CHAPTER 6

GRAPH THEORY

HOMEWORK

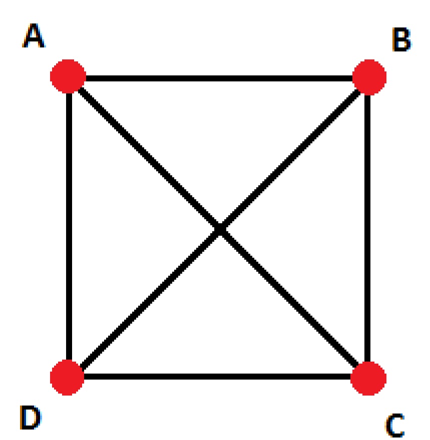
- Again, all homework is from the Exercises
  - No problems are from the Review Exercises
- Section 6.1, (p. 271), #5-10, 17-18, 22, 27-28, 46-48
- Section 6.2, (p. 281), #20-21, 28-38, 39, 41
- Section 6.3, (p. 296), #1-7
- Section 6.5, (p. 300), #1-3, 7-9, 13-14, 24-25
- Section 6.6, (p. 305), #1-7
- Section 6.7, (p. 311), #6-9, 18-24

HAMILTONIAN CYCLES AND THE TRAVELING SALESMAN PROBLEM

- Some reminders
- A cycle is a path that
  - Starts and ends at the same vertex, and
  - Has no repeated edges
- A Hamiltonian cycle is a cycle that
  - Contains each vertex of the graph, and
  - Contains each "internal" vertex exactly once
- The traveling salesman problem shows an example of a problem that a Hamiltonian cycle can solve

AN EXAMPLE-HAMILTONIAN CYCLES VS. EULER CYCLES

- Euler cycles and Hamiltonian cycles might seem to be the same
  - They're not!
- The graph pictured here has a Hamiltonian cycle (A, B, C, D, A)
- It does not have an Euler cycle since every vertex has odd degree



GRAY CODES

- **An n-bit Gray code is a list of binary numbers where**
  - **Each n-bit number is in the list**
  - **Each element of the list is exactly one bit different from the next element**
  - **The first and last elements also differ in exactly one bit**

FINDING THE GRAY CODES-BEGINNING STEPS

- There is an n-bit Gray code for every value of n
- The proof is by induction on n
- Base Step: n=1
  - The sequence 0, 1 is a 1-bit Gray code

FINDING THE GRAY CODES-THE INDUCTIVE STEP

- Inductive Step
  - Assume there is an n bit Gray code
  - Show that there is an n+1 bit Gray code
- Create two new sequences
  - The original sequence, but add a 0 to the front of each element
  - The reverse of the original sequence, but add a 1 to the front of each element
- Put the second after the first
- This is an n+1 bit Gray code

THE KNIGHT'S TOUR ON A CHESSBOARD-AN INTRODUCTION

- A knight is a chess piece that makes L-shaped moves
- A tour means that the knight can move to every square on the board exactly once
- A normal chessboard is 8x8
  - We will allow any size board, but the board must be square
- GK_n will be used to denote the tour on an nxn chessboard
- The question is
  - Can we find a tour for every value of n?

THE KNIGHT'S TOUR ON A 2X2 BOARD

- **We start with a 2x2 board**
  - **It's easy to see that a tour here is impossible**

