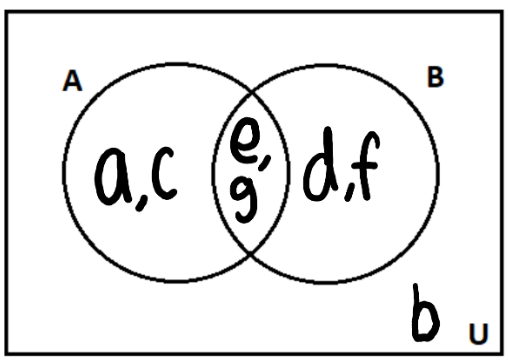
Period 1
3/2/2021
Chapter 2

Chapter 2 Review of some ideas

1. Prove that if
   A = {6m+12 | m is an integer}
   B = {3n | n is an integer}
   then $A \subseteq B$.
   - To show that a is a subset of b, you need to show that x is an element of A, then show that x is an element of B.
   - Let x be an element of A. Then let x = 6m+12 for some integer m.
   - Then let x = 3(2m+4).
   - 2m+4 is an integer, so it is essentially just x = 3n, for n is an integer.
   - Pick something in A (format of 6m+12).
   - So, x is an element of B (Now, we know that A is a subset of B because B contains all integers, and A contains these integers × 2, which are a subset of the integers in B.)

2.
   a.) Suppose U = {a, b, c, d, e, f, g}, and A = {a, c, e, g}, and B = {d, e, f, g}. Draw a Venn Diagram to illustrate this.



   b.) Are A and B disjoint?
   They are not disjoint, because they share elements e and g.

3. Suppose $t_n = 6n - 1$, for n = 1, 2, 3, …
   a.) Write t_1, t_2, t_3.
   $t_1 = 5$, $t_2 = 11$, $t_3 = 17$
   b.) What is
   $$\sum_{n=1}^{4} 2t_n$$
   10 + 22 + 34 + 46 = 112

4. Suppose $m_1 = 3$ and $m_n = m_{n-1} + 5$, n = 2, 3, 4, …
   a.) Write m_2, m_3, m_4.
   $m_2 = 8$, $m_3 = 13$, $m_4 = 18$
   b.) What is

$$5+\sum_{n=1}^{4} m_n$$

5 + 3 + 8 + 13 + 18 = 47

5. For the relation on the integers xRy (x in relation to y) if 2 ÷ (y-x)
   a.) Determine if the relation is reflexive.
       The relation is reflexive.
   b.) Determine if the relation is symmetric.
       The relation is symmetric.
   c.) Determine if the relation is transitive.
       The relation is transitive.
   d.) Determine if the relation is antisymmetric.
       The relation is not antisymmetric.
   e.) Is the relation an equivalence relation? If so, write the equivalence classes.
       Yes, the relation is an equivalence relation because it is reflexive, symmetric, and transitive.
       Equivalence classes:
       [1] = {x ϵ X | 2 divides into 1-x} = {1, 3, 5, 7, 9, …}
       [2] = {x ϵ X | 2 divides into 2-x} = {2, 4, 6, 8, 10, …}
       [3] = [1]
       [4] = [2]
       …

6. Repeat Problem 5 if the relation (again on the integers) is xRy iff (x in relation to y if and only if) x = y.
   a.) Determine if the relation is reflexive.
       The relation is reflexive.
   b.) Determine if the relation is symmetric.
       The relation is symmetric.
   c.) Determine if the relation is transitive.
       The relation is transitive.
   d.) Determine if the relation is antisymmetric.
       The relation is antisymmetric.
   e.) Is the relation an equivalence relation? If so, write the equivalence classes.
       Yes, the relation is an equivalence relation because it is reflexive, symmetric, and transitive.
       Equivalence classes:
       [1] = {1}
       [2] = {2}
       [3] = {3}
       …
       (There are an infinite number of equivalence classes.)

7. Is the relation {(2, 2), (2, 7), (4, 4), (4, 5), (5, 4), (5, 5), (7, 2), (7, 7)} on {2, 4, 5, 7} reflexive? Is it symmetric? Is it transitive? Is it antisymmetric?

Reflexive: Yes. {(2,2), (4, 4), (5, 5), (7, 7)}

Symmetric: Yes. {(2, 7), (7, 2)}, {(4, 5), (5, 4)}

Transitive: Yes.

Antisymmetric: No.

Notes on (x, x) Pairs

- (x, x) pairs do not mean anything for symmetric and antisymmetric, meaning that they are irrelevant for testing for symmetry, because they are both symmetric and antisymmetric.
- (x, x) pairs are only useful for testing for reflexive.
- Relations can either be symmetric or antisymmetric, not both, disregarding (x, x) pairs.

8. Here is a relation on ordered pairs of integers. (a, b) R (c, d) if ad – bc = 0. Note: Elements of the relation are ordered pairs of ordered pairs! (For some contrast, if the relation is xRy if both x and y are odd OR both x and y are even, then elements of the relation are ordered pairs, even though x and y are both single integers.)
   a.) Write down five elements of the relation.
      {(0,0),(0,0)}, {(1,2),(3,6)}, {(2,4),(2,4)}, {(6,4),(3,2)}, {(2,6),(1,3)}
   b.) What do you need to test to tell if this relation is symmetric?
      You just need to prove that a/b = c/d in order to show that this is symmetric.

Question 8 Notes:

- Assume that (a, b) is in the relation, and then show your relation to (c, d).
- They are individual pairs, versus the individual integers we are used to, so it is trickier.
- If you have the pair (a, b) relating to the pair (c, d), then you can also get the pair (c, d) relating to the pair (a, b).
- We then have ad - bc = 0, as well as cb - da = 0, but remember to prove how you ended up here.
- According to that, cb = da, and you end up with a/b = c/d.
- You cannot have {(0,0),(0,0)}, because a/b = c/d, but this should have been specified in the question.

9. Consider the function f:Z to Z given by $f(x) = x^2 + 1$.
   a.) Is the function one-to-one?
      No.
   b.) Is the function onto?
      No.
   c.) Does the function have an inverse? If so, find it.
      No, if the function is not one-to-one and not onto, it does not have an inverse.

10. Do the sets {1, 2, 3, …} and {10, 11, 12, 13, ...} have the same cardinality?
Yes
f(x)=x+9
$f^{-1}(x) = x-9$

11. Do the sets {1, 3, 5, 7, …} and {3, 7, 11, 15, …} have the same cardinality?
Yes
f(x)=2x+1
$f^{-1}(x) = (x-1)/2$

12. Do the sets {1, 2, 3, 4, …} and {-1, -2, -3, -4, …} have the same cardinality?
Yes
f(x)=-x
$f^{-1}(x) = -x$

Notes on Cardinality:

- Because I am able to find the inverse, the function is one to one and onto, so therefore the two sets have the same cardinality.
- Find inverse of the test to prove one-to-one and onto relation.
- You use the range to check if a function is onto.