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ELEC3310:
Electrical Energy Conversion and Utilisation

Topic 3: Magnetic circuits

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Magnetic Circuits

- Sources of Magnetic Field/Flux
- **Amperes Law**
- Ferromagnetism
- Analogy to Electrical Circuits
- **Faraday's Law**
- Inductance and Magnetic Energy
- Core losses

Magnetic Circuits

Most electric machines convert energy by use of magnetic field. e.g.

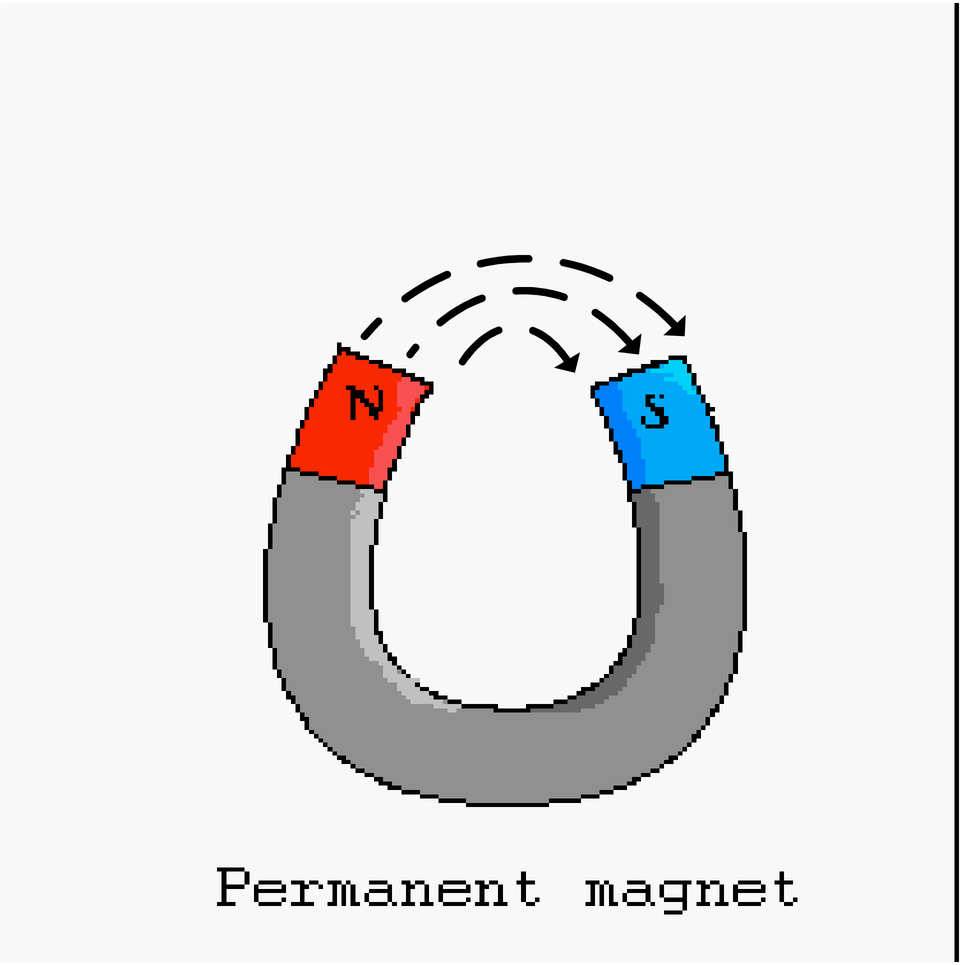
- Synchronous Generators
- Induction Motors
- DC Machines

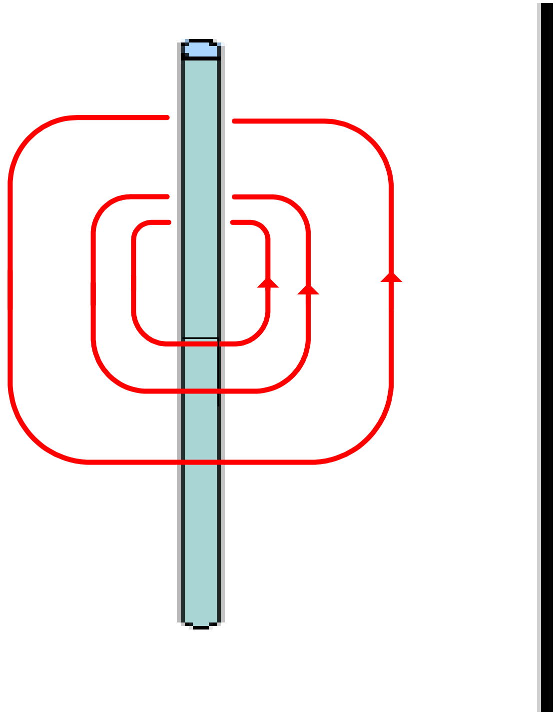
Need to understand basic magnetic circuit concepts

- Especially analogy to electric circuits

Application of magnetic circuit concepts to determine the magnetic fields in practical devices fundamental in determination of equivalent circuits for AC machines

Source of Magnetic Fields/Flux





Current carrying wire

Source of Magnetic Fields

Permanent Magnetic

- North pole is the pole from which the field lines emanate, and the south pole is the end to which the field lines return

Current carrying wire

- If a wire is grasped with the thumb pointing in the (conventional) current direction, the fingers encircle the wire in the direction of the magnetic field

Source of Magnetic Fields

Summary

- Magnetic fields are produced by electric current
  - Macroscopic current in wires
  - Microscopic current associated with electrons in atomic orbit

Ampere's Law

The line integral of the magnetic field intensity around a closed path is equal to the algebraic sum of the currents flowing through the area enclosed by the path.

$$\oint \mathbf{H} \cdot d\mathbf{l} = I$$

H – Magnetic field intensity [amperes/metre] [A/m]
- measure of magnetic field strength

I – Current producing magnetic field

Ampere's Law

Magnetic field intensity produced by coil of N turns

$$\oint \mathbf{H}_c \cdot \mathbf{dl} = NI$$

NI – magneto motive force
Units – ampere.turns
Dimension – amperes

Presence of magnetic field will produce lines of *magnetic flux* with density

$$B = \mu H \qquad [\text{Tesla}] \text{ or } [\text{Wb/m}^2]$$

- Distribution and strength of magnetic field will depend upon path taken by flux produced by magneto-motive force (MMF)

