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ELEC3310:
Electrical Energy Conversion and Utilisation

Topic 2: Three Phase System

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Three phase fundamentals

Three Phase AC Circuits

- Phasor Notation
- Single Phase Power
- Complex Power
- Direction of Power Flow
- Wye Connected Load
- Delta Connected Load
- Power in Balanced 3-Phase Load
- Per-Unit systems

System Parameters

Frequency

The standard frequency of operation of power systems either 50 or 60 Hz (cycles per second).

In Australia, the frequency is 50 Hz,

Voltages

The line (line – line) voltages is **ALWAYS** used when discussing three phase systems.

Power Rating

Total 3 phase power rating **ALWAYS** assumed

Phasor Notation

All electrical parameters can be expressed in form shown

$$V(t) = V_m \cos(\omega t + \phi)$$

This is equivalent to a vector with a magnitude of $V_m/\sqrt{2}$ (i.e. rms value of V(t)) rotated clockwise by an angle of Φ radians from the reference vector as described by:

$(V_m/\sqrt{2})*[\cos(\Phi) + j*\sin(\Phi)]$

This is commonly represented as $V \angle\Phi$

rms: "root mean square"

$$V_m - peak\ value$$

$$\omega - angular\ \ frequency = 2\pi f$$

$$f - 50Hz\ in\ Australia$$

$$\phi - phase\ shift$$

$$1/f = T - period\ of\ waveform$$

RMS of Effective Value

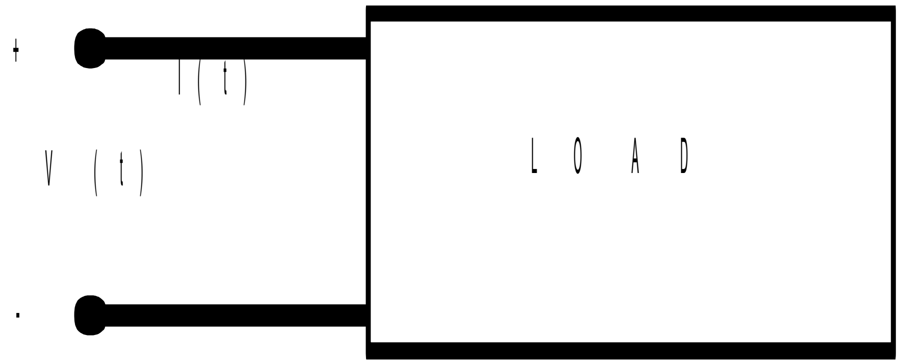
$$V_{rms} = \sqrt{\frac{1}{T}\int v^2(t)dt} \qquad V(t) = V_m \cos(\omega t + \phi)$$

$$= \sqrt{\frac{V_m^2}{2\pi/\omega}\int_0^{2\pi/\omega} \cos^2(\omega t + \phi)dt} = \frac{V_m}{\sqrt{2}}$$

V_{rms} – effective voltage of AC quantity, would produce same heating effect in resistor as DC voltage with same magnitude

Power in Single Phase Circuit

$V(t) = V_m\cos(\omega t+\Phi)$
$I(t) = I_m\cos(\omega t+\theta)$



Instantaneous power

$$p(t) = V(t)I(t)$$
$$= V_m I_m \cos(\omega t+\Phi)\cos(\omega t+\theta)$$
$$= 0.5V_m I_m\cos(2\omega t+\Phi+\theta)+0.5V_m I_m\cos(\Phi-\theta)$$

- Two components

  $0.5V_m I_m\cos(2\omega t+\Phi+\theta)$ – double frequency component
  $0.5V_m I_m\cos(\Phi-\theta)$ – time invariant component

Power in Single Phase Circuit

Real power

- of greater interest is average power dissipated in load, or mean value of p(t),
  $P = 0.5V_m I_m \cos(\Phi - \theta)$
  $= V_{rms} I_{rms} \cos(\Phi - \theta)$ [watts], [W]
- P – real power, active power
  - “useful” energy dissipated in load

Apparent power

- $S = V_{rms} I_{rms}$ [volt-amperes], [VA]

Reactive power

- $Q = V_{rms} I_{rms} \sin(\Phi - \theta)$ [volt-amps reactive], [VAr]

Power factor

Power factor = $\cos(\Phi - \theta) = P/VI$

- proportion of apparent power capable of doing “useful” work
- $(\Phi - \theta)$ – power factor angle
  - +ve – *lagging* power factor – mainly inductive load
  - - ve – *leading* power factor – mainly capacitive load
- Power factor lies between 0 - 1

Complex Power

Consider arbitrary load

- $v(t) = V_m \cos(\omega t + \theta_v)$ → $\boldsymbol{V} = V\angle\theta_v$
- $i(t) = I_m \cos(\omega t + \theta_i)$ → $\boldsymbol{I} = I\angle\theta_i$

"Complex" power – i.e. vector quantity

- $\boldsymbol{S} = \boldsymbol{VI}^* = (V\angle\theta_v)\,(I\angle{-\theta_i}) = VI\angle(\theta_v - \theta_i)$
- Magnitude given by apparent power – VI
- Phase angle given by phase angle difference between voltage and current, i.e. the power factor angle

