PSTAT 126 - Assignment 4

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Kayla Katakis

Note: ***Submit both your `Rmd` and generated pdf file to Canvas.*** *Use the same indentation level as* ***Solution*** *markers to write your solutions. Improper indentation will break your document.*

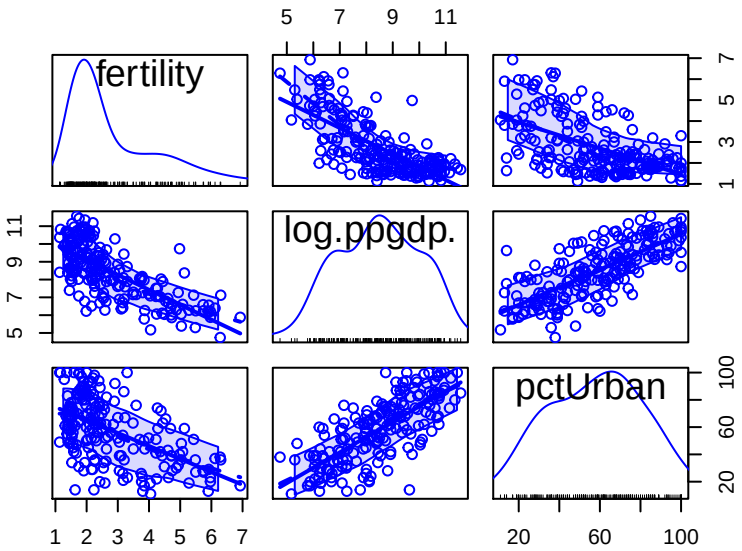
```
library(alr4)
library(ggplot2)
data(UN11)
```

1. This problem uses the data set `UN11` from the `alr4` package.

   (a) Examine the figure generated by using `scatterplotMatrix` function for attributes (`fertility`, log(`ppgdp`), `pctUrban`), and comment on the marginal relationships.

   Solution: There are positive linear relationships between log(ppgdp) / pctUrban, and negative linear relationships between fertility / log(ppgdp) and fertility /pctUrban

```
attributes = cbind(UN11[3],log(UN11[4]),UN11[6])
colnames(attributes) <- c("fertility","log(ppgdp)","pctUrban")
scatterplotMatrix(attributes)
```



   (b) Fit the two simple regressions `fertility` $\sim$ log(`ppgdp`) and `fertility` $\sim$ `pctUrban`, and verify that the slopes are significantly different from zero at any conventional level of significance.

   Solution: Both slopes are significant at any confidence level, as they both have p-values of approximately 0, which is less than 0.01, 0.05, and 0.1

```
ppgdp_lm <-lm(fertility~log(ppgdp),UN11)
pctUrban_lm <- lm(fertility~pctUrban,UN11)
summary(ppgdp_lm) #slope is -0.62009
```

```
##
## Call:
## lm(formula = fertility ~ log(ppgdp), data = UN11)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.16313 -0.64507 -0.06586  0.62479  3.00517
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.00967    0.36529   21.93   <2e-16 ***
## log(ppgdp)  -0.62009    0.04245  -14.61   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9305 on 197 degrees of freedom
## Multiple R-squared:   0.52,  Adjusted R-squared:  0.5175
## F-statistic: 213.4 on 1 and 197 DF,  p-value: < 2.2e-16
```

