A Four Component Fitness-Fatigue Model for Athletic Preparedness

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Abstract

This paper proposes a four component model for the impact of athletic training on preparedness for competition. The model accounts for increases in fitness and skill during training as well as the accumulation and dissipation of fatigue. Fatigue is considered to have two components of its own: a fast changing component that reflects burning of fuel substrate during training and a slow changing component that accumulates due to repetitive microtraumas incurred during training. The model is applied to study training plans designed to maximize preparedness for a major event at the end of a ten week training cycle. Results suggest an optimal structure for reducing training load in order to maximize preparedness. The sensitivity of results is analyzed with respect to model parameters.

1 Introduction

Whether an athlete aspires to be a little league champion or an olympic medalist, they are always filled with the desire to make the most of the time, energy, and thought invested in their training. While athletic training provides benefits to overall fitness and allows an athlete to develop specific skills, there are also bumps, bruises, and fatigue that the athlete must fight along the way. Athletes generally get stronger as their training continues, but performance at a championship event is not a function of strength or fitness alone.

When designing a training plan, it is important for a coach or athlete to consider their goals throughout the season. If performance at a particular event is paramount, the training plan should reflect that. It is common practice for teams and individual athletes to reduce their training load immediately before a major event, a strategy referred to as 'tapering.' This strategy allows an athlete to recover from accumulated fatigue in order to better take advantage of fitness gains accumulated during the training cycle. The effectiveness of tapering as a strategy for achieving peak preparedness and performance highlights the need to consider fatigue alongside fitness when building a model for athletic training.

The deterministic fitness-fatigue model suggested by Chiu[6] provides some insight into the mechanisms of training using a two component model. Here we suggest an extension on the fitness-fatigue framework by providing a four component model. The model is applied to a ten week training cycle built to mimic the typical training plan of a high school or club level athlete, and it is used to analyze trends in overall preparedness across different taper structures and an ideal structure is suggested.

2 Description of the Model

The following model is proposed for describing the overall training effect on preparedness of an athlete during a training cycle. The overall impact of training is resolved into four components: overall fitness (p_1), development of skills (p_2), fatigue due to burning of fuel substrate (glycogen, creatine-phosphate, etc.) (n_1), and fatigue due to repetitive microtrauma (sore muscles, bruises, etc.) during training (n_2). The overall impact of training on the physical preparedness of the athlete is taken to be the sum of the positive influences p_1 and p_2 minus the negative influences n_1 and n_2.

$$\text{Preparedness} = p_1 + p_2 - n_1 - n_2 \tag{1}$$

Changes in these components are modeled with the differential equations

$$\frac{dp_1}{dt} = w(t)\left(1 - \kappa(n_1 + n_2)\right) - \frac{p_1}{\tau_1} \tag{2}$$

$$\frac{dp_2}{dt} = \frac{1}{\tau_2} w(t)\left(1 - \frac{p_2}{M}\right) \tag{3}$$

$$\frac{dn_1}{dt} = w(t) - \frac{n_1}{\tau_3}(1 + \kappa p_1) \tag{4}$$

$$\frac{dn_2}{dt} = K w(t) - \frac{n_2}{\tau_1}(1 + \kappa p_1)) \tag{5}$$

Here $w(t)$ is a time dependent parameter that describes the training schedule of the athlete. This paper focuses on a realistic training schedule for an intermediate level athlete, so it is assumed that workouts are assigned for 2 hours per day, six days per week during the training cycle. The further assumption is made that the training load of each individual workout remains constant throughout most of the training cycle. During the bulk of the training cycle, we take $w(t) = 1$ during a training session and $w(t) = 0$ otherwise. We will also consider the effects of reduced training load ($w(t) < 1$) during a taper period leading up to a major event at the end of the training cycle.

