# Disagreement of Disagreement

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**Abstract**

We find remarkably low correlation among major investor disagreement proxies, making it challenging to infer how disagreement impacts prices. We develop a unified framework that not only offers new theoretical support for existing measures of disagreement taken individually, but also provides guidance for the relationships among these proxies via their incorporation into a novel nonlinear composite disagreement measure. We show empirically that this new measure is more predictive of returns in the cross section than existing measures. Our model also provides a disagreement-based rationale for the idiosyncratic volatility puzzle, which is absorbed by our composite disagreement in empirical tests.

*Keywords:* disagreement, difference of opinion, analyst forecast dispersion, idiosyncratic volatility, idiosyncratic skewness, short interest, volume, financial distress, heterogeneous agents, beliefs, expectations
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## 1. Introduction

Disagreement is ubiquitous in financial markets but fundamentally unobservable. Many proxies have been proposed in the literature, making it a challenge to decide which proxy to use and difficult to conclude how different measures are expected to relate with each other and with subsequent returns. Our paper proposes a unified framework to theoretically link these various proxies to the unobserved disagreement. We propose and implement a composite measure of disagreement.

We construct empirical proxies of investor disagreement corresponding to three major categories of thought as guided by the literature based on (a) analyst forecast dispersion for individual firms, (b) idiosyncratic volatility (IVOL), or (c) trading volume.[1] Ironically, the level of disagreement across these categories is relatively high. The average correlation among these measures is under 0.15. This low correlation among disagreement proxies means that empirical results likely depend on the proxy chosen. Moreover, many disagreement candidates can also double as proxies for other phenomena or lead to mixed evidence regarding the predictability of disagreement for returns.[2] Therefore, if one proxy indicates a high level of disagreement yet another indicates a low level of disagreement, then how do we decide when to favor one proxy over another or whether to temper the indication of one proxy by another, given that we do not observe the true level of disagreement?

To establish baseline expectations of behavior among the major disagreement variables, we seek a theoretical framework for examining connections between the (unobservable) scale of disagreement and other market variables. A good starting point for a model is to make as

[1] These three categories are among the most common firm-specific measures in the literature and are based on the least stale information for a broad cross-section of stocks (Huang, Li, and Wang, 2021). For example, Boehme, Danielsen, and Sorescu (2006) and Berkman et al. (2009) build on prior theory relating belief heterogeneity to volatility to propose IVOL as a firm-level measure of disagreement, which is applied in many studies. See Appendix B for a more detailed discussion of measures within these categories as well as other categories of disagreement proxies employed in the literature.

[2] For example, volume-based measures could also proxy for liquidity (e.g., Benston and Hagerman, 1974; Petersen and Fialkowski, 1994) and volatility-based measures such as IVOL could also proxy for risk (e.g., Goyal and Santa-Clara, 2003) or firm transparency (e.g., Ferreira and Laux, 2007).

few and as mild of assumptions as possible and see how much it explains without invoking further restrictions. Accordingly, we start with something simple: standard investor preferences, no parametric assumptions on the payoff distribution, and no trading frictions. We build from here to generate useful predictions and find that frictions are not necessary to obtain our insights.

More specifically, we analyze a two-period market for one risky asset in which the equilibrium price is determined endogenously by a standard market-clearing approach across a continuum of expected-utility maximizing investors having differences of opinion.[3] The pertinent assumptions on investor utility are standard: investors prefer more to less, are risk averse, and have demand for precautionary savings or positive intensity of prudence—properties exhibited, for example, by power utility. Sources for differences of opinion could be, for example, that investors perceive information differently or view the information of other investors as unreliable or ambiguous. We are agnostic to the specific source of investor disagreement and take their heterogeneous beliefs as given.

We model investor beliefs via two components: the relative distribution of beliefs and the scale of those beliefs. This approach allows us to not only consider many possible distributional shapes but also easily express different overall levels of disagreement. For example, investor beliefs might be polarized: half of investors have relatively high expectations and the other half have relatively low expectations. The scale of these polarized beliefs determines the magnitude of the difference between high and low expectations; e.g., beliefs clustering at $\pm 1\%$ to fundamental value is much different from beliefs clustering at $\pm 25\%$ despite both cases being polarized. Alternatively, investor beliefs might be evenly distributed across a

[3]Price influences investor trades but not beliefs, consistent with the paradigm of differences of opinion as delineated in the studies by Varian (1985), Harris and Raviv (1993), Shalen (1993), Kandel and Pearson (1995), and Cao and Ou-Yang (2008). Differences of opinion are consistent with investors not conditioning their decisions on prices (Banerjee, 2011; Banerjee, Davis, and Gondhi, 2024). Alternatively, insensitivity of beliefs to price can be approximately true under slow information diffusion, in which conditioning on price would only diffuse information gradually among the investor population (Hong and Stein, 1999), or with the addition of noise traders, in which conditioning on price would only transmit partial information (e.g., Grossman and Stiglitz, 1980; Diamond and Verrecchia, 1981).

range, in which case the scale determines the magnitude of the span of expectations; e.g., beliefs evenly spread across $[-1\%, 1\%]$ relative to fundamental value is a lot different from beliefs evenly spread across $[-25\%, 25\%]$. Likewise, we can consider beliefs largely concentrated in a spike or asymmetrically dispersed around fundamental value with various spans of the belief range.

We also model analyst forecast dispersion as a separate source influencing investor disagreement. A predominant assumption in past research is that analyst forecast dispersion can directly substitute investor heterogeneity (e.g., Rees and Thomas, 2010; Dittmar and Thakor, 2007; Park, 2005). Such a view ignores the prospect that investors' beliefs might be downstream from analysts' forecasts, as pointed out by Goulding (2017, 2018), so analyst forecast dispersion and investor disagreement may be distinct (but potentially correlated) constructs. In our model, dispersion in analyst forecasts can serve to amplify the magnitude of already existing differences of opinion. This amplification is via the scale of investors beliefs rather than via the relative distribution of beliefs.

We make no parametric assumptions on the payoff distribution, but its shape is relevant for studying disagreement. First, note that local convexity of demand is important for disagreement to impact equilibrium prices. Such convexity implies that positive and negative demands associated with symmetric deviations of price from fundamental value do not offset on average, thereby injecting a pricing bias. Accordingly, all else equal, disagreement that leads some investors to long positions but other investors to short positions, requires a price biased relative to fundamental value in order to clear the market. Without demand convexity, long and short positions would offset at a price equal to fundamental value.

Although local convexity can result from short-sales frictions—commonly invoked to explain the impact of disagreement on expected returns—because our model is frictionless, skewness is important as an alternative source of local convexity of demand. A common property of standard utility functions is a preference for positively skewed payoffs (all else equal), which leads to asymmetric demand responses to upside opportunities and downside risks.[4] For example, the payoff distribution of a short position in a positively skewed asset is akin to that of a long position in a negatively skewed asset, which under the common preference for skewness would be a lower-magnitude long position than for a positively skewed asset.

Because most stocks are ex-ante positively skewed (Boyer, Mitton, and Vorkink, 2010, among others), skewness is a relevant source of demand convexity for disagreement to impact equilibrium stock prices in our empirical application. Moreover, skewness is a mild assumption common to the literature. An example of a positively skewed payoff distribution is the lognormal distribution, which is very common in the literature for its useful and realistic properties, such as taking only positive values and having asymmetric tails, and appears in some of the most influential models in finance such as that of Black and Scholes (1973).

Our framework not only provides new theoretical support for existing measures of disagreement taken individually, but also generates new predictions as well as a new composite measure of investor disagreement, which is a nonlinear function of model variables corresponding to different disagreement proxies. Note that candidate proxies for investor disagreement could be any variable that positively co-moves with disagreement, within some framework. In our model, we find positive partial derivatives of our composite disagreement measure with respect to each variable corresponding to the individual existing proxies, thereby providing new theoretical support for these proxies as measures of disagreement. Our composite measure also incorporates the equilibrium interplay of these variables with each other as a function of model disagreement and, therefore, reflects amplifying or tempering interactions among individual disagreement proxies.

Our model predicts disagreement to be negatively related to expected returns for most stocks. If disagreement is a proxy for risk, however, then one might expect a positive relationship under the fundamental paradigm of the risk and expected return trade-off. An

[4]This property is true of any utility function having non-increasing absolute risk aversion. Examples of such utility functions include negative exponential and power utility.

investor whose holdings are not well-diversified requires compensation for the idiosyncratic risk of those holdings (Merton, 1987) and, therefore, stocks with higher disagreement should provide higher expected returns and disagreement should possess explanatory power beyond the standard risk factors. However, Diether, Malloy, and Scherbina (2002) and a large literature that followed have documented the opposite and proposed various explanations for this very puzzling result.[5]

Short-selling constraints are common ingredients in existing models. Our model differs in that trading is frictionless. There are arguably many relevant frictions in markets and we do not assert that frictions are irrelevant. Rather, in the interest of parsimony mentioned earlier, we exclude any ingredient not strictly necessary to obtain our results, and thereby can discern whether the impact of frictions might layer upon underlying phenomena intrinsic to the market mechanism. Accordingly, we predict that the disagreement–expected return effect can manifest even for stocks having less short-sales frictions. To test this hypothesis, following the literature, we use institutional ownership (IO) to capture the effects of short-sale constraints, which are most likely to bind among stocks with low IO (Nagel, 2005). We split our sample into five IO quintiles and report that our disagreement measure remains a significant negative predictor of expected returns in all IO quintiles, even the highest IO quintile, which represents the stocks with the most institutional investors and, hence, the least short-sales frictions.

We construct an empirical composite disagreement proxy via simple mappings of model variables to observable data counterparts to test the model's predictions. The (negative) correlation of our disagreement measure with future returns is strongly supported in the data. First, its predictability is economically significant. Higher disagreement is associated with lower expected returns and shorting such stocks to buy lower disagreement stocks translates to substantial returns in our historical analysis. For example, a value-weighted decile spread portfolio sorted on our composite measure of disagreement generates over 12% annualized

[5] Chang et al. (2022) provide a recent overview of this literature. See also Appendix B.

excess returns and over 21% annualized CAPM alpha. The size of the multifactor alphas are similar. These remarkably large alphas arise because stocks with higher disagreement tend not only to have lower expected returns but also higher betas in our empirical tests (see Section 4.2.3): as disagreement increases, expected returns increasingly diverge from those predicted by the CAPM, generating large magnitude alphas for upper disagreement deciles. The alpha is not driven only by the short leg, as the lowest disagreement decile contributes a significant portion to the alpha. Moreover, the CAPM alpha is larger than that generated by the major individual disagreement proxies: 8.9% higher than the annualized CAPM alpha of value-weighted decile spread portfolio sorted on analyst forecast dispersion (Diether, Malloy, and Scherbina, 2002); 6.1% above an IVOL decile spread portfolio (Ang et al., 2006); and 13.1% above a short-interest (Chen, Da, and Huang, 2022) decile spread portfolio.