Ferromagnetism

When iron, iron alloys or other special materials situated in magnetic field, *flux density* produced within metal much greater than corresponding value in free space

FERROMAGNETISM

- Property of producing higher flux densities (than free space)

Flux density in free space

$B = \mu_o H$, μ_o – permeability of free space, $4\pi \times 10^{-7}$ [Wb/A.m]

Flux density in ferromagnetic material

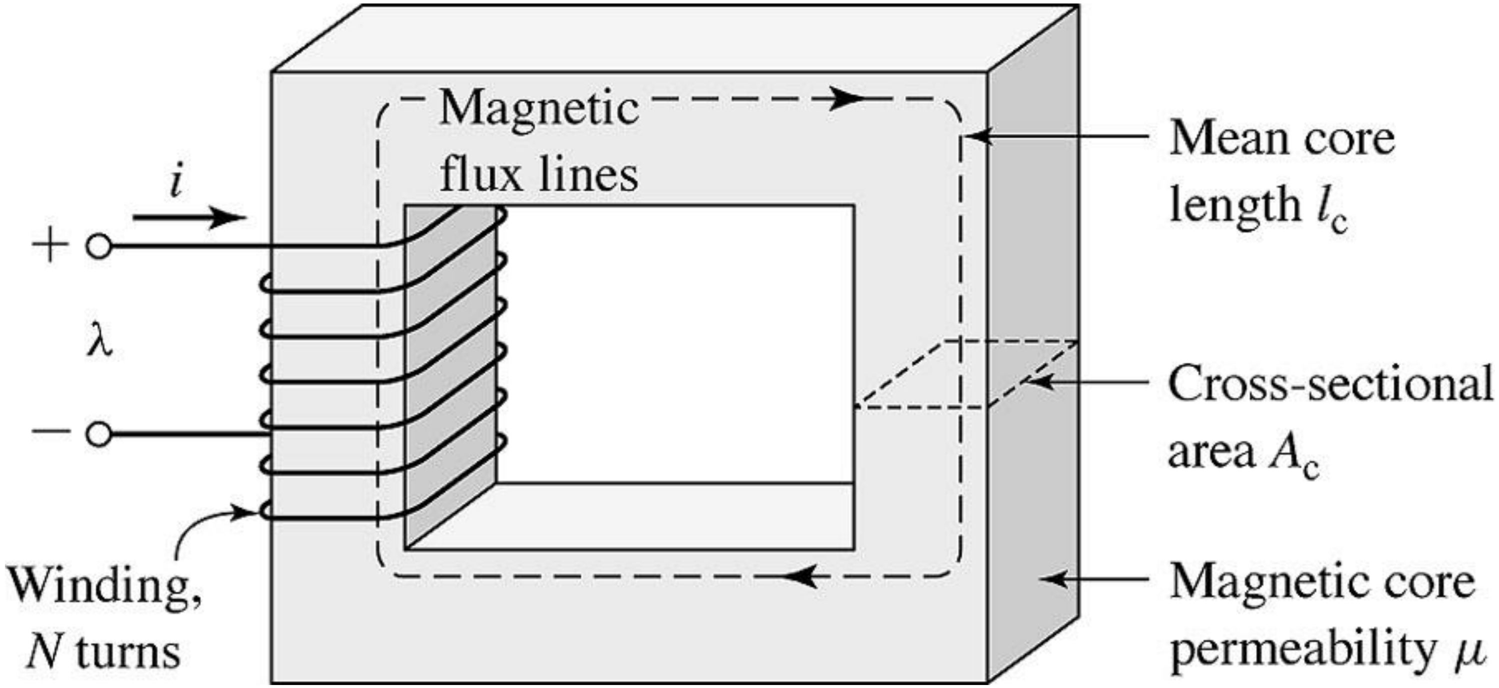
$B = 400000\mu_o H$, $= \mu_r \mu_o H$ μ_r – relative permeability

Magnetic Circuits

A magnetic circuit then consists of magnetic structure built mainly of high permeability material.

- Magnetic flux confined to paths presented by high permeability material
- Similar to confinement of electric current to conductors in electric circuit

Magnetic Circuit



Magnetic motive force (MMF), produces magnetic field

Magnetic field, H_c, approximately constant

Produces magnetic flux, Φ, within the material

- Dependent upon permeability of material

Magnetic Flux

$$\phi = \int_S B.dS$$

Magnetic flux, Φ, [webers] or [Wb]
– sum across a surface of density of magnetic flux passing through it (at perpendicular)

Important concept when flux density uniform across surface through piece of magnetic material