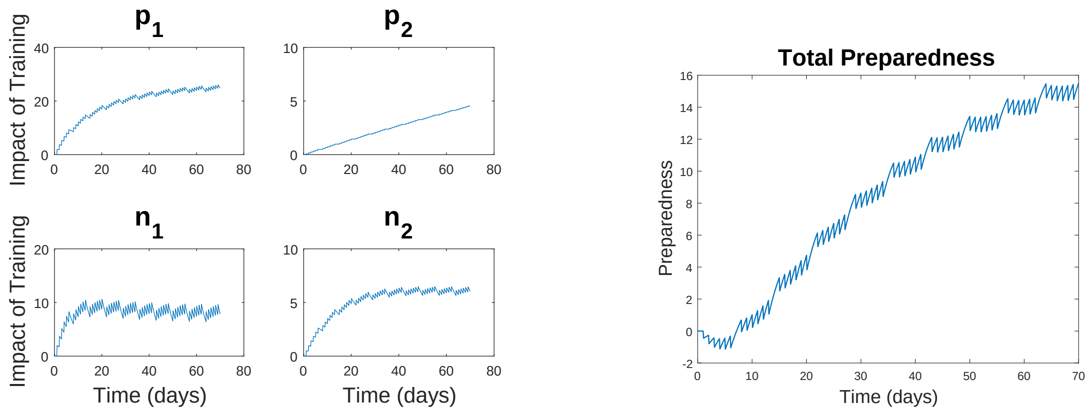
In general, all four components will increase during a training session. Fitness (p_1) increases at a rate dependent on workout intensity and the current fatigue level. Generally a more fatigued athlete will derive less fitness benefit than a well rested one. The constant κ described the extent to which fatigue and fitness effect one another directly. There is some uncertainty in determining a realistic size for κ. We initially take $\kappa = 0.02$ and investigate the model's sensitivity to changes in this parameter as part of a later section. Skill (p_2) will increase at a rate proportional to the difference of the current skill level and a 'mastery' level with time constant τ_2. For the remainder of this paper, we take $\tau_2 = 24$ hours and $M = 25$. These parameter choices result in significant skill development over the course of a training cycle while assuring that overall fitness is still the dominant contributor to overall preparedness for competition. Each fatigue component increases at a rate propotional to the workout intensity. Fatigue due to use of fuel substrate builds quickly whereas fatigue due to repetitive microtrauma builds more slowly, so the constant of propotionality $K < 1$. For the remainder of this paper, we take $K = \frac{1}{4}$.

Outside of training sessions, fitness and fatigue will dissipate. Time constant τ_1 describes the rate at which fitness gains will be lost. We take this to be $\tau_1 = 3$ weeks, which provides a realistic timeline for loss of fitness if training is stopped. The body will generally replenish fuel on a faster time scale[3, 4], so we take $\tau_3 = 1$ week. Because fitness loss and recovery from microtrauma both involve physiological change and adaptation, we model dissipation of mictrotraumatic fatigue using the same time constant τ_1. Again, there is some crossover effect as a fitter athlete will generally be quicker to adapt and recover from a training session. The dissipation of both fatigue components occurs at a faster rate for an athlete with greater overall fitness. Finally, it is assumed that skills gained during a training session are persistent, so p_2 does not decay between or after training sessions.

The model described is used to study the impact of training for an intermediate athlete during a ten week training cycle. This is a duration consistent with the competitive season of many high school and club level sports teams. In order to focus on the impact of a single cycle or season of training, we take the initial values of all four components (p_1, p_2, n_1, n_2) to be zero. The author is particularly interested in investigating preparedness for a major event at the end of the ten week training cycle. It is common practice to taper workout intensity as competitions approach[1, 7], so attention is focused on changes to workout loading during the final weeks of the training cycle.

3 Analysis

Solution curves were created for this model using Matlab's ode45 function.



(a) (b)

Figure 1: Modeled behavior with standard workout load.

