CHAPTER 4

COUNTING AND THE PIGEONHOLE PRINCIPLE

HOMEWORK

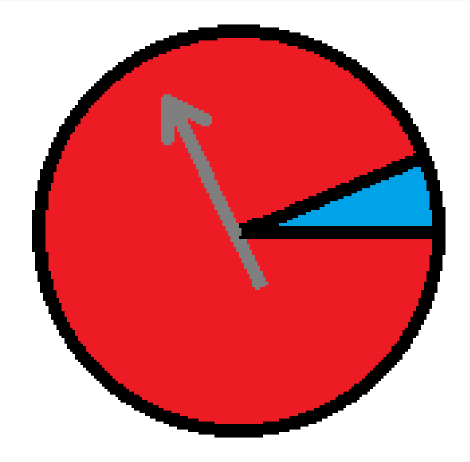
- Again, all homework is from the Exercises
  - No problems are from the Review Exercises
- Section 4.1 (p. 170), #5, 17-20, 28-30, 34-37, 42-46, 60, 62
- Section 4.2 (p. 182), #10-14, 25-29, 31-34, 58-62
- Section 4.4 (p. 194), #11-17, 30-33
- Section 4.5 (p. 204), #1-5, 22-26, 42-45
- Section 4.6 (p. 210), #1-3, 7-9, 15-17, 22-24
- Section 4.7 (p. 215), #1, 3-5, 10-11
- Section 4.8 (p. 219), #1-10

A PROBABILITY FUNCTION

- **A Probability Function is a function P(x) from a sample space to [0, 1]**
- **We read P(x) as "P of x"**
- **This means that $0 \leq P(x) \leq 1$ for all x in the sample space**
- **Also,** $\sum_{x \in S} P(x) = 1$

CALCULATING THE PROBABILITY OF ANY EVENT

- Until now, all elements of the sample space were equally likely
  - That means they had the same chance of happening
- If E is an event, P(E) = the sum of the probabilities of the outcomes in E
- Example: #4 on p. 204
- Another example:
  - For the spinner pictured, can you land in the blue?
  - Would it be easy?
  - The spinner is 1/8 of the circle
  - So, P(red) = 0.875, or 87.5% and P(blue) = 0.125, or 12.5%



RULES FOR CALCULATING PROBABILITIES

- **There are several rules for calculating probabilities**
- **These are like the counting shortcuts**
  - **They allow us to avoid long detailed additions and multiplications**

THEOREM 4.5.5 (OFTEN CALLED THE COMPLEMENT RULE)

- The complement of an event E is the "rest" of the sample space after E is removed
  - I will denote the complement of E as E'
  - In our set notation E' = S – E, where S is the sample space
- This rule relates the probability of an event to the probability of its complement
- The rule says P(E) + P (E') = 1
- We often use the rule in the format P (E') = 1 – P (E)

AN EXAMPLE: THE BIRTHDAY PROBLEM

- The Problem is
  - How many people do you need in order to guarantee that P(two identical birthdays) > 0.5?
- We have a group of people
- We ask each person to write down their birthday (month and day only, not the year)
- We want to find two identical birthdays
- In terms of probability, this is
  - What is P(two identical birthdays)?
- This is too hard to computer
- Let’s check a tree

ACTUALLY SOLVING THE BIRTHDAY PROBLEM

- That's too hard
- It's easier to compute P(E')
  - This is
    - P (No two people have the same birthday)
- P (two identical birthdays out of n people) = 1 - P (No two people have the same birthday)
- We calculate P (No two people have the same birthday)
- Trying numbers shows that with 22 people, P (Two equal birthdays) = (about) 0.475965,
- We can also calculate that with 23 people, P (Two equal birthdays) = (about) 0.507297
- So, if you have 23 people, your chances are slightly better than ½ for two equal birthday

THEOREM 4.5.9-VERSION 1 (OFTEN CALLED THE ADDITION RULE)

- If $E \cap F = \emptyset$, the events E and F are called mutually exclusive events
- The theorem says
  - If E and F are mutually exclusive events, $P(E \cup F) + P(E) + P(F)$

- Let's try #16, 17, 18, p. 205. Also, let's find $P(E_2 \cup E_3)$
- First, we need to list the elements of the sample space

- You can also read this as
  - The probability of E or F happening is the probability of E + the probability of F
  - Again, for this to hold, E and F must be disjoint
  - Otherwise, you counted some outcomes more than once

THEOREM 4.5.9-VERSION 2

- What do you do if $E \cap F \neq \emptyset$?
- The theorem actually says
  - For any events E and F, $P(E \cup F) + P(E) + P(F) - P(E \cap F)$
- Notice that this is always true
  - it doesn't matter what E and F are
- Let's try #10 on p. 205

DEFINITION 4.5.13 (OFTEN CALLED THE CONDITIONAL PROBABILITY RULE)

- This rule computes the probability of one event, given that another event has already occurred
- The rule is $P(E \mid F) = \dfrac{P(E \cap F)}{P(F)}$
- We read that as P of E given that F has occurred, or P of E given F
- It should be easy to understand the formula
  - Everything is based on assuming that F is a certainty
- Let's try #25, 26 on p. 205

INDEPENDENT EVENTS

- Two events E and F are independent if knowing that one occurred tells you no information about the second
- Let's try #28, 29 on p. 205
- For example, flipping two coins
  - The two flips are independent of each other
  - If the first coin come up heads, can you tell what the second will be?
- If $P(E \cap F) = P(E)P(F)$, then E and F are independent
- $P(E \mid F) = P(E)$ is the same test
- There are two reasons why that is true
  - One is from the formula for conditional probability; the other is from understanding

THE MULTIPLICATION RULE FOR INDEPENDENT EVENTS—AN EXAMPLE

- The rule from the previous slide says that if E and F are independent events, then
  - P (E AND F) = P (E) P (F)
- Roll a die
- What is P(Odd number)?
- Small means 1, 2, or 3
- What is P (Odd | Small)?
  - Knowing Small changes P(Odd)
  - So we know that odd and small are not independent

HOMEWORK

- You should now be able to do the homework from Sections 4.4, 4.5
- Section 4.4 (p. 194), #11-17, 30-33
- Section 4.5 (p. 204), #1-5, 22-26, 42-45