Worksheet: Congruences

1. Simplify.

a.) 4 (mod 3) = 1	e.) 18 (mod 4) = 2	i.) 2 (mod 5) + 3 (mod 5) = 0
b.) 11 (mod 2) = 1	f.) (-2) (mod 5) = 3	j.) -1 (mod 5) +3 (mod 5) = 2
c.) 7 (mod 5) = 2	g.) (-3) (mod 12) = 9	k.) 15 (mod 5) + 17 (mod 5) = 2
d.) 15 (mod 5) = 0	h.) 2 (mod 5) + 2 (mod 5) = 4	l.) 12 (mod 5) + 13 (mod 5) = 0

2. See if you can solve each equation below. Note: Some of these have no answers; some have one answer; some have multiple answers. (What's up with that? Aren't these linear equations?) (The equations have multiple solutions because different numbers can reduce to the same mod because there are only so many possible solutions.)

a.) $4x \equiv 3 \pmod 8$ Not possible, 4n is always 0 or 4 in mod 8.

b.) $3x \equiv 9 \pmod{10}$ x=10n+3, n is an int

c.) $3x \equiv 7 \pmod 8$ x=8n+5, n is an int

d.) $2x \equiv 6 \pmod{12}$ x=6n+3, n is an int

3. Try to calculate these expressions, using ideas similar to those I discussed in class.

a.) $6^4 \pmod 8 \equiv 0$

- $(6^2)^2 \pmod 8$
- $36^2 \pmod 8$, $36 \pmod 8 \equiv 4$
- $4^2 \pmod 8$
- $16 \pmod 8$
- $\equiv 0$

b.) $8^9 \pmod 7 \equiv 1$

- $8 \pmod 7 \equiv 1$
- $1^9 = 1$
- $\equiv 1$

c.) $8^{15} \pmod 5 \equiv 2$

- $8 \pmod 5 \equiv 3$
- $3^{15} \pmod 5 = (3^3)^5 \pmod 5$
- $27^5 \pmod 5$, $27 \pmod 5 \equiv 2$
- $2^5 \pmod 5$
- $32 \pmod 5$
- $\equiv 2$