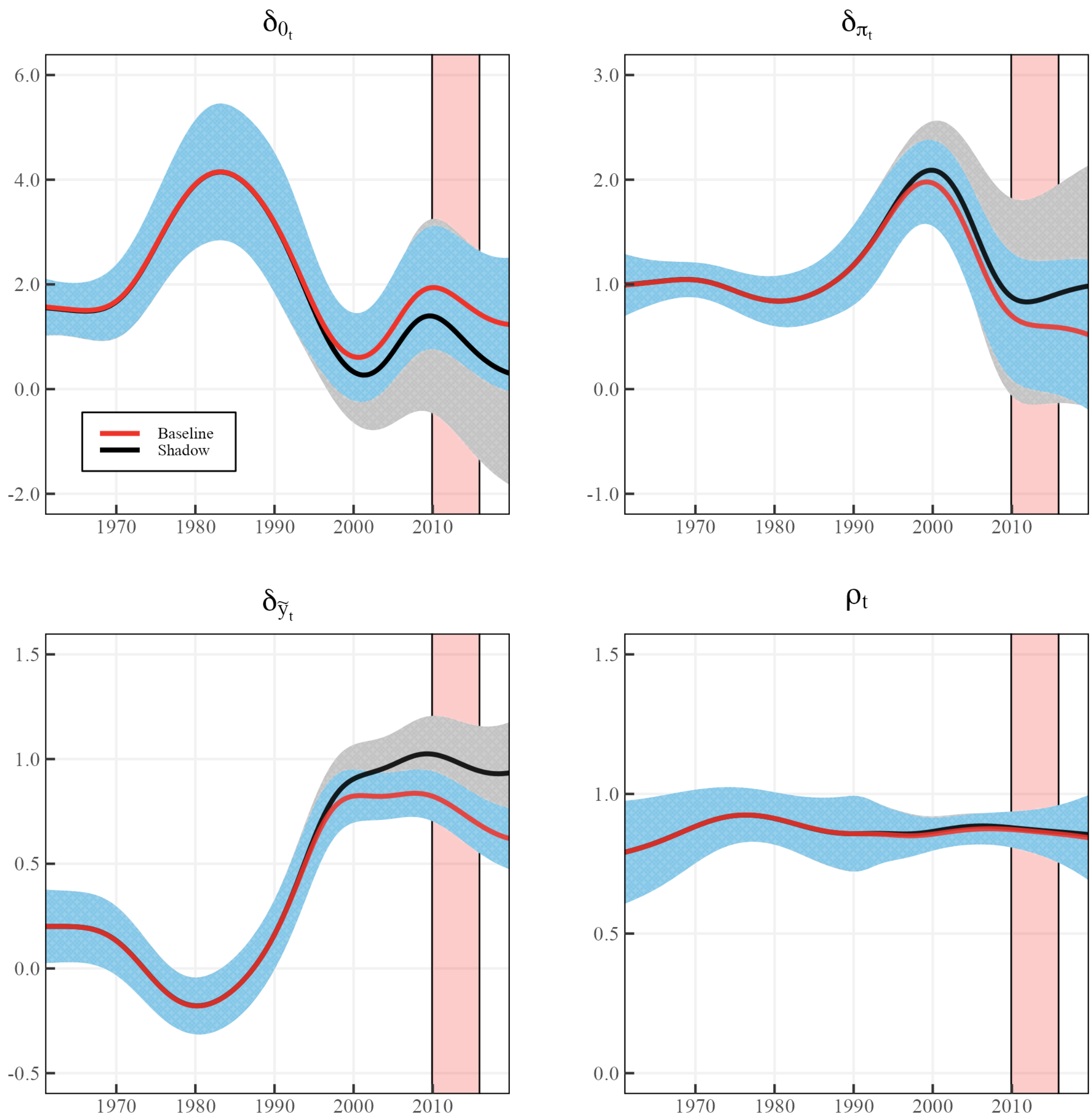

Figure 5. ZLB Consistent Parameter Estimates

Note: Time-varying parameter estimates of the Taylor policy rule by TV CU-GMM. K(x): Gaussian. h = 0.6. Baseline estimates are presented in red. Estimates using Wu-Xia (2016) shadow rates are presented in black. Shaded vertical regions in red correspond to periods in which the lower bound is binding between 2008:4-2015:4. Shaded regions in blue (baseline) and grey (shadow rates) correspond to the 90% confidence band.

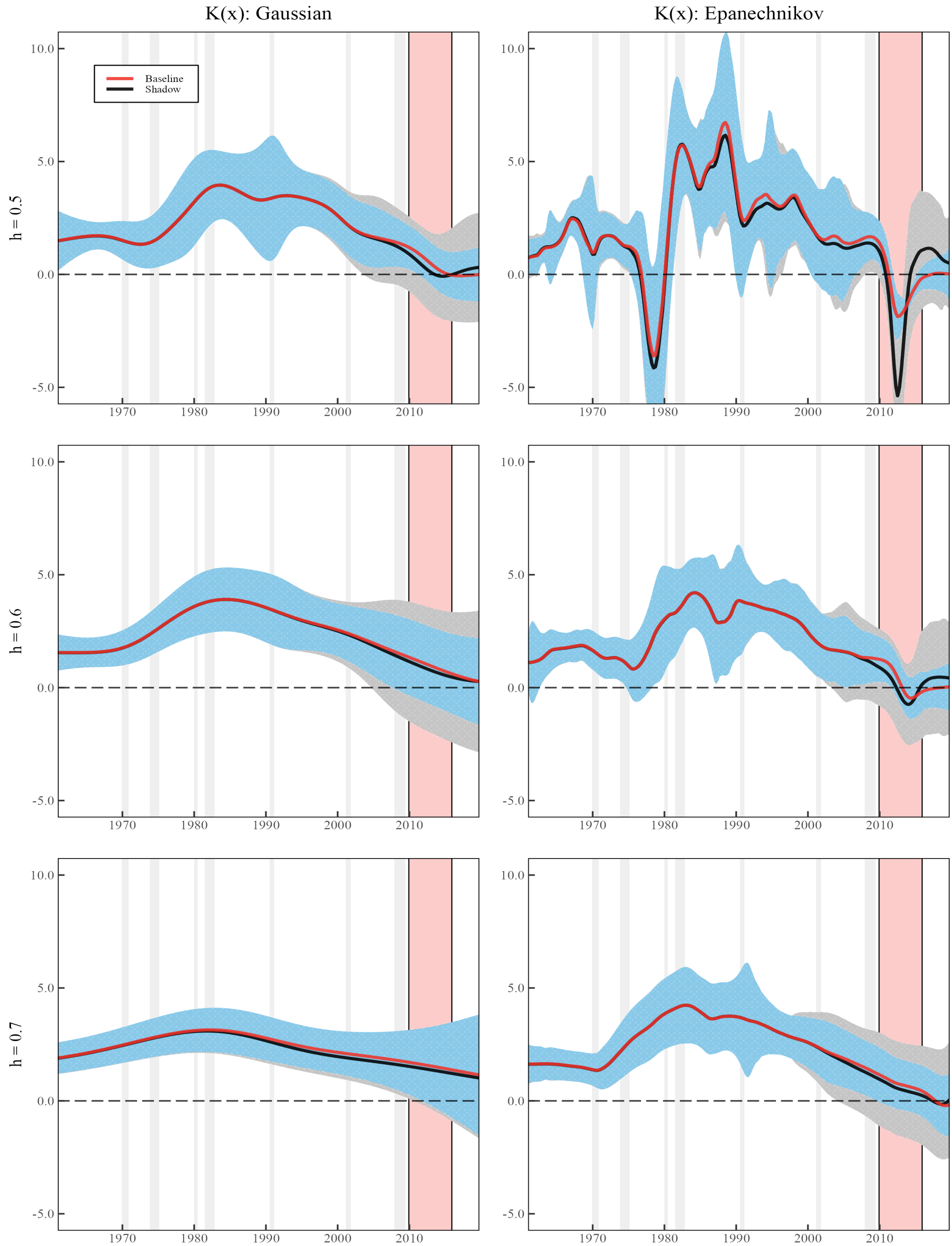

Figure 6. ZLB Consistent Implied Rates

Note: Implied natural rates of interest derived from estimated time-varying coefficients of the Taylor rule. Baseline estimates are presented in red. Estimates using Wu-Xia (2016) shadow rates are presented in black. Shaded vertical regions in red correspond to periods in which the lower bound is binding between 2008:4-2015:4. Shaded regions in blue (baseline) and grey (shadow rates) correspond to the 90% confidence band.

