CHAPTER 6

GRAPH THEORY

HOMEWORK

- Again, all homework is from the Exercises
  - No problems are from the Review Exercises
- Section 6.1, (p. 271), #5-10, 17-18, 22, 27-28, 46-48
- Section 6.2, (p. 281), #20-21, 28-38, 39, 41
- Section 6.3, (p. 296), #1-7
- Section 6.5, (p. 300), #1-3, 7-9, 13-14, 24-25
- Section 6.6, (p. 305), #1-7
- Section 6.7, (p. 311), #6-9, 18-24

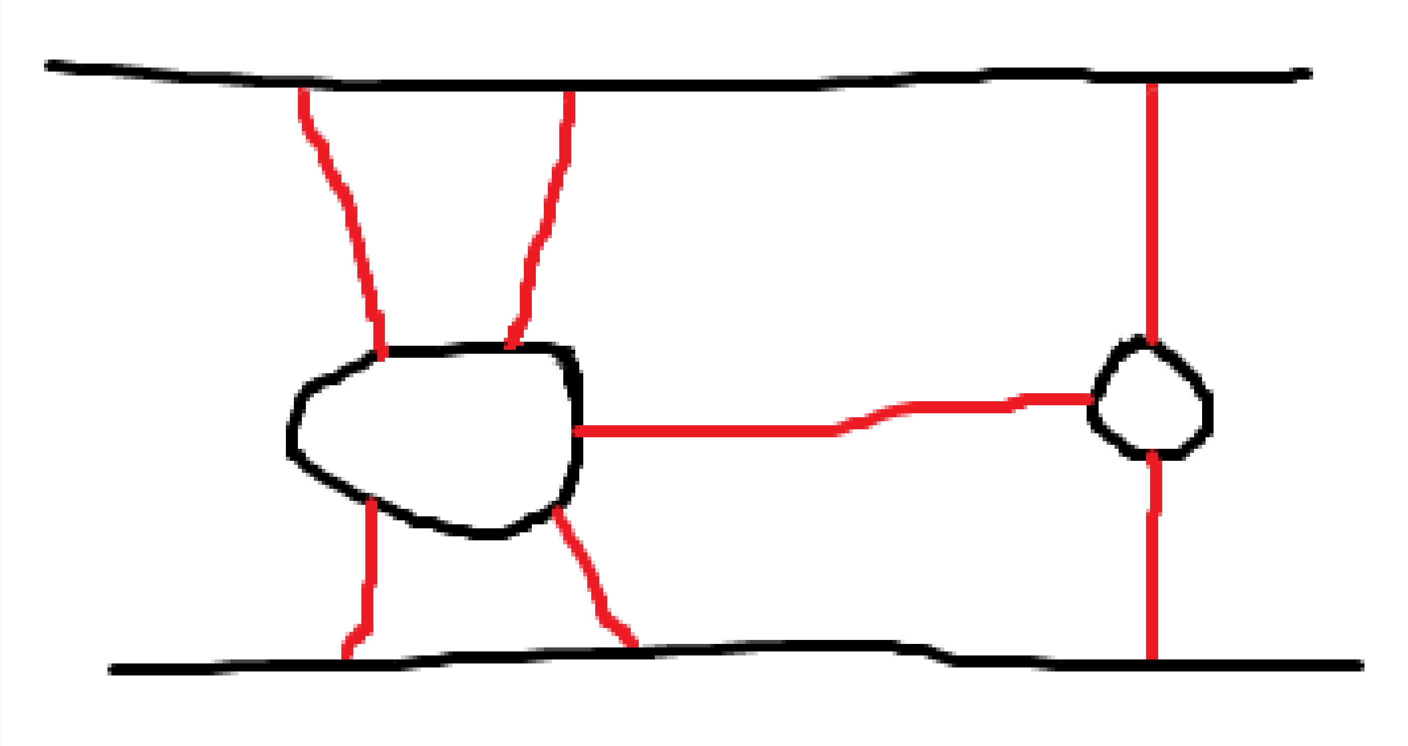
APPLICATIONS OF GRAPHS

- There are lots of ideas that come from or lead to applications of graphs
- Here are a few