Magnetic Flux

For magnetic circuit shown previously

$\phi = B_c A_c \qquad = \mu H_c A_c$

From Amperes Law – $\qquad NI = H_c L_c$

$\phi = \mu A_c (NI/L_c)$

$\phi [L_c/(\mu A_c)] = NI = \mathcal{F}$

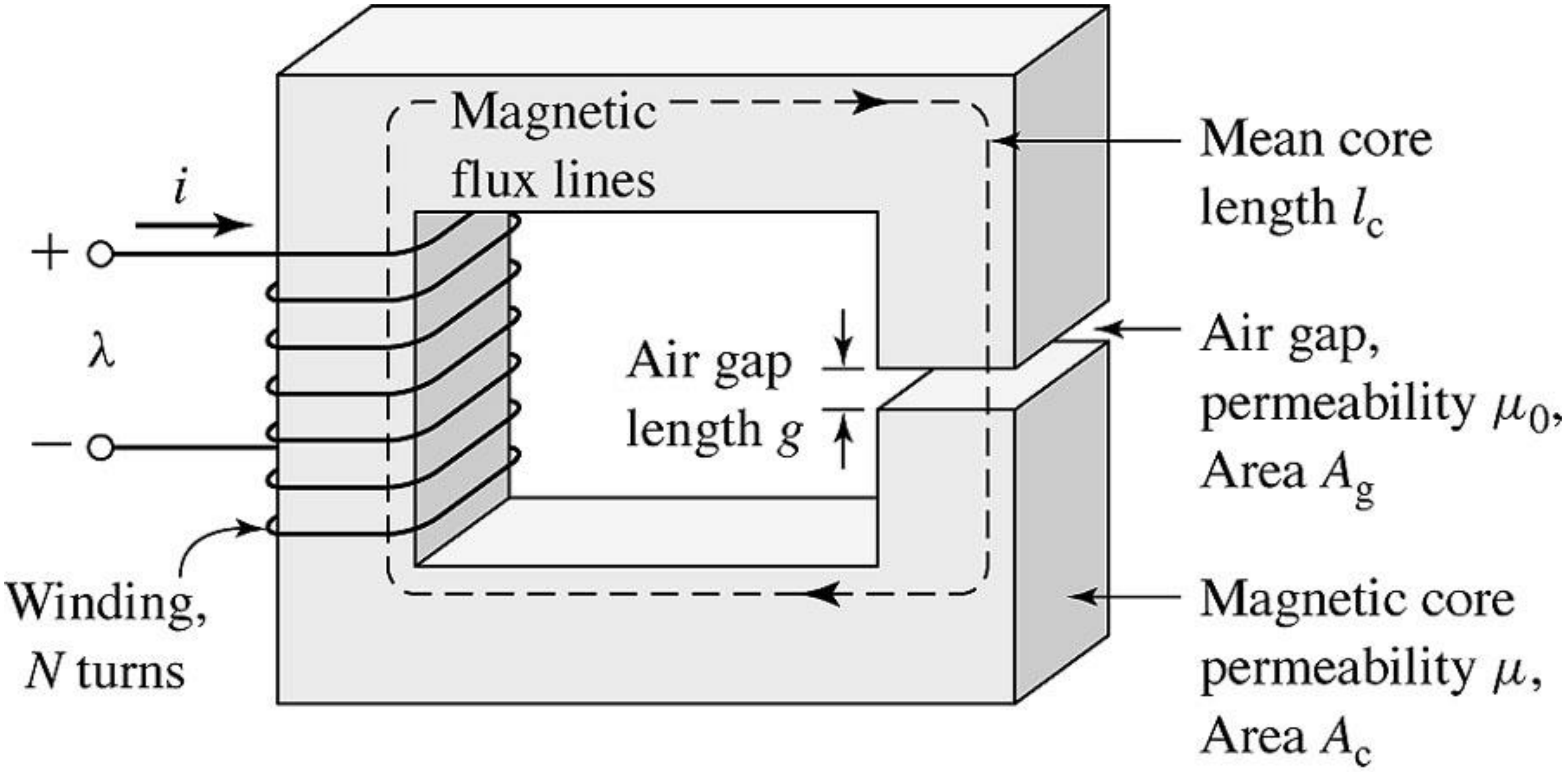
This can be re-written as

$R \phi = NI = F$, where R = reluctance of the magnetic circuit in (A-t)/Wb.

Electric – Magnetic Circuits

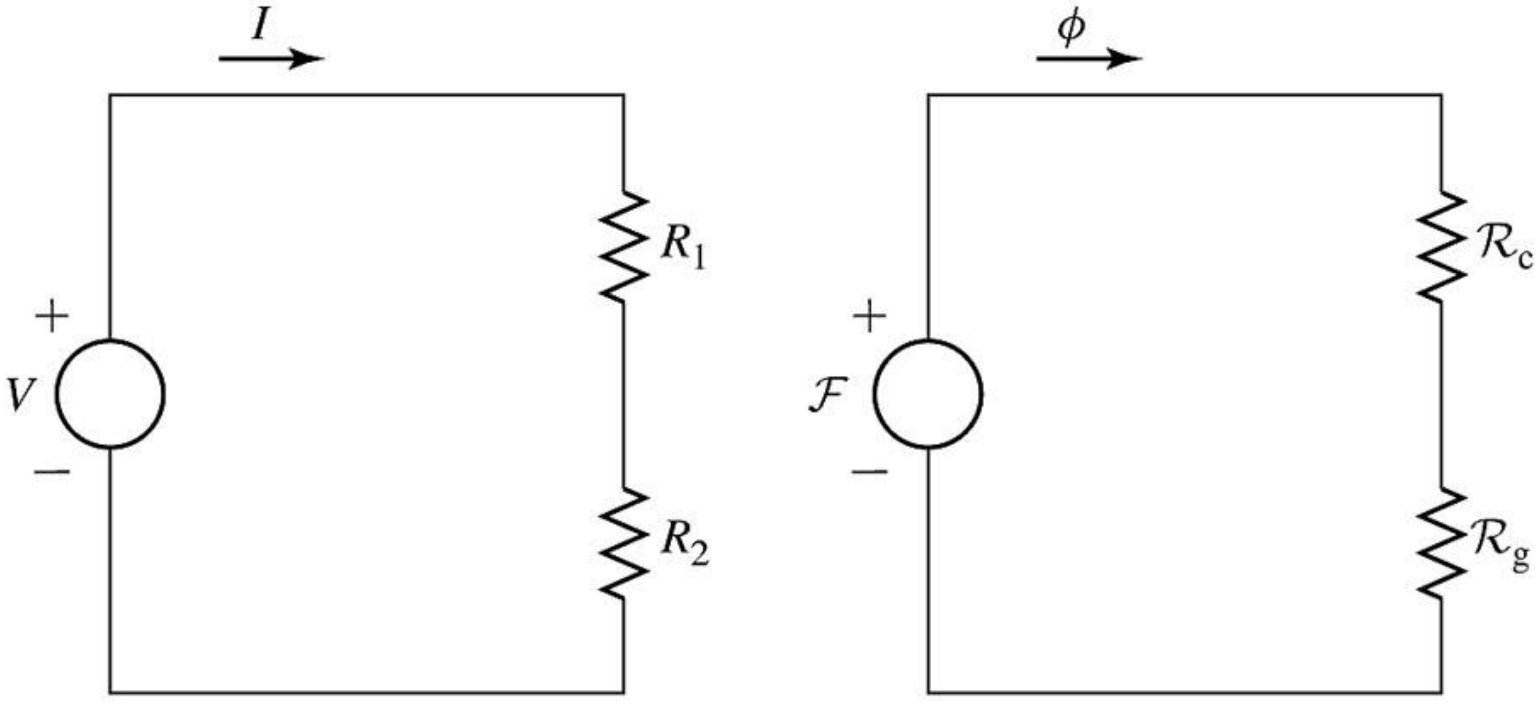
Electric Circuit	Magnetic Circuit
I – current (A)	Φ – flux (Wb)
V – emf (V)	*F* – MMF (A-t)
R – resistance (Ω)	*R* – reluctance (A-t/Wb)
Σ – conductivity (S/m)	μ - permeability (H/m)