Second, our composite proxy is a statistically significant and robust predictor of returns that also partially absorbs the predictability of some prominent cross-sectional predictors, such as IVOL. In predictive Fama and MacBeth (1973) regressions, in which we control for other disagreement measures as well as common cross-sectional predictors, our composite proxy significantly negatively predicts subsequent returns ($t$-statistic $= -3.27$). Ang et al. (2006) document a strong negative relationship between IVOL and subsequent stock returns, which has been difficult to rationalize. For similar reasons to the dispersion effect, this result is very puzzling under traditional asset pricing theories of risk and reward, which predict either no impact on returns because idiosyncratic risk is not priced, or a positive relationship with returns as a compensation for risk. A large literature has followed attempting to explain the IVOL puzzle.[6] We find that after controlling for our composite measure of disagreement, the negative predictability of IVOL for future returns weakens and is no longer statistically significant. Our results suggest that our composite disagreement measure captures additional disagreement effects not transmitted by IVOL. This finding is consistent with the predictions

[6] Hou and Loh (2016) provide a comprehensive overview and analysis of different approaches and conclude that none of them fully explain the IVOL puzzle.

of our framework, which provides a disagreement-based rationale for the role of IVOL—an explanation not appealed to in the large literature following Ang et al. (2006).[7] The model implies that IVOL is closely associated with disagreement and IVOL factors in prominently in our composite proxy.

Our disagreement measure is a nonlinear function of major disagreement proxies. To investigate the importance of this complexity, we employ two distinct methods for deriving alternative linear disagreement measures for comparison purposes. First, we calculate straightforward, equally weighted disagreement scores across different collections of individual disagreement proxies. Second, we leverage principal component analysis to identify underlying patterns across these collections and extract common components that summarize the variations within these proxies. Our nonlinear measure remains a robust negative predictor of expected returns even after controlling for each of these linear disagreement measures, which is consistent with the nonlinear structure implied by the model.

Finally, we apply our new disagreement measure to study the characteristics of firms sorted along the disagreement dimension. We document that higher disagreement stocks are smaller,[8] have higher beta and more lottery-like behavior. Higher disagreement is also more characteristic of less liquid firms and value firms. Moreover, higher disagreement firms tend to have higher levels of financial distress such as Altman's (1968) Z-Score, Ohlson's (1980) O-Score, and Kaplan and Zingales's (1997) KZ-Index, consistent with financial distress being linked to disagreement effects on prices (Avramov et al., 2009).

Furthermore, our findings also indicate that elevated levels of disagreement are associated with younger firms, increased net income idiosyncratic volatility, heightened concentration of institutional investors (as measured by the Herfindahl-Hirschman Index), intensified research and development expenditures, augmented net equity issuance, greater information asymmetry, elevated standardized unexplained volume, increased turnover, higher change

[7]See Hou and Loh (2016).

[8]Higher disagreement stocks are smaller among those stocks followed by multiple analysts, which tend to be larger compared to all stocks.

in market-adjusted turnover, greater opinion divergence among mutual funds (Chen, Hong, and Stein, 2002), wider bid-ask spreads, and reduced volumes of Google searches, both overall and specifically investment-related. Several of these characteristics correspond to one of many disagreement proxies employed in the literature, each demonstrating the anticipated relationship with our measure of disagreement, thereby providing additional credibility for our measure. See Appendix B for further details.

### *1.1. Related Literature*

It is generally acknowledged that differing opinions—heterogeneous investors with different beliefs—play a crucial role in asset pricing. A rich literature studies price formation under heterogeneous beliefs and offers a variety of views on the impact of disagreement on expected returns; see, e.g., Chen, Hong, and Stein, 2002; Diether, Malloy, and Scherbina, 2002; Yu, 2011; Carlin, Longstaff, and Matoba, 2014; Fischer, Kim, and Zhou, 2022 and the surveys of Curcuru et al. (2010) and Bielecki et al. (2004) for numerous references.

The use of different proxies to measure investor heterogeneity leads to varying conclusions regarding the correlation of disagreement and expected returns. For instance, some studies adopting analyst forecast dispersion as a measure of investor disagreement report a negative correlation (e.g., Diether, Malloy, and Scherbina, 2002; Hong and Sraer, 2016). Using a measure based on regressions of analyst forecasts against each other, Fischer, Kim, and Zhou (2022) report a positive relation. In the context of agency mortgage-backed securities, Carlin, Longstaff, and Matoba (2014) also find analyst forecast dispersion has a positive correlation. Studies employing metrics based on trading volume have reported mixed results, with most documenting a positive relation (e.g., Garfinkel and Sokobin, 2006; Chen et al., 2022). In Appendix B, we enumerate a vast variety of disagreement proxies employed in the literature in top tier journals across accounting, economics, finance, and management studies, and provide additional background on the three major categories of disagreement measures.

The theoretical role of disagreement in financial markets has been the focus of a large literature. Barry and Brown (1985), Merton (1987), and Varian (1986) show that investor heterogeneity can be seen as an additional risk factor and thus is associated with higher expected returns. Miller (1977), Jarrow (1980), Diamond and Verrecchia (1987), and Chen, Hong, and Stein (2002) explore static models while Harrison and Kreps (1978), Scheinkman and Xiong (2003), and Hong, Scheinkman, and Xiong (2006) study dynamic models and speculative bubbles. Miller (1977) demonstrates that when investors disagree and market frictions are in place (i.e., short selling constraints), the most optimistic investors determine the price. Thus, investor heterogeneity should result in lower future returns. In this spirit, disagreement may explain speculative bubbles and, hence, lower subsequent returns (Scheinkman and Xiong, 2003).

Alternative explanations appeal to market frictions such as illiquidity as trading costs (Sadka and Scherbina, 2007), unpriced information risk that interacts with leverage (Johnson, 2004), or variables related to financial distress (Avramov et al., 2009). Goulding (2017, 2018) provides equilibrium models that explain the disagreement effect from a noisy rational expectations approach. Connections with trading volume, price volatility, and informed trading, are studied by Harris and Raviv (1993), Kandel and Pearson (1995), Cao and Ou-Yang (2008), Dumas, Kurshev, and Uppal (2009), Banerjee and Kremer (2010), Ottaviani and Sørensen (2015), Atmaz and Basak (2018), Banerjee, Davis, and Gondhi (2018), and Chabakauri and Han (2020) among others. See Hong and Stein (2007) and Xiong (2013) for surveys. Our paper is different from these studies in that we do not rely on firm characteristics or market frictions, such as short-selling constraints, which are commonly featured in these models, and we do not impose parametric assumptions on the risky asset's payoff distribution. We also establish linkages among different disagreement measures.

Our pricing model is most related to that of Goulding, Santosh, and Zhang (2024), who study expected return as a function of a given arbitrary disagreement measure in a frictionless market of risk averse investors with differences of opinion. They show that demand convexity arising from skewness (Goulding, Santosh, and Zhang, 2023) interacts with disagreement to generate variation in expected returns and document that a particular measure, analyst forecast dispersion, has a negative relationship with expected returns for positively skewed stocks. We depart from their approach in our model of investor disagreement, impacting how equilibrium expected returns relate to volatility. Moreover, rather than taking disagreement to be analyst forecast dispersion to study its effects on expected returns, we develop our own measure of disagreement by introducing derived quantities such as trading volume to reverse engineer how the (unobservable) level of disagreement is influenced by other variables corresponding to existing proxies of disagreement, including analyst forecast dispersion, volatility, and volume. Our new measure of disagreement exhibits more explanatory power for expected returns and its construction provides a new rationale for these other proxies, taken individually, as well as new insights on how these proxies temper each other in conveying the underlying level of disagreement. We also explore implications of market size on equilibrium expected return, showing that the disagreement–expected-return relationship can be expected to be stronger for smaller firms.[9]

Our paper contributes to the study of how differing indications of different disagreement measures can be reconciled. Huang, Li, and Wang (2021) study the problem at the *aggregate* level, empirically aggregating information across disagreement measures to construct a disagreement index for the stock market as a whole. They use this index to demonstrate predictability of disagreement for the overall stock market return via the linear partial least squares method of Kelly and Pruitt (2013, 2015). In contrast, our approach generates a model-based nonlinear disagreement measure applicable to *individual* stock returns, that in empirical tests exhibits cross-sectional return predictability beyond that of linear-based composite measures such as principal components.

## 2. Model

Consider a two-period market having dates $t \in \{0, 1\}$ with two assets: a risk-free asset with its price and payoff normalized to one; and one risky asset with price $p$ at date $t = 0$ and

[9] See Appendix F.

payoff $\widetilde{y}$ at date $t = 1$.[10] We follow a standard market-clearing approach to characterize the equilibrium price $p$. Specifically, payoff $\widetilde{y}$ is exogenously given and the equilibrium price $p$ will be endogenously determined by market clearing of the demands of a continuum of expected-utility maximizing investors, whose utility functions and differences of opinion about $\widetilde{y}$ are also exogenously given.

### *2.1. Payoff Distribution*

We model the payoff $\widetilde{y}$ as a random variable having cumulative distribution function (cdf) denoted by $F(y) = \Pr[\widetilde{y} \leq y]$, with idiosyncratic statistics of mean, variance, and skewness denoted by $\mu = \mathbf{E}[\widetilde{y}]$, $\sigma^2 = \mathbf{Var}[\widetilde{y}]$, and $s = \mathbf{Skew}[\widetilde{y}] = \frac{\mathbf{E}[(\widetilde{y}-\mu)^3]}{\sigma^3}$, respectively. We denote the corresponding net return random variable by $\widetilde{r}$, where $\widetilde{r} = \frac{\widetilde{y}-p}{p}$, with its variance denoted by $\sigma_r^2 = \mathbf{Var}[\widetilde{r}]$.[11] Note that the return mean satisfies $\mathbf{E}[\widetilde{r}] = \frac{\mu-p}{p}$, the return variance satisfies $\sigma_r^2 = \frac{\sigma^2}{p^2}$, and the return skewness is the same as the payoff skewness. We assume $\mu$ is finite and $\sigma$ is strictly positive to avoid a trivial solution.

