Kayla Mitchell
ESS 212
Homework 2
2/9/2024

1) $L(n+2) = L(n+1) + L(n)$

initial conditions $L(0) = 2$ $L(1) = 1$

a. $L(2) = 1 + 2$

for $n = 0$

$L(2) = 3$

$L(3) = 3 + 1$

for $n = 1$

$L(3) = 4$

$L(4) = 4 + 3$

for $n = 2$

$L(4) = 7$

b. find A & B such that

$$L(n) = A\left(\frac{1+\sqrt{5}}{2}\right)^n - B\left(\frac{1-\sqrt{5}}{2}\right)^n$$

satisfies diff EQ + initial conditions

for $n = 0$,

$$L(0) = A\left(\frac{1+\sqrt{5}}{2}\right)^0 - B\left(\frac{1-\sqrt{5}}{2}\right)^0$$

$$2 = A - B$$

$$\Rightarrow \quad A = 2 + B$$

for $n=1$, plug in $A = 2 + B$,

$$L(1) = 1 = A\left(\frac{1+\sqrt{5}}{2}\right)^1 - B\left(\frac{1-\sqrt{5}}{2}\right)^1$$

$$1 = (2+B)(\quad) - B(\quad)$$

$$2 = 2 + B(1+\sqrt{5}) - B(1-\sqrt{5})$$

(multiply both sides by 2)

$$2 = 2 + 2\sqrt{5} + B + B\sqrt{5} - B + B\sqrt{5}$$
$$0 = 2\sqrt{5} + B\sqrt{5} + B\sqrt{5}$$
$$0 = 2 + 2B$$
$$B = -1$$

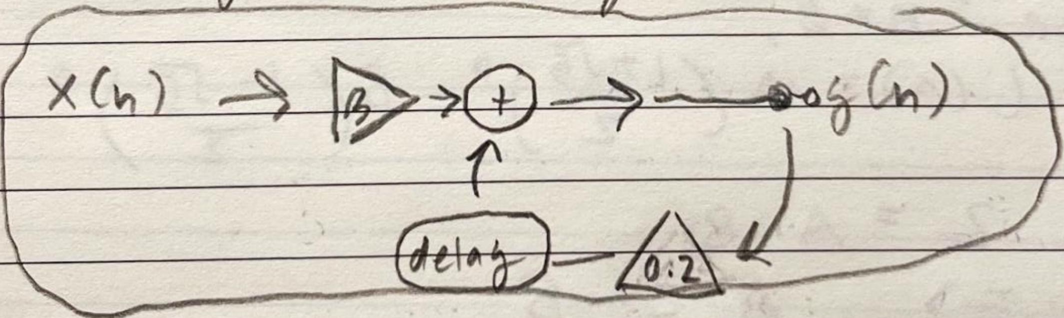
$$A = 2 + B = 2 - 1 = 1$$
$$A = 1$$

2) a. $y(n) = \alpha \cdot (y(n-1) + x(n))$

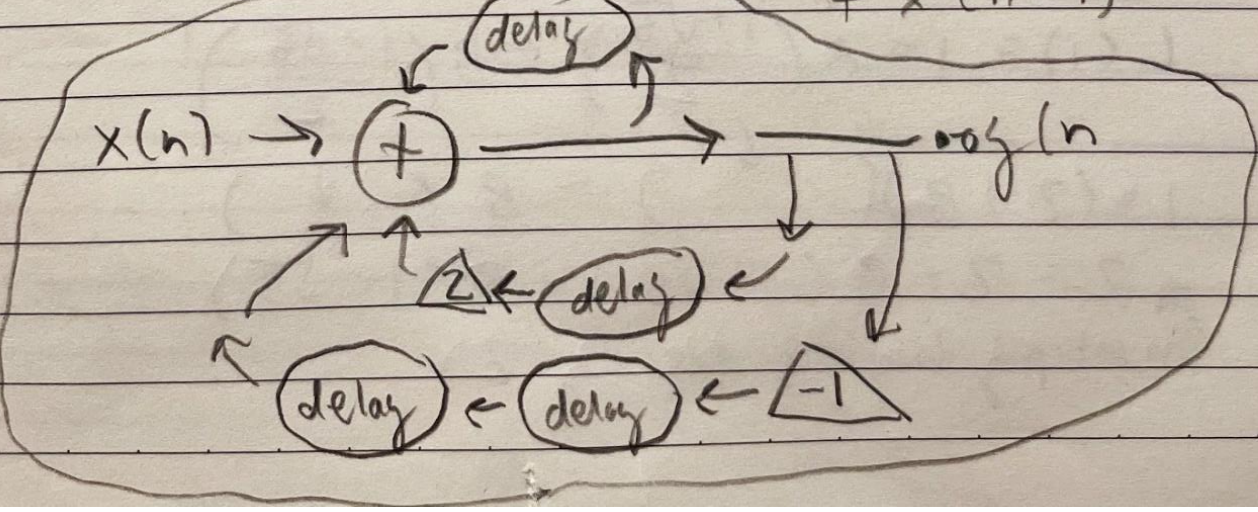
b. $y(n) = \alpha\left(\dfrac{y(n-1)}{\alpha} + y(n-2) + x(n)\right)$