Electric – Magnetic Circuits



A magnetic circuit with an air gap

Electric vs Magnetic Circuits Comparison



$$I = \frac{V}{(R_1 + R_2)}$$

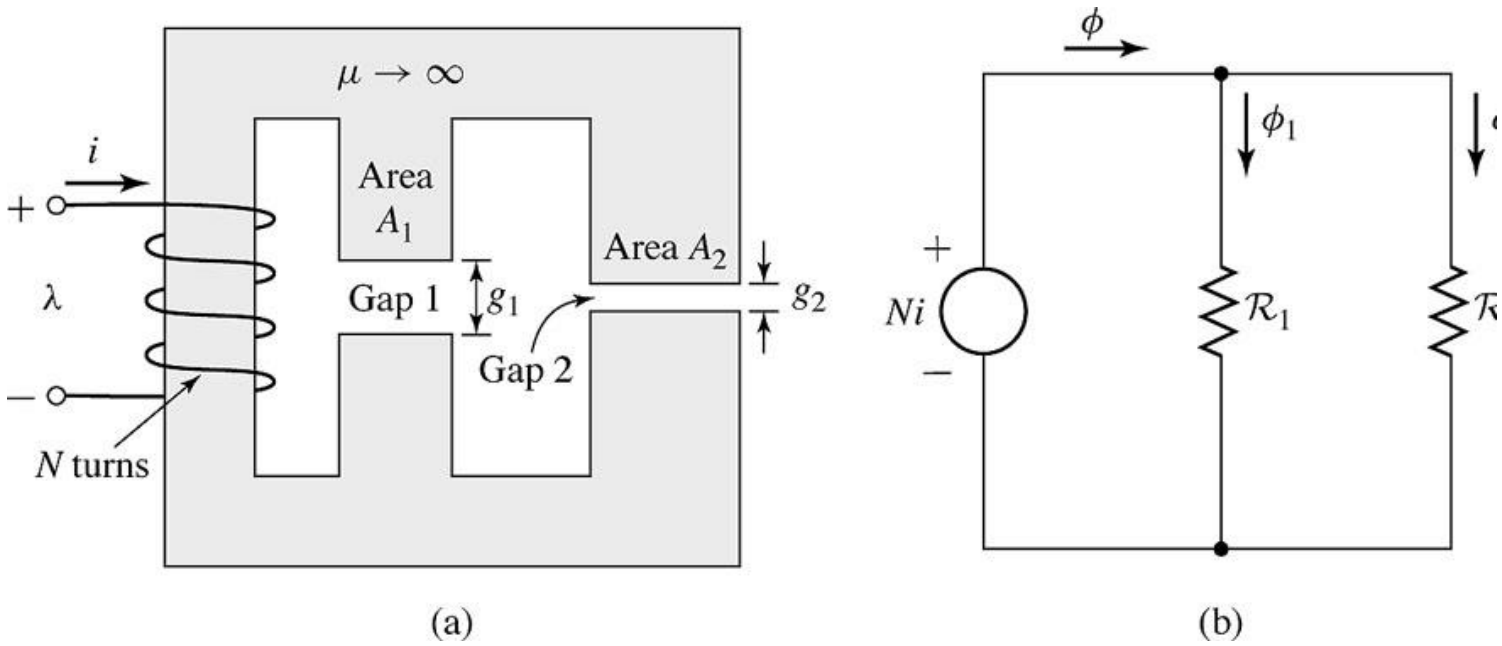
(a)

$$\phi = \frac{\mathcal{F}}{(\mathcal{R}_c + \mathcal{R}_g)}$$

(b)

Equivalent circuit (with air-gap)

Electric-Magnetic Circuits



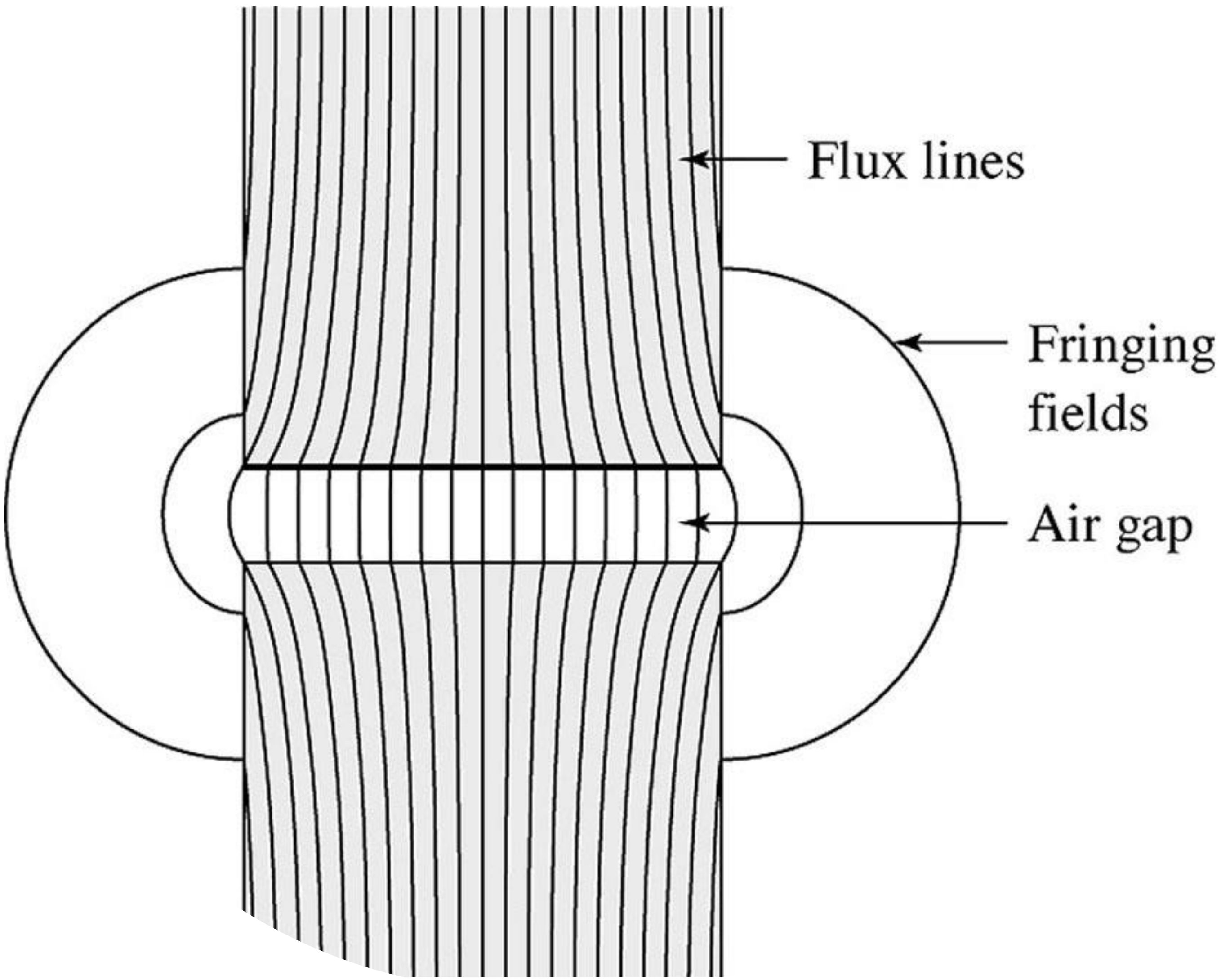
A magnetic circuit with two air gaps and equivalent circuit

