

Review for Chapter 4 Exam in CS 113

1. One major topic in this chapter was counting. Be able to count the number of ways something can happen.

Some examples:

How big is the power set of a finite set? Be able to write it.

Given an 8-bit byte, how many different 0,1 patterns are there?

How many different three-letter strings can be made from the letters A, B, C?

Be able to answer this question if repetition is not allowed.

Also, be able to answer this question if repetition is allowed.

If there are a small number of permutations or combinations, be able to list them all.

Also, if there are a small number of permutations or combinations, be able to draw a tree.

Be able to use the multiplication and addition principles.

Know the notation and what it means: $P(n,r)$, $C(n,r)$

Be able to use Pascal's triangle and/or the definition of the combinatorial coefficients and/or the Binomial Theorem to verify combinatorial identities.

Be able to expand the expression $(x + y)^n$ using the Binomial Theorem. x and y don't have to be just letters.

The Pigeonhole Principle

Version 1: At least $k+1$ pigeons, k holes implies that some hole has ≥ 2 pigeons

Version 2: $f: X \rightarrow Y$, both X, Y finite with $|X| > |Y|$. Then $f(a) = f(b)$ for some a, b

There is another version in the book that would be worth checking out.

3. The final major topic in the chapter was about probability. When writing an answer giving a probability, be sure to write $P(\text{some event}) = \dots$

Be able to find the sample space for an experiment

Know that an event is a subset of the sample space. Be able to use this knowledge.

Be able to describe experiments (sample spaces and events) for dice, spinners, coins, and a deck of playing cards

When solving probability problems, always find the sample space first. Then, find the event as a subset of the sample space.

The probability of an event is defined to be $P(E) = \frac{|E|}{|S|}$ (What are E, S ?)

Also, be able to use the Complement Rule. (Theorem 4.5.5) and the Addition Rule (Theorem 4.5.9)

Be able to answer questions about probability.

For example You take a true/false test for which you have not studied. The test has 10 questions. What is the probability that you get all ten questions right? What is the probability that you get one question right? Again, remember to write the answer as $P(\text{getting one question correct}) = \dots$

Be able to use the Conditional Probability Rule (Definition 4.5.13) It has two uses. One is to calculate conditional probabilities. The other is to see if two events are independent or not. Events E and F are independent if $P(E|F) = P(E)$. Since $P(E|F) = \frac{P(E \cap F)}{P(F)}$, events E and F are independent if $P(E \cap F) = P(E)P(F)$.

Problems to practice

p. 171, #8-13, 21-25, 34-38

p. 183, #20-22, 31-35, 41-45

p. 204, #6-11, 22-29, 42-45

p. 211, #15-18, 35-37

p. 214, #3, 4, Example 4.7.7

p. 219, #1, 2, 5, 6, 11-13

The problems in the handout

Here are some more counting problems to practice.

1. Determine how many different computer passwords are possible if:
 - a) digits and letters can be repeated
 - b) digits and letters cannot be repeatedand the password is 3 digits followed by 4 letters.
2. Determine how many different computer passwords are possible if:
 - a) digits and letters can be repeated
 - b) digits and letters cannot be repeatedand the password is 2 digits followed by 5 letters.
3. How many eight-digit numbers can be formed if the leading digit cannot be a zero and the last number cannot be 1?
4. How many 4 digit odd numbers can be formed if no digit can be repeated?
5. How many even 3-digit positive integers can be written using the digits 1, 2, 4, 7, and 8?
6. How many 3-letter code words can be formed if at least one of the letters is to be chosen from the vowels a, e, i, o, and u?
7. Suppose you have totally forgotten the combination to your bike lock. There are three numbers in the combination, and you're sure each number is different. The numbers on the lock's dial range from 0 to 35. If you test one combination every 12 seconds, how long (in days to the nearest hundredth) will it take to test all possible combinations?
8. A men's department store sells 3 different suit jackets, 6 different shirts, 8 different ties, and 4 different pairs of pants. How many different suits consisting of a jacket, shirt, tie, and pants are possible?
9. A baseball manager is determining the batting order for the team. The team has nine players, but the manager definitely wants the pitcher to bat last. How many batting orders are possible?
10. A single die is rolled. How many ways can you roll a number less than 3, then an even, and then an odd?
11. From a standard deck of cards, how many ways can you pick an even numbered card, then a spade? Provided you put the first card back?
12. In how many different ways can a 10-question true-false test be answered if every question must be answered?

13. How many license plates of 3 symbols (letters and digits) can be made using at least 2 letters for each?

14. If a license plate is 3 letters followed by 3 digits, how many different license plates are possible if letters and digits can be repeated? How many if they cannot be repeated?