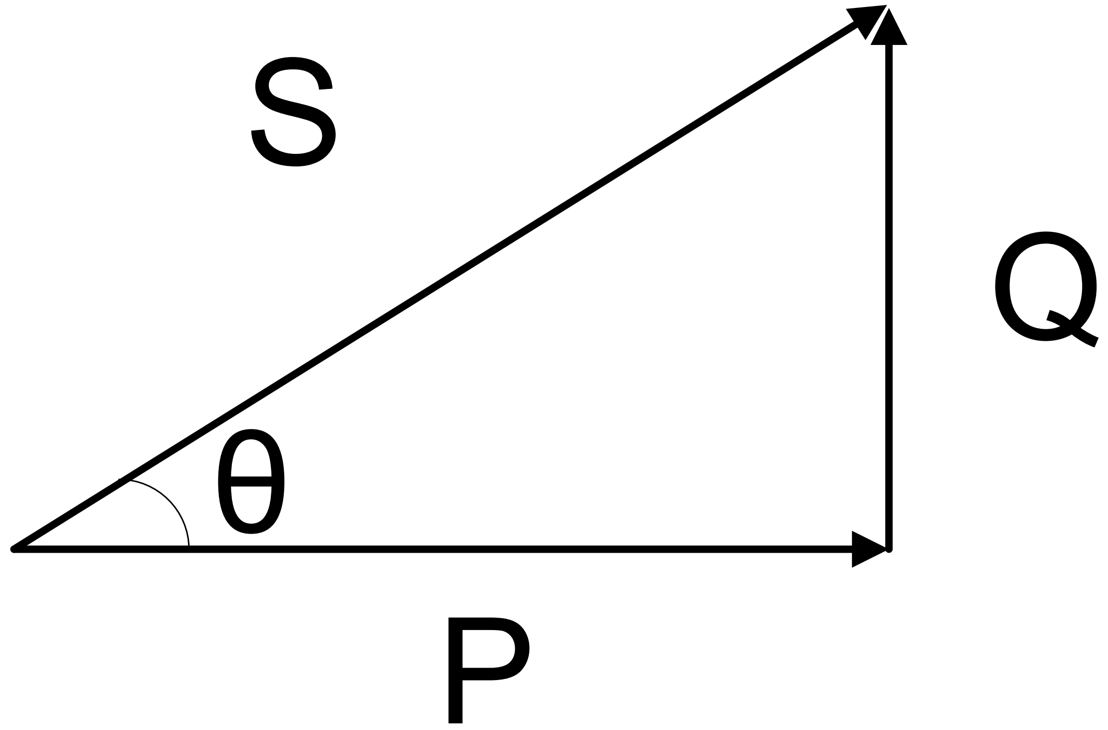
Power Triangle

If $\theta_v > \theta_{i,}$, power factor angle +ve, Q also +ve

- Current lagging voltage as in inductive load

If $\theta_v < \theta_{i,}$, power factor angle -ve, Q also -ve

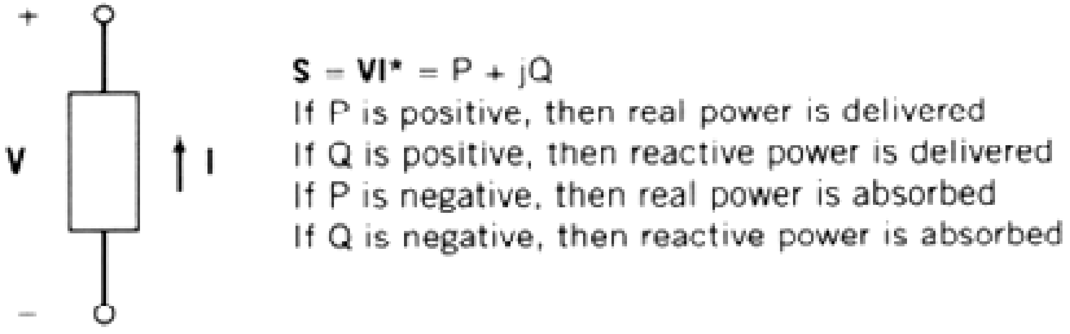
- Current leading voltage as in capacitive load



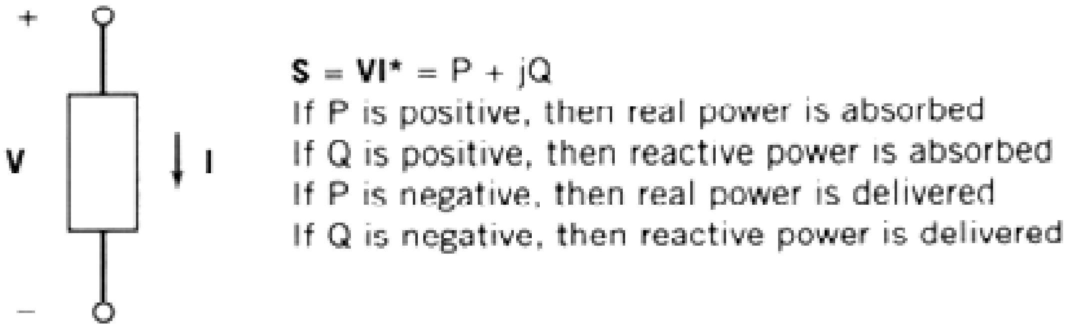
Direction of Power Flow

Direction of current flow assumed to be different for generator or load

- Generator
  - Current assumed to flow out in direction of voltage rise
- Load
  - Current assumed to flow into direction of voltage drop



(*a*) Generator condition



(*b*) Load condition

Tutorial #1

Q1. The instantaneous voltage *v(t)* across an electrical device and the instantaneous current *i(t)* entering the positive terminal of the circuit element are given by the following expressions:

$$v(t) = 110\cos(\omega t + 65^0)$$

$$i(t) = 15\sin(\omega t - 20^0)$$

Determine:

(a) The maximum or peak value of the voltage and current

(b) The rms value of the voltage and current

(c)The phasor expression for the voltage and current

3 Phase Systems

Bulk power generation, transmission and distribution accomplished using 3 phases

- Lightning and small motors can be connected single phase
  - Assigned equally to three phase distribution systems to maintain balance