(parallel reluctances)

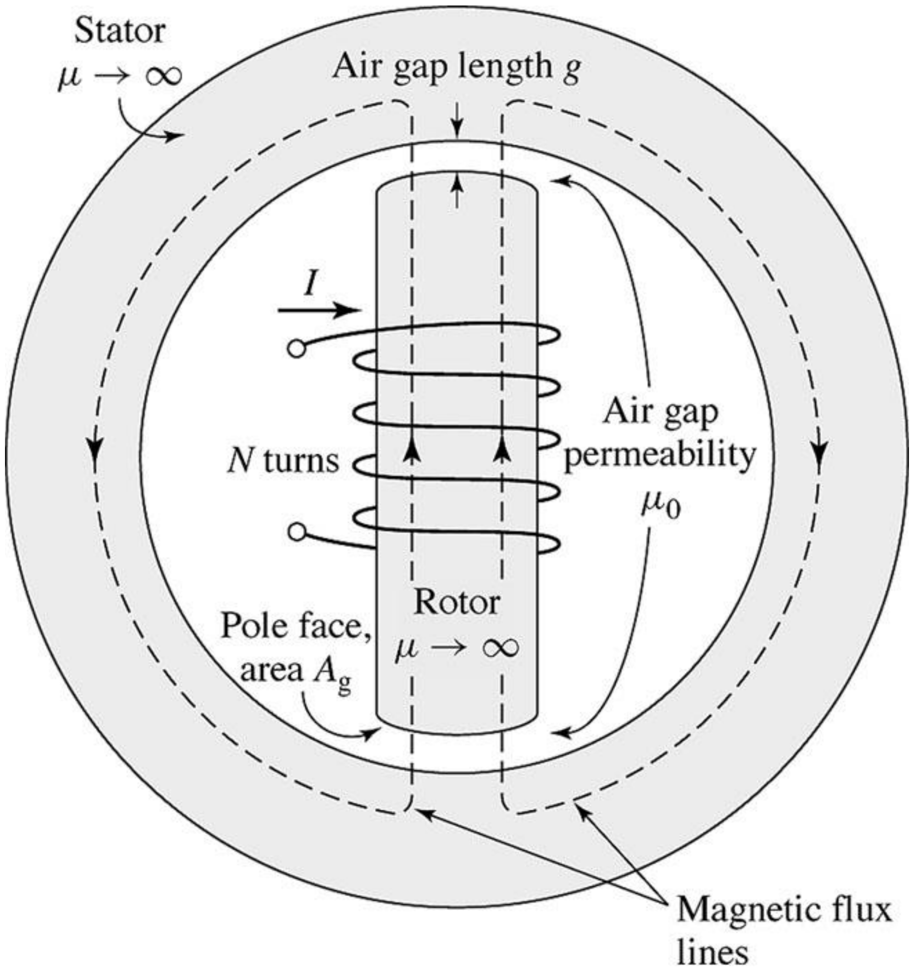
Air Gap Fringing

The flux across an air-gap will not be confined to the surface area of the iron core. This is called “fringing”

- Usually accounted for by ignoring effect or adding air gap length to both dimensions of cross section of air gap



Simple Synchronous Machine



Faraday's Law

"A time-varying magnetic field induces an electromotive force that produces a current in a closed circuit. This current flows in a direction such that it produces a magnetic field that tends to oppose the changing magnetic flux of the original time varying field."

Mathematically

$$emf = d\lambda/dt$$

- λ - total flux linkages of closed path = N ϕ

$$emf = dN\Phi/dt$$

Produced by:

- Time varying flux linking stationary path – transformer
- Relative motion between steady flux and stationary path – synchronous generator under no-load conditions
- A combination of previous two cases – induction

A Simple Inductor

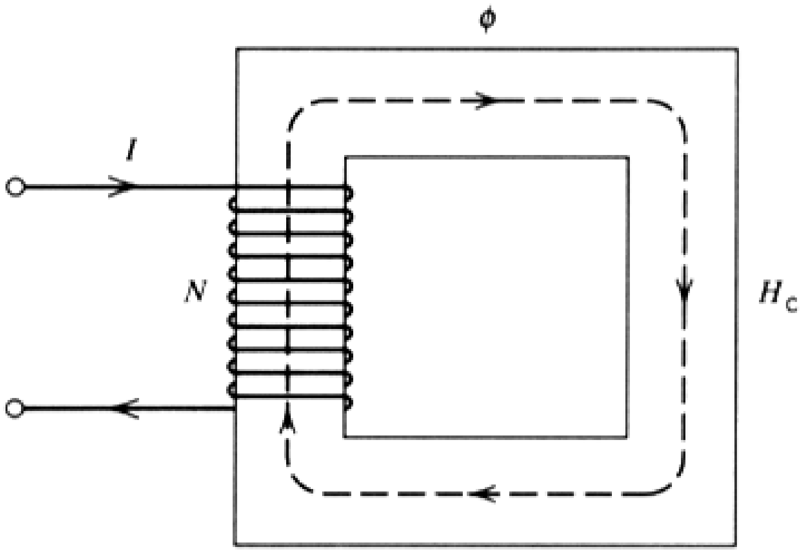


FIGURE 4.1 A magnetic circuit.

