DP(merchants):

Rough Problem Description:

There are n merchants in a line.

There are 3 items A, B, C.

You start with any of the items and go through the merchants in order. At each merchant, you sell the merchant the item you have buy an item of your choice that you then take to the next merchant. Each merchant values the items different and merchant m will buy/sell item i for $V_m(i)$

The goal is to maximize total profits after going through all n merchants.

Solution:

this is a slightly different bellman than ben's but still works

$A[i][m]$ = the maximal profit attainable through the first m merchants where we end with item i.(i.e. buy item i from merchant m)

$$
\begin{aligned}
&\textbf{Base Cases:} \\
&\qquad A[i][0] = 0 \qquad\qquad\qquad\qquad i \in \{A, B, C\} \\
&\textbf{Reccursive:} \\
&\qquad A[i][m] = \max \begin{cases} A[A][m-1] + V_m(A) - V_m(i) \\ A[B][m-1] + V_m(B) - V_m(i) \\ A[C][m-1] + V_m(C) - V_m(i) \end{cases} \\
&\textbf{Return:} \\
&\qquad \max_{i \in \{A,B,C\}} A[i][n]
\end{aligned}
$$

Fill forward left to right(increasing order of m)

Runtime $\mathcal{O}(9n) = \mathcal{O}(n)$

Intractiblity(Rectangle Tiling):

Rough Problem Description:

You are given a target rectangle T and a bunch of smaller rectagles $\{r_i\}$ Is it possbile to exactly cover T with the smaller rectangle with no overlap?

Prove that the above problem is NP-Hard.

Solution:

Subset Sum $\leq_P$ Rectangle Filling:

For an instance of subset sum $\{x_i\}, k$

Create target rectangle T w/dims: $1 \times k$

For each element create rectangle r_i: $1 \times x_i$

The target is tillable if and only if the lengths sum to k. This happens if and only if the substance sum of was feasible.
this proof of the reduction is very brief and on an exam you should do the standard proof by showing both directions of the if and only if separately