CS 113 DISCRETE STRUCTURES

Chapter 1: Logic and Proof

PROPOSITIONS

- A proposition is a statement that is either true or false
  - “I am wearing shoes” IS a proposition
  - “It’s raining” IS a proposition
  - “This sentence is false” is NOT a proposition
  - “Is it 9:00?” is NOT a proposition

- We will use p, q, r, s to represent propositions

COMBINING PROPOSITIONS

- **There are three main ways we can combine propositions**

- **Method 1: <u>Conjunction</u>**
  - **We normally call this AND**
  - **We write $p \wedge q$**
  - **$p \wedge q$ is true only if both p and q are true. Otherwise, it's false.**
  - **We could write out the four cases, but there's an easier way**

TRUTH TABLES

- **Truth tables are a simple way to express and/or determine the truth or falsity of a proposition**
- **We write all propositions across the top**
- **We fill in T (for true) and F (for false) under them**
- **We then build the rest of the table until we see the entire proposition**
- **The list of Ts and Fs at then end is called the truth value of the proposition**

EVEN MORE WAYS TO COMBINE PROPOSITIONS

- **We can negate a proposition**
- **This is its opposite and is called NOT**
- **Here is a truth table for NOT**

p	NOT p
T	T
F	T

MORE WAYS TO COMBINE PROPOSITIONS

- **We can form the disjunction of two propositions**
- **<u>Disjunction</u> means OR**
- **Here is a truth table for OR**

p	q	p OR q
T	T	T
T	F	T
F	T	T
F	F	F

MORE WAYS TO COMBINE PROPOSITIONS

- **We can form a conditional proposition**
- **A <u>conditional proposition</u> is what you mean when you say "if p then q"**
- **It is also read as "p implies q"**
- **Here is a truth table for that**
- **It is written as $p \rightarrow q$**

- **The table comes from the idea of lying:**
  - **$p \rightarrow q$ is true not based on either p or q taken separately, but on the truth value of the entire implication**

p	q	IF p THEN q
T	T	T
T	F	F
F	T	T
F	F	T

EXAMINING THE CONDITIONAL PROPOSITION

- To build the truth table for $p \to q$
- Let p stand for "You are good"
  - That is a proposition
- Let q stand for "You get a candy bar"
  - That is also a proposition
- Then $p \to q$ is "If you are good, then you will get a candy bar"
- To examine the truth value of $p \to q$, we check if $p \to q$ is a lie or not based on the truth value of p and q

A TRUTH TABLE FOR THE CONDITIONAL PROPOSITION-PART 1

- Someone says "If you are good, then you will get a candy bar"
- We want to build a truth table for this
- Suppose p and q are both true
  - This means you are good AND you get a candy bar
  - Is the speaker a liar? No
  - Therefore, if p is true and q is true, then $p \rightarrow q$ is true (It's not a lie.)
- Suppose p is true and q is false
  - This means you are good AND you don't get a candy bar
  - Is the speaker a liar? Definitely!
  - Therefore, if p is true and q is false, then $p \rightarrow q$ is false (It's a lie.)

A TRUTH TABLE FOR THE CONDITIONAL PROPOSITION-PART 2

- **Remember, someone said "If you are good, then you will get a candy bar"**
- **Suppose p is false and q is true**
  - **This means you are not good AND you get a candy bar**
  - **Is the speaker a liar? No**
  - **Therefore, if p is false and q is true, then p → q is true (It's not is a lie.)**
- **Suppose p and q are both false**
  - **This means you are not good AND you don't get a candy bar**
  - **Is the speaker a liar? No**
  - **Therefore, if p is false and q is false, then p → q is true (It's not a lie.)**

Logical Equivalence

- **When the truth value of two propositions is the same, we say the propositions are <u>logically equivalent</u>**
- **We use ≡ to denote logical equivalence**
- **Notice that his only looks at the end results, not the steps we take to get them**
  - **The steps may be different**
  - **One propositions may be much longer than the other**

Checking Logical Equivalence

- **Is** $p \to q$ **logically equivalent to** $\overline{p} \vee q$**?**

Creating New Propositions from Old Propositions

- **Negation, conjunction, and disjunction are three ways to create new propositions from old ones**
- **There are three ways to create a new proposition from a conditional proposition**
  - **The <u>converse</u>. This is $q \rightarrow p$**
  - **The <u>inverse</u>. This is $\overline{p} \rightarrow \overline{q}$**
  - **The <u>contrapositive</u>. This is $\overline{q} \rightarrow \overline{p}$**
- **The contrapositive is logically to the original proposition**
  - **Remember that this means they have the same truth value**
  - **Both are true or both are false**

The Biconditional

- **There is still another way to create a new proposition from a conditional**
- **This is the biconditional**
- **It is written p $\leftrightarrow$ q**
- **It is a combination of p $\rightarrow$ q and q $\rightarrow$ p**
- **The <u>biconditional</u> actually means both p $\rightarrow$ q and q $\rightarrow$ p**
- **We read it as p if and only if q**
  - **It is sometimes abbreviated as p iff q**
- **Let's create a truth table for the biconditional**

A Final Note about AND and OR

- Checking the truth table for OR shows that
  - p OR q
    - If p is true, p OR q = true
    - If p is false, p OR q = q
- Similarly, for AND
  - p AND q
    - If p is false, p AND q = false
    - If p is true, p AND q is q

Adding Variables to Propositions

- We will now add variables to propositions
- We are not really creating anything new
- This just allows us to put variables into propositions
  - For example, is “x is even” a proposition?
    - That, of course, depends on x
  - How about “2z is even”?
    - This is a proposition for all integers z
    - It’s always true
  - What if I allow non-integers for x and z?
- There are two many issues because the variables are unclear
- We have gained a lot of flexibility, though

Propositional Functions and the Domain of Discourse

- **A proposition with a variable is called a <u>propositional function</u>**
- **A propositional function needs to have restrictions on the variable**
- **For example, suppose x is an even number**
  - **Then the proposition "x is even" is true**
- **Now suppose x can be any integer**
  - **Then the proposition "x is even" is only true sometimes**
- **So, the list of possible choices is needs to be specified**
  - **The list of possible choices is called the <u>domain of discourse</u>**
- **The domain of discourse can be specified or implied**
  - **It it's implied, it's the largest possible set of x for which the propositional function makes sense**

Back to Propositional Functions

- **We would like to be able to easily say that something is true**
  - **for all choices of x vs.**
  - **for some choices of x**
- **We call "for all" and "for some" <u>quantifiers</u>**
- **The phrase "For all x" is written as** $\forall x$
  - **This is called the <u>universal quantifier</u>**
- **The phrase "For some x" is written as** $\exists x$
  - **This is called the <u>existential quantifier</u>**

The Truth Value of the Quantifiers

- Think of this as lying or telling the truth
- The phrase "For all x, P(x)" is true if it's true for all x
  - It's false if there is at least one x that makes it false
- The phrase "For some x, P(x)" is true if it's true for at least one x
  - It's false if it's false for all x

QUESTIONS

- **Any questions?**