### *2.2. Investor Utility*

There is a continuum of investors, indexed by the unit interval $[0, 1]$, each with common utility function of wealth $u(w)$ and initial wealth $w_0 > 0$. In our baseline analysis, we analyze a simple constant relative risk aversion (CRRA) utility function of the form,

$$u(w) = \begin{cases} \frac{w^{1-\gamma}-1}{1-\gamma}, & \gamma \neq 1, \\ \ln(w), & \gamma = 1, \end{cases} \tag{1}$$

[10] Like Barberis and Huang (2008) and Goulding, Santosh, and Zhang (2023), we employ the modeling device that considers a single risky asset whose payoff is independent of the market portfolio. It can be shown under commonly used utility functions such as those with constant absolute or relative risk aversion that an investor's optimal choice in a setting of a single risky asset coincides approximately with the optimal choice for that same asset taken as independently distributed in a setting of multiple risky assets (Goulding, Santosh, and Zhang, 2023). Analysis of a single stock can also approximate features in a portfolio with a limited number of holdings, which is typical of most investors because most are not fully diversified. Accordingly, our pricing results reflect the asset's idiosyncratic statistics relative to a fuller market.

[11] Division by $p$ is well-defined, for example, under the natural condition of limited liability that entails the payoff $\widetilde{y}$ has only nonnegative support such that $p > 0$ in equilibrium.

where $\gamma > 0$ is a scalar that governs the degree of relative risk aversion.[12] While there is no widely accepted estimate in the large literature on estimating risk aversion, the more common values for the coefficient of relative risk aversion are between 1 and 3, among the wider range of estimates just above zero to 10 and higher. In numerical analyses, we use $\gamma = 2$ for illustration and, in empirical applications, we analyze a range to establish robustness to the choice of $\gamma$ within the accepted range from the literature.

*2.3. Investor Disagreement*

Investors have differences of opinion about the expected payoff of the risky asset and, accordingly, also disagree about the expected return. To model their disagreement, we specify separately the relative distribution of beliefs and the scale of those beliefs. This framework encompasses many possibly shapes of the distribution of beliefs around the fundamental value. We outline here a few examples. First, investor beliefs could be polarized: one half concentrated above fundamental value and the other half below with the scale determining how far apart are the poles. Second, investor beliefs could be evenly distributed across a range with the scale determining the span of the range. Third, investor beliefs could be concentrated in a spike with the scale determining the heaviness of the tails around the spike. Fourth, investor beliefs could exhibit asymmetry in which a majority have expectations near fundamental value and a few have more extreme expectations, again with the scale determining the span. Scale is important in quantifying the level of disagreement across

[12]This utility function, also known as power utility or isoelastic utility, has the feature of constant relative risk aversion: $-w\frac{u''(w)}{u'(w)} = \gamma$. Power utility functions are very common in the literature because of several useful and realistic properties: the utility function is infinitely differentiable; is strictly increasing, which entails investors prefer more to less; is strictly concave, which entails investors are risk averse; and exhibits a positive third derivative, which entails investors have a demand for precautionary savings and a positive intensity of prudence: $u'(w) = \frac{1}{w^{\gamma}} > 0$, $u''(w) = -\frac{\gamma}{w^{\gamma+1}} < 0$, $u'''(w) = \frac{\gamma(\gamma+1)}{w^{\gamma+2}} > 0$, for all $w > 0$. Leland (1968) and Sandmo (1970) connect a positive third derivative of a utility function to a preference for precautionary savings. Kimball (1990) defines $-\frac{u'''(w)}{u''(w)}$ as the intensity of prudence, which indicates the strength of the precautionary saving motive.

This specific choice of utility is not important for our results. What matters is that investors prefer more to less ($u'(w) > 0$), are risk averse ($u''(w) < 0$), and have a demand for precautionary savings or positive intensity of prudence ($u'''(w) > 0$).

different belief shapes and corresponds to the standard deviation of beliefs.

Specifically, regarding the relative distribution, each investor's belief corresponds to an independent realization of the random variable $\widetilde{\Delta}$ at date $t = 0$, which is a unitless standarized variable governing belief heterogeneity. Specifically, $\widetilde{\Delta}$ is a discrete mean-zero random variable with variance normalized to one, which takes $i = 1, 2 \ldots, n$ possible values: $\widetilde{\Delta} = \Delta_i$ with probability $\pi_i$. Belief heterogeneity characterizes $n$ types of investors, with fraction $\pi_i$ of investors being of type $i$, and models the concentration or dispersion of beliefs relative to each other holding the overall variance fixed to one unit.

Regarding the scale of these beliefs, each type–$i$ investor believes the expected payoff is $\mu_i = \mu + D\Delta_i$, where $D$ is a positive scalar, in units of currency, that expands or contracts the relative distribution of beliefs to their absolute levels. Equivalently, each type–$i$ investor believes that $\widetilde{y}$ is drawn from cdf $F(y - D\Delta_i)$.[13] We refer to $D$ when discussing the role of disagreement in prices or payoffs, which are also denominated in units of currency.

Likewise, each type–$i$ investor believes the expected return is $\mathbf{E}[\widetilde{r}] + d\Delta_i$, where $d = \frac{D}{p}$ is a unitless scalar. We refer to $d$, when discussing the role of disagreement in the expected return, which is also unitless. The higher that $D$ (or $d$) is above zero, the greater the scale of disagreement is within the investor population, all else equal.

### *2.4. Investor Demand and the Equilibrium Price*

Each investor of type $i = 1, 2, \ldots, n$ demands $x_i(p)$ units of the risky asset given price $p$, where $x_i(p)$ maximizes investor $i$'s expected utility of date–1 wealth,

$$x_i(p) = \arg\max_x \mathbf{E}_i[u(w_0 + x(\widetilde{y} - p))], \tag{2}$$

[13]For example, suppose the expected payoff is $\mu = 10$. Consider the case of $n = 2$ with $\Delta_1 = -1$ and $\Delta_2 = 1$ equally likely. These $\Delta_i$'s are zero on average with unit variance, representing that type–1 investors are relatively "pessimistic" and type–2 investors are relatively "optimistic." If the scale of disagreement is $D = 0.5$, then each pessimistic investor believes the payoff distribution of the risky asset is shifted downward by $-0.5$ such that $\mu_1 = 9.5$ and therefore that $\widetilde{y}$ is drawn from cdf $\mathbf{P}[\widetilde{y} - 0.5 \leq y] = \mathbf{P}[\widetilde{y} \leq y + 0.5] = F(y + 0.5)$. Similarly, each optimistic investor believes the payoff distribution of the risky asset is shifted upward by 0.5 such that $\mu_2 = 10.5$ and therefore that $\widetilde{y}$ is drawn from cdf $\mathbf{P}[\widetilde{y} + 0.5 \leq y] = \mathbf{P}[\widetilde{y} \leq y - 0.5] = F(y - 0.5)$; i.e., so more probability mass is put on above $\mu$.

where $\mathbf{E}_i$ denotes the expectation is taken with respect to investor $i$'s beliefs: $F(y - D\Delta_i)$. Under type–$i$ investor's belief, the expected payoff is $\mu_i = \mathbf{E}_i[\widetilde{y}] = \mu + D\Delta_i$. For notational simplicity, we denote the variance under this investor's belief by $\sigma_i^2 = \mathbf{E}_i[(\widetilde{y} - \mu_i)^2]$ and the skewness by $s_i = \frac{\mathbf{E}_i[(\widetilde{y}-\mu_i)^3]}{\sigma_i^3}$. Because investors only disagree about the mean payoff via a shift of the cdf, $F(y - D\Delta_i)$, it can be shown that variance and skewness do not depend on $i$, so we drop the subscripts and write $\sigma = \sigma_i = \mathbf{SD}[\widetilde{y}]$ and $s = s_i = \mathbf{Skew}[\widetilde{y}]$.

**Proposition 1** (Demand)**.** *The Taylor's expansion of type–$i$'s demand for the risky asset around $p = \mu_i$ is*

$$x_i(p) = -\frac{w_0}{\gamma}\frac{1}{\sigma^2}(p - \mu_i) + \frac{1}{2}\frac{w_0}{\gamma}\left(1 + \frac{1}{\gamma}\right)\frac{s}{\sigma^3}(p - \mu_i)^2 + h_i(p)(p - \mu_i)^2, \tag{3}$$

*where $h_i(p)$ is an unknown function that converges to 0 as $p \to \mu_i$ by Taylor's theorem.*[14]

*Proof of Proposition 1.* See Appendix C. □

Proposition 1 characterizes the demand of a type-$i$ investor as a function of price—obtained from a Taylor expansion of $x_i(p)$ around $p = \mu_i$. Intuitively, the slope of the demand at $p = \mu_i$ is negative, $x_i'(\mu_i) = -\frac{w_0}{\gamma\sigma^2}$, and risk aversion implies zero demand at the price equal to the investor's belief, $p = \mu_i$. Depending on skewness through the quadratic term in (3), however, demand assumes different shapes—all anchored at zero for $p = \mu_i$.

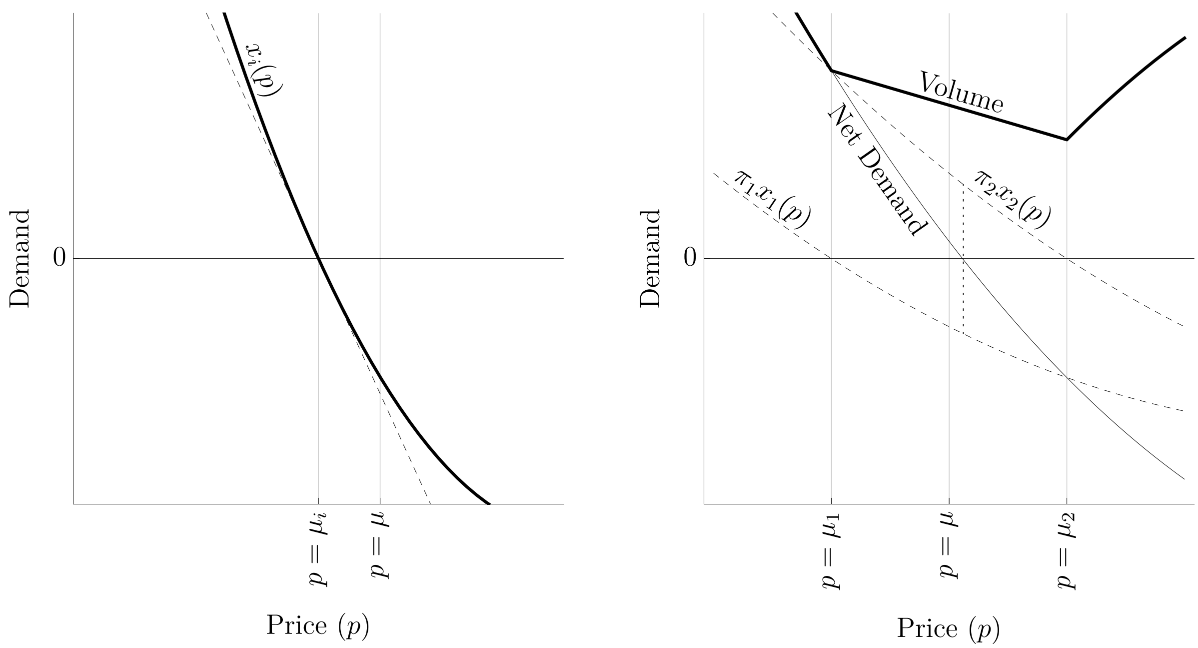
### *2.4.1. Skewness and Local Demand Convexity*

Figure 1a illustrates the main properties of the demand function in the case of positive skewness: $s > 0$. Positive skewness is not only appropriate for empirical stock analysis, but also a mild assumption common to the literature. Positive skewness produces nonlinear

[14] A second-order Taylor expansion is sufficient to capture first- and second-order price sensitivity as well as generate the property of local convexity of demand generated by payoff skewness. For prices outside the range of investor beliefs, higher-order expansions that reflect higher moments of the payoff distribution, such as kurtosis, can take on more relevance and reduce approximation error—at the expense of increased complexity. Our equilibrium price presented in Proposition 2, however, falls within the span of beliefs for disagreement not too large. Moreover, expected returns have limited sensitivity empirically to fourth or higher moments of the distribution. Therefore, the additional complexity of higher-order expansions is not material for this study.

