NP

NP Hard	NP Comp	
	NP	P

Reduction: $Y \leq_P X$ ← Turn this into X

Prove NP Complete: Prove NP = Show polynomial time certifier (Can check for a yes instance)

Prove NP Hard = Show reduction & Show that if S_Y is for Y then S_X is yes & if S_X is yes then S_Y is yes (→ / ←)

Known NP Problems:

3SAT

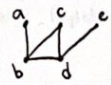
$X = \{x_1, \dots, x_n\}$ $C = C_1 \wedge C_2 \wedge \dots \wedge C_k$

Each clause has 3 X literals ∨ together (they can be noted $\bar{x}_i$)

Ex: $(x_1 \vee x_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee x_2 \vee x_3)$

Independent Set (Goal: Get as many V w/ out violating edge constraint)

A subset of graph is independent if contains no adjacent vertecies

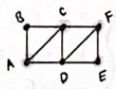


{a, c, e} } Examples of independent sets
{b}

maximum independent set Vertex # = 3

Vertex Cover (Goal: cover all E w/ as few V as possible)

Subset of vertices is vertex cover if every edge is incident / connects to a vertex in S & k = max vertex to use



k = 4

{A, C, F, E} Example of nodes that cover all edges

Set Cover

U: $S_1, \dots, S_m$ universe w/ set of subsets

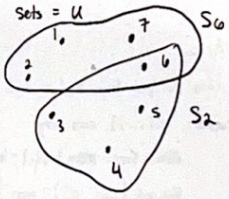
K = max # sets want to find such that the entire universe is covered such that union of sets = U

$U = \{1, 2, 3, 4, 5, 6, 7\}$ $k = 2$

$S_1 = \{3, 7\}$ $S_2 = \{3, 4, 5, 6\}$

$S_3 = \{1\}$ $S_4 = \{2, 4\}$ Use S_2 & S_6 to cover all of U

$S_5 = \{5\}$ $S_6 = \{1, 2, 6, 7\}$ S_2 union $S_6 = \{1, 2, 3, 4, 5, 6, 7\}$



3D Matching

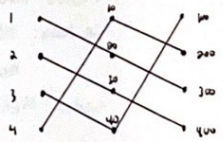
Given 3 sets X, Y, Z

T = subset of X × Y × Z and contains triples (x, y, z)

For any 2 distinct triples (x_1, y_1, z_1) and (x_2, y_2, z_2) must have $x_1 \neq x_2$ $y_1 \neq y_2$ $z_1 \neq z_2$

$X = \{1, 2, 3, 4\}$ $Y = \{10, 20, 30, 40\}$ $Z = \{100, 200, 300, 400\}$

Want each X Y Z to appear in exactly 1 element: {(1, 20, 300) (2, 30, 400) (3, 40, 100) (4, 10, 200)}



VC ≤ IS ≤ SP

3SAT ≤ IS ≤ VC ≤ SP

VC ≤ SC Given G = (V, E) & k

Reduction:
- U = E
- S_v = Subset w/ all edges incident to V
- k = k

SC size ≤ k ↔ VC size ≤ k

3SAT ≤ IS

Reduction: x_i Conflict $\bar{x}_i$ ... Each triangle represents an clause in 3SAT

See TA notesheet for template to write out answer for proof of NP Hard