3) a. block diagrams for:

$y(n) = 0.2\, y(n-1) + B x(n)$



b. $y(n) = 2y(n-1) - y(n-2) + x(n) \ldots$
$+ x(n-1)$



4a) a. given $y(n) = 0.2y(n-1) + \beta x(n)$
assume for $n=1$, $x(1) = A$ &
$y(1) = B$
plug this into the eqn:
$y(1) = 0.2y(\emptyset) + \beta x(1)$
$B = 0.2y(\emptyset) + \beta A$
we cant solve for $n=1$ because
we dont know $y(\emptyset)$.
we can't compute for $n = 1, 2, \ldots, N$

b. for $n > N$,
$y(n) = 0.2\, y(n-1) + \beta(\emptyset)$
$y(n) = 0.2y(n-1)$
when $n > N$, $y(n)$ decays by
factor of 0.2 each timestep

5) a. upload to Github
b. given $S(n) = Aa^n + Bb^n$,
& initial conditions : $S(0) = 0$, $S(1) = 1$,
& having calculated $S(2) = 2$ & $S(3) = 6$,
we can solve for A, a, B & b.

$S(\emptyset) = Aa^\emptyset + Bb^\emptyset = A + B$
$\underline{A = -B}$

$S(1) = 1 = Aa^1 + Bb^1 = Aa + Bb$ sub in
$A_a = -Ba + Bb$
$= B(b-a)$
$\underline{B = \dfrac{1}{b-a}}$

