

1 **Cross Validation with `curydice` Yields Reliable Planet Mass Constraints in the HD 63433 System**

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ABSTRACT

12 Understanding young exoplanets is critical for constraining models of planetary evolution, yet obtaining
13 precise masses from radial velocity (RV) measurements is challenging due to high stellar variability
14 in young stars. Recent progress has been made using Gaussian Process (GP) models to mitigate these
15 stellar signals, but assessing the predictive reliability of these models remains difficult, limiting their
16 effectiveness. We introduce `curydice`, an open-source software package designed to evaluate the pre-
17 dictive performance of GP models using cross-validation techniques. We apply `curydice` to new and
18 previously published RV datasets of the young planetary system HD 63433, demonstrating that our
19 GP models produce planet mass estimates consistent with previously published results. By testing
20 model reliability via cross-validation, `curydice` supports more reliable mass measurements for active
21 young stars. We release `curydice` publicly to encourage its use in the community as a tool for future
22 RV studies.

23 *Keywords:* exoplanets, radial velocity, stellar activity

1. INTRODUCTION

25 Planet properties are thought to be largely shaped
26 during the earliest stages of their evolution. Core ac-
27 cretion models predict that planets that form with ex-
28 tended volatile envelopes may be eroded by mass loss
29 processes over \sim 100 Myr, eventually contracting into
30 the older population of planets observed today (see e.g.,
31 J. L. Bean et al. 2021, H. E. Schlichting 2018, and ref-
32 erences within). However, many details of this evolu-
33 tionary process remain unclear, including the dominant
34 mechanisms driving envelope loss. Mechanisms like pho-
35 toevaporation (S. Ginzburg et al. 2018), core-powered
36 mass loss (J. E. Owen & Y. Wu 2013), and boil-off (J. E.
37 Owen & Y. Wu 2016) may all play a role in shaping the
38 mature exoplanet population, though they likely oper-
39 ate on different timescales. Moreover, planetary systems
40 experience significant dynamical evolution early in their

41 history, including orbital migration (e.g., T. J. David
42 et al. 2016; A. W. Mann et al. 2016; M. G. Barber et al.
43 2024; D. R. Alves et al. 2025) and planet–planet scatter-
44 ing or collision events (e.g., R. Frelih et al. 2019), yet
45 the relative importance of these processes in shaping the
46 final characteristics of mature exoplanet populations are
47 not well understood.

48 A promising way forward is to observe young plan-
49 etes that are still experiencing the processes that sculpt
50 mature exoplanet populations. Several surveys are pur-
51 suing this approach, including the TESS Hunt for Young
52 and Maturing Exoplanets Survey (THYME; E. R. New-
53 ton et al. 2019), Cluster Difference Imaging Photometric
54 Survey (CDIPS; L. G. Bouma et al. 2019), and PSF-
55 based Approach to TESS High Quality Data Of Stellar
56 Clusters (PATHOS; D. Nardiello et al. 2020). Searching
57 for planets around stars younger than 1 Gyr and study-
58 ing how their properties change as a function of stellar
59 age can reveal the key timescales on which the dynam-

60 ical and atmospheric properties of planets evolve (J. E.
 61 Owen 2020).

62 Nevertheless, detecting and characterizing planets
 63 around young stars remains observationally challenging,
 64 particularly with the radial velocity (RV) method. The
 65 RV signals of young stars are dominated by spot-induced
 66 variability, which can be up to hundreds of times larger
 67 than the variability of Gyr-old stars (e.g., B. L. Cale
 68 et al. 2021, A. Suárez Mascareño et al. 2021). Historically,
 69 these large activity signals have hindered RV
 70 searches for young planets, as even the RV signal of a
 71 giant, close-in planet can be masked by the stellar noise.
 72 As a result, young and/or active stars were often avoided
 73 in RV surveys (see C. K. Harada et al. 2024 for empirical
 74 evidence of this). Furthermore, the limited sample
 75 of young RV planets remains controversial, with some
 76 cases challenging the reported planetary signals (e.g.,
 77 V830 Tau b in M. Damasso et al. 2020; CI Tau b in
 78 J.-F. Donati et al. 2024).

79 Despite these observational challenges, the RV com-
 80 munity is beginning to explicitly target young stars
 81 in RV campaigns as statistical and observational tech-
 82 niques for managing stellar activity improve (e.g., Q. H.
 83 Tran et al. 2021). For example, advances in transit
 84 search algorithms in the presence of stellar noise have
 85 marked a major improvement (e.g., M. G. Barber et al.
 86 2024). Rather than blind searches, RV studies can fol-
 87 low up young planets that have been independently val-
 88 idated and inform orbit models based on planet proper-
 89 ties measured by transits.

90 Gaussian process (GP) regression is becoming a
 91 widespread technique for modeling Keplerian RV sig-
 92 nals in the presence of stellar activity signals (R. D.
 93 Haywood et al. 2014, S. Aigrain & D. Foreman-Mackey
 94 2023). By treating stellar activity as correlated noise
 95 with known or expected covariance properties, we can
 96 bypass the need for a functional model for stellar vari-
 97 ability. GPs have been used very successfully on well-
 98 sampled and relatively well-understood datasets, such
 99 as Sun-as-a-star observations (B. Klein et al. 2024).

100 Nevertheless, as with any modeling approach, extending
 101 existing GP methods to a new context (in this case,
 102 to young star RV time series) requires some caution. S.
 103 Blunt et al. (2023) found evidence for severe overfitting
 104 in a GP model of the V1298 Tau system, a \sim 20 Myr
 105 star with a \sim 500 m/s stellar activity signal and 4 tran-
 106 siting planets, raising concerns about the reliability of
 107 the model's inferred planet masses. Subsequent analyses
 108 of this system have validated these concerns by deriving
 109 significantly lower planet masses than initially reported
 110 (S. Barat et al. 2024, Livingston et al. *in press*). In
 111 light of these issues, S. Blunt et al. (2023) emphasized

112 the need for developing tools to assess the predictive-
 113 ness of all RV models, including but not limited to GP
 114 models, a gap we aim to address here.

115 HD 63433 is a young (414 ± 23 Myr; J. Jones et al.
 116 2015) nearby (\sim 22 pc) planetary system hosting three
 117 planets orbiting a Sun-like star (spectral type G5V)
 118 in the Ursa Major moving group (B. K. Capistrant
 119 et al. 2024). Planets HD 63433 b and c, first dis-
 120 covered by A. W. Mann et al. (2020), are estimated
 121 to be sub-Neptunes, both with radii between 2 and
 122 $3R_{\oplus}$. The innermost planet HD 63433 d was recently
 123 reported by B. K. Capistrant et al. (2024) with a size
 124 of $1.073^{+0.046}_{-0.044} R_{\oplus}$ on a short, 4.2-day orbit. As such,
 125 HD 63433 d is the closest Earth-size planet orbiting a
 126 young star, offering a unique opportunity to understand
 127 a young Earth analog.

128 Given its brightness ($V \approx 6.9$ mag), HD 63433 is an at-
 129 tractive target for ground-based RV campaigns aimed at
 130 measuring planetary masses. Independent RV analyses
 131 by M. Damasso et al. (2023) and M. Mallorquín et al.
 132 (2023) have provided mass estimates for the outer plan-
 133 etes HD 63433 c and d using GP frameworks to model
 134 stellar activity, but these estimates have not been as-
 135 sessed for reliability using held-out datasets. Given the
 136 high-amplitude activity signal, it is important to assess
 137 the predictive ability of the GP models, and therefore
 138 how reliable the inferred masses may be.

139 In this work, we revisit the analyses of M. Damasso
 140 et al. (2023) and M. Mallorquín et al. (2023) and add ad-
 141 ditional data to place further constraints on the masses
 142 of HD 63433 b and c, present the first mass estimates
 143 for planet d, and test the reliability of the results.

144 To support this analysis and larger efforts to eval-
 145 uate GP models in RV studies, we introduce **eurydice**, a
 146 Python package developed to evaluate GP models using
 147 cross-validation techniques. As the usage of GP models
 148 continues to grow in the exoplanet community, so too
 149 does the need for standardized tools to assess their pre-
 150 dictive performance across different kernel choices and
 151 modeling assumptions. **eurydice** provides a consistent
 152 framework for evaluating fitted models on held-out data,
 153 helping to improve the reliability of planetary signal de-
 154 tections. We use HD 63433 as the first test case for
 155 **eurydice**, demonstrating its utility in validating GP
 156 models for RV-based mass estimation.

157 In Section 2, we present the radial velocity data used
 158 in our analysis of HD 63433, using both previously re-
 159 ported data from M. Damasso et al. (2023) and M. Mal-
 160 lorquín et al. (2023) along with new EXPRES observa-
 161 tions. In Section 3, we detail our GP and model-fitting
 162 framework. We introduce the **eurydice** package and
 163 suite of cross-validation tests in Section 4 and discuss

164 modeling choices, the impacts on inferred parameters,
 165 and future work with `eurydice` in Section 5. We end
 166 with our conclusions in Section 6.

167 2. DATA

168 In this work, we use three RV time series derived from
 169 spectra taken by three different spectrographs. All RV
 170 data are shown in Figure 1 and each dataset is detailed
 171 below.

172 2.1. HARPS-N

173 We used the published dataset from M. Damasso et al.
 174 (2023) of HD 63433 observations from the HARPS-N
 175 spectrograph (R. Cosentino et al. 2012, 2014) installed
 176 at the Telescopio Nazionale Galileo (TNG) at the Ob-
 177 servatory of the Roque de los Muchachos in La Palma,
 178 Spain. HARPS-N is a fiber-fed, cross-dispersed echelle
 179 spectrograph with an average spectral resolution of $\mathcal{R} =$
 180 115,000 and covers a wavelength range of 383-690 nm. A
 181 total of 103 spectra were collected between 26 February
 182 2020 and 17 April 2022, and the derived RV time series
 183 was calculated from the DRS pipeline (version 3.7.1).

184 2.2. CARMENES

185 We used the published dataset from M. Mallorquíñ
 186 et al. (2023) of HD 63433 observations from the high-
 187 resolution CARMENES spectrograph (A. Quirrenbach
 188 et al. 2014, 2016) located on the 3.5 m telescope at
 189 the Calar Alto Observatory in Almería, Spain. A total
 190 of 157 spectra were collected between 19 September
 191 2020 and 23 February 2022 using both the visual
 192 and near-infrared channels of CARMENES. The spec-
 193 tra were reduced using CARACAL (J. A. Caballero et al.
 194 2016) and the RVs were derived using SERVAL (M. Zech-
 195 meister et al. 2018). The near-infrared RVs exhibited a
 196 mean uncertainty of 14.3 m/s, significantly higher than
 197 the 3.9 m/s mean uncertainty measured in the visual
 198 channel. Consequently, we excluded the near-infrared
 199 arm RVs from this analysis and only utilized the 150
 200 RV points from the visual arm. The visual light arm
 201 has a measured spectral resolution of $\mathcal{R} = 93,400$ and
 202 spans the wavelength range of 520-960 nm.

203 2.3. EXPRES

204 Between 16 November 2021 and 30 October 2022, we
 205 obtained 52 spectra using the EXtreme PREcision Spec-
 206 trograph (EXPRES; C. Jurgenson et al. 2016) on the
 207 4.3-m Lowell Discovery Telescope (LDT; S. E. Levine
 208 et al. 2012) located in Flagstaff, Arizona. EXPRES
 209 is a high-resolution ($R \sim 137,500$) optical (380 - 822
 210 nm) stabilized spectrograph designed for extreme pre-
 211 cision RV. The spectrograph is equipped with both a

212 ThAr lamp and Laser Frequency Comb (LFC) for pre-
 213 cise wavelength calibration. Exposures of both calibra-
 214 tor sources were obtained every 30 minutes during our
 215 observations. The data were reduced using the standard
 216 EXPRES pipeline as described in R. T. Blackman et al.
 217 (2020) and telluric removal was applied using SELEN-
 218 ITE (C. Leet et al. 2019). Wavelength calibration was
 219 performed using the *excalibur* method (L. L. Zhao et al.
 220 2021) and the RVs were computed using the pipeline de-
 221 scribed in R. R. Petersburg et al. (2020). All EXPRES
 222 RVs used in this work are provided in Table A1.

