A Cautionary Tale on the Marriage of Mathematics and Physics

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Class of 2024

Submitted for:

Undergraduate Essay in the Natural Sciences

Epitome: This piece attempts to dispel the notion that the usefulness of mathematics in physics is a mystical property of the universe and emphasizes the dangers of holding this viewpoint.

Word Count: 6357

Introduction

The marriage between mathematics and physics is a tale as old as time. Today, mathematics has solidified its role as both the language and rational mode of argumentation in the field of physics. The union between mathematics and physics is so deeply entrenched in intellectual discourse that we may forget to scrutinize it or envision alternative possibilities.

The history of physics is brimming with examples of the often prophetic nature of mathematics in describing the natural world. In 1928, physicist Paul Dirac sought to resolve the problem that the relativistic Klein-Gordon equation yielded negative probability densities. He introduced the use of 4×4 matrices as coefficients, solving the probability density problem but generating yet another enigma: negative energy solutions to the equation. It turned out that these negative energy solutions, which some looked at as a mere mathematical artifact, predicted the positron and the existence of antimatter several years before they were ever detected. A few years before that, Albert Einstein developed the Theory of General Relativity, drawing upon the mathematical formalism of Riemann curvature tensors and differential geometry. After his seminal 1905 paper, he spent the rest of his life wavering about the predictions made by the theory, particularly about its prediction of gravitational waves. Nearly 100 years later, two 4 kilometer-long lasers detected the Earth stretching by a thousandth of the width of a proton, successfully confirming the existence of gravitational waves [1]. When used in physics, mathematics can seem to take on a life of its own, sometimes defying the intuition of the physicists directing its usage. It also works with astonishing precision in some cases. Quantum Electrodynamics (QED) predicts the magnetic moment of the electron with up to 11 decimal places of agreement with experimental measurements.

For centuries, physicists and mathematicians have puzzled at the enigmatic connections between physics and mathematics. In 1623, Galileo Galilei proclaimed that "the book of nature is written in the language of mathematics." The mysterious interplay between physics and mathematics received perhaps its most famous expression nearly three centuries later in physicist Eugene Wigner's 1960 essay entitled, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [2]. In this essay, Wigner sheds light on the idea that "the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it." He provides three illuminating examples of the unreasonable effectiveness at work: the use of second derivatives in describing planetary motion, the use of matrix mechanics in quantum mechanics, and the use of pure mathematics in the theory of the Lamb Shift. The near floating point accuracy in the predictions of some mathematical physics theories illustrates that it is difficult to think of mathematics as merely a handy notation in physics; rather, Wigner believes that "it is, in a very real sense, the correct language." Toward the conclusion of the essay, Wigner appeals to the mystery. He says, "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve."

Wigner's essay prompted important philosophical discourse about the nature of mathematics, physics, and their dynamic relationship with each other. But, the field of physics has evolved significantly since the time of Wigner's writing. With the advent of the computer, the field of computational physics has burgeoned into an important field that allows us to perform some of the less elegant physics calculations that would have been unimaginable in Wigner's time. With 60 years of physics and mathematics between us, it is worth reevaluating "the unreasonable effectiveness" problem and the impact it has had on the field of physics.

In this paper, my aim is to demystify the effectiveness of mathematics in physics. To achieve this, I will delve into the major schools of thought that have emerged in the philosophy of mathematics – namely Pythagoreanism, Platonism, and Formalism – to argue that mathematics derives its inspiration from the physical world, and accordingly, it is reasonable that it should be effective at describing it. Furthermore, I will contend that an affinity for beauty animates mathematicians to work on particular problems that are relevant to physics. Throughout this all, I describe what the dangers are of oversubscribing to the "unreasonable effectiveness of mathematics." We should indeed marvel at the cases where math is so effective in physics, possibly due to symmetries in nature or other rational explanations. But, by looking so intently for agreement between mathematics and the physical world, we may well be turning the "unreasonable effectiveness of mathematics" into a self-fulfilling prophecy. This pattern of looking restrictively for scientific phenomena that affirm our current conception of reality is well-documented in the history and philosophy of science. Finally, we should bear in mind that physics at its core, is a field driven by humans. Exploring the innate desire for feelings of rationality in science illuminates why the "unreasonable effectiveness of mathematics" lures physicists to use *more* mathematics in physics when this may be counterproductive. Ultimately, I will argue that we should avoid the temptation of overindulging in mathematics in our quest to understand the physical world. We should look with open eyes for new scientific paradigms that will allows us to make great strides in answering the most elusive problems in modern physics.

