Chapter 4 Written Homework

Section 4.1 (p. 170): 5, 17-20, 28-30, 34-37, 42-46, 60, 62

5. With 5 interior colors, six exterior colors, two types of seats, three types of engines, and three types of radios, there are 540 possibilities available to the consumer.

17. There are 50 routes from Oz to Fantasy Island passing through Mid Earth.

18. There are 2500 (50×50) round-trip routes from Oz to Fantasy Island back to Oz.

19. There are 1800 (10×5×4×9) round-trip routes from Oz to Fantasy Island back to Oz which do not reverse the original route from Oz to Fantasy Island.

20. With the requirement of three letters followed by two digits on a license plate, you can construct 1757600 (26×26×26×10×10) license plates if repetitions are allowed, and 1404000 (26×25×24×10×9) license plates if repetitions are not allowed.

28. There are 60 (6×5×4) selections which exclude Connie.

29. There are 24 (4×3×2) selections in which neither Ben nor Francisco is an officer.

30. There are 24 (3×2×4) selections in which both Ben and Francisco are officers.

34. With the letters ABCDE used to form strings of length 3, we can form 125 (5×5×5) strings if repetitions are allowed.

35. With the letters ABCDE used to form strings of length 3, we can form 60 (5×4×3) strings if repetitions are not allowed.

36. With the letters ABCDE used to form strings of length 3, we can form 25 (1×5×5) strings which begin with A if repetitions are allowed.

37. With the letters ABCDE used to form strings of length 3, we can form 12 (1×4×3) strings which begin with A if repetitions are not allowed.

42. In the set of integers from 5 to 200, inclusive, there are 196 numbers.

43. In the set of integers from 5 to 200, inclusive, there are 98 even numbers.

44. In the set of integers from 5 to 200, inclusive, there are 98 odd numbers.

45. In the set of integers from 5 to 200, inclusive, there are 40 (196/5 = 39.2, round to 40) numbers which are divisible by 5.

46. In the set of integers from 5 to 200, inclusive, there are 128 (200-72) numbers which are greater than 72.

60. If X is an n-element set and Y is an m-element set, there are m×n functions from X to Y.

62. There are 36 (2×3×3×2) terms in the expansion of (x+y)(a+b+c)(e+f+g)(h+i).

Section 4.2 (p. 182): 10-14, 25-29, 31-34, 58-62

10. By ordering the letters ABCDE, 6 (3!) strings can be formed containing the substring ACE.

11. By ordering the letters ABCDE, 36 (3!×3!) strings can be formed containing the letters ACE in any order.

12. By ordering the letters ABCDE, 6 (3!) strings can be formed containing the substrings DB and AE.

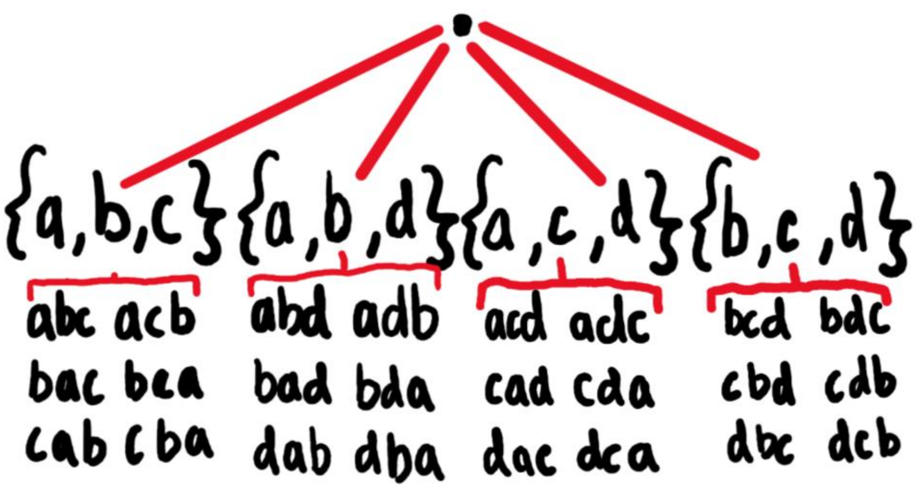
13. By ordering the letters ABCDE, 48 (2×4!) strings can be formed containing the letters ACE in any order.

14. By ordering the letters ABCDE, 60 (C(5,2)×3! = 5!/(3!×2!)*3!) strings can be formed where A appears before D.

25. With X = {a, b, c, d}, there are 4 (4!/(3!1!)) 3-combinations of X.

26. With X = {a, b, c, d}, the 3-combinations of X are abc, abd, bcd, cda.

27. Visual representation of the relationship between the 3-permutation and the 3 combinations of X, X = {a, b, c, d}



28. There are 165 (11!/(3!(8!))) ways to select a committee of three from a group of 11 people.

29. There are 495 (12!/(4!(8!))) ways to select a committee of four from a group of 12 people.

31. In a club of 6 distinct men and seven distinct women, there are 1287 ways we can select a committee of five persons.

32. In a club of 6 distinct men and seven distinct women, there are 700 (6!/(3!3!)×7!/(4!3!)) ways we can select a committee of three men and four women.