**Figure 1:** Investor demand and trading volume as a function of price.

(a) Investor type $i$ demand (b) Net demand and trading volume




**Notes:** Figure (a) plots the 2nd-order Taylor polynomial of the demand $x_i(p)$ of a type–$i$ investor with belief $\mu_i = \mu + D\Delta_i$ as a function of price (solid curve) and its tangent line at $p = \mu_i$ (dashed line) over the domain $p \in [\mu_i - \sigma, \mu_i + \sigma]$, where $\Delta_i = -1$. Figure (b) plots the trading volume (bold solid curve), the 2nd-order Taylor polynomials of the total demand functions of type–1 and type–2 investors with beliefs $\mu_1 = \mu + D\Delta_1$ and $\mu_2 = \mu + D\Delta_2$ (dashed curves), respectively, as well as the net demand (solid curve) over the domain $p \in [\mu - 0.5\sigma, \mu + 0.5\sigma]$, where $\Delta_1 = -1$, $\Delta_2 = 1$, $\pi_1 = \pi_2 = 0.5$. The vertical dotted line to the right of $\mu$ marks the equilibrium price at which type–1 and type–2 demands are equidistant from the price axis, so that positive and negative demands offset to clear the market. The figures have the following parameter values in common: $D = 0.05$, $\mu = 1$, $\sigma = 0.2$, $s = 0.6$, $\gamma = 2$, $w_0 = 1$.

demand and, in particular, locally strictly convex demand around the investor's belief (solid curve in Figure 1a). This convexity generates asymmetric demand responses (to symmetric deviations in price) for all investor types. For example, for prices below the investor's belief, $p < \mu_i$, the investor has increasingly positive demand that accelerates as price decreases; whereas, for prices above, $p > \mu_i$, the investor has negative demand or short interest in the risky asset, which increases in magnitude at a diminishing rate as price increases because of the convexity.[15] Without skewness, demand would be a linear function (dashed tangent line

[15] This demand behavior reflects the preference for precautionary savings, for example, represented by CRRA utility of a positively skewed payoff.

in Figure 1a)—which is the norm in the classic set-up of a normally-distributed payoff—and both positive demand and short interest would increase in magnitude symmetrically away from $p = \mu_i$ at a uniform rate. In Section 2.4.4, we discuss how local demand convexity is important for the relationship between the equilibrium price and disagreement.

### *2.4.2. Demand and Expected Return*

The Taylor polynomial of Proposition 1 can be recast in terms of expected returns. Suppose that aggregate initial wealth of all investors, $w_0 = \int_{i=0}^{1} w_0 di$ is proportional to the market price of the asset, $p$, with unit proportionality constant for simplicity: i.e., $w_0 = p$. Using the second-order polynomial of type–$i$ demand in (3) of Proposition 1, and the identities $\sigma = \sigma_r p$ and $\mathbf{E}_i[\widetilde{r}] = \frac{\mu_i - p}{p}$, we can approximate investor $i$'s demand per unit of currency in terms of investor $i$'s expected return, $\mathbf{E}_i[\widetilde{r}]$:

$$\frac{1}{\gamma}\frac{1}{\sigma_r^2}\mathbf{E}_i[\widetilde{r}] + \frac{1}{2}\frac{1}{\gamma}\left(1 + \frac{1}{\gamma}\right)\frac{s}{\sigma_r^3}(\mathbf{E}_i[\widetilde{r}])^2. \tag{4}$$

For a positively-skewed asset, investor $i$'s demand is increasing in the investor's mean belief and decreasing in both the coefficient of risk aversion and volatility—aligning with intuitive properties of asset demand.

### *2.4.3. Market Clearing Equilibrium Price and Expected Return*

Combining the demand functions of all investor types in a market-clearing analysis determines the equilibrium price. Because investors of type $i$ each trade $x_i(p)$ and are of size $\pi_i$ in the population, we can express the net demand of all investors as

$$\int_0^1 x_i(p)di = \pi_1 x_1(p) + \pi_2 x_2(p) + \cdots + \pi_n x_n(p), \tag{5}$$

where $\pi_i > 0$ and $\sum_{i=1}^{n} \pi_i = 1$. We assume that the net supply of the risky asset is $X = 0$ and determine the equilibrium price from market clearing: $\int_0^1 x_i(p)di = X$. Note that we do not require noisy supply to motivate trade as in market models with investors having common expectations. Differences in beliefs are sufficient to generate trade.

Figure 1b illustrates the net demand (5) in a market with two types of investors who disagree: $\pi_1 = \frac{1}{2}$ of investors are type 1 investors, who are relatively pessimistic about the expected payoff, $\mathbf{E}_1[\widetilde{y}] = \mu - D$, and $\pi_2 = \frac{1}{2}$ of investors are type 2 investors, who are relatively optimistic about the expected payoff, $\mathbf{E}_2[\widetilde{y}] = \mu + D$. Figure 1b plots the case of a positively skewed payoff ($s = 0.6$). The total demands of each investor type, $\pi_1 x_1(p)$ and $\pi_2 x_2(p)$, respectively, are decreasing and convex in price $p$ in a neighborhood overlapping their beliefs, by Proposition 1. In Figure 1b, this overlapping region of prices includes $p = \mu$, which is between $p = \mu_1$ and $p = \mu_2$, and a price above $p = \mu$ that perfectly offsets demands to clear the market. Again, because investors are risk averse, they are unwilling to trade a deterministic amount $p$ for an uncertain payoff that equals $p$ in expectation. Hence, demand is zero for all type–1 investors at price $p = \mu_1$. At $p = \mu_1$, type–2 investors, who believe the asset is worth more than $p$ in expectation, demand a positive quantity, and therefore, net demand is positive. As price increases above $p = \mu_1$, type–1 demand becomes negative to offset the positive demand of type–2, increasing short interest and reducing net demand.

More generally, setting net demand (5) equal to zero determines the equilibrium price that offsets demands to clear the market. Because each $x_i(p)$ depends on the scale of disagreement, $D$, through belief $\mu_i = \mu + D\Delta_i$, the equilibrium price also depends on $D$. In Proposition 2, we use implicit differentiation of price in the market-clearing condition with respect to the $D$ to obtain a Taylor expansion in terms of the scale of disagreement, for $D$ not too large.[16]

**Proposition 2** (Equilibrium price)**.** *The Taylor expansion of the equilibrium price $p$ as a function of the scale of disagreement, around $D = 0$, is given by:*

$$p(D) = \mu + \frac{1}{2}\left(1 + \frac{1}{\gamma}\right)\frac{s}{\sigma}D^2 + h(D)D^2, \tag{6}$$

*where $h(D)$ is an unknown function that converges to 0 as $D \to 0$ by Taylor's theorem.*[17]

[16] The phrase "for $D$ not too large" means that there exists a $\overline{D} > 0$ such that the approximation is reasonable for all $D < \overline{D}$. See Appendix D for additional discussion in the context of equilibrium volume and price relationships.

[17] The second-order Taylor polynomial approximation of equilibrium price is reasonable for $D$ not too large and captures all disagreement sensitivity inherited from the second-order demand polynomials of Proposition 1 because of its determination via market clearing of such demands. That is, higher-order expansions

*Proof of Proposition 2.* See Appendix C. □

Proposition 2 shows how the equilibrium price depends on the scale of disagreement. Because we are examining one risky asset, it is this asset's individual statistics such as mean ($\mu$), variance ($\sigma^2$), and skewness ($s$) that are relevant for pricing rather than, say, covariances with the market.[18] If $s > 0$ and $D$ is not too large, then equilibrium price increases in $D$, is larger than expected payoff ($p > \mu$), and by the 2nd-order Taylor polynomial in (6), the expected return satisfies

$$\mathbf{E}[\widetilde{r}] = -\frac{1}{2}\left(1 + \frac{1}{\gamma}\right)\frac{s}{\sigma_r}d^2, \tag{7}$$

which is decreasing in $d$. By inspection of (6) and (7), we find that more disagreement means higher prices, and, therefore, lower expected returns for positive skewness, all else equal. This prediction of a negative relationship between disagreement and subsequent returns is consistent with the findings of Diether, Malloy, and Scherbina (2002) and related studies. In Section 2.4.4, we discuss how local demand convexity is important for this relationship.

Note that because of the simplifying standard assumption of zero supply, the expected return (7) is zero in the case of zero skewness. This result reflects the fact that positive expected returns typically are the result of a risk premium necessary to induce investors to have net positive demand for the asset in equilibrium. Therefore, with zero supply, we have isolated the impact of disagreement on expected returns net of the risk premium rather than gross of the risk premium. In Section F, we explore further the implications of positive net supply, which allows us to analyze the role of market size in the disagreement effect.

#### *2.4.4. Demand Convexity and the Disagreement–Price Relationship*

Local demand convexity has important implications for the market-clearing price. As pointed out in Section 2.4.1, local demand convexity (around the price equal to an investor's belief) implies asymmetric demand responses to symmetric price deviations. Specifically,

for equilibrium price are relevant only for higher-order expansions of demand.

[18]See footnote 10.

for a price one unit below an investor's belief, the investor will have positive demand of a certain magnitude. For a price one unit above, the investor will have negative demand or short interest of a smaller magnitude. To clear the demand of several investors with different beliefs, the price will need to be below some beliefs, to generate positive demand, and above other beliefs to generate short interest. Because the distribution of investor beliefs is centered around fundamental value, a price at fundamental value cannot offset asymmetric postive and negative demand responses. Instead, the price must be biased upward with respect to fundamental value. For example, consider Figure 1b. Because of demand convexity, at price $p = \mu$, the net demand remains positive. Price must increase above $\mu$ to induce net demand that clears the market. For higher scales of disagreement, the magnitude of asymmetry in demand responses grows, and the market-clearing price deviates from fundamental value.