223 3. RADIAL VELOCITY ANALYSIS

224 In this analysis, we have opted to exclude the EX-
 225 PRES data from our modeling due to its smaller sam-
 226 ple size (52 observations) compared to the HARPS-N
 227 and CARMENES datasets (103 and 150 observations,
 228 respectively). The EXPRES data was instead held as a
 229 test set for later cross-validation tests.

230 3.1. Rotation Period Analysis

231 Although the stellar rotation period of host star HD
 232 63433 has been well characterized in previous studies
 233 (see A. W. Mann et al. 2020 and B. K. Capistrant et al.
 234 2024), we performed an independent check to assess
 235 the power of the rotation signal in our combined RV
 236 dataset. We computed a Lomb-Scargle periodogram us-
 237 ing the `astropy` LombScargle class (J. T. VanderPlas &
 238 Ž. Ivezić 2015; J. VanderPlas et al. 2012), shown in Fig-
 239 ure 2. While the fundamental rotation period at 6.4
 240 days was only weakly recovered, its first and second
 241 harmonics at 3.2 and 2.1 days appear more prevalent.
 242 This behavior is consistent with results from A. Van-
 243 derburg et al. (2016), who found that RV signals from
 244 stellar activity often manifest most strongly at the ro-
 245 tation period and its first two harmonics. Interestingly,
 246 CARMENES and HARPS-N detect different har-
 247 monics, with the former recovering the 2.1 day second har-
 248 monic and the latter recovering the 3.2 day first har-
 249 monic. This discrepancy may be due to different obser-
 250 vation cadences that are sensitive to different parts of
 251 the stellar rotation signal. Based on this analysis, we
 252 conclude that the true rotation period is 6.4 days, con-
 253 sistent with previous studies (A. W. Mann et al. 2020;
 254 B. K. Capistrant et al. 2024) and use this as a prior in
 255 the GP models described below.

256 3.2. Gaussian Process Modeling Choices

257 GPs are flexible, non-parametric models that use a co-
 258 variance function, or kernel, to describe the correlation
 259 between pairs of observations. For further discussion on
 260 GPs, a broad review is provided in C. E. Rasmussen

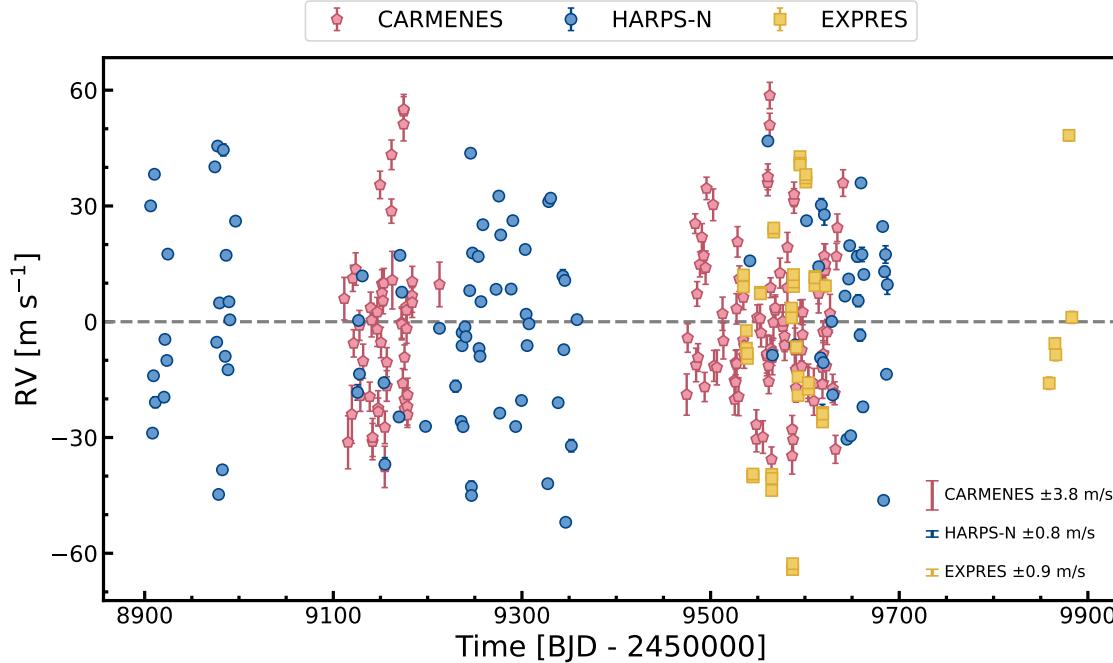


Figure 1. A survey of the RVs used in this analysis. The CARMENES and HARPS-N RVs are published in M. Mallorquí *et al.* (2023) and M. Damasso *et al.* (2023), respectively, while the EXPRES RVs are newly introduced in this work. Across all datasets, the RVs show a large spread despite low observational uncertainties (median error bars for each dataset are shown in the bottom right), highlighting the strong stellar variability of HD 63433 present in this data.

& C. K. I. Williams (2006) and applications of GPs to astronomical timeseries are detailed in S. Aigrain & D. Foreman-Mackey (2023).

As HD 63433 is a Sun-like star (A. W. Mann *et al.* 2020), we expect its p-mode oscillation, granulation, and supergranulation activity amplitudes to be similar to the Sun's, i.e., below ~ 1 m/s (see N. K. O'Sullivan *et al.* 2025 and references therein). Therefore, the stellar activity-induced RVs are expected to be dominated by rotation or the evolution of active regions. In this regime, a quasi-periodic kernel is commonly used in GP models of RVs (S. Aigrain *et al.* 2012; R. D. Haywood *et al.* 2014):

$$k(t, t') = \eta_1^2 \exp\left(-\frac{(t - t')^2}{2\eta_2^2} - \frac{2 \sin^2(\frac{\pi(t-t')}{\eta_3})}{\eta_4^2}\right) \quad (1)$$

where t and t' are the times of the two observations, η_1 is the GP amplitude, η_2 is the evolution timescale of active regions, η_3 is the stellar rotation period, and η_4 is the harmonic complexity, which dictates how the signal varies within a rotation period (B. A. Nicholson & S. Aigrain 2022).

When combining RV measurements from different spectrographs, an unresolved question is whether or not to treat datasets from different instruments as correlated measurements (see S. Blunt *et al.* 2023 section

4.1, as well as Figures 11 and 12). Since stellar activity arises from physical processes, we can predict that multiple instruments taking data at the same time will observe the same stellar signal, with potentially distinct amplitudes due to the chromaticity of stellar activity (S. Blunt *et al.* 2023; B. L. Cale *et al.* 2021). This assumption motivated our two approaches in GP modeling: a joint covariance matrix framework with a shared kernel functional form and hyperparameters that assume a correlated signal across instruments, and an added log-likelihood approach, commonly used in literature, that models the datasets with separate independent GPs but shares the same hyperparameters.

The joint covariance matrix framework uses a multi-instrument quasi-periodic Gaussian Process kernel in which the GP hyperparameters are shared across the model, while allowing for separate amplitude hyperparameters (η_1) for each instrument i :

$$k(t, t') = \eta_{1,i} \eta_{1,j} \exp\left(-\frac{(t - t')^2}{2\eta_2^2} - \frac{2 \sin^2(\frac{\pi(t-t')}{\eta_3})}{\eta_4^2}\right). \quad (2)$$

Here, $\eta_{1,i}$ is the GP amplitude associated with the instrument that took the observation at time t , and $\eta_{1,j}$ is the amplitude associated with the instrument that took the observation at time t' . All other hyperparameters retain the same definitions as in Equation 1.

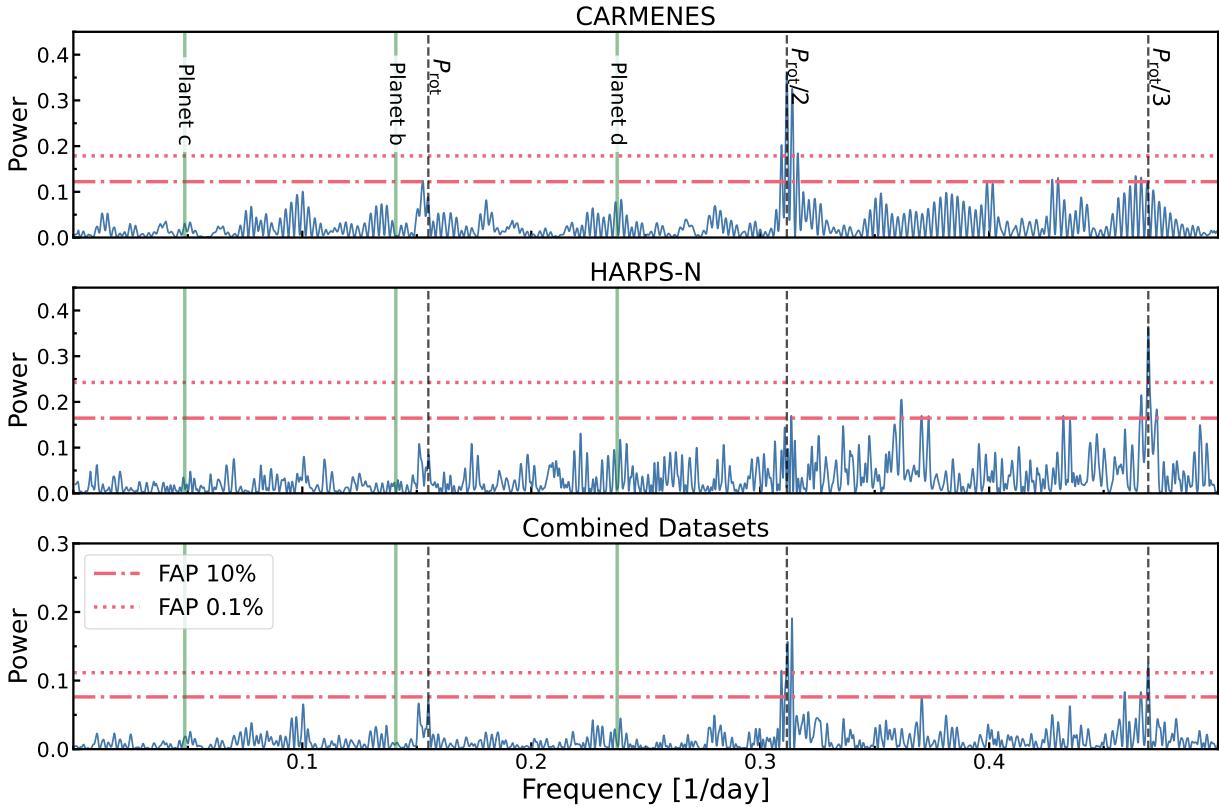


Figure 2. Lomb-Scargle periodograms of the CARMENES and HARPS-N RVs. **Top:** CARMENES RVs; **middle:** HARPS-N RVs; **bottom:** combined RVs. Black dashed lines indicate HD 63433’s rotation period (6.4 days) and its harmonics, while green lines mark the orbital periods of the three planets. Orange dotted and dash-dotted lines correspond to false-alarm probabilities (FAPs) of 0.1% and 10%, respectively. None of the periodograms show a strong signal at the 6.4 day rotation period, but both the first and second harmonics (3.2 and 2.1 days) are detected.

