Chapter 2 Written Homework

Section 2.1 (p. 62): 1-11 (odds only), 25, 26, 31, 78 or 82

1. A set is a collection of objects.

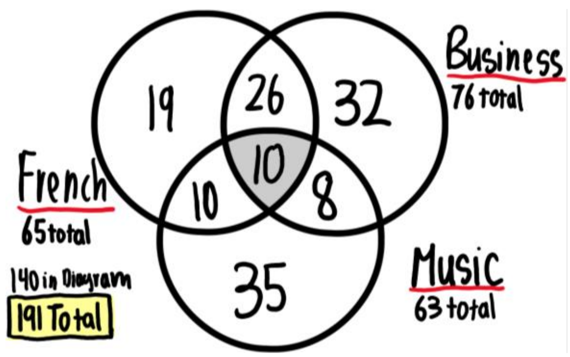
3. If X is a finite set, |X| is equal to the cardinality, or how many elements are in the finite set X.

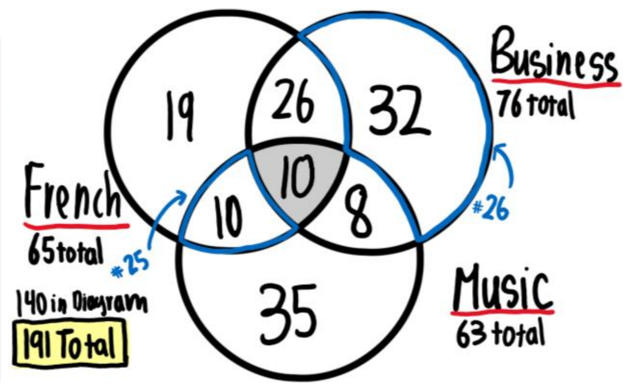
5. We denote that x is not an element of the set X by using $\notin$.

7. If X = Y, and X and Y are both sets, this means that whenever x is in the set X, then x is in the set Y, and whenever x is in the set Y, x is in the set X as well.

9. X is a proper subset of Y if X is a subset of Y and X does not equal Y.

11. If X has n elements, the power set of X (P(X)) has 2^n elements (as demonstrated by Theorem 2.1.4).





25. There are 10 students taking French and music but not business.

26. There are 32 students taking business and neither French nor music.

31. X = {1, 2} and Y = {a, b, c}; X × Y, the Cartesian Product of X and Y, is equal to {(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c)}.

82. P denotes a set of integers greater than 1, and for $i \geq 2$, $X_i = \{ik \mid k \geq 2, k \in P\}$ is defined as for $i \geq 2$, the set X_i equals the set of all ik such that k is greater than or equal to 2, and k is in the set P (a set of integers greater than 1). P - ∞⋃i=2 X_i means that the set P and the union of the sets X_i, from i equals 2 to ∞, are disjoint.

Section 2.2 (p. 69): 4, 9, 43, 45, 88, 89

4. For $t_n = 2n-1$, $n \geq 1$; $t_3 = 5$.

9. For $t_n = 2n-1$, $n \geq 1$; 7∑i=3 t_i = 5+7+9+11+13 = 45.

43. For the sequence x defined by $x_1=2$, $x_n = 3 + x_{n-1}$, $n \geq 2$; 3∑i=1 x_i = 2+5+7 = 14.

45. For the sequence x defined by $x_1=2$, $x_n = 3 + x_{n-1}$, $n \geq 2$; the formula for the sequence c defined by $c_n = n\sum i=1\ x_i = 2 + (n-1)3$.

