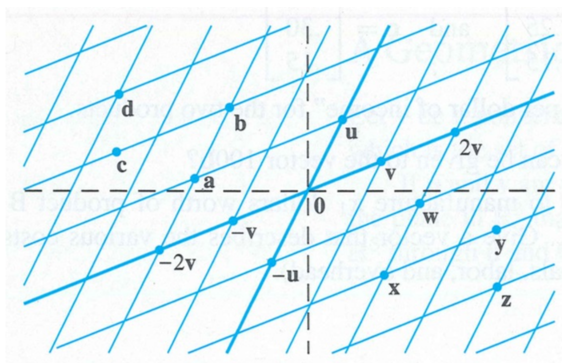
Homework 1

Due: Sunday, September 17
at 5:00pm

Please give complete, well-written solutions to the following exercises.

1.

Using the following figure, determine the linear combination of $\mathbf{u}$ and $\mathbf{v}$ that results in the following vectors: $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$.



2.

Give a geometric description of the linear combinations of $\begin{bmatrix} 8 \\ 2 \\ -6 \end{bmatrix}$ and $\begin{bmatrix} 12 \\ 3 \\ -9 \end{bmatrix}$

3.

A mining company has two mines. One day's operation at mine # 1 produces ore that contains 30 metric tons of copper and 600 kilograms of silver, while one day's operation at mine # 2 produces ore that contains 40 metric tons of copper and 380 kilograms of silver. Let $\mathbf{v}_1 = \begin{bmatrix} 30 \\ 600 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 40 \\ 380 \end{bmatrix}$. Then $\mathbf{v}_1$ and $\mathbf{v}_2$ represent the "output per day" of mine # 1 and mine # 2, respectively.

a) What physical interpretation can be given to the vector $5\mathbf{v}_1$?

b) Suppose the company operates mine # 1 for x_1 days and mine # 2 for x_2 days. Write a vector equation whose solution gives the number of days each mine should operate in order to produce 240 tons of copper and 2824 kilograms of silver. Do not solve the equation.

4.

Let $\mathbf{v}_1, \ldots, \mathbf{v}_k$ be points in $\mathbb{R}^3$ and suppose that for $j = 1, \ldots, k$ an object with mass m_j is located at point $\mathbf{v}_j$. Physicists call such objects *point masses.* The total mass of the system of point masses is

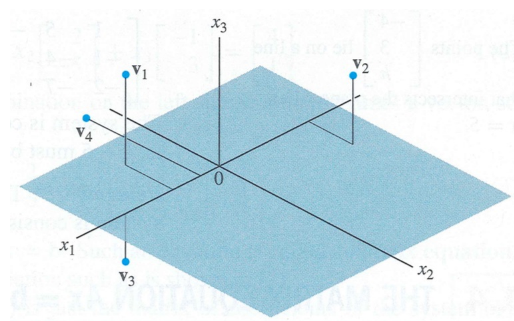
$$m = m_1 + \cdots + m_k.$$

The *center of gravity*(or center of mass) of the system is

$$\bar{\mathbf{v}} = \frac{1}{m}[m_1\mathbf{v}_1 + \cdots + m_k\mathbf{v}_k].$$

Compute the center of gravity of the system consisting of the following point masses:

Point	Mass
$\mathbf{v}_1 = (2, -2, 4)$	4 g
$\mathbf{v}_2 = (-4, 2, 3)$	2 g
$\mathbf{v}_3 = (4, 0, -2)$	3 g
$\mathbf{v}_4 = (1, -6, 0)$	5 g



5.

Given the vectors $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -3 \\ -1 \\ -5 \end{bmatrix}$, compute the following quantities:

a) $\frac{\mathbf{v}\cdot\mathbf{u}}{\mathbf{u}\cdot\mathbf{u}}$

b) $\left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}$

c) $\frac{1}{\mathbf{w}\cdot\mathbf{w}}\mathbf{w}$

d) Find the distance between $\mathbf{u}$ and $\mathbf{v}$.

e) Find a unit vector in the direction of $\mathbf{w}$.

6.

All vectors are in $\mathbb{R}^n$. Determine if each statement is true or false and justify each answer.

a) $\mathbf{v}\cdot\mathbf{v} = ||\mathbf{v}||^2$

b) For any scalar c, $\mathbf{u}\cdot(c\mathbf{v}) = c(\mathbf{u}\cdot\mathbf{v})$

c) If the distance from $\mathbf{u}$ to $\mathbf{v}$ equals the distance from $\mathbf{u}$ to $-\mathbf{v}$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

d) $\mathbf{u}\cdot\mathbf{v} - \mathbf{v}\cdot\mathbf{u} = 0$

e) If $||\mathbf{u}||^2 + ||\mathbf{v}||^2 = ||\mathbf{u}+\mathbf{v}||^2$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.