Longer-Run Equilibrium Interest Rates: Evidence From The United Kingdom*

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Abstract

The natural rate of interest (r*) has recently been in decline, yet little is known about its historical variation. This paper estimates r* over more than three centuries in the United Kingdom between 1700-2019. Results suggest that the natural rate of interest rose persistently during much of the eighteenth and nineteenth century, and only began its decline around the turn of the twentieth. Historical variance decompositions suggest structural changes in productivity, demography and risk are largely responsible for this reduction. I argue that secular stagnation is unique to contemporary history, insofar as r* was ascending across the Late Modern Era.

Keywords: Natural rate of interest, Secular stagnation, Kalman filter, Vector Autoregression

JEL classification: C32, E43, E52, N13, N33, N43

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1 Introduction

The natural rate of interest (r*) has received significant attention over recent years. As the long-run equilibrium interest rate that is consistent with output and inflation stability, it has understandably played an increasingly important role in the conduct of monetary policy. The concept of r* is first introduced by Wicksell (1898), broadly defined as the rate of interest associated with stable prices. Despite its profundity, the Wicksellian rate faded into obscurity for many years before being popularised by Woodford (2003) within the New Keynesian framework. Since its reintroduction into modern mainstream macroeconomic theory, r* has become a key criterion in the setting of policy rates, such that they are neither overly contractionary nor expansionary.

The importance of measuring the natural rate of interest ex-post is clear. Understanding how it has evolved in the past gives us a deeper insight into long-run trends and structural changes in its determinants such as shifts in productivity, demography and savings and investment decisions, each relating crucially to the state of the macroeconomy. Estimates of the natural rate of interest also allow us to assess the historical conduct of monetary policy and the extent to which policy-makers are effectively regulating output and inflationary pressures, yielding an important tool for policy evaluation and optimal policy rate determination.

Given its theoretical nature, the long-run equilibrium real interest rate must be estimated rather than directly observed. Furthermore, persistent declines in time-varying estimates of r* have been widely documented across the advanced world, significantly limiting both the scope and conduct of monetary policy. However, much of the existing literature has focused exclusively on trends in the natural rate of interest and its core drivers across contemporary history, passing little judgment on the evolution of the equilibrium interest rate prior to this.

A number of significant debates concerning long-run growth pertain to these periods and the natural rate of interest, particularly surrounding theories of secular stagnation, in which economies exhibit persistently low rates of growth in output due to anaemic aggregate demand. Most notably, Summers (2014a; 2014b; 2015a; 2015b) connects this hypothesis first proposed by Hansen (1939) to the concept of r*, arguing that within a low neutral rate environment, savings exceed investment, generating shortfalls in demand and stifling growth. In this regard, declining r-stars are considered a key symptom of secular stagnation. Rather interestingly, whilst Summers (2015a) refers mainly to the experience of advanced economies after the Great Recession, Hansen's (1939) reference to the period following the Great Depression has yet to be connected to equilibrium interest rates. Whilst r* is being used to gauge this hypothesis, it is surprising that few have estimated its variation prior to the contemporary era, in which theories of secular stagnation have been revived.

Many of these discussions concerning secular stagnation have primarily concerned the United States. However, the experience of the United Kingdom along with the industrialised world more generally has also been of interest to the literature recently (e.g. Crafts, 2014; Jimeno et al. 2014; Rachel and Summers, 2019), particularly due to common experiences felt after the Great Recession and similar long-run trends that are exerting downward pressures on the equilibrium interest rate across the advanced world (Blanchard et al. 2014). However, these trends are recent phenomena and have not persisted for centuries, suggesting r* may not have always been in decline.

Given the availability of rich historical data in the United Kingdom, estimating the natural rate of interest over an extended sample allows us to determine the uniqueness of its variation over recent years. Appreciating the historical trend would therefore better inform us of the wider context in which declines in r* are taking place and, perhaps more importantly, if this has always been the case. In addition, popular theories that attempt to explain these trends in terms of long-run shifts in the supply and demand for savings such as the secular stagnation hypothesis, would benefit from further historical scrutiny in the context of the United Kingdom, allowing us to determine if such a phenomenon is truly restricted to the contemporary period.

This study is therefore primarily concerned with the estimation of the natural rate of interest and its drivers. As for the former, the existing literature is somewhat divided on how best to estimate r*, which can be broadly categorised into nonstructural, structural, and semi-structural models. Nonstructural approaches involve time-series techniques ranging from simple univariate models to more sophisticated multivariate models. For instance, Lubik and Matthes (2015) use Bayesian estimation methods to measure the equilibrium interest rate within a time-varying parameter vector autoregression (TVP-VAR) model, which is arguably the most widely cited in its class of models to estimate the equilibrium interest rate. These methods generally impose minimal restrictions and are largely agnostic about relationships between macroeconomic fundamentals, yet still conform to the traditional Wicksellian conception of estimating the real interest rate that would prevail over the long-run. Notwithstanding, structural relationships for which clear theoretical evidence exists are clearly not reflected in these estimates of the natural rate of interest.

Structural models involve theoretically motivated restrictions that are imposed in the specification of the model. Given the identification of deep parameters and mechanisms that either partially or totally describe preferences and constraints, they provide counterfactual predictions to allow for policy evaluation. For instance, Del Negro et al. (2017) define the natural rate of interest as the real rate of interest that prevails in the absence of rigidities in the market. Insofar as these frictions are the reason for policy effectiveness, r* is interpreted as the counterfactual rate that would be observed devoid of monetary policy. Building upon Christiano et al. (2005) and Smets and Wouters (2007), the authors incorporate several frictions and structural shocks outlined in Bernanke et al. (1999), Justiano et al. (2010), and Christiano et al. (2014), in addition to the zero lower bound and forward guidance. The authors estimate a dynamic stochastic general equilibrium (DSGE) model using standard Bayesian techniques, which is also arguably perhaps the mostly widely cited in its class of models to estimate the short-run natural rate of interest.

Such methods generally impose numerous restrictions on decision-making and often involve the estimation of highly parameterised models. The resulting estimates of r* are therefore sensitive to assumptions and may not be robust to model uncertainty. Whilst consistent with the Wicksellian definition, this approach tends to focus on the higher frequency components of r* over the business cycle. Although general equilibrium models recover the precise counterfactual that define r* as the rate achieving price stability period-by-period, it is often modelled as a stationary linear combination of transitory shocks to preferences and technology. However, parameters typically considered fixed in the steady state to derive log-linear approximations, such as the rate of time-preferences or growth rate of technology, may themselves be subject to highly persistent shifts.

Semi-structural models identify only a subset of these parameters and mechanism rather than the complete counterfactuals. In doing so, they impose relatively more restrictions than nonstructural models, but are far less parameterised than fully specified structural models. The advantage of semi-structural approaches lies in neither imposing too many restrictions nor too few, rather what is sufficiently necessary to estimate the natural rate of interest. In doing so, these flexible models are able to account for some aspect of both lower and higher frequency components of the equilibrium interest rate that might ordinarily be overlooked by nonstructural and structural approaches respectively. This medium frequency conception or r* strikes an optimal balance between the two extreme approaches and can account for a range of underlying shifts in the natural rate that may otherwise be overlooked by alternative models.

Perhaps the most widely celebrated semi-structural approach used to measure the natural rate of interest is the Laubach and Williams (2003) and Holston, Laubach and Williams (2017) model, estimated in a multistage maximum likelihood procedure wherein the likelihood function is computed by the Kalman (1960) filter. This particular model focuses primarily on the medium-long run fluctuations in the equilibrium interest rate that are influenced by low-frequency nonstationary processes. In what are now a seminal set of results, the authors find that the natural rate of interest has been in secular decline across the advanced world since the 1960s.

These empirical findings are robust to a number of extensions (e.g. Clark and Kozicki, 2005; Mésonnier and Renne, 2007; Trehan and Wu, 2007; Berger and Kempa, 2014; Lewis and Vazquez-Grande, 2018). However, a consistent theme across research inspired by this particular methodology are imprecise estimates due to filter and parameter uncertainty. Despite a range of alternative models and estimation techniques, including more structural approaches (e.g. Barsky, Justiniano and Melosi, 2014; Kiley, 2015; Lubik and Matthes, 2015; Hamilton et al., 2016; Johannsen and Mertens, 2016; Pescatori and Turunen, 2016; Christensen and Rudebusch, 2019; Krustev, 2019) no other dominant alternative presents itself in the existing literature, and as such, the model has become a key point of reference in estimating the natural rate of interest.

It is clear from the existing literature that regardless of methodology, r* is seldom estimated prior to the twentieth century. Amongst the few studies that have, Jordà et al. (2022) investigate the long-run equilibrium real interest rate across Europe over a sample stretching back as far as the fourteenth century. The crux of their analysis is on the consequences of pandemics, which, unlike wars, can depress the natural rate for decades. Whilst not the focus of their research, r* is modelled as a simple random walk, argued to be flexible enough to capture secular trends. Whilst this may be tractable over many centuries, time variation in the medium-long run frequency component of the equilibrium interest rate would require a more intricate approach. Hamilton et al. (2016) also conduct an extensive study of real interest rates in the United States, albeit this does not extend beyond the 1800s, nor seek to filter out the latent natural rate from the historical data. Instead, the authors use moving averages of ex-post rates to gauge r*, whilst acknowledging this to be a noisy measure of the theoretical rate consistent with output and inflation stability.

In a related strand of literature, a number of studies have attempted to measure the real interest rate over an even longer horizon using similar techniques. Perhaps of most relevance to this paper is the extensive work of Schmelzing (2020), in which moving averages of real rates are shown to exhibit 'suprasecular' decline since the fourteenth century, maintaining what is argued to be their true historical trend and calling into question the narrative that secular stagnation is a uniquely recent phenomenon affecting the advanced world. Whilst informative, there are important caveats to this methodology. Moving averages of real rates used to infer upon the natural rate of interest by allowing for persistent change are often characterised by large shifts that last for decades. In other words, this approach struggles to separate long-term from short-term variation in the natural rate of interest during periods of harsh volatility. In particular, movements in inflation and output are not explicitly accounted for by reduced-form univariate approaches and are therefore not entirely consistent with the Wicksellian interest rate that prevails during medium-long run pressures, which are particularly important throughout the historical sample. In this regard, there are strong reasons to use multivariate filtering techniques to estimate r* over a longer horizon.

Given its centrality, I maintain and adapt the HLW (2017) framework to estimate the longer-run equilibrium interest rate using a broad series of macroeconomic data for the United Kingdom. In particular, this paper recasts the state-space system into a form more suitable for annual data whilst preserving the fundamental relationship between state variables of the model. These modifications relate entirely to the lag structure found in the dynamic Investment-Savings (IS) curve and Phillips Curve (PC), which is assumed to be adaptive as opposed to a linear combination of multiple lagged quarters. Using this adjusted empirical specification, I estimate the natural rate of interest across a rich sample that ranges more than three centuries in the United Kingdom.

As for the causes of such decline in the natural rate of interest, existing research has emphasised the importance of particular macroeconomic trends that have recently influenced the long-run supply and demand for savings (e.g. Carvalho et al., 2016; Gordon, 2016; Rachel and Smith, 2017; Eggertsson et al., 2019; Rachel and Summers, 2019; Gagnon et al., 2021). These particular trends include but are not restricted to; adverse changes in demography, slowing technological progress and innovation, falling (relative) prices of investment goods, an increasing scarcity of safe assets, stronger risk aversion and greater capital and income inequality. This paper seeks to explain the historical variation of r* in terms of factors for which data is available in the United Kingdom. Incidentally, these variables are also at the centre of recent literature concerning the primary drivers of the neutral rate such as demography, productivity, debt, and risk.

Existing research suggests that the natural rate of interest has been in decline over much of the last century, but have these trends historically been the case? What behaviour does the equilibrium interest rate exhibit prior to this period? I find that r* has not always been in secular decline and was in fact persistently rising between much of the eighteenth and nineteenth century, reversing its trend around the turn of the twentieth due to structural shifts in its core drivers. I argue secular stagnation is restricted to contemporary history, insofar as it contrasts itself to the Late Modern Era in terms of the evolution of the natural rate. In doing so, I provide evidence that recent declines in r* are unique and reflect deeper structural shifts that do indeed warrant concern.

The remainder of this article is structured as follows. Section 2 outlines the theoretical framework used to estimate the longer-run equilibrium interest rate. Section 3 outlines the historical data used in the estimation. Section 4 discusses particulars concerning the empirical implementation. Section 5 presents and discusses the main findings of the paper. Finally, Section 6 concludes.

2 Theoretical Framework

This paper uses the HLW (2017) model to estimate r*. In particular, the canonical New Keynesian model is relaxed by specifying reduced equations of the Investment-Savings (IS) curve and Phillips Curve (PC) that permit shocks to the output gap $\tilde{y}_t = y_t - y_t^*$ and inflation π_t given as follows:

$$\tilde{y}_t = \phi_y(L)\tilde{y}_t + \phi_r(L)\tilde{r}_t + \epsilon_{\tilde{y},t} \tag{1}$$

$$\pi_t = \varphi_\pi(L)\pi_t + \varphi_y(L)\tilde{y}_t + \epsilon_{\pi,t} \tag{2}$$

where $\tilde{r}_t$ is the real interest rate gap given by deviations in the real interest rate from the natural rate of interest $r_t - r_t^*$, and the error terms $\epsilon_{\tilde{y},t}$ and ϵ_{π_t} capture any transitory shocks to output and inflation respectively. The law of motion for natural rate of interest is given by the following:

$$r_t^* = \theta_g g_t + z_t \tag{3}$$

where g_t is the trend growth rate of the natural rate of output y_t^* and z_t is the error term capturing unobserved factors driving the real equilibrium interest rate. This relation follows directly from the textbook Ramsey optimal growth model.[1] The transition equations of the system are as follows:

$$y_t^* = (L)y_t^* + (L)g_t + \epsilon_{y*,t} \tag{4}$$

$$g_t = (L)g_t + \epsilon_{g,t} \tag{5}$$

$$z_t = (L)z_t + \epsilon_{z,t} \tag{6}$$

where equation (4) defines potential output as a random walk with drift g_t, which itself is defined as a random walk in equation (5), as is the unobserved component of the natural rate of interest in equation (6). It is assumed that the error terms in transition equations (4) to (6) are contemporaneously uncorrelated and normally distributed, with the following variance-covariance matrix:

$$\begin{bmatrix} \sigma_{\tilde{y}}^2 & 0 & \cdots & \cdots & 0 \\ 0 & \sigma_\pi^2 & \ddots & & \vdots \\ \vdots & \ddots & \sigma_{y*}^2 & \ddots & \vdots \\ \vdots & & \ddots & \sigma_g^2 & 0 \\ 0 & \cdots & \cdots & 0 & \sigma_z^2 \end{bmatrix} \tag{7}$$

The general structure is therefore similar to the HLW (2017) system except for the order of the lag polynomials within the state equations given the use of lower frequency data. These adjustments to the lag structure of the IS and PC equations are further detailed in the specification of the model.

[1] This result is contingent on the assumption of a representative household with a CES utility function and constant relative risk aversion, which is the inverse of the intertemporal elasticity of substitution in consumption. In steady state, optimisation yields r* as a function of the growth rate of per capita consumption and the rate of time preference.