88. Compute the given quantity using the strings α = baab, β = caaba, γ = bbab.

a. $\alpha\beta$ = baabcaaba
b. $\beta\alpha$ = caababaab
c. $\alpha\alpha$ = baabbaab
d. $\beta\beta$ = caabacaaba
e. $|\alpha\beta| = 9$
f. $|\beta\alpha| = 9$
g. $|\alpha\alpha| = 8$
h. $|\beta\beta| = 10$
i. $\alpha\lambda = \alpha$ = baab
j. $\beta\lambda = \beta$ = caaba
k. $\alpha\beta\gamma$ = baabcaababbab
l. $\beta\beta\gamma\alpha$ = caabacaababbabbaab

89. All strings over X = {0, 1} of length 2 are 00, 01, 10, and 11; there are a total of four such strings.

Section 2.3 (p. 76): Any 6 problems from 1-31

6. $110111011011_2 = 3547$

8. $61 = 111101_2$

16. 101101 + 11011 = 1001000

19. $3A_{16} = 58$

24. $4B07A_{16} = 307322$

28. $4A_{16} + B4_{16} = 74 + 180 = 254$

32. $82054_{16} + AEFA3_{16} = 532564 + 716707 = 1249271$

Section 2.4 (p. 83): 19-22, 34

Refer to the relation R on the set {1, 2, 3, 4, 5} defined by the rule (x, y) ∈ R if 3 divides x – y (R = {(x, y)|3 divides x -y})

19. The elements of R are {(1, 1), (1, 4), (2, 2), (2, 5), (3, 3), (4, 1), (4, 4), (5, 2), (5, 5)}.

20. The elements of R^{-1} (set {(b, a)|(a, b) ∈ R}) are {(1, 1), (1, 4), (2, 2), (2, 5), (3, 3), (4, 1), (4, 4), (5, 2), (5, 5)}.

21. The domain of R is equal to {1, 2, 3, 4, 5}

22. The range of R is equal to {1, 2, 3, 4, 5}

34. X = nonempty set, $A = P(x)$, and $R = \{(B, C) \mid B \subseteq C\}$
Since a set is always a subset of itself, this implies that R is reflexive. However, since there are cases where B will not be a subset of C, as in cases where the power set is the entire set and bigger than other elements in the power set, this would not me symmetric, thus antisymmetric. Since this is transitive as well, we have a partial order.

Section 2.5 (p. 89): 9, 11, 31

Determine whether the given relation is an equivalence relation on the set of all people.

9. $\{(x, y) \mid x \text{ and } y \text{ are the same height}\}$. Reflexive, because a person always has the same height as themselves, symmetric, because both people have the same height, and piggybacking off of that, transitive as well because everyone will be the same height. Therefore, since it is reflexive, symmetric, and transitive, it is an equivalence relation.

11. $\{(x, y) \mid x \text{ and } y \text{ have the same first name}\}$. Reflexive, because the person always has the same first name as themselves, and symmetric and transitive, because both x and y have the same name. Therefore, since it is reflexive, symmetric, and transitive, it is an equivalence relation.

31. Let $X = \{1, 2, \ldots, 10\}$. Define a relation R on $X \times X$ by $(a, b)R(c, d)$ if $ad = bc$.
a. $ab = ba$ for all a and b which are an element of X, which implies that $(a, b)R(a, b)$ for all (a, b) in the Cartesian product of $X \times X$, so R is reflexive. If we assume $(a, b)R(c, d)$, by definition of R, $ad = bc$; this also implies that $(c, d)R(a, b)$, meaning that R is symmetric. If we let $(a, b)R(c, d)$ and $(c, d)R(e, f)$, $ad = bc$ and $cf = de$, thus $(a, b)R(e, f)$, meaning that R is transitive. Since R is reflexive, symmetric, and transitive, R is an equivalence relation.
b. $ad = bc$ implies $a/b = c/d$, so all elements in the 10 different equivalence classes have different ratios. An example would be $\{(8, 1), (8, 2), (8, 3), (8, 4), (8, 5), (8, 6), (8, 7), (8, 8), (8, 9), (8, 10)\}$.
c. All elements in R contain two ordered pairs that have the same ratio between two terms.