309 The added log-likelihood approach applies a separate
 310 GP to each dataset, and the model is optimized using
 311 the sum of the individual log-likelihoods. While this
 312 framework still assumes a stellar noise model with the
 313 same GP hyperparameters across the instruments, it
 314 assumes measurements taken by different instruments
 315 are independent. Following the discussion in S. Blunt
 316 et al. (2023), modeling the different datasets as inde-
 317 pendent effectively introduces more free parameters to
 318 this model, which may increase the risk of overfitting.
 319 As this aspect is not fully understood in the literature,
 320 we sought to explore it in our analysis.

321 3.3. Model Fitting

322 To model both stellar activity and planetary signals,
 323 we defined the GP mean function as a sum of Keplerian
 324 orbits using `jaxplanet` (S. Hattori et al. 2024) and
 325 constructed the GP models with `tinygp` (D. Foreman-
 326 Mackey et al. 2024). For transparency, our GP fitting
 327 pipeline code is publicly available on GitHub.⁸

328 We explored several model configurations, including
 329 two-planet and three-planet fits. The two-planet model
 330 was first implemented to reproduce previous analyses
 331 (M. Damasso et al. 2023; M. Mallorquín et al. 2023),
 332 which were carried out before the discovery of HD
 333 63433 d, and to test whether our joint modeling of
 334 CARMENES and HARPS-N data would result in con-
 335 sistent planetary parameters. Once this baseline was
 336 established, we expanded to a three-planet model. The
 337 expected RV semi-amplitude of HD 63433 d is small (<
 338 1 m/s for an Earth-sized planet of a comparable rocky
 339 composition at a 4.2-day orbit). Thus, its signal is be-
 340 low the median RV uncertainties of our data, but we in-
 341 cluded it in the three-planet model to evaluate its effect
 342 on the derived parameters of the two previously known
 343 planets.

344 In addition to testing two- and three-planet fits and
 345 different joint modeling strategies for the combined
 346 datasets, we tested both circular and eccentric orbital
 347 models, resulting in a total of eight GP model config-
 348 urations. For comparison, we also constructed a three-
 349 planet circular model assuming only white noise without

⁸ <https://github.com/kayleebarrera/HD63433>

³⁵⁰ a GP component. A summary of all models is presented
³⁵¹ in Table 1.

³⁵² As this work did not utilize photometric data, we
³⁵³ adopted narrow Gaussian priors on the orbital periods
³⁵⁴ and transit times of the three planets, based on pub-
³⁵⁵lished values from B. K. Capistrant et al. (2024). As M.
³⁵⁶ Mallorquín et al. (2023) found no evidence of transi-
³⁵⁷ting timing variations, we consider these priors reliable for
³⁵⁸ our analysis. Uniform priors were used for all remaining
³⁵⁹parameters and summarized in Table A2. For the eccen-
³⁶⁰tric orbital models, we used the $\sqrt{e} \cos(\omega)$ and $\sqrt{e} \sin(\omega)$
³⁶¹parameterization of eccentricity and argument of peri-
³⁶²astron.

³⁶³ Parameter distributions were sampled using the affine
³⁶⁴invariant Markov chain Monte Carlo (MCMC) sampler
³⁶⁵implemented in emcee (D. Foreman-Mackey et al. 2019).
³⁶⁶ For each model, the number of walkers was set to twice
³⁶⁷the number of free parameters. Circular models were
³⁶⁸run for 10^6 steps, while eccentric models were run for
³⁶⁹ 2×10^6 to ensure adequate sampling of the larger para-
³⁷⁰meter space. The walkers were initialized at para-
³⁷¹meter values near those reported by B. K. Capistrant et al.
³⁷², M. Damasso et al. (2023), and M. Mallorquín et al.
³⁷³ (2023). Burn-in was discarded based on the maxi-
³⁷⁴mum autocorrelation time (τ) across all parameters, and
³⁷⁵chains were thinned according to the minimum τ . Con-
³⁷⁶vergence was assessed by visually inspecting the walker
³⁷⁷trace plots and checking that the chain lengths exceeded
³⁷⁸50 τ for all parameters, as recommended by emcee.

379 4. EURYDICE: A SOFTWARE PACKAGE FOR 380 CROSS-VALIDATION

³⁸¹ As investigated in S. Blunt et al. (2023) and M.
³⁸² MacLeod et al. (2025), GP models applied to RV
³⁸³datasets, like all models, are susceptible to overfitting.
³⁸⁴ Overfitting occurs when a model reproduces noise or lo-
³⁸⁵calized trends present in its training data rather than
³⁸⁶the true underlying signal. Cross-validation is a method
³⁸⁷for evaluating model generalization by testing how well
³⁸⁸a model can predict unseen data.

³⁸⁹ To evaluate our suite of GP models, we developed
³⁹⁰eurydice,⁹ a new open-source Python package designed
³⁹¹to perform cross-validation on GP models applied to ra-
³⁹²dial velocity (RV) data. This package provides a stream-
³⁹³lined and standardized framework for comparing differ-
³⁹⁴ent GP configurations and assessing their predictive per-
³⁹⁵formance to improve the reliability of modeling plan-
³⁹⁶etary signals.

⁹ <https://eurydice.readthedocs.io/en/latest/>

³⁹⁷ Currently, eurydice supports a simple hold-out cross-
³⁹⁸validation method,¹⁰ implemented as follows:

- ³⁹⁹ 1. Divide the dataset into a training set and a test set
⁴⁰⁰ (e.g., by instrument or using a predefined split).
- ⁴⁰¹ 2. Condition the GP on the training set.
- ⁴⁰² 3. Evaluate the GP predictions on the held-out test
⁴⁰³ set.
- ⁴⁰⁴ 4. Compare the scatter of the residuals in the test set
⁴⁰⁵ to the predicted uncertainty distribution.

⁴⁰⁶ Predictive performance is quantified using normalized
⁴⁰⁷residuals, which measure the deviation between model
⁴⁰⁸predictions and the actual RV measurements, scaled by
⁴⁰⁹observational errors, GP uncertainties, and instrumental
⁴¹⁰jitters:

$$\text{norm. residuals} = \frac{\text{RV}_{\text{obs}} - \text{RV}_{\text{model}}}{\sqrt{\sigma_{\text{obs}}^2 + \sigma_{\text{GP}}^2 + \sigma_{\text{jit}}^2}} \quad (3)$$

⁴¹² We report the standard deviation of these normalized
⁴¹³residuals as metrics for our cross-validation tests in Ta-
⁴¹⁴ble 1. We distinguish between the standard deviation of
⁴¹⁵observations used to condition the GP ($\sigma_{\text{condition}}$) and
⁴¹⁶the observations used to test the model (σ_{test}). Assum-
⁴¹⁷ing an ideal model that perfectly captures all correlated
⁴¹⁸noise, the normalized residuals should follow a standard
⁴¹⁹normal distribution with $\sigma_{\text{test}} \approx 1$. If the residuals from
⁴²⁰the test set are much broader than the expected stan-
⁴²¹dard normal distribution with $\sigma_{\text{test}} \gg 1$, this may indi-
⁴²²cate that the model is overfitting to the training data.

⁴²³ Importantly, eurydice itself does not perform any pa-
⁴²⁴rameter training or optimization. Instead, it operates
⁴²⁵using the results of an external GP model parameter es-
⁴²⁶timation procedure (such as posterior inference from an
⁴²⁷MCMC sampler). Users first optimize the GP model
⁴²⁸parameters on the training set using their preferred
⁴²⁹parameter-fitting tools, and eurydice takes these fitted
⁴³⁰parameters to generate predictions on the test set. Con-
⁴³¹sequently, eurydice offers flexibility in working with a
⁴³²wide range of GP models and fitting approaches. Be-
⁴³³cause it does not perform parameter optimization itself,
⁴³⁴eurydice is not only compatible with different inference
⁴³⁵methods, but can evaluate any GP model with arbitrary
⁴³⁶kernel choices. There are a handful of excellent tools for

¹⁰ Although termed “cross-validation,” we recognize that this is a single hold-out validation rather than full k-fold cross-validation. It assesses generalization on one held-out subset but does not provide the repeated-resampling statistics of formal cross-validation.

437 building and fitting GP models that can be used along-
 438 side `eurydice`, including `tinygp` (D. Foreman-Mackey
 439 et al. 2024, used in this work), `george` (S. Ambikasaran
 440 et al. 2015), `celerite` (D. Foreman-Mackey et al. 2017),
 441 and `pyaneti` (O. Barragán et al. 2022), which is specif-
 442 ically designed for jointly modeling transits and radial
 443 velocity data.

444 Often, GP models in the literature are selected us-
 445 ing Bayesian model comparison techniques, such as the
 446 Bayesian log-evidence ($\ln \mathcal{Z}$) or the Bayesian Infor-
 447 mation Criterion (BIC). While these criteria provide a re-
 448 lative measure of model quality based on model fit and
 449 complexity, they do not offer an absolute assessment
 450 of how well a model predicts unseen data. In con-
 451 trast, cross-validation provides a robust complementary
 452 method for model selection by directly evaluating gen-
 453 eralization on held-out data. Although cross-validation
 454 is not yet widely adopted in the radial velocity commu-
 455 nity, we aim to promote its broader use by providing
 456 tools like `eurydice` for accessible model evaluation of
 457 GPs for RV datasets. We hope that `eurydice` can be
 458 seamlessly integrated into existing RV analysis pipelines,
 459 enabling cross-validation tests without requiring heavy
 460 modification.

461 4.1. Cross-Validation Tests on HD 63433

462 Our cross-validation tests focused on three key model-
 463 ing choices: (1) circular versus eccentric orbital assump-
 464 tions, (2) two-planet versus three-planet system config-
 465 urations, and (3) the treatment of instrument systematics
 466 using either the joint GP covariance matrix model or an
 467 added likelihood approach across instruments.

468 During training, all models were fit using the com-
 469 bined HARPS-N and CARMENES datasets to infer
 470 global model parameters. In our cross-validation tests,
 471 however, we conditioned the model on data from only
 472 one instrument and validated predictions against the
 473 other. For example, in test CV1 (see Table 1) for model
 474 M1, the GP would be conditioned solely on HARPS-N
 475 data using the median of the globally trained parameter
 476 posteriors to predict the CARMENES observations, and
 477 vice versa for CV2. In this context, training refers to de-
 478 termining the overall GP model structure using MCMC,
 479 while conditioning uses the trained model to generate
 480 predictions for a particular dataset. A model can gener-
 481 ate predictions for both the data it has been trained on
 482 and the data it has not. Although both the HARPS-N
 483 and CARMENES datasets were used in training, condi-
 484 tioning on only one dataset isolates whether the trained
 485 GP can reproduce the signal observed by the other
 486 instrument, allowing for instrument-specific differences

487 such as unique jitter values. This tests the model’s abil-
 488 ity to generalize across instruments without retraining.

489 Additionally, the EXPRES dataset, which was not in-
 490 cluded in the training set, was used for further cross-
 491 validation tests of the joint covariance matrix approach.
 492 To account for instrumental differences between the EX-
 493 PRES data and the other two datasets, we fit for an RV
 494 zero-point offset and amplitude that minimized the neg-
 495 ative log-likelihood with respect to the GP model pre-
 496 dictions to minimize the sum of normalized residuals.
 497 We did not perform cross-validation with the EXPRES
 498 data in the added likelihood case, since there is currently
 499 no straightforward way to combine their predictive dis-
 500 tributions to generate predictions for a third unseen in-
 501 strument. To evaluate how well the GP mitigated cor-
 502 related noise from stellar activity, we also performed a
 503 cross-validation test of a white-noise-only model.