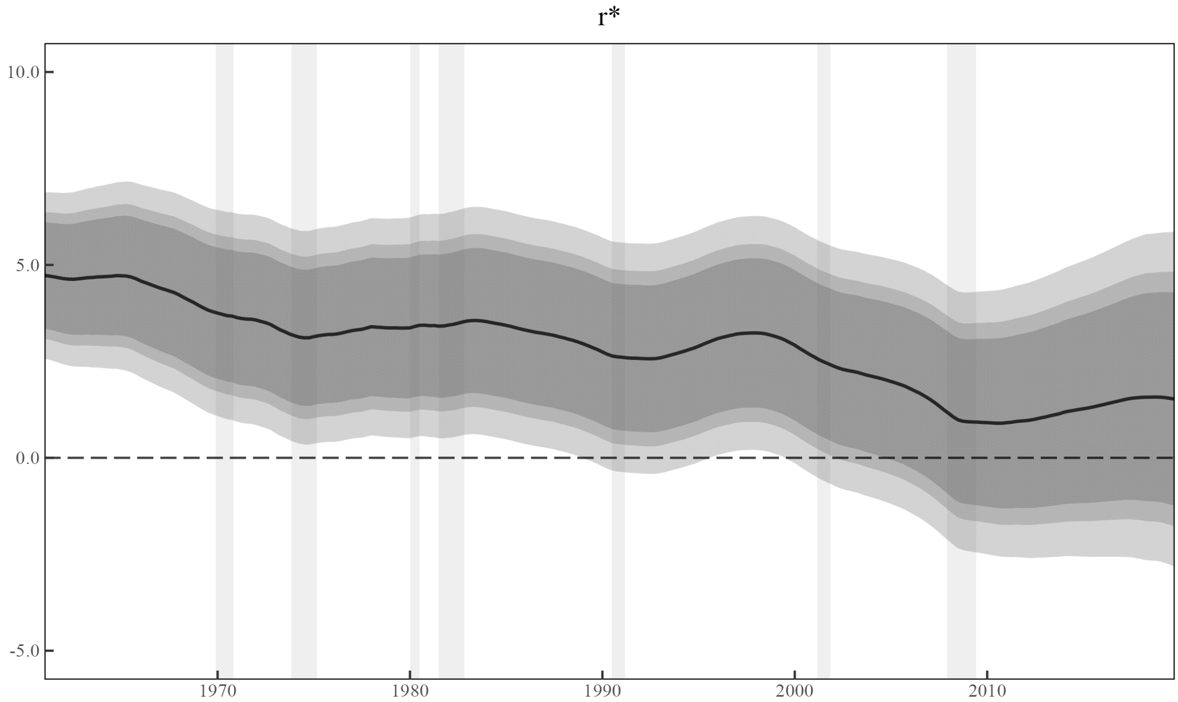

Figure 7. Actual Natural Rate of Interest

Note: Two-sided estimates of the natural rate of interest computed by the Kalman smoother in the Holston, Laubach and Williams (2017) model. Shaded regions represent the 90/95/99% confidence bands. Standard errors are derived using Hamilton's (1986) procedure accounting for both parameter and filter uncertainty.

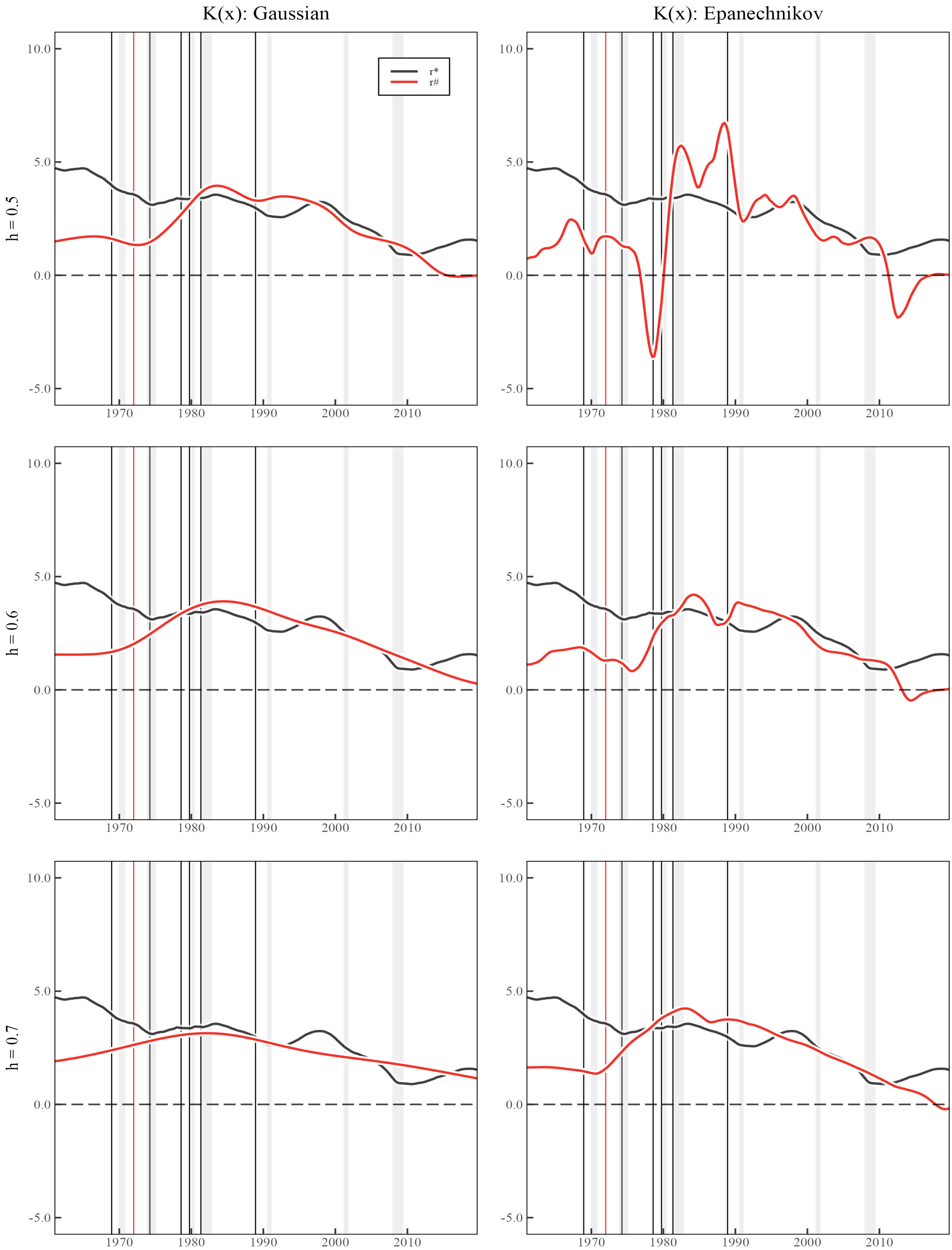

Figure 8. Equilibrium Interest Rates

Note: Time-varying estimates of r-star in black and r-hash in red for all combinations of kernel function K(x) and bandwidth parameter h. Vertical bars in black and red correspond to negative and positive exogenous monetary policy shocks respectively, outlined across research by Romer and Romer (1989; 1994; 2023).