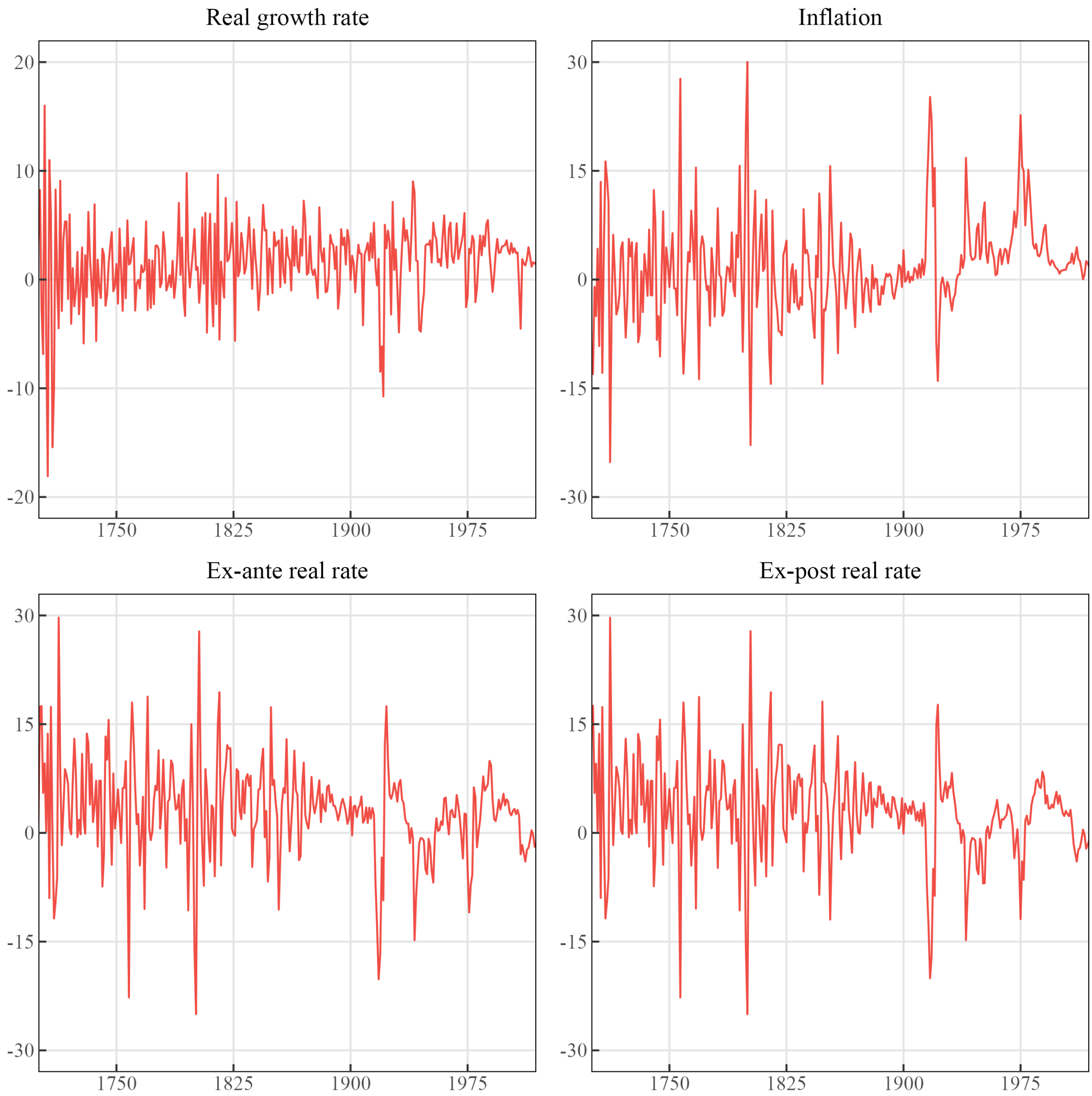

Figure 1. Historical growth, inflation and interest rates

Note: Annual historical data for the United Kingdom between 1700-2019 in red. The real growth rate of output is the rate of change in real gross domestic product. Inflation is the rate of change in the consumer price index. Ex-post real rates are computed by subtracting the realised inflation rate from nominal interest rates. Ex-ante real rates are computed using the past lag of inflation as a proxy for inflation expectations.

3 Data

Given data used in this paper spans over three centuries, it is useful to categorise the sample into periods of significance that facilitate our analysis. In this regard, the years prior to 1750 shall be referred to as the Early Modern Era and the years after this shall be referred to as the Late Modern Era. Within the Late Modern Era, the First Industrial Revolution takes place between 1760 to 1840 and the Second Industrial Revolution is dated between 1870 to 1914. As is known, World War I is dated between 1914 to 1918, the Great Depression occurs between 1929 to 1939, and World War II between 1939 to 1945. The Contemporary Era begins at the end of our sample, from 1945, and is a period in which r* has been well documented (see Holston et al., 2017).

It is worth noting that this analysis does not extend beyond 2019 due to the sharp effects of the Great Lockdown that was enforced in reaction to the coronavirus pandemic. Holston, Laubach and Williams (2023) correctly argue that such an unprecedented extreme-tailed event may violate the Kalman filter, in which stochastic innovations are assumed Gaussian in distribution. Furthermore, transitory shocks in the Phillips curve are also assumed serially uncorrelated, which is inconsistent with responses to the pandemic. Despite methods to account for these unique features of the data as outlined in Holston, Laubach and Williams (2023),[2] there has been considerable uncertainty in estimates of the natural rate of interest immediately following the Great Lockdown (refer to Baker et al. 2023; Benigno et al. 2024). Given that the primary focus of this research is on the historical evolution of r* and its long-run drivers, this period is omitted from the sample.

This paper exploits a broad series of annual data published by the Bank of England to estimate the equilibrium interest rate between 1700 to 2019. Nominal short-term interest rates are sourced from the Official Bank Rate (OBR) series published by the Bank of England. The Consumer Price Index (CPI) is equated to the Shumpeter-Gilboy Index between 1700 to 1750 (see Mitchell, 1988), sourced from Crafts and Mills (1994) between 1750 to 1770, Feinstein (1991; 1998) between 1770 to 1914 and the Office for National Statistics (ONS) between 1914 to 2019 (see O'Donoghue et al., 2004). Finally, Real GDP is sourced from the geographically-consistent real headline series; a chained volume measure constructed from an array of sources (in particular, refer to Feinstein, 1972; Solomou and Weale, 1991; Sefton and Weale, 1995; Broadberry et al., 2015). This data is collated meticulously in a central database by Thomas and Dimsdale (2017).[3]

I use the past lag of inflation as a proxy for inflation expectations to derive the ex-ante interest rate. This is consistent with the approach adopted in Holston et al. (2017), albeit my results are also robust to using one year ahead inflation expectations in Thomas and Dimsdale (2017), composed largely of NIESR inflation forecasts. As this time-series only extends to 1959, expectations prior to this are simply equal to their outturn for that year. This implies that for much of the sample, ex-post interest rates using realised inflation were computed instead. Given the long horizon of this sample and low frequency of the annual data, I did not find significant discrepancies between estimates of r* using these inflation-adjusted interest rates and thus choose to use the lag of inflation to compute ex-ante interest rates within the measurement equations of the model.

Figure 1 plots the historical series for real GDP, CPI inflation, ex-post interest rates, and ex-ante interest rates (refer to Appendix F for moving averages). These series clearly exhibit substan-

[2] In particular, the adjustment includes a persistent albeit temporary supply shock that captures the effects of lockdown restrictions due to the pandemic, which is equated to quarterly averages of the Government Response Stringency Index taken from the Oxford COVID-19 Government Response Tracker (OxCGRT). The adjustment to the natural rate of output is given by $y_t^* + \zeta \frac{d_t}{100}$, where ζ is an estimated coefficient that captures the effects of the COVID-19 indicator d_t on output. The output gap is then expressed using the adjusted natural rate of output: $100 \cdot (y_t - y_t^*) - \zeta d_t$.

[3] Refer to the '*Millennium of Macroeconomic Data for the United Kingdom*' OBRA dataset published by the Bank of England (Thomas and Dimsdale, 2017). Much of the data used to estimate the longer-run equilibrium interest rate is found within this historical dataset, which the reader may consult for further details on construction and sources.

tial volatility and cyclicality. WWI and WWII separate themselves as events associated with strong volatility, albeit given the extensive sample size under scrutiny, they are certainly not unique. For instance, inflation at the start of the 1800s was clearly unprecedented in the history of the United Kingdom, which is largely attributed to the strain on aggregate supply due to the Napoleonic Wars and upward pressure on aggregate demand due to the Industrial Revolution (Gilboy, 1936).

In the post-war period, real rates seem to begin trending down to very low and even negative levels. Given the natural rate of interest has been in stagnation over much of the last half-century, this suggests a subsequent rise and secular decline until the present (Jordà et al., 2019). However, focusing our discussion on such a narrow period can be potentially misleading. Contrary to what most studies have concluded over the Contemporary Era, it is unclear what trend real interest rates follow across the wider Late Modern Era in the United Kingdom. Given the presence of transitory shocks across this period, there is strong motivation to estimate the long run equilibrium real rate of interest associated with inflation and output stability.

4 Empirical Specification

To motivate a multivariate model, it is useful to first consider univariate approaches to estimate the natural rate of interest. In the absence of variation, sample means of ex-post real interest rates may be sufficient to pass judgment on r*. However, factors influencing the supply and demand for savings are seldom constant, shifting the equilibrium interest rate across time. If these movements are sufficiently large and persistent, sample averages would be a poor estimate of r*. One method to allow for persistent changes is to compute moving averages of historical real rates.

In addition to simple ten-year moving averages, Figure 2 presents other examples of common univariate methods that separate medium or long-term trends from short-term variations such as the band-pass filter, the Hamilton (2018) filter, and the Hodrick-Prescott filter. In principle, these techniques would be sufficient to extract the natural rate of interest when both inflation and output are stable. However, given the sample length and our knowledge of periods during which this is unlikely to be the case, univariate approaches may yield unreliable estimates by ascribing movements in the real rate to its underlying trend (Hamilton et al., 2016). One potential solution is to use forward rates on longer-term securities that capture market perceptions of the natural rate of interest. However, term premia may be a harsh source of variation in forward rates, also rendering them an unreliable proxy for future long-run equilibrium real interest rates.

Despite these issues, we still observe that most estimates suggest a downward trend beginning around the end of the nineteenth century, followed by large movements in real rates across the first half of the twentieth century. However, long-run trends in the real rate are unclear prior to this. In this regard, equations (1) to (6) may be cast into a state-space system to estimate the parameters by the maximisation of a likelihood function provided by the Kalman filter (refer to Appendix A for the full representation of the model). Given observation and transition equations, this recursive algorithm for updating a projection for a dynamic system yields linear unbiased estimates of the state variables and provides a way to compute their uncertainty. This is not without calibration. In particular, initialisation of the state vector and estimation of the error variance are central issues.

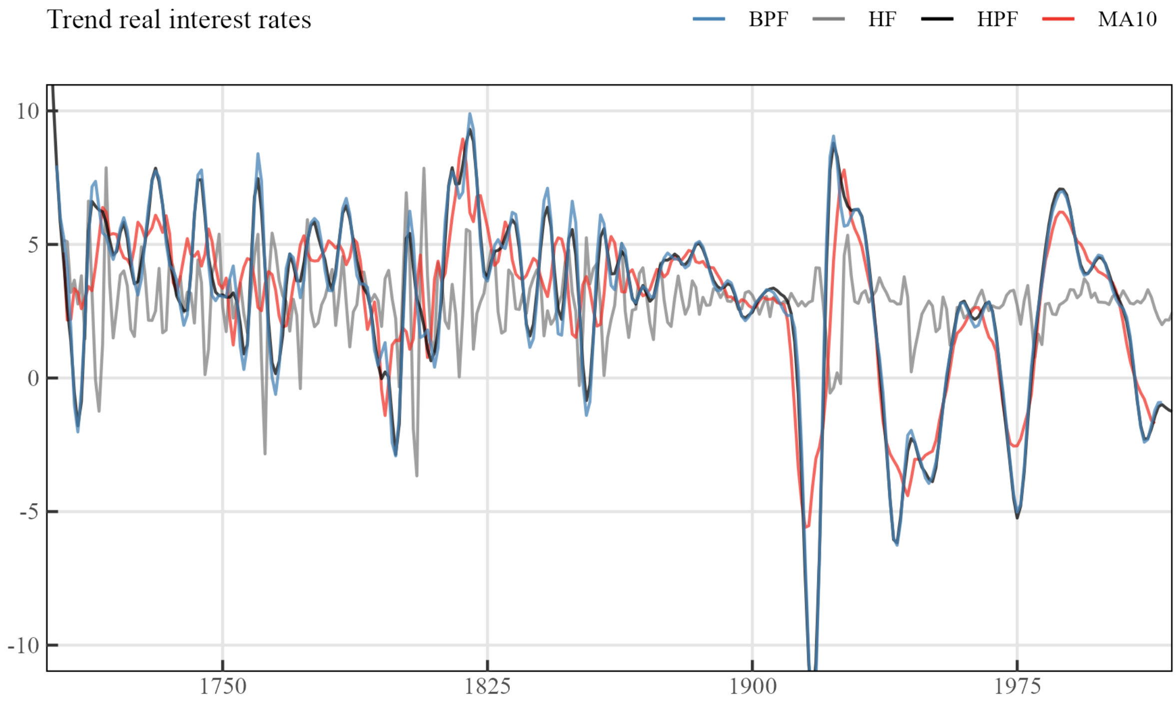

Figure 2. Trend real interest rates

Note: Univariate estimates between 1700-2019. Band-pass (Baxter and King, 1999) filter (BPF) estimates are presented in blue. Hamilton (2018) filter (HF) estimates are presented in grey, with the recommended look-ahead period of $h = 2$. Hodrick-Prescott filter (HPF) estimates are presented in black, with $\lambda = 6.25$ as in Ravn and Uhlig (2002) given annual data. Ten year moving averages (MA10) are presented in red.

As for the state vector, it is relatively straightforward to derive the unconditional mean and autocovariance of the autoregressive processes as a function of model parameters. However, this task is made difficult for the output gap due to unspecified dynamics of the nominal policy rate. The typical solution that is implemented in this paper is to apply the Hodrick-Prescott filter to initialise the state vector. As for the latter, given real growth rates and interest rates are often influenced by highly persistent shifts, maximum likelihood estimation would likely return estimates of the standard deviations of innovations σ_g and σ_z that are biased towards zero. To fix this 'pile-up' problem (Stock, 1994), the median unbiased estimator is used to derive signal-to-noise ratios $\lambda_g = \sigma_g/\sigma_{y*}$ and $\lambda_z = \phi_r \sigma_z/\sigma_{\tilde{y}}$ that are imposed as restrictions on the remaining model parameters (Stock and Watson, 1998). This procedure follows directly from HLW (2017). In particular, the natural rate of output is first estimated barring the real rate gap and assuming constant trend growth. The median unbiased estimate for the ratio between the standard deviations of innovations for the natural rate of output and its trend growth rate is then derived and subsequently imposed as a restriction in the second stage of the estimation that includes the real rate gap. Finally, the ratio for the component unrelated to the trend growth rate is derived in a similar manner and imposed in the third stage to estimate the remaining parameters of the model.[4]

[4] Confidence intervals for these estimates and their corresponding standard errors are calculated using a constraint Monte Carlo procedure, which accounts for filter and parameter uncertainty (see Hamilton (1986) for further details).

Given quarterly data is unavailable for the historical period in question, the model is specified to reflect annual time series data. The main alteration is in the order of the lag polynomials within the state equations, where the output gap is related to one autoregressive lag in addition to the real rate gap, and inflation is related to one autoregressive lag and the output gap.[5] The three transition equations are otherwise identical to those specified in HLW (2017). The final system of equations to be estimated by maximum likelihood are therefore specified concisely as follows:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1-\phi_y & 1-\phi_r & -\phi_r \\ -\varphi_y & 0 & 0 \end{bmatrix} \begin{bmatrix} y^*_{t-1} \\ g_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \phi_y & 0 & \phi_r \\ \varphi_y & \varphi_\pi & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{\tilde{y},t} + \epsilon_{y^*,t} \\ \epsilon_{\pi,t} \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} y^*_t \\ g_t \\ z_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y^*_{t-1} \\ g_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{y^*,t} \\ \epsilon_{g,t} \\ \epsilon_{z,t} \end{bmatrix} \tag{9}$$

Where (8) is the measurement equation involving the dynamic IS and Phillips curve in addition to the law of motion for the natural rate of interest, and (9) is the corresponding transition equation.[6]

It is worth noting that the implementation of the Kalman filter in a maximum likelihood framework to estimate r* across such an extensive horizon poses a number of unique challenges. Perhaps of most significance is the effect of extreme-tailed phenomena that undermine the assumption that all the stochastic innovations are Gaussian in distribution. One potential solution to this particular issue is the inclusion of omitted factors in the measurement equations that control for the presence and extent of these harsh shocks to the output gap. This could accommodate sharp and persistent changes in output and thus allow the model to parse other influences into transitory and permanent components. However, given the extensive size of our sample, it would be difficult to identify, let alone control for shocks strong enough to potentially violate this assumption. Whilst it is probable that certain historical events such as both World Wars and the Great Depression distort estimates of r*, this methodology may still reveal general trends in the equilibrium interest rate during periods of such volatility. The Great Recession is analogous in this regard over which much of the existing literature continues to estimate the natural rate of interest, despite the fact that it likely exhibited similar pressures on the assumptions of the Kalman filter.

Another unique challenge that transpires from the use of historical data is that of identification due to potentially flat IS and Phillips curves. In particular, the low frequency of annual data may lack the information needed to estimate the relationship between the output gap, real rate gap and inflation. In addition, the longer horizon may involve structural breaks in the relationship between

[5] Lag selection in the state equations is justified given the use of lower frequency data in which it is found that most of the feedback in inflation and the output gap is accounted for within the year and that the interest rate affects output in one period, which subsequently affects inflation in the next (see Bank of England (1999) for a further discussion).