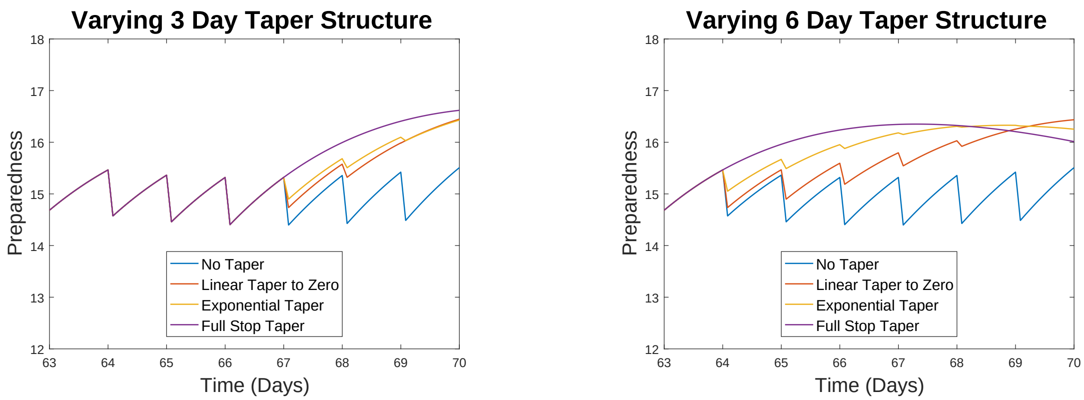
In Figure 1(a), we see the general behavior of each component of the training impact for an athlete that trains consistently throughout the ten week season. Figure 1(b) shows the total preparedness, obtained by summing the positive and negative impacts of the training cycle. The jagged nature of the solution curves reflects the non-smooth training parameter $w(t)$. Over the course of the season, total preparedness for competition increases, but seems to do so at a decreasing rate. It is interesting to note that there is a short period of time near the start of the training cycle where the overall impact of training is negative[1].

While preparedness tends to increase over time with a standard workout load, many coaches and athletes employ a tapering of workout intensity in order to allow accumulated fatigue to dissipate during the days before a major competition. Research by Bosquet et al.[2] suggests a progressive decrease of workout load over a one to two week period before competition in order to provide the largest benefit to preparedness. By varying workout load, $w(t)$, during the final weeks of the training cycle, we are able to study the impact of tapers with different duration and structure. A workout session during which $0 \leq w < 1$ reflects a reduced training intensity.

We consider three distinct taper structures. A linear taper decreases workout intensity at a constant rate over the duration of the taper, with the final day reaching intensity $w(t) = 0$ (a day off). An exponential taper decreases workout intensity by a constant percentage each day throughout the taper. For our example we decrease intensity by 50% each day. Lastly, we consider a full stop taper, where workouts are stopped entirely during the taper period. Figure 2, on the following page, shows the impact of these taper structures over a 3-day and a 6-day taper period.

In the case of each taper structure, 3 day and 6 day tapers result in an overall increase in preparedness for competition on the last day of the training cycle. As shown in Figure 2(b), for a 6 day taper the progressive structures (linear and exponential) produce better end results than the full stop taper, which is consistent with suggestions by Bompa and Nolte[1, 7]. On the other hand, Figure 2(a) shows that a full stop taper will actually produce a better end result than either progressive structure for a 3 day taper. Note that a 6 day taper is effectively the same as a one week taper, as athletes are not assigned a workout on the seventh weekday.

[1]Negative training impact on this interval may correspond to muscle soreness and energy demands of starting a new training program.



(a) (b)

Figure 2: Comparison of linear, exponential, and full stop tapers.

The results shown in Figure 2 suggest that a coach or athlete seeking the ideal taper duration must also account for the structure of the taper. Figure 3, at right, shows the overall preparedness of an athlete at the end of the training cycle following tapers of varying length and structure. The behavior in the previous figure is reflected at the 3 and 6 day time points. It is clear that the highest preparedness is obtained after 3 days of a full stop taper. This duration is shorter than those suggested in the literature[2, 1, 7], but it is worth noting that the best possible duration for a progressive taper appears to be longer than the best possible full stop taper. In particular, note that a 6 day linear taper has an overall impact on preparedness that is not far from the best possible impact for that taper structure. Such a taper is in line with the suggestions of Bosquet et al[2].

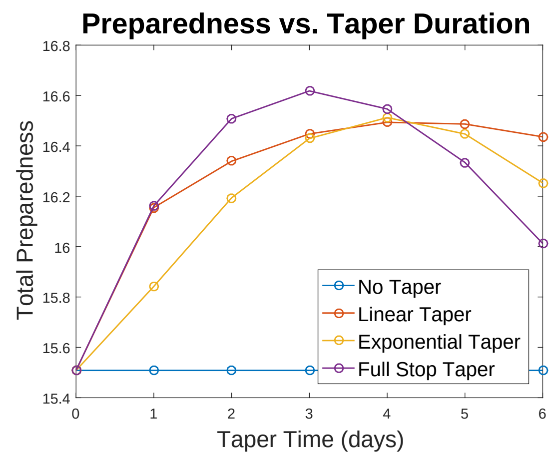


Figure 3: Maximum preparedness is achieved with a three day full stop taper.