The Inspiration of Modern Mathematics

"How can it be that mathematics, being after all a product of human thought, independent of experience, is so admirably appropriate to the objects of reality?" - Albert Einstein

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The nature of mathematics and its relation to the world have been richly debated since antiquity. One of the most profound epistemological debates that has emerged in the discourse on the philosophy of mathematics is the invention-discovery dilemma. This debate seeks to determine if mathematics is invented – in the way that language is invented – or discovered – in the way that scientific laws are uncovered through observation and experimentation. While the field of mathematics is unlikely to achieve any sort of consensus on this question, the two word invention-discovery dichotomy oversimplifies a more nuanced debate. Albert Einstein's suggestion that mathematics is "independent of experience" is one such intricacy worth exploring. Importantly, without having to resolve the broader debate, we can delve into the question of from where mathematics, either invented *or* discovered, derives its inspiration.

By investigating the inspiration behind mathematical developments, we can challenge the belief of Wigner and others that mathematics' usefulness in the natural sciences is a mystical quality of the universe. If modern mathematics draws its inspiration from the physical world, it is certainly less remarkable that it is effective at modeling it. There are two fundamental reasons to believe that mathematics *is* intrinsically linked to the physical world. First, much of early mathematics was born directly out of an interest in comprehending nature. To understand how modern mathematics turned away from its original focus, we must consider the development and rise to popularity of the formalist view of mathematics, which will shed light on the *reasonability* of the effectiveness of mathematics in physics. The second point is that even if we believe modern mathematics emanates purely from the human mind, our minds are implicitly shaped by the world in which we live.

There have been several prominent philosophical camps that have advanced distinct explanations about how mathematics is developed. The Pythagoreans in the 6th Century B.C. were the first to connect the nature of the physical world to mathematics. They famously adopted the mantra, "all is number." In other words, they believed everything in the universe was composed of "the elements of number." This marked the first time in recorded history that the practice of mathematics transcended its typical role as a practical tool and was treated as more of a divine pursuit intended to understand our place in the universe.

The Pythagorean tradition was influential on the work of Plato, who developed the philosophical theory now known as Platonic Realism. Platonic Realism posits that our physical reality is a mere reflection of a transcendent realm of Forms. These Forms represent the true, abstract manifestations of the concepts and ideas that we encounter in the world. While the scope of Platonic Realism is not limited to mathematical objects, Platonism has become an important doctrine in the philosophy of mathematics. Platonists have imbued concepts and structures in mathematics with ontological status, suggesting they have an objective reality that exists independently and outside of the human mind. This perspective is consistent with the view that mathematics is discovered, not invented. The distinction between Pythagoreanism and Platonism is subtle but important. The Pythagoreans believe that the structure of nature is fundamentally mathematical whereas Platonists believe that the true mathematical forms exist in an external realm – separate from nature – but that we can use these abstract forms to describe nature.

Platonism stands in direct contravention to the formalist or nominalist belief that mathematical concepts are mental constructs, lacking any metaphysical existence outside of human cognition. This view is consistent with the belief that mathematics is invented, not discovered. Importantly, formalism at its core does not exclude the possibility that the development of mathematics is shaped by inspiration from the physical world, although formalists tend to argue that mathematics is an emanation from the mind, unmarred by external influences. Indeed, my objection to Einstein's position is that even if we accept the formalist idea that mathematics is a "product of human thought," it is *not* "independent of experience." Since a formalist view of mathematics is the fundamental underpinning of Wigner's belief in the "unreasonable effectiveness of mathematics," it is essential to untangle how the formalist position emerged.