504 In addition to these cross-validation tests, we also
 505 derived posterior distributions of planetary masses for
 506 each model configuration to illustrate how these differ-
 507 ent models impact the inferred planet properties. These
 508 posteriors are presented in Figures 3-4 and discussed
 509 in detail in the following section. All model-predicted
 510 planetary masses, including the white noise model, were
 511 consistent with each other within approximately 1σ .

512 5. RESULTS AND DISCUSSION

513 5.1. Reliability of GP Models

514 Across all GP configurations, the models reproduce
 515 the radial velocity signals similarly, with no single ap-
 516 proach strongly favored by cross-validation metrics. All
 517 inferred planetary properties remain largely unchanged,
 518 with only a minor sensitivity to model assumptions.

519 Adding a third planet to the model does not substan-
 520 tially improve the overall fit or change the other two
 521 planetary parameters. However, only one of the three
 522 planets, HD 63433 c, is confidently detected by any of
 523 our models; we are only able to place upper limits on the
 524 masses of the other two planets. Since the small third
 525 planet does not significantly affect the derived proper-
 526 ties of the system, we adopt the three-planet model to
 527 reflect the current state of knowledge of the system.

528 The cross-validation tests show that circular and ec-
 529 centric orbital models perform similarly, with no clear
 530 preference for either. The eccentricity posterior for HD
 531 63433 c is roughly Gaussian and consistent with zero,
 532 reflecting a moderately well-constrained orbit. Yet, the
 533 posteriors for planets b and d indicate that their ec-
 534 centricities are extremely under-constrained, with broad
 535 distributions spanning nearly the full allowed range and
 536 favoring unphysical values near $e \approx 0.9$ (Figures A1-A2).
 Given the closely-packed architecture of the system and

Table 1. Cross-validation results and derived planetary masses for each model. All models were trained on both HARPS-N AND CARMENES data. **CV1:** Model conditioned on HARPS-N data and tested on CARMENES data. **CV2:** Model conditioned on CARMENES data and tested on HARPS-N data. **CV3:** Model conditioned on both HARPS-N and CARMENES data and tested on EXPRES data. Note that CV3 is only available for the joint covariance matrix and white noise models. For all CV tests, $\sigma_{\text{condition}}$ and σ_{test} describe the standard deviations of the residual distributions of the conditioning and test sets, respectively (as defined in Equation 3). A σ_{test} value closer to 1 indicates a more predictive model. All upper limits for m_b and m_d are reported at 3σ . The adopted GP model (M3) is bolded.

Model	Description	$\sigma_{\text{condition}}$	σ_{test}	Derived Planetary Masses
		(CV1 / CV2 / CV3)	(CV1 / CV2 / CV3)	(M_{\oplus})
M1	Joint covariance, 2-planet circular	0.34 / 0.10 / 0.33	0.92 / 1.07 / 1.05	$m_b < 22.85$ $m_c = 18.60^{+4.45}_{-4.26}$
M2	Joint covariance, 2-planet eccentric	0.35 / 0.11 / 0.34	0.92 / 1.08 / 1.03	$m_b < 31.96$ $m_c = 27.42^{+7.37}_{-6.65}$
M3	Joint covariance, 3-planet circular	0.34 / 0.11 / 0.32	0.92 / 1.06 / 1.04	$m_b < 23.61$ $m_c = 18.78^{+4.44}_{-4.26}$ $m_d < 10.33$
M4	Joint covariance, 3-planet eccentric	0.34 / 0.12 / 0.34	0.94 / 1.03 / 0.99	$m_b < 34.83$ $m_c = 27.45^{+7.64}_{-6.34}$ $m_d < 38.49$
M5	Added likelihood, 2-planet circular	0.34 / 0.08 / -	1.34 / 1.02 / -	$m_b < 19.39$ $m_c = 16.71^{+4.28}_{-4.14}$
M6	Added likelihood, 2-planet eccentric	0.35 / 0.09 / -	1.35 / 1.03 / -	$m_b < 34.59$ $m_c = 24.95^{+9.08}_{-6.59}$
M7	Added likelihood, 3-planet circular	0.34 / 0.08 / -	1.35 / 1.01 / -	$m_b < 20.22$ $m_c = 16.71^{+4.29}_{-4.15}$ $m_d < 10.04$
M8	Added likelihood, 3-planet eccentric	0.34 / 0.08 / -	1.35 / 1.02 / -	$m_b < 37.31$ $m_c = 24.77^{+8.83}_{-6.47}$ $m_d < 37.44$
M9	White-noise only, 3-planet circular	- / - / 0.98	- / - / 30.53	$m_b < 13.75$ $m_c = 18.55^{+7.85}_{-7.72}$ $m_d < 19.72$

constraints from B. K. Capistrant et al. (2024), such extreme eccentricities are likely unstable over the system lifetime. Instead, including eccentricity in the model provides additional degrees of freedom in the model, allowing the fits to absorb noise or sparse sampling by artificially inflating the semi-amplitude.

For models in which e is allowed to vary, we observe a degeneracy between e and K , resulting in slight but systematic increases in the inferred planetary masses. This degeneracy is most noticeable for planets c and d, where the posterior distributions shift toward higher masses and develop longer right-hand tails relative to the circular fits (Figures 3 and 4). As the eccentricities of the weaker signals cannot be reliably measured and allowing them to vary introduces small but consistent

mass biases, we consider the circular model to provide the more reliable and physically motivated description of the system. We expect that more precise eccentricity measurements from future transits or transit timing variations could break this degeneracy and allow the planetary masses to be determined more accurately.

For the same planetary configurations, models using a joint covariance matrix are generally more predictive compared to those using an added likelihood approach. When predicting HARPS-N data conditioned on CARMENES observations, the training data standard deviation remains unchanged between the two strategies while the test data standard deviation decreases for the joint covariance models (~ 1.3 for added

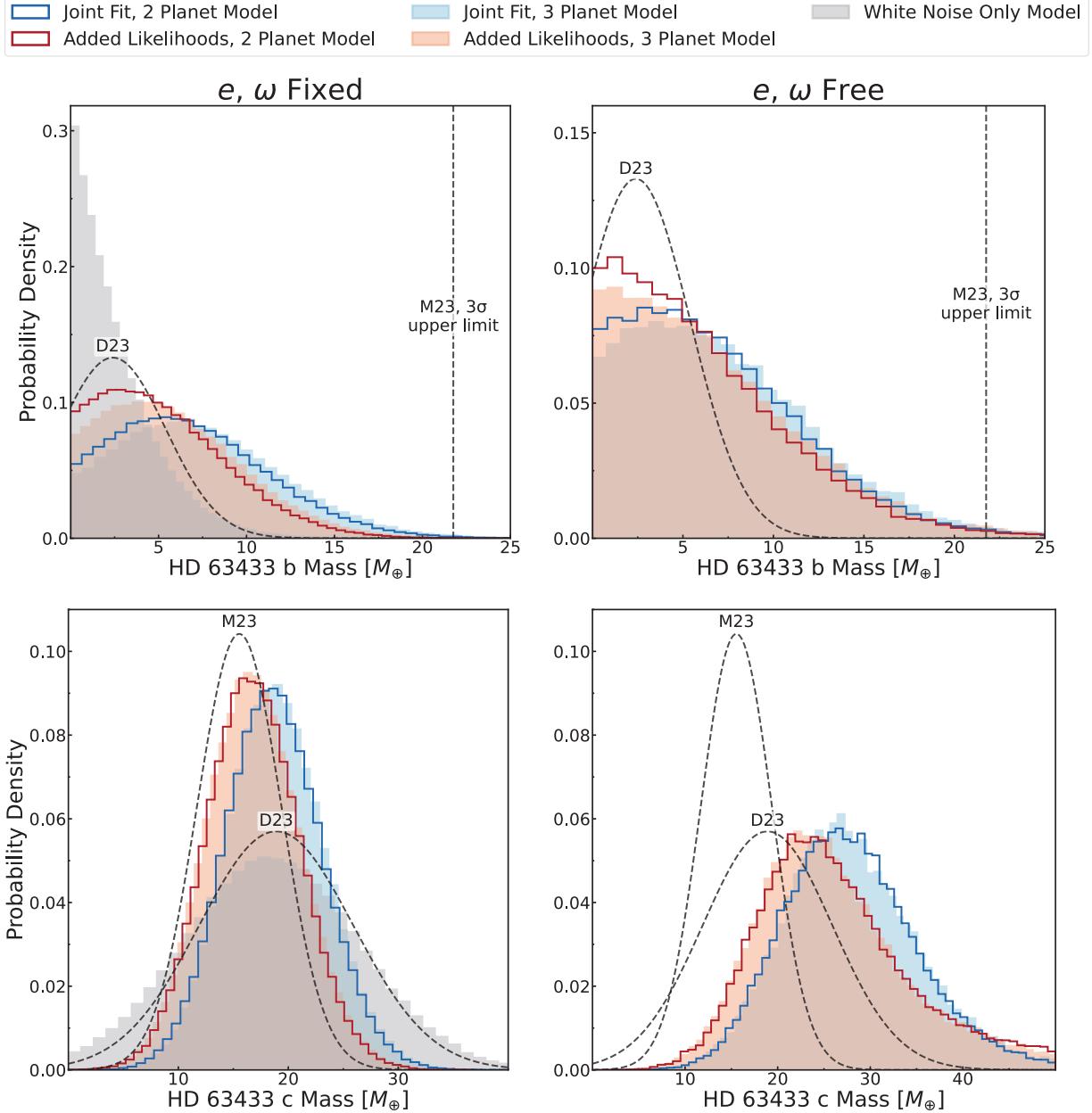


Figure 3. Mass posteriors for HD 63433 b (**top**) and c (**bottom**). Circular models are shown in the left column, and eccentric models are shown in the right. The dashed curves labeled M23 and D23 show the mass estimates from [M. Mallorquín et al. \(2023\)](#) and [M. Damasso et al. \(2023\)](#), respectively, plotted as Gaussians using the reported median and $\pm 1\sigma$ uncertainties for illustration. The derived planetary masses from our models remain consistent across all tested models within their 1σ uncertainties. The circular models agree better with literature estimates from M23 and D23, as the eccentric models tend to shift the inferred masses higher. (See planetary mass comparisons in Table 1).

567 likelihoods versus ~ 0.9 for the joint covariance matrix),
568 indicating slightly improved predictive performance.

569 When predicting CARMENES data from HARPS-
570 N observations, the test data standard deviation is
571 marginally higher for the joint covariance models. In
572 all cases, however, conditioning on HARPS-N alone ap-
573 pears to under-constrain the model, with the training
574 data standard deviation for all configurations shrink-

575 ing to ~ 0.1 compared to ~ 0.3 for the CARMENES-only
576 cases. Under this scenario, the joint covariance models
577 show slightly better predictive accuracy in the training
578 data standard deviations (~ 0.11 versus ~ 0.08 for the
579 added likelihoods).