The model suggests that a 3 day full stop taper is ideal for maximizing preparedness at the end of the training cycle. Though this model considers aggregated training impacts, the effectiveness of the full stop taper can be best explained by considering the impact of an individual workout as time progresses. In Figure 4, at left, we see that the immediate impact of a workout is negative (dominated by fatigue) but it becomes positive at some time afterward as fitness and skill benefits outlast fatigue generated during the session.

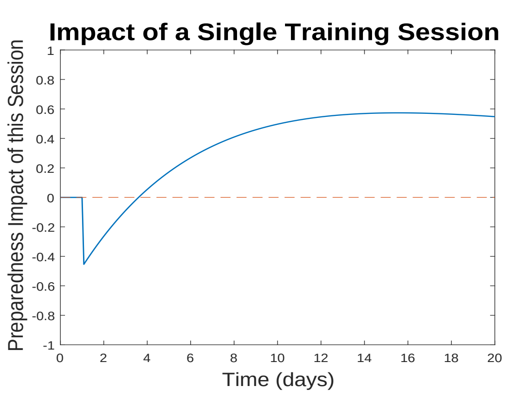


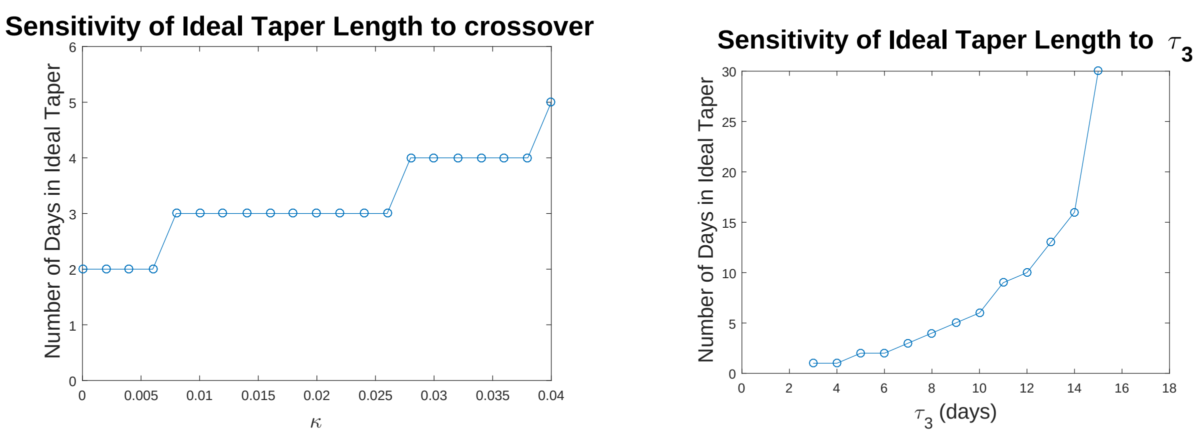
Figure 4: The impact of a single training session. Here a workout is performed on day 1 and its impact tracked over the following days.

There is a critical point where the individual impact curve becomes positive, and this point occurs roughly 3 days after the training session. Athletes that use a 3 day full stop taper will not train at all within 3 days of their major event, thereby eliminating the negative impacts of those training sessions. On the other hand, they will continue to train with standard intensity right up until this point, thereby maximizing the positive benefits from any earlier sessions. Each of the progressive taper structures requires training sessions within three days of the major event, thereby exposing the athlete to negative training impact in the days right before the event. In addition, progressive tapers of more than 3 days will assign less than standard training load for workouts that have a net positive impact on the day of the event, so athletes following these plans will also miss out on some positive benefit. Thus, the superiority of the full stop taper seems reasonable.