Jeremy Gray, a historian of mathematics, presents an insightful explanation in his book *Plato's Ghost: The Modernist Transformation of Mathematics* of how 20th-century mathematicians and physicists, including Albert Einstein, became invigorated by the spirit of mathematical formalism. He contextualizes this transformation as belonging to a broader cultural shift toward modernism [3]. At the outset of the book, Gray provides a general definition of modernism as "an autonomous body of ideas, having little or no outward reference, placing considerable emphasis on formal aspects of the work and maintaining a complicated – indeed, anxious – rather than a naïve relationship with the day-to-day world." He mentions how modernist transformations first took place in art and other non-scientific fields before it extended to mathematics. The success of this transformation in mathematics can be attributed to the efforts, primarily led by David Hilbert at the beginning of the 20th century, to axiomatize mathematics. This had the effect of detaching modern mathematics from its long history. Gray also argues that the increased specialization of mathematicians and physicists, a consequence of mathematicians and physicists starting to operate in separate spheres at universities in the 20th century, created the false impression that the two fields exist in isolation when, in reality, they had been mutually enriched by a long history of developing in tandem.

The successful axiomatization of mathematics in certain fields allowed for the reconstruction of the entire field from its basic axioms, disregarding the history of its development. For some, this adds fuel to the fire of formalism. Although the mathematics of the day *could* be distilled to a set of axioms, the axioms themselves were distilled from a mathematics that derived its inspiration from the natural world. Had it not, the history of mathematics could have taken a different path, and the mathematics they inherited could have been very different. It is impossible to entirely divorce one's thinking from the world that surrounds us even though axiomatic frameworks give the illusion that complex mathematics can be developed without an awareness of the outside world. Mathematician Michael Atiyah posed a powerful thought experiment on this topic. He said, "... let us imagine that intelligence has resided, not in mankind, but in some vast solitary and isolated jelly-fish, buried deep in the depths of the Pacific Ocean. It would have no experience of individual objects only with the surrounding water. Motion, temperature and pressure would provide its basic sensory data. In such a pure continuum the discrete would not arise and there would be nothing to count" [4]. This thought experiment illuminates the idea that even our most abstract mathematical thoughts are products of the environment in which we live. While mathematics can transcend the world in which it was conceived – continuous mathematics and discrete mathematics are just as true in the depths of the ocean as they are on land – it is still a product of that world, and this should not be overlooked.

The more one champions the formalist interpretation of mathematics, the more "unreasonable" the effectiveness of mathematics in physics appears. How can human minds independently devise concepts that describe reality when the concepts' instantiations were not grounded in reality? This is the sentiment of many 20th century physicists. In recent years, one reaction to the applicability of abstract mathematics in physics has been to revive a Neo-Pythagoreanist spirit. A striking example of this is MIT cosmologist Max Tegmark's Mathematical Universe Hypothesis [5]. In his book on the theory, he posits that the physical universe *itself* is a mathematical structure. As a result, the Tegmarkian resolution to the "unreasonable effectiveness" problem is that mathematics is effective in physics because the physical world is at its core mathematical.

I will not attempt to refute the Mathematical Universe Hypothesis on physical grounds, although several works have attempted to do so. Instead, I will contend that if Tegmarkian mathematical creationism or even moderate forms of it turn out to be inaccurate representations of the universe, adhering to them may blind us to our physical reality in a way that parallels our past acceptance of flawed scientific theories, such as geocentrism, and impede our ability to gain a true understanding of the natural world.

A Mathematical Fishing Net

Sir Arthur Eddington had a famous parable about a fisherman, who day after day, went out to sea with his net to catch fish. After years of fishing, he proclaimed to his best friend that he had discovered a new and important law of nature: all fish are bigger than five centimeters! His friend, surprised by this law, proceeded to ask him what the size of the holes in his fishing net were. To the fisherman's surprise, they measured to be exactly five centimeters.