VOCABULARY TO KNOW

- A graph is made up of
  - Vertices
  - Edges connecting the vertices

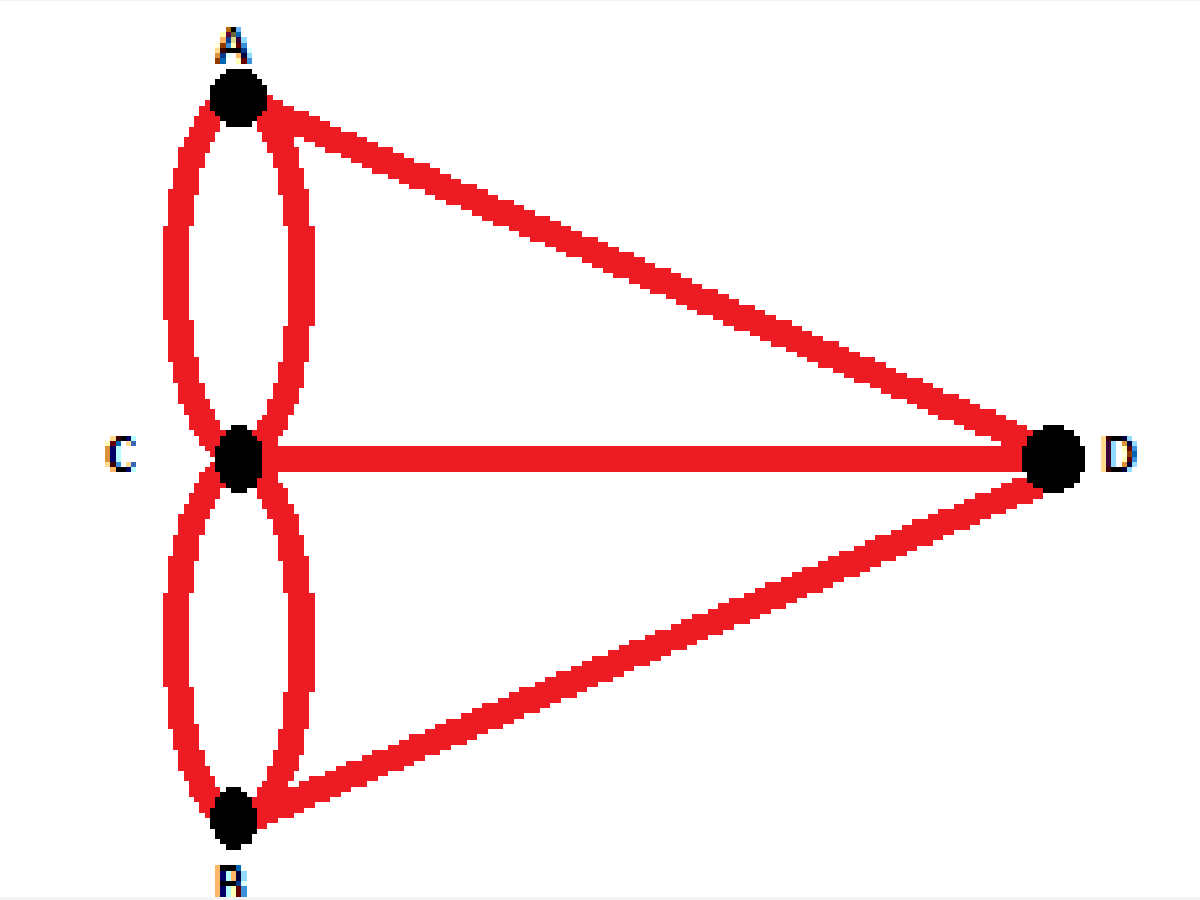
THE KÖNIGSBERG BRIDGES



THE KÖNIGSBERG BRIDGES AND GRAPHS

- Here is the connection to graphs
  - We label the top bank A
  - We label the bottom bank B
  - We label the left island C
  - We label the right island D
- We connect the vertices as in the picture