3.1 Sensitivity Analysis

We now consider the sensitivity of this result to changes in two parameters. First, we consider sensitivity to the constant κ, which reflects the extent to which fitness and fatigue impact changes in each other. Next, we consider sensitivity to the time constant τ_3, which reflects the athlete's ability to replenish fuel substrate that has been used during a workout. While this parameter may differ naturally between individuals, it is also one that can be directly effected by the actions of each athlete. By incorporating a well planned diet into the training plan, an athlete may be able to speed up the process of recovery with respect to fuel based fatigue. On the other hand, an athlete who eats poorly may take longer to replenish certain key fuel substrates.

In figure 5(a), below, we see that the ideal taper duration increases with increased κ. It is uncertain how much crossover is typical between fitness and fatigue for athletes; while the constant should certainly be positive, it is unknown how large it should be in order to reflect actual training. As our model is formulated, large κ values have the potential to cause instability in the model since for large enough κ, $\frac{dp_1}{dt} < 0$ even during a training session. We thus restrict the parameter to small values, $0 \leq \kappa \leq 0.04$. Though the constant is small, its impact within the model is significant. An athlete with accumulated fatigue of $n_1 + n_2 = 10$ will gain fitness 40% slower during training when $\kappa = 0.04$. While there is an increase in ideal taper duration as κ increases, that increase is slow on this interval, and the figure shows that the ideal duration of 3 days is consistent in the neighborhood of our earlier assignment of $\kappa = 0.02$.



(a) (b)

Figure 5: Sensitivity of Ideal Taper Duration to changes in κ and τ_3.

In figure 5(b), above, we see that the ideal taper duration increases with increasing τ_3. This reflects the fact that an athlete that is slower to recover from fatigue will require a longer time to pass before the positive impacts of training overcome the negative impacts. If τ_3 is very long, the ideal taper length increases dramatically. In the extreme case, where $\tau_3 \approx \tau_1$, the negative impacts of training will always be greater than the positive impacts. The research of Busso et al.[3] suggests that this parameter typically falls within the range of 3 to 15 days, and that it may vary over time even for a given individual.

It should not be forgotten that the effectiveness of the taper hinges mainly on the dissipation of fatigue, and that with a full stop structure, there is no fitness gained after the start of the taper. Bosquet et al.[2] points out that lost fitness will be more apparent after a longer taper. Therefore, while Figure 5(b) suggests that there is an ideal taper length for each value of τ_3, it is worth considering the overall preparedness of athletes at the end of the training cycle as a function of τ_3.

Figure 6, at right, shows the total preparedness after full stop tapers of various duration given three different τ_3 values. The figure shows that an athlete with $\tau_3 = 5$ days will achieve a higher level of preparedness after tapering than an athlete with $\tau_3 = 7$, and an athlete with $\tau_3 = 9$ will achieve a still lower level of preparedness. This emphasizes the importance of diet considerations for an athlete. If an athlete can make conscious decisions to aide their rate of fuel recovery, the overall impact of their training plan will leave them more ready to compete.

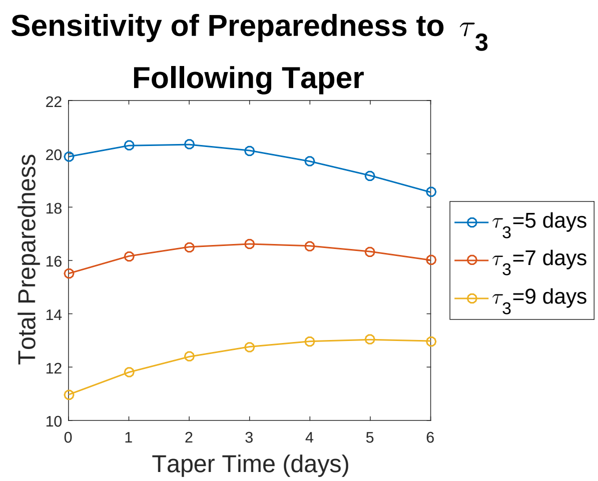


Figure 6: Faster recovery from fatigue leads to overall higher preparedness.