When we approach physics with mathematics as our net, it is no surprise that we begin to believe we live in a mathematical universe. Eddington's tale emphasizes that our understanding of the world is inherently limited by the lenses through which we look at it. This story specifically highlights the limitations of physical tools – the size of the holes in the fishing net in this case. The history of science has demonstrated that often, scientific progress has been hindered by the limitations of our physical tools. In these cases, technological innovation gives us access to understanding previously elusive phenomena, and perhaps some of the open questions that persist in physics today, too, will be resolved when we build more powerful detectors, computers, microscopes, or telescopes. More troublingly, history has also shown that at times our inability to solve scientific problems stems from flawed intellectual frameworks that impede us from perceiving the world as it truly is. Identifying these barriers is more challenging as mental blindspots are difficult to recognize in real-time, without the benefit of hindsight. In this context, I will explore how mysticism surrounding the usefulness of mathematics in physics may be fueling our generation's own mental blindspot. By acknowledging that many phenomena in nature cannot be described well with mathematics, I will argue that we should be wary of our infatuation with mathematics and its ability to explain the physical world. By selectively working on problems in which we can build elegant mathematical theories, the physicists of today may be constructing the same types of anthropocentric blinders that have obscured scientific reality from societies of the past.

The two-body problem is a canonical problem in classical mechanics where given two masses and some initial parameters one solves for the motion of the two masses. In elementary physics courses, it is common for students to solve the two-body problem over and over again with a wide variety of set ups. However, when students of physics take the seemingly innocuous step of adding a third mass to the problem, they see the beautiful mathematical nature of the physical world collapse before their eyes because, in general, there is no closed form mathematical solution to the the three-body problem. If we focus on the triumphs of abstract mathematics in leading us to an understanding of general relativity and the curvature of spacetime, we forget the simple examples of where mathematics fails us.

Over the past century, no area of physics has experienced greater abandonment than the field of fluid mechanics. I posit that this trend is not a coincidence but rather that this can be attributed, in large part, to the construction of our mathematical fishing net. Fluids, like air and water, are ubiquitous in our everyday lives and their enigmatic behavior often defy our understanding. Fluid mechanics also has no shortage of open problems. Richard Feynman once called turbulence, "the most important unsolved problem in classical physics." With its relevance to everyday life, abundance of unsolved problems, and relatively low cost for experimentation, it is worth questioning why fluid mechanics has gone so out of fashion. Fluid mechanics is governed by a non-linear set of partial differential equations, requiring powerful supercomputers to solve or, more often, simulations to find approximate solutions. Indeed, one of the six remaining unsolved Millennium Prize Problems posed by the Clay Mathematics Institute is to prove the existence of smooth solutions to the Navier-Stokes Equation [6]. Physicist Werner Heisenberg who launched his physics career working on fluid mechanics before transitioning to quantum mechanics once said, "When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first." Fluid mechanics gives rise to many problems that do not lend themselves well to elegant mathematical frameworks, and this quotation reflects Heisenberg's belief that fluid mechanics somehow slipped through the cracks of the mathematical creation of the universe. Despite the wealth of computational and experimental tools that we now have at our disposal, this is a paralyzing notion, and unfortunately, this has made fluid mechanics one of the many casualties of the mathematization of physics. If physicists direct their attention *en masse* to problems that conform well with abstract mathematical frameworks and neglect the ones that do not, it should be no surprise that we find great agreement between physics and mathematics: it is merely a selection effect.

This example of how the current framework in physics often selects problems that can be described effectively through mathematics and brings them to the forefront connects to another intriguing aspect of Wigner's essay: its title. One cannot overlook the fact that Wigner titled the essay "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" when all of his examples are drawn from physics. Typically, the natural sciences refers to physics, astronomy, chemistry, biology, and geology, and yet all of Wigner's examples about the unreasonable effectiveness of mathematics are culled from physics. Perhaps this was a mindless editorial choice, but few would argue that mathematics is unreasonably effective in any of the natural sciences aside from physics. There may be some practical reasons for this. For example, we typically distinguish physics from other fields based on its high degree of fundamentality. This alone could provide a reason why the "unreasonable effectiveness of mathematics" is most conspicuous in physics. In theoretical physics, we often deal with abstract concepts and principles that are independent of any particular experiment or observation. It is possible that in other fields, such as biology or chemistry, there is no need for such a high degree of abstraction and generality. More saliently, I believe that we have used the "unreasonable effectiveness of mathematics" to redefine the central questions, demarcate the boundaries of physics, and shape its priorities around finding agreement between the natural world and elegant mathematics. This may be well-justified if we believe that our current mathematical proficiency makes approaching physics with mathematics the most tractable way to make progress. As great successes with the use of mathematics in physics suggest, there is merit to this approach, but we should be transparent with ourselves about whether mathematics *truly* describes the inherent structure of nature or if this is simply a consequence of our chosen approach.