CONVERTING THE PROBLEM TO GRAPH THEORY

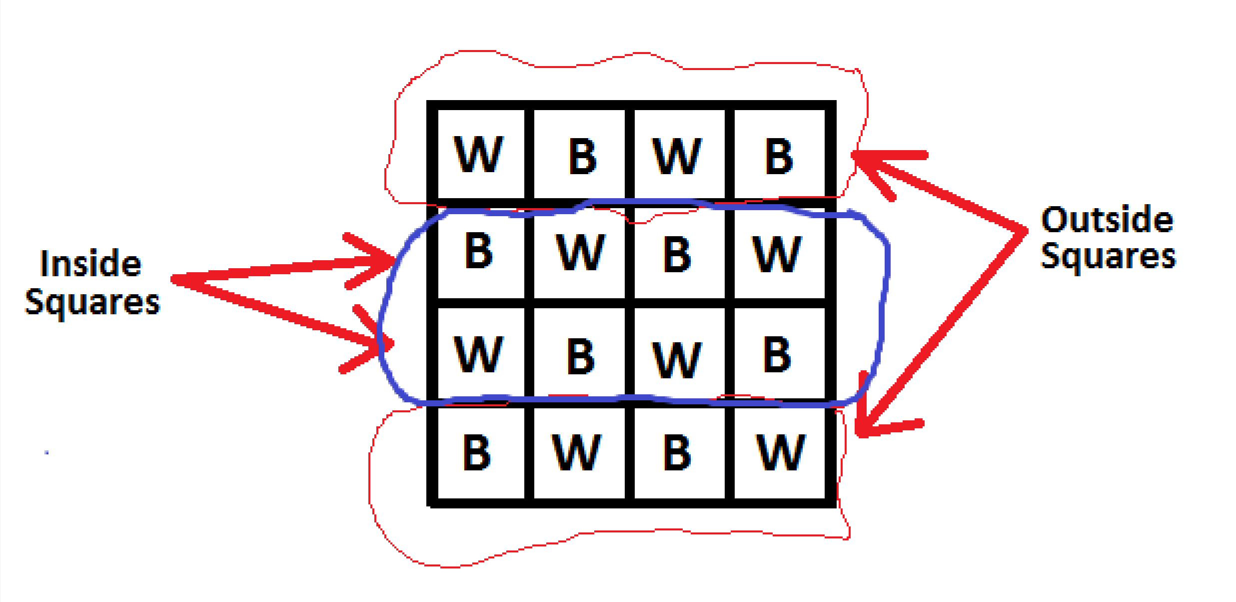
- Change the squares to vertices
- Connect two vertices with an edge if the knight can move form one square to another
- The problem now becomes
  - Can we find a Hamiltonian circuit?

LIMITING THE PROBLEM

- In fact, we can only find a tour on a 2n x 2n board
- To see this, break the vertices into two sets
  - The first set (W) is the white squares and the second is the black (B) squares
- The knight's move always goes from a white square to a black or vice-versa
- So now we see that the graph is bipartite
- Since any tour must alternate between a vertex in W and a vertex in B, the total number of vertices must be even

CHECKING GK_4

- It is not possible to get a tour on a 4x4 chessboard
- Here is a picture of the chessboard



FOLLOWING THE KNIGHT'S MOVES

- We list the knight's 16-square tour as S1, S2, …, S16
- Assume S1 is the upper left square
  - This is not really a limitation
  - A similar version of the proof holds for any starting square
- There are two notes about the knight's moves
  - The knight must get to an outside square from an inside square
  - The knight must move from an outside square to an inside square

LISTING THE 16 SQUARES-PART 1

- **Draw 16 blanks. Put I or O in each blank**
- **Since there are eight outside and eight inside squares, the comments from the previous slide force the sequence to be**
  - **O,I,O,I,O,I,O,I,O,I,O,I,O,I,O,I**

LISTING THE 16 SQUARES-PART 2

- But, the knight's moves alternate colors
  - This means that the only outside squares that are visited are the white squares
  - Similarly, the only inside squares that are visited are the black squares
- This has led to a contradiction
- The cycle is not a Hamiltonian cycle
- You can check that a 2nx2n graph does have a Hamiltonian cycle

HOMEWORK

- **Again, all homework is from the Exercises**
  - **No problems are from the Review Exercises**
- **Section 6.1, (p. 271), #5-10, 17-18, 22, 27-28, 46-48**
- **Section 6.2, (p. 281), #20-21, 28-38, 39, 41**
- **Section 6.3, (p. 296), #1-7**
- **Section 6.5, (p. 300), #1-3, 7-9, 13-14, 24-25**
- **Section 6.6, (p. 305), #1-7**
- **Section 6.7, (p. 311), #6-9, 18-24**