3.2 Tapering for Multiple Events

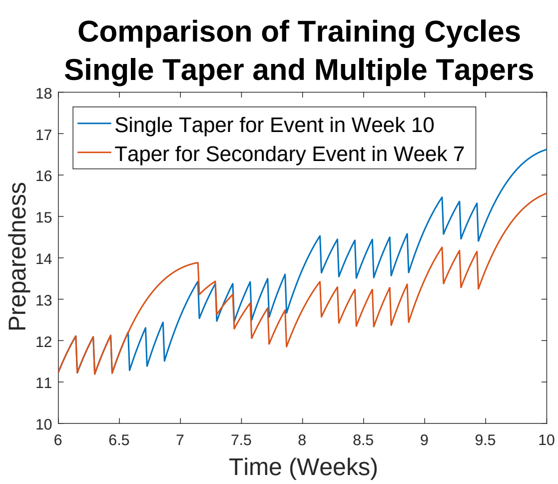


Figure 7: Tapering for an event earlier in the training cycle leads to decreased readiness at the end of the cycle.

Earlier analysis was restricted to preparedness response generated by a taper implemented at the end of a ten week training cycle. In many competitive sports, athletes will train for and compete in more than one event. While the end of the training cycle tends to coincide with the primary event of the season (a championship, rivalry game, etc.), there may be other important events that occur earlier in the cycle (qualifiers or exhibitions).

Figure 7, at left, shows the overall preparedness curve resulting from tapering for secondary event in week 7 before tapering again for the primary event at the end of week 10. While the tapers still produce the characteristic improvement in preparedness before each of these events, there is a distinct difference in the final preparedness of an athlete that tapers for two events and that of an athlete that tapers only once. This can again be explained by the underlying concept that the taper's impact is primarily based on dissipation of fatigue. While a taper in week 7 will increase an athlete's preparedness in the short term, fitness will not increase during that interval as it would otherwisse, and as a result they will not be in the same shape heading into the taper in week 10.

4 Discussion

Analysis of this model demonstrates that gains in overall athletic preparedness should be expected after a taper period at the end of a 10 week training cycle. These gains are primarily driven by the rapid decrease in fatigue compared to the relatively slow decrease in fitness as a result of reduced training. In a broad sense, the model agrees with the literature[2, 1, 7] that tapering is an effective method for increasing preparedness for a major event. The ideal structure and duration that the model suggests, however, is distinctly different from those suggested in the literature. While Bosquet et al. describe a progressive taper of one to two weeks, we find here that this model suggests a full stop taper of only 3 days. This difference suggests the need for further study of model performance and parameters.

It is possible that there are reasons for maintaining some level of training throughout the taper period that are not handled by the model. In particular, consideration of psychological or tactical impacts of training might change the behavior of the model under taper conditions. Extending the model to reflect these impacts could bring the model's predictions closer in line with descriptions in the literature. It is also possible that duration of the training cycle will have an impact. Studies cited often track preparedness of elite athletes with longer training cycles, and it may be true that the best possible taper structure will be different for less experienced athletes on shorter training plans. Full stop tapers are not often seen in common practice, but perhaps there is some daring coach or athlete that will incorporate this structure into a more thorough study of athletic performance.

Our analysis also showed that the rate of fuel replenishment will have an effect on overall performance as well as an effect on the duration of the ideal taper. It should be expected that a faster recovering athlete will, in general, be more ready to perform, but our analysis strongly suggests that a well fueled athlete will be ready to perform at a higher peak following training.

Lastly, the suggestion that multiple tapers during a season will impact preparedness for the final major event is an important result. It is common for high school and club level teams to have many competitions throughout a single season, and this result implies that an athlete simply cannot taper to achieve maximum preparedness at every single event. Coaches and athletes should be careful in choosing which events are major enough to require a taper, and they should try to minimize the number of secondary events in their schedule if they wish to maximize preparedness for a particular event.

5 Conclusion

Athletic performance is a topic of interest to many. From youth level sports through the elite levels, coaches and athletes make frequent decisions about which training plan is best for them. When designing a training plan, a coach will consider several goals. If performance at a particular event is among these goals, then the training plan should be built around that event and this model suggests a structure for that. This model suggests that a three day full stop taper in the days before a major event is best for athletes training in a 10 week cycle. The model further suggests that tapers for secondary events should be minimized, and thatan athlete should take care during the season to maximize the rate of fuel recovery after training sessions. Empirical studies of recovery and crossover parameters as well as sensitivity analysis with respect to training cycle length remain as possible directions for extension of the model.

References

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