CHAPTER 4

COUNTING AND THE PIGEONHOLE PRINCIPLE

HOMEWORK

- Again, all homework is from the Exercises
  - No problems are from the Review Exercises
- Section 4.1 (p. 170), #5, 17-20, 28-30, 34-37, 42-46, 60, 62
- Section 4.2 (p. 182), #10-14, 25-29, 31-34, 58-62
- Section 4.4 (p. 194), #11-17, 30-33
- Section 4.5 (p. 204), #1-5, 22-26, 42-45
- Section 4.6 (p. 210), #1-3, 7-9, 15-17, 22-24
- Section 4.7 (p. 215), #1, 3-5, 10-11
- Section 4.8 (p. 219), #1-10

COUNTING

- Counting can mean finding the number of things in a set
  - The set can be a regular set containing generic objects
  - Or it can contain objects constructed using a rule
  - For example, it can be the power set of a given set
- Counting can also mean finding the number of ways something can happen
  - For example, given an 8-bit byte, how many different 0,1 patterns are there?

COUNTING TECHNIQUES

- The idea of counting is to use shortcuts to count things
- For example
  - Suppose you have 20 rows of objects
  - There are 29 objects in a row
  - Someone asks how many objects there are in total

  - You can count the 580 objects: 1, 2, 3, …
    - You will soon tire of this
    - We seek shortcuts

THERE'S COUNTING AND THEN THERE'S COUNTING

- When we say "counting", we don't mean listing all the objects and counting them
- We mean finding a pattern and using it to tell the number of objects
- This chapter then, will be a search for patterns that "count" the number of objects
- The pattern for the example is this
  - We notice that all rows contain the same number of object (29 objects)
  - We also notice that there are 20 rows
  - We also know we can multiply 29 x 20 to get the total number of objects
  - This is the kind of pattern we are looking for

PATTERNS

- So, in this chapter
  - BE ON THE LOOKOUT FOR **PATTERNS**
- Every problem in the chapter will use some kind of pattern
- It's your job to find the pattern
- Keep this in mind as we read the chapter

ANOTHER EXAMPLE

- How many three-letter strings can be made from the letters A, B, C?
  - Sometimes people say "word" instead of "string"
  - Be careful! A "word" may not be an actual word!
- You can start to count them:
  - AAA, BBB, ABA, CAA, …
  - This gets tedious after a while
- You start over, listing the strings in order
- Your new list might look something like this
  - AAA, AAB, AAC, ABA, ABB, ABC, ACA, ACB, ACC, …
  - The list is getting too long, and you still haven't listed all strings

FINDING THAT PATTERN

- We look at the changing letters in the string
  - We look at individual positions
  - We notice that there are three letters for the first position, three for the second, and three for the third
  - We see that there are two-letter strings on the left, followed by a single letter
- This then gives us the pattern
  - The number of three-letter strings is 3 x the number of two-letter strings
- We notice that the two-letter strings follow the same pattern:
  - A two-letter string is a single letter followed by another single letter
  - So the number of two letter strings must be 3 x the number of one-letter strings
  - There are 3 single letters

