CS 113
DISCRETE STRUCTURES

Chapter 2: The Language of Mathematics

HOMEWORK

- Section 2.1: 1-10, 25-29, 31, 33, 35, 78, 82
- Section 2.2: 4, 6, 8, 10, 43, 45, 74, 82, 88-92, 96-100
- Section 2.3: 1-31 (odd), 35-44
- Section 2.4: 19-24, 29-34
- Section 2.5: 9-14, 30-31
- Section 2.6: 1-6, 8-17, 19, 23, 24
- Section 2.8: 10-15, 19-23, 26-27, 36-37, 56, 78-84

SETS

- A set is a collection of objects
- We use { and } to denote sets
- The order of the objects doesn't matter
- So {1, 2, 3} and {2, 1, 3} are the same set
- Also, we usually don't allow the same thing to be in a set more than once
- So the set {1, 2, 1, 3, 1} doesn't usually make sense
  - We write {1, 2, 3} instead

THE EMPTY SET

- **The empty set is a set with nothing in it**
- **It is also called the null set**
- **We can write it as {}**
- **We can also write it as ϕ**
- **We CANNOT write it as {ϕ}**
  - **Why not?**

BELONGING

- **A common thing to do is to see if something is in a set or not**
- **For example, 1 is in the set {1, 2, 3}, but 6 is not**
- **We write $1 \in \{1, 2, 3\}$**
  - **We can read that as**
    - **"1 is a member of the set {1, 2, 3}", or**
    - **"1 is an element of the set {1, 2, 3}"**
- **We can also write $6 \notin \{1, 2, 3\}$**

COMPARING SETS

- **There are several ways to compare sets**
- **One way is to check if one set is contained in another**
- **The set {1, 2} is contained in {1, 2, 3}**
- **The set {1, 4} is not contained in the set {1, 2, 3}**
- **We write {1, 2}$\subseteq$ {1, 2, 3}**
  - **Question: True or False. {2, 1}$\subseteq$ {1, 2, 3}?**
  - **Question: True or False. {1}$\subseteq$ {1, 2, 3}?**
  - **Question: True or False. {1, 6}$\subseteq$ {1, 2, 3}?**
- **We read $\subseteq$ as "is a subset of"**
  - **It almost looks like $\leq$**

COMPARING SETS- VERSION 2

- To show that a set A $\subseteq$ B, we have to show that
  - Every element of A is also an element of B
- Sometimes this is obvious:
  - $\{1, 2, 3\} \subseteq \{1, 4, 2, 3, 6\}$
- Is {all chairs} $\subseteq$ {furniture}?
- Is {square roots of positive numbers} $\subseteq$ {real numbers}?

MORE COMPARISONS

- **We can check if two sets are equal**
- **If the sets are small, we can tell by looking**
- **If the sets are large, we need a better method**
- **The correct way to determine that two sets are equal is to verify that each is a subset of the other**
- **We write A = B if A$\subseteq$ B and B$\subseteq$ A**
- **Notice that every set is a subset of itself**
- **If a set A is a subset of a set B, but A is not equal to B, we say A is a proper subset of B**

THE EMPTY SET IS UNUSUAL

- **Fact: The empty set is a subset of EVERY set, including even itself!**
  - **This means, among other things, { } $\subseteq$ { }**
- **In symbols, { } $\subseteq$ A for any set A**
- **Question: True or false. { } $\in$ A for any set A?**

THE **POWER** SET

- The power set of a set is the set of all subsets of the set
- For example, if A is the set {1, 2, 3} then the power set of A is
  - { { }, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3} }
- Question: What is the power set of {a, b}?
- We usually write P(A) to mean the power set of A

THE CARDINALITY OF A SET

- **The cardinality of a set is the number of elements in the set**
  - **This only makes sense if you can count the number of elements**
  - **That means the number of elements is finite**
    - **We could try to make sense of this if the set has an infinite number of elements, but to do that, we would have to discuss what infinity means**
- **We write it like absolute value**
- **So, if A = {2, 4, 5, 7}, then |A|= 4**
- **An interesting idea**
  - **If B is a (finite, in our case) set, then |P(A)| = $2^{|A|}$**

THE WORD “INFINITE”

- **Many people use infinite to mean large**
- **Infinite means that no number can describe the quantity**
  - **It is larger than any number**
- **A billion is NOT infinite**
- **The number of grains of sand on the earth is NOT infinite**
- **The word “finite” then means not infinite**
- **If a quantity is finite, you can count it**
  - **0 is finite too**

COMBINING SETS

- **One way to combine sets is to form the union**
- **The union of two sets is the set of things contained in both sets**
- **For example**
  - **Suppose A = {1, 2, 6}**
  - **Suppose B = {1, 4, 7}**
  - **Then the union is {1, 2, 4, 6, 7}**
  - **We write that as** $A \cup B = \{1,2,4,6,7\}$

COMBINING SETS, PART 2

- **Another way to combine sets is to form the intersection**
- **The intersection of two sets is the set of things that are common to both sets**
- **For example**
  - **Suppose A = {1, 2, 6}**
  - **Suppose B = {1, 4, 7}**
  - **Then the intersection is {1}**
  - **We write that as** $A \cap B = \{1\}$

Vocabulary

- **If two sets have nothing in common, we say they are disjoint**
  - **This means** $A \cap B = \{\,\}$
- **If you have many sets, and each pair has nothing in common, we say the sets are pairwise disjoint**