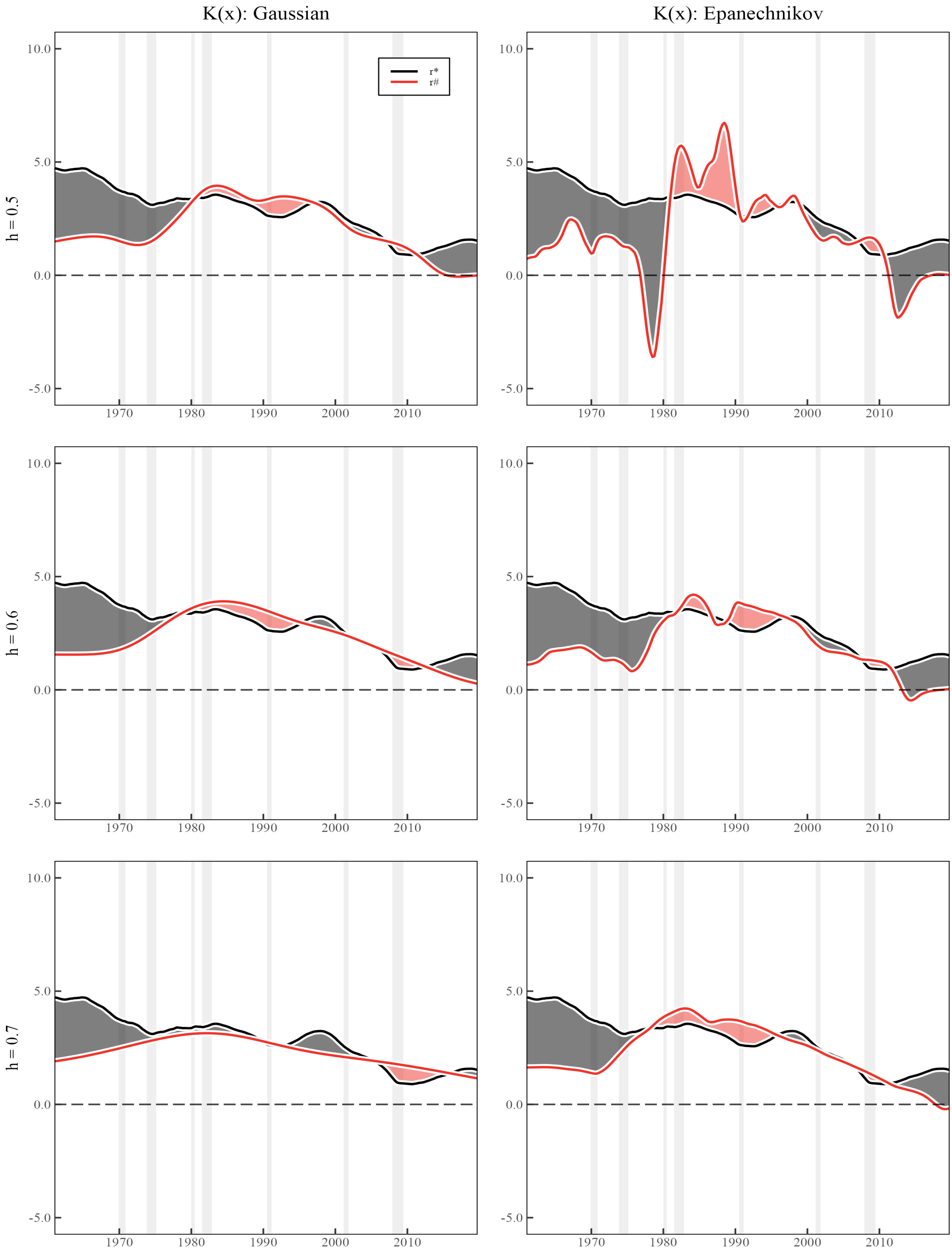

Figure 9. Monetary Policy Misperceptions

Note: Area of the deviations in r-hash (red) from r-star (black). Shaded regions in grey are associated with periods of underestimation (overly expansionary monetary policy), where r-hash < r-star. Shaded regions in red are associated with periods of overestimation (overly contractionary policy), where r-hash > r-star.