Following slide shows positive phase sequence connection, or *a-b-c* sequence

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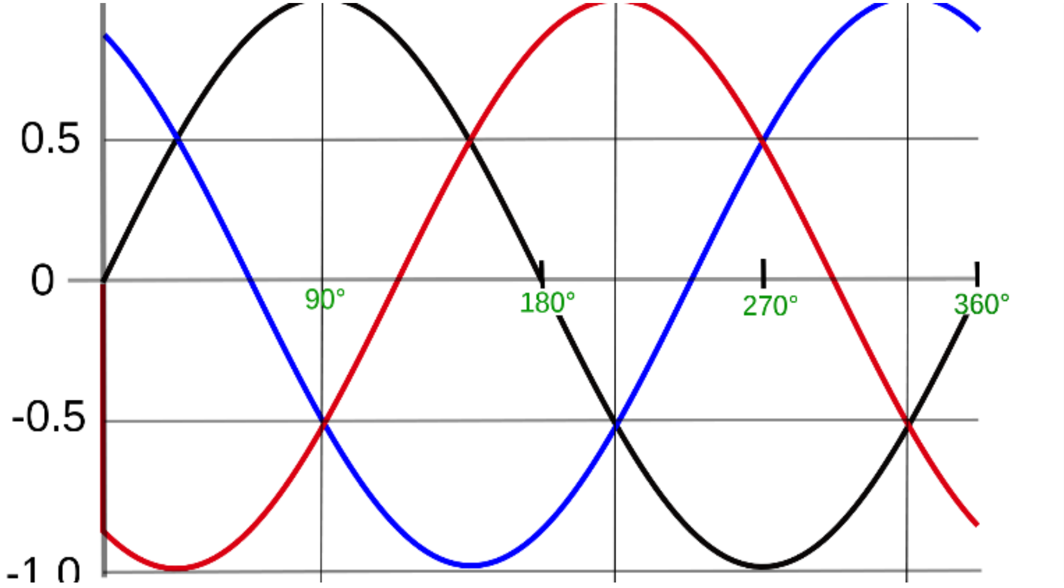
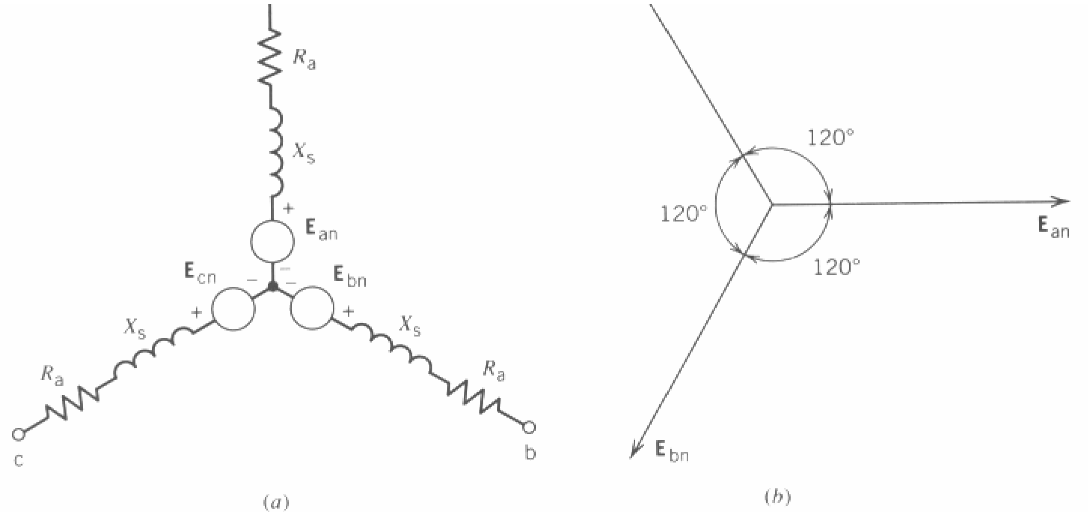
3 Phase Systems





va(t) = $V_m$cos(ωt) - Black

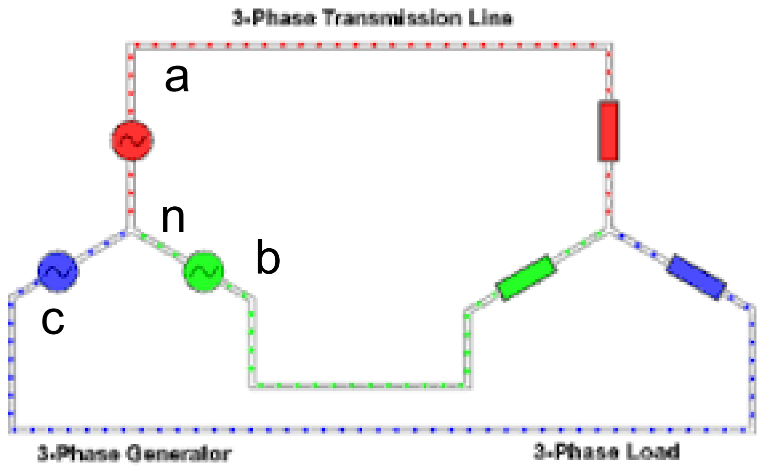
vb(t) = $V_m$cos(ωt-120) - Red

vc(t) = Vmcos(ωt-240) - Blue

3 Phase Systems

Three Phase Wye Source

- Terminals a, b, c called line terminals
- n – neutral terminal



Source balanced if voltages V_{an}, V_{bn}, V_{cn}, i.e. *phase voltages*, have same magnitude and sum (vector sum) to zero

- $|V_{an}| = |V_{bn}| = |V_{cn}|$
- $V_{an} + V_{bn} + V_{cn} = 0$

Wye connected load

$V_{an} = V\angle 0°$
$V_{bn} = V\angle\text{-}120°$
$V_{cn} = V\angle\text{-}240°$