COMBINING SETS, PART 3

- **There is still another way to combine sets**
- **It is the set difference**
- **It is the set of all things that are in the first but not in the second**
- **An example**
  - **Suppose A = {1, 2, 5, 7, 8}**
  - **Suppose B = {1, 2, 3}**
  - **Then** $A - B = \{5,7,8\}$
- **Set difference is written with a minus sign**
- **A formula:** $A - B = \{a \in A \mid a \notin B\}$

DRAWING PICTURES OF SETS AND THEIR RELATIONSHIPS

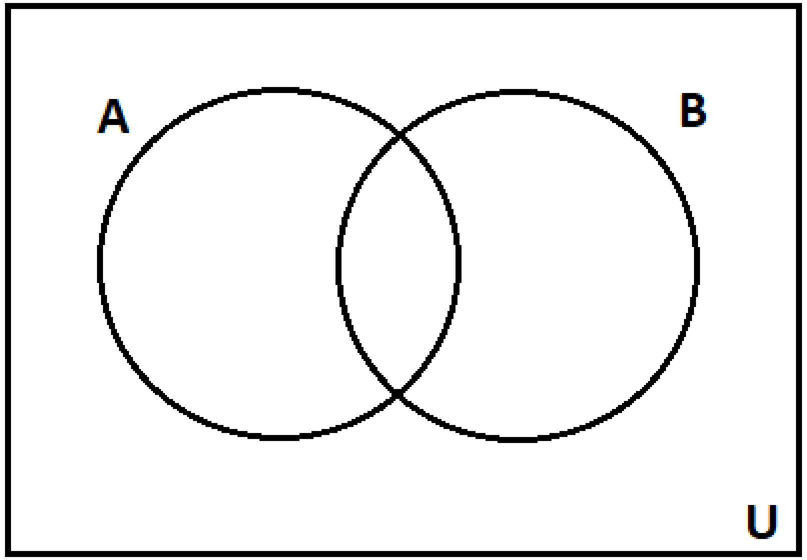
- **To do this, we must decide on a universal set**
- **First note, that there is no such thing as "the biggest set"**
  - **Bertrand Russell discovered some weird things if we move in that direction**
  - **For example, should we allow a set to belong to itself?**
    - **This would certainly be bigger than the set itself**
    - **Following this path leads to all sorts of confusion**
- **So, a universal set is somewhat artificial**
- **We choose it ourselves**
- **We don't always need a universal set; only in some settings**

GETTING READY TO DRAW THOSE PICTURES

- **So, we first need to choose a universal set**
  - **Let's call it U**
- **This will be our "biggest set"**
- **Then we get a new idea: the complement of a set**
  - **It is the set of all things in U that are not in X**
- **An example**
  - **Suppose U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}**
  - **Suppose A = {1, 2, 6, 7, 8}**
  - **Then the complement of A, written $\overline{\mathrm{A}}$, is {3, 4, 5, 9 10}**

ACTUALLY DRAWING THOSE PICTURES

- **This picture is called a Venn Diagram**
- **We start with something like this**



AN EXAMPLE

- Let's try an example
- Can we draw the picture for this situation?
  - U = {1, 2, 3, 4, 5, 6, 7,8, 9}
  - A = {1, 2, 3, 5, 7, 9}
  - B = {1, 3, 6, 8}
- We can use these pictures to solve problems
- For example, p. 62, #30

ANOTHER EXAMPLE

- Let’s try another example
- Students can take math and/or history and/or chemistry
- 15 students are taking all three
- 25 students are taking math and history
- 25 students are taking math and chemistry
- 35 students are taking history and chemistry
- 75 students are taking math
- 85 students are taking history
- 100 students are taking chemistry

SET LAWS

- There are some laws that pertain to sets
- They are listed on p. 59

PARTITIONS

- Sometimes we need to break a set into parts
- Also, we require that the parts don't overlap
- And, we also require that if you put the parts back together, you didn't leave anything out
- We call this a partition
- For example
- The sets {1, 3, 5} and {2, 4, 6} ARE a partition of {1, 2, 3, 4, 5, 6}
- The sets {1, 3} and {2, 4, 6} are NOT a partition of {1, 2, 3, 4, 5, 6}
- The sets {1, 3, 5}, {1, 2}, and {2, 4, 6} are NOT a partition of {1, 2, 3, 4, 5, 6}

EXTRA SYMBOLS

- There are some symbols to represent unions and intersections of many sets
- For example, to denote the union of five sets, A_1, A_2, A_3, A_4, and A_5, we can write

$$\bigcup_{i=1}^{5} A_i$$

- We can do something similar for the intersection

COMBINING SETS, PART 4

- There is still another way to combine sets
- It is called the Cartesian product
- If A and B are two sets, the Cartesian product of A and B
  - Is the set of all ordered pairs (x, y) with x in A, y in B
  - Is written A x B
  - Is read as “A cross B”
- Notice that order matters
  - A x B is not the same as B x A
- If we have three sets, then A x B x C is { (x, y, z) with x in A, y in B, z in C}
- We call (x, y, z) and ordered triple
- In general, we can have an ordered n-tuple

SOME SET IDEAS FORMALLY

- **Here are some formal ideas**
- $x \in A \cap B \to x \in A \text{ and } x \in B$
- $x \in A \cup B \to x \in A \text{ or } x \in B$
- $A = B \to A \subseteq B \text{ and } B \subseteq A$
- $x \in \overline{A} \to x \notin A$
- $A \subseteq B \to (x \in A \to x \in B)$
- **Actually, the arrows should point both ways**
  - **These are really definitions**

QUESTIONS

- **Any questions?**