

CHAPTER 9

**BOOLEAN ALGEBRAS AND
COMBINATORIAL CIRCUITS**

LOGIC (COMBINATORIAL) CIRCUITS

- Now we will view logic from the hardware's viewpoint
- We will also look at other similar ways to design circuits
- Taking the hardware's viewpoint
- From now on, everything is 0 or 1
 - 0 represents false
 - 1 represents true

BASIC GATES



AND gate



OR gate



NOT gate (inverter)

LOGIC TABLES

- These gates work like our old logic
- A truth table will now be called a logic table
- We use 0, 1 instead of false, true
- For example, here is the logic table for **AND**

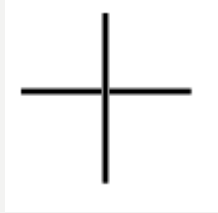
p	q	P AND q
0	0	0
0	1	0
1	0	0
1	1	1

COMBINATORIAL CIRCUITS

- The key kind of circuit we are concerned with is a combinatorial circuit
- This is a circuit in which an output is defined for each combination of inputs
- Combinatorial circuits can be built from **AND, OR, NOT**

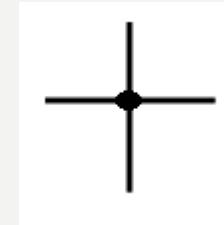
PICTURES WITH WIRES

- Here is a picture of two wires



- This is ambiguous; are these wires connected or is one just on top of the other?

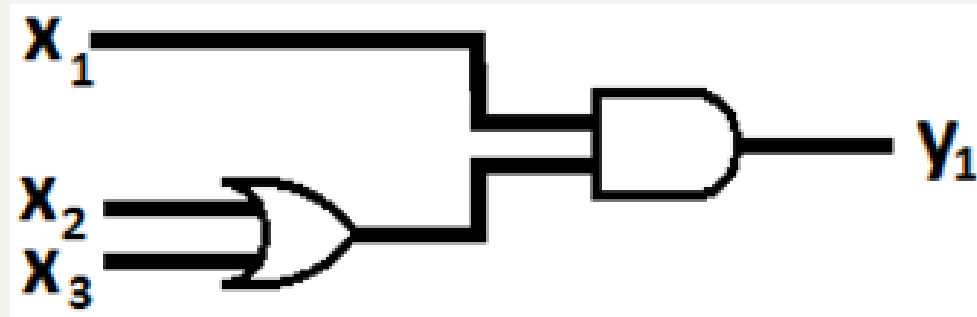
- If we mean that the wires are connected, we will write it as (with a dot)



- If we mean that the wires are just one on top of another, we will write it as



A PICTURE OF A CIRCUIT



LET'S CREATE A LOGIC TABLE FOR IT

- We create a table with columns for each input and each intermediate step
- We then fill it in
- For example, we have columns for X_1 , X_2 , X_3 , $X_2 \text{ OR } X_3$, $X_1 \text{ AND } (X_2 \text{ or } X_3)$
- Then we fill it in

THE LOGIC TABLE FOR THE CIRCUIT

X1	X2	X3	X2 OR X3	X1 AND (X2 OR x3)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

WRITING A BOOLEAN EXPRESSION

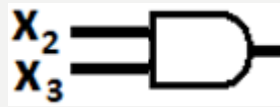
- We could write a Boolean expression for the circuit
- We work backward from the output
- $Y1 = \text{ ______ } \text{ AND } \text{ ______ }$
- $Y1 = X1 \text{ AND } \text{ ______ }$
- $Y1 = X1 \text{ AND } (X2 \text{ OR } X3)$

A BOOLEAN EXPRESSION

- **What is a Boolean expression?**
 - I used that term on the previous slide
 - I assumed we would guess at the meaning
- **Here is a formal definition**
 - 0 and 1 are Boolean expressions
 - X_1, X_2, \dots, X_n are Boolean expressions
 - If (BE) is a Boolean expression
 - NOT (BE) is a Boolean expression
 - BE1 AND BE2 is a Boolean expression
 - BE1 OR BE2 is a Boolean expression
- **This definition is recursive?**

CREATING A CIRCUIT FROM A BOOLEAN EXPRESSION

- To do this, you first work with what's inside parentheses, and then work outward
- Suppose the expression is $Y_1 = X_1 \text{ OR}(X_2 \text{ AND } X_3)$
- We draw the circuit
- Start with $X_2 \text{ AND } X_3$



- Then connect in X_1 or <that result>



SWITCHING CIRCUITS

- A switching circuit is a circuit made only of switches
- I will usually add a light bulb and a battery to show the point of the circuit
 - We do not draw these in an actual circuit
- In a switching circuit
 - You can use a letter more than once
 - But, if you use A in two different places, that means those two switches move together
 - And, if you use A and -A, that means those two switches move oppositely
- Do #25, p. 422

EQUIVALENT CIRCUITS

- Equivalent circuits are circuits where the same input values give the same outputs
- An example is $(X1 \text{ OR } X2) \text{ AND } X3$ and $(X1 \text{ AND } X2) \text{ OR } X3$
- Are these two circuits equivalent?

LAWS OF CIRCUITS- PART 1

- **Associative laws**
 - $(a \text{ OR } b) \text{ OR } c = a \text{ OR } (b \text{ OR } c)$
 - $(a \text{ AND } b) \text{ AND } c = a \text{ AND } (b \text{ AND } c)$
- **Commutative Laws**
 - $a \text{ OR } b = b \text{ OR } a$
 - $a \text{ AND } b = b \text{ AND } a$
- **Distributive Laws**
 - $a \text{ AND } (b \text{ OR } c) = (a \text{ AND } b) \text{ OR } (a \text{ AND } c)$
 - $a \text{ OR } (b \text{ AND } c) = (a \text{ OR } b) \text{ AND } (a \text{ OR } c)$

LAWS OF CIRCUITS- PART 2

- Identity Laws

- $x \text{ OR } 0 = x$
- $x \text{ AND } 1 = x$
- The identity laws are quite useful in assembly language programming

- Complement Laws

- $x \text{ OR } x' = 1$
- $x \text{ AND } (x') = 0$