[6] This specification imposes $\theta_g = 1$ in equation (3). Holston, Laubach and Williams (2017) also impose a one-for-one relationship between r^* and g following Laubach and Williams (2003) in which estimates of this parameter yield a value close to unity. For robustness, I estimate θ_g over the sample to be 0.953 (0.419), which is also close to unity.

these variables. In such instances, one would also expect more uncertainty around estimates of the natural rate of interest due to a lack of observability (Fiorentini et al., 2018). Few studies related to r* have focused their analysis on the United Kingdom to gauge the true extent of these problems. The one notable exception is HLW (2017), whose estimates suggest that whilst the Phillips curve is well identified ($\varphi_y = 0.49$), the elasticity of the current output gap to past real rate gaps is close to zero ($\phi_r = -0.01$). However, this result relies on higher frequency data over a narrow horizon and it is unclear whether it is true in lower frequencies over the historical sample. With this caveat in mind, I proceed to estimate the natural rate of interest and return to the issue of identification in the following section given estimates of the model parameters.

5 Results

5.1 Parameter Estimates

Parameter estimates of the model are reported in Table 1. Median unbiased estimates of the signal-to-noise ratios λ_g and λ_z indicate variation in the trend growth rate and natural rate of interest and are relatively higher than those in HLW (2017), likely due to greater volatility over the historical sample. Whilst the output gap seems to be well identified ($\varphi_y = 0.452$), the slope coefficient of the IS curve ($\phi_r = -0.041$) suggests that it is considerably flat, and given large standard errors, the natural rate of interest is narrowly identified. However, it is worth noting that this parameter is marginally higher than typical estimates using quarterly data over a much shorter horizon. Finally, inflation reacts strongly to variation in the output gap evidenced by the coefficient φ_y, which itself is fairly persistent as seen by the coefficient ϕ_y associated with its lag. In respect to the standard deviations of these estimates, clearly inflation is poorly explained by its own lag and the lagged output gap, indicated by a high value for σ_π. Variation in the output gap $\sigma_{\tilde{y}}$, natural rate of output σ_{y*}, and its trend growth rate σ_g is also higher when compared to analogous estimates using higher frequency data over the contemporary period as reported in HLW (2017).

This uncertainty is reflected in higher average standard errors for the natural rate of interest, natural rate of output and the trend growth rate, which are otherwise stable across the sample and rising marginally as indicated by slightly higher point estimates of the final standard errors relative to the average. It is worth noting that the magnitude of error alone does not entirely invalidate these results; imprecision due to parameter and filter uncertainty is a well known issue that even affects quarterly estimates of r* over shorter horizons. HLW (2017) rightly point out that this imprecision is a mixture of both the ordinary uncertainty associated with parameter estimation had they been constant, in addition to the uncertainty due to time-variation in these variables.

It is important to examine the stability of these parameters in light of the extensive historical sample. To achieve this, I implement structural break tests for unknown change points outlined in Andrews (1993) and Andrews and Ploberger (1994), on the slope coefficients of the measurement equations, in addition to the natural rate of interest. F-statistics and associated dates are presented in Table 2. These results indicate a structural break in the slope of the IS curve in 1836 and in the slope of the Phillips curve in 1913. The equilibrium interest rate exhibits several structural breaks across the sample; however, upon closer inspection (Figure E.1), only two appear to correspond to meaningful changes in the trend. I re-estimate the model over various subsamples that omit parts of the data associated with these structural breakpoints (Table E.1) in order to check the robustness of my estimates and find that the slope coefficients do not change significantly between periods.[7]

Table 1. Parameter estimates

Parameter			Standard error	
λ_g	0.035		$\mathrm{r}^*_{\mathrm{avg}}$	3.527
λ_z	0.028		$\mathrm{g}_{\mathrm{avg}}$	0.743
ϕ_y	0.875		$\mathrm{y}^*_{\mathrm{avg}}$	1.685
ϕ_r	-0.041	(1.133)		
φ_y	0.452	(1.925)		
$\sigma_{\tilde{y}}$	0.363		$\mathrm{r}^*_{\mathrm{fin}}$	3.874
σ_π	2.174		$\mathrm{g}_{\mathrm{fin}}$	0.782
σ_{y*}	0.946		$\mathrm{y}^*_{\mathrm{fin}}$	1.703
σ_g	0.132			
σ_z	0.248			
σ_{r*}	0.281		Sample	1700 - 2019

Note: Estimated parameters of the Holston, Laubach and Williams (2017) model for the United Kingdom between 1700 to 2019. Average and final standard errors of the natural rate of interest, the natural rate of output and trend growth rate are reported in the last column. σ_{r*} is computed as the square root of $\sigma_g^2 + \sigma_z^2$.

Table 2. Structural breakpoints

	sup$F$	ave$F$	exp$F$	Date	95% Interval
r^*	232.42	93.33	112.58	1750	[1747, 1753]
				1795	[1793, 1797]
				1843	[1841, 1845]
				1875	[1873, 1877]
				1918	[1917, 1919]
				1967	[1965, 1969]
$\phi_{\tilde{r}}$	33.16	20.78	14.70	1836	[1818, 1854]
$\varphi_{\tilde{y}}$	37.84	27.85	17.16	1913	[1906, 1920]

Note: Aggregated F-statistics for unknown change points (Andrews, 1993; Andrews and Ploberger, 1994) in the natural rate of interest (r^*) and slope parameters of the IS ($\phi_{\tilde{r}}$) and Phillips curve ($\varphi_{\tilde{y}}$). Breakpoints are associated with periods where the probability that the supremum of the F-statistic exceeds $\alpha = 0.05$.

[7] To further investigate parameter stability, I employ a non-parameteric time-varying kernel-weighted least squares estimator to trace the time-paths across the sample (see Figures E.2-E.3). These results corroborate the stability in these slope coefficients over time and around a similar magnitude to that of the point estimates from the state space model.

5.2 Longer-Run Equilibrium Interest Rates

Given the size of the historical sample, I separate this analysis into parts by isolating three phases in the evolution of the natural rate of interest. The aggregate series is shown in Figure B.1. Much of the sample is characterised by wars in which Britain played a large role. The number of business cycles, their duration, and the volatility in estimates of the natural rate of interest and trend growth rate are a reflection of the shocks that took place over this period.[8] This is particularly relevant, as large sums of long-term debt resulting from wartime finance places upward pressure on rates due to crowding out, in addition to greater risk-premiums and potential capital scarcity.

With this in mind, the first phase is presented in Figure 3 between 1700 to 1750. My estimates reveal sizeable time variation in r* during the initial part of the half-century, which coincides with the War of the Spanish Succession (1701-1714) and sharp declines in output associated with the failure of harvest due to the Great Frost (1709). By the end of this period the natural rate declines by approximately 250-300 basis points before recovering during much of the subsequent decade. Britain was straddled with unprecedented long-term public debt after the war (Carlos et al., 2013), which may have contributed to upward pressures on r*. Despite the number of recessions that later ensued in Britain, the natural rate of interest remains relatively stable over the rest of this period, averaging roughly 150-200 basis points across significant events such as the Great Northern War (1717-1720) and the war of the Austrian Succession (1740-1748).

The second phase is shown in Figure 4 between 1750 to 1875. In the latter half of the eighteenth century, the natural rate and trend growth rate begin to rise. This was the case despite major events such as the Seven Years' War (1756-1763), the American Revolutionary War (1775-1783), the French Revolutionary War (1792-1802) and the Napoleonic War (1802-1813). Estimates suggest that this ascent persists right until the last quarter of the nineteenth century, pushing r* and trend growth to unprecedentedly high levels. It is clear that across much of the nineteenth century, the number of cycles and their duration diminish substantially. However this was certainly not an era free from war, rather Britain was largely unstrained by most of the conflicts she was involved in and bathed in the rewards of many of her victories in Asia, the Pacific, and Africa.

This long stretch of 'peace' that persisted through to the early twentieth century is often known as *Pax Britannica* (1815-1914) and may explain why r* saw a relatively unperturbed rise. The First Industrial Revolution also played an important role in Britain closer to the end of this period and its contribution to the rise in r* shall be discussed in relation to changes in total factor productivity growth. By the end of this particular phase, the natural rate of interest increases by approximately 600 basis. This secular ascent over such a vast horizon provides clear evidence of long-run drivers that transcend transitory shocks to the British economy. Finally, the trend growth rate follows a similar trajectory over this period, suggesting an era of expansion that persisted for a number of decades before peaking towards the latter part of the nineteenth century, which largely corroborates existing research on this period and the Industrial Revolution (see Crafts and Mills, 1996).

[8] The consistency of the output and real rate gap with these business cycles are presented in Figure C.1 and Figure C.2 respectively. Results suggest that these series coincide well with dates outlined in Broadberry et al. (2023) across the sample. In addition, the output gap seems to be largely consistent with estimates in Thomas and Dimsdale (2017).

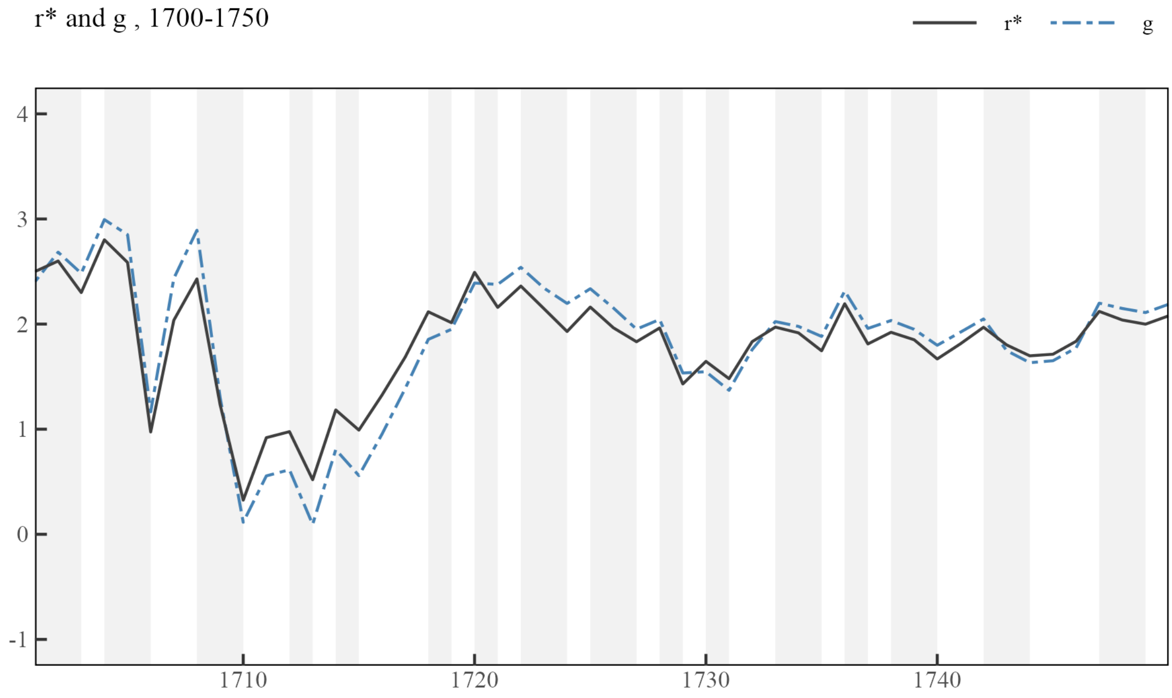

Figure 3. Prelude: 1700-1750

Note: Estimated series for the natural rate of interest (r) in black and trend growth rate (g) in blue between 1700-1750 in the United Kingdom. Recession bands in grey are constructed from Broadberry et al. (2023).*

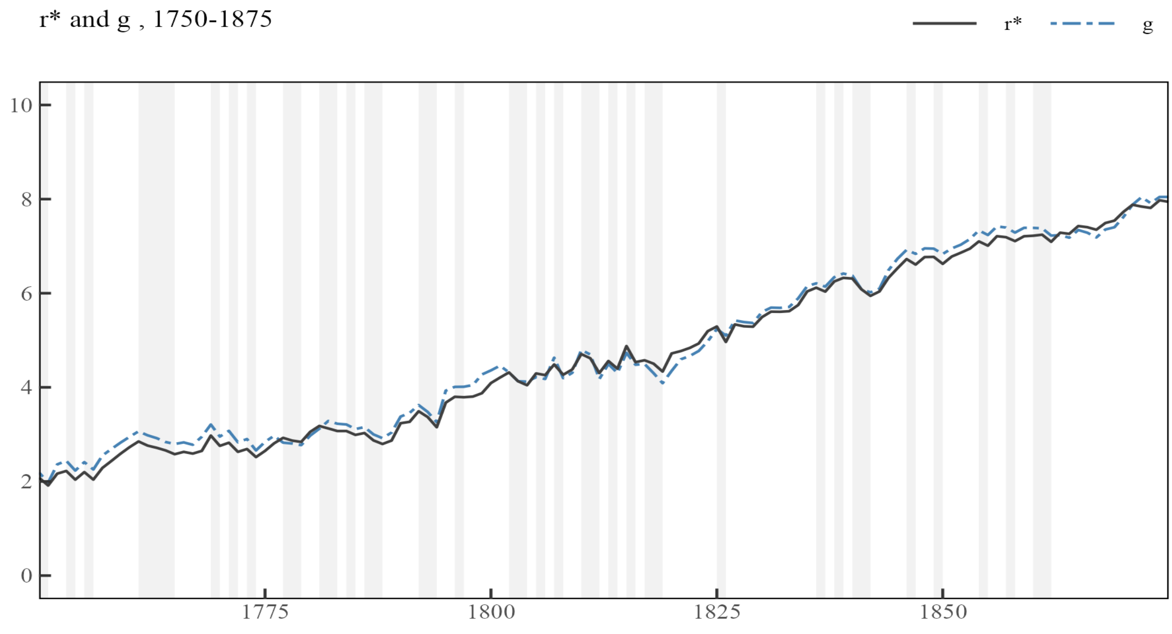

Figure 4. Ascent: 1750-1875

Note: Estimated series for the natural rate of interest (r) in black and trend growth rate (g) in blue between 1750-1875 in the United Kingdom. Recession bands in grey are constructed from Broadberry et al. (2023).*

The third phase is presented in Figure 5 between 1875-2019. In the last quarter of the nineteenth century, the natural rate of interest and the trend growth rate exhibit signs of secular decline. This trend seems to stabilise around the turn of the century until the First World War. These estimates suggest that the implications of WWI on the natural rate of interest and trend growth rate were extensive. In particular, the post-war recession coincides with a reduction of approximately 200-250 points. For context, this is more than twice the estimated reduction in r* that was experienced during the Great Recession. The equilibrium interest rate remains permanently depressed until the start of the Great Depression, which coincides with a further reduction of roughly 50-100 basis points, albeit soon after which it recovers up until the Second World War.

The post-WWII recession also has a negative impact on the natural rate of interest and trend growth rate. In particular, r* falls by roughly 50-100 basis points during this period and continues to decline thereafter until the end of the sample. Estimates over the contemporary era corroborate the existing literature, exhibiting a prolonged reduction exacerbated by the Great Recession. However, these results suggest that secular decline actually began just before the turn of the twentieth century and was sharply affected by the First World War. The Great Depression, Second World War, and Great Recession also play a large role in depressing the natural rate of interest and trend growth rate to unprecedented lows. Whilst these are significant events, this secular decline clearly begins well before they occur. By the end of this period, r* falls by around 600 basis points.