If coil connected to voltage source, *v*, current *i* will flow.

Current will produce magnetic flux, Φ

Total flux linkages of the coil containing N turns $\lambda = N\Phi$

Inductance

For simple coil

$$\Phi = Ni/\mathcal{R} = Ni \div (L_c/\mu A_c)$$

Self-Inductance of coil defined as

$$L = \frac{\text{total flux linkage of the coil}}{\text{current producing the flux}} = \frac{\lambda}{i}$$

This can be re-written as

$$L = \frac{\lambda}{i} = \frac{N^2 \mu A}{l}$$

Implies inductance is a property of the coil energized

Mutual Inductance

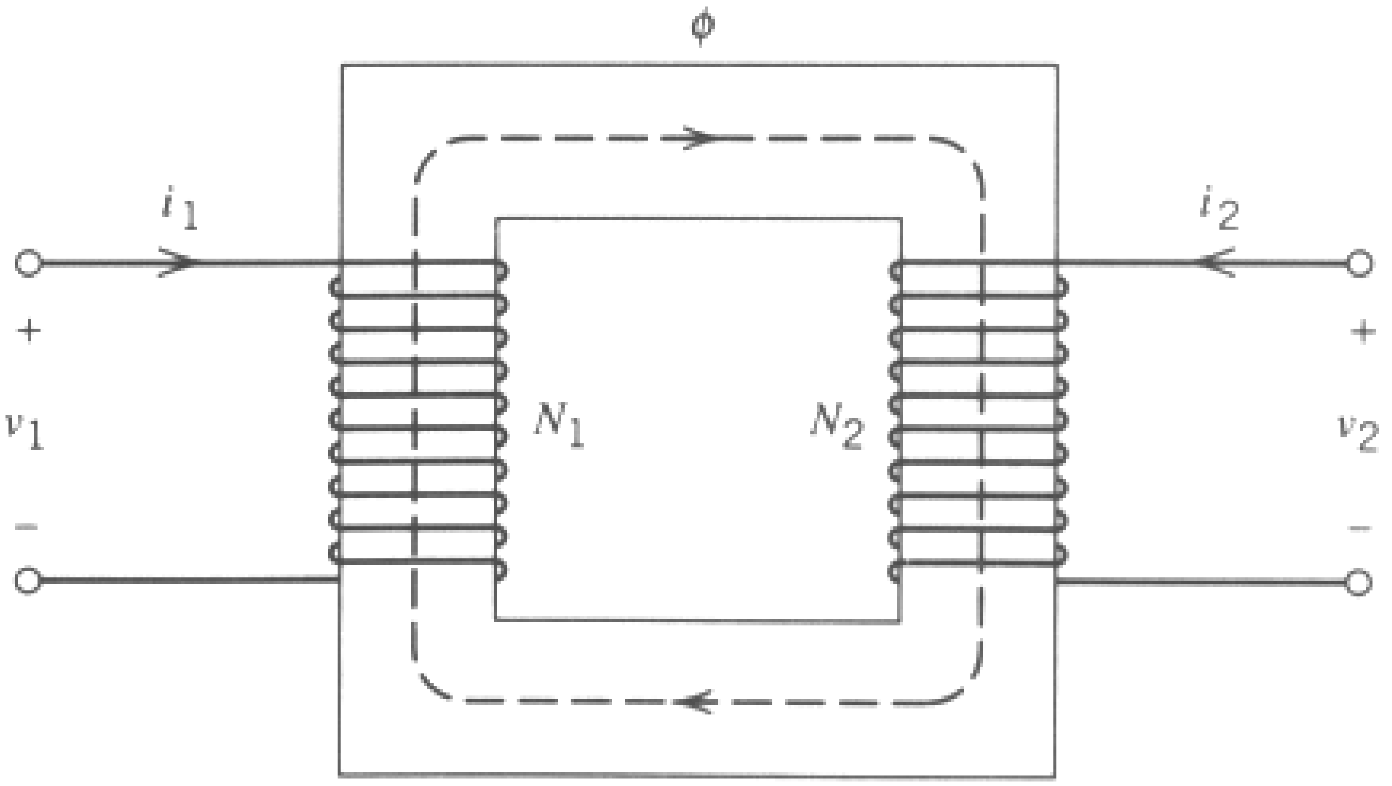


FIGURE 4.7 Mutual inductance.

Mutual Inductance

Flux produced by coil 1 will link coil 2.

Define mutual inductance as

$$L_{21} = \frac{\text{total flux linking coil 2}}{\text{current flowing through coil 1}} = \frac{\lambda_{21}}{i_1}$$

Assume no "leakage" of flux, so that all flux generated by coil 1 reaches coil 2

Mutual Inductance

$$\lambda_{21} = N_2\phi_{21} = N_2\phi, \quad \phi = \frac{N_1 i_1}{\Re} = \frac{N_1 i_1}{\left(l \big/ \mu A \right)}$$

$$\therefore \lambda_{21} = N_2\phi = \frac{N_1 N_2 i_1}{\Re}, \quad L_{21} = \frac{\lambda_{21}}{i_1} = \frac{N_1 N_2}{\Re}$$

Mutual Inductance

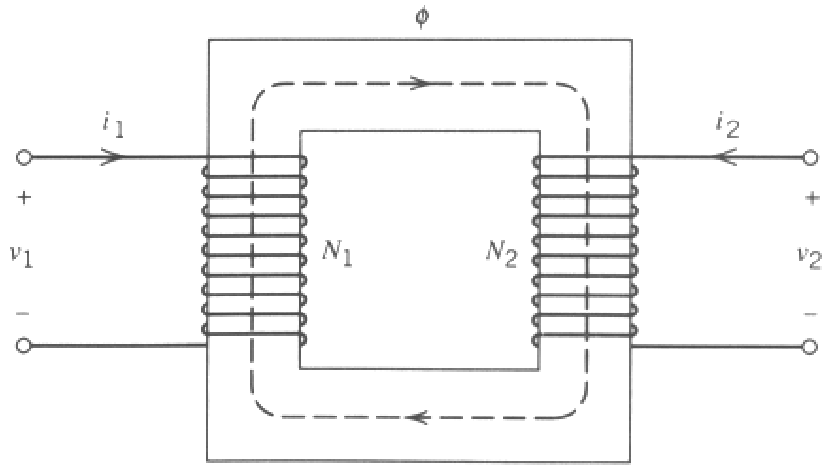


FIGURE 4.7 Mutual inductance.

