CHAPTER 4

COUNTING AND THE PIGEONHOLE PRINCIPLE

HOMEWORK

- Again, all homework is from the Exercises
  - No problems are from the Review Exercises
- Section 4.1 (p. 170), #5, 17-20, 28-30, 34-37, 42-46, 60, 62
- Section 4.2 (p. 182), #10-14, 25-29, 31-34, 58-62
- Section 4.4 (p. 194), #11-17, 30-33
- Section 4.5 (p. 204), #1-5, 22-26, 42-45
- Section 4.6 (p. 210), #1-3, 7-9, 15-17, 22-24
- Section 4.7 (p. 215), #1, 3-5, 10-11
- Section 4.8 (p. 219), #1-10

LISTING COMBINATIONS AND PERMUTATIONS

- C(n,k) tells the number of ways of choosing k items out of a list of n items
- Suppose you choose values for n and k
  - For example, n=5, k=3
- You know there are C(5,3), which is 10, different ways of choosing 3 items out of 5
- You want to list those 10 combinations
  - They are ABC, ABD, ABE, ACD, ACE, ADE, ADE, BCD, BCE, CDE
- What if you choose n=6, k=2?
- The list has C(6,2), or 15 items
- They are (using ABCDEF as the items)
  - AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, EF

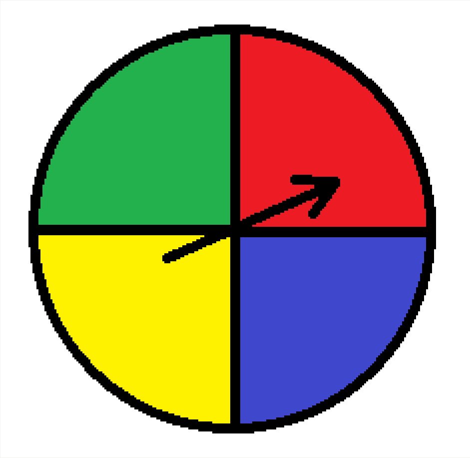
LISTING COMBINATIONS AND PERMUTATIONS

- To build a list like that, you have to put the combinations in some kind of order
- See if you can find a way to do that
  - That is, choose permutation or combinations
  - Choose a value for n and one for k
  - Calculate C(n,k) or P(n,k) to tell how many things you need to list
  - Find a way to list the things in order

DISCRETE PROBABILITY

- Probability is a branch of mathematics that uses theory to try to predict outcomes of experiments
- We will perform an experiment
- To look at experiments and outcomes from those experiments, we need some tools
- Coins, dice, spinners, playing cards, roulette wheels—the tools of probability
  - You could flip a coin
  - You could roll a die
    - You could roll two dice
  - You could spin a spinner
  - You could choose a card from a standard deck of playing cards
  - You could spin a roulette wheel

EXAMINING THE TOOLS



- For a coin, there are two outcomes: heads and tails
- For a die, there are six outcomes: 1, 2, 3, 4, 5, 6
  - We can look at rolling numbers less than 3 or more than 3, or even or odd, etc.
- For a pair of dice there are 36 outcomes
  - We can concern ourselves with the individual dice or the sum
- A spinner is a circle that is divided into numbered parts, with a needle that can spin around
  - Usually all the parts are the same size
  - I will refer to a spinner that has the circle divided into n equal parts as an n-spinner
  - A 4-spinner is pictured

A STANDARD DECK OF PLAYING CARDS

- A deck of cards has 52 cards
- The cards are divided into four groups, called suits
  - The suits are clubs, spades (both black), hearts, and diamonds (both red)
- Each suit contains 13 cards:
  - 2, 3, 4, …, 10, Jack, Queen, King, Ace
  - Ace can be high or low

A ROULETTE WHEEL

- A roulette wheel is a circular wheel that can spin
- The wheel has 38 slots, numbered from 1 to 36
- Also, there is one 0 slot and one 00 slot
- A ball is sent spinning around the top of the wheel and the wheel is spun in the opposite direction
- Eventually, the wheel will stop and the ball will land in one of the slots
- Half of the (1-36) numbers are black; half are red
- The number-color matching scheme is a bit unusual
  - 1 – 10: odd numbers are red and even are black
  - 11 – 18: even numbers are red and odd are black
  - 19 – 28: odd numbers are red and even are black
  - 29 – 36: even numbers are red and odd are black
- See https://www.youtube.com/watch?v=u9hESWMykp0