33. In a club of 6 distinct men and seven distinct women, there are 700 (13!/(4!9!)×6!/(4!2!)) ways we can select a committee of four persons that has at least one woman.

34. In a club of 6 distinct men and seven distinct women, there are 245 (6×7!/(3!4!)×7!/(4!3!)) ways we can select a committee of four persons that has at most one man.

58. When a coin is flipped 10 times, 1024 (2^{10}) outcomes are possible.

59. When a coin is flipped 10 times, 45 (10!/(2!8!))outcomes have exactly three heads.

60. When a coin is flipped 10 times, 176 (120+45+10+1, C(10,0)+C(10,1)+C(10,2)+C(10,3)) outcomes have at most three heads.

61. When a coin is flipped 10 times, 512 (2^9) outcomes have a head on the fifth toss.

62. When a coin is flipped 10 times, 252 (10!/(5!5!)) outcomes have as many heads as tails.

<u>Section 4.4 (p. 194): 11-17, 30-33</u>

11. The probability of getting a 5 on a fair die is 1/6.

12. The probability of getting an even number on a fair die is 3/6, or ½.

13. The probability of not getting a 5 on a fair die is 5/6.

14. The probability of randomly selecting the ace of spaces from a deck of cards is 1/52.

15. The probability of randomly selecting a jack from a deck of cards is 4/52, or 1/13.

16. The probability of randomly selecting a heart from a deck of cards is 13/52, or ¼.

17. The probability of rolling two fair die and have the numbers sum to 9 is _ (out of 6×6, 36 possible options, then you have (3,6), (4,5), (5,4), and (6,3), which gives you 4/36 or 1/9).

30. On a 10-question true-false quiz where an unprepared student guesses the answer to every question, the probability of the student answering every question correctly is 0.5^{10}.

31. On a 10-question true-false quiz where an unprepared student guesses the answer to every question, the probability of the student answering every question incorrectly is 0.5^{10}.

32. On a 10-question true-false quiz where an unprepared student guesses the answer to every question, the probability of the student answering exactly one question correctly is 0.5^{10}.

33. On a 10-question true-false quiz where an unprepared student guesses the answer to every question, the probability of the student answering exactly five questions correctly is $C(10,5)/2^{10}$.

<u>Section 4.5 (p. 204): 1-5, 22-26, 42-45</u>

1. The probability of getting a 5 with a weighted die is 1/8.

2. The probability of getting an even number with the weighted die is 3/8.

3. The probability of not getting a 5 with the weighted die is 7/8.

4. Even numbers each have a 1/12 probability, while odd numbers each have a 3/12, or 1.4 probability.

5. The probability of getting a 5 is 1/12.

22. The probability of all girls is 0.5^4, or 0.0625.

23. The probability of exactly two girls are 6/16, or 3/8.

24. The probability of at least one boy and at least one girl 14/16, or 7/8.

25. The probability of all girls given that there is at least one girl is 1/15.

42. P(A) = 0.55, P(D) = 0.1, P(N) = 0.35.

43. P(B|A) = 1/55, P(B|D) = 0.3, P(B|N) = 3/35.

44. P(A|B) = 1/7, P(D|B) = 3/7, and P(N|B) = 3/7.

45. P(B) = 0.07.

Section 4.6 (p. 210): 1-3, 7-9, 15-17, 22-24

1. The number of strings that can be formed by ordering GUIDE is 5!, or 120 strings.

2. The number of strings that can be formed by ordering SCHOOL is 6!/1!1!1!2!1!, or 360 strings.

3. The number of strings that can be formed by ordering SALESPERSONS is 12!/(4!1!1!2!1!1!1!1!), or 9979200 strings.

7. With selections among Action Comics, Superman, Captain Marvel, Archie, X-Man, and Nancy comics, there are 462 (C(6+6-1,6), 11!/(6!5!)) ways to select six comics.

8. With selections among Action Comics, Superman, Captain Marvel, Archie, X-Man, and Nancy comics, there are 5005 (C(10+6-1,10), 15!/(6!9!)) ways to select 10 comics.

9. With selections among Action Comics, Superman, Captain Marvel, Archie, X-Man, and Nancy comics, there are 126 (C(6+4-1,6), 9!/(5!4!)) ways to select 10 comics if we choose at least one of each book.

15. Referring to piles of identical red, blue, and green balls where each pile contains at least 10 balls, there are 66 (C(10+3-1,10), 12!/(10!2!)) ways that 10 balls can be selected.