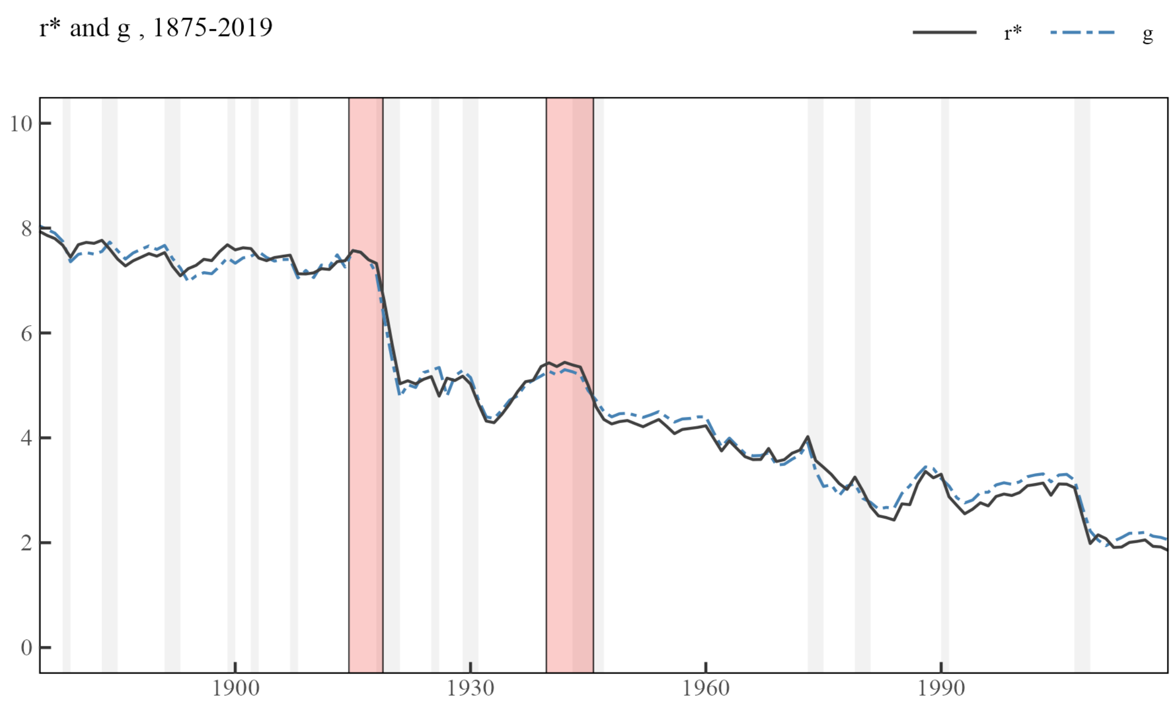

Figure 5. Decline: 1875-2019

Note: Estimated series for the natural rate of interest (r) in black and trend growth rate (g) in blue between 1875-2019 in the United Kingdom. Recession bands in grey are constructed from Broadberry et al. (2023). Shaded regions in red correspond to the First World War (left) and Second World War (right) respectively.*

Whilst my estimates of the natural rate of interest reveal periods of long run ascension and stagnation, they capture a relatively short part of an even longer history of real interest rates. Shmelzing (2020) offers perhaps the most authoritative documentation of this history and shows, using seven year moving averages, that ex-post real rates locally and globally have been in decline over the last eight centuries (refer to Figure 6). Although useful over longer horizons given time series which exhibit clear trends with minimal noise, moving averages often struggle to parse short-term and long-term variation, particularly during times of volatility. In this regard, the work of Shmelzing (2020) does well to explain the general trend of real rates, but the historical evolution of the equilibrium rate consistent with output and inflation stability may perhaps differ.

Estimating the natural rate of interest over such an extensive horizon is a major challenge for economic historians. Barring data and measurement issues, the imposition of theoretical structure becomes increasingly untenable as the sample extends further into history; these periods predate mechanisms and institutions that facilitate the transmission between macro-fundamentals necessary for the estimation of r* as conceived by New Keynesian models. Furthermore, the frequency and magnitude of crises across this broad history continue to threaten key assumptions underlying the Kalman filter. In this regard, nonstructural time-series models may well be more appropriate. My choice of horizon, coinciding roughly with the inception of the Bank of England, and by extension the transmission that modern macroeconomic models rest upon, is therefore no coincidence, and I leave the challenge of estimating r* over even longer horizons to future research.

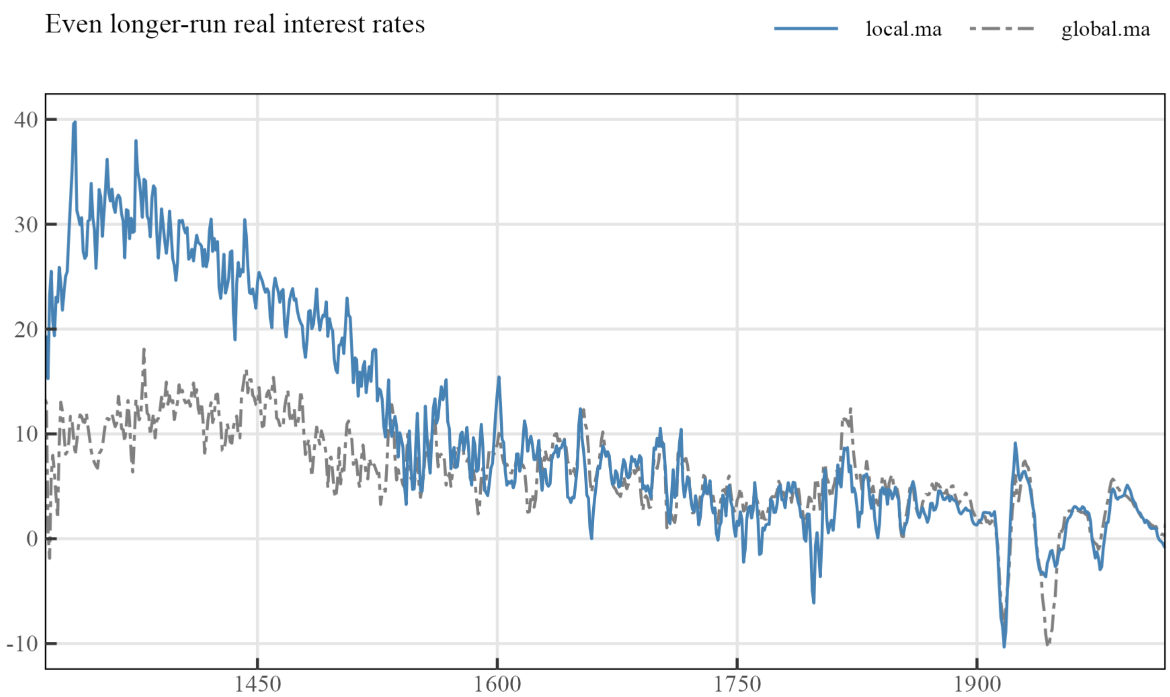

Figure 6. Even longer-run real interest rates

Note: Seven year moving averages of (local) ex-post real rates for the United Kingdom in blue and (global) ex-post rates in grey between 1317-2019. Global ex-post rates are calculated using weighted averages of real rates across advanced economies. Data used to compute this series is taken from Schmelzing (2020).

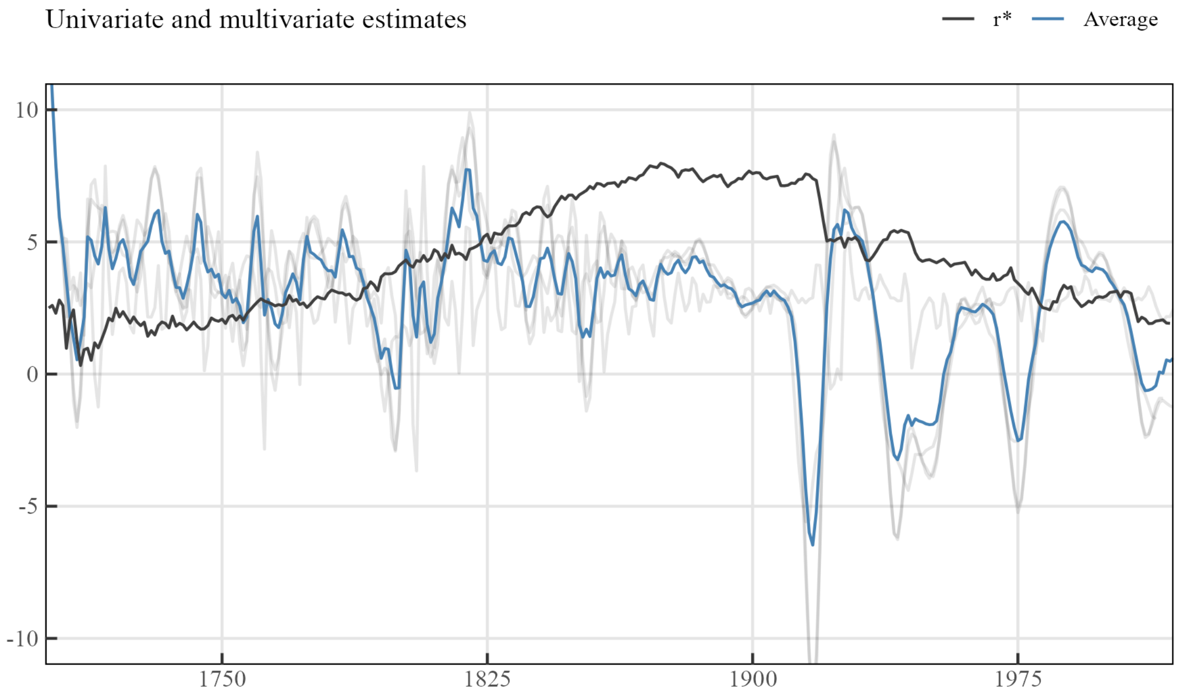

Figure 7. Univariate and multivariate estimates

Note: One-sided Kalman filter estimates are presented in black. The average of univariate measures such as the band-pass filter, Hamilton filter, and Hodrick-Prescott filter, in addition to seven year moving averages sourced directly from Schmelzing (2020), are presented in blue between 1700-2019 for the United Kingdom.

Given this investigation does not attempt to pass judgment on the natural rate of interest prior to the eighteenth century, Figure 7 presents one-sided Kalman filter estimates of r* alongside the average of the band-pass filter, Hamilton filter, and Hodrick-Prescott filter, in addition to seven year moving averages sourced directly from Schmelzing (2020). Although not directly comparable, this period of ascension estimated in the natural rate of interest beginning around the mid-eighteenth century can also be observed in averages of univariate measures. Whilst this does not persist for as long as in r*, the decline does seem to coincide with the end of the nineteenth century and lasts until the end of the sample. Large discrepancies between these series transpiring primarily at the beginning of the nineteenth century suggest that univariate measures of the equilibrium interest rate fail to capture pressures reflected in multivariate estimates. In addition, they clearly exhibit large swings that in many instances persist for decades, which is a known symptom of their difficulty in parsing underlying variation in the real interest rate (see Laubach and Williams, 2016).

Why is secular stagnation widely considered a surprising phenomenon, given our knowledge of the historical trend in real rates of interest? This paper offers a simple explanation; our ignorance of the historical trend in the equilibrium interest rate. Recent developments in r* relative to its longer-run trend might be an explanation as to why the secular stagnation hypothesis has once again resurfaced (Summers, 2015a). As one-sided Kalman filter estimates clearly demonstrate, the decline in r* is an unprecedented phenomenon over the Late Modern and Contemporary Era. Yet our understanding of neutral rates consistent with inflation and output stability prior is still unclear, rendering recent developments in r* even more startling. In this regard, if moving averages of real rates are any indication of long-run trends in the natural rate of interest over such an extensive horizon (as in Schmelzing, 2020), its behaviour across recent history is indeed far from unique. Notwithstanding, my estimates indicate a clear shift in the long-run trend of r* by the close of the nineteenth century. What are the reasons for this reversal and to what extent can they be explained by changes in long-run drivers proposed by existing research? What follows is an explicit attempt to understand these shifts in the long-run supply and demand for savings beyond the last century.

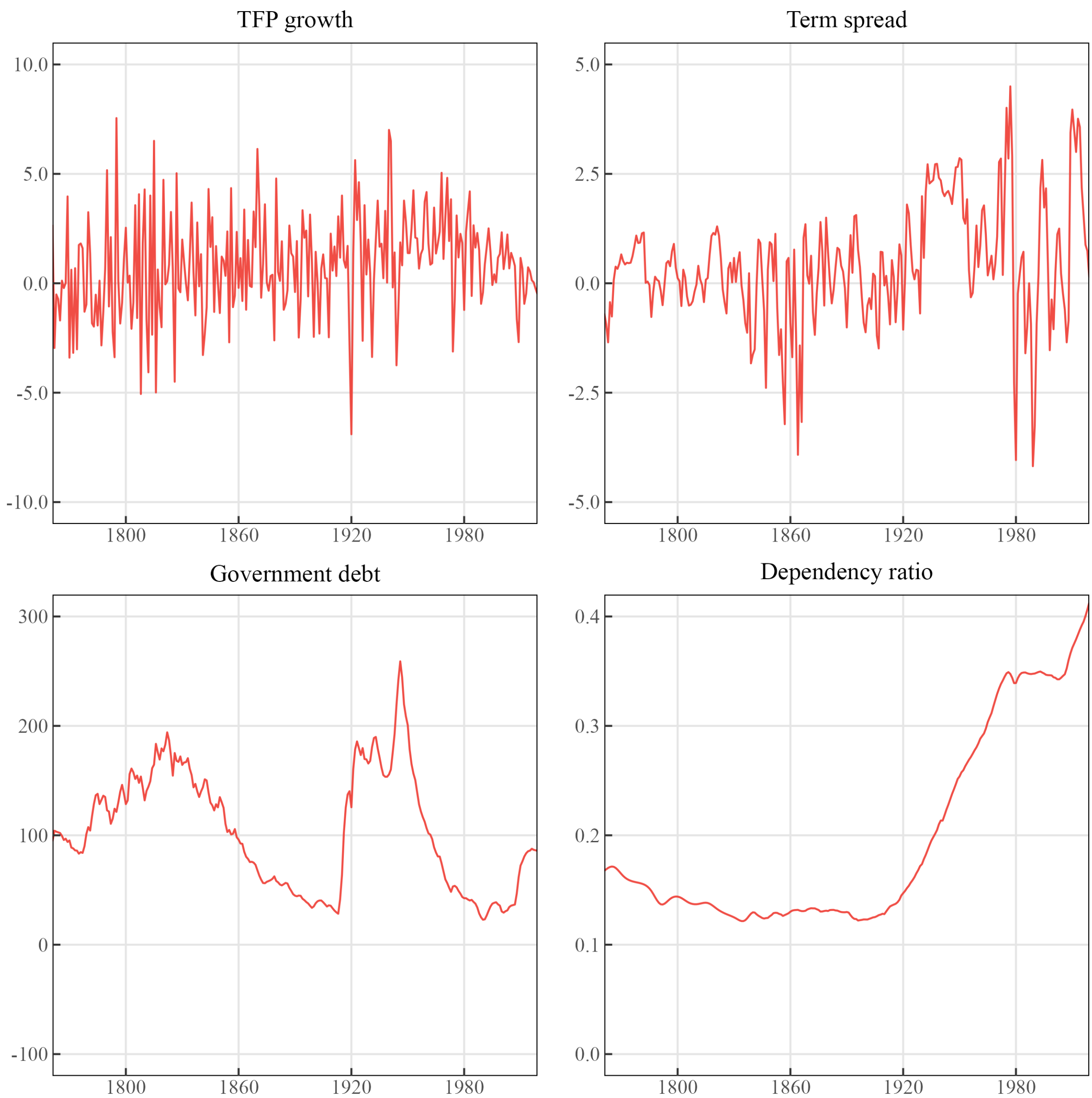

Figure 8. Historical productivity, risk, debt and demography

Note: Annual time series for the United Kingdom between 1761 to 2019 in red. The term spread measures the difference between interest rates on long-term (consol) and short-term (3 month) bonds as a proxy for risk preferences. Public sector debt is measured as a percentage of nominal GDP. The demographic old-age dependency ratio is calculated as the ratio of old-age dependents (60+) to the working age population.

5.3 Historical Drivers of the Natural Rate

This section investigates core drivers of the natural rate of interest established in the existing literature and for which historical data is available. In particular, I investigate the long-run relationship between r*, productivity, demography, public policy, and risk preferences. The decline in growth rates of total factor productivity and their contributions to reductions in r* is widely documented in the existing literature (refer to Rachel and Smith, 2017; Rachel and Summers, 2019). In the Ramsey model, lower productivity growth reduces expected future income, and greater savings needed to sustain future consumption results in higher rates of capital accumulation, which subsequently implies a higher capital-to-output ratio and a lower marginal product of capital. As the real interest rate equates to the marginal product of capital, reductions in productivity growth create downward pressures on the natural rate of interest.

However, total factor productivity growth, whilst slower than expected (as suggested ex-post in Crafts and Harley, 1992), grew steadily during the Industrial Revolution. In the Ramsey model, we would expect this to sustain a ceteris paribus increase in the long-run equilibrium interest rate. In reality, it was only until the 1870s that Britain experienced reductions in labour and total factor productivity, which persisted through to the ‘80s, before further albeit more modest reductions in the decade preceding the First World War (Crafts and Mills, 2020). The historical series for total factor productivity growth is collected from Thomas and Dimsdale (2017) between 1761-2017, which I extend to 2019 with data from the Penn World Table (see Feenstra et al. 2015), which is presented in Figure 8.[9] Moving averages of this time series (see Figure F.5) reveal a gradual rise in the growth rate of productivity from the start of the sample, peaking in the 1870s before strong reductions that seem to persist for decades. Total factor productivity then struggles to recover and declines even further in the 1900s. These trends confirm much of the existing literature in regard to the history of productivity in the United Kingdom around the end and beginning of the nineteenth and twentieth century respectively. In light of the textbook Ramsey growth model discussed, these trends in total factor productivity suggest a decline in r* during this period.