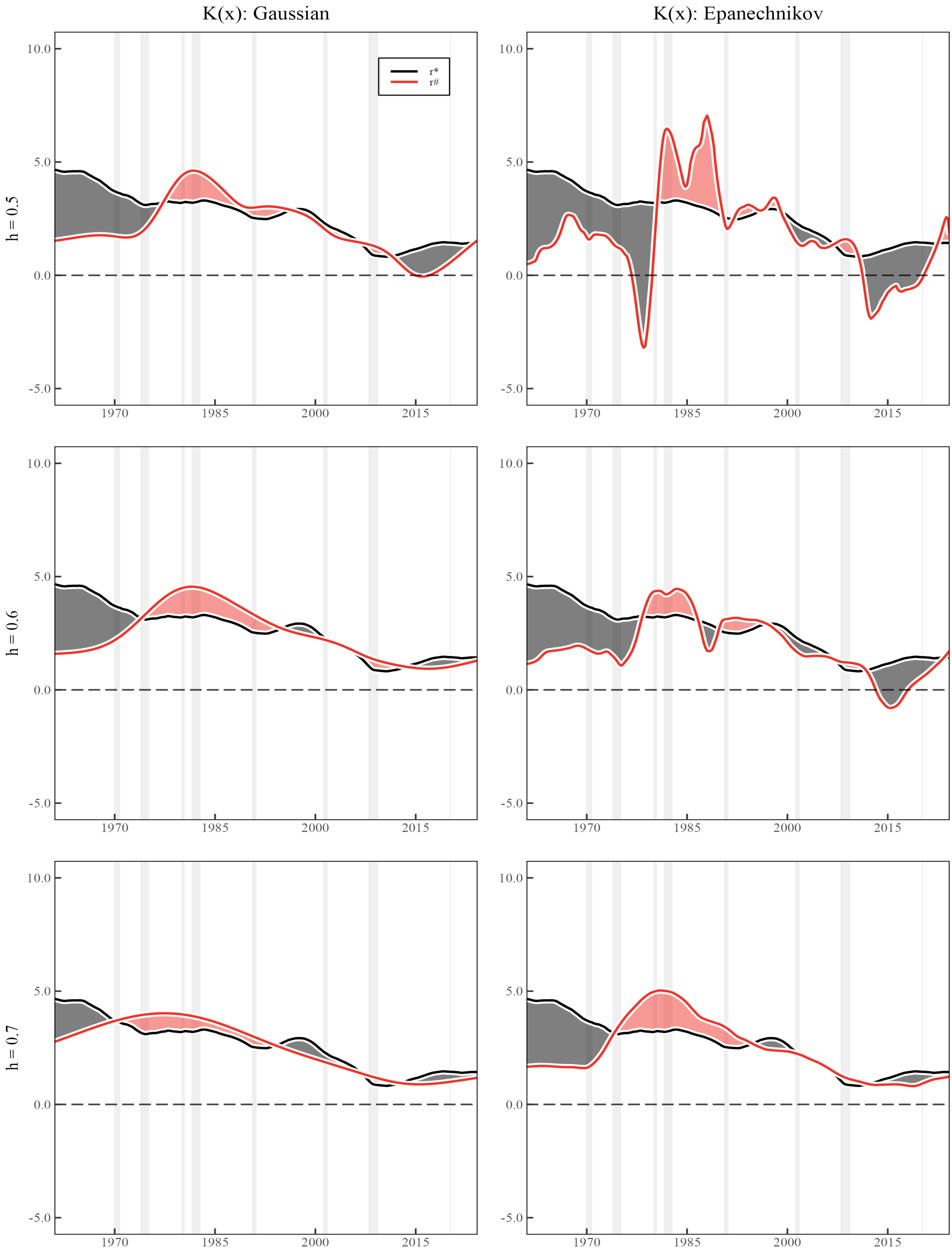

Figure 10. Extended Sample Estimates

Note: Deviations in r-hash (red) from r-star (black) over an extended sample size between 1961:1-2024:2. Shaded grey regions reflect periods of underestimation (overly expansionary policy), where r-hash < r-star. Shaded red regions reflect periods of overestimation (overly contractionary policy), where r-hash > r-star.

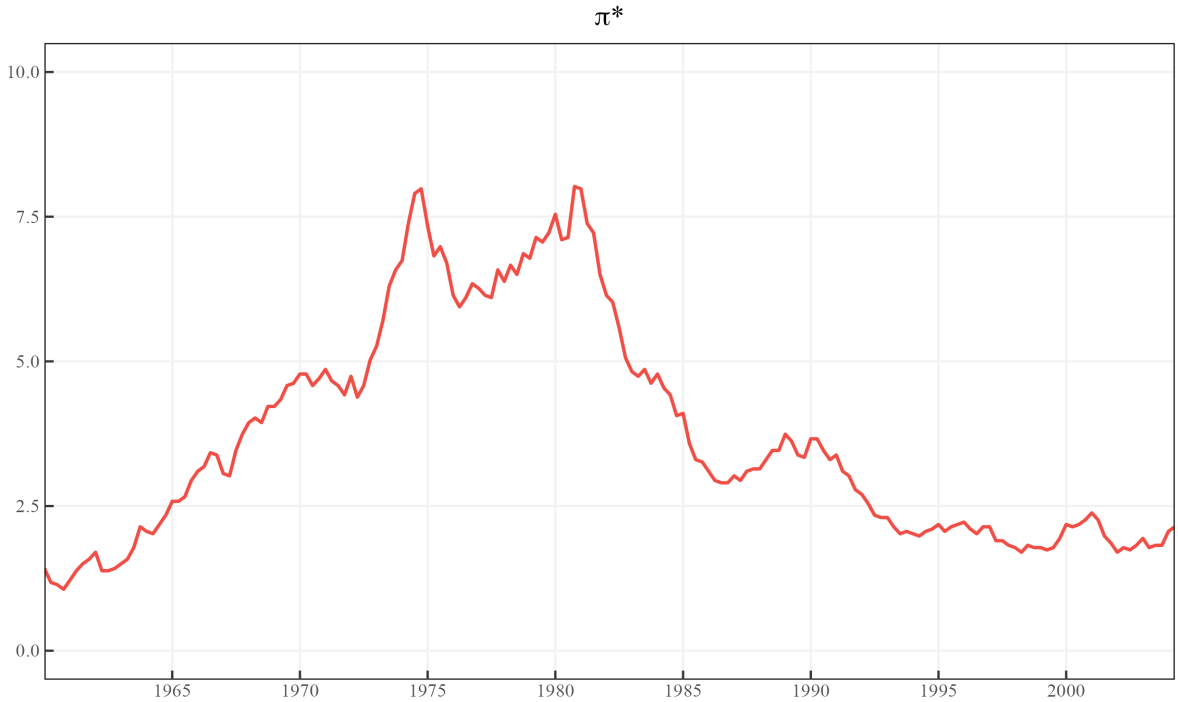

Figure 11. Time-Varying Inflation Target

Note: Estimated time-varying inflation target arising from the standard New Keynesian model as computed in Ireland (2007). The time series presented ranges between 1961:1-2004:2. The Federal Reserve's inflation target is subsequently assumed to be constant at 2% until 2012:1, after which it is observable with certainty.

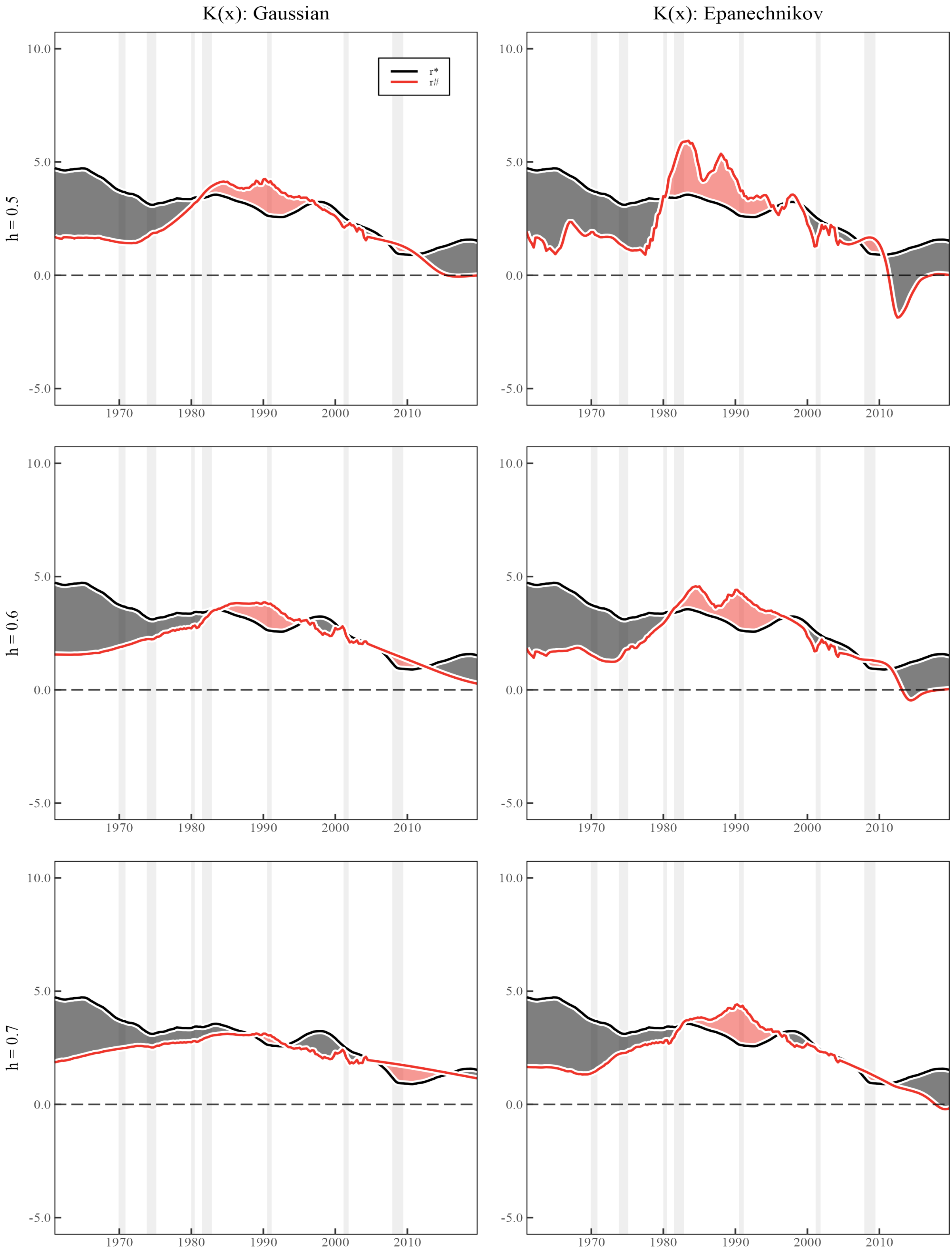

Figure 12. Time-Varying Target Consistent Estimates

Note: Deviations in r-hash given time-varying inflation targets (red) between r-star (black) 1961:1-2004:2. Shaded grey regions reflect periods of underestimation (overly expansionary policy), where r-hash < r-star. Shaded red regions reflect periods of overestimation (overly contractionary policy), where r-hash > r-star.