16. Referring to piles of identical red, blue, and green balls where each pile contains at least 10 balls, there are 10 (C(9+2-1,9), 10!/(9!1!)) ways that 10 balls can be selected if at least one red ball must be selected.

17. Referring to piles of identical red, blue, and green balls where each pile contains at least 10 balls, there are 15 (C(4+3-1,4), 6!/(4!2!)) ways that 10 balls can be selected if at least one red ball, at least two blue balls, and at least three green balls must be selected.

22. The number of integer solutions of $x_1+x_2+x_3=15$ when $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$ is equal to C(17, 15).

23. The number of integer solutions of $x_1+x_2+x_3=15$ when $x_1 \geq 1$, $x_2 \geq 1$, $x_3 \geq 1$ is equal to C(14, 12).

24. The number of integer solutions of $x_1+x_2+x_3=15$ when $x_1 = 1$, $x_2 \geq 0$, $x_3 \geq 0$ is equal to C(15, 14).

Section 4.7 (p. 215): 1, 3-5, 10-11

1. Expanding $(x + y)^4$ using the Binomial Theorem would give $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$.

3. The coefficient of x^4y^7 when $(x + y)^{11}$ is expanded is 330.

4. The coefficient of s^6t^6 when $(2s-t)^{12}$ is expanded is 59136.

5. The coefficient of $x^2y^3z^5$ when $(x + y + z)^{10}$ is expanded is 45.

10. The number of terms in the expansion of $(x + y + z)^{10}$ is equal to C(10+3-1, 10).

11. The number of terms in the expansion of $(w + x + y + z)^{12}$ is equal to C(12+4-1, 12).

Section 4.8 (p. 219): 1-10

1. Thirteen persons have the first names Dennis, Evita, and Ferdinand and last names Oh, Pietro, Quine, and Rostenkowski. There are 13 possible names for the 12 people, through the Pigeonhole Principle, there is going to be one name assigned to at least 2 people.

2. Eighteen persons have the first names Alfie, Ben, and Cissi and last names Dumont and Elm. Because there are 6 possible names for 18 people, through the Pigeonhole Principle, there is going to be one name assigned to at least 3 people.

3. Professor Euclid is paid every other week on Friday. In some month, she is paid three times, because assuming the month begins on a Friday, Professor Euclid would be paid on the 1st, then the 15th, and then the 29th. It will not matter if the month is that of a February on a leap year, or a regular 30- or 31-day month – in order for this to make sense, the Friday would have to fall on the first day of the month.

4. It is possible to interconnect five processors so that exactly two processors are directly connected to an identical number of processors. You need to connect processors 1 and 2, 2 and 3, 2 and 4, 3 and 4. Processor 5 is not connected to any processors. Now only processors 3 and 4 are directly connected to the same number of processors.

5. With an inventory consisting of 115 items, each marked available or unavailable, and 60 available items, there are at least two available items in the list exactly four items apart. This is because if we number the list from 1 to 115, there are 119 possible numbers for the items in the sequence and 120 numbers in the two sequences. Because 119 is less than 120, that means there are at least two numbers in the two sequences that are the same number by the Pigeonhole Principle.

6. With an inventory of 100 items, each marked available or unavailable, and 55 available items, there are at least two available items in the list exactly nine items apart. This is because if we number the items in the list from 1 to 100 and have two sequences the first one incrementing through the list by 1 until 55, and the second one incrementing through the list by 9 until 55, we end up with 109 possible numbers in the sequence and 110 numbers in the two sequences.

Because 109 is less than 110, that means there are at least two numbers in the two sequences that are the same number by the Pigeonhole Principle.

7. With an inventory of 80 items, each marked available or unavailable, and 50 available items, there are at least two available items in the list either three or six items apart. This is because the 90 numbers range in value from 1 to 86, and by the second form of the Pigeonhole Principle, two of these numbers are the same. If the sequence is incremented by 3, two are three apart. If the sequence is incremented by 6, two are six apart. If the incremented by 3 and incremented by 6 sequences are equal, however, that means that the two are three apart.

8. If the pairs are dissimilar, there are three pictures that are mutually similar or mutually dissimilar. This is because each of them (i, j, k), differ in average brightness by more than c from P1. If those photos are similar, then these three pictures are mutually similar. If any pair in those photos is dissimilar, then these two pictures along with P1 are mutually dissimilar. If all three pairs are dissimilar, then these three pictures are mutually dissimilar. There are three pictures that are mutually similar or mutually dissimilar in each case.

9. Given less than 6 pictures, the conclusion from question 8 is not necessarily true, as we can pick two pictures which are close in their average brightness with the third picture having an average brightness that is more than the fixed value.

10. Given more than 6 pictures, the conclusion stays true, as we know that there are at least three mutually similar or mutually dissimilar pictures among the 6. This also implies that three mutually similar or mutually dissimilar pictures are also mutually dissimilar and similar among the n pictures, with n being larger than 6, and the example is always true when there are more than 6 pictures.