Without demand convexity, long and short positions corresponding to deviations of the price from investors' respective beliefs can perfectly offset at a price equal to the average of their beliefs or fundamental value. For example, in Figure 1b, if the demand functions of type-1 and type-2 investors were linear, then their net demand would offset precisely at the midpoint price between their beliefs, which is the same as fundamental value. We would reach the same conclusion of no price bias even for higher scales of disagreement, in which investors' beliefs are further spread out.

#### *2.4.5. Short-Sales Restrictions and Demand Convexity*

Local demand convexity could arise from positive skewness—per our discussion in Section 2.4.1—or, alternatively, from short-sales frictions. For example, suppose that skewness is zero but short sales are not permitted. The demand function of Figure 1a would equal the linear tangent line for prices below the investor's belief, $p < \mu$, and be flat (zero) for prices above the investor's belief, $p > \mu$, with a kink at $p = \mu$. Likewise, if short sales are permitted, but costly—say, proportional to quantity traded—then for prices above the investor's belief, $p > \mu$, demand would decrease but at a slower pace (flatter slope), still with a kink at $p = \mu$. In either case, the demand function would be locally convex. Intuitively, positive skewness and short-sales frictions could simultaneously contribute to local demand convexity.

Although local demand convexity can result from short-sales frictions, which are a likely source of pricing effects, our model implies that they are not strictly necessary to generate the negative relationship between disagreement and expected returns. In our frictionless model, positive skewness serves as an alternative source. In Section 4.2.4, we present empirical support for both sources playing a role.

### *2.5. Trading Volume*

Denote trading volume as a function of price $p$ by $V(p) = \int_0^1 |x_i(p)|di$; i.e., the sum of the absolute value of investors' trades. Because investors of type $i$ each trade $x_i(p)$ and are of size $\pi_i$ in the population, we have trading volume

$$V(p) = \pi_1|x_1(p)| + \pi_2|x_2(p)| + \cdots + \pi_n|x_n(p)|, \tag{8}$$

where $\pi_i > 0$ and $\sum_{i=1}^n \pi_i = 1$. Figure 1b shows trading volume as a function of price for a market with two investor types. There are two kinks in the trading volume curve: one occurs where type–1 investors' demand crosses zero (at $\mu_1$) and the other occurs where type–2 investors' demand crosses zero (at $\mu_2$).

We now write $x_i$ and $V$ to denote the equilibrium demand of investor type $i$ and equilibrium volume, respectively, in which each of $x_i(p)$ and $V(p)$ are evaluated at equilibrium price $p(D)$, using the 2nd-order Taylor polynomials for demand, volume, and price:

$$x_i = \tfrac{w_0}{\gamma} D(\tfrac{1}{2}(1+\tfrac{1}{\gamma})\tfrac{s}{\sigma} D - \Delta_i)[-\tfrac{1}{\sigma^2} + \tfrac{1}{2}(1+\tfrac{1}{\gamma})\tfrac{s}{\sigma^3} D(\tfrac{1}{2}(1+\tfrac{1}{\gamma})\tfrac{s}{\sigma} D - \Delta_i)], \tag{9}$$

$$V = \pi_1|x_1| + \pi_2|x_2| + \cdots + \pi_n|x_n|, \tag{10}$$

where $\pi_i > 0$ and $\sum_{i=1}^n \pi_i = 1$.

Relatedly, short interest is the absolute value of all negative demands. In the model, short interest is determined in equilibrium as a function of all other parameters and variables. It ends up being in constant proportion ($\frac{1}{2}$) to equilibrium volume because of our simplifying assumption of zero net demand, but it varies according to model parameters. Specifically, $\sum_{x_i<0} \pi_i |x_i| = -\sum_{x_i<0} \pi_i x_i = \sum_{x_i>0} \pi_i |x_i|$, so short interest is volume divided by two, and therefore, volume and short interest will have common sensitivities to other variables in the model. In our model, it is negative demand, or short interest, that translates the convexity of demand for a positively-skewed asset into an equilibrium price above fundamental value. So, although our analysis focuses on overall volume for simplicity of exposition, the sensitivity of volume in the model is interchangeable with that of short interest. Accordingly, we use a measure of short interest as our volume proxy in our empirical implementation.

Note that $x_i$ in (9) depends on disagreement both via the underlying source of heterogeneity, $\Delta_i$, and the overall scale of disagreement, $D$. Likewise, $V$ depends on the scale of disagreement. To understand the equilibrium relationship between volume and the scale of disagreement, consider Figure 1b. At the market-clearing price, denoted by the vertical dotted line, trading volume is decreasing in price; i.e., an increase in price results in lower overall trading activity. If an increase in equilibrium price is the result of a higher scale of disagreement, however, demands expand away from the price axis at a rate faster than the trading volume decreases in price. The result is that equilibrium trading volume (as well as short interest) increases in the scale of disagreement. This result is consistent with other models in the literature, but we arrive at this result based on investors with different beliefs trading a skewed asset in a frictionless model, which to our knowledge is a novel mechanism. See Proposition 3 and Appendix D for a rigorous analysis.

**Proposition 3.** *Equilibrium trading volume, $V$, is increasing in the scale of disagreement, $D$, for $D$ not too large:*

$$\frac{\partial V}{\partial D} > 0.$$

*Proof of Proposition 3.* See Appendix C. □

Proposition 3 indicates that trading volume (and short interest) is a proxy for disagreement, consistent with the implications of earlier literature and the use of volume-based measures as proxies for disagreement, but arrived at under different assumptions and, hence, providing new support for the volume-based category of proxies. Higher levels of disagreement are associated with higher levels of trading volume and short interest, all else equal. See Appendix D for more discussion of the intuition for this result.

## 3. A Composite Proxy of Disagreement

In this section, we apply the model of Section 2 to develop a novel composite proxy of investor disagreement as a function of all model variables plus variables related to analyst forecast dispersion.

### 3.1. Two Symmetric Investor Types

First, we analyze the special case of disagreement between $n = 2$ symmetric investor types. Let $\pi_1 = \pi_2 = \frac{1}{2}$, $\Delta_1 = -1$, and $\Delta_2 = 1$, so that $\mathbf{E}[\widetilde{\Delta}] = 0$ and $\mathbf{Var}[\widetilde{\Delta}] = 1$. Because of market clearing, we have $x_1 < 0$ and $x_2 > 0$, and so $V = -\frac{1}{2}x_1 + \frac{1}{2}x_2$. Substituting (9) for $x_1$ and $x_2$ and simplifying, we find trading volume equals

$$V = \frac{w_0}{\gamma}\frac{D}{\sigma^2}\left[1 - \frac{1}{2}\left(1 + \frac{1}{\gamma}\right)^2 \frac{s^2}{\sigma^2} D^2\right], \tag{11}$$

which is well-defined (i.e., positive) for $D$ not too large. Equation (11) relates all key model variables in a simple expression.

### 3.2. Analyst Dispersion

We now augment our main model with a simple auxiliary model that incorporates the impact of analyst forecasts on the scale of disagreement. Recall that investors agree about the payoff variance ($\sigma^2$) and, therefore, any impact of analyst forecast information on investor beliefs manifests in the variation of their means and not in their perception of risk. This implication is consistent with Diether, Malloy, and Scherbina (2002), who document a puzzle that disagreement is not a risk factor. Also, recall that the distribution of expected payoffs corresponds with $\mu + D\widetilde{\Delta}$, where $\widetilde{\Delta}$ is a standardized random variable that reflects relative differences and $D$ governs scale. Analyst forecast dispersion thus impacts the scale of disagreement via a public information effect that does not alter the common payoff volatility belief, $\sigma$, and does not necessarily alter the underlying relative investor heterogeneity captured by $\widetilde{\Delta}$. Accordingly, we model the variance of means indexed to the common volatility level, ($\mathbf{Var}[(\mu + D\widetilde{\Delta})/\sigma]$), as proportional to the information conveyed by analyst forecasts via the posterior variance of the mean payoff given analyst forecasts. With this approach, analyst forecast dispersion will merge conveniently with our main model.

To fix ideas, let $j = 1, 2, \ldots, m$ denote financial analysts who each obtain a noisy signal about the true expected payoff and publish a corresponding forecast. These forecasts are observed by investors at date $t = 0$. Let $\widetilde{\mu} \sim \mathcal{N}(\mu, \sigma_\mu^2)$ be a random variable denoting the unknown expected payoff of the risky asset—i.e., $\widetilde{\mu}$ is normally distributed around the payoff's true mean $\mu$ with variance $\sigma_\mu^2$. Via their research process, analysts obtain noisy observations of $\widetilde{\mu}$. Suppose analyst $j$ publishes a forecast $A_j$, which is a realization of the random variable $\widetilde{A}_j = \widetilde{\mu} + \widetilde{a}_j$, where $\widetilde{a}_j \sim \mathcal{N}(0, \sigma_a^2)$ is independent, normally distributed noise with variance $\sigma_a^2$. Hence, analyst forecasts are correlated with each other because their signals partially reflect the true payoff $\widetilde{\mu}$, but each forecast differs because of analyst-specific noise; i.e., $\widetilde{a}_j$. Analysts publish forecasts conveying their noisy inferences about $\mu$ and the precision of their information is governed by $\frac{1}{\sigma_a^2}$.

**Lemma 4.** *Suppose the variance of investor beliefs about the expected payoff per unit of volatility is proportional to the variance of the expected payoff conditional on analysts forecasts:* $\mathbf{Var}[(\mu + D\widetilde{\Delta})/\sigma] \propto \mathbf{Var}\left[\widetilde{\mu}\middle|\widetilde{A}_1, \widetilde{A}_2, \ldots, \widetilde{A}_m\right]$. *Then,*

$$\frac{D^2}{\sigma^2} = \left(\frac{m}{\sigma_a^2} + \frac{1}{\sigma_\mu^2}\right)^{-1}, \tag{12}$$

*when the proportionality constant is one.*

Under the simplifying assumptions of analysts' signals, the scale of disagreement depends on the number of analysts, $m$, in Lemma 4. Because we model each analyst's signal as having independent noise for simplicity, analysts' collective forecasts reduce the posterior variance of the true payoff. If instead the noise in analyst signals were positively correlated across analysts or if the number of analysts increased with the uncertainty around the mean payoff—both plausible alternative assumptions—then the role of the number of analysts would drop out or be diminished. In our empirical implementation, we scale the number of analysts each month by the cross-sectional average number of analysts to allow for some variation across firms but to moderate the impact coming from our simplifying assumptions as well as to keep our disagreement measure unitless (see Section 4).