If we do not live in a mathematical universe – a hypothesis that has several weaknesses to address if it is to be taken seriously – accepting that the universe has a fundamental mathematical nature could profoundly impact the progress of physics. Consider the Copernican

Revolution, when the prevailing view was that Earth was the center of our universe. This was not because the scientists made flawed logical inferences based on the data available at the time. As Thomas Kuhn, the author of the seminal philosophy of science book *The Structure of Scientific Revolutions* pointed out, the intellectual and cultural traditions in which scientists are immersed play a crucial role in shaping how the facts and data they collect are interpreted [7]. He describes scientific revolutions as non-linearities in a rational process, occurring only when enough experimental or observational anomalies have built up *and* an alternative paradigm can account for all of them well. This has been true not only for geocentrism but also for beliefs about a flat Earth and the existence of the ether. In hindsight, we can see the folly of the beliefs of our predecessors. However, their oversights were not merely due to a lack of sophisticated scientific instruments or logical reasoning ability, but more importantly, they were not yet culturally prepared for the revolutionary thinking or the implications of the new science.

The course of intellectual history has been defined by the constant process of revising and reevaluating our understanding of the world. Unless we are entering a new post-truth era, the scientists of the future will likely look back on the present day and identify flaws in our current ways of thinking. In the realm of physics, we already recognize the shortcomings of our current theories having seen physics beyond the standard model of particle physics and being thus far unsuccessful in unifying general relativity and quantum theory. As such, we should be critical of our infatuation with using elegant mathematics in physics to ensure that it is not impeding progress.

Beauty and Simplicity in Mathematics and Physics

"The chief forms of beauty are order and symmetry and definiteness, which the mathematical sciences demonstrate in a special degree." -Aristotle

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Mathematics is often regarded as the embodiment of rationality – rationality distilled into its purest form. Unlike other fields of scientific or intellectual inquiry, it is uniquely insulated from the emotions and prejudices of its practitioners. A mathematical proof is a mathematical proof regardless of who developed it. It cannot be fabricated or impacted by factors outside of our control in the way that experiments can, and if a statement is proven true on solid mathematical grounds, it will remain true foreseeably for the indefinite future. However, emotions can still infiltrate mathematics in one way. Pure mathematicians from every era have reported an overwhelming sense of beauty and aesthetic pleasure while engaged in their work. The search for beautiful mathematics often attracts the attention of mathematicians to certain problems and deters them from tackling others. If this same sense of mathematical beauty also influences physicists, they may implicitly be building their mathematical fishing net not because they are looking for physics that align with mathematics but because they are looking for “beautiful" physics.

The affinity pure mathematicians have for the beauty of their work connects to the "unreasonable effectiveness" problem in two ways. First, theoretical physicists report the same appreciation for the beautiful nature of certain physical theories, and it is a quality for which they strive. If mathematicians and physicists share the same notion of beauty, whether innate or learned, the questions that physicists pursue for aesthetic reasons may intersect with existing mathematical developments. This point can be completely independent of whether or not mathematics derives its inspiration from the physical world, as was discussed in a previous section. If mathematicians are driven to pursue certain areas of math purely for aesthetic reasons, and mathematicians and physicists share a singular set of qualities with respect to what constitutes beauty, they may converge upon ideas that turn out to be useful in physics. In this way, the beauty we find in certain physical theories may be connected to the beauty found in the mathematics used to describe them.

The second point concerns the possibility that evolutionary or cultural forces have implicitly shaped the human mind's conception of beauty to have a relation to the natural world. If this is indeed the case that our definition of beauty draws its inspiration from the physical world, then the mathematical structures developed for their beauty may turn out to have unforeseen applications in physics. Philosophers have attempted to characterize the qualities of the types of problems and proofs that tend to evoke a sense of aesthetic pleasure among mathematicians. This is a notoriously hard endeavor. Henri Poincaré illuminates why in his nebulous attempt to define mathematical beauty as "a real aesthetic feeling that all true mathematicians recognize." That is to say, mathematical beauty is difficult to articulate in words, but mathematicians know when they are in the presence of it. Some common themes that have emerged about the qualities of "beautiful" mathematics are simplicity, symmetry, and unity.