580 When comparing the derived mass distributions, the
581 joint covariance approach produces marginally higher
582 masses for planets b and c, although the differences re-

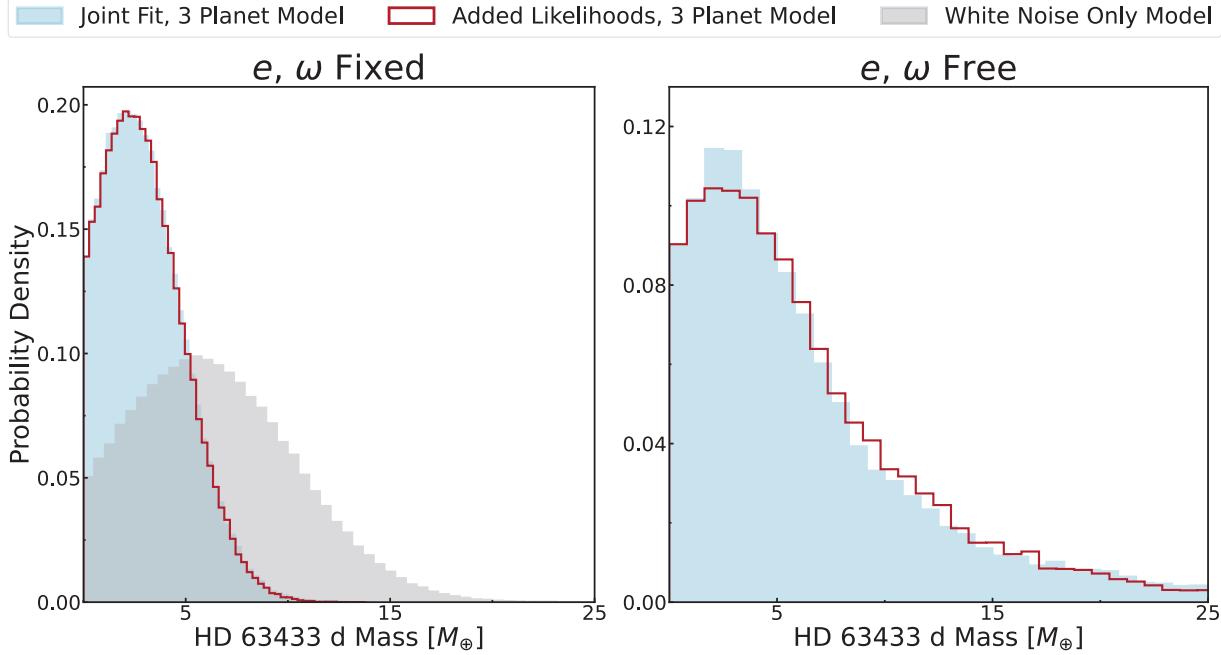


Figure 4. Mass posteriors for HD 63433 d. The joint covariance model and the added-likelihood model produce nearly identical distributions. Both predict lower masses than the white-noise-only model, indicating that modeling stellar noise with GPs can push the detection barrier to smaller masses. As with the other planets in Figure 3, allowing eccentricity (right) shifts the posterior toward higher masses, producing an extended right-hand tail.

main well within 1σ between the two methods. For the smaller planet d, the mass posteriors are essentially unchanged, reflecting that neither method has a clear advantage in detecting its low amplitude.

Both the joint-fit and added-likelihood modeling approaches produce reasonable results, but we ultimately adopt the three-planet circular model using the joint covariance matrix approach (M3). The model and its predictions are presented in Figure 5, and a histogram of the normalized residuals in Figure 6. The best-fit parameters for M3 are reported in Table 2. We believe that treating data points from different instruments taken around the same time as physically correlated provides a more realistic representation of the underlying noise structure, though this assumption should be further tested. Overall, the consistency of cross-validation results across all model configurations reinforces our confidence in the robustness of the inferred planetary parameters.

As an additional test, we compared the adopted GP model to the white-noise only model. The jitter values of both instruments increase significantly in the white noise case, from $3.52^{+2.46}_{-2.34}$ to $23.76^{+1.78}_{-1.59}$ m/s for HARPS-N, and from $0.73^{+0.78}_{-0.51}$ to $19.15^{+1.22}_{-1.11}$ m/s for CARMENES. As shown in Figure 7, it is clear that the jitter in the white-noise only model is dominated by stellar activity and dominates over the phase-folded Keplerian signal

of HD 63433 c. However, the mass distribution for HD 63433 c derived from the white-noise model agrees with that found with a GP, albeit with broader uncertainties. Without modeling the stellar noise, the white-noise model also inflates the mass of HD 63433 d and detects a limited signal from HD 63433 b, although both are also consistent with the results of the GP model. Comparisons to a white-noise model mainly serve as a check that managing stellar activity with a GP is appropriate for properly estimating the masses of this system.

5.2. Properties of HD 63433 Planets

With our new fit to the full set of radial velocity observations of HD 63433, we are now able to update our knowledge of the planets' properties. In general, we find results that are consistent with those from previous studies by M. Damasso et al. (2023) and M. Mallorquín et al. (2023).

We detect the radial velocity signal of the outermost planet HD 63433 c with a mass of $18.78^{+4.44}_{-4.26} M_{\oplus}$ at approximately 4.2σ confidence. Combined with the known radius of $2.521 \pm 0.1 R_{\oplus}$ (B. K. Capistrant et al. 2024), this yields a density of $\rho_c = 6.43^{+1.79}_{-1.58} \text{ g/cm}^3$, considerably higher than Neptune ($\rho \approx 1.64 \text{ g/cm}^3$; R. Helled et al. 2020). Observations of atmospheric escape from HD 63433 c revealed excess Ly α absorption, suggesting that some hydrogen may still be escaping, although additional searches in H α and He I show much weaker

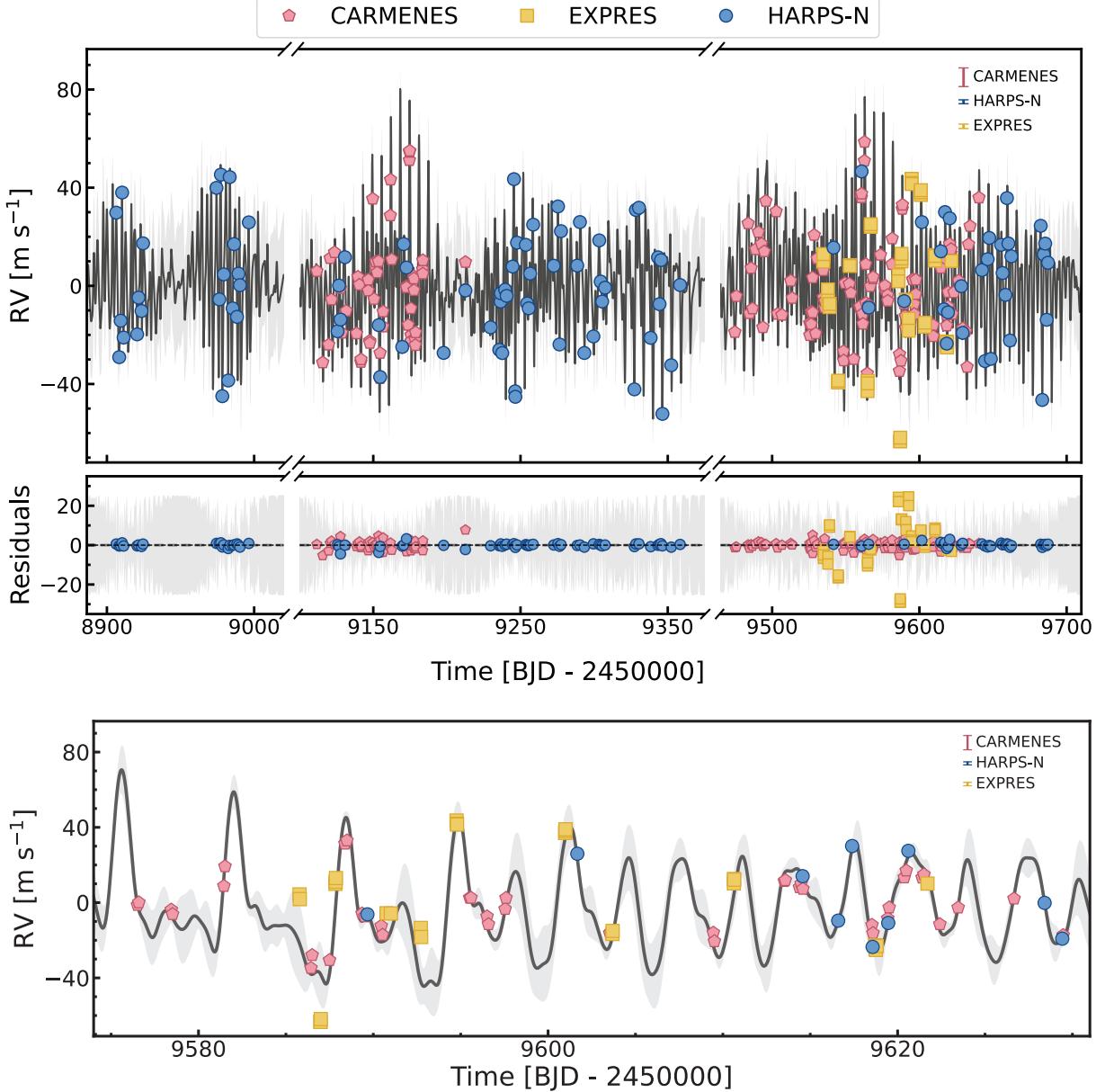


Figure 5. Top: The full GP model M3 (see Table 1 and Section 5.1), which we adopt for our analysis. The black line shows the mean GP prediction, and the shaded grey region corresponds to the 1 σ model uncertainty. The bottom panel displays the residuals between the observed data and the GP prediction. **Bottom:** Zoomed-in view of the adopted GP model (M3). The GP, which is conditioned only on the HARPS-N and CARMENES data, accurately reproduces the structure of those datasets, but some features of the held-out EXPRES data are not fully captured. Although the model performs reasonably well, the GP may still miss patterns in new data. Overall, the GP provides a reliable fit while maintaining good predictive performance, even if some small-scale variability in the test set is not reproduced.

signals, with 3 σ upper limits of 0.4–0.5% excess He I absorption (M. Zhang et al. 2022; J. Orell-Miquel et al. 2024). While the current escape is minimal, the planet's higher density at a young age suggests that HD 63433 c must have lost the majority of its primordial H/He earlier in its history or never had a significant atmosphere. According to theoretical structure models (L. Zeng et al. 2019), HD 63433 c falls in regimes consistent

with compositions of 50% H_2O and 50% rock or a mostly Earth-like core with H_2 envelopes (Figure 8). This scenario is consistent with evolution models in which a lower-density progenitor planet underwent rapid loss of its primordial envelopes within the first \sim 100 Myr after formation from high-intensity X-ray or extreme UV radiation (A. P. Jackson et al. 2012; J. E. Owen & Y. Wu 2017). We conclude that HD 63433 c is likely a sub-

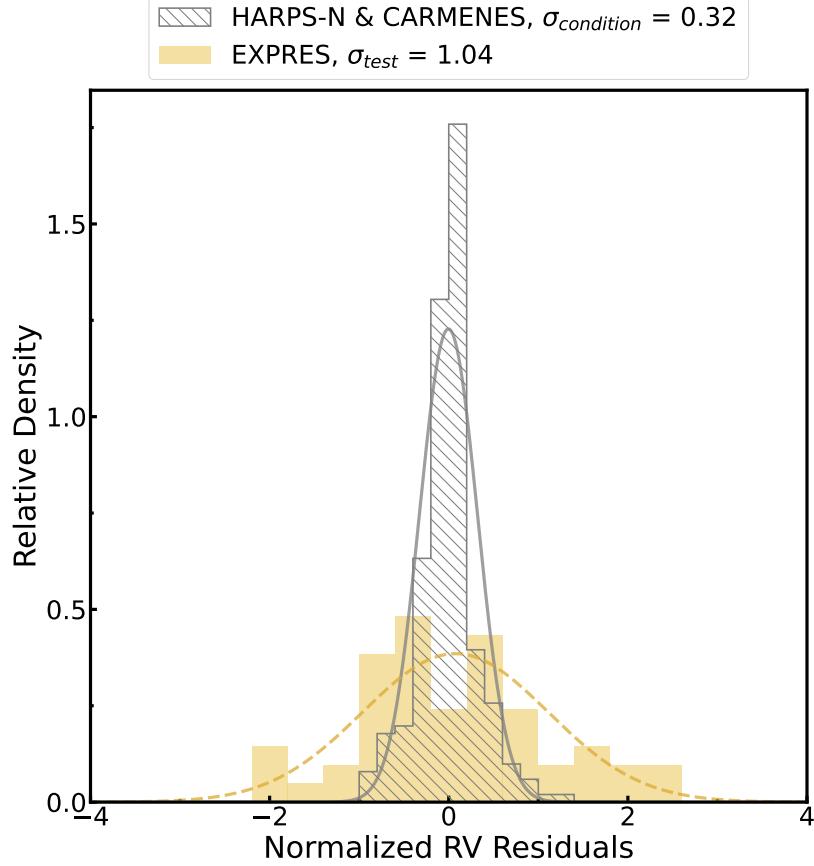


Figure 6. Histogram of cross-validation results for the adopted GP model (M3). The conditioning data (HARPS-N and CARMENES) histogram is narrow, with a standard deviation of $\sigma_{\text{condition}} = 0.32$, reflecting that the GP is closely fitting to the training points. The testing data (EXPRES) histogram, modeled with a Gaussian, has a standard deviation of $\sigma_{\text{test}} = 1.04$, signaling that the model generalizes reasonably to unseen data.

