Workshop on fast food restaurant

When we go to a fast food restaurant, we perhaps care more about how quickly the food is served than the quality of food (as long as it is above some standard). If a restaurant offers a guarantee that, if the wait time is over some limit, the food will be free, intuitively it will attract more customers. If you were a restaurant owner, would you consider offering such a guarantee?

We shall investigate this question with a queuing model. Let's consider a single-queue (for orders waiting to be filled) and single-server (the kitchen) system. We shall start the timer when an order has been placed and stop it when the customer gets his/her food.

1. What are the relevant processes and data to collect?

2. Suppose you know on average how often a customer comes, what kind of distribution would you assume for the customer arrival? What about the service time?

3. We shall assume the service time is exponentially distributed (please criticize this choice). Suppose you know the price of the food item (say a burger) and the cost (labor plus material) of producing it, and you have the policy that the customer will get the food for free if the wait time exceeds some threshold. Investigate whether that is a profitable policy by modifying the queuing.m code on canvas. Considering the following questions:

a. Let a and b be the average number of arrivals and customers served per unit time for a typical restaurant under consideration. What is the expected total time (wait time plus the service time)? In queuing theory, there is an analytical result that the probability that total time T exceeds a value t is given by

$$\Pr(T > t) = \exp(-(b-a)t)$$

where a and b are the average number of arrivals and customers served per unit time. For a derivation, see the file "Derivation for the probability distribution of total time.pdf" on Canvas. Can you verify this result with the code queuing.m or queuing.ipynb? Note that the histogram values plotted in the code are proportional to $\Pr(t < T < t + \Delta t)$ where Δt is the bin size used.

b. How should we choose the guaranteed time? How should we model how the inter-arrival time of the customers changes with the guaranteed time (also known as a behavior response function)?

c. After you have decided on a behavior response function, think about how you would get the expected profit given the guaranteed time (either as a function or a procedure), assuming that you already know the industry average arrival and service time and the profit margin. If you get stuck on this part, consult the document on Canvas named "An analytical approach to the fast food workshop".

4. The fast food industry is known to have small profit margins, heavy throughput, and high expectation of speedy services. With these in mind, can you evaluate whether it makes sense to offer such a time guarantee? One way to think about this is what the service time needs to be compared to the industry standard (a typical restaurant) for the guarantee to increase profit. To approach this do the following:

a. Modify the code fastfood.m or fastfood.ipynb so that the inter-arrival time is a function of the time guarantee based on what you decided in part 3.

b. Knowing the cost and profit, plot the profit per unit time as a function of the time guarantee and find the maximum. Remember this is a stochastic system so you need to run many experiments/trials to draw useful conclusions.

c. Modify the service time to see how much faster the kitchen needs to be compared the industry standard for the policy to increase profit. Note that guarantee is designed to increase the number of arrivals per unit time and will further require faster service.

d. Explore the dependence of your conclusions on the profit margin and the industry standard inter-arrival and service time.

e. You may find the analytical effort posted on Canvas a useful guide.

Turn in a brief write-up of what you find and your conclusions, along with the codes.