By Lemma 4, in absence of the influence of analyst forecasts ($m = 0$), risk-adjusted disagreement is captured by the fundamental uncertainty about $\mu$: $D/\sigma = \sigma_\mu$. Thus, total heterogeneity among investors has a source that does not depend solely on heterogeneity of analysts (via $\sigma_a^2$) and instead reflects the total volatility of returns factoring in both idiosyncratic volatility (i.e., $\sigma$) and fundamental uncertainty about the mean (i.e., $\sigma_\mu$). When there are two or more analysts, however, dispersion in analysts' forecasts can serve to amplify the magnitude of already-existing differences in investors' beliefs.[19] Specifically, based on the method-of-moments estimate of the variance of analyst forecasts, $\widehat{\sigma}_a^2$, the expression (12) characterizes explicitly the relationship between the scale of disagreement among investors, $D$, and analyst forecast dispersion, $\widehat{\sigma}_a$, as measured by the standard deviation of analyst forecasts. Although in this simple model, $D$ and $\widehat{\sigma}_a^2$ are functionally related, analyst dispersion in the data is an imperfect proxy for disagreement. We can combine this result with our main model to capture interactions with trading-related components.

### *3.3. Composite Disagreement Measure*

We now merge this simple auxiliary analyst dispersion model with our main trading model to develop a composite proxy of disagreement. Substituting equation (12) for $D^2/\sigma^2$

[19]Theoretically, this amplification is true for $m = 1$, but empirically we need $m \geq 2$ to estimate $\sigma_a^2$.

into the second term of (11) yields

$$V = \frac{w_0}{\gamma}\frac{D}{\sigma^2}\left[1 - \frac{1}{2}\left(1 + \frac{1}{\gamma}\right)^2 s^2 \left(\frac{m}{\sigma_a^2} + \frac{1}{\sigma_\mu^2}\right)^{-1}\right]. \tag{13}$$

In our stylized model, supply and, therefore, shares outstanding are zero. In empirical applications of our model, however, shares outstanding are positive for each firm. To aid empirical adaptation, let $v$ denote volume traded as a fraction of shares outstanding (which is unitless) and suppose that aggregate initial wealth of all investors, $w_0$, is proportional to the market value of the risky asset, $w_0 = pX$, with unit proportionality constant. Applying identities $\sigma_r p = \sigma$ and $d = D/p$ and rearranging (13) to isolate $d$ yields

$$d = \gamma\sigma_r^2 v\left[1 - \frac{1}{2}\left(1 + \frac{1}{\gamma}\right)^2 s^2 \left(\frac{m}{\sigma_a^2} + \frac{1}{\sigma_\mu^2}\right)^{-1}\right]^{-1}, \tag{14}$$

which is a single equation relating our unitless disagreement variable to all quantities of interest. Define $g \equiv \frac{1}{2}\left(1 + \frac{1}{\gamma}\right)^2 s^2 \left(\frac{m}{\sigma_a^2} + \frac{1}{\sigma_\mu^2}\right)^{-1}$. Note that the term $[1 - g]^{-1}$ in (14) should be positive in order for $d$ to be well-defined. This condition will be met as long as the term $g$ is between 0 and 1. Note that $g \geq 0$ because the number of analysts is nonnegative ($m \geq 0$), the variance of analyst forecasts is positive ($\sigma_a^2 \geq 0$) and the variance of the true payoff is positive ($\sigma_\mu > 0$). However, depending on the scale of the quantities $\gamma$, $s$, $m$, $\sigma_a$, and $\sigma_\mu$, then $g$ can potentially exceed 1. To aid empirical adaptation and ensure the condition $g \in [0, 1]$, we apply the strictly monotonic transformation $g \to 1 - e^{-g}$, which is increasing in $g$, equal to 0 for $g = 0$, and equal to 1 in the limit as $g \to \infty$. Applying this to (14), after simplification we obtain the modified composite disagreement score:

$$d^* = \gamma\sigma_r^2 v \cdot \exp\left[\frac{\frac{1}{2}\left(1 + \frac{1}{\gamma}\right)^2 s^2}{\frac{m}{\sigma_a^2} + \frac{1}{\sigma_\mu^2}}\right]. \tag{15}$$

We will use (15) to construct a unitless disagreement proxy for our main empirical analyses. Specifically, we use DIS $= \ln(1 + d^*)$, where $d^*$ is as in (15), which helps to attenuate the skewness of $d^*$ exhibited in our empirical implementation.

### *3.4. Sensitivities*

The composite proxy (15) is increasing in relative risk aversion, idiosyncratic volatility, idiosyncratic skewness, trading volume, analyst forecast dispersion, and in the scale of uncertainty about the mean payoff, all else equal; and is decreasing in the precision of analysts' information and the number of analysts, all else equal.[20] Moreover, this composite proxy reflects model interactions among these variables—something missing from individual measures and otherwise difficult to account for in empirical analyses.

Disagreement $d^*$ in (15) is a function of $\gamma$ (coefficient of relative risk aversion), $\sigma_r$ (idiosyncratic return volatility), $v$ (short interest), $s$ (idiosyncratic return skewness), $m$ (number of analyst forecasts), $\sigma_a$ (dispersion of analyst forecasts), and $\sigma_\mu$ (fundamental uncertainty) with the following sensitivities to key variables with observable proxies:

$$\frac{\partial d^*}{\partial \sigma} = 2\frac{d^*}{\sigma_r} > 0, \tag{16}$$

$$\frac{\partial d^*}{\partial v} = \frac{d^*}{v} > 0, \tag{17}$$

$$\frac{\partial d^*}{\partial \sigma_a} = d^* \frac{g}{\left(\frac{m}{\sigma_a^2} + \frac{1}{\sigma_\mu^2}\right)} \frac{2m}{\sigma_a^3} > 0. \tag{18}$$

Hence, each of idiosyncratic volatility, trading volume, and analyst forecast dispersion are proxies for disagreement, all else held equal. Moreover, each of the sensitivities (16) and (17) are consistent with those implied by (11) of the main model. In Appendix E, we explore higher-order sensitivities.

## 4. Empirical Analysis

### *4.1. Variable Selection and Methodology*

To empirically test our model, we utilize well-established disagreement measures from existing literature. Each measure represents a distinct aspect of disagreement. Our aim is to

[20] Uncertainty about mean returns is not observable in the data. In our empirical analyses, we use trailing volatility as a proxy, which overstates the uncertainty in the mean but can represent additional uncertainties absent in our simplified auxiliary model of analyst forecasts.

consolidate these various measures into a unified and unitless composite disagreement metric, which can then be applied in empirical analysis to evaluate its predictive capability and for suitable cross-sectional comparisons. In this section, we briefly outline each constituent of this composite measure. Our primary dataset is the intersection of CRSP and COMPUSTAT for all common stocks traded on the NYSE, AMEX, or NASDAQ. We apply the following standard filters. To minimize the influence of smaller, less liquid stocks, we exclude those with a month-end price below $5 (McInish and Wood, 1992). Additionally, we require each stock, denoted as stock $i$, to have a minimum of 15 valid observations in any given month, $t$, and we exclude stock-months with 10 or more zero daily returns.

Given that our composite measure is partially based on analyst forecast dispersion, our analysis is confined to stocks covered by at least two analysts. Data for these measures is sourced from the I/B/E/S unadjusted details file. Using analyst data also restricts our sample start year to 1994 as analyst forecast dates are less reliable before that (Clement and Tse, 2005; Kirk, Reppenhagen, and Tucker, 2014). The endpoint of our dataset is 2022, coinciding with the last year of available data for analyst forecasts and for our expected idiosyncratic skewness measure (obtained from the authors of Boyer, Mitton, and Vorkink (2010), who extended the measure through 2022). Consequently, our main dataset contains $720,519$ monthly observations spanning from January 1994 to December 2022.

#### *4.1.1. Risk Aversion*

The existing literature has yet to establish a consensus regarding the measurement of risk aversion or the identification of an appropriate proxy for it. The utilization of the VIX as a proxy was suggested (e.g., Bekaert and Hoerova, 2014), but incorporating time-varying risk aversion would introduce additional complexity into our model. In addition, the potential distortion of interactions with other input variables is a concern, particularly given the contentious nature of the VIX as a proxy. Consequently, in alignment with the prevalent approach in the literature, we have chosen to adopt constant risk aversion, setting $\gamma = 2$. Our results are robust to the choice of $\gamma$; see Appendix G.

*4.1.2. Idiosyncratic Volatility*

The first component of our composite measure which is not a constant is $\sigma_r$ or idiosyncratic volatility (IVOL). Theoretical evidence suggests that IVOL and divergent investor opinions are positively correlated (Shalen, 1993; Wang, 1998). Hence, IVOL should act as a proxy for disagreement (e.g., Dierkens, 1991; Boehme, Danielsen, and Sorescu, 2006; Berkman et al., 2009). One of the most frequently used methods to derive IVOL is calculating the standard deviation of the residuals from a market model, for instance, Fama and MacBeth (1973), Brown and Warner (1985), or Fama and French (1993) (e.g., Danielsen and Sorescu, 2001; Ang et al., 2006; Jiang and Sun, 2014). Here, we compute IVOL at the firm-level for stock $i$ and month $t$ by obtaining the residuals from a Fama and French (1993) daily three factor model as in Ang et al. (2006). We use IVOL expressed in percentage: $\sigma_r = \text{IVOL}_{i,t}$. See Appendix A for variable definitions.

*4.1.3. Trading Volume*

Further, numerous studies demonstrate that trading volume—short interest in particular—is indicative of the degree of disagreement between investors regarding the value of a security, based on either different information or divergent opinions (Bamber, 1987; Bessembinder, Chan, and Seguin, 1996; Bamber, Barron, and Stober, 1999; Banerjee, 2011; Boehme, Danielsen, and Sorescu, 2006; Blocher, Reed, and Van Wesep, 2013; Chang et al., 2022; Varian, 1986). For this study, volume is measured as monthly short interest, SIR, from the COMPUSTAT short interest file, which is the ratio of shares shorted to shares outstanding for stock $i$ during month $t$ (Chen, Da, and Huang, 2019; Karpoff and Lou, 2010), plus one to ensure our measure of volume is always strictly positive. We supplement our data with the short interest data for NASDAQ stocks prior to 2003 (Chen, Da, and Huang, 2019; Chen, Da, and Huang, 2022). Hence, we use: $v = 1 + \text{SIR}_{i,t}$. See Appendix A for variable definitions.

#### 4.1.4. Expected Idiosyncratic Skewness

Our model suggests that idiosyncratic skewness can be interpreted as a measure of disagreement. Directly integrating skewness into our model is challenging, however, primarily due to its lack of significant persistence (Harvey and Siddique, 1999; 2000). This lack of persistence indicates that the time-frame selected for measuring skewness influences its predictive power regarding returns. To address this issue in our analysis, we have chosen to employ the expected idiosyncratic skewness metric, ISKEW, developed by Boyer, Mitton, and Vorkink (2010). This metric, which is derived from 60-month data on firm characteristics, has been demonstrated to be predictive of idiosyncratic skewness. Hence, $s = \text{ISKEW}_{i,t}$. See Appendix A for variable definitions.