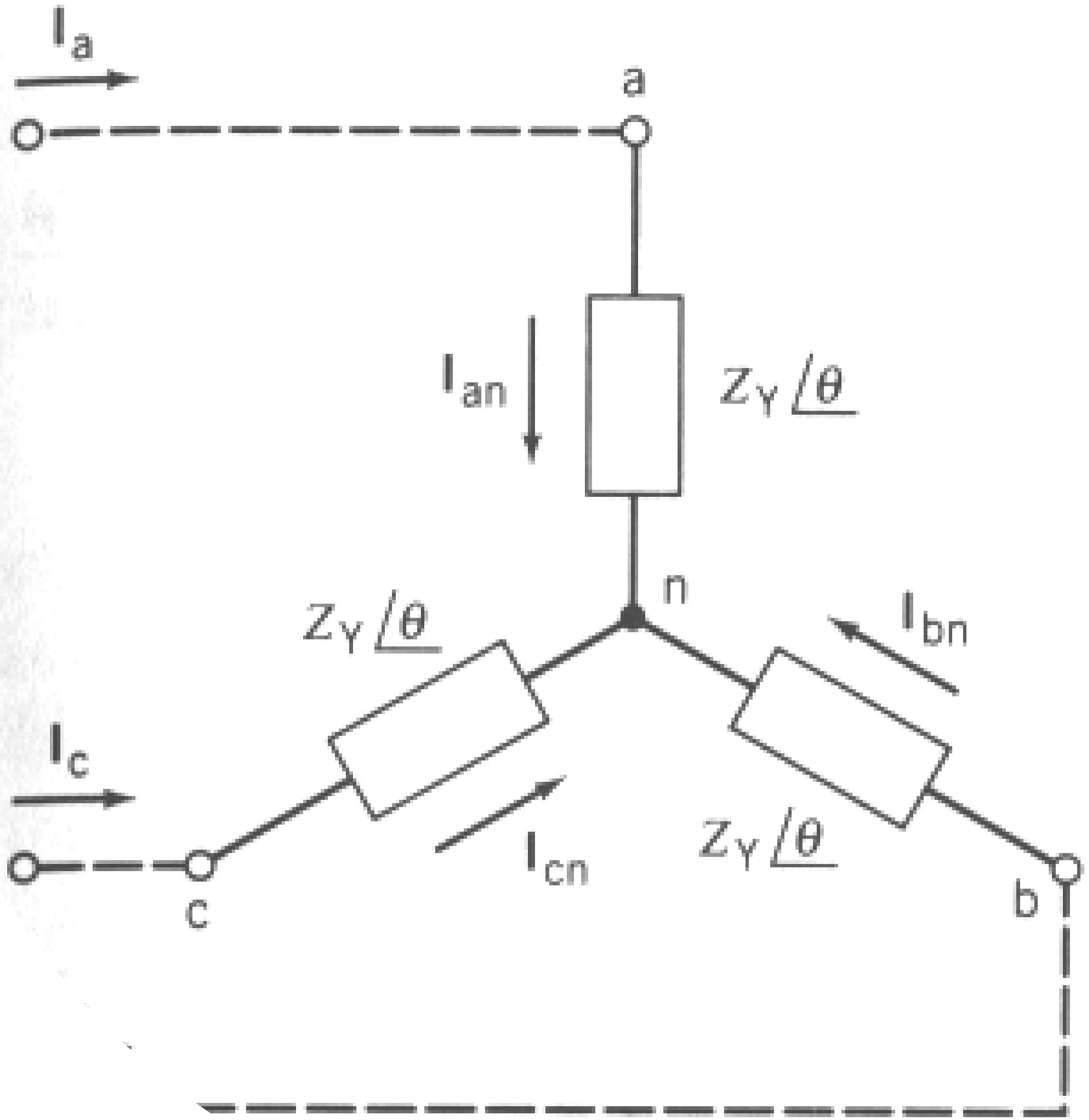
$I_{an} = V\angle 0°/Z_Y\angle\boldsymbol{\theta}$
$I_{bn} = V\angle\text{-}120°/Z_Y\angle\boldsymbol{\theta}$
$I_{cn} = V\angle\text{-}240°/Z_Y\angle\boldsymbol{\theta}$

Phase currents and line current are identical

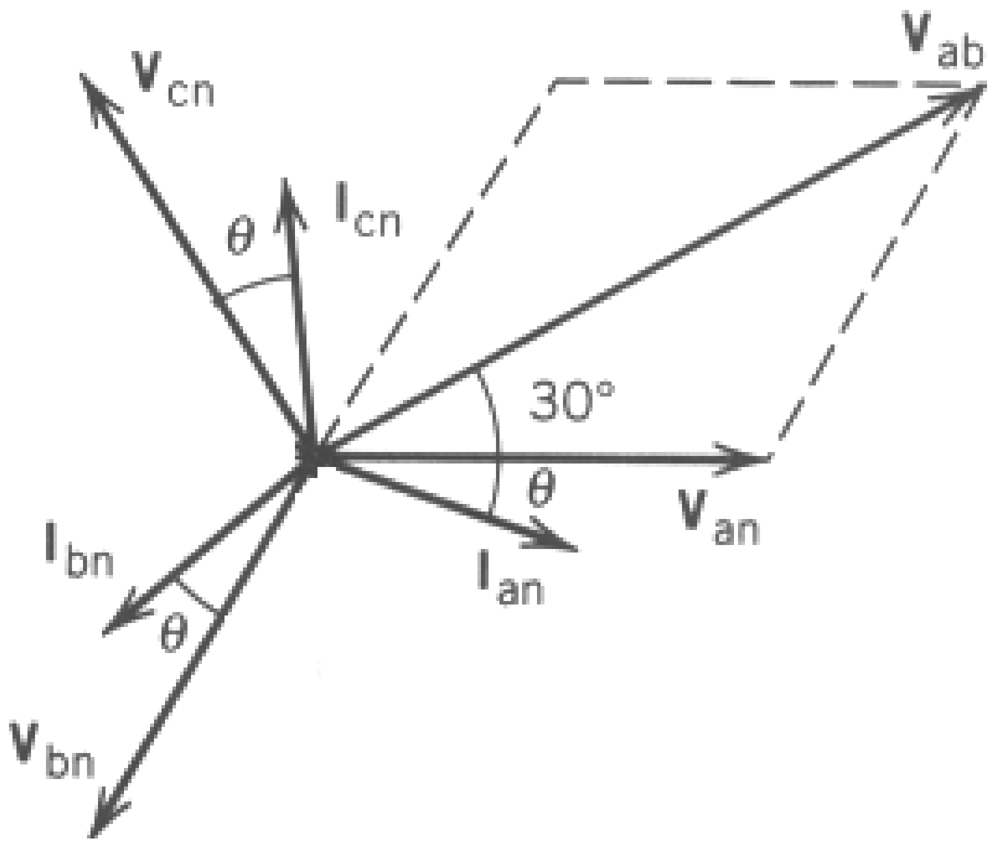
- Both determined by applied phase voltages and load impedances