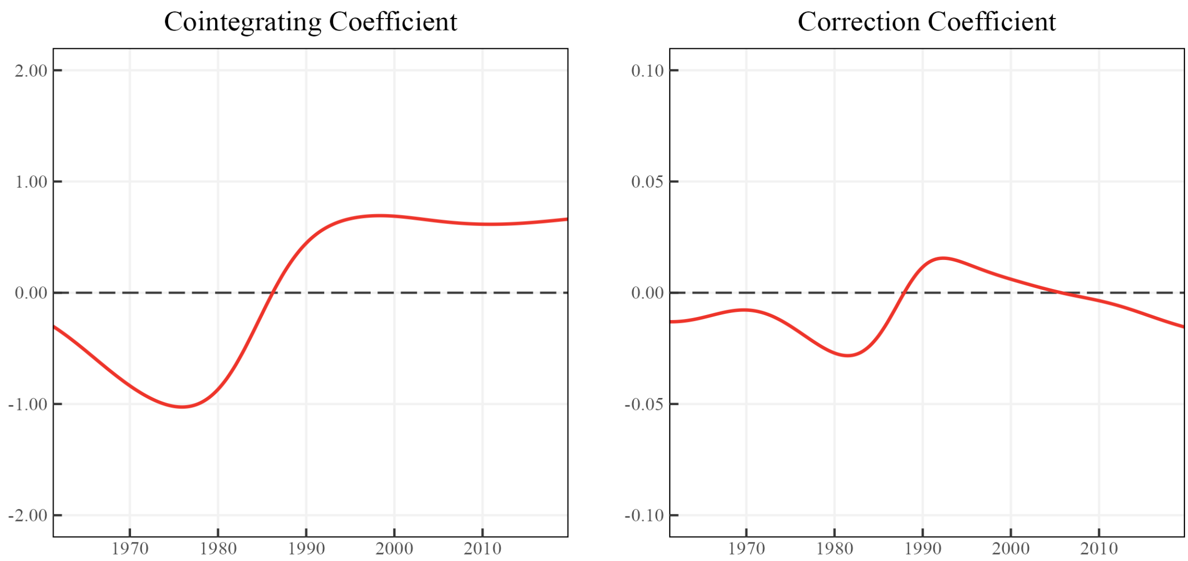

Figure 13. Time-Varying ECM Estimates

Note: Time-varying estimates of the cointegrating relation and error correction coefficient. K(x): Gaussian. h = 0.6. See Appendix F for estimates using all combinations of kernel function and bandwidth parameter.

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APPENDIX

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A Natural Rate of Interest

Stage I Specification:

$$\mathbf{y}_t = \begin{bmatrix} y_t, \pi_t \end{bmatrix}' \quad \mathbf{x}_t = \begin{bmatrix} y_{t-1}, y_{t-2}, \pi_{t-1}, \pi_{t-2,4} \end{bmatrix}' \quad \xi_t = \begin{bmatrix} y_t^*, y_{t-1}^*, y_{t-2}^* \end{bmatrix}'$$

$$\mathbf{H}' = \begin{bmatrix} 1 & -\phi_{y,1} & -\phi_{y,2} \\ 0 & -\varphi_y & 0 \end{bmatrix} \quad \mathbf{A}' = \begin{bmatrix} \phi_{y,1} & \phi_{y,2} & 0 & 0 \\ \varphi_y & 0 & \varphi_\pi & 1-\varphi_\pi \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} \sigma_{y*}^2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_1 = \begin{bmatrix} \phi_{y,1}, \phi_{y,2}, \varphi_\pi, \varphi_y, g, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*} \end{bmatrix}$$

Stage II Specification:

$$\mathbf{y}_t = \begin{bmatrix} y_t, \pi_t \end{bmatrix}' \quad \mathbf{x}_t = \begin{bmatrix} y_{t-1}, y_{t-2}, r_{t-1}, r_{t-2}, \pi_{t-1}, \pi_{t-2,4}, 1 \end{bmatrix}' \quad \xi_t = \begin{bmatrix} y_t^*, y_{t-1}^*, y_{t-2}^* \end{bmatrix}'$$

$$\mathbf{H}' = \begin{bmatrix} 1 & -\phi_{y,1} & -\phi_{y,2} & \phi_g \\ 0 & -\varphi_y & 0 & 0 \end{bmatrix} \quad \mathbf{A}' = \begin{bmatrix} \phi_{y,1} & \phi_{y,2} & \frac{\phi_r}{2} & \frac{\phi_r}{2} & 0 & 0 & \phi_0 \\ \varphi_y & 0 & 0 & 0 & \varphi_\pi & 1-\varphi_\pi & 0 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} \sigma_{y*}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (\lambda_g \sigma_{y*})^2 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_2 = \begin{bmatrix} \phi_{y,1}, \phi_{y,2}, \phi_r, \phi_0, \phi_g, \varphi_\pi, \varphi_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*} \end{bmatrix}$$

Stage III Specification:

$$\mathbf{y}_t = \begin{bmatrix} y_t, \pi_t \end{bmatrix}' \quad \mathbf{x}_t = \begin{bmatrix} y_{t-1}, y_{t-2}, r_{t-1}, r_{t-2}, \pi_{t-1}, \pi_{t-2,4} \end{bmatrix}'$$

$$\xi_t = \Big[y_t^*, y_{t-1}^*, y_{t-2}^*, g_{t-1}, g_{t-2}, z_{t-1}, z_{t-2}\Big]'$$

$$\mathbf{H}' = \begin{bmatrix} 1 & -\phi_{y,1} & -\phi_{y,2} & \frac{-\phi_r}{2} & \frac{-\phi_r}{2} & \frac{-\phi_r}{2} & \frac{-\phi_r}{2} \\ 0 & -\varphi_y & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{A}' = \begin{bmatrix} \phi_{y,1} & \phi_{y,2} & \frac{\phi_r}{2} & \frac{\phi_r}{2} & 0 & 0 \\ \varphi_y & 0 & 0 & 0 & \varphi_\pi & 1-\varphi_\pi \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad \mathbf{Q} = \begin{bmatrix} (1+\lambda_g^2)\sigma_{y*}^2 & 0 & 0 & (\lambda_g \sigma_{y*})^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (\lambda_g \sigma_{y*})^2 & 0 & 0 & (\lambda_g \sigma_{y*})^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (\frac{\lambda_z \sigma_{\tilde{y}}}{\phi_r})^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_3 = \Big[\phi_{y,1}, \phi_{y,2}, \phi_r, \varphi_\pi, \varphi_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*}\Big]$$