THE KÖNIGSBERG BRIDGE GRAPH



THE ROAD INSPECTOR PROBLEM

- A person is responsible for keeping the roads in an area in good working (driving?) order
- We draw a map of the area
- This is the connection to graphs
  - The cities are vertices
  - The roads are edges

THE TRAVELING SALESPERSON PROBLEM

- This is similar to the road inspector problem
- This is the connection to graphs
  - The vertices in the graph are cities
  - The edges are roads connecting the cities
- The graph could be weighted
  - The edges might be weighted based on the cost of travel
  - The edges might be weighted based on the number of miles

A DRILL PRESS

- You have a sheet of metal
- You have a drill press that can be programmed
- You want to drill a series of holes in the metal
- What's the shortest path connecting all holes?
- This is the connection to graphs
  - The holes are vertices
  - Thee graph is made up of edges connecting each pair of holes

ERDÖS NUMBERS

- Paul Erdös was a mathematician
- His Erdös number is 0
- If you wrote a paper with him, your Erdös number would be 1
- If you wrote a paper with someone who wrote a paper with him, your Erdös number would be 2
- This continues

CONNECTING ERDÖS NUMBERS TO GRAPHS

- This is the connection to graphs
  - Erdös is one vertex in the graph
  - People who wrote papers with him are connected to him
  - People who wrote papers with those people are connected to them
- The process continues

SIMILARITY GRAPHS

- We want to know how complicated a program is
- We need some criteria
- For example, we could use
  - The number of lines
  - The number of calls to methods
  - The number of return statements

ARE TWO PROGRAMS SIMILAR?

- We can then tell if two programs are similar
- Let's call s the dissimilarity function
  - $S(P1, P2) = |L2-L1| + |M2-M1| + |R2-R1|$
- This is the connection to graphs
  - Each program is a vertex
  - We connect programs that have the same similarity
  - This graph may be quite disconnected

THE KNIGHT'S TOUR

- A knight is a chess piece
- It has a specific L-shaped move
- Chess boards are 8 x 8
  - We will allow any size, but the board must be square
- Can we place the knight somewhere so that it can, in successive moves, visit every square on the board?
- We want to know which sizes of boards allow this kind of knight's tour

CONNECTING THE KNIGHT'S TOUR TO GRAPH THEORY

- This is the connection to graphs
  - The squares are vertices
  - Edges are added if the knight can move from one square (vertex) to another square (vertex)

PATHS

- A path connects two vertices
  - It is nothing more than a list of the edges in the path
- There are many special types of paths
  - One type of path is a cycle
  - A cycle starts and ends at the same point
  - We usually don’t allow a cycle to repeat an edge
- Two important kinds are of cycles are
  - Euler cycles
  - Hamiltonian cycles

CONNECTED GRAPHS

- Often, I will assume the graph comes in one piece
- We call this kind of graph **connected**
- You can adapt most of what I do to graphs that are not connected
  - Just apply it to each piece
  - Those pieces are called **components**

EULER CYCLES

- An **Euler cycle** is a path that traverses each edge exactly once
  - No edges are omitted
  - No edge is traversed twice (or more!)
- Not every graph has an Euler cycle
  - The Königsburg Bridge graph doesn't

FINDING AN EULER CYCLE

- If a graph has an Euler cycle, every vertex has an even number of edges incident on it
- Interestingly, the reverse is true also
  - If in a (connected) graph every vertex has an even number of edges incident on it, then the graph has an Euler cycle

HAMILTONIAN CYCLES

- A **Hamiltonian cycle** is a path that
  - Starts and ends at the same place
  - Visits every vertex exactly once

HOMEWORK

- Again, all homework is from the Exercises
  - No problems are from the Review Exercises
- Section 6.1, (p. 271), #5-10, 17-18, 22, 27-28, 46-48
- Section 6.2, (p. 281), #20-21, 28-38, 39, 41
- Section 6.3, (p. 296), #1-7
- Section 6.5, (p. 300), #1-3, 7-9, 13-14, 24-25
- Section 6.6, (p. 305), #1-7
- Section 6.7, (p. 311), #6-9, 18-24