Elaborating on the example of symmetry, there is evidence that a taste for symmetry is a universal human attribute. This observation is evident in both our attraction to symmetry in human faces and art as well as in mathematics. Emmy Noether, a renowned mathematician and physicist, famously linked the symmetries of a physical system to conservation laws [8]. This result had far-reaching consequences in physics given the numerous symmetries we observe in nature, including time translation invariance. However, it is worth noting that an excessive preoccupation with the mathematics of symmetries can blind us too. Numerous anomalous asymmetries in nature exist, such as matter-antimatter asymmetry and CP violation, among others. Understanding these asymmetries may necessitate the use of less aesthetically pleasing mathematics or alternative paradigms. Paul Dirac took the search for beauty in physics farther than perhaps any other physicist, saying "it is more important to have beauty in one's equations than to have them fit experiment" [9]. However, while mathematicians and physicists share a similar proclivity for beauty in their work, the use of aesthetic criteria can be more problematic for physicists. Unlike mathematics, physics is ultimately beholden to nature – not logic, beauty, or simplicity – as the ultimate arbiter. Its strongest mandate is to empirical evidence, gleaned through experiments and observations. That mathematics has reached a level of sophistication and abstraction that it can wander off in any directions it pleases does not mean that physics should try to – nor has to – follow suit. Although elegant mathematics has benefited physics in the past, physicists must be acutely aware of how their mandate to nature ought to limit the extent to which they go searching for elegant mathematics to solve new problems just because it has worked in the past. Physicists must be careful not to let their pursuit of beauty distract them from the need to develop their intuitions and remain focused on empirical evidence. String theory serves as a cautionary example of how an excessive focus on aesthetics can lead physicists astray. Therefore, while aesthetics may have a role in physics, it must be tempered by a commitment to empirical evidence and practicality.

In the history of physics, looking for mathematical beauty in the natural world has also led scientists astray. The most storied example of this was Johannes Kepler's proposal in his *Mysterium Cosmographicum* [10]. Kepler aimed to describe the orbits of the planets, and he developed a theory that was inspired by the beauty of platonic solids. He posited that the geometry of the orbits of the planets could be described by nesting the five ideal Platonic solids within each other. His theory supported Copernicus' heliocentric model and explained why there were 6 planets, as was believed at the time. From a geometrical standpoint, it is hard to imagine a more exquisite theory. However, as we now know, this theory is fundamentally flawed. Kepler's theory serves as a reminder that nature is under no obligation to conform to what we find beautiful, even if, in some instances, the two may align.

Trends of Human Rationality in Physics

To better clarify the relationship between physics and mathematics, it is important to recognize that physics is ultimately driven by humans whose internal experiences and cognitive processes are inexorably intertwined with their scientific work. Importantly, our inner experiences of rationality and discovery can distort our perceptions of our own work. Understanding this process within the field of physics may shed light on why physicists grip onto mathematics so tightly. The reason we practice physics is surely motivated by a desire to demystify and simplify the world in which we find ourselves, to uncover its underlying order, and to understand our place. This quest for simplicity is beautifully captured by psychologist William James in his *Sentiment of Rationality* where he writes, "Our pleasure at finding that a chaos of facts is the expression of a single underlying fact is like the relief of the musician at resolving a confused mass of sound into melodic or harmonic order. The simplified result is handled with far less mental effort than the original data" [11]. Although James was not explicitly discussing physics in his essay, his insights highlight why we do physics and why we are so pleased when it agrees with mathematics. Physics *itself* is organized around simplifying and abstracting things we see in the universe, so much so that we can develop a core set of principles that explain more complex phenomena. That mathematics might be the language for doing this is tempting, not only because it pretends there is a even greater order to the universe but also because it gives us an organizing principle for making progress in understanding it.