B Optimal Bandwidth Selection

Leave-One-Out Cross-Validation Approach

Consider the following error criterion for the estimator $\hat{g}(\cdot)$:

$$\text{ISE}[\hat{g}(\cdot)] = \int (\hat{g}(x) - g(x))^2 f(x) dx \tag{B.1}$$

where the objective function to be minimised is defined as:

$$\text{MISE}[\hat{g}(\cdot)|X_t] = \mathbb{E}[\text{ISE}[\hat{g}(\cdot)]|X_t] = \int \mathbb{E}[(\hat{g}(x) - g(x))^2|X_t] f(x) dx \tag{B.2}$$

The idea here is to use the sample twice in a cross-validatory way; to construct the estimator and to evaluate its performance, such that data used for the former is not used for the latter. The simplest approach is to compare Y_t with the *leave-one-out* estimate of g, that is computed barring the t-th datum (X_t, T_t), yielding:

$$\text{CV}(h) = \frac{1}{T} \sum_{t=1}^{T} \big(Y_t - \hat{g}_{-t}(X_t)\big)^2 \tag{B.3}$$

Each time one observation is omitted, the remaining observation points are used to fit the data and predict the omitted value. Cross-validation is therefore a method to estimate the prediction error, which approximates the error criterion. As this may yield several minima, the loss curve is graphed to identify a global solution.

C Time-Varying Parameter Estimates

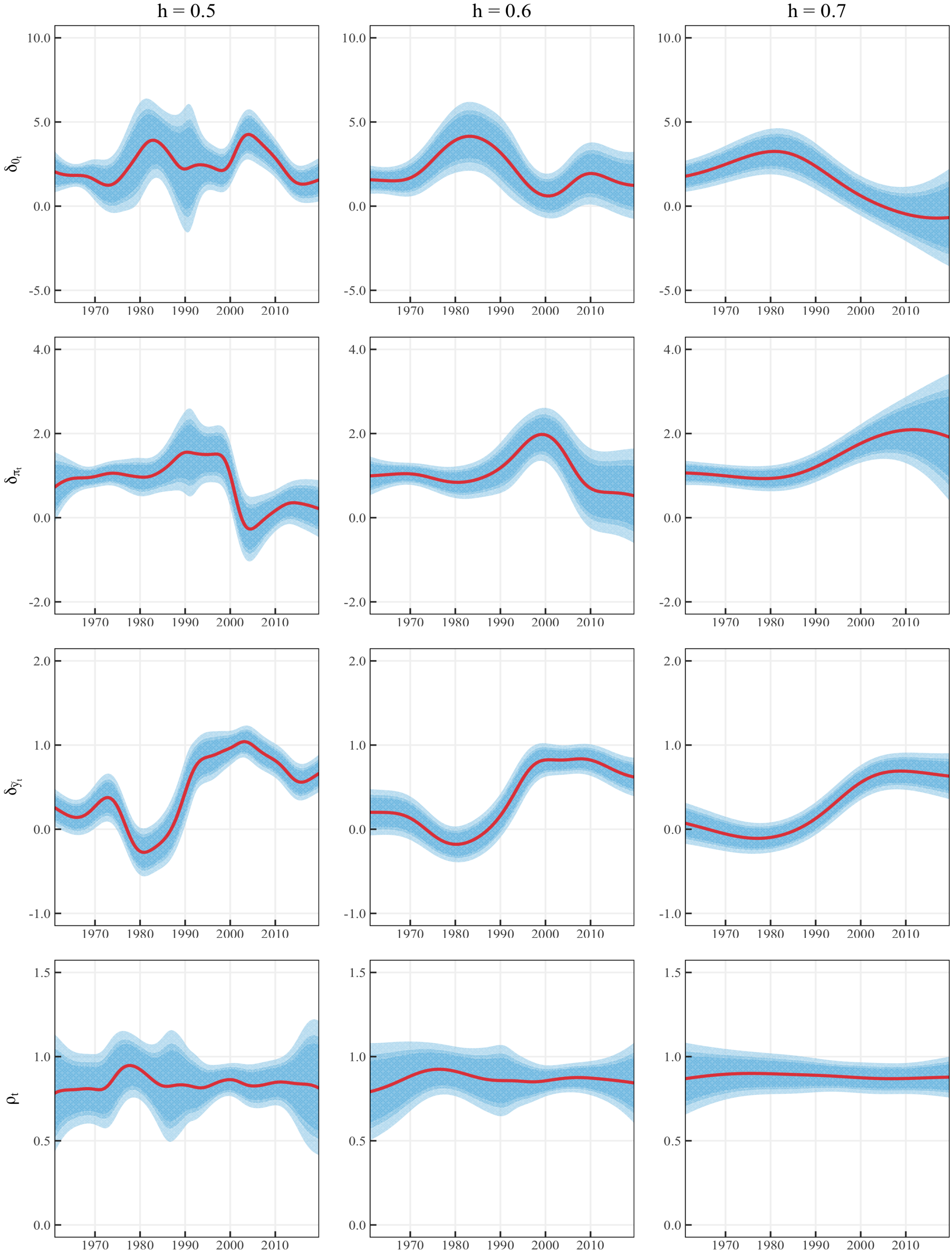

Figure C.1. Time-Varying Parameters, K(x): Gaussian

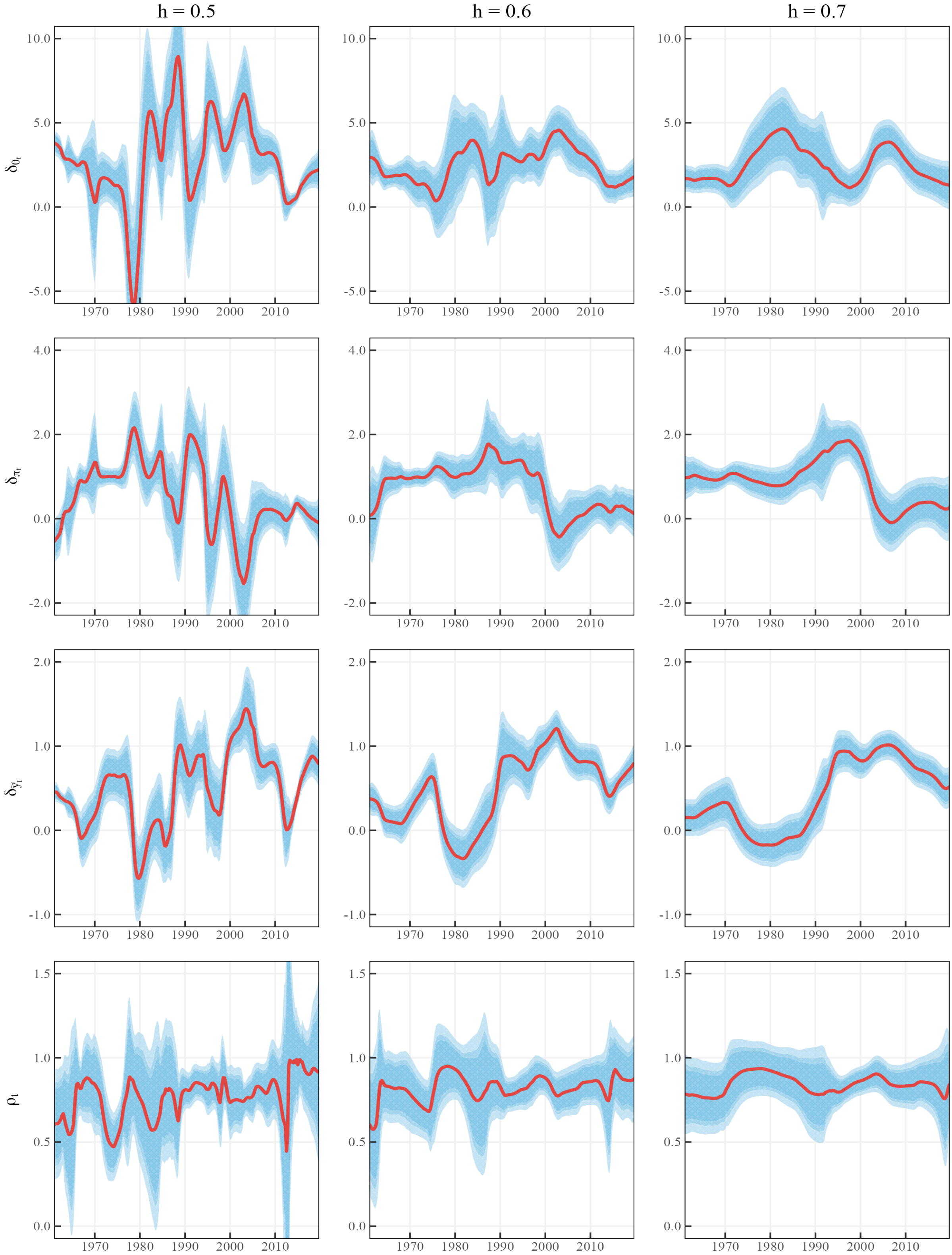

Figure C.2. Time-Varying Parameters, K(x): Epanechnikov

D Time-Varying J-Test Statistics

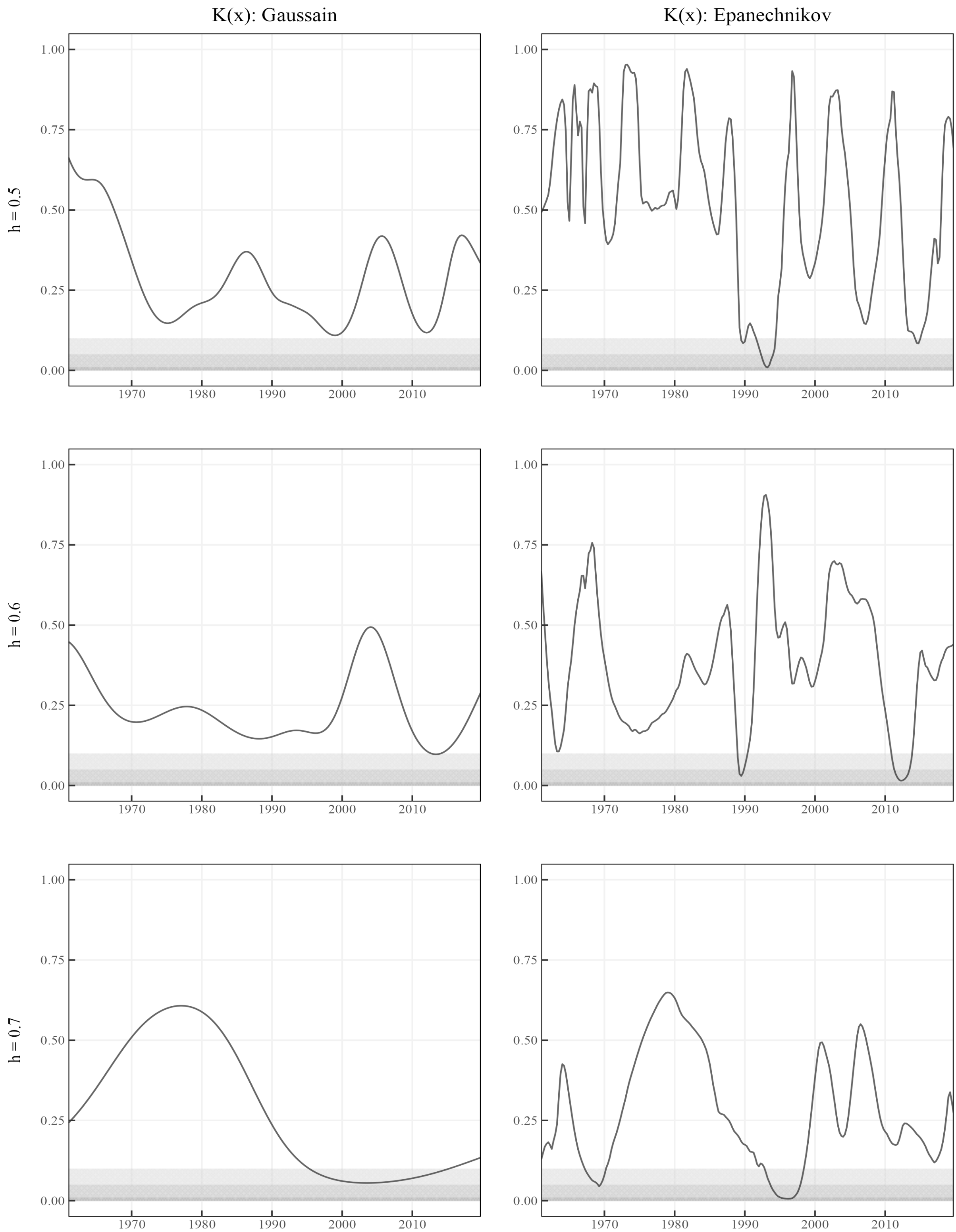

Figure D.1. Sargan-Hansen J-Test Statistics (p-value)

E Subsample Averages

Table E.1. Subsample Averages, K(x): Gaussian

	Pre-Volcker 1960:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2017:4	Yellen-Powell 2014:1-2019:4
			Bandwidth Parameter: $h = 0.5$		
$\bar{\delta}_{\pi}$	0.96 (0.15)	1.05 (0.27)	0.76 (0.28)	0.20 (0.21)	0.30 (0.23)
$\bar{\delta}_{\tilde{y}}$	0.18 (0.10)	0.50 (0.11)	0.80 (0.10)	0.73 (0.08)	0.59 (0.08)
$\bar{\rho}$	0.84 (0.09)	0.84 (0.07)	0.84 (0.06)	0.84 (0.08)	0.83 (0.13)
$\bar{J}$	0.37	0.24	0.22	0.27	0.35
			Bandwidth Parameter: $h = 0.6$		
$\bar{\delta}_{\pi}$	0.98 (0.12)	1.38 (0.23)	1.34 (0.29)	0.72 (0.37)	0.57 (0.40)
$\bar{\delta}_{\tilde{y}}$	0.07 (0.04)	0.37 (0.09)	0.65 (0.08)	0.77 (0.08)	0.67 (0.08)
$\bar{\rho}$	0.87 (0.08)	0.87 (0.06)	0.86 (0.05)	0.87 (0.05)	0.85 (0.07)
$\bar{J}$	0.37	0.33	0.34	0.29	0.27
			Bandwidth Parameter: $h = 0.7$		
$\bar{\delta}_{\pi}$	0.99 (0.11)	1.39 (0.15)	1.72 (0.21)	2.06 (0.36)	2.00 (0.49)
$\bar{\delta}_{\tilde{y}}$	0.04 (0.08)	0.26 (0.08)	0.48 (0.08)	0.68 (0.09)	0.65 (0.10)
$\bar{\rho}$	0.89 (0.06)	0.88 (0.04)	0.88 (0.04)	0.87 (0.04)	0.88 (0.04)
$\bar{J}$	0.48	0.24	0.11	0.08	0.11

