

CS113 –Review for Final Exam

Note: You must show work for each problem. Any answer, even if it's correct, will receive no points if there is no work to support it.

1. Create a truth table for the expression $(p \wedge q) \vee (q \wedge r)$

2. Write the inverse of

If the sum of two even numbers is even, then the sum of two odd numbers is even.

3. Using induction, prove that, for $n > 0$,

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} = 1 - \frac{1}{2^n}, \text{ for } n = 1, 2, 3, \dots$$

4. Suppose X and Y are two sets. Using the properties given below, prove that

$$X' \cap Y' \subseteq (X \cup Y)'$$

Here the apostrophe represents the complement. You can assume there is a universal set containing each of X and Y .

$$x \in A \cap B \rightarrow x \in A \text{ and } x \in B$$

$$x \in A \cup B \rightarrow x \in A \text{ or } x \in B$$

$$A \subseteq B \rightarrow (x \in A \rightarrow x \in B)$$

$$A = B \rightarrow A \subseteq B \text{ and } B \subseteq A$$

$$x \in \bar{A} \rightarrow x \notin A$$

5. What is the power set of $\{1, 2, 3, 4\}$?

6. Convert the numbers 18 and 35 to base 2. Then add them. Then convert the answer back to base 10 to verify that you performed the various operations correctly. Make sure you do each step.

7. Convert the numbers as indicated.

a.) $16_{16} = ?_{10}$

b.) $25F_2 = ?_{16}$.

8. Suppose aRb if $5 \mid (b - a)$. The domain for the relation is the integers. Show that this is an equivalence relation. Then determine the equivalence classes.

Solution: To show something is an equivalence relation, you

9. What is a bijection? What can it be used for?

10. A committee of five people is to be formed from 10 women and eight men. The committee should have at least one man and one woman. How many different committees are there?

11. An event is rolling two dice to decide how many squares a piece should move in a game. You want to avoid the square 5 spaces ahead. What is the probability that a random roll avoids the bad square?

12. Are the two events rolling an even number on a die and rolling a number less than 4 on a die independent?

13. Solve the recurrence relation

$$a_0 = 3$$

$$a_1 = 2$$

$$a_n = 2a_{n-1} - 4a_{n-2}, n \geq 2$$

14. Draw K_3 .

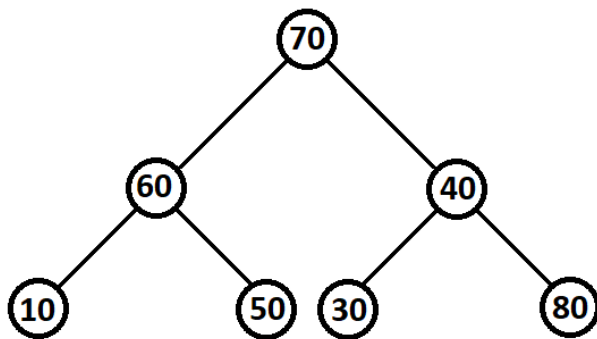
15. Can you find an Euler cycle in the graph pictured?



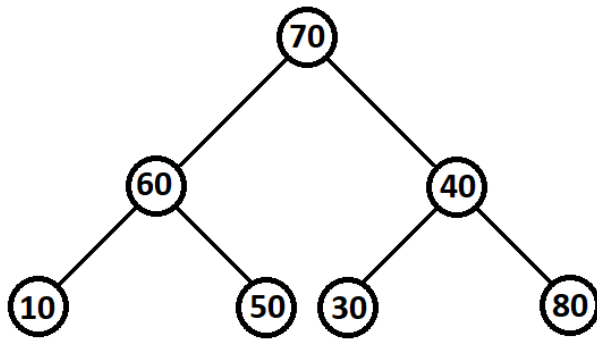
16. State Euler's formula and verify it for each graph below.



17. For the tree below, identify the root, a leaf, two nodes that are siblings, two nodes that are a parent and a child. Also, tell the height of the tree.



18. Show a pre-order traversal of the tree pictured.



19. Give one example or applications of trees.

20. Know the four equivalent conditions of Theorem 7.2.3. Also know what it means to say “the following are equivalent” in a theorem.

21. Explain what a spanning tree for a graph is, and which graphs have spanning trees..

22. Draw the circuit for a half adder