CS113 – Review for Exam over Chapter 1

Studying for this exam: This exam is a little deeper than those we had in CS102 (C++). You should write examples of each term, not just those in the table directly below, but those listed everywhere in this review. Practice doing what each section tells you to know, actually writing out examples.

Vocabulary—Be able to define (explain) each term AND give an example

Propositions	Definitions	Theorem	Corollary	Instantiating a propositional function
Axioms	Undefined terms	Lemma		The Domain of Discourse

Truth tables
Know about truth tables and be able to create one, either to tell the truth value of a logical expression or to determine if two expressions are logically equivalent.
Conjunction (AND)
Negation (NOT)
Disjunction (OR)
The conditional proposition (IF—THEN)

Creating New Propositions from Old Propositions
Again, know about truth tables here and be able to create one, either to tell the truth value of a logical expression or to determine if two expressions are logically equivalent.
The converse (Be able to tell how this relates to the original, both in form and in truth value.)
The inverse (Same comment as above)
The contrapositive (Same comment as above)
The biconditional (Same comment as above)

Quantifiers
Know what they mean. Be able to instantiate them, determine the truth value of an instantiated proposition, and be able to negate a universally or existentially quantified statement.
The quantifiers we know are: For all x and For some x

General Methods of Proof
Be able to use the three methods (direct proof, indirect proof, proof by contrapositive) to prove something. Also, be able to tell if a proof is direct, indirect, or uses the contrapositive.

The formal methods of logical verification
Know these and be able to use them either to derive proofs or to verify that a proof is valid.

Modus ponens	Addition	Conjunction	Disjunctive syllogism
Modus tolens	Simplification	Hypothetical syllogism	Resolution

Mathematical induction
Be able to prove a statement using mathematical induction. The proof has to be clear, correctly indicating and displaying the base and inductive steps.

CS113 – Some Sample Questions

1. Explain AND give an example of each term.

Propositions	Definitions	Theorem	Corollary	Instantiating a propositional function
Axioms	Undefined terms	Lemma		The Domain of Discourse

2. Create a truth table for each propositional expression below.
   A.) $p \vee (q \wedge r)$
   B.) $\overline{p} \vee q \vee r$
   C.) $\overline{p} \vee p$ This is always true. It's sometimes called the law of the excluded middle.
   D.) $\overline{p} \wedge p$ This is never true. It's sometimes called a contradiction.

3. Verify that DeMorgan's Law is valid.
   $(p \vee q) \equiv \overline{p} \wedge \overline{q}$ Note: There should be a bar over the expression in parentheses

4. Is $\overline{p} \vee p \vee q$ logically equivalent to q?

5. If $p \rightarrow q$ is the original statement, is $p \wedge \overline{q}$ logically equivalent to the converse?

6. Show that $p \rightarrow q$ is logically equivalent to its contrapositive.

7. Show that OR distributes on the left over AND. That is,
$A \vee (D \wedge B) \equiv (A \vee D) \wedge (A \vee B)$

8. Show that $p \leftrightarrow q$ is logically equivalent to $(p \oplus q)$. Note: The second expression should be a bar over the expression in parentheses

9. The original statement is: If today is Wednesday, school will start late. Label each part of the question with "converse", "inverse", or "contrapositive".
______ A.) If today is not Wednesday, school will not start late.
______ B.) If school will start late, then today is Wednesday.
______ C.) If school will not start late, today is not Wednesday.

10. If "$\forall$ mammals x, (x is warm-blooded)" is true, and a cow is a mammal, then is it valid to infer that a cow is warm-blooded?

11. If "$\exists$ planet x, (x is bigger than Neptune)" is true, and earth is a planet, then is it valid to infer that earth is bigger than Neptune?

12. Write the negation of $\forall$ x, (x is even).

13. Here is a proof of: If n is an even integer greater than 2, n is not prime.
Classify the proof as a direct proof, an indirect proof, or a proof by contraposition.

Theorem: If n is an even integer greater than 2, n is not prime.
Proof:
Let n be an even integer greater than 2. Then n = 2k for some integer k. Therefore, 2 divides into n. Therefore, n is not prime.

14. Here is a proof of: $2+4+6+\cdots+2n=n(n+1)$. Fill in each blank with "Theorem", "Lemma" or "Corollary".

________________ $1+2+3+\cdots+n=\frac{n(n+1)}{2}$.

Proof: by induction:

Base step: n=1. Then the left side is 1. The right side is $\frac{1(1+1)}{2}=\frac{1(2)}{2}=\frac{2}{2}=1$. Since the left side equals the right side, the base case is proven.

Inductive step: Assume $1+2+3+\cdots+n=\frac{n(n+1)}{2}$.

We need to show that $1+2+3+\cdots+n+1=\frac{(n+1)(n+1+1)}{2}$. The fraction is $\frac{(n+1)(n+2)}{2}$

$1+2+3+\cdots+n+1$

$=1+2+3+\cdots++n+(n+1)$ because n is the integer immediately before n+1

$=\frac{n(n+1)}{2}+n+1$ by the inductive hypothesis

$=\frac{n^2+n}{2}+\frac{2(n+1)}{2}$ distributing in the fraction and getting a common denominator

$=\frac{n^2+n}{2}+\frac{2n+2}{2}$ by the inductive hypothesis

$=\frac{n^2+3n+2}{2}$ combining the fractions

$=\frac{n^2+3n+2}{2}$ combining the fractions

$=\frac{(n+1)(n+2)}{2}$ factoring the numerator. This is what we need to show.

________________ Proof that $2+4+6+\cdots+n=n(n+1)$

$2+4+6+\cdots+2n$

$=2(1+2+3+\cdots+n)$ factoring out 2

$=2\frac{n(n+1)}{2}$ by the statement above. (Is it a lemma, theorem, or corollary?)

$=n(n+1)$ This is what we need to show.

Note: The problem above also contains a proof by induction. This shows how you should write up a proof by induction.

15. General Methods of Proof
Be able to use the three methods (direct proof, indirect proof, proof by contrapositive) to prove something.

A.) Prove that an even number added to an even number gives an even sum.

B.) Prove that the product of two even numbers is even.

16. The formal methods of logical verification

Label each step in this proof with the correct formal method of logical verification.