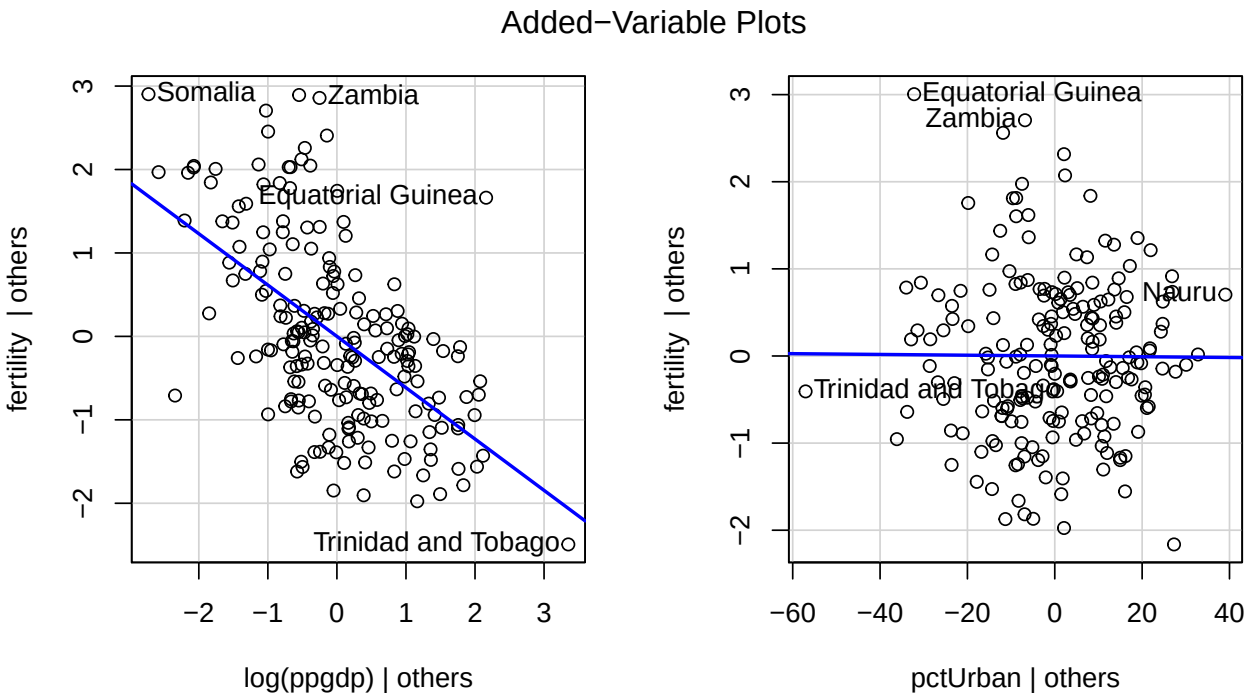
```
summary(pctUrban_lm) # slope is -0.031045
```

```
##
## Call:
## lm(formula = fertility ~ pctUrban, data = UN11)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.4932 -0.7795 -0.1475  0.6517  2.9029
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.559823   0.213681  21.339   <2e-16 ***
## pctUrban    -0.031045   0.003421  -9.076   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.128 on 197 degrees of freedom
## Multiple R-squared:  0.2948, Adjusted R-squared:  0.2913
## F-statistic: 82.37 on 1 and 197 DF,  p-value: < 2.2e-16
```

(c) Obtain the added-variable plots for both predictors. Based on the added-variable plots, does log(`ppgdp`) seem to be useful after adjusting for `pctUrban`, and similarly, does `pctUrban` seem to be useful after adjusting for log(`ppgdp`)?

Solution: Log(ppgdp) appears to be useful, as in still correlates with, fertility after adjusting for pctUrban, but pctUrban is not useful on its own (after adjusting for log(ppgdp)).

```
MLR_fertility <- lm(fertility~log(ppgdp)+pctUrban,UN11)
avPlots(MLR_fertility)
```



2. Consider a multiple linear regression model with two continuous predictors:

$$Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i, \qquad \varepsilon_i \overset{iid}{\sim} \mathcal{N}(0, \sigma^2).$$

(a) Suppose that x_{i1} and x_{i2} are exactly related in that $x_{i1} = 2.2x_{i2}$ for all i. For example, x_{i2} could be weight in kilograms and x_{i1} weight in pounds for the i-th individual. Describe the appearance of the added variable plot for x_{i2} after adjusting for x_{i1}.

Solution:
After adjusting for x_{i1}, the added variable plot for x_{i2} would look the same, except for a slight downward shift in the intercept.

b) Suppose that x_{i1} and x_{i2} are not perfectly correlated, but that $Y_i = 3x_{i1}$, i.e. Y_i is perfectly correlated with x_{i1}. Describe the added-variable plot for x_{i2}.

Solution: The added variable plot for x_{i2} would have a slope of 0, as $Y_i = 3x_{i1}$ implies that x_{i2} does not predictc Y_i.

c) (**Bonus**): Simulate some data for each of the situations in parts a) and b) and create an added-variable plot to confirm you answers.

Solution: