Chapter 5 Written Homework

Section 5.1 (p. 232): 4-8, 18-20, 37-40

4. Assuming a person invests \$2000 at 14% interest compounded annually, and A_n represents the amount at the end of n years, the recurrence relation for the sequence A_0, A_1…. is $A_n = (1.14)A_{n-1}$.

5. Assuming a person invests \$2000 at 14% interest compounded annually, and A_n represents the amount at the end of n years, the initial condition for the sequence A_0, A_1…. is \$2000.

6. Assuming a person invests \$2000 at 14% interest compounded annually, and A_n represents the amount at the end of n years, A_1 is equal to \$2280, A_2 is equal to \$2599.2, and A_3 is equal to \$2963.088.

7. Assuming a person invests \$2000 at 14% interest compounded annually, and A_n represents the amount at the end of n years, the explicit formula for A_n is equal to $(1.14)^n (2000)$.

18. With S_n denoting the number of n-bit strings that do not contain the pattern 000, the recurrence relation and initial conditions for the sequence $\{S_n\}$ is $S_n = S_{n-1} + S_{n-2} + S_{n-3}$ for $n > 3$, This is because if we begin with 1, the remaining (n-1)-bit strong will not contain 000 and neither will the n-bit string itself, leaving us to conclude that there are S_{n-1} such (n-1)-bit strings. If we begin with 0, there will be two cases to consider. If we begin with 01, the remaining (n-2) bit string does not contain 000, and neither will the n-bit string, leaving us to conclude that there are S_{n-2} such (n-2)-bit strings. Next, if we begin with 00, the third bit must be a 1 and if the remaining (n-3)-bit string does not contain 000, neither will the n-bit string itself, leading us to conclude that there are S_{n-3} such (n-3)-bit strings. Because theses cases are mutually exclusive and cover all n-bit strings not containing 000, we are able to end up with the resulting sequence.

19. Referring to the sequence S where S_n denotes the number of n-bit strings that do not contain the pattern 00, the recurrence relation and initial conditions for the sequence $\{S_n\}$ are $S_n = S_{n-1}+S_{n-2}$, with the initial conditions of $S_1 = 2$ and $S_2 = 3$. This is because there are S_{n-1} n-bit strings that begin 1 and do not contain the pattern 00 and there are S_{n-2} n-bit strings that begin 0 since the second bit must be 1 and do not contain the pattern 00.

20. Referring to the sequence S where S_n denotes the number of n-bit strings that do not contain the pattern 00, S_n denotes the Fibonacci sequence, albeit with different initial conditions. While the initial conditions of S_n are $S_1 = 2$ and $S_2 = 3$, the initial condition of the Fibonacci sequence are $F_0 = 1$, $F_1 = 1$.

37. Referring to Ackermann’s function A(m, n), computing A(2, 2) and A(2, 3) would result in 7 and 9, respectively.

38. Referring to Ackermann’s function A(m, n), the formula for A(1, n) = n + 2, for n = 0, 1, ….. This is because A(1, 0) = A(0, 1) = 2, A(2, 0) = A(1, 1) = A(0, A(1, 0)) = 1 + A(1, 0) = 3. Therefore, A(1, n) = A(0, A(1, n-1)) = 1 + A(1, n-1), and by recursion, A(1, n) = n + A(1, 0) = n + 2.

39. Referring to Ackermann's function A(m, n), the formula for A(2, n) = 3n + 2, for n = 0, 1, ….. This is because A(2, n) = A(1, A(2, n-1)) = 2 + A(2, n-1), and by recursion, A(2, n) = 2n + (A(1, n-n)) = 2n + 3.

40. Referring to Ackermann's function A(m, n), I guess that the formula for A(3, n) would be $2^{n+3} - 3$. This is because A(3, n) = A(2, A(3, n-1)) = 3 + 2A(3, n-1). Next, A(3, n) + 3 = 2(3 + A(3, n-1)), and by recursion, A(3, n) + 3 = 2^n(A(3, n-n) + 3) = 2^n(A(3, 0) + 3) = 2^n(A(2, 1) + 3) = 2^{n+3}, since A(2, 1) = 5.

Section 5.2 (p. 244): 14-18, 34-36

Chapter 5 Homework

Section 5.2: Page 244, #14,15,16,17,18,34,35,36

14. $a_n = 2^n a_{n-1}, n>0 \quad a_0 = 1$

$a_1 = 2^1 \cdot 1 = 2^1$
$a_2 = 2^2 \cdot 2 = 2^3$
$a_3 = 2^3 \cdot 2^3 = 2^6$
$a_4 = 2^4 \cdot 2^6 = 2^{10}$
$a_5 = 2^5 \cdot 2^{10} = 2^{15}$
$a_6 = 2^6 \cdot 2^{15} = 2^{21}$

$\boxed{2^{\frac{n(n+1)}{2}}}$

15. $a_n = 6a_{n-1} - 8a_{n-2}; \quad a_0 = 1, a_1 = 0$
$r^n = 6r^{n-1} - 8r^{n-2}$
$r^n - 6r^{n-1} + 8r^{n-2} = 0 \rightarrow r^{n-2}(r^2 - 6r + 8) = 0$
$(r-4)(r-2) = 0 \rightarrow$ roots: $r = 2, r = 4$

$\boxed{a_n = 2^{n+1} - 4^n}$

16. $a_n = 7a_{n-1} - 10a_{n-2}; \quad a_0 = 5, a_1 = 16$
$r^n = 7r^{n-1} - 10r^{n-2}$
$r^{n-2}(r^2 - 7r + 10) \rightarrow (r-2)(r-5) = 0 \rightarrow$ roots: $r = 2, r = 5$

$\boxed{a_n = 2^{n+1} - 5^n}$

17. $a_n = 2a_{n-1} + 8a_{n-2}; \quad a_0 = 4, a_1 = 10$
$r^n = 2r^{n-1} + 8r^{n-2}$
$r^{n-2}(r^2 - 2r - 8) \rightarrow (r-4)(r+2) = 0 \rightarrow$ roots: $r = 4, r = -2$

$4 = R + S$
$10 = 4R - 2S$

$S = 4 - R$
$10 = 4R - 2(4 - R)$
$10 = 4R - 8 + 2R$
$18 = 6R, \; R = 3$

$S = 4 - 3$
$S = 1$

$\boxed{a_n = 3(4)^n + 1(-2)^n}$

18. $2a_n = 7a_{n-1} - 3a_{n-2}; \quad a_0 = a_1 = 1$
$2r^n = 7r^{n-1} - 3r^{n-2}$
$r^{n-2}(2r^2 - 7r + 3) \rightarrow (2x-1)(x-3) = 0 \rightarrow$ roots: $r = \frac{1}{2}, r = 3$
$r^2 - 7r + 6$
$(r+$

$a_n = R(\frac{1}{2})^n + S3^n$

$\left.\begin{array}{l} a_0 = 1 = R + S \\ a_1 = 1 = \frac{1}{2}R + 3S \end{array}\right\} \begin{array}{l} R = \frac{4}{5} \\ S = \frac{1}{5} \end{array}$

$a_n = \frac{4}{5}(\frac{1}{2})^n + \frac{1}{5}(3)^n$

$\boxed{a_n = \frac{1}{5}(\frac{1}{2}^n + 3^n)}$

34. $\sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}}$ $\quad a_0 = a_1 = 1$ $\quad$ substitute $b_n = \sqrt{a_n}$

$b_n = b_{n-1} + 2b_{n-2} \rightarrow r^n = r^{n-1} + 2r^{n-2}$ $\qquad b_n = R(-1)^n + S(2)^n$

$r^n - r^{n-1} - 2r^{n-2} = 0$

$r^{n-2}(r^2 - r - 2) = 0 \rightarrow (r-2)(r+1) \rightarrow$ roots: $r = -1,\ r = 2$

$b_0 = 1 = R + S$

$b_1 = 1 = -R + 2S$

$1 = R + S$

$+\ 1 = -R + 2S$

$2 = 3S$

$\frac{2}{3} = S$

$R = 1 - S$

$R = 1 - \frac{2}{3}$

$R = \frac{1}{3}$

$b_n = \frac{1}{3}(-1)^n + \frac{2}{3}(2)^n$

$\boxed{b_n = \frac{1}{3}\left((-1)^n + 2(2)^n\right)}$

35. $a_n = \sqrt{\frac{a_{n-2}}{a_{n-1}}}$ $\quad a_0 = 8,\ a_1 = \frac{1}{2\sqrt{2}} = \frac{1}{\sqrt{8}}$ $\quad$ substitute $b_n = \lg a_n$

$a_n = \left(\frac{a_{n-2}}{a_{n-1}}\right)^{\frac{1}{2}}$

$\ln a_n = \frac{1}{2}(\ln a_{n-2} - \ln a_{n-1})$

$b_n = \frac{1}{2}(b_{n-2} - b_{n-1}) = b_n = \frac{1}{2}b_{n-2} - \frac{1}{2}b_{n-1} \rightarrow r^n = \frac{1}{2}r^{n-2} - \frac{1}{2}r^{n-1}$

$r^n + \frac{1}{2}r^{n-1} - \frac{1}{2}r^{n-2} = 0 \rightarrow r^{n-2}(r^2 + \frac{1}{2}r - \frac{1}{2}) = 0 \rightarrow 2r^2 + r - 1 = 0 \rightarrow (2r-1)(r+1) \rightarrow$ roots: $r = \frac{1}{2},\ r = -1$

$b_n = R(\frac{1}{2})^n + S(-1)^n$

$b_0 = \ln 8$

$b_2 = \ln(8^{-\frac{1}{2}}) = -\frac{1}{2}\ln 8$

$\ln 8 = R + S$

$-\frac{1}{2}\ln 8 = \frac{1}{2}R - S$

$\ln 8 = R + S$

$-\frac{1}{2}\ln 8 = \frac{1}{2}R - S +$

$\frac{1}{2}\ln 8 = \frac{3}{2}R$

$\frac{1}{2}\ln 8 \cdot \frac{2}{3} = R \rightarrow \frac{1}{3}\ln 8 = R$

$S = \ln 8 - R$

$S = \ln 8 - \frac{1}{3}\ln 8$

$S = \frac{2}{3}\ln 8$

$b_n = \frac{1}{3}\ln 8(\frac{1}{2})^n + \frac{2}{3}\ln 8(-1)^n$

$\boxed{b_n = \frac{1}{3}\ln 8\left((\frac{1}{2})^n + 2(-1)^n\right)}$

36. $a_n = -2na_{n-1} + 3n(n-1)a_{n-2};\ a_0 = 1\ \ a_1 = 2$

$b_n = \frac{a_n}{n!}$ $\quad b_n = -2b_{n-1} + 3b_{n-2}$

$\boxed{a_n = n!\,b_n = \frac{n!}{4}(5 - (-3)^n)}$