Review for Exam 2 in CS 113

Sets

1.) Given two (small) sets, be able to find their union, intersection, and set difference.

2.) Be able to find the power set of a set. (As a hint, you can count in base 2. We now know how to do that!)

3.) Be able to use the symbols $\in$ and $\subseteq$ correctly.

4.) Know what the empty set is and that it's a subset of every set. You can write it as {} or ϕ

$A \cap B = A \rightarrow A \subseteq B$

5.) Be able to use the formal rules about sets.

A. $x \in A \cap B \rightarrow x \in A$ and $x \in B$

B. $x \in A \cup B \rightarrow x \in A$ or $x \in B$

C. $A \subseteq B \rightarrow (x \in A \rightarrow x \in B)$

D. $A = B \rightarrow A \subseteq B$ and $B \subseteq A$

E. $x \in A' \rightarrow x \notin A$ (Here I have used ' for complement.)

A typical problem might be to show that $A \cap B = A \rightarrow A \subseteq B$

6.) Given a universal set, be able to draw a Venn diagram and use it to get information.

7.) Given a set, be able to tell if a list of sets is a partition of the original set.

Sequences

Be able to write the first few terms of a sequence given either a recursive formula or a non-recursive one.

Math in Various Bases

1.) Be able to convert numbers between bases (2, 10, 16.)

2.) Be able to add binary numbers

Relations

1.) Given a relation, be able to write its matrix

2.) Given a relation, be able to draw its graph

3.) Given a relation, be able to tell if it's reflexive, symmetric, transitive, and/or antisymmetric.

4.) Given two relations, be able to compose them.

5.) Given a relation, be able to determine if it's an equivalence relation or not
If it is an equivalence relation, be able to find the equivalence classes

6.) Given a relation, be able to determine if it's a partial order or not

Functions

1.) Be able to determine if a function is 1-1 (injective), and/or onto (surjective)

2.) Know that if there is a bijection (1-1 and onto function) between two sets they are the same "size". (We actually say that the sets "have the same cardinality")

3.) Be able to determine if a function has an inverse or not

Some Sample Questions

1. The universal set is U. U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. A = {1, 2, 3, 4, 5}. B = {1, 3, 5, 7, 9}. C = {1, 3, 6, 9}.

   a.) Find $A \cap B$.

   b.) Find $A \cup C$

   c.) Find $B - C$

   d.) Find P(C), the power set of C.

   e. True or False.

      i.) $\phi \subseteq S$, for every set S

      ii.) $\phi \in S$, for every set S

      iii.) $A \subseteq B$

      iv.) $1 \subseteq B$

2. Show $A \cap B = A \rightarrow A \subseteq B$ using the formal rules about sets. (The rules will be given on the exam.)

   First, notice that we need to show that $x \in A \rightarrow x \in B$

   $A \cap B = A$ means that $A \subseteq A \cap B$ by the second part of Property D

   So, now suppose that $x \in A$

   Then $x \in A \cap B$ by Property C applied to the second line above

   Then also $x \in B$ by Property A

   So now I have shown that, if $x \in A$ then $x \in B$, which means $A \subseteq B$

3. A school with 50 students offers classes in Spanish and French. Students can take Spanish or French or both languages or neither. There are 12 students taking both languages. There are 20 students taking French. There are 18 students taking Spanish. How many students are not taking a language?

4. The formulas below represent sequences. Write the first four terms of each sequence.

   a.) $a_n = 3n + 2$, n = 1, 2, 3, 4, …

   b.) $a_1 = 1$, $a_n = 2a_{n-1} + 1$, n = 2, 3, 4, …

5. Convert the numbers below as indicated.

   a.) $37_{10} = ?_{16}$

   b.) $10110_2 = ?_{10}$

   c.) $10111001_2 = ?_{16}$ (You will be given the hex-binary conversion table.)

6. Add the binary numbers 10111 and 1011.

7. For the relation (a,b) R (c,d) if 7 divides ad – bc, on the integers x the integers, tell if it's reflexive, symmetric, transitive, and/or antisymmetric.

8. For the relation {(a,b) | 13 divides into b – a} on the integers, tell if it's an equivalence relation or not. If it is an equivalence relation, find the equivalence classes.

9. For the relation {(1,1), (1,2), (2,2), (3,3), (3,4), (3,5), (4,4), (4,5), (5,5)} on the set {1, 2, 3, 4 5}, tell if it's a partial order or not.

10. For the relation in Problem 9:
    a.) Draw the graph of the relation.
    b.) Write the matrix of the relation

11. For each function, determine if it's one-to-one and/or onto.
    a.) $f(x) = x^3$, Domain = Real numbers, Range = Real numbers
    b.) $f(x) = \sqrt{x}$, Domain = Non-negative real numbers, Range = Non-negative real numbers
    c.) $f(x) = \frac{1}{x}$, Domain = Positive real numbers, Range = Positive real numbers

12. If any function above has an inverse, find the inverse.

13. Do the two sets {1, 2, 3, 4, …} and {3, 6, 9, 12, …} have the same cardinality?