1. Write the characteristic polynomial for the recurrence relation.

$$a_n = a_{n-1} - 2a_{n-2} + 3a_{n-3}$$

$r^n = r^{n-1} - 2r^{n-2} + 3r^{n-3}$

- This is our assumption that linear homogeneous recurrence relation with constant coefficient (LHRRCC)
- We get to assume that the answer looks like $a_n = r^n$, $a_{n-1} = r^{n-1}$, $a_{n-2} = r^{n-2}$, and $a_{n-3} = r^{n-3}$

$r^n - r^{n-1} + 2r^{n-2} - 3r^{n-3} = r^{n-1} - 2r^{n-2} + 3r^{n-3} - r^{n-1} + 2r^{n-2} - 3r^{n-3}$

$r^n - r^{n-1} + 2r^{n-2} - 3r^{n-3} = 0$

- We then take out the smallest exponent, which is $3r^{n-3} = 0$

$r^{n-3} (r^3 - r^2 + 2r - 3) = 0$

- Ignore r^{n-3} because 0 will not work in this homogenous recurrence relation
- $(r^3 - r^2 + 2r - 3) = 0$ is what we are left with
- Cubic polynomial means that there will be three roots

Characteristic polynomial: $0 = (r^3 - r^2 + 2r - 3)$

2. Solve the recurrence relation.

$$a_n = -7a_{n-1} + 12a_{n-2}$$

Characteristic polynomial: $x^2 + 7x - 12 = 0$

- See how the factors of 12 in any combination, be it (1, 12), (2, 6), or (3, 4) cannot add to + 7
- We need to use the quadratic equation to solve

a = 1, b = 7, c = - 12

Plug into the quadratic formula: $x = \frac{-7 \pm \sqrt{7^2 - 4\times1\times-12}}{2\times1}$, $x = \frac{-7\pm\sqrt{49+48}}{2}$, $x = \frac{-7\pm\sqrt{97}}{2}$

We end up with $x = \frac{-7+\sqrt{97}}{2}$ and $x = \frac{-7-\sqrt{97}}{2}$

$a_n = A(\frac{-7+\sqrt{97}}{2})^n + B(\frac{-7-\sqrt{97}}{2})^n$