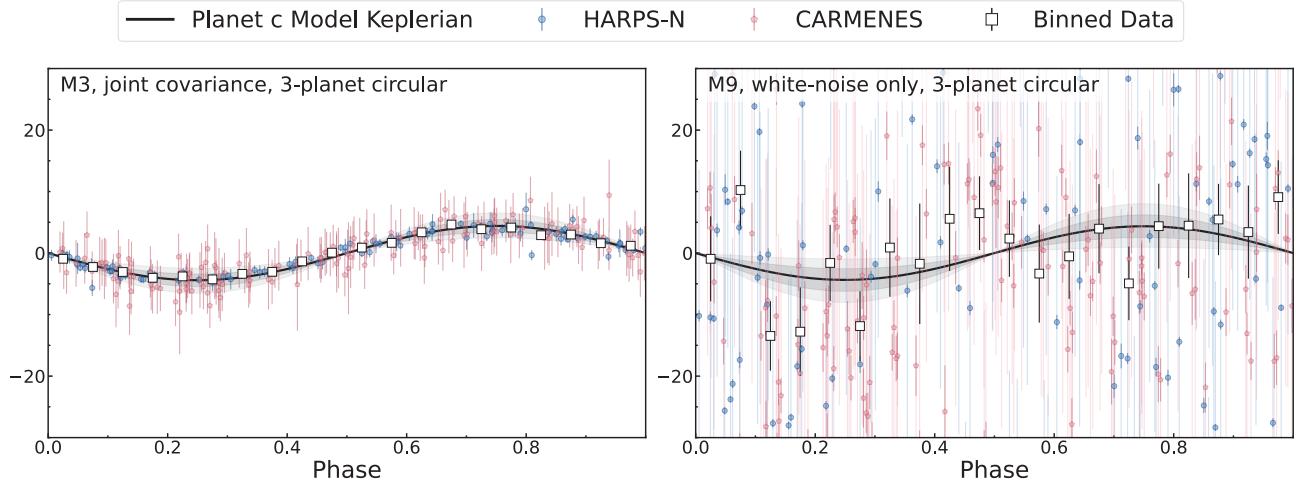


Figure 7. Phase-folded RV models of HD 63433 c, with (left) and without (right) a GP. The data are binned in the white squares, and the best-fit Keplerian model is plotted in black with 1σ and 2σ uncertainties in grey. On the left, the stellar activity component of the adopted GP model (M3) is subtracted from the data, greatly improving the agreement between the observations and the Keplerian model. The larger, lighter error bars on the right represent the model jitter added in quadrature to the data uncertainties, displaying the increased scatter (~ 50 m/s) when stellar activity is not accounted for.

653 Neptune that has reached its final radius and will not
654 undergo significant contraction in the future.

655 We are unable to confidently detect the RV signal of
656 the middle planet HD 63433 b. We are able to place a

Table 2. Best-fit (median) parameters for adopted model M3.

Parameter	Best-fit value ^a
Fitted	
<i>GP terms:</i>	
$\eta_{1,HARPS-N}$ [m s ⁻¹]	$25.44^{+2.10}_{-1.96}$
$\eta_{1,CARMENES}$ [m s ⁻¹]	$18.71^{+1.62}_{-1.39}$
η_2 [d]	$19.31^{+2.00}_{-1.66}$
η_3 [d]	6.39 ± 0.02
η_4	0.21 ± 0.02
<i>Instrumental terms:</i>	
$\sigma_{ jit,HARPS-N}$ [m s ⁻¹]	$3.52^{+2.46}_{-2.34}$
$\gamma_{HARPS-N}$ [m s ⁻¹]	$-15805.51^{+3.18}_{-3.16}$
$\sigma_{ jit,CARMENES}$ [m s ⁻¹]	$0.73^{+0.78}_{-0.51}$
$\gamma_{CARMENES}$ [m s ⁻¹]	$2.62^{+2.40}_{-2.39}$
<i>Planet b parameters:</i>	
$T_{\text{conj,b}}$ [BJD - 2450000]	8916.45286 ± 0.00042
P_b [d]	7.107934 ± 0.000005
K_b [m s ⁻¹]	< 7.90
<i>Planet c parameters:</i>	
$T_{\text{conj,c}}$ [BJD - 2450000]	8844.05971 ± 0.00052
P_c [d]	20.543784 ± 0.000016
K_c [m s ⁻¹]	$4.41^{+1.04}_{-1.00}$
<i>Planet d parameters:</i>	
$T_{\text{conj,d}}$ [BJD - 2450000]	9373.82337 ± 0.00103
P_d [d]	4.209078 ± 0.000022
K_d [m s ⁻¹]	< 4.12
Derived	
m_b [M_\oplus]	< 23.61
ρ_b [g cm ⁻³]	$< 15.18^b$
m_c [M_\oplus]	$18.78^{+4.44}_{-4.26}$
ρ_c [g cm ⁻³]	$6.43^{+1.79}_{-1.58} b$
m_d [M_\oplus]	< 10.33
ρ_d [g cm ⁻³]	$< 50.60^b$

^aThe uncertainties are given as the 16th and 84th percentiles of the posterior distributions and upper limits are reported at 3σ .

^bMean densities derived using radii reported in B. K. Capistrant et al. (2024).

timescale of 80 Myr for this planet, suggesting that its primordial H/He atmosphere may have already evaporated at its current age of 414 Myr (J. Jones et al. 2015; B. K. Capistrant et al. 2024).

L. A. Rogers (2015) argues that most planets larger than $\sim 1.6R_\oplus$ are too low-density to be composed only of iron and silicates. As HD 63433 b has a radius of $2.112^{+0.093}_{-0.086} R_\oplus$ (B. K. Capistrant et al. 2024), it is still large enough to be expected to have a gaseous envelope. Given our current observational constraints, we are unable to distinguish between a pure rock composition and a small H/He atmosphere. Thus, additional RV observations are needed to provide better constraints on the mass in order to rule out rocky compositions.

Finally, we place the first constraints on the mass of HD 63433 d, the innermost rocky planet recently detected by B. K. Capistrant et al. (2024). This planet is slightly larger than Earth at $1.073^{+0.046}_{-0.044} R_\oplus$, and we place a 3σ upper limit on the mass of $10.33M_\oplus$. As visualized in Figure 8, the current 3σ upper limit places HD 63433 beyond the mass limits of a pure iron planet. A pure iron planet with HD 63433 d's radius would have a mass less than $2.72M_\oplus$ based on models from L. Zeng et al. (2019), which would induce a signal with a semi-amplitude of at most 1.23 m/s, compared to the 3σ upper limit semi-amplitude we found of 4.12 m/s.

Given its near Earth-like size and proximity to its host star, HD 63433 d is unlikely to host any substantial atmosphere (L. A. Rogers 2015). Our mass limit is consistent with a terrestrial composition of this planet, but actually detecting the RV amplitude of HD 63433 d with this composition will be challenging, since the semi-amplitude is only expected to be about ~ 55 cm/s. Placing further constraints would require a coordinated observing campaign with dense sampling and advanced stellar activity mitigation methods.

With an age of 414 Myr (J. Jones et al. 2015; B. K. Capistrant et al. 2024), the HD 63433 planets are old enough to no longer be rapidly contracting, and should be roughly their “final” radii; for example, J. E. Owen 2020 calculates that 500 Myr sub-Neptunes should contract by $\sim 5\%$ before achieving their \sim Gyr radii. However, the difference between expected current and final radii is significant enough for future extreme precision RV campaigns to measure and use precise mass measurements to further constrain planetary evolution models. Although the radius difference for these moderately young sub-Neptunes relative to their Gyr radii is smaller than for very young (< 10 Myr) planets, their lower stellar activity makes precise RV measurements more feasible. Stellar activity, and not RV precision, remains the major hurdle our field will need to overcome to make

657 3σ upper limit on the mass of this planet of $23.61 M_\oplus$, in
658 agreement with the upper limit of $21.76 M_\oplus$ found in M.
659 Mallorquín et al. (2023). Similar to planet c, HD 63433
660 b has non-detections of atmospheric escape (M. Zhang
661 et al. 2022; J. Orell-Miquel et al. 2024; M. K. Alam
662 et al. 2024). Hydrodynamical models from M. Zhang
663 et al. (2022) estimated a short atmospheric mass-loss

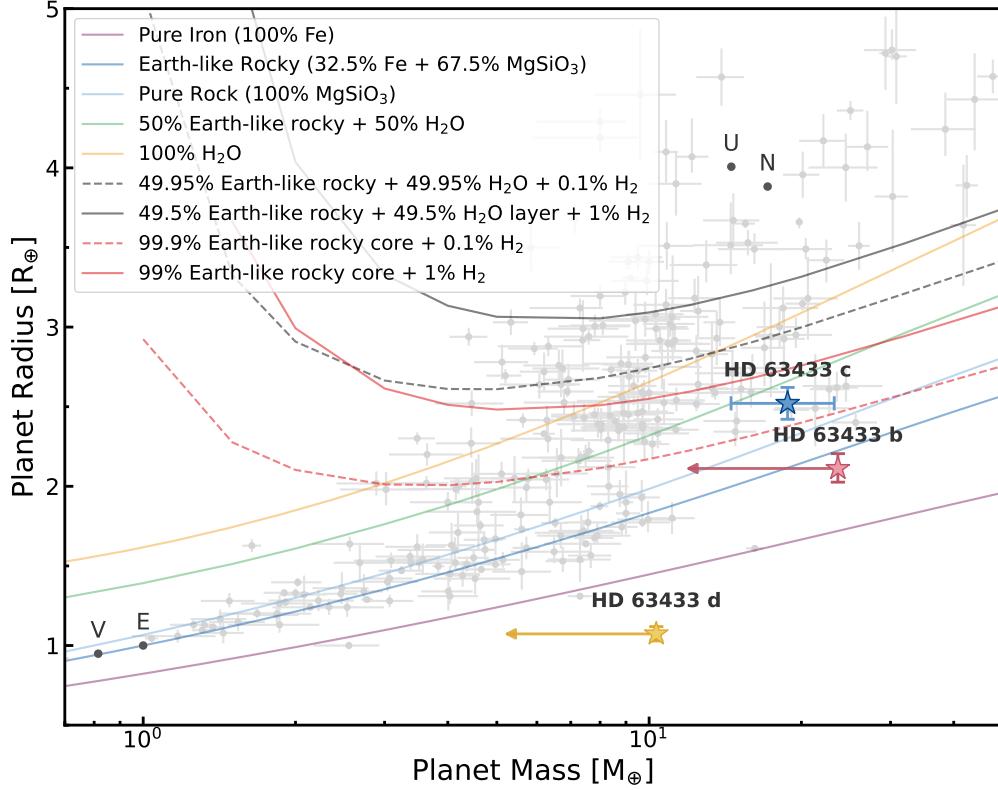


Figure 8. Mass-radius diagram for small exoplanets. The HD 63433 planets are shown as colored stars and labeled. Other planets from the NASA Exoplanet Archive (J. L. Christiansen et al. 2025) are shown as grey points, and composition curves from L. Zeng et al. (2019) are shown as solid and dashed curves. HD 63433 c has a reliably measured mass incompatible with an Earth-like rocky composition, suggesting it has retained a low-density volatile envelope. HD 63433 b and c have only upper limits on their masses that cannot uniquely constrain their compositions.

these measurements. This work, which has studied the activity of the star HD 63433, as well as the impact of various modeling assumptions on the inferred planet properties, is a necessary first step toward making planetary mass measurements with sufficient precision to distinguish between these evolution models.