$S(2) = 2 = Aa^2 + Bb^2$

$2 = -Ba^2 + Bb^2 = B(-a^2 + b^2)$

$= \frac{1}{b-a}(b^2 - a^2) = \frac{b^2 - a^2}{b-a}$

Sub in

$\Rightarrow \quad 2 = \frac{(b-a)(b+a)}{b-a} = b + a$

$\underline{a = 2 - b}$

$S(3) = 6 = Aa^3 + Bb^3$

$6 = -Ba^3 + Bb^3 = B(b^3 - a^3)$

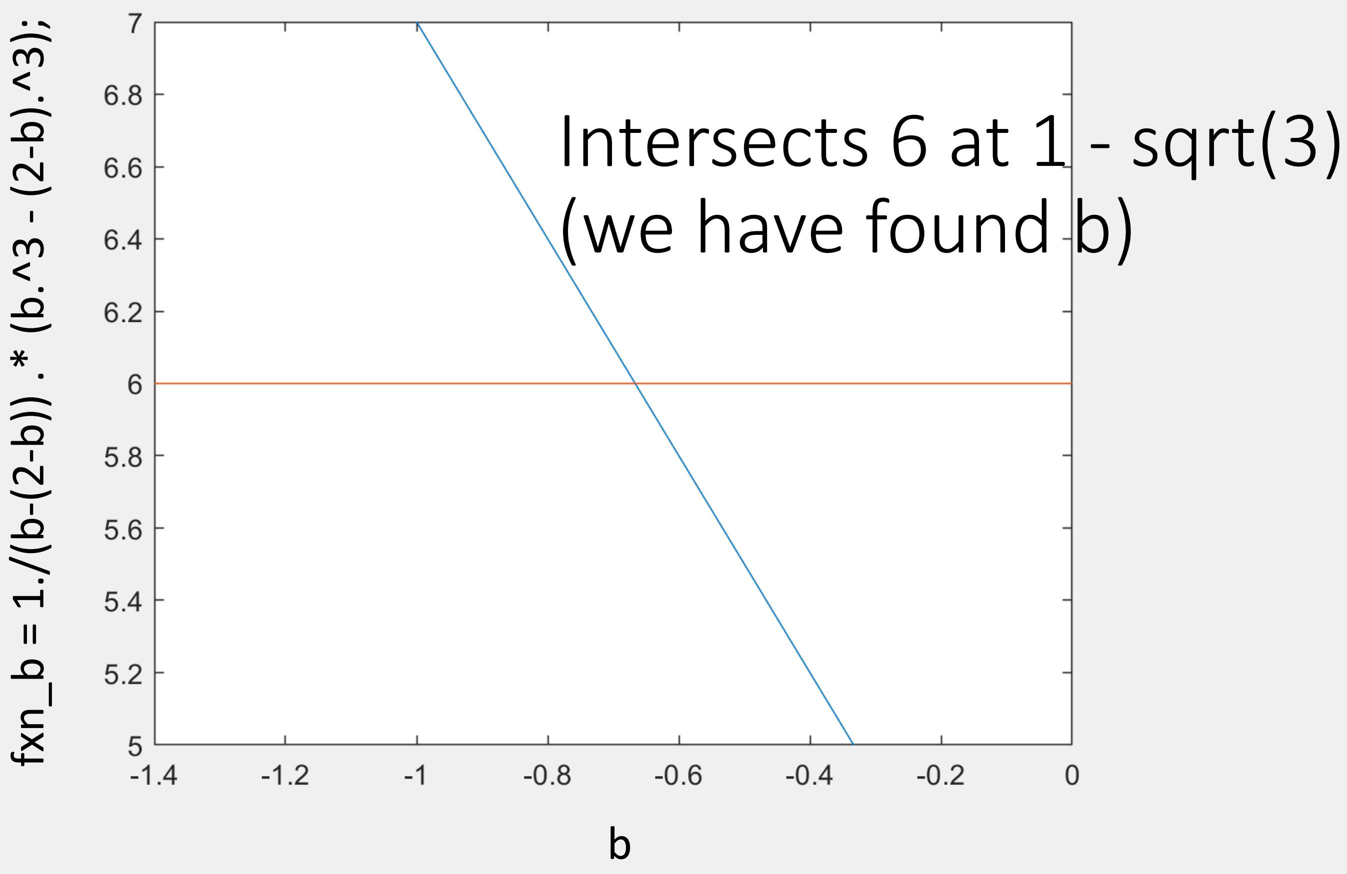
$= \frac{1}{b-(2-b)}(b^3 - (2-b)^3)$

plot this as a fxn of 6 to solve for b, find where it intersects 6.

$b = 1 - \sqrt{3}$, so $a = 1 + \sqrt{3}$,

$A = \frac{1}{2\sqrt{3}}$ & $B = \frac{-1}{2\sqrt{3}}$

5. In my Github Code:
a. compute the sequence S(n) for n = 0,1,...,N
c. verify computer program produces same sequence as closed form solution from part b (part c starts on line 49)
My code fprintfs “Sequences are the same!” After doing checks.



6a) a. given recurrence relation
$$x(n+1) = \frac{1}{2}\left(x(n) + \frac{c}{x(n)}\right)$$

can be used to find c, then:
$$x(n+1) = \sqrt{c}$$

plug this into next ~~time~~step $x(n+2)$:

$$x(n+2) = \frac{1}{2}\left(x(n+1) + \frac{c}{x(n+1)}\right)$$

$$= \frac{1}{2}\left(\sqrt{c} + \frac{c}{\sqrt{c}}\right) = \frac{1}{2}\left(\sqrt{c} + \frac{\sqrt{cc}}{\sqrt{c}\sqrt{c}}\right)$$

$$= \frac{1}{2}\left(\sqrt{c} + \frac{\sqrt{cc}}{c}\right) = \frac{1}{2}\left(\sqrt{c} + \sqrt{c}\right)$$

$$= \frac{1}{2}\left(2\sqrt{c}\right)$$

$$\boxed{x(n+2) = \sqrt{c}}$$

by plugging $x(n+1) = \sqrt{c}$ to get, the relation will continue to return $\sqrt{c}$ + can be used to compute $\sqrt{c}$