Table E.2. Subsample Averages, K(x): Epanechnikov

	Pre-Volcker 1960:1-1979:2	Volcker-Greenspan 1979:3-2005:4	Greenspan-Bernanke 1987:3-2013:4	Bernanke-Yellen 2006:1-2017:4	Yellen-Powell 2014:1-2019:4
			Bandwidth Parameter: $h = 0.5$		
$\bar{\delta}_\pi$	0.89	0.62	0.29	0.10	0.10
	(0.21)	(0.43)	(0.38)	(0.16)	(0.14)
$\bar{\delta}_{\tilde{y}}$	0.33	0.57	0.77	0.55	0.64
	(0.13)	(0.19)	(0.17)	(0.11)	(0.08)
$\bar{\rho}$	0.70	0.76	0.79	0.84	0.94
	(0.12)	(0.10)	(0.10)	(0.16)	(0.13)
$\bar{J}$	0.66	0.49	0.42	0.34	0.38
			Bandwidth Parameter: $h = 0.6$		
$\bar{\delta}_\pi$	0.91	0.93	0.62	0.20	0.22
	(0.16)	(0.29)	(0.28)	(0.20)	(0.20)
$\bar{\delta}_{\tilde{y}}$	0.24	0.54	0.83	0.68	0.60
	(0.09)	(0.13)	(0.12)	(0.08)	(0.07)
$\bar{\rho}$	0.79	0.83	0.83	0.85	0.87
	(0.10)	(0.07)	(0.06)	(0.08)	(0.11)
$\bar{J}$	0.33	0.44	0.42	0.30	0.36
			Bandwidth Parameter: $h = 0.7$		
$\bar{\delta}_\pi$	0.96	1.17	0.94	0.22	0.20
	(0.14)	(0.24)	(0.26)	(0.25)	(0.20)
$\bar{\delta}_{\tilde{y}}$	0.12	0.46	0.76	0.79	0.60
	(0.10)	(0.10)	(0.09)	(0.08)	(0.09)
$\bar{\rho}$	0.84	0.85	0.84	0.84	0.82
	(0.08)	(0.06)	(0.06)	(0.06)	(0.10)
$\bar{J}$	0.30	0.28	0.23	0.26	0.20

F Cointegration

Table F.1. ECM Estimates, K(x): Gaussian

Bandwidth (h)	0.5	0.6	0.7
Cointegrating vector (S.E.)	0.46 (0.06)	0.41 (0.06)	0.30 (0.03)
Error correction coefficient (S.E.)	-0.02 (0.01)	-0.04 (0.02)	-0.01 (0.01)
Granger test (p)	0.03	< 0.01	0.05
ADF test (p)	0.58	0.41	0.10

Table F.2. ECM Estimates, K(x): Epanechnikov

Bandwidth (h)	0.5	0.6	0.7
Cointegrating vector (S.E.)	0.51 (0.10)	0.40 (0.06)	0.41 (0.05)
Error correction coefficient (S.E.)	0.01 (0.01)	-0.04 (0.01)	-0.05 (0.02)
Granger test (p)	0.02	< 0.01	0.04
ADF test (p)	0.48	0.84	0.75

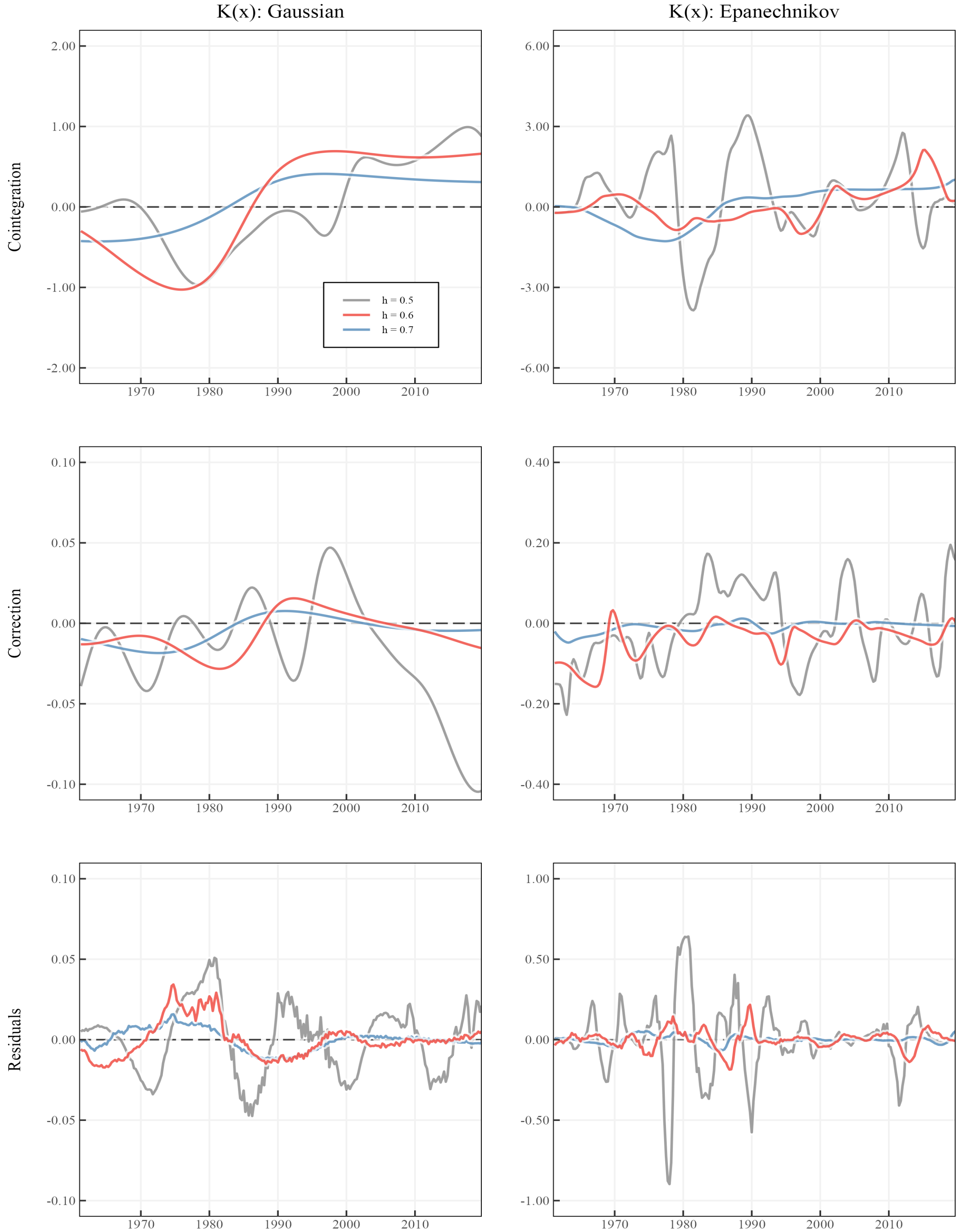

Figure F.1. Time-Varying ECM Estimates

G Critical Dates

Table G.1. NBER Dates

Peak	Trough
April 1960	February 1961
December 1969	November 1970
November 1973	March 1975
January 1980	July 1980
July 1981	November 1982
July 1990	March 1991
March 2001	November 2001
December 2007	June 2009

Table G.2. RR Dates

Shock	Sign
October 1947	- ve
August 1995	- ve
September 1958	- ve
December 1968	- ve
January 1972	+ ve
April 1974	- ve
August 1978	- ve
October 1979	- ve
May 1981	- ve
December 1988	- ve