Currents form balanced set of phasors which sum to zero

$$I_{an} + I_{bn} + I_{cn} = 0$$



Wye connected load – line-to-line voltages

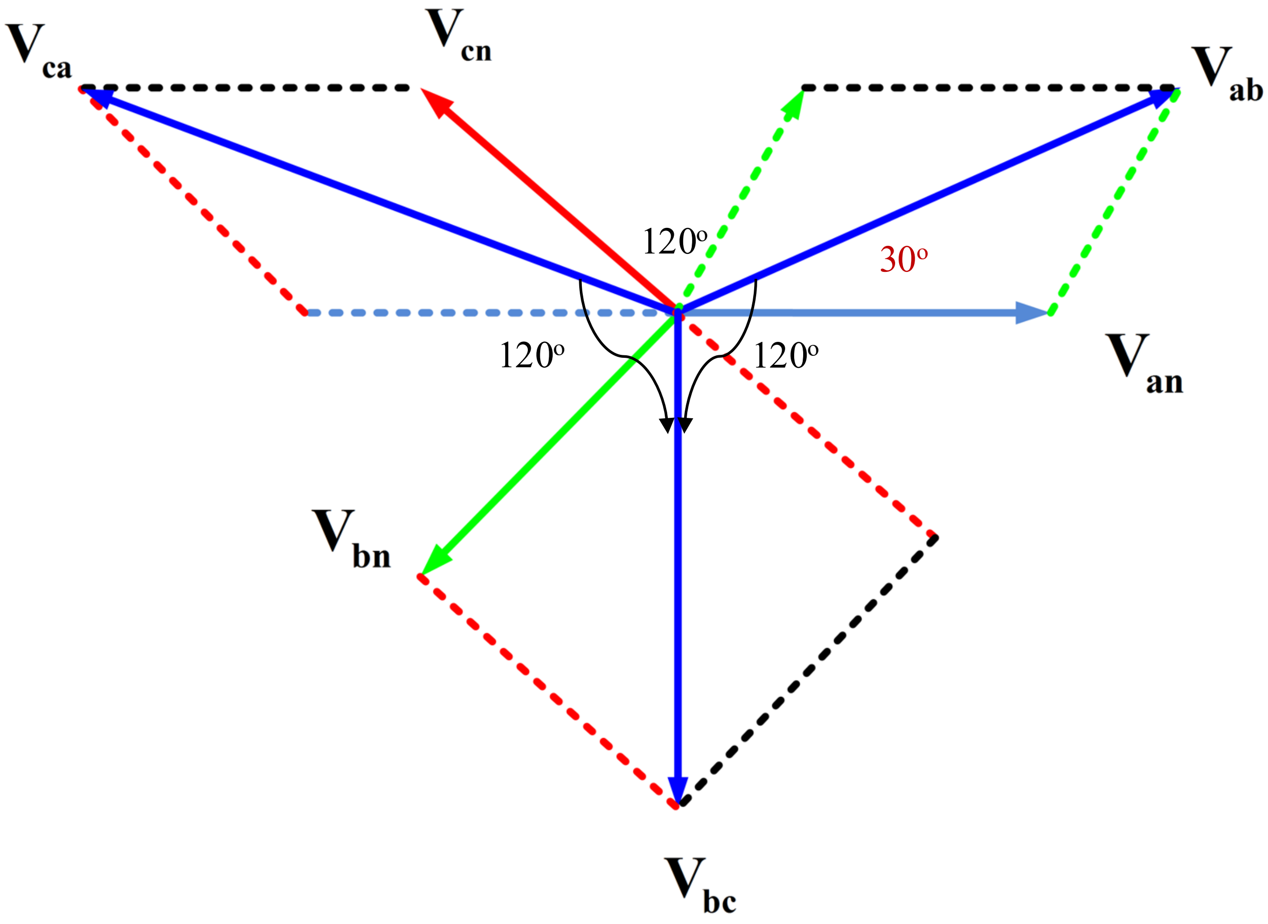


$$
\begin{aligned}
V_{ab} = V_{an} + V_{nb} &= V_{an} - V_{bn} \\
&= |V_{an}|*[1- 1\angle\text{-}120°] \\
&= \sqrt{3}|V_{an}| \angle{+30°}
\end{aligned}
$$

Line-to-line voltage:

- Has magnitude √3 times magnitude of phase voltage
- Leads corresponding phase voltage by 30°