#### 4.1.5. Analyst Forecast Dispersion

We further supplement our model with a measure of respective analyst forecast dispersion. Banerjee, Kaniel, and Kremer (2009) as well as Verardo (2009) make the case that professional analysts' forecasts are the result of their underlying assumptions and information-processing models. Additionally, forecast dispersion is also likely to be free of any liquidity, trading costs, and/or size-related bias (Alexandridis, Antoniou, and Petmezas, 2007). Analyst forecasts are also frequently released, creating a near-constant flow of information (Ajinkya, Atiase, and Gift, 1991). As such, dispersion can be a suitable proxy for disagreement, provided that analysts express their unbiased opinion and that investors base their opinions on those of analysts (Garfinkel, 2009).

Diether, Malloy, and Scherbina (2002) propose a now widely-used disagreement measure based on the dispersion of analysts' forecasts, computed as the standard deviation across analysts divided by the mean absolute value of EPS forecasts (e.g., Ali et al., 2019; Barinov, 2013; Daniel, Klos, and Rottke, 2023). This measure has been used with various scaling factors, including the stock price (e.g., Banerjee, 2011; Cheong and Thomas, 2011), the book value of equity (e.g., Cen, Wei, and Yang, 2017; Dittmar and Thakor, 2007) or assets (e.g., Golez and Goyenko, 2022; Johnson, 2004), and actual earnings (Park, 2005). According to Cen, Wei, and Yang (2017), this measure is relatively robust to the selection of the scaling factor, suggesting that the standard deviation in the numerator reflects the degree of disagreement.[21] We, therefore, construct our composite disagreement measure with the forecast dispersion measure proposed by Diether, Malloy, and Scherbina (2002) for fiscal-year end forecasts, expressed in percentage:[22] $\sigma_a = \mathrm{DISP}_{i,t}$. See Appendix A for variable definitions.

*4.1.6. Number of Analysts*

Next, we use a unitless metric for the number of analyst forecasting EPS for the fiscal year-end. Aligning with our objective to minimize the introduction of units into our composite measure, we construct the unitless metric $\mathrm{ANALYST}_{\mathrm{ADJ}}$ by dividing the number of analysts with valid forecasts for stock $i$ in month $t$ by the average number of analysts following in the same month. To accurately compute this average, we omit stocks that are followed by fewer than two analysts. See Appendix A for variable definitions.

*4.1.7. Return Volatility*

The final component of our disagreement measure is the variance of the expected payoff. Identifying an appropriate empirical proxy for this variance is challenging due to the inherent difficulty of predicting payoff variance in advance. While using measures like return volatility or the volatility of a firm's profitability may seem reasonable as proxies for the expected payoff variance, there are limitations to consider. Utilizing profitability metrics could lead to a reduction in sample size, as a sufficient number of data points on profitability are required to calculate such a measure. Additionally, the choice of profitability metric could introduce sensitivity issues, affecting the robustness of the measure. In contrast, simple one-month ex-

[21] Cen, Wei, and Yang (2017) scale the standard deviation across analysts' forecasts by the absolute value of mean EPS forecast, the book value of assets, and the book value of equity. They find that the results in Diether, Malloy, and Scherbina (2002) are consistent regardless of the choice of the dispersion measure.

[22] Following Hribar and McInnis (2012), we exclude observations with mean absolute forecast of less than \$0.10 EPS to avoid effects due to small scaling. We further do not include stopped and excluded estimates in our analysis and carry forward estimates that were released up to 105 days before the earnings announcement day.

post return volatility based on daily data, which is widely available for virtually all stocks, offers a more accessible and direct reflection of the risk as perceived by investors. Therefore, for the construction of our composite measure, we opt to use RVOL (Return Volatility) expressed in percentage so that $\sigma_\mu = \text{RVOL}_{i,t}$. See Appendix A for variable definitions.

#### *4.1.8. Composite Disagreement*

Our composite disagreement measure is $\text{DIS} = \ln(1 + d^*)$, where $d^*$ is as defined in (15), with the above unitless empirical proxies for each of its components. We use the natural logarithm transformation of $d^*$ to retain the nonnegativity property of $d^*$ and to help attenuate the skewness of $d^*$ exhibited in our empirical implementation.

#### *4.1.9. Other Variables*

In addition to measuring disagreement and its impact on subsequent returns, we examine several firm- and stock-specific characteristics that explain the cross-sectional variation in returns which are also used as control variables in later analyses. We highlight the most important ones in this section. SIZE is computed as the natural logarithm of market capitalization and BTM as the book-to-market ratio as in Davis, Fama, and French (2000). Moreover, to account for the variations in returns as shown in the empirical asset pricing literature, we include MOM (Carhart, 1997) in our analysis defined as the cumulative returns from month $t - 12$ to $t - 1$. Notably, we omit the lagged return from MOM, in line with Jegadeesh and Titman (1993). BETA is the coefficient derived from regressing a stock's daily returns in month $t$ on the market return. ILLIQ is based on Amihud (2002) illiquidity measure which averages the ratio of absolute daily returns to daily trading volume. Lastly, to account for the payoff of lottery stocks, we use RMAX, which is the average of the five highest daily returns in a given month $t$ (Bali, Brown, and Tang, 2017). All variables, including our composite disagreement measure, are evaluated at a monthly level. A comprehensive overview of all variables utilized in this paper is provided in Appendix A.

### 4.2. Results

#### 4.2.1. Descriptive Statistics and Correlation Analysis

**Table 1: Descriptive Statistics**

| Variable | Code | Mean | SD | p10 | p25 | p50 | p75 | p90 |
|---|---|---|---|---|---|---|---|---|
| Composite Disagreement | DIS | 11.99 | 34.17 | 1.04 | 1.79 | 3.48 | 8.78 | 24.08 |
| Short Interest | SIR | 0.05 | 0.06 | 0.00 | 0.01 | 0.03 | 0.06 | 0.11 |
| Idiosyncratic Volatility | IVOL | 2.10 | 1.46 | 0.81 | 1.14 | 1.71 | 2.62 | 3.84 |
| Exp. Idiosyncratic Skewness | ISKEW | 0.80 | 0.58 | 0.15 | 0.45 | 0.76 | 1.07 | 1.46 |
| Analyst Forecast Dispersion | DISP | 13.18 | 64.07 | 0.74 | 1.51 | 3.58 | 9.61 | 25.24 |
| Market Size ($\times 10^{-9}$) | SIZE | 7.86 | 36.68 | 0.19 | 0.43 | 1.25 | 4.12 | 14.25 |
| Book-to-Market Ratio | BTM | 0.55 | 0.47 | 0.13 | 0.25 | 0.44 | 0.72 | 1.04 |
| Momentum | MOM | 19.02 | 66.62 | −33.79 | −12.32 | 9.75 | 34.83 | 70.80 |
| Market Loading | BETA | 1.08 | 0.95 | 0.11 | 0.55 | 1.00 | 1.52 | 2.15 |
| Illiquidity | ILLIQ | 0.10 | 1.90 | 0.00 | 0.00 | 0.00 | 0.01 | 0.06 |
| Maximum Return | RMAX | 3.27 | 2.22 | 1.32 | 1.82 | 2.68 | 4.04 | 5.92 |

**Notes:** This table presents the descriptive statistics for the relevant cross-sectional variables of interest. It includes the following statistical measures for each variable: mean, standard deviation, and percentiles at the 10th, 25th, 50th (median), 75th, and 90th levels. The sample ranges from January 1994 to December 2022 and comprises all common stocks (identified as shrcd = 10, 11) traded on the NYSE, AMEX, or NASDAQ. Stocks with month-end prices below $5 per share are excluded from the analysis. The data is collected on a monthly basis. For detailed definitions of the variables, please refer to Appendix A.

First, Table 1 provides a descriptive overview of our composite disagreement measure and selected characteristics of our sample. Focusing on our key variable of interest, DIS, the sample exhibits an average disagreement of 11.99 with an average standard deviation of 34.17. Notably, the distribution of this measure is right-skewed, with the 10th percentile at 1.04 and 90th percentile at 24.08. Most stocks exhibit positive ISKEW and each of the variables in Table 1 is itself positively skewed (i.e., its mean above its median). In particular, analyst forecast dispersion, DISP, a widely-used measure of disagreement, is especially right-skewed.

To further investigate the interplay between our empirical proxies for the components of our disagreement measure, Panel A of Table 2 presents pooled correlations of DIS with its individual components and various stock and firm characteristics, pooling all firm-months. In line with our model's predictions, there is a positive correlation between the measure and each of its individual components. DIS comoves relatively strongly with IVOL (0.47) and relatively weakly with DISP (0.10) and SIR (0.12). To the extent that DIS is an accurate proxy of disagreement, of the major disagreement proxies, IVOL is then the most

**Table 2: Pooled Correlations**

| Panel A: Correlations with DIS | |
|---|---|
| SIR | 0.12 |
| IVOL | 0.47 |
| ISKEW | 0.51 |
| DISP | 0.10 |
| SIZE | −0.05 |
| BTM | 0.08 |
| MOM | −0.05 |
| BETA | 0.12 |
| ILLIQ | 0.04 |
| RMAX | 0.46 |

| Panel B: Correlations Among Major Disagreement Proxies | | | |
|---|---|---|---|
| | DISP | IVOL | SIR |
| DISP | 1.00 | | |
| IVOL | 0.08 | 1.00 | |
| SIR | 0.07 | 0.14 | 1.00 |

**Notes:** Panel A presents pooled correlations between our disagreement measure (DIS) and stock characteristics. Panel B presents pooled correlations among major disagreement proxies, DISP, IVOL, and SIR. The sample ranges from January 1994 to December 2022 and comprises all common stocks (identified as shrcd = 10, 11) traded on the NYSE, AMEX, or NASDAQ. For detailed definitions of the variables, please refer to Table 1 and Appendix A.

aligned with disagreement and, therefore, we should expect DIS to absorb some of the return predictability of IVOL in empirical tests. DIS also comoves relatively strongly with ISKEW (0.51), reflecting the important role of skewness in our model. The correlation observed with IVOL also suggests that our measure, in part, may be capturing effects associated with firm transparency (Ferreira and Laux, 2007) and information asymmetry (Berkman et al., 2009). Panel B of Table 2 presents correlations among the major disagreement proxies, DISP, IVOL, and SIR. These cross-correlations are relatively low, highlighting the frequency of disagreement among different disagreement proxies. In Appendix G, we report time-series average correlations corresponding to those of Table 2, from which we draw similar conclusions.