If coil 2 excited first and coil 1 left un-energized flux produced by coil 2 will link coil 1

$$L_{12} = \frac{\text{total flux linking coil 1}}{\text{current flowing through coil 2}} = \frac{\lambda_{12}}{i_2}$$

Mutual Inductance

$$\lambda_{12} = N_1\phi_{12} = N_1\phi, \quad \phi = \frac{N_2 i_2}{\Re} = \frac{N_2 i_2}{\left(l \middle/ \mu A \right)}$$

$$\therefore \lambda_{12} = N_1\phi = \frac{N_1 N_2 i_2}{\Re}, \quad L_{12} = \frac{\lambda_{12}}{i_2} = \frac{N_1 N_2}{\Re}$$

Mutual Inductance

Flux (and flux leakages) are symmetrical implying that mutual inductance also symmetrical

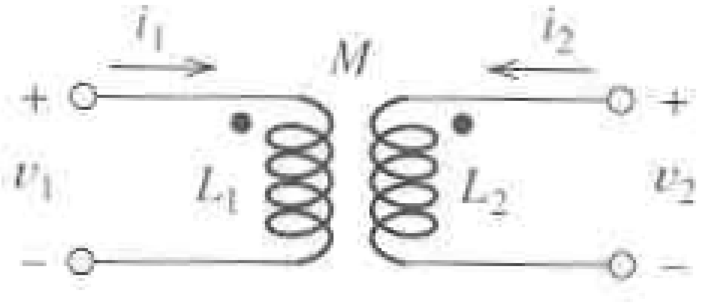
$$L_{pq} = \frac{N_p N_q}{\Re} k_{pq} \qquad k_{pq} = 0 \rightarrow 1$$

$$k_{pq} - leakage\ factor$$

Mutual inductance is a property of the magnetic circuit being considered and is controlled by

- Core shape and composition
- Configuration of energizing coils

Mutual Inductance - Dot convention

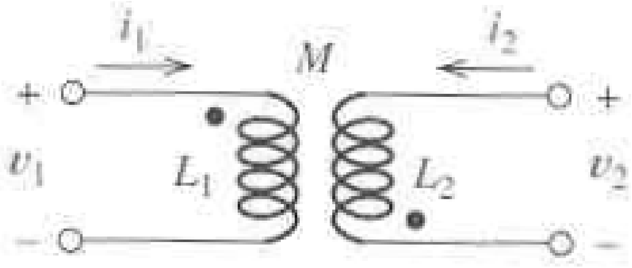


$$\lambda_1 = L_1 I_1 + M I_2, \quad \lambda_2 = M I_1 + L_2 I_2$$

or

$$\frac{d\lambda_1}{dt} = v_1 = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt}$$

$$\frac{d\lambda_2}{dt} = v_2 = M \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt}$$



$$\lambda_1 = L_1 I_1 - M I_2, \quad \lambda_2 = -M I_1 + L_2 I_2$$

or

$$\frac{d\lambda_1}{dt} = v_1 = L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt}$$

$$\frac{d\lambda_2}{dt} = v_2 = -M \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt}$$

Mutual inductance - Dot convention

Flux produced by one coil can either aid or oppose the flux produced by other coil

- Dots on the ends of the coils indicate whether field aiding or opposing

Sign of the mutual terms in equations for voltages depend on how the currents are referred with respect to dots

- If one current enters dotted terminal and other current leaves, fields oppose one another
- If both currents are enter (or both current leave) the dotted terminals, the mutual term is positive.

Energy delivered to or stored in Inductor

$$W = \int_{t_0}^{t_1} p\,dt = \int_{t_0}^{t_1} ie\,dt = \int_{t_0}^{t_1} i\left(\frac{d\lambda}{dt}\right)dt$$

$$W = \int_{\lambda_0}^{\lambda_1} i\,d\lambda = \frac{1}{L}\int_{\lambda_0}^{\lambda_1} \lambda\,d\lambda = \left(\frac{1}{2L}\right)\lambda_1^2$$

$$W = L\int_{i_0}^{i_1} i\,di = \frac{1}{2}Li_1^2$$

Calculated from integral of power over time period of operation

- Inductor considered to be energized at t = 0, consequently, $\lambda_0 = i_o = 0$, but this not always the case