Wye connected load – line-to-line voltages

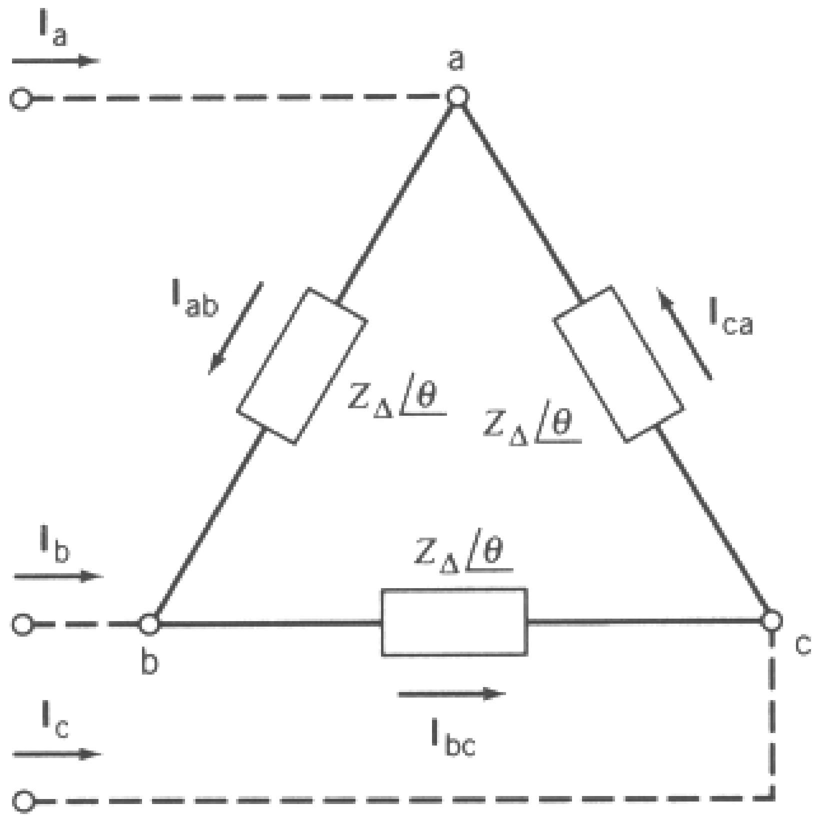


Delta Connected Load

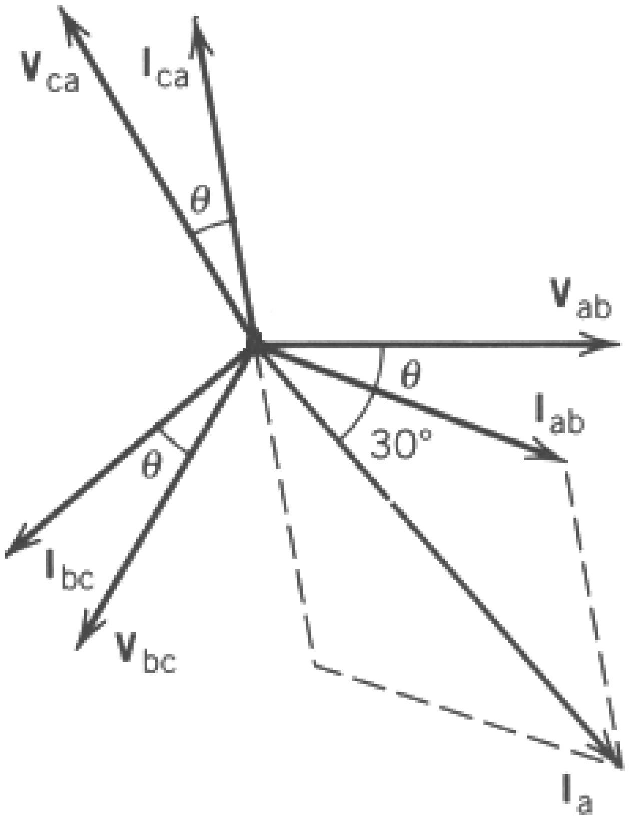
Line-to-line voltages identical to voltage across each load

Phase currents determined according to applied voltage and load. E.g.

$I_{ab} = V_{ab}/Z_{\Delta}\angle\theta = I\angle{-\theta}$
$I_{bc} = V_{bc}/Z_{\Delta}\angle\theta = I\angle{-\theta-120°}$
$I_{ca} = V_{ca}/Z_{\Delta}\angle\theta = I\angle{-\theta-240°}$



Delta Connected Load - Line currents



To calculate line current, apply KCL at any terminal

E.g. at point "a"

- $- I_a + I_{ab} - I_{ca} = 0$

  $I_a = I_{ab} - I_{ca}$

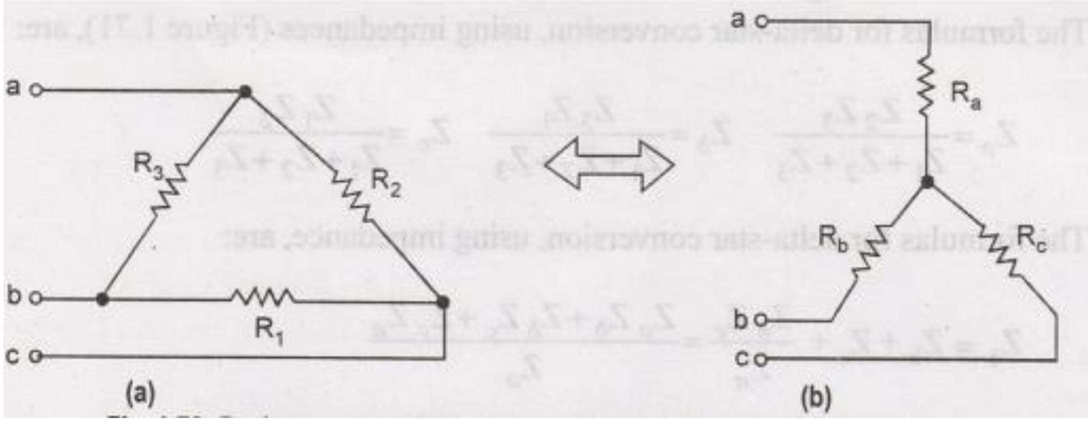
  $I_a = I_{ab} - I_{ab} \angle -240°$

  $= \sqrt{3} I_{ab} \angle -30°$

Line current

- Has magnitude √3 times phase current
- Lags corresponding phase current by 30°

STAR to Delta Conversion



$$R_a = \frac{R_2 R_3}{R_1 + R_2 + R_3} \quad R_b = \frac{R_3 R_1}{R_1 + R_2 + R_3} \quad R_c = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_1 = R_b + R_c + \frac{R_b R_c}{R_a} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

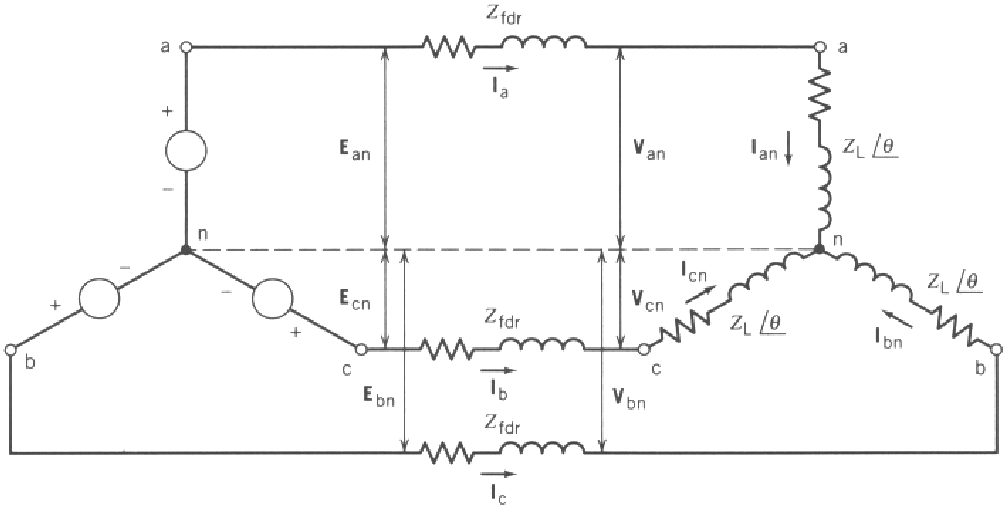
$$R_2 = R_c + R_a + \frac{R_c R_a}{R_b} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

$$R_3 = R_a + R_b + \frac{R_a R_b}{R_c} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

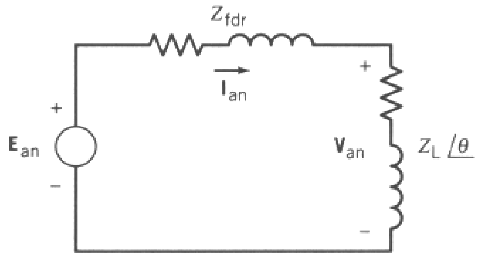
For a Balanced Load

$$R_{STAR} = \frac{1}{3} R_{DELTA}$$

Analysis of Balanced 3 Phase Systems



(a)



(b)

FIGURE 3.9 (a) Three-phase power system; (b) single-phase equivalent circuit.