Section 2.6 (p. 96): 4, 8, 9, 12

4. $R = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$; ordering of X: 1, 2, 3, 4, 5

8. The relation, R, is equal to $\{(a, w), (a, y), (c, y), (d, w), (d, x), (d, y), (d, z)\}$.

9. The relation, R, is equal to $\{(1, 1), (1, 3), (2, 2), (2, 3), (2, 4)\}$.

10. The relation, R, is equal to $\{(w, w), (w, y), (y, w), (y, y), (z, z)\}$.

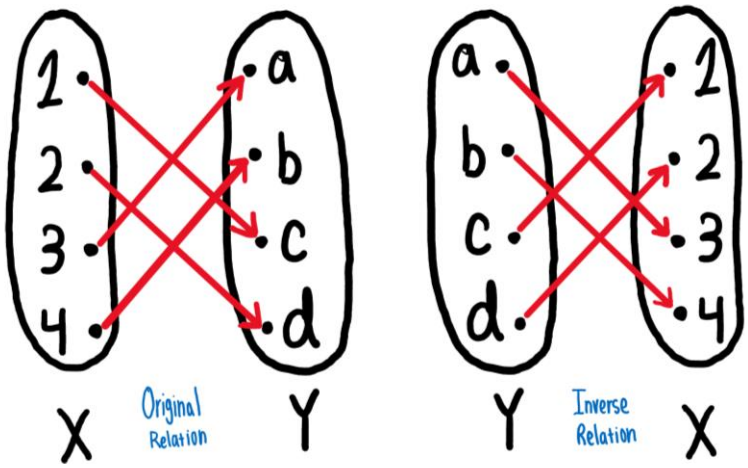
12. The relation in Exercise 10 is reflexive, symmetric, transitive, is an equivalence relation.

Section 2.8 (p. 111): 2, 3, 10, 11, 13, 19, 56

Determine whether each relation is a function from $X = \{1, 2, 3, 4\}$ to $Y = \{a, b, c, d\}$. If it is a function, find its domain and range, draw its arrow diagram, and determine if it is one-to-one or onto. If it is both one-to-one and onto, give the description of the inverse function as a set of ordered pairs, draw its arrow diagram, and give the domain and range of the inverse function.

2. {(1, c), (2, a), (3, b), (4, c), (2, d)}: This relation is not a function, as when X = 2, it points to both a and d in Y.

3. {(1, c), (2, d), (3, a), (4, b)}: This relation is both a one-to-one and an onto function, the conditions for there to be an inverse function. The ordered pairs for the inverse are {(c, 1), (d, 2), (a, 3), (b, 4)}. The domain of the inverse function is {a, b, c, d}, while the range of the inverse is from {1, 2, 3, 4}.



Determine whether each function is one-to-one. The domain of each function is the set of all real numbers. If the function is not one-to-one, exhibit distinct numbers a and b with f(a) = f(b). Also, determine whether f is onto the set of all real numbers. If f is not onto, exhibit a number y for which $f(x) \neq y$ for all real x.

10. Because f(x) = 6x -9 is a linear function, it is classified as both one-to-one and an onto function.

11. $f(x) = 3x^2 -3x +1$ is not a one-to-one function, as it does not pass the horizontal line test. a = 0, b = 1; f(0) = f(1) = 1.

13. $f(x) = 2x^3 -4$ is an one-to-one function, as it passes the horizontal line test. f(x) is also an onto function, as all elements in the range are used.

19. $f(x) = 4x+2$, $f^{-1}(x) = (x-2)/4$

56. Let g = {(1, a), (2, c), (3, c)} be a function from X = {1, 2, 3} to Y = {a, b, c, d}. Let S = {1}, T = {1, 3}, U = {a}, and V = {a, c}. Find:
g(S) = {a}
g(T) = {a, c}
$g^{-1}(U)$ = {1}
$g^{-1}(V)$ = {1, (2, 3)}