VOCABULARY

- An <u>experiment</u> is a process that yields an outcome
- A <u>sample space</u> is a set containing all possible outcomes
  - You could flip a coin (2 outcomes): S ={H,T}
  - You could roll a die (6 outcomes): S={1, 2, 3, 4, 5, 6]
- An <u>event</u> is a (non-empty) subset of the sample space
  - A <u>simple event</u> is an event with only one outcome
- The <u>probability</u> of an event
  - This is defined to be $P(E) = \frac{|E|}{|S|}$ (Here E is an event, and S is the sample space.)
  - This is sometimes called the "classical method"
- If E is an event, then P(E) is read as "The probability of E"
- Notice that we are assuming that all outcomes are equally likely
  - For example, the 1-4 spinner has four equally sized areas

A PROBLEM TO TRY: P. 194, #18

- The sample space is
- S = { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
  (2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
  (3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
  (4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
  (5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
  (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }
- This is the standard sample space for rolling a pair of dice

CONTINUING THE PROBLEM

- Here E is the event:
  - The sum on the dice is odd
    - Notice that all sums are between 2 and 12
    - The odd numbers are 3, 5, 7, 9, 11
- This means E is the set { (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (3,6), (4,1), (4,3), (4,5), (5,2), (5,4), (5,6), (6,1), (6,3), (6,5) }

- |E| = 2 (If the sum = 3) + 4 (Sum = 5) + 6 (Sum = 7) + 4 (Sum = 9) + 2 (Sum = 11)
- = 18
- |S| = 36
- So, then $P(E) = \frac{|E|}{|S|} = \frac{18}{36} = \frac{1}{2} = 0.5$

ANOTHER PROBLEM: P. 194, #30

- I changed the problem from a 10 question test into 16 question test
- Here, S is a set containing all possible outcomes of the test
- Each question could be true or false
- A tree shows us that there are 2^{16}, or 65,536 possibilities
  - If there were only three questions, S would be the set {TTT, TTF, TFT, TFF, FTT, FTF, FFT, FFF}
  - Remember, only one of those outcomes is correct
    - So, E has only one element
    - For example, in this case, E could be {TTF}
- Getting back to the original question,
- $P(E) = \frac{|E|}{|S|} = \frac{1}{65536}$, which is 0.0000 to 4 decimal places

YET ANOTHER PROBLEM: P. 194, #32

- S is the same as in #30
- Again, I am assuming there are 16 questions on the test
- There are 16 elements in E
- You can see this two ways.
  - You get only one question right
    - Then the other 15 must be wrong
    - There are C(16,15) ways to do that
    - If you notice that each element of E has one true and 15 falses, you find C(16,1) ways

YET ANOTHER PROBLEM: P. 194, #32—A TREE IDEA

- Suppose the correct answers are all false and you get only one question correct
  - Suppose you get the first question right
    - Then you have to get the other 15 wrong
    - This gives one element of E
    - It's (F,T,T, …,T)
  - Repeat this thinking: Suppose you get the second question correct
    - Then you get the other 15 wrong
    - Again, this is one element of E
    - It's (T,F,T,T, …, T)
  - Repeating this thinking shows there are 16 possible choices.

YET ANOTHER PROBLEM: P. 194, #32 —THE SOLUTION

- P(Getting only one correct) = $\frac{16}{2^{16}} = \frac{16}{65536} = \frac{1}{4096}$, or about 0.0002, which is 0.02%

A FINAL WORD ABOUT PROBABILITY

- Suppose you flip a coin
  - Then S = {H,T}
- The probability of getting a head is 0.5
  - We are assuming a regular, fair coin
- You repeat this 10 times
  - You expect 5 heads, 5 tails
  - You get 6 heads, 4 tails
- What went wrong?

PROBABILITY IS APPROXIMATE. IT'S AN "IN THE LONG RUN" IDEA

- This just means you didn't flip the coin enough times
- If you flip the coin 1000 times, you will get approximately 500 heads and 500 tails
- Similarly, if you flip it 10,000 times, you will get approximately 5000 heads and 5000 tails

- Probability predicts what will happen with many trials
- You can't use it directly for a small number of trials

HOMEWORK

- **You should be able to do the homework from Section 4.4**
- **Section 4.4 (p. 194), #11-17, 30-33**