Analysis of Balanced 3 Phase Systems

3 Phase circuits analysed using single phase equivalents

- Voltage/Currents in each phase identical to corresponding parameters in other phases, except shifted by 120°
- Neutral wire contains sum of return current from each phase $I_a + I_b + I_c = I_n = 0$ – for balanced systems neutral wire can be ignored

Delta connected loads can be transformed into wye connected loads for which line-to-line voltages and line currents unchanged by assuming:

$$Z_Y = (1/3)*Z_\Delta$$

Power in Balanced Three Phase Systems

Total average power absorbed by a three-phase balanced load (or delivered by three phase generator) equal to the sum of the powers in each phase.

Power absorbed in phase A

$$P_a = |V_{an}||I_{an}| \cos(\theta_p)$$

where θ_p is the phase angle between the voltage and the current determined by the load

$$|V_{an}| = |V_{bn}| = |V_{cn}| = |V_p|, \qquad |I_{an}| = |I_{bn}| = |I_{cn}| = |I_p|$$

$$P_a = |V_p||I_p| \cos(\theta_p) = P_b = P_c = P_p$$

Total three-phase real power:

$$P_T = 3P_p = 3|V_p||I_p| \cos(\theta_p)$$

Total three-phase reactive power:

$$Q_T = 3Q_p = 3|V_p||I_p| \sin(\theta_p)$$

Power in Balanced Three Phase Systems

Total 3 Phase apparent power

- $S_T = 3S_p = 3\sqrt{(P_p^{\ 2} + Q_p^{\ 2})} = \sqrt{(P_T^{\ 2} + Q_T^{\ 2})}$

For wye connected load

- $\sqrt{3}|V_p| = |V_{l\text{-}l}|$ - line-to-line voltage $\sqrt{3}$ time larger than phase voltage
- $|I_p| = |I_L|$ - line current and phase current identical

- Thus

$$P_T = \sqrt{3}\ |V_{l\text{-}l}|\ |I_L| \cos(\theta_p)$$
$$Q_T = \sqrt{3}\ |V_{l\text{-}l}|\ |I_L| \sin(\theta_p)$$
$$S_T = \sqrt{3}\ |V_{l\text{-}l}|\ |I_L|$$

For delta connected load

- $|V_{l\text{-}l}|$ - line-to-line voltage across load
- $\sqrt{3}|I_p| = |I_L|$ - line current $\sqrt{3}$ time larger than phase current

- Thus

$$P_T = 3P_p = \sqrt{3}\ |V_{l\text{-}l}|\ |I_L| \cos(\theta_p)$$
$$Q_T = 3Q_p = \sqrt{3}\ |V_{l\text{-}l}|\ |I_L| \sin(\theta_p)$$
$$S_T = \sqrt{3}\ |V_{l\text{-}l}|\ |I_L|$$

Power in Balanced Three Phase Systems

Irrespective of load type

- $P_T = 3|V_p||I_p| \cos(\theta_p) = \sqrt{3}\ |V_{l\text{-}l}|\ |I_L| \cos(\theta_p)$
- $Q_T = 3|V_p||I_p| \sin(\theta_p) = \sqrt{3}\ |V_{l\text{-}l}|\ |I_L| \sin(\theta_p)$
- $S_T = 3|V_p||I_p| = \sqrt{3}\ |V_{l\text{-}l}|\ |I_L|$

where θ_p is the phase angle of the load impedance of each phase

Instantaneous Power in Balanced Three Phase Systems

For a balanced three phase load, total instantaneous power equal to sum of the powers of three phases.

- $P_T = P_a + P_b + P_c = v_{an}\, i_a + v_{bn}\, i_b + v_{cn}\, i_c$

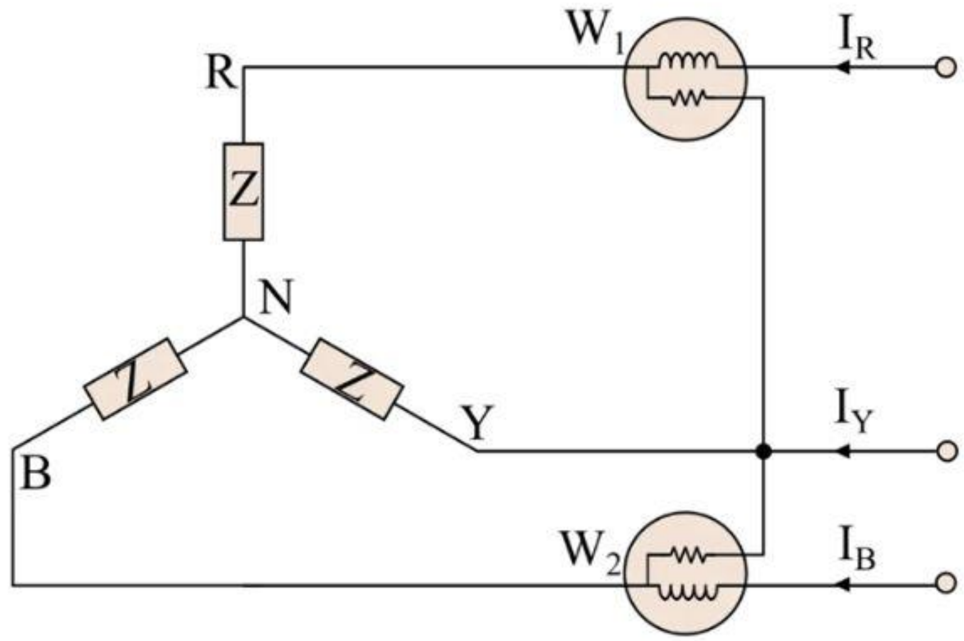
$$= 1.5*[V_m\, I_m\, \cos(\theta_P)\,]$$
$$+\, 0.5*V_m\, I_m\, [\cos(2\omega t-\theta_P) + \cos(2\omega t- \theta_P + 120^o)$$
$$+ \cos(2\omega t-\theta_P - 120^o)]$$

Second term reduces to zero, thus

$$P_T = (3/2)*[V_m\, I_m\, \cos(\theta_P)\,]$$

Total instantaneous power is time invariant.