6 b-d

(in my Github code)

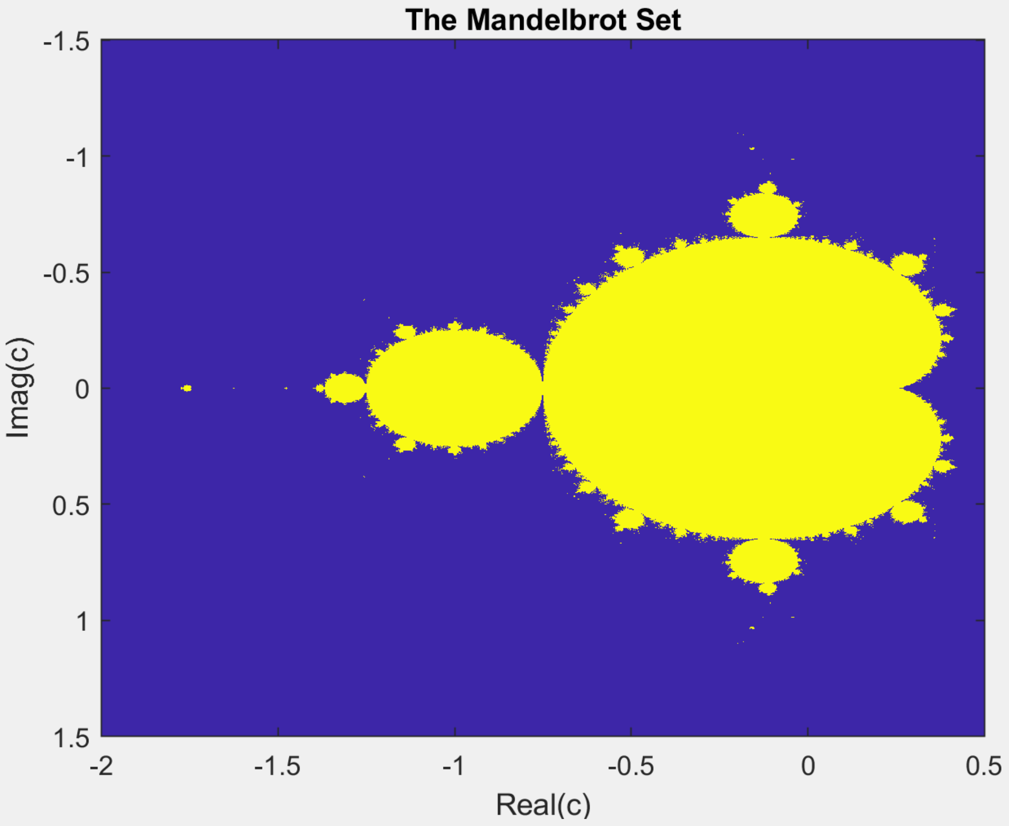
Problem 7

Compute the set of points c = x + iy with −2 < x < 0.5 and −1.5 < y < 1.5 that belong

to the Mandelbrot set. Make a plot of the points belonging to the Mandelbrot set. Upload

your code to your GitHub repository and include your plot in the HW2 pdf that you will

upload to Canvas :



8) a. Computed wille-E's height at time 1 * delta t and 2 * delta t. Tried to compute when Wille-E reaches bottom of canyon, determined whether the law of conservation of energy was satisfied (it was not). **In my attached matlab code is where I compute z values and find he never reaches the ground!** The z values I computed stay the same (he stays at the top of the cliff in the air forever?)

Scheme:

8) a. $z(t+\Delta t) = z(t) - \Delta t\sqrt{2g(z-z_0)}$

initial conditions: $z(\emptyset) = z_0$

the next steps @ $t \to \emptyset$, $t = \Delta t$, $t = 2\Delta t$
timestep 1 timestep 2
are:

$z(\emptyset + \Delta t) = z(\Delta t) = z(\emptyset) - \Delta t\sqrt{2g(z-z_0)}$

$z(\Delta t) = z_0 - \Delta t\sqrt{2g(z-z_0)}$ ← $z(\Delta t)$

$z(2\Delta t) = z(\Delta t) - \Delta t\sqrt{2g(z-z_0)}$
$= z_0 - \Delta t\sqrt{2g(z-z_0)}$