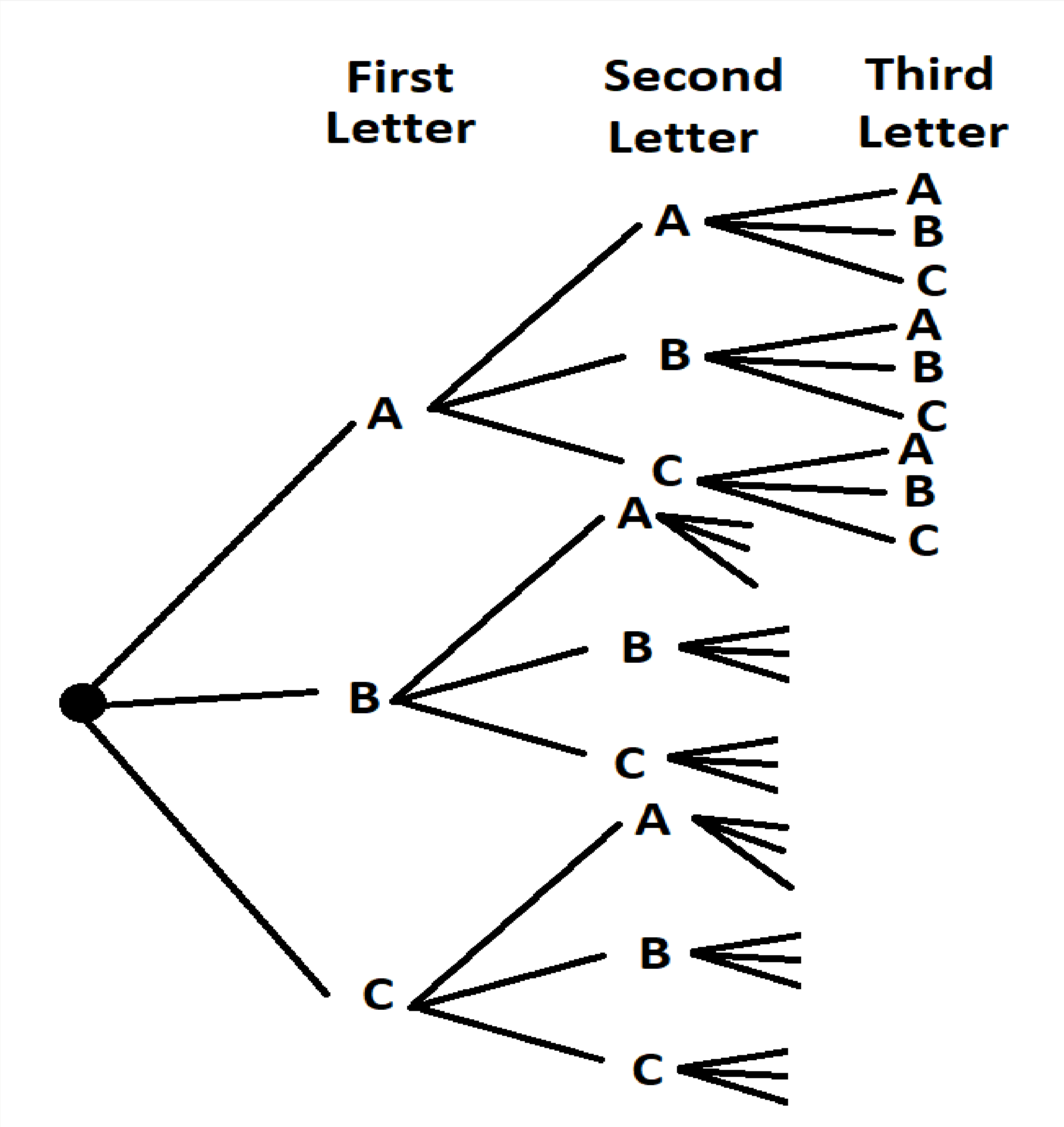
THE PATTERN

- Putting this all together, we discover that
  - The number of three-letter strings is 3x3x3, or 27
- You probably have known this counting technique for a long time

THE PATTERN-A TREE APPROACH

- We again try to find a pattern in how we list the strings
- One idea is to draw a tree
- Notes about trees:
  - Trees are "sideways"
    - They can also be upside down
  - The root (a real technical word) is on the left
  - The branches (another real technical word!) spread out to the right
  - The leaves (yet another real technical word) are the ends
- The point of drawing a tree is to count the number of leaves

THE TREE



First Letter
Second Letter
Third Letter
A
B
C
A
B
C
A
B
C
A
B
C
A
B
C
A
B
C
A
B
C

USING THAT TREE

- Remember, the point of the tree was to determine the number of three-letter strings
- From the tree we can see that there are 3 x 3 x 3, or 27three-letter strings
- Probability students will probably recognize trees
- I have often used trees to sort out difficult problems
- You should get comfortable with them too

- Can we use these ideas to answer this question:
  - What if we don’t allow duplicate letters?

A FIRST KEY SHORTCUT- THE MULTIPLICATION PRINCIPLE

- **Both of these problems use what is called the Multiplication Principle**
- **If an activity can be broken down into parts, and**
  - **The first part can be done in n_1 ways, and**
  - **The second part can be done in n_2 ways, and**
  - **The third part can be done in n_3 ways, and, …, until**
  - **The kth part can be done in n_k ways**
- **Then**
  - **The activity can be done in $n_1n_2n_3\ldots n_k$ ways**

A SECOND KEY SHORTCUT- THE ADDITION PRINCIPLE

- Suppose there are pairwise disjoint sets $X_1, X_2, \ldots, X_k$
- Suppose X_i has n_i items
- Then
- The number of elements that can be selected from X_1 or X_2 or … or Xn is

$$n_1 + n_2 + \ldots + n_k$$

- It's important that the sets be pairwise disjoint
- Notice that $n_1 + n_2 + \ldots + n_k$ is just the cardinality of the union of the sets

AN EXAMPLE OF THE ADDITION PRINCIPLE

- A die is shaped like a cube or a box
- It has the numbers 1 to 6 on it
- We will roll two dice
  - Dice is the plural of die
- Example 1: How many ways we can get a 2 or a 3 or a 4 using two dice?

ANOTHER EXAMPLE OF THE ADDITION PRINCIPLE

- It’s important that the sets be pairwise disjoint
- Example 2: How many ways we can get an even total or a total smaller than 6 using a pair of dice?

- If the sets are not pairwise disjoint, we will count some things multiple times
- We just subtract out all of those extra numbers
- The usual way of writing this for two sets X anY is

$$|X \cup Y| = |X| + |Y| - |X \cap Y|$$

- The formula is much harder for more than two sets

HOMEWORK

- You should now be able to complete the homework from Section 4.1
- Section 4.1 (p. 170), #5, 17-20, 28-30, 34-37, 42-46, 60, 62