5.3. Future Work with `eurydice`

Future work with `eurydice` aims to expand the framework to handle multi-dimensional GPs that jointly model photometry or activity indicators. These multi-dimensional GPs are expected to provide an alternative treatment of correlated stellar noise and are already implemented in other RV modeling frameworks like `pyaneti` (O. Barragán et al. 2022), so bridging cross-validation to those more elaborate models will be an important path forward. Additionally, `eurydice` currently implements a simple hold-out cross-validation method. In the future, more advanced cross-validation techniques, such as k-fold or leave-one-out cross-validation, will be implemented to validate models at different levels of robustness.

737

6. CONCLUSIONS

In this work, we introduced `eurydice` and applied it to both existing and new observations of the HD 63433 system. Our main conclusions can be summarized as follows:

742 1. `eurydice` is an open-source Python package for
743 performing hold-out cross-validation on GP models
744 of stellar activity. It provides a standardized,
745 flexible framework to evaluate how well these
746 models predict unseen RV data, making it an effective
747 tool for assessing predictive performance and iden-
748 tifying signs of overfitting.

749 2. We applied `eurydice` to models of the young HD
750 63433 planetary system. We derived mass mea-
751 surements for the outermost planet HD 63433 c
752 at $19^{+4}_{-4} M_{\oplus}$, a 4.2σ detection. We placed 3σ up-
753 per limit masses for inner planets HD 63433 b and
754 d at $< 22 M_{\oplus}$ and $< 10 M_{\oplus}$, respectively. These
755 masses are consistent with past work reported in
756 M. Damasso et al. (2023) and M. Mallorquín et al.

(2023), and cross-validation affirms the reliability of these measurements.

3. We directly compared two GP approaches for handling multi-instrument RV data. In the first, a joint covariance matrix encodes the assumption of correlated signals across instruments, while the added likelihood approach, much more prevalent in the literature, treats each instrument independently. Our cross-validation tests found marginally better performance for the joint covariance matrix models, but there is no significant difference in the planetary parameters derived. We adopt the joint covariance matrix model as a more physically motivated description of stellar activity, but future work should rigorously test this assumption.

GP modeling is becoming increasingly standard for RV studies, yet the reliability of GP fits remains a source of uncertainty in understanding young exoplanet systems. Going forward, we hope cross-validation becomes a standard tool in the exoplanet community, providing more reliable measurements of planet properties and guiding future studies toward a clearer understanding of how young planets mature and evolve.

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APPENDIX

Table A1. RVs derived from EXPRES spectra.

Time [BJD]	RV [m s^{-1}]	$\sigma_{RV} [\text{m s}^{-1}]$
2459534.810552	10.760	0.886
2459534.823394	11.530	0.755
2459534.837843	9.019	0.731
2459534.853980	12.103	0.752
2459537.900158	-2.315	0.846
2459537.914665	-6.865	0.931
2459538.961198	-9.500	0.808
2459538.969824	-8.240	0.730
2459544.845671	-40.111	1.075
2459544.860033	-40.224	0.907
2459544.874006	-39.475	0.885
2459552.789574	7.608	0.772
2459552.797160	7.251	0.845
2459564.828333	-43.729	0.804
2459564.842154	-39.574	0.758
2459564.856901	-40.602	0.786
2459566.953363	23.235	0.945
2459566.967272	24.278	0.866
2459585.779930	3.534	0.962
2459585.794421	1.015	0.843
2459586.973151	-64.115	0.862
2459586.983498	-64.193	0.789
2459586.993271	-62.701	0.839
2459587.832862	9.138	0.868
2459587.846746	10.292	0.888
2459587.861261	12.213	1.033
2459590.757694	-6.478	1.064
2459591.006895	-6.619	0.954
2459592.721409	-14.282	0.999
2459592.738293	-19.110	1.002
2459594.762732	42.774	1.064
2459594.777590	41.057	1.156
2459594.793006	40.632	1.205
2459600.976240	36.196	0.857
2459600.986720	37.261	0.810
2459600.998656	38.135	0.887

Table A1 *continued*

Table A1 (*continued*)

Time [BJD]	RV [m s^{-1}]	$\sigma_{RV}[\text{m s}^{-1}]$
2459603.683994	-17.507	0.771
2459603.693087	-15.851	0.771
2459610.630149	9.387	0.781
2459610.638326	11.722	0.742
2459610.646550	11.372	0.792
2459618.713927	-23.526	0.800
2459618.726495	-25.140	0.731
2459618.739595	-23.704	0.819
2459618.752535	-25.968	0.830
2459618.803671	-23.913	0.796
2459621.705842	9.282	1.687
2459858.997210	-15.924	1.603
2459865.014861	-5.659	1.423
2459866.004055	-8.548	1.572
2459879.951915	48.288	1.436
2459882.973990	1.128	1.579

Table A2. Prior parameters of all models.

Parameter	Prior
$\eta_{1,HARPS-N}$ [m s ⁻¹]	$\mathcal{U}(0, 500)$
$\eta_{1,CARMENES}$ [m s ⁻¹]	$\mathcal{U}(0, 500)$
η_2 [d]	$\mathcal{U}(0, 500)$
η_3 [d]	$\mathcal{U}(5.4, 6.6)$
η_4	$\mathcal{U}(0, 1)$
$\sigma_{ jit,HARPS-N}$ [m s ⁻¹]	$\mathcal{U}(0, 3\sigma_{HARPS-N})$
$\gamma_{HARPS-N}$ [m s ⁻¹]	$\mathcal{U}(-16000, -15500)$
$\sigma_{ jit,CARMENES}$ [m s ⁻¹]	$\mathcal{U}(0, 3\sigma_{CARMENES})$
$\gamma_{CARMENES}$ [m s ⁻¹]	$\mathcal{U}(-3\sigma_{CARMENES}, 3\sigma_{CARMENES})$
$T_{0,b}$ [BJD]	$\mathcal{N}(2458916.45286, 0.00042)^a$
P_b [d]	$\mathcal{N}(7.1079342, 0.0000049)^a$
$\sqrt{e} \sin(\omega)_b$	$\mathcal{U}(-1, 1)$
$\sqrt{e} \cos(\omega)_b$	$\mathcal{U}(-1, 1)$
K_b [m s ⁻¹]	$\mathcal{U}(0, 30)$
$T_{0,c}$ [BJD]	$\mathcal{N}(2458844.05971, 0.00052)^a$
P_c [d]	$\mathcal{N}(20.543784, 0.000016)^a$
$\sqrt{e} \sin(\omega)_c$	$\mathcal{U}(-1, 1)$
$\sqrt{e} \cos(\omega)_c$	$\mathcal{U}(-1, 1)$
K_c [m s ⁻¹]	$\mathcal{U}(0, 30)$
$T_{0,d}$ [BJD]	$\mathcal{N}(2459373.82337, 0.00104)^a$
P_d [d]	$\mathcal{N}(4.209078, 0.000022)^a$
$\sqrt{e} \sin(\omega)_d$	$\mathcal{U}(-1, 1)$
$\sqrt{e} \cos(\omega)_d$	$\mathcal{U}(-1, 1)$
K_d [m s ⁻¹]	$\mathcal{U}(0, 10)$

^aTransit timing and periods taken from B. K. Capistrant et al. (2024)

Table A3. Best-fit (median) parameters for two planet models.

Parameter	Joint-fit ($e=\omega=0$)	Joint-fit (e,ω free)	Added-likelihoods ($e=\omega=0$)	Added-likelihoods (e,ω free)
$\eta_{1,HARPS-N}$ [m s ⁻¹]	$25.50^{+2.11}_{-1.94}$	$25.43^{+2.17}_{-2.05}$	$24.03^{+2.12}_{-1.87}$	$23.93^{+2.10}_{-1.87}$
$\eta_{1,CARMENES}$ [m s ⁻¹]	$18.71^{+1.60}_{-1.37}$	$18.75^{+1.66}_{-1.42}$	$20.11^{+2.02}_{-1.74}$	$20.05^{+1.98}_{-1.74}$
η_2 [d]	$19.50^{+2.15}_{-1.74}$	$19.48^{+1.97}_{-1.66}$	$20.59^{+2.07}_{-1.73}$	$20.39^{+2.06}_{-1.71}$
η_3 [d]	6.38 ± 0.02	6.39 ± 0.02	6.38 ± 0.02	6.38 ± 0.02
η_4	0.21 ± 0.02	0.22 ± 0.02	0.22 ± 0.02	0.23 ± 0.02
$\sigma_{ jit,HARPS-N}$ [m s ⁻¹]	$3.27^{+2.38}_{-2.18}$	$3.53^{+2.32}_{-2.31}$	$1.73^{+2.17}_{-1.23}$	$1.77^{+2.28}_{-1.26}$
$\gamma_{HARPS-N}$ [m s ⁻¹]	$-15805.40^{+3.18}_{-3.15}$	-15804.90 ± 3.25	$-15806.30^{+3.51}_{-3.50}$	$-15806.04^{+3.51}_{-3.52}$
$\sigma_{ jit,CARMENES}$ [m s ⁻¹]	$0.76^{+0.82}_{-0.53}$	$0.81^{+0.87}_{-0.57}$	$0.72^{+0.78}_{-0.50}$	$0.73^{+0.76}_{-0.51}$
$\gamma_{CARMENES}$ [m s ⁻¹]	$2.60^{+2.38}_{-2.39}$	$2.72^{+2.39}_{-2.48}$	$1.89^{+3.62}_{-3.68}$	$1.92^{+3.69}_{-3.68}$
$T_{\text{conj,b}}$ [BJD - 2450000]	8916.45286 ± 0.00042	$8916.45261^{+0.00064}_{-0.00063}$	$8916.45286^{+0.00042}_{-0.00041}$	$8916.45262^{+0.00065}_{-0.00064}$
$\sqrt{e} \sin(\omega)_b$	0 (fixed)	$-0.18^{+0.50}_{-0.48}$	0 (fixed)	$-0.25^{+0.57}_{-0.46}$
$\sqrt{e} \cos(\omega)_b$	0 (fixed)	$0.04^{+0.42}_{-0.77}$	0 (fixed)	$0.09^{+0.44}_{-0.80}$
K_b [m s ⁻¹]	$2.17^{+1.64}_{-1.33}$	$2.08^{+3.32}_{-1.45}$	$1.60^{+1.44}_{-1.06}$	$1.83^{+3.30}_{-1.32}$
$T_{\text{conj,c}}$ [BJD - 2450000]	8844.05971 ± 0.00052	$8844.05973^{+0.00059}_{-0.00060}$	8844.05971 ± 0.00052	8844.05971 ± 0.00060
P_c [d]	20.543784 ± 0.000016	20.543791 ± 0.000018	20.543784 ± 0.000016	20.543791 ± 0.000018
$\sqrt{e} \sin(\omega)_c$	0 (fixed)	$0.41^{+0.21}_{-0.34}$	0 (fixed)	$0.42^{+0.28}_{-0.52}$
$\sqrt{e} \cos(\omega)_c$	0 (fixed)	$-0.30^{+0.26}_{-0.17}$	0 (fixed)	$-0.20^{+0.46}_{-0.25}$
K_c [m s ⁻¹]	$4.37^{+1.04}_{-1.00}$	$5.33^{+1.14}_{-1.13}$	$3.93^{+1.01}_{-0.97}$	$4.88^{+1.33}_{-1.15}$

Table A4. Best-fit (median) parameters for three planet models.