Demographic shifts across the advanced world have also driven recent trends in r*. Declining rates of fertility and rising rates of average life expectancy are thought to be important contributors (see for instance Carvalho et al., 2016; Gagnon et al., 2021). According to this particular strand of the literature, shifts in the age distribution across the advanced world due to the mid-twentieth century baby boom has created asymmetries in the distribution of savings behaviour. In the standard overlapping-generations framework, subsequent bulges in the middle-aged demographic relative to the rest of the population have meant greater savings and lower expected returns (see Eggertsson et al., 2019). The United Kingdom experienced a sharper boom in fertility rates post-WWI, yet it was short-lived relative to the sustained increase post-WWII.

However, these demographic trends are recent phenomena, and in contrast to the wider historical narrative. In particular, it was only until the early twentieth century, that the United Kingdom experienced structural shifts in old-age demography. To demonstrate this, I temporally interpolate

[9] To the best of my knowledge, data prior to this period is scarce for the United Kingdom. Given this is the shortest time series among the selected historical drivers of the natural rate of interest, we are forced to restrict the sample to this horizon. As the secular ascent in r* begins around the mid-eighteenth century, this should not undermine our analysis.

quinquennial demographic data from Wrigley and Schofield (1981) and derive an annual series for the old-age dependency ratio in the United Kingdom between 1761-1841, which I then extend to 2019 using data from the Human Mortality Database (HMD).[10] The historical series is presented in Figure 8. The data suggests that the ratio of old-age dependents to the working age population had actually been in gradual decline since the beginning of the sample until the early 1900s, rising sharply thereafter. These structural trends in demography suggest upward pressures on r* followed by an accelerated reduction around the turn of the twentieth century.

Shifts in private risk preferences are also thought to have contributed to recent variation in the natural rate of interest. Risk premium shocks, which capture precautionary savings due to greater uncertainty, in addition to scarcities in the supply of safe assets, have driven down r* across the advanced economies (see for instance Caballero and Farhi, 2017; Del Negro et al., 2017; Borio et al., 2019). These pressures on the equilibrium real interest rate follow directly from changes in the supply and demand for savings. However, historical trends in risk preferences over the late modern era suggest an alternative narrative. To highlight this, I compute the spread between long and short-term bonds as a proxy for the term premium to measure risk preferences. It is important here to emphasise that whilst this may not be a perfect proxy, data necessary to construct alternative measures of risk preferences is limited over such an extensive sample. In this regard, I compute the term spread between 1761 to 2017 using data from Thomas and Dimsdale (2017), which I extend to 2019 using data from the OECD. This series is presented in Figure 8.[11] Moving averages (see Figure F.7) reveal a relatively stable, if not declining term spread until the twentieth century, rising thereafter. Providing this is a good proxy for risk preferences, we expect marginal contributions to the rise in the equilibrium interest rate followed by downward pressure over its decline.

Finally, recent strands within the existing literature suggest that public policy may have worked to prevent sharper reductions in the natural rate of interest in the advanced world (see Rachel and Summers, 2019). In particular rising public debt, increasing old-age payments, and greater public provision of credit have likely increased the natural rate of interest over recent years. The reasons for this arise from a number of models. The flow-based IS-LM framework suggests that permanent increases in government deficits lead to higher equilibrium real interest rates in the long-run. Following similar logic, temporary shifts to the deficit that reverse any shift in the IS schedule would therefore have only temporary consequences on the real rate. In the neoclassical model assuming complete markets and infinitely-lived agents, the Ricardian Equivalence suggests that representative households offset any changes to fiscal policy by consumption smoothing with private savings.

[10] Ratios calculated using demographic data from the Human Mortality Database define old-age dependents as being above 60 to remain consistent with data from Wrigley and Schofield (1981). The old-age dependency ratio is therefore harmonised between data sets and defined as the ratio of old dependents (60+) to the working age population (15-59). Time variation in the quinquennial demography data provided by Wrigley and Schofield (1981) is relatively negligible during this period of the historical sample, rendering cubic spline interpolation somewhat uncontroversial.

[11] Long-term bonds are the averaged monthly spliced consol yields between 1761-2017 (constructed using data on consol bonds from a variety of sources detailed in Thomas and Dimsdale, 2017), extended to 2019 using data on 10-year bond yields from the OECD Main Economic Indicators. Short-term bonds are averaged monthly spliced series for the discount rate on prime short-term paper between 1761-2017 (constructed using data on 3-month bonds from a variety of sources also detailed in Thomas and Dimsdale, 2017), extended to 2019 using data on 3-month bond yields from the OECD Main Economic Indicators. These time series are constructed in a similar way to Jordá et al. (2017).

Government borrowing therefore has no impact on the equilibrium interest rate. However, the neoclassical framework does allow a channel for government policy to influence interest rates through its pressure on the private capital stock. Greater expenditure tightens resource constraints and expands labour supply; as the marginal product of capital rises, investment is more profitable. Given the capital-to-labour ratio is fixed, interest rates are therefore only higher transitionally.

Additionally, modern micro-founded macroeconomic models in which the Ricardian equivalence does not hold, also suggest a link between government policy and the natural rate of interest. Given a finite planning horizon, agents adjust their consumption and savings in response to transfers due to their knowledge that future generations have to service what they do not. Furthermore, heterogeneity in the Marginal Propensity to Consume (MPC) across agents imply that any transfers away from low-MPC towards high-MPC agents will increase the aggregate propensity to consume and decrease the aggregate desire to save, thereby increasing real rates. Finally, public policy that reduces risk by increasing the provision of safe assets also pushes real rates higher. This particular channel of precautionary saving has been well documented in the literature and is the main driver considered in this paper (see Caballero et al., 2016; Caballero and Farhi, 2018).

In this regard, were it not for the positive influence of fiscal policy in recent years, equilibrium interest rates may well have been far lower. However, such trends in government policy are also a recent phenomenon that do not necessarily inform us of the wider historical narrative. I therefore collect data from Thomas and Dimsdale (2017) on government debt between 1761 to 2016, which I extend to 2019 using data from the IMF World Economic outlook, as arguably the most significant indicator of public policy.[12] Data presented in Figure 8 reveals a sustained rise in government debt at the start of the sample followed by a prolonged decline across much of the nineteenth century. The start of the twentieth century is met with unprecedented increases in public debt coinciding with the First and Second World War. These trends suggest that shifts in public policy in terms of debt likely exerted downward pressure on the natural rate of interest over much of its ascension and upward pressure during much of its decline. What follows in the remainder of this paper is an empirical attempt to explain longer-run trends in the long-run equilibrium interest rate in terms of these core drivers established by the existing literature.

Augmented Dickey Fuller (ADF) tests indicate non-stationarity in each driver barring Total Factor Productivity at the 5% confidence level. I find evidence of at least two cointegrating vectors using the Johansen test for cointegration and estimate the following Vector Error Correction Model (VECM) to characterise the joint relationship between the equilibrium interest rate and its drivers:

$$\Delta r_t^* = \alpha(r_{t-1}^* - \beta' X_{t-1}) + \gamma' \Delta X_t + \epsilon_t \tag{10}$$

where X_t is the vector of drivers of the natural rate of interest. Estimation results are presented in Table 3 for a VECM with one and two cointegrating vectors, in addition to Granger causality tests. Optimal lag selection of the model ($k = 2$) is determined using the Akaike Information Criterion.

[12] Government debt is the spliced measure of consolidated debt as a percentage of gross domestic product constructed by Thomas and Dimsdale (2017) between 1761-2016, extended to 2019 using data on the Public debt-to-GDP ratio from the International Monetary Fund World Economic Outlook Database.

Table 3. Cointegration

	r*	TFP	Spread	Debt	Ratio
ADF (p)	0.88	0.01	0.23	0.47	0.95
$r = 1, k = 2$					
Cointegrating vector (S.E.)	1 (-)	0.35 (0.17)	-0.64 (0.11)	0.57 (0.31)	-0.41 (0.08)
Error correction coefficient (S.E.)	-0.04 (0.03)	-0.03 (0.02)	-0.06 (0.01)	-0.05 (0.02)	-0.02 (0.03)
Granger test (p)	0.38	0.03	< 0.01	0.04	0.02
$r = 2, k = 2$					
Cointegrating vector (S.E.)	1 (-)	0 (-)	-0.43 (0.08)	0.27 (0.11)	-0.34 (0.15)
	0 (-)	1 (-)	-0.36 (0.17)	0.32 (0.13)	-0.39 (0.10)
Error correction coefficient (S.E.)	-0.02 (0.04)	-0.09 (0.05)	-0.06 (0.02)	-0.05 (0.01)	-0.01 (0.03)
Granger test (p)	0.32	< 0.01	0.03	0.05	0.09

Note: Estimates of a vector error correction model with one and two cointegrating vectors. r is chosen by the Johansen test. k is chosen by the Akaike Information Criterion. Standard errors reported in parentheses.

It is worth noting here that the natural rate is generated from an estimated model with its own uncertainty. In this regard, caution must be exercised when interpreting results from any estimated model in which it features; the objective here is therefore to simply characterise its comovement. With this in mind, estimates of the cointegrating vector suggest a long-run relationship between the equilibrium interest rate and its core drivers that conform to theory. In particular, there exists a positive relationship with total factor productivity growth and public debt. In addition, there exists a negative relationship with the term spread and old-age dependency ratio. The loading coefficients associated with all drivers barring demography are also negative and withing range, suggesting a slow correction back to equilibrium. To summarise these relationships further, Figure 9 presents the forecast error variance decompositions for the natural rate of interest in terms of shocks to its drivers using a Cholesky decomposition with the ordering: Demography, Debt, Risk, Productivity, and R*. For robustness, I present results assuming both one and two cointegrating vectors.

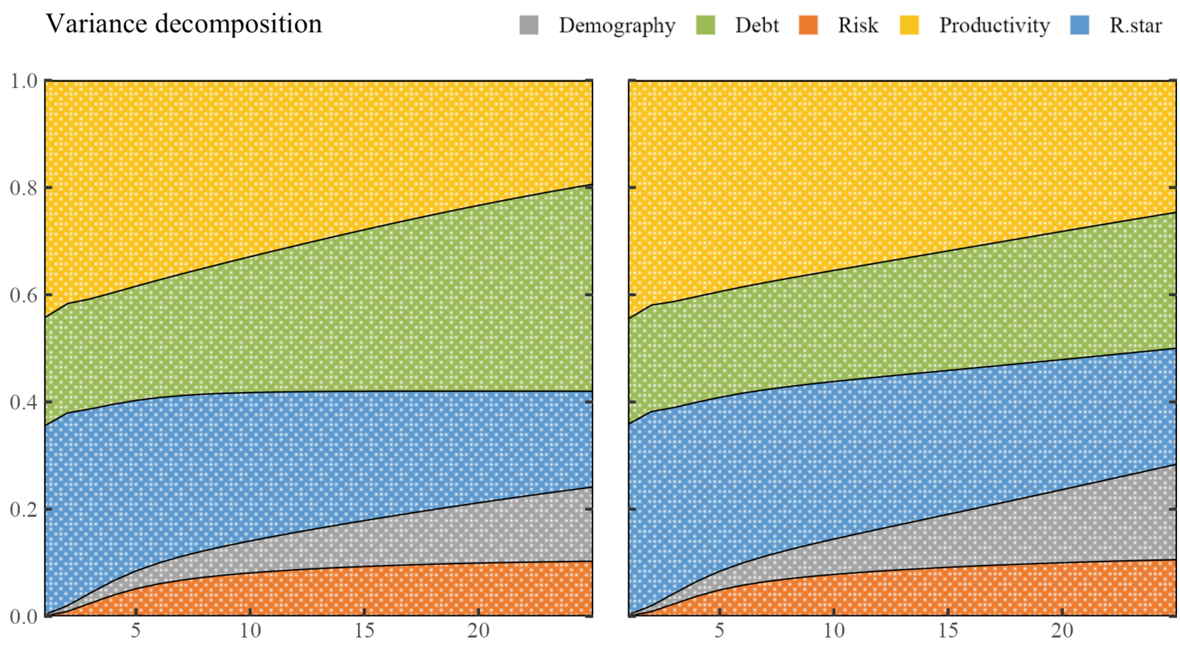

Figure 9. Variance decomposition of r*

Note: Variance decomposition of the natural rate of interest (r) over 25 periods. Areas represent individual contributions of exogenous shocks to the error variance of r-star. Left: VECM (r=1). Right: VECM (r=2).*

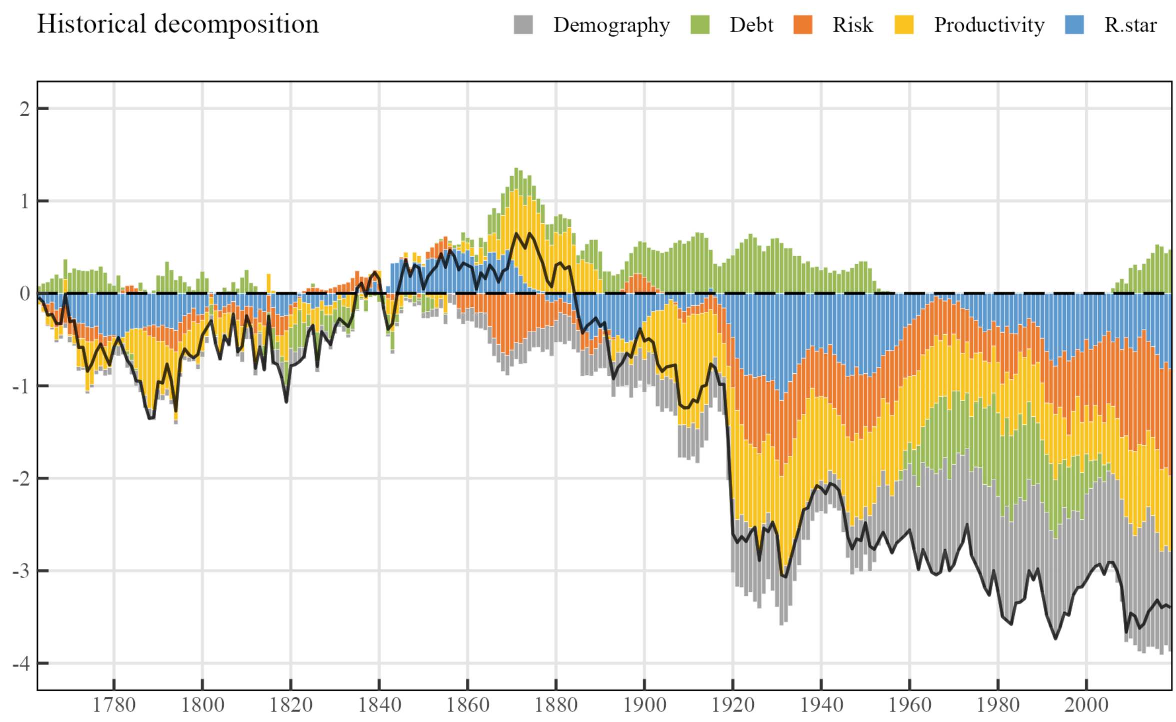

Figure 10. Historical decomposition of r*

Note: Historical VAR decomposition of the natural rate of interest (r) between 1761-2019. The stochastic component of the long-run equilibrium interest rate to which all shocks aggregate is represented in black.*

Error variance decompositions arising from the VECM in 10 suggest that some drivers put forward by the literature explain relatively more of the variation in r* than others.[13] In particular, shocks to productivity growth seem to have a large influence, explaining around 40% of the variation at short horizons; diminishing to around 20% over longer horizons. Shocks to debt also play an important role, explaining approximately 20% of this variation at shorter horizons and growing to account for roughly 20-40% at longer horizons. In contrast, risk and demography initially play a relatively smaller role, but later increase to explain around 10% and 15% of the variation respectively. The autoregressive component of the model clearly plays a sizeable role at short horizons, accounting for almost 40% of the variation, which diminishes to around 20% at longer horizons.

These results suggest productivity growth and public debt play a greater role than documented in the existing literature. Additionally, demography and private risk are thought to be key contributors, yet my estimates suggest they play a lesser role in explaining variation in r*. This could be explained by the fact that these variance decompositions capture variation over a historical sample that is characterised by unique pressures to the United Kingdom economy. These pressures, arising from phenomena such as multiple wars, the industrial revolution and various economic crises, may have affected the contributions of each driver in non-standard ways relative to recent history.

To parse these pressures on the equilibrium interest rate, further analysis is therefore required. Whilst heterogeneous overlapping-generation (OLG) approaches seem tempting, strong structural restrictions imposed on such an extensive sample is to some extent inappropriate, particularly due to the unique challenges in calibration and potential volatility in each of these underlying drivers. In this regard, I let the data do most of the talking and compute the historical decomposition from a recursively identified structural VAR model with zero contemporaneous restrictions. This identification scheme is once again motivated by the simplicity of the restrictions, which are perhaps better suited to the historical data as compared to more sophisticated approaches that require relatively stringent assumptions. In addition, short-run restrictions are also empirically consistent with the defining frequency characteristics of the natural rate of interest estimated in this article, which reflect medium- to long-run pressures within the United Kingdom's economy.

