Proofs

Some Preliminary Ideas

- 1. What do odd and even mean?
  - n odd means that $n = 2k+1$, and n even $n = 2k$ for some integer k.
  - Find k for 15, 18.
- 2. $x > y$ and $z > t$ implies that $x+z > y+t$.
  - Based on properties of real numbers.
- 3. Another idea: Positives multiplied by positives give positives
  - That is, $a > 0$ and $b > 0$ implies that $ab > 0$
- 4. Still one more idea:
  - $x > y > 0$ and $z > t > 0$ means that $xz > yt$.

Direct Proof

- Theorem A: If a and b are two positive numbers, and $a > b$, then $a^2 > b^2$.
- Theorem B. If n is an odd integer then n^2 is also odd.

Indirect Proof

- Theorem C. If n^2 is an even integer then n is also even.
- Theorem D. The square root of 2 is irrational; that is, it can't be written as a fraction of two integers.

Proof Using the Contrapositive

- Theorem E. If n^2 is an even integer then n is also even.
- Theorem F. If a product of two positive real numbers is greater than 100, then at least one of the numbers is greater than 10.

Proof Using Cases

- Also called Proof by Exhaustion
- Theorem G. Lim as (x->4) of x-3 = 1
  - This is a two-sided limit
- Theorem H. Prove that if n is any integer not divisible by 3, then n^2 leaves a remainder of 1 when it is divided by 3.