

CHAPTER 9

**BOOLEAN ALGEBRAS AND
COMBINATORIAL CIRCUITS**

APPLICATIONS OF CIRCUIT IDEAS

- Now we will look at some applications of these ideas

FUNCTIONALLY COMPLETE SETS

- A set of gates is functionally complete if it can be used to build any combinatorial circuit
- {AND, IOR, NOT} is functionally complete
- A new gate to learn is the NAND gate
- NAND means “not and”: $x \text{ NAND } y$ means $\neg(x \text{ AND } y)$
- The gate symbol is a combination of the AND and the NOT

THE SET {NAND} IS FUNCTIONALLY COMPLETE

- $\text{NOT } x = x \text{ NAND } x$ (We have added the NOT gate)
- $x \text{ AND } y = \text{NOT } (x \text{ NAND } y)$ (We already have NOT, NAND)
- $x \text{ OR } y = \neg (\neg x \text{ AND } \neg y)$ (Again, we already have NOT, NAND)
- So, we can build all logic circuits from only (perhaps a lot of!) NAND gates

SIMPLIFYING LOGIC - A SIMPLE DEMONSTRATION

- $\neg ABC + A(\neg B)C + ABC$
- $= BC(\neg A) + A(\neg B)C + BC(A)$
- $= BC(\neg A) + BC(A) + A(\neg B)C$
- $= (BC)(\neg A + A) + A(\neg B)C$
- $= (BC)(I) + A(\neg B)C$
- $= BC + A(\neg B)C$
- $= \dots = (B + A(\neg B))C$

A HALF ADDER

- A half adder is a circuit that can add two bits
- It takes the two bits as input and produces a sum bit and a carry bit
 - The actual sum of the two bits is carry, sum
- Here is a picture of a half adder
- The picture is on p. 444

A FULL ADDER

- The problem with a half adder is that it needs more circuitry to be useful
 - A full adder can add two bits plus the carry bit from the addition “on the right”
 - It also produces a sum and a carry
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- The name for the bit in a byte that act as 2^0 is the least significant bit
 - To make adders useful for adding two bytes, for example, you need
 - A half adder for the least significant bits
 - A full adder for the rest of the bits