Historical decompositions presented in Figure 10 suggest that selected core drivers explain far more variation in r* over its descension than its ascension. In particular, public debt, private risk, and old-age demography explain little of the variation in r* during the latter eighteenth and early nineteenth century. Shocks to productivity play a slightly larger, albeit negligible role in driving r* during this period. However, both productivity and risk clearly account for a substantial proportion of this historical variation around the trend reversal in the equilibrium interest rate, suggesting that they may be largely to blame for the rise and fall in r* over the latter part of the nineteenth century. This is perhaps intuitive, given the peak and subsequent decline in total factor productivity growth around the 1870s that persisted towards the WWI and trends in the term spread.

Shocks to demography explain a negligible fraction of variation in the natural rate of interest during its ascension. However, it becomes increasingly relevant after the mid-nineteenth century,

[13] The baseline ordering is justified by the fact that the old-age dependency ratio is perhaps the least influenced by other variables of the model, whilst the natural rate is assumed to be most influenced by its drivers. Whilst the precise ordering of Debt, Risk, and Productivity may be debated, these results are largely robust to permutations between them.

which coincides with a stagnant and subsequently rising old-age dependency ratio towards the turn of the century. Similarly, shocks to risk seem to explain only a minor fraction of the variation in r* prior to the mid-nineteenth century, after which their contributions become substantial around the point of the trend reversal in r* and during the early half of the twentieth century. These pressures also coincide with well documented shifts in the term spread over the same period.

As with shocks in productivity growth, changes in risk preferences and demography account for a large amount of the long-run stagnation in the natural rate of interest within the United Kingdom, particularly after the First World War. These findings corroborate a large body of empirical research that suggests such factors are of particular relevance in driving down the long-run equilibrium interest rate over the last half-century across the advanced world (see for instance Del Negro et al., 2019; Rachel and Summers, 2019; Kiley, 2020; Cesa-Bianchi et al., 2022). Such pressures are clearly evident during the contemporary era, where the variation in all drivers are unanimously driving down the natural rate of interest. It is worth noting that the stochastic component to which all shocks aggregate turns negative well before this period and around the turn of the twentieth century, indicating that the decline in r* began much earlier in history.

Finally, this decomposition suggests that were it not for changes in public policy, the natural rate of interest would have declined further over the latter nineteenth and early twentieth century. This is clearly captured by positive pressures during periods that coincide with sharp increases in government debt around the start of the twentieth century. Such movements in public policy also coincide with both World Wars, which would have exerted upward pressures on the natural rate of interest, insulating it from harsher downward adjustments due to variation in other drivers. These findings are also consistent with recent studies on the role of public policy in determining r* over the contemporary era (see Rachel and Summers, 2019).

6 Conclusion

This paper estimates the natural rate of interest in the United Kingdom between 1700-2019. Findings reveal a sustained upward trend in r* across much of the Late Modern period before the start of secular stagnation around the turn of the twentieth century. This sustained decline in the natural rate of interest is further exacerbated by both World Wars, the Great depression, and Great Recession. These results suggest that the long-run equilibrium interest rate is now being pressured back to the historical lows of the early eighteenth century in the United Kingdom. In other words, whilst secular stagnation is relatively unique to contemporary history, the present era of low natural rates of interest is not an entirely unfamiliar one to monetary policymakers.

I estimate the long-run cointegrating relationship between the natural rate of interest and theoretical drivers established within the existing literature. In particular, I investigate the link between the equilibrium interest rate, productivity, demography, public policy, and private risk preferences. Empirical results suggest a long-run relationship between r* and its drivers that conforms to theory. To determine their individual contributions, I estimate a structural vector autoregression model that provides the historical error variance decomposition of the long-run equilibrium interest rate in terms of its drivers. Findings reveal strong pressures from productivity in addition to moderate pressures from demography and private risk around the trend reversal in r* and over much of its subsequent decline. In addition, had it not been for the rise in public debt around the World Wars, the natural rate of interest may well have declined even further than estimated.

There are caveats to this analysis that ought to be emphasised. As for the measurement of the natural rate of interest, estimates from a seminal semi-structural framework suggest that prevailing issues within the existing literature of parameter and filter uncertainty are also present across the historical sample. In addition, whilst this approach attempts to strike a balance in the imposition of theoretical structure between nonstructural time-series and structural general equilibrium models, improving identification within the measurement equations would benefit from further research. However, to the extent that it can inform us of general long-run trends in the natural rate of interest, particularly given lower frequency annual data, this methodology is clearly useful.

As for drivers of the natural rate of interest, whilst they explain much of its decline, core factors put forward by existing literature struggle to explain the ascent of r*, estimated to have persisted for more than a century. Further analysis is therefore required around shifts in the long-run supply and demand for savings during the Late Modern period within the United Kingdom. This sustained ascension may be explained by potentially omitted factors that have not been investigated in this paper, such as inequality, as well as alternative measures of core drivers for which data was scarce over the historical sample. In addition, time-series methods such as those used in this article may lack the theoretical rigour oft found in more structural models when teasing out the contributions of these drivers. Given this was a period characterised by numerous wars and macroeconomic crises, volatility may also be undermining the long-run relationship between r* and drivers established in the existing literature. I leave the inclusion of potentially omitted drivers, structural approaches to the longer-run equilibrium interest rate, and the use of econometric techniques to control for such historical instability to future extensions of this work.

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APPENDIX

Longer-Run Equilibrium Interest Rates:
Evidence From The United Kingdom

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This document provides supplementary material for the paper "*Longer-Run Equilibrium Interest Rates: Evidence From The United Kingdom*." Appendix A outlines the annually adapted Holston, Laubach and Williams (2017) model used to estimate the natural rate of interest (r*). Appendix B presents aggregate estimates of the long-run equilibrium interest rate and trend growth rate. Appendix C presents estimates of the output and real interest rate gap. Appendix D presents historical business cycles dates. Appendix E presents empirical results concerning structural stability. Finally, Appendix F presents five-year and ten-year moving averages of the historical time series used to estimate the natural rate of interest and investigate its drivers.

A Adapted HLW System

State-space representation:

$$\mathbf{y}_t = \mathbf{A}'\mathbf{x}_t + \mathbf{H}'\xi_t + \mathbf{w}_t$$

$$\xi_t = \mathbf{F}\xi_{t-1} + \mathrm{v}_t$$

A.1 First Stage Specification:

$$\mathbf{y}_t = \begin{bmatrix} y_t, \pi_t \end{bmatrix}' \qquad \mathbf{x}_t = \begin{bmatrix} y_{t-1}, \pi_{t-1} \end{bmatrix}' \qquad \xi_t = \begin{bmatrix} y_t^*, y_{t-1}^* \end{bmatrix}'$$

$$\mathbf{A}' = \begin{bmatrix} \phi_y & 0 \\ \varphi_y & \varphi_\pi \end{bmatrix} \qquad \mathbf{H}' = \begin{bmatrix} 1 & -\phi_y \\ 0 & -\varphi_y \end{bmatrix} \qquad \mathbf{F} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \qquad \mathbf{Q} = \begin{bmatrix} \sigma_{y*}^2 & 0 \\ 0 & 0 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_1 = \begin{bmatrix} \phi_y, \varphi_\pi, \varphi_y, g, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*} \end{bmatrix}$$

A.2 Second Stage Specification:

$$\mathbf{y}_t = \begin{bmatrix} y_t, \pi_t \end{bmatrix}' \qquad \mathbf{x}_t = \begin{bmatrix} y_{t-1}, r_{t-1}, \pi_{t-1}, 1 \end{bmatrix}' \qquad \xi_t = \begin{bmatrix} y_t^*, y_{t-1}^*, g_{t-1} \end{bmatrix}'$$

$$\mathbf{A}' = \begin{bmatrix} \phi_y & \phi_r & 0 & \phi_0 \\ \varphi_y & 0 & \varphi_\pi & 0 \end{bmatrix} \qquad \mathbf{H}' = \begin{bmatrix} 1 & -\phi_y & \phi_g \\ 0 & -\varphi_y & 0 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{Q} = \begin{bmatrix} \sigma_{y*}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\lambda_g \sigma_{y*})^2 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_1 = \begin{bmatrix} \phi_y, \phi_r, \phi_0, \phi_g, \varphi_\pi, \varphi_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*} \end{bmatrix}$$

A.3 Third Stage Specification

$$\mathbf{y}_t = \begin{bmatrix} y_t, \pi_t \end{bmatrix}' \qquad \mathbf{x}_t = \begin{bmatrix} y_{t-1}, r_{t-1}, \pi_{t-1} \end{bmatrix}' \qquad \xi_t = \begin{bmatrix} y_t^*, y_{t-1}^*, g_{t-1}, z_{t-1} \end{bmatrix}'$$

$$\mathbf{A}' = \begin{bmatrix} \phi_y & \phi_r & 0 \\ \varphi_y & 0 & \varphi_\pi \end{bmatrix} \qquad \mathbf{H}' = \begin{bmatrix} 1 & -\phi_y & -\phi_r & -\phi_r \\ 0 & -\varphi_y & 0 & 0 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{Q} = \begin{bmatrix} (1+\lambda_g^2)\sigma_{y*}^2 & 0 & (\lambda_g \sigma_{y*})^2 & 0 \\ 0 & 0 & 0 & 0 \\ (\lambda_g \sigma_{y*})^2 & 0 & (\lambda_g \sigma_{y*})^2 & 0 \\ 0 & 0 & 0 & \left(\frac{\lambda_z \sigma_{\tilde{y}}}{\phi_r}\right)^2 \end{bmatrix}$$

Vector estimated by maximum likelihood:

$$\theta_1 = \begin{bmatrix} \phi_y, \phi_r, \varphi_\pi, \varphi_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*} \end{bmatrix}$$

B One-Sided Kalman Filter Estimates

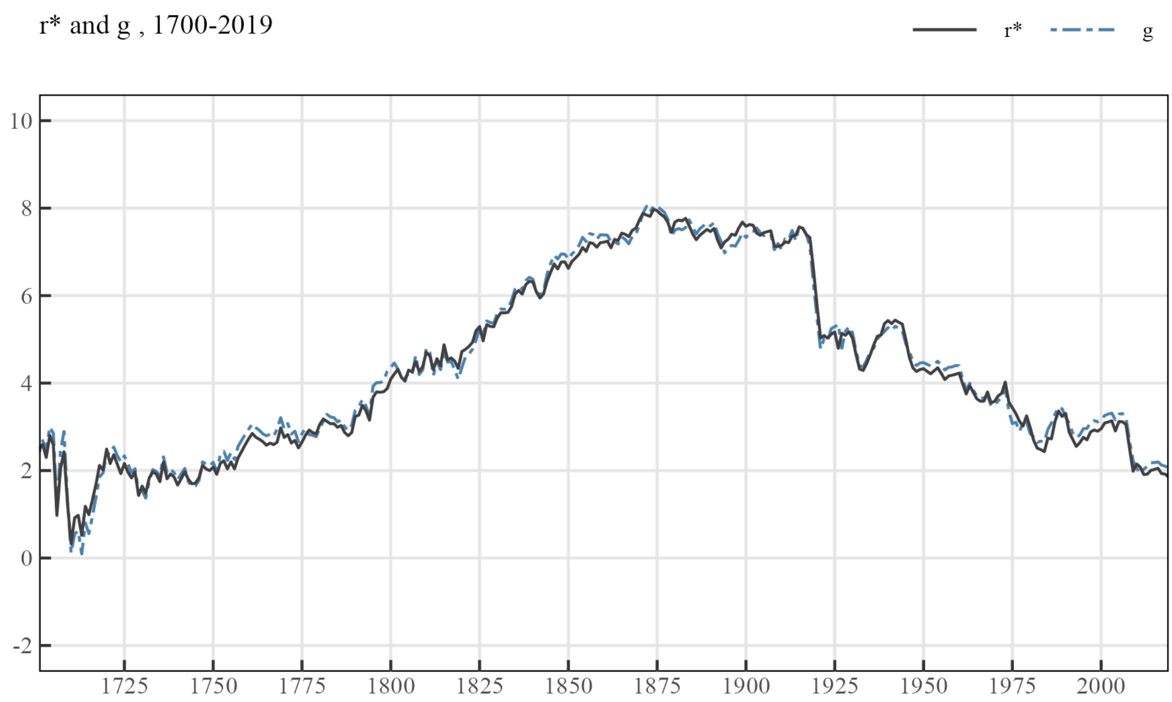

Figure B.1. Complete one-sided estimates for the natural rate of interest and trend growth rate

C Output and Interest Rate Gaps

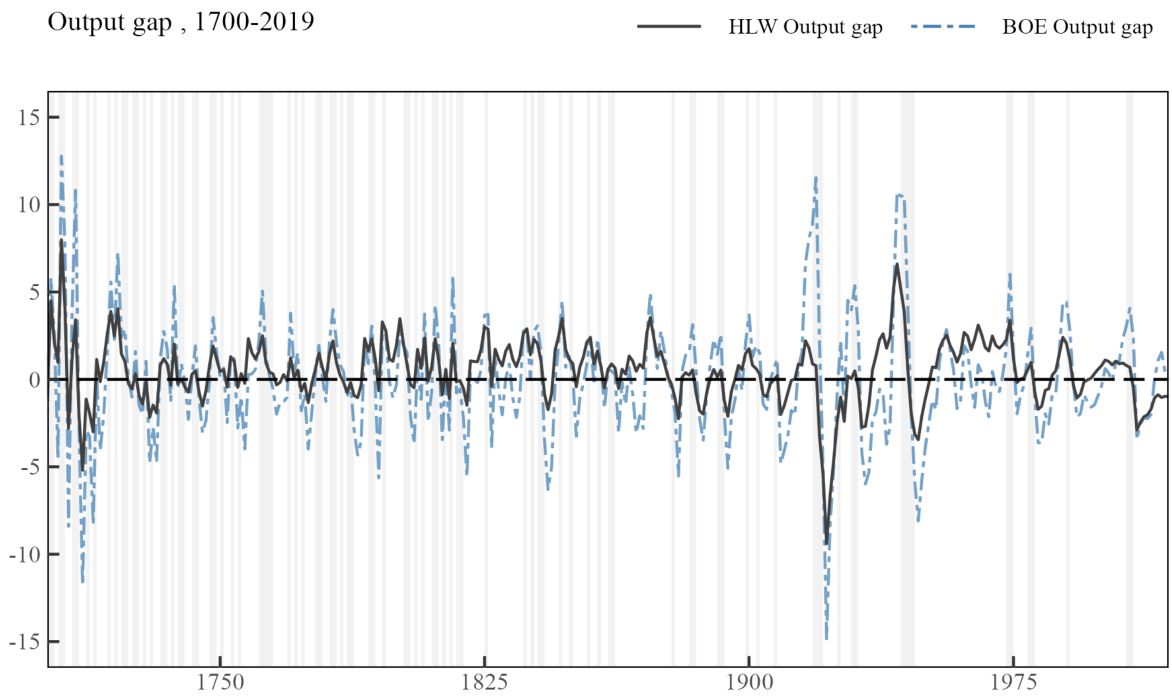

Figure C.1. Estimated output gap from the HLW model and Thomas and Dimsdale (2017)

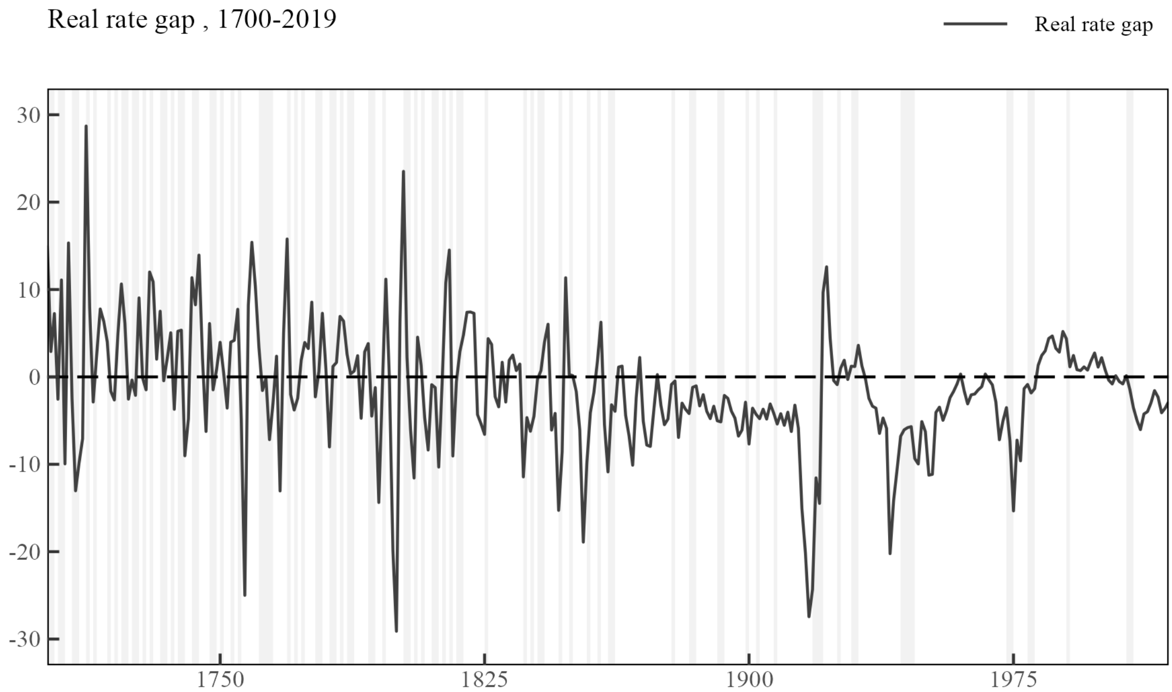

Figure C.2. Estimated real interest rate gap ($r_t - r_t^*$) from the anually adapted HLW model

D Business Cycle Dates

Table D.1. Turning Points: 1700-2019

Peak	Trough	Peak	Trough
1701	1703	1802	1804
1704	1706	1805	1806
1708	1710	1807	1808
1712	1713	1810	1812
1714	1715	1813	1814
1718	1719	1815	1816
1720	1721	1817	1819
1722	1724	1825	1826
1725	1727	1836	1837
1728	1729	1838	1839
1730	1731	1840	1842
1733	1735	1846	1847
1736	1737	1849	1850
1738	1740	1854	1855
1742	1744	1857	1858
1747	1749	1860	1862
1750	1751	1878	1879
1753	1754	1883	1885
1755	1756	1891	1893
1761	1765	1899	1900
1769	1770	1902	1903
1771	1772	1907	1908
1773	1774	1918	1921
1777	1779	1925	1926
1781	1783	1929	1931
1784	1785	1943	1947
1786	1788	1973	1975
1792	1794	1979	1981
1796	1797	1990	1991
1802	1804	2007	2009

Note: Business cycle dates (annual turning points) reported in Broadberry et al. (2023).