Simplification serves as a means of achieving a sense of fluency in our understanding, which mediates rational sentiments according to James. He wrote, "As soon, in short, as we are enabled from any cause whatever to think with perfect fluency, the thing we think of seems to us pro tanto rational." Our desire for fluency in physics is reflected in our efforts to show the self-consistency and unity of physics. This desire for unity is exemplified by the calls for the axiomatization of physics over a century ago. In 1900, David Hilbert, known for axiomatizing various areas of mathematics, published a list of 23 problems that he hoped would define the trajectory of mathematics in the 20th century [12]. The sixth problem on his list specifically called for "A Mathematical Treatment of the Axioms of Physics." The description of this problem states that, "The investigations on the foundations of geometry suggest the problem: To treat in the same manner, by means of axioms, those physical sciences in which already today mathematics plays an important part." This is one example of how the modernist movement in mathematics encroached its way toward physics but did not succeed, as it was perhaps an ill-founded idea in physics in the first place.

For any field, building an axiomatic framework would certainly give a sense of absolute fluency and rationality. By exploring the innate psychological desire to simplify and build coherent narratives out of our research, it is easier to understand our grip to specific paradigms like mathematics in physics. The "Unreasonable Effectiveness of Mathematics in the Natural Sciences" is, *itself*, a narrative about the organization of nature. With examples like fluid mechanics, as described in a previous section, that do not conform to this narrative, it begs the question why we might want to glorify mathematics at all. In an article titled "Lessons from Einstein's 1915 Discovery of General Relativity," physicist Lee Smolin attempts to give a more historically accurate account of the way in which Einstein developed General Relativity that sheds light on what Einstein had to gain from promoting the use of more mathematics in physics. In particular, his article tries to debunk the myth that "Einstein was a lone genius who followed beautiful mathematics to discover his great theory. Genius, inspired by aesthetics. Mathematics as a tool of prophecy" [13].

To start, Smolin challenges Kuhn's portrayal of scientific revolutions by highlighting the lack of anomalous experimental evidence that motivated the development of General Relativity. Einstein's groundbreaking theory was driven purely by his physical intuition, rather than external factors. The physical phenomena that General Relativity provides explanations for, like black holes, gravitational waves, and gravitational lensing, were not known about *at all* at his time. Furthermore, Smolin clarifies that Einstein did not develop most of the mathematics underpinning General Relativity, but rather relied on his imaginative thought experiments and ability to conceptualize extraordinarily new physical paradigms and let others do the mathematical work. He regards Einstein's rhetoric about the usefulness of mathematics in physics after developing the celebrated Theory of General Relativity as a sort of "propaganda." After General Relativity, Einstein transitioned to working on unified field theories in which he found much less success. Smolin says, "In the absence of ideas and insights about nature, Einstein fell back on mathematics as his guide. He constructed a myth about how mathematical beauty had been prophetic for his invention of general relativity and he attempted to use it to justify his forays into unified field theory." Harvard Professor Howard Georgi reports a similar sociological phenomenon that he notes in theoretical particle physics. When the experimental side of the field enters into a more stagnant period, in the absence of anomalous experimental results, theoretical physicists tend to retreat into a world of more abstract mathematics. The idea of the "unreasonable effectiveness of mathematics" has thus been co-opted as a narrative to elevate the work of physicists, with some even praising the divine beauty of their own work as demonstrating the fundamentality of mathematics in nature. However, this type of rhetoric can serve to elevate the use of physics in mathematics beyond its original purpose as a tool for modeling.

Finally, gripping onto mathematics can lead us to make flawed conclusions about the world, particularly in the areas of physics where our intuitions fail us the most. The field of quantum mechanics is an important example of where the desire for fluency in the way we discuss the microscopic world, a world we do not experience, has led some to deduce ontological truths about nature from pure mathematical formalism. Currently, there is broad philosophical debate in the philosophy of physics literature about mathematical instrumentalism versus mathematical realism in quantum theory. In particular, various mathematical objects like wavefunctions appear in the mathematics of quantum theory. Although they help us to make accurate predictions about the world, they are a large source of contention as some believe wavefunctions are objects that truly exist while others believe they are a calculational tool. This has led to various different interpretations of quantum mechanics that each say many different things about what the nature of reality truly is. The quest to make sense of even the things we cannot see is admirable, but by ascribing new aspects of reality to useful pieces of mathematics, we may be taking the relationship between mathematics and physics too far.