(Ferro)Magnetic materials

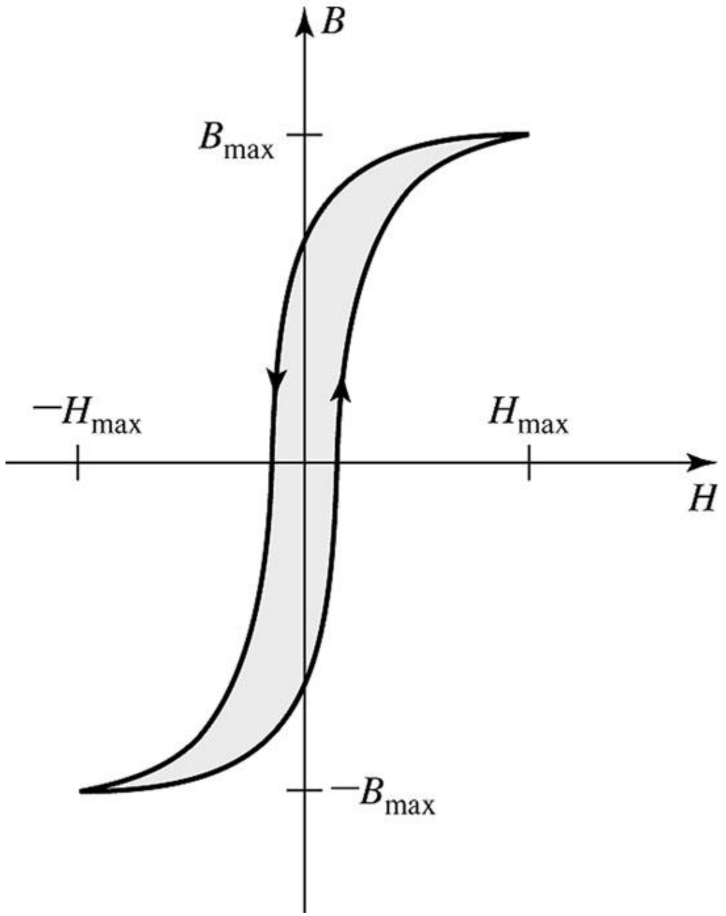
Made up of many domains

- All atoms are in parallel – give a net magnetic moment for the domain
- With no magnetisation – all domains randomly orientated – net magnetic flux is zero
- When magnetising force is applied, domains line up to the magnetic field – add to the applied field
  - Gives a much greater flux than from the magnetising force alone

Hysteresis effect

Only a finite number of magnetic domains, however

- Eventually, all domains will have lined up
- No further increase in flux (or flux density) with increasing magnetic field intensity



Hysteresis effect

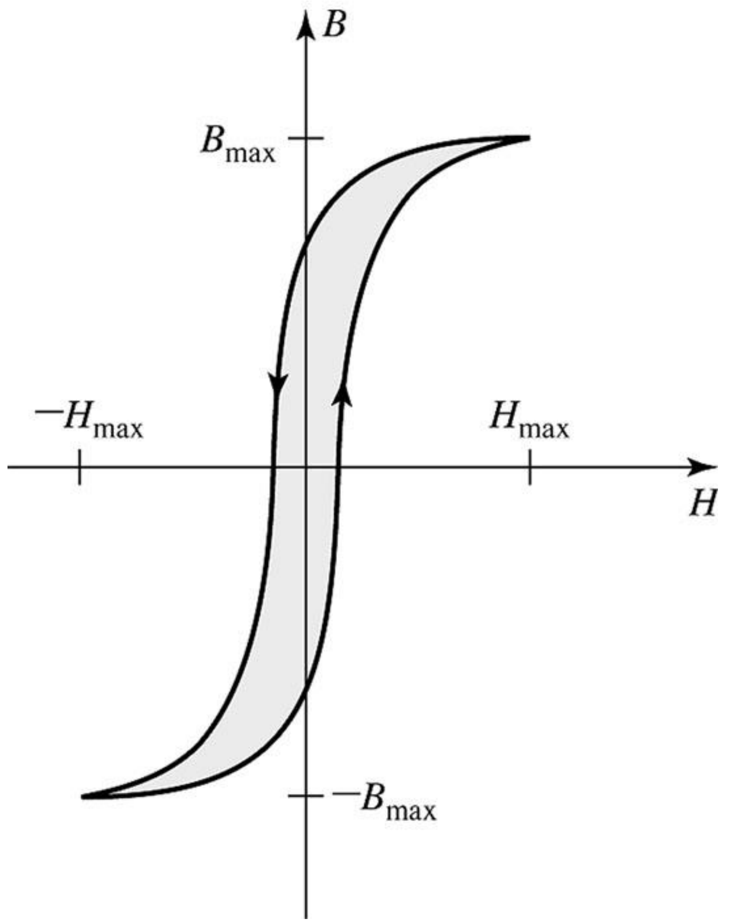
When magnetic field is removed or reduced:

- Domains will no longer align randomly – rather they will settle into *axes of easy magnetisation*
- Energetically favourable crystal lattice regions for domains to relax into
- Therefore, residual flux density will still reside in a material with no magnetic field applied

Hysteresis effect

Leads to hysteresis effect

- B-H curve not analytically describable (i.e. can't write a precise equation for it)
- Called the hysteresis effect/loop – leads to magnetising losses
- Stronger the "fatter" the loop



Eddy Current Losses

Consider Faraday's law again:

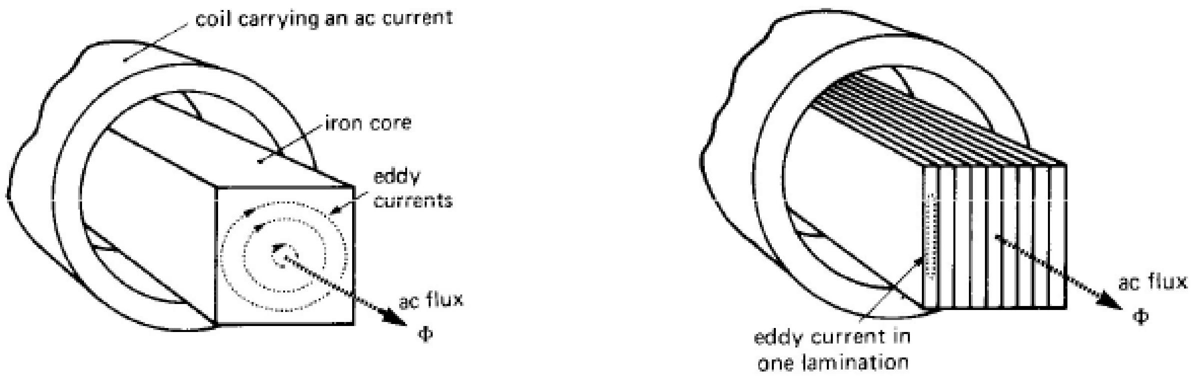
A time-varying magnetic field induces an electromotive force that produces a current in a closed circuit

In the core, we have a time varying flux density – therefore a time varying magnetic field

This induces an emf within the core

- Ideally core will have infinite resistance
- However, it has a finite resistance, therefore circulating currents flow within the core
- Eddy currents – leads to I^2R losses

Eddy Current Losses



These eddy current losses can be reduced (but not eliminated) by core laminations

Recap

Have examined:

- Magnetic fields – where they come from, how they work
- Flux
- Electric – Magnetic circuit equivalences
- Inductance – Mutual inductance
- Core losses
  - Hysteresis effect
  - Eddy currents

Questions?

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