E Parameter Stability

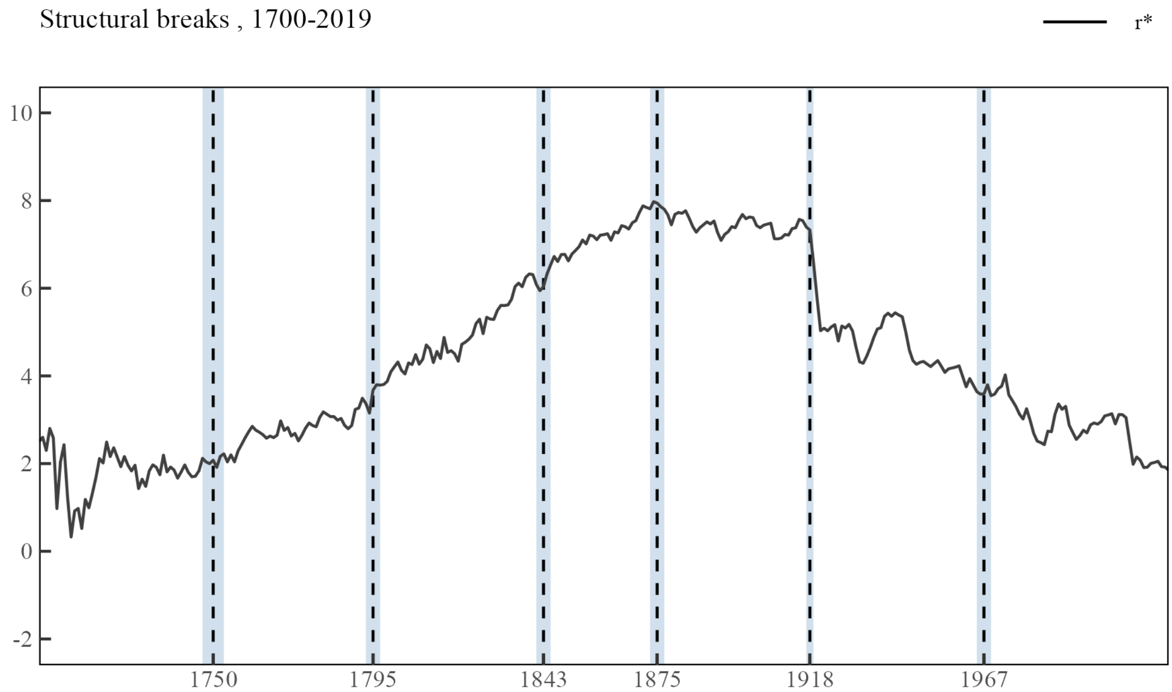

Figure E.1. Estimated structural breakpoints in the natural rate of interest (see Table 2).

Table E.1. Subsample Estimation

	Ascent 1750-1875	Decline 1875-2019	Late Modern 1750-1945	Contemporary 1945-2019
λ_g	0.029	0.035	0.041	0.025
λ_z	0.021	0.029	0.025	0.026
ϕ_y	0.824	0.876	0.711	0.913
ϕ_r	-0.042	-0.021	-0.036	-0.014
φ_y	0.487	0.522	0.563	0.491
$\sigma_{\tilde{y}}$	0.315	0.172	0.294	0.157
σ_π	3.173	2.914	3.689	2.968
σ_{y*}	0.892	0.853	0.915	0.872
σ_g	0.103	0.119	0.150	0.087
σ_z	0.158	0.238	0.204	0.292
σ_{r*}	0.189	0.266	0.253	0.305
$\mathrm{r}^*_{\mathrm{avg}}$	3.321	3.504	3.477	3.619
$\mathrm{g}_{\mathrm{avg}}$	0.574	0.591	0.620	0.524
$\mathrm{y}^*_{\mathrm{avg}}$	1.289	1.114	1.512	1.087

Note: Estimated parameters of the annually adapted HLW (2017) model across sub-samples.

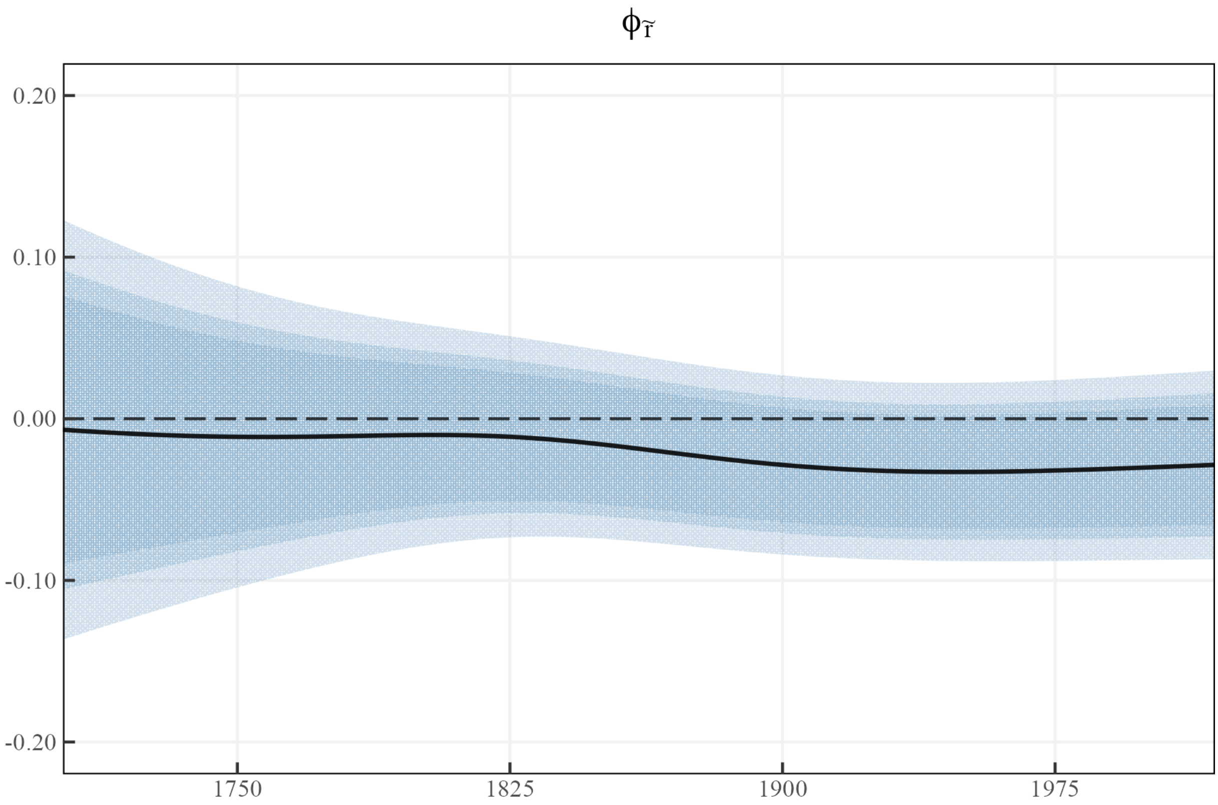

Figure E.2. Kernel-weighted (Gaussian) least squares estimates of the IS slope coefficient.

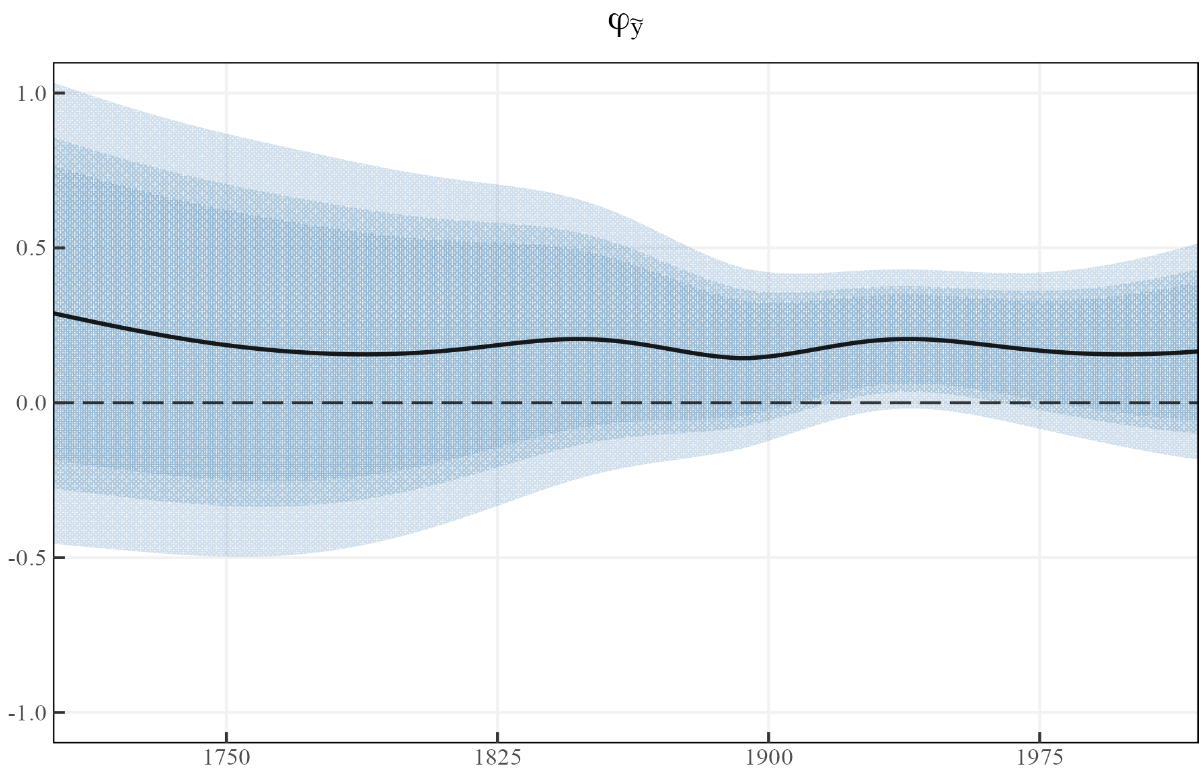

Figure E.3. Kernel-weighted (Gaussian) least squares estimates of the PC slope coefficient.

F Moving Averages

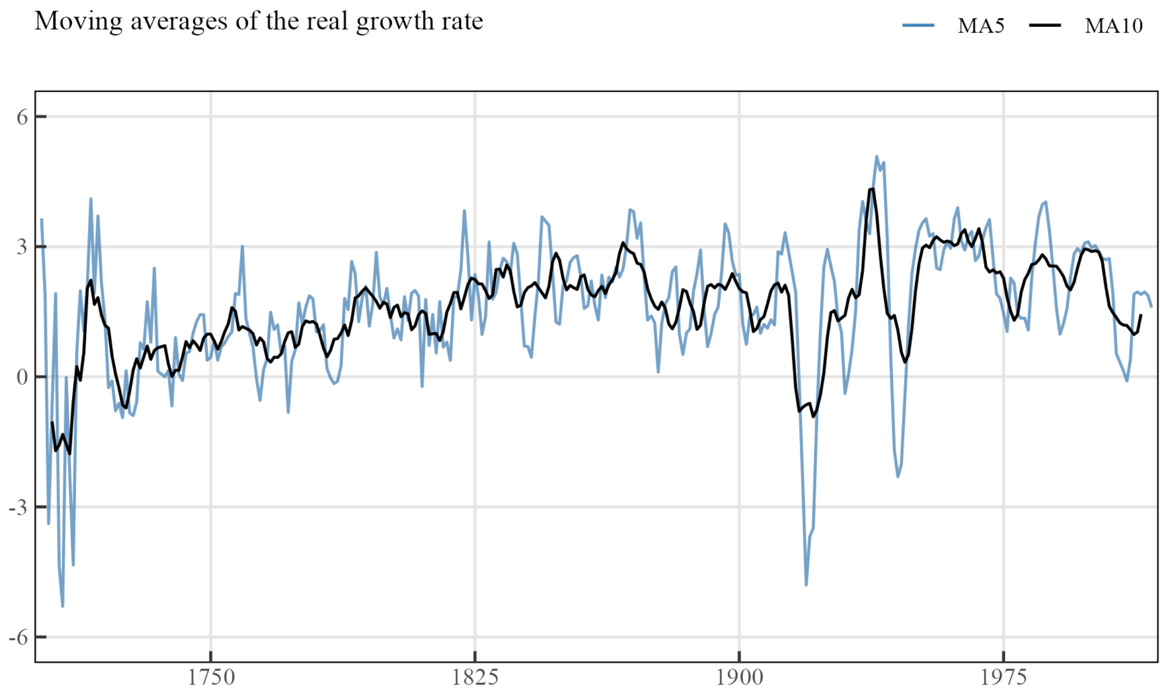

Figure F.1. Moving averages of the real growth rate.

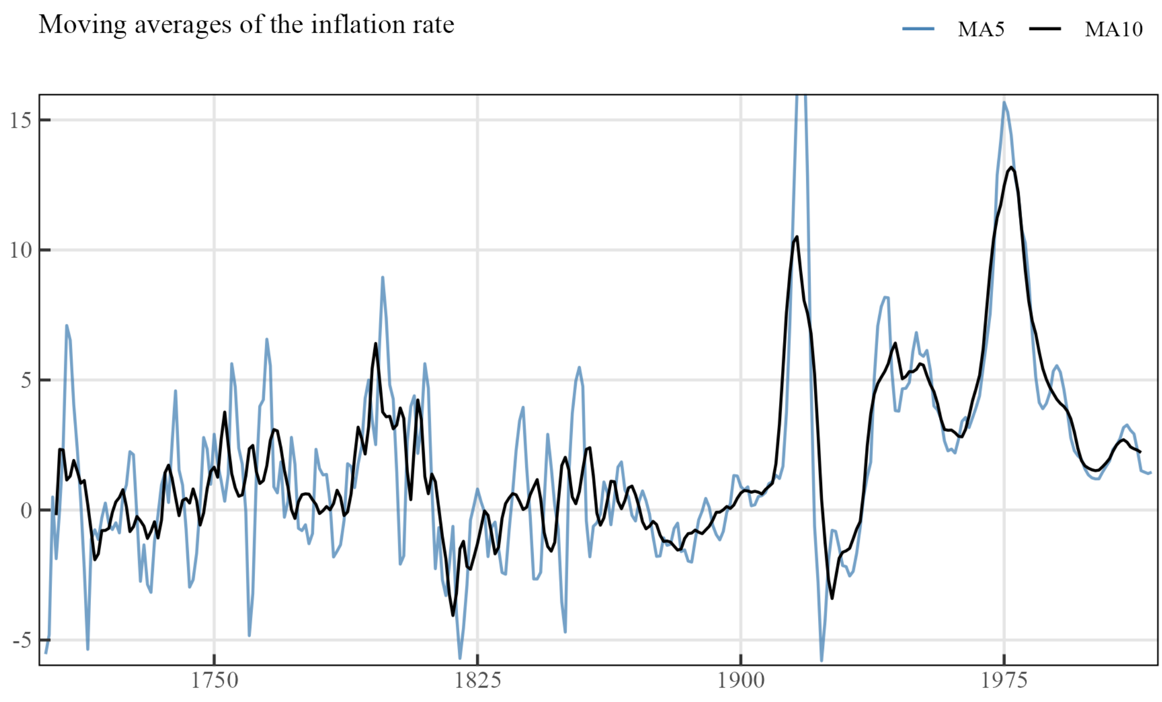

Figure F.2. Moving averages of the inflation rate.