Parameter	Joint-fit (e, ω free)	Added-likelihoods ($e=\omega=0$)	Added-likelihoods (e, ω free)
$\eta_{1,HARPS-N}$ [m s ⁻¹]	$25.36^{+2.16}_{-1.99}$	$24.20^{+2.15}_{-1.91}$	$24.01^{+2.16}_{-1.89}$
$\eta_{1,CARMENES}$ [m s ⁻¹]	$18.65^{+1.68}_{-1.39}$	$20.00^{+2.01}_{-1.73}$	$19.94^{+2.03}_{-1.74}$
η_2 [d]	$19.30^{+2.03}_{-1.63}$	$20.44^{+1.99}_{-1.69}$	$20.29^{+2.10}_{-1.71}$
η_3 [d]	6.39 ± 0.02	6.38 ± 0.02	6.39 ± 0.02
η_4	0.22 ± 0.02	0.22 ± 0.02	0.23 ± 0.02
$\sigma_{ jit,HARPS-N}$ [m s ⁻¹]	$3.53^{+2.38}_{-2.35}$	$1.74^{+2.25}_{-1.25}$	$1.71^{+2.30}_{-1.22}$
$\gamma_{HARPS-N}$ [m s ⁻¹]	$-15804.98^{+3.27}_{-3.19}$	$-15806.25^{+3.56}_{-3.50}$	$-15805.97^{+3.55}_{-3.48}$
$\sigma_{ jit,CARMENES}$ [m s ⁻¹]	$0.79^{+0.87}_{-0.56}$	$0.70^{+0.77}_{-0.49}$	$0.73^{+0.81}_{-0.51}$
$\gamma_{CARMENES}$ [m s ⁻¹]	$2.71^{+2.48}_{-2.34}$	$1.76^{+3.60}_{-3.68}$	$1.90^{+3.65}_{-3.61}$
$T_{\text{conj,b}}$ [BJD - 2450000]	$8916.45263^{+0.00065}_{-0.00064}$	8916.45286 ± 0.00042	8916.45261 ± 0.00065
P_b [d]	7.107938 ± 0.000005	7.107934 ± 0.000005	7.107938 ± 0.000005
$\sqrt{e} \sin(\omega)_b$	$-0.19^{+0.50}_{-0.51}$	0 (fixed)	$-0.24^{+0.54}_{-0.46}$
$\sqrt{e} \cos(\omega)_b$	$0.07^{+0.41}_{-0.74}$	0 (fixed)	$0.11^{+0.39}_{-0.80}$
K_b [m s ⁻¹]	$2.23^{+3.62}_{-1.53}$	$1.81^{+1.49}_{-1.16}$	$1.89^{+2.84}_{-1.34}$
$T_{\text{conj,c}}$ [BJD - 2450000]	$8844.05972^{+0.00059}_{-0.00061}$	$8844.05971^{+0.00052}_{-0.00051}$	$8844.05971^{+0.00061}_{-0.00059}$
P_c [d]	20.543792 ± 0.000018	20.543784 ± 0.000016	20.543791 ± 0.000018
$\sqrt{e} \sin(\omega)_c$	$0.44^{+0.20}_{-0.33}$	0 (fixed)	$0.39^{+0.31}_{-0.52}$
$\sqrt{e} \cos(\omega)_c$	$-0.28^{+0.25}_{-0.17}$	0 (fixed)	$-0.23^{+0.45}_{-0.23}$
K_c [m s ⁻¹]	$5.34^{+1.14}_{-1.08}$	$3.93^{+1.01}_{-0.97}$	$4.89^{+1.35}_{-1.16}$
$T_{\text{conj,d}}$ [BJD - 2450000]	$9373.82349^{+0.00112}_{-0.00114}$	9373.82337 ± 0.00103	$9373.82347^{+0.00109}_{-0.00113}$
P_d [d]	$4.209075^{+0.000023}_{-0.000022}$	4.209078 ± 0.000022	$4.209075^{+0.000022}_{-0.000023}$
$\sqrt{e} \sin(\omega)_d$	$0.27^{+0.50}_{-0.71}$	0 (fixed)	$0.10^{+0.57}_{-0.66}$
$\sqrt{e} \cos(\omega)_d$	$-0.26^{+0.71}_{-0.49}$	0 (fixed)	$-0.28^{+0.70}_{-0.43}$
K_d [m s ⁻¹]	$1.88^{+2.47}_{-1.24}$	$1.12^{+0.89}_{-0.71}$	$1.70^{+1.83}_{-1.12}$

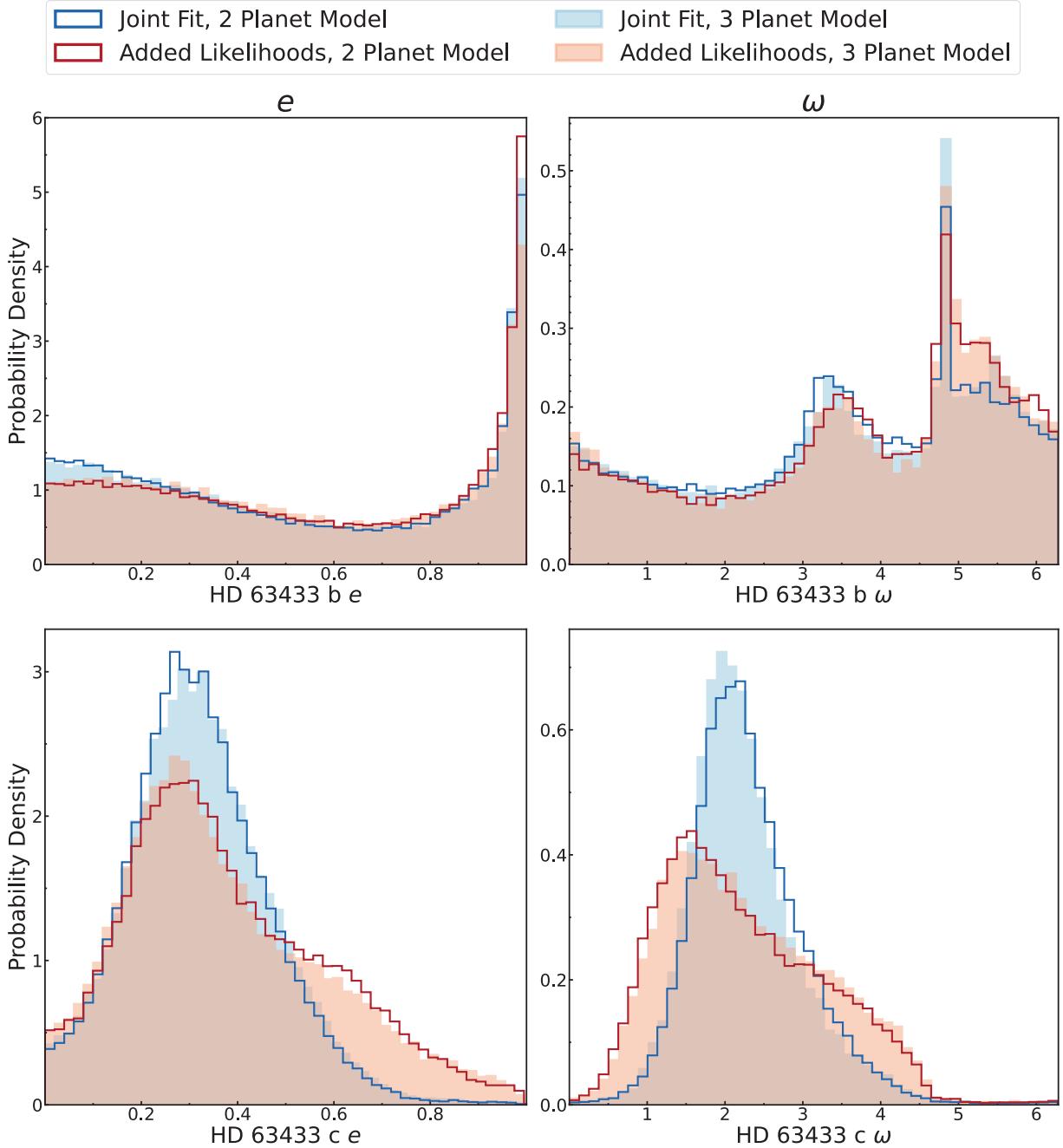


Figure A1. e and ω posterior distributions for planets HD 63433 b and c.

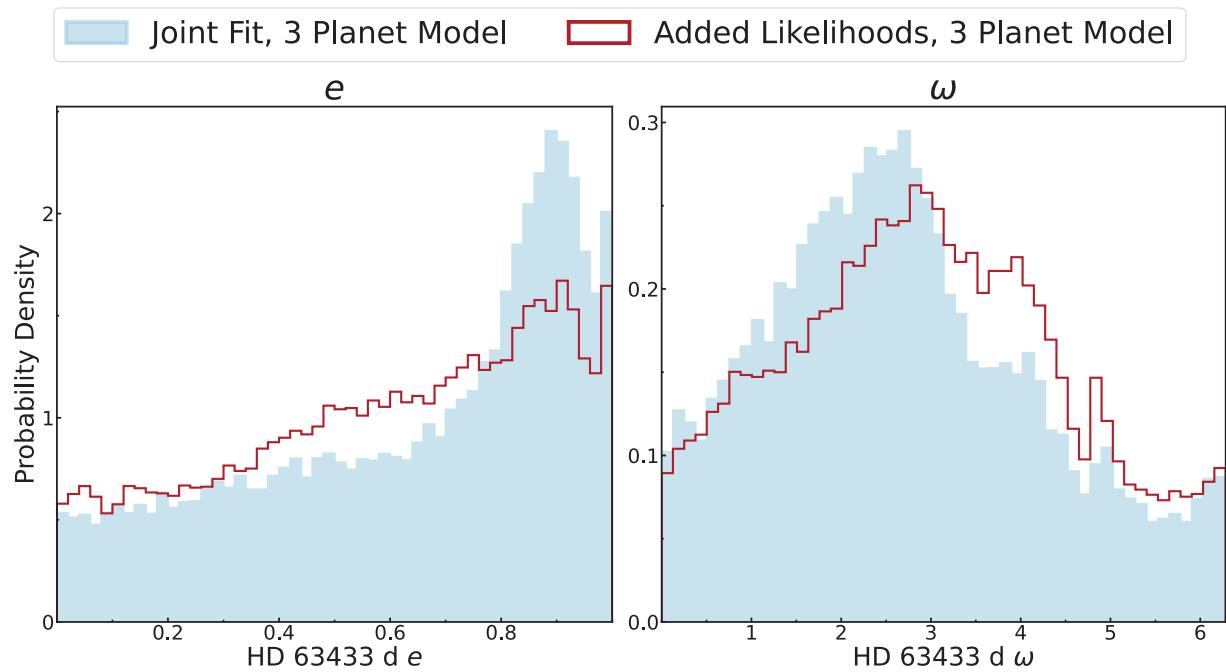


Figure A2. e and ω posterior distributions for planet HD 63433 d.