Conclusions & A Post-Mathematical Paradigm for Physics

When physicists, who have dedicated their professional lives to refuting supernatural explanations of the complex phenomena we see in the world, praise the mystical properties mathematics in physics – as Wigner did in his famous 1960 essay – it is worth examining the problem from many angles. While it is true that abstract mathematics has helped achieve many feats in physics, the most groundbreaking results have always come from new sparks of physical intuition rather than from mathematics alone. Of course there are physical reasons that may help rationalize the "effectiveness of mathematics," like the symmetries that exist in nature and the distinct physical scales that phenomena can be separated into that make physics amenable to approximations. Leaving these aside, in this paper, I have argued that modern mathematics derives its inspiration from the world around us, and the shared affinity for beauty among mathematicians and physicists brings the problems they work on into alignment with each other. With this understanding of mathematics, it is less surprising, then, that it proves to be useful in physics. I have also discussed the human aspect to this issue and tried to explain why it is enticing to believe in the "unreasonable effectiveness of mathematics."

More broadly, I have argued that we should be critical of the blind use of complex mathematics in physics by resisting the temptation to find problems that fit neatly into elegant mathematical frameworks while abandoning the ones that do not. Not only does the creation of "a mathematical fishing net" contribute to an illusion of the "unreasonable effectiveness of mathematics" in physics, but the history and philosophy of science have shown the high stakes of adopting such dogmatic frameworks in science. This is not to say that we should throw out the previous success of mathematics in physics. Importantly, a post-mathematical paradigm does not mean that mathematics will not be used at all but rather that its role will be different. There is a certain extent to which essential mathematics, like calculus, linear algebra, and geometry will always be relevant to physics so long as our experiments continue to be conducted in a quantitative manner. The emergence of computational physics is now allowing us to reimagine our understanding of the world through complex models. Indeed, neural networks and machine learning are among the potential paradigms that have begun to enter the field of physics that I am most excited about. Although we have had great success with our use of mathematics in the past, it is crucial to open our eyes to new paradigms for physics that promote experimentally guided theory. As such, we must avoid being mystified by the "unreasonable effectiveness of mathematics."

Bibliography

[1] B. P. Abbott et al. "Observation of Gravitational Waves from a Binary Black Hole Merger". In: *Phys. Rev. Lett.* 116 (6 Feb. 2016), p. 061102. DOI: 10.1103/PhysRevLett.116.061102. URL: https://link.aps.org/doi/10.1103/PhysRevLett.116.061102.

[2] Eugene P. Wigner. "The Unreasonable Effectiveness of Mathematics in the Natural Sciences." In: *Communications on Pure and Applied Mathematics* 13.1 (1960), pp. 1–14. DOI: 10.1002/cpa.3160130102.

[3] Jeremy Gray. *Plato's Ghost: The Modernist Transformation of Mathematics.* Princeton University Press, 2008.

[4] "Book Review by Sir Michael Atiyah: Conversation on Mind, Matter, and Mathematics". In: *European Mathematical Society Newsletter* 36 (June 2000), pp. 40–41. URL: https://ems.press/content/serial-issue-files/13610.

[5] Max Tegmark. *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality.* Vintage Books, 2015.

[6] *Millennium problems.* URL: https://www.claymath.org/millennium-problems.

[7] Thomas S. Kuhn. *The Structure of Scientific Revolutions.* University of Chicago Press, 1970.

[8] Emmy Noether. “Invariant variation problems”. In: *Transport Theory and Statistical Physics* 1.3 (1971), pp. 186–207. DOI: 10.1080/00411457108231446.

[9] P. A. Dirac. “The evolution of the physicist’s picture of nature”. In: *Scientific American* 208.5 (1963), pp. 45–53. DOI: 10.1038/scientificamerican0563-45.

[10] Johannes Kepler and Peter Michael Schenkel. *Mysterium cosmographicum*. Bayerische Akademie der Wiss., 1993.

[11] William James. *The Sentiment of Rationality*. Longmans, Green and Co., 1905.

[12] David Hilbert. “Mathematical problems”. In: *Bulletin of the American Mathematical Society* 37.4 (2000), pp. 407–436. DOI: 10.1090/s0273-0979-00-00881-8.

[13] Lee Smolin. *Lessons from Einstein’s 1915 discovery of general relativity*. 2015. DOI: 10.48550/ARXIV.1512.07551.