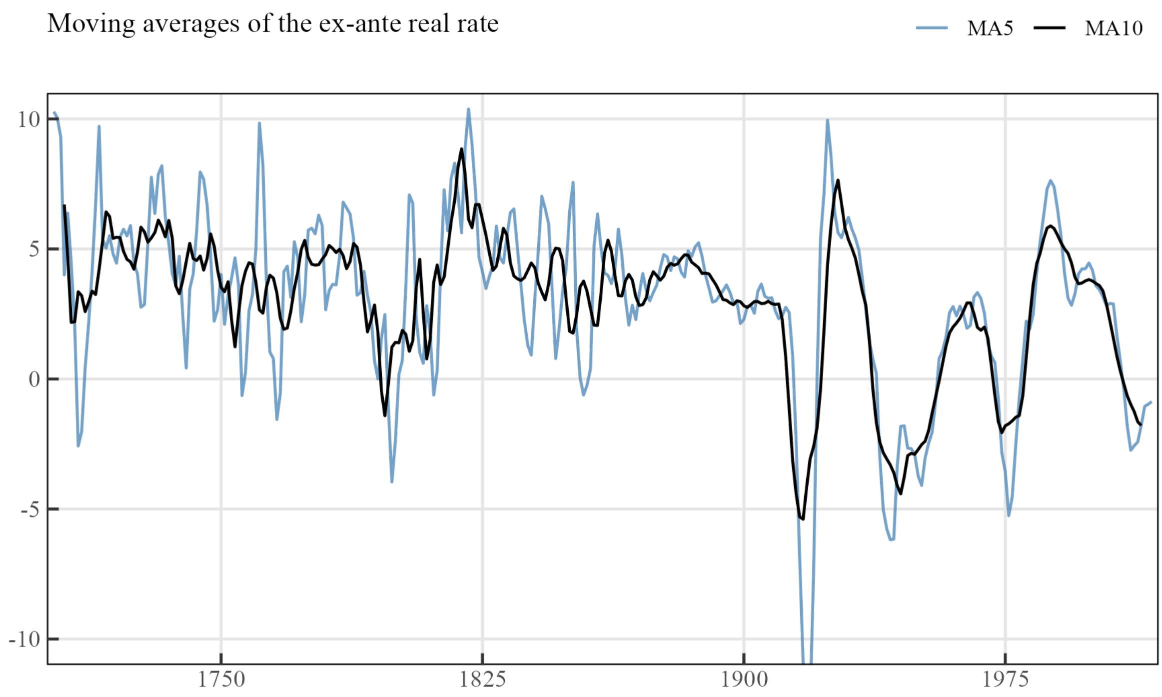

Figure F.3. Moving averages of the ex-ante real rate.

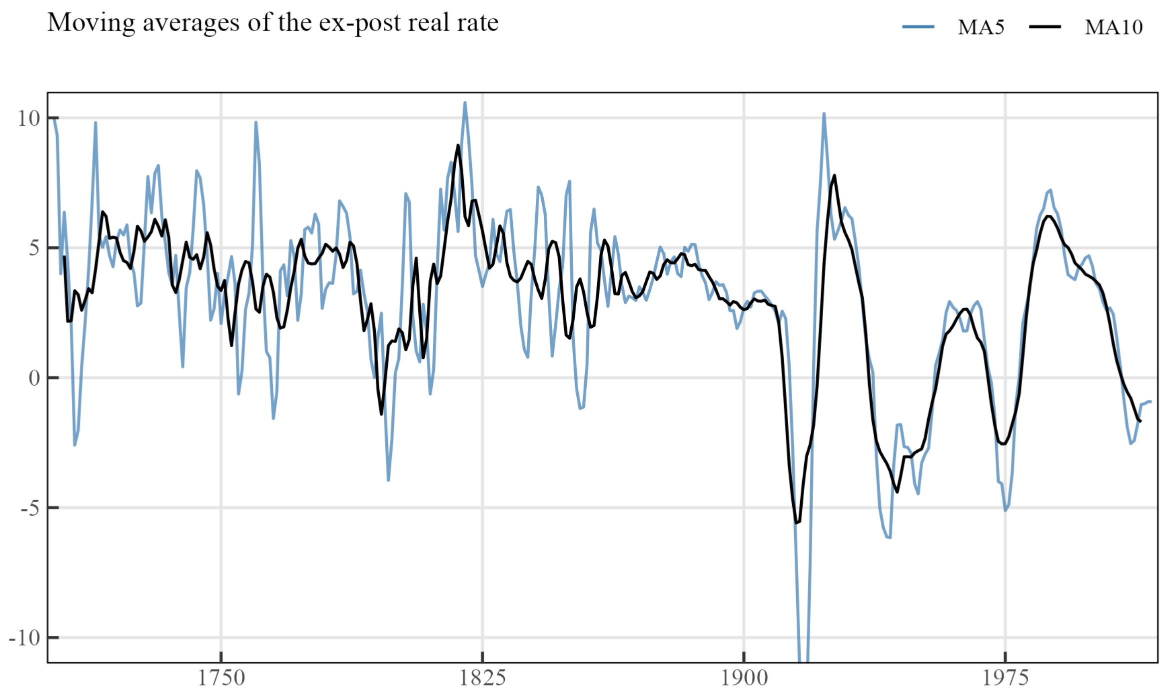

Figure F.4. Moving averages of the ex-post real rate.

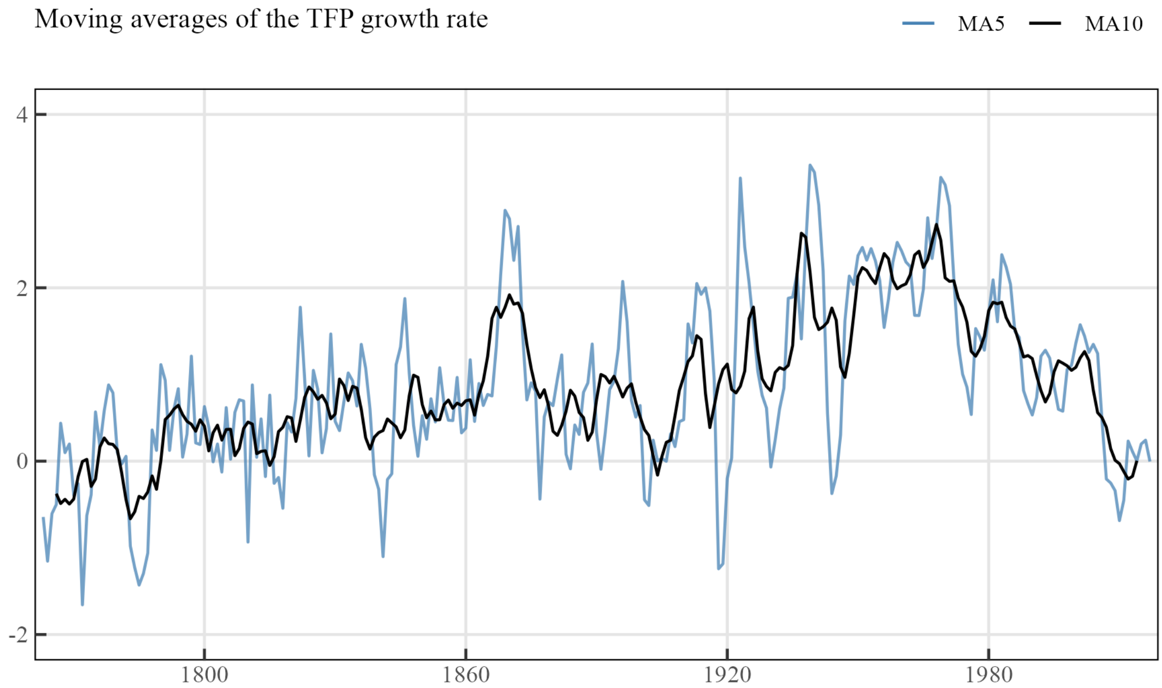

Figure F.5. Moving averages of the TFP growth rate.

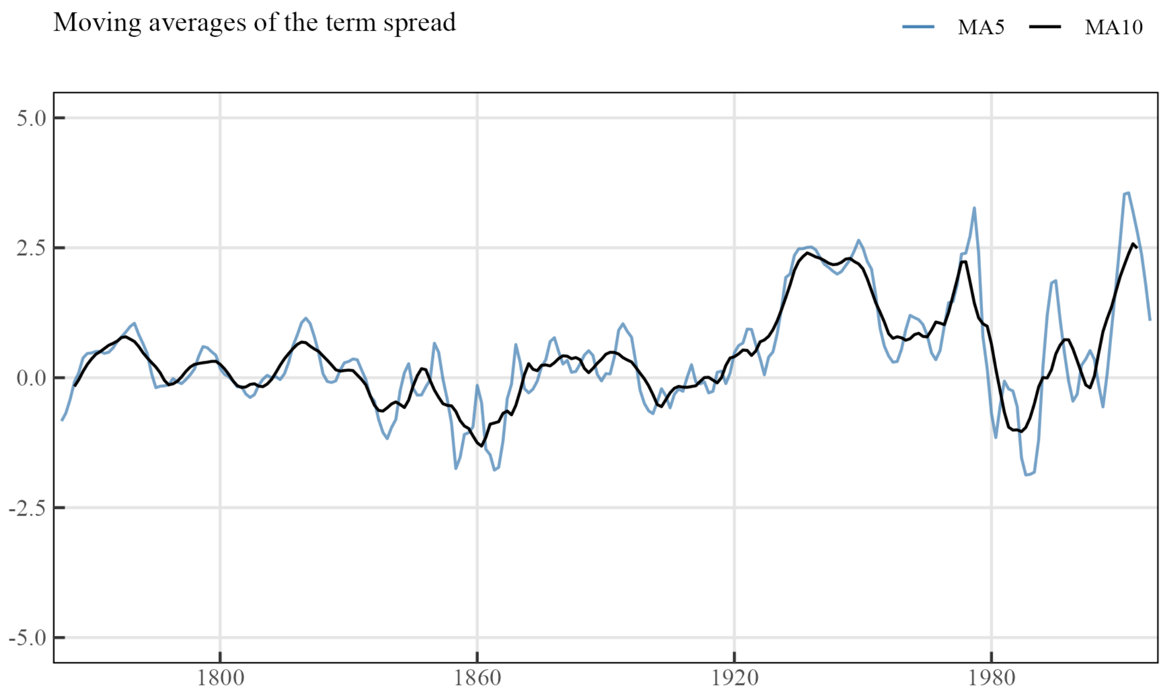

Figure F.6. Moving averages of the term spread.

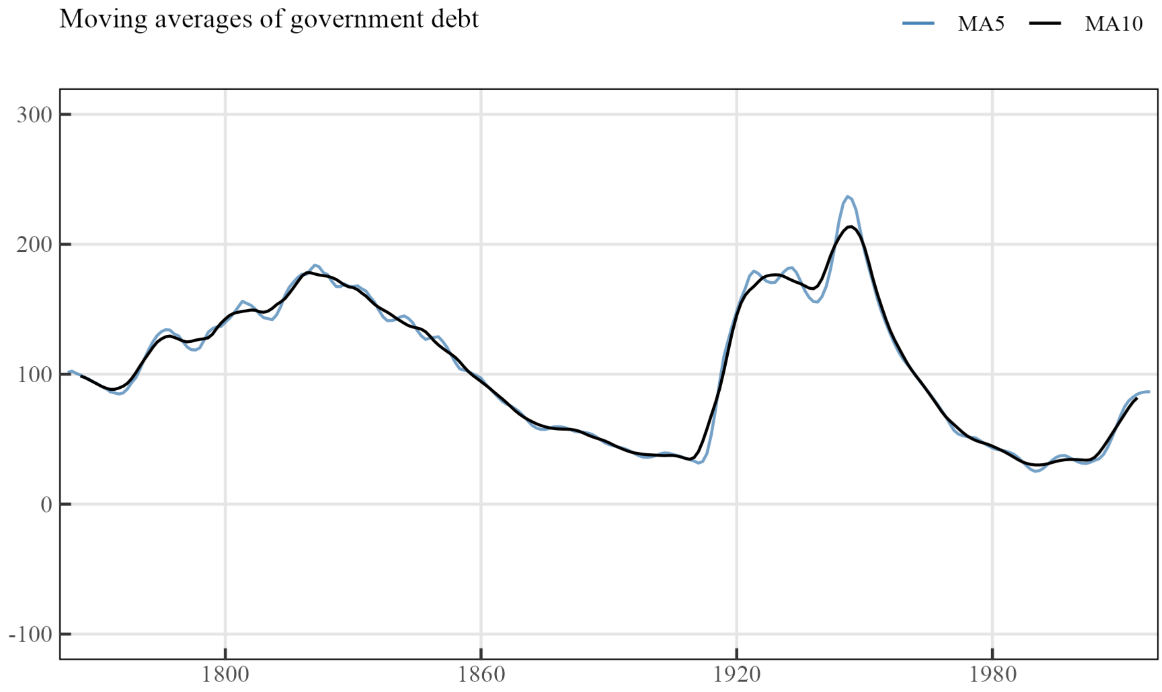

Figure F.7. Moving averages of government debt.

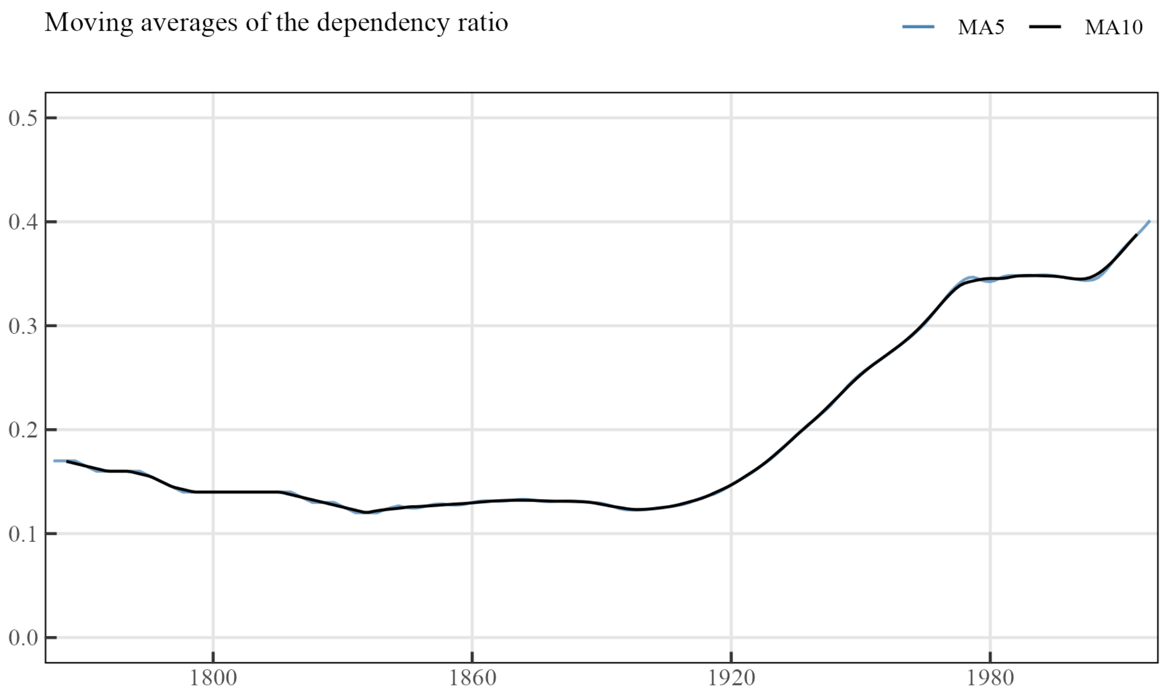

Figure F.8. Moving averages of the dependency ratio.