$z(2\Delta t) = z(0) - \Delta t\sqrt{2g(z-z_0)}$
$= z_0 - \Delta t\sqrt{2g(z-z_0)} - \Delta t\sqrt{2g(z-z_0)}$
↓

$z(2\Delta t) = z_0 - 2\Delta t\sqrt{2g(z-z_0)}$

Wille-E reaches bottom of canyon @ $z = \emptyset$

$z(t+\Delta t) = z(t) - \Delta t\sqrt{2g(\emptyset - z_0)}$

Negative # in sqrt
he turns an imaginary #
for $z(t+\Delta t)$ → NOT POSSIIBLE

~~t was multiplied by velocity~~ ✗
instead of acceleration ✓

This scheme DOES NOT SATISFY (in part a.) ✓ LAW OF CONSERVATION OF ENERGY

8b) I converted the second order equation into two first order equations, dz/dt and dv/dt. Then I applied these to the Forward Euler Scheme. Does it conserve energy? I plotted the scheme with Matlab to figure this out.

8) b. $m \frac{d^2z}{dt^2} = -mg$

EULER FORWARD SCHEME

Convert to 2 first order equations, dz/dt + dv/dt

$\frac{dz}{dt}$ (change in distance over time) is equal to velocity, v.

$\frac{dz}{dt} = v$

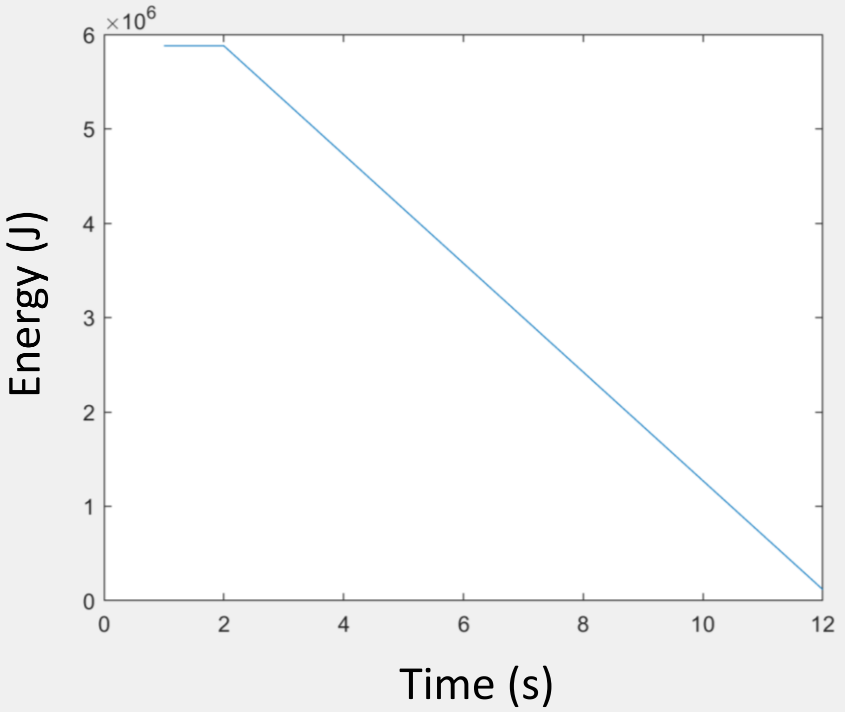
$\frac{dv}{dt}$ (change in velocity over time),

→ $\frac{dv}{dt}$ is same as $\frac{d^2z}{dt^2}$!