Power Measurements in Three-Phase System



Assuming line-to-line voltage is V and line current is I for a balanced system Power factor angle is ϕ

$$W_1 = VIcos(30 + \emptyset)$$

$$W_2 = VIcos(30 - \emptyset)$$

$$W_1 + W_2 = \sqrt{3}\,\mathrm{V}I\cos(\emptyset)$$

$$W_2 - W_1 = VIsin(\emptyset)$$

Advantages of 3 Phase Systems

1. Each separate single-phase system requires both forward and return conductors to have current capacity at least as high as load current
    - Balanced three phase system can deliver same power to three single phase loads but requires only half number of conductors
        - Line losses potentially reduced by 50%
        - Reduced capital and operating costs of transmission and distribution, along with better voltage control (regulation)

Advantages of 3 Phase Systems

2. Total instantaneous power delivered under balanced conditions nearly constant
   - Three phase generator will have nearly constant mechanical input power and consequently near constant shaft torque
   - Instantaneous power in single phase systems has constant component **AND** double frequency component
     - High frequency component causes variation mechanical power to generator and shaft torque leading to shaft vibration and noise, and even shaft failure

Recap

Have reviewed three phase fundamentals

- AC power
- Line and phase voltages and currents
- Wye and delta connected loads and generators
- Three phase power

Now will finally look at per-unit quantities

Per unit fundamentals

Per Unit Quantities

- Definition
- Advantages
- Formulation
- Change of Base
- Examples

Per-Unit Quantities

Any electrical quantity may be expressed in "per-units" as a ratio of actual quantity to a chosen base value of that quantity

E.g.

$$per-unit\ quantity = \frac{actual\ quantity}{base\ value\ quantity}$$

- Actual quantity
  – value of quantity in actual units (such as volts, amps)
- Base quantity
  – reference value with same units as actual quantity

Base Quantity

Reference level

Always has same units as actual quantity being measured

- PER-UNIT QUANTITY IS DIMENSIONLESS

Always a real number – e.g. 100, or 1.5

- Phase angle of per-unit quantity always the same as the phase-angle of the actual quantity being measured

Advantages of Per-Unit

Eliminates need for conversion of voltages, current and impedances across every transformer

- Per-unit quantities same on both sides of transformer
  - Reduces chance of computational error

Many network quantities lies within narrow numerical bounds when expressed in per-units

- Nominal voltage or rated voltage of system usually chosen as voltage base ➔ per-unit value of voltage usually ~ 1.0 p.u.
  - Per-unit data can be checked rapidly for gross errors
- Manufacturers usually specify impedance of machines and transformers in per-unit or percent based on name-plate ratings

Per-Unit Quantities – Single Phase Systems

Network behaviour characterised by 4 base quantities

- Power (apparent power)
- Voltage
- Current
- Impedance

Base quantities must satisfy electrical laws

- $S_{base} = V_{base} I_{base}$
- $V_{base} = I_{base} Z_{base}$

Necessary to select two base values from which remaining quantities will be specified

Per-Unit Quantities – Single Phase Systems

Usual to specify Power and Voltage bases

- These parameter often determined according to rated values or nominal values network
  E.g. – for transmission line nominal voltage 132 kV
  and power rating 100 MVA

Current and Impedance bases calculated from Power and Voltage bases

$$I_{base} = \frac{S_{base}}{V_{base}}, \quad Z_{base} = \frac{V_{base}}{I_{base}} = \frac{(V_{base})^2}{S_{base}}$$

Per-Unit Quantities – Single Phase Systems

Per-unit electrical quantities calculated as:

$$S_{per-unit} = \frac{P + jQ}{S_{base}} = P_{per-unit} + jQ_{per-unit}$$

$$V_{per-unit} = \frac{V\angle\phi_v}{V_{base}} = \frac{|V|}{V_{base}}\angle\phi_v$$

$$I_{per-unit} = \frac{I\angle\phi_i}{I_{base}}$$

$$Z_{per-unit} = \frac{Z\angle\phi_z}{Z_{base}}$$

Per-Unit Quantities – Three Phase Systems

For 3 phase systems,

- Base power is total 3 phase
- Base voltage is line-to-line voltage
- Base line current assumed equal to base phase current
  - Assumption is that network wye connected
- Base impedance is the same per-phase base quantity

Per-Unit Quantities – Three Phase Systems

For 3 Phase Systems

- Base power: $S_{T,base} = 3S_{p,base}$
- Base voltage: $V_{L,base} = \sqrt{3}V_{p,base}$
- Base current: $I_{p,base} = (S_{T,base})/(\sqrt{3}V_{L,base})$

  $= (S_{p,base})/(V_{p,base})$
- Base impedance: $Z_{p,base} = (V_{L,base})^2/(S_{T,base})$

  $= (V_{p,base})^2/(S_{p,base})$

Change of base formula

Impedance characteristics of electrical equipment usually expressed as percentage based on machine ratings

- Machine ratings may be different from system voltage or power bases
- Need formula to convert per-unit impedance or percentage impedance of machine ratings to per-unit impedance for new base

Change of base formula

$$Z_{per-unit,old} = \frac{Z_{actual}}{Z_{base,old}} = \frac{Z_{actual} \times S_{base,old}}{\left(V_{base,old}\right)^2}$$

$$Z_{per-unit,new} = \frac{Z_{actual}}{Z_{base,new}} = \frac{Z_{actual} \times S_{base,new}}{\left(V_{base,new}\right)^2}$$

$$Z_{per-unit,new} = Z_{per-unit,old} \frac{Z_{base,old}}{Z_{base,new}} = Z_{per-unit,old} \frac{\left(V_{base,old}\right)^2}{\left(V_{base,new}\right)^2} \times \frac{S_{base,new}}{S_{base,old}}$$

Recap

Had looked at:

- Energy sources and conversion
- Single phase phasors, power etc.
- Three phase voltages, currents, loads, power
- Per-unit quantities

Coming Up…

- Magnetic circuits

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Questions?

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