# Chapter 5 Homework

## Section 5.2: Page 244, #14, 15, 16, 17, 18, 34, 35, 36

14. $a_n = 2^n a_{n-1}$, $n>0$ $a_0=1$

$a_1 = 2^1 \cdot 1 = 2^1$
$a_2 = 2^2 \cdot 2 = 2^3$
$a_3 = 2^3 \cdot 2^3 = 2^6$
$a_4 = 2^4 \cdot 2^6 = 2^{10}$
$a_5 = 2^5 \cdot 2^{10} = 2^{15}$
$a_6 = 2^6 \cdot 2^{15} = 2^{21}$

$\boxed{2^{\frac{n(n+1)}{2}}}$

15. $a_n = 6a_{n-1} - 8a_{n-2}$; $a_0=1$, $a_1=0$
$r^n = 6r^{n-1} - 8r^{n-2}$
$r^n - 6r^{n-1} + 8r^{n-2} = 0 \rightarrow r^{n-2}(r^2-6r+8)=0$
$(r-4)(r-2)=0 \rightarrow$ roots: $r=2, r=4$

$\boxed{a_n = 2^{n+1} - 4^n}$

16. $a_n = 7a_{n-1} - 10a_{n-2}$; $a_0=5$, $a_1=16$
$r^n = 7r^{n-1} - 10r^{n-2}$
$r^{n-2}(r^2-7r+10) \rightarrow (r-2)(r-5)=0 \rightarrow$ roots: $r=2, r=5$

$\boxed{a_n = 2^{n+1} - 5^n}$

17. $a_n = 2a_{n-1} + 8a_{n-2}$; $a_0=4$, $a_1=10$
$r^n = 2r^{n-1} + 8r^{n-2}$
$r^{n-2}(r^2-2r-8) \rightarrow (r-4)(r+2)=0 \rightarrow$ roots: $r=4, r=-2$

$4 = R+S$
$10 = 4R-2S$

$S = 4-R$
$10 = 4R - 2(4-R)$
$10 = 4R - 8 + 2R$
$18 = 6R$, $R=3$

$S = 4-3$
$S=1$

$\boxed{a_n = 3(4)^n + 1(-2)^n}$

18. $2a_n = 7a_{n-1} - 3a_{n-2}$; $a_0 = a_1 = 1$
$2r^n = 7r^{n-1} - 3r^{n-2}$
$r^{n-2}(2r^2 - 7r + 3) \rightarrow (2x-1)(x-3)=0 \rightarrow$ roots: $r=\frac{1}{2}, r=3$
$r^2-7r+6$
$(r+$

$a_n = R(\frac{1}{2})^n + S3^n$

$\left.\begin{array}{l} a_0 = 1 = R+S \\ a_1 = 1 = \frac{1}{2}R + 3S \end{array}\right\}$ $R=\frac{4}{5}$, $S=\frac{1}{5}$

$a_n = \frac{4}{5}(\frac{1}{2})^n + \frac{1}{5}(3)^n$

$\boxed{a_n = \frac{1}{5}(\frac{1}{2}^n + 3^n)}$

34. $\sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}} \quad a_0 = a_1 = 1$ subsitute $b_n = \sqrt{a_n}$

$b_n = b_{n-1} + 2b_{n-2} \rightarrow r^n = r^{n-1} + 2r^{n-2}$ $\quad b_n = R(-1)^n + S(2)^n$

$r^n - r^{n-1} - 2r^{n-2} = 0$

$r^{n-2}(r^2 - r - 2) = 0 \rightarrow (r-2)(r+1) \rightarrow$ roots: $r = -1, r = 2$

$b_0 = 1 = R + S$

$b_1 = 1 = -R + 2S$

$1 = R + S$

$+\ 1 = -R + 2S$

$2 = 3S$

$\frac{2}{3} = S$

$R = 1 - S$

$R = 1 - \frac{2}{3}$

$R = \frac{1}{3}$

$b_n = \frac{1}{3}(-1)^n + \frac{2}{3}(2)^n$

$\boxed{b_n = \frac{1}{3}((-1)^n + 2(2)^n)}$

35. $a_n = \sqrt{\frac{a_{n-2}}{a_{n-1}}} \quad a_0 = 8,\ a_1 = \frac{1}{2\sqrt{2}} = \frac{1}{\sqrt{8}}$ subsitute $b_n = \lg a_n$

$a_n = \left(\frac{a_{n-2}}{a_{n-1}}\right)^{\frac{1}{2}}$

$\ln a_n = \frac{1}{2}(\ln a_{n-2} - \ln a_{n-1})$

$b_n = \frac{1}{2}(b_{n-2} - b_{n-1}) = b_n = \frac{1}{2}b_{n-2} - \frac{1}{2}b_{n-1} \rightarrow r^n = \frac{1}{2}r^{n-2} - \frac{1}{2}r^{n-1}$

$r^n + \frac{1}{2}r^{n-1} - \frac{1}{2}r^{n-2} = 0 \rightarrow r^{n-2}(r^2 + \frac{1}{2}r - \frac{1}{2}) = 0 \rightarrow 2r^2 + r - 1 = 0 \rightarrow (2r-1)(r+1) \rightarrow$ roots: $r = \frac{1}{2}, r = -1$

$b_n = R(\frac{1}{2})^n + S(-1)^n$

$b_0 = \ln 8$

$b_2 = \ln(8^{-\frac{1}{2}}) = -\frac{1}{2}\ln 8$

$\ln 8 = R + S$

$-\frac{1}{2}\ln 8 = \frac{1}{2}R - S$

$\ln 8 = R + S$

$-\frac{1}{2}\ln 8 = \frac{1}{2}R - S\ +$

$\frac{1}{2}\ln 8 = \frac{3}{2}R$

$\frac{1}{2}\ln 8 \cdot \frac{2}{3} = R \rightarrow \frac{1}{3}\ln 8 = R$

$S = \ln 8 - R$

$S = \ln 8 - \frac{1}{3}\ln 8$

$S = \frac{2}{3}\ln 8$

$b_n = \frac{1}{3}\ln 8(\frac{1}{2})^n + \frac{2}{3}\ln 8(-1)^n$

$\boxed{b_n = \frac{1}{3}\ln 8((\frac{1}{2})^n + 2(-1)^n)}$

36. $a_n = -2na_{n-1} + 3n(n-1)a_{n-2};\ a_0 = 1 \quad a_1 = 2$

$b_n = \frac{a_n}{n!} \quad b_n = -2b_{n-1} + 3b_{n-2}$

$\boxed{a_n = n!b_n = \frac{n!}{4}(5 - (-3)^n)}$