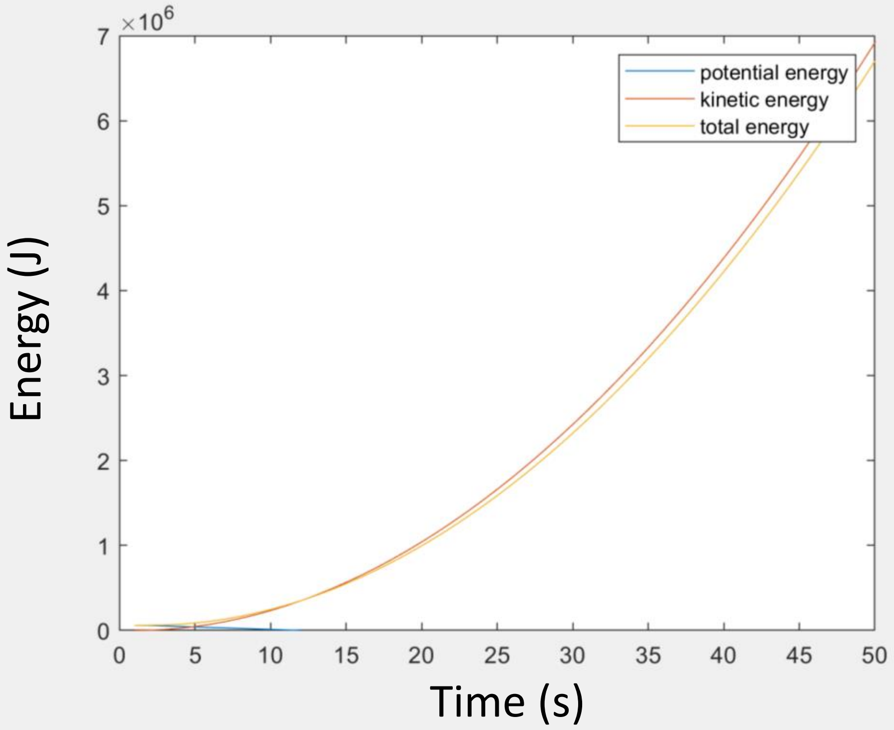
Using, $\cancel{m}\frac{d^2z}{dt^2} = -\cancel{m}g$ So, $\frac{dv}{dt} = -g$

Apply to FORWARD EULER SCHEME:

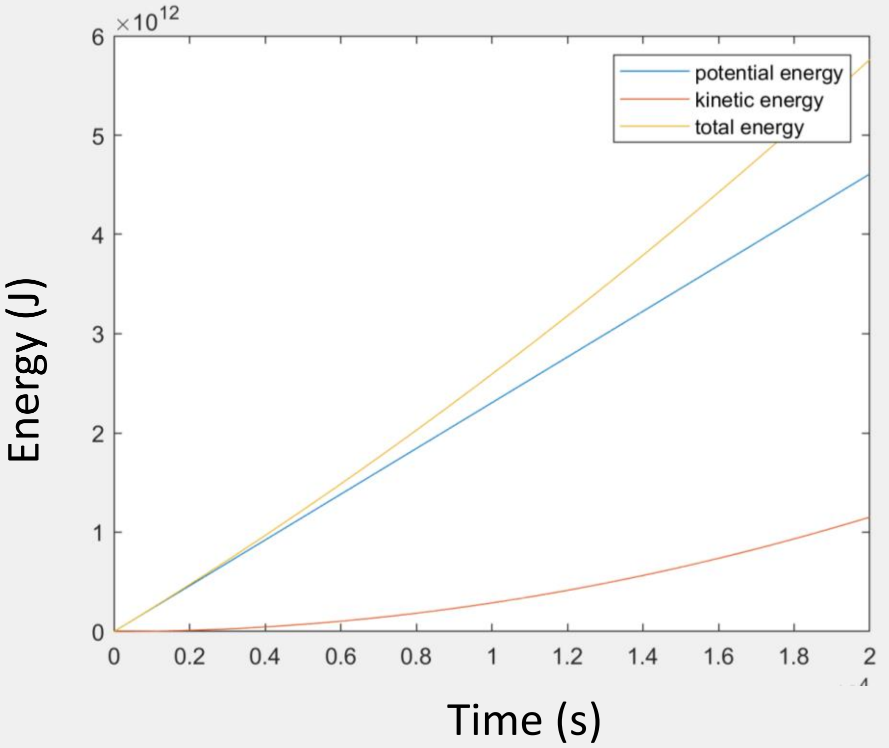
$z(t+\Delta t) = z(t) + \Delta t v = z(t) - \Delta t g$



8b. **Euler forward scheme**
← close up on potential energy- decreases with time and eventually goes negative (I cut it off after zero because negative energy isn't a thing).



← potential, kinetic, and total energy on same chart**. I think 8b, Euler Forward scheme is the one that works/conserves energy. It was derived using Newton's 2nd Law.**



8c. **Leap Frog Scheme**
← Both kinetic and potential energy increase with time? Doesn't seem possible.