*4.2.2. Univariate Analysis*

To gain insights into the relationship between our disagreement measure and future returns, we carry out a series of portfolio analyses, as detailed in Table 3. In this analysis,

**Table 3: Portfolio Sorts on Disagreement, DIS**

| DIS | Raw Return | Excess Return | CAPM-$\alpha$ | FF3-$\alpha$ | FFC-$\alpha$ |
|---|---|---|---|---|---|
| Low | 0.97 | 0.79 | 0.28 | 0.26 | 0.19 |
| 2 | 0.97 | 0.79 | 0.19 | 0.16 | 0.15 |
| 3 | 0.93 | 0.75 | 0.06 | 0.05 | 0.05 |
| 4 | 0.84 | 0.66 | −0.10 | −0.11 | −0.07 |
| 5 | 0.79 | 0.61 | −0.20 | −0.20 | −0.16 |
| 6 | 0.93 | 0.75 | −0.11 | −0.13 | −0.02 |
| 7 | 0.66 | 0.48 | −0.45 | −0.46 | −0.31 |
| 8 | 0.65 | 0.47 | −0.55 | −0.55 | −0.36 |
| 9 | 0.30 | 0.12 | −1.03 | −0.99 | −0.78 |
| High | −0.04 | −0.22 | −1.49 | −1.43 | −1.23 |
| Low-High | 1.01 | 1.01 | 1.78 | 1.69 | 1.42 |
| $t$-stat. | (2.09) | (2.09) | (4.95) | (5.53) | (4.41) |
| $p$-value | 0.038 | 0.038 | 0.000 | 0.000 | 0.000 |

**Notes:** This table reports the mean values of value-weighted monthly returns at month $t + 1$, based on univariate portfolios sorted according to disagreement (DIS) at date $t$, excluding firm-months with negative ISKEW. The sample ranges from January 1994 to December 2022. The row labeled "Low-High" presents the differences in monthly returns between the 10th decile (representing high disagreement) and the 1st decile (representing low disagreement). "Raw returns" refer to one-month-ahead monthly returns, whereas "Excess returns" are defined as one-month-ahead monthly returns minus the risk-free rate. The alphas (CAPM-$\alpha$, FF3-$\alpha$, FFC-$\alpha$) represent intercepts from a time-series regression of monthly excess returns at month $t + 1$ against the market return (CAPM), further adjusted for size and value factors (FF3), and additionally a momentum factor (FFC) at month $t$. $t$-statistics, adjusted according to Newey and West (1987), are presented in parentheses along with their corresponding $p$ values.

stocks are categorized monthly into deciles based on their level of disagreement (DIS), excluding firm-months with negative ISKEW to be consistent with Proposition 2, which predicts a negative disagreement–expected return relationship for positively skewed stocks.[23] For each decile, we calculate the value-weighted average returns using five different one-month-ahead return measures. These include raw returns, excess returns (defined as the difference between the monthly returns of stock $i$ and the risk-free rate), CAPM-$\alpha$ (which includes a market factor), the Fama-French 3-factor model alpha (FF3-$\alpha$, which additionally considers size and value factors), and FFC-$\alpha$ (which includes an additional momentum factor as per Carhart, 1997). Value-weighted long-short portfolio returns are then computed for each return metric, and their statistical significance is reported.

A decreasing trend, although not monotonic, is observed in all return and alpha metrics

[23]Because the vast majority of stocks are positively skewed, as reported in Table 1, our inferences are robust to the inclusion of negative ISKEW stocks and we include all stocks in subsequent analyses. See Table G.2 in the appendix for the version of Table 3 that includes all stocks.

across increasing disagreement deciles, especially in the upper deciles. All long-short L–H portfolios exhibit positive returns, which are statistically significant at the 5% level or lower.[24] For instance, buying low disagreement stocks and selling high disagreement stocks results in annualized returns of 12.1% for raw and excess ($t$-statistic $= 2.09$), 21.3% for risk-adjusted CAPM-$\alpha$ ($t$-statistic $= 4.95$), 20.3% for risk-adjusted FF3-$\alpha$ ($t$-statistic $= 5.53$), and 17.0% for risk-adjusted FFC-$\alpha$ ($t$-statistic $= 4.41$). These results suggest that disagreement encompasses more than the usual risk elements.

The magnitudes of the returns and alphas of these spread portfolios are economically quite large. Because stocks with higher disagreement tend to have lower expected returns but also higher betas (see Section 4.2.3), expected returns diverge from those predicted by the CAPM as disagreement increases, generating large alphas for the upper disagreement deciles. The L–H spread CAPM alpha is not driven only by the short leg, however, as the lowest disagreement decile contributes 3.4%, which is statistically significant with $t$-statistic 2.88.

These magnitudes are consistent with recent evidence of disagreement return predictability of Bali et al. (2024), who develop a machine-learning-based measure of disagreement. Bali et al. (2024) report annualized excess return of 13.8% and CAPM-$\alpha$ of 17.6% for a value-weighted decile spread portfolio sorted on their disagreement measure, which are comparable to the 12.1% and 21.3%, respectively, that we document using our composite disagreement measure. However, we arrive at our disagreement measure using a distinct and parsimonious approach. We first developed a pricing model founded on ordinary assumptions in a frictionless market, and then took our formula (15) to the data and applied simple mappings from a small set of model variables to their empirical counterparts to construct and test our disagreement proxy. Accordingly, our empirical results are consistent with the core theoretical insights.

[24] The $p$-values are unadjusted for multiple tests and should be interpreted with caution. That said, the empirical work is based on implications from a theoretical model. See Harvey (2017).

Although none of these portfolio results take transaction costs into account, these portfolios are value-weighted and only rebalanced monthly and, therefore, most of these portfolios would be expected to retain much of their alpha after accounting for trading frictions. Alphas for deciles 9 and 10 are likely overstated, however, given that stocks with higher disagreement are less liquid (see Section 4.2.3) and shorted in the L–H spread portfolio, entailing significant trading costs.

### *4.2.3. Characteristics*

**Table 4: Average Stock Characteristics of Disagreement-Sorted Portfolios**

| Panel A | | | | | | | |
|---|---|---|---|---|---|---|---|
| DIS | DIS | SIR | IVOL | ISKEW | DISP | SIZE | BTM |
| Low | 1.00 | 0.02 | 0.83 | 0.45 | 3.03 | 24.80 | 0.52 |
| 2 | 1.59 | 0.03 | 1.16 | 0.51 | 4.15 | 15.69 | 0.50 |
| 3 | 2.12 | 0.04 | 1.41 | 0.55 | 5.47 | 11.33 | 0.50 |
| 4 | 2.76 | 0.04 | 1.64 | 0.60 | 7.19 | 8.36 | 0.50 |
| 5 | 3.63 | 0.05 | 1.87 | 0.65 | 9.24 | 6.12 | 0.51 |
| 6 | 4.93 | 0.05 | 2.10 | 0.73 | 11.61 | 4.81 | 0.53 |
| 7 | 7.12 | 0.05 | 2.33 | 0.85 | 14.88 | 3.48 | 0.55 |
| 8 | 11.19 | 0.06 | 2.58 | 1.00 | 19.51 | 2.28 | 0.59 |
| 9 | 20.25 | 0.06 | 2.96 | 1.19 | 24.73 | 1.50 | 0.62 |
| High | 67.67 | 0.07 | 4.13 | 1.54 | 33.15 | 0.79 | 0.66 |
| High-Low | 66.67 | 0.04 | 3.30 | 1.09 | 30.12 | −24.01 | 0.14 |
| $t$-stat. | 8.64 | 13.79 | 34.86 | 13.08 | 22.01 | −10.91 | 4.97 |

| Panel B | | | | | | | |
|---|---|---|---|---|---|---|---|
| DIS | MOM | BETA | ILLIQ | RMAX | Z-SCORE | KZ | OHLSON |
| Low | 15.07 | 0.74 | 0.01 | 1.60 | 2.91 | 3.37 | 0.05 |
| 2 | 17.39 | 0.86 | 0.01 | 2.06 | 3.20 | 3.78 | 0.05 |
| 3 | 20.03 | 0.93 | 0.02 | 2.39 | 4.38 | 4.09 | 0.06 |
| 4 | 21.79 | 0.99 | 0.03 | 2.68 | 3.77 | 4.29 | 0.08 |
| 5 | 23.52 | 1.05 | 0.04 | 2.97 | 4.29 | 4.52 | 0.09 |
| 6 | 24.20 | 1.12 | 0.05 | 3.28 | 4.67 | 4.81 | 0.11 |
| 7 | 22.17 | 1.18 | 0.07 | 3.58 | 4.36 | 4.77 | 0.14 |
| 8 | 19.41 | 1.23 | 0.11 | 3.91 | 4.96 | 4.94 | 0.18 |
| 9 | 14.50 | 1.30 | 0.21 | 4.43 | 6.17 | 5.40 | 0.25 |
| High | 8.08 | 1.42 | 0.46 | 5.94 | 7.93 | 6.21 | 0.35 |
| High-Low | −6.99 | 0.69 | 0.45 | 4.34 | 5.03 | 2.84 | 0.31 |
| $t$-stat | −1.77 | 15.25 | 5.18 | 26.94 | 5.16 | 7.76 | 18.24 |

**Notes:** This table reports the average values of various stock characteristics across univariate, equally weighted portfolios sorted based on disagreement (DIS). The sample ranges from January 1994 to December 2022. The categories "Low" and "High" refer to the portfolios within the lowest and highest disagreement deciles, respectively. Newey and West (1987) adjusted $t$-statistics for assessing the statistical difference between the highest and lowest disagreement decile are shown in the bottom row. For detailed definitions of the variables, please refer to Table 1 and Appendix A.

Next, we analyze in Table 4 monthly average characteristics of decile portfolios sorted by our composite disagreement measure. Consistent with our model's predictions, we observe an increase in individual components with rising levels of disagreement. Moreover, stocks associated with higher disagreement are characterized by lower market capitalization (SIZE) and are consistent with potential market undervaluation (indicated by higher BTM ratios). Stocks with higher disagreement also exhibit less momentum, although the relationship is statistically weaker than those other characteristics. The relationship between disagreement and future returns implies that high levels of disagreement may signal potential turning points in momentum (Goulding, Harvey, and Mazzoleni; 2023). Additionally, our findings indicate that stocks with high disagreement are riskier (as evidenced by higher beta values) and exhibit higher levels of illiquidity, correlating with observations on stock size. 'Lottery' stocks, identified by heightened uncertainty (Zhang, 2006), also display increased disagreement. Lastly, we explore the connection between our composite disagreement measure and a firm's financial health, utilizing the Altman Z-Score (Altman, 1968) and the Ohlson O-Score (Ohlson, 1980) as bankruptcy risk indicators and the Kaplan-Zingales index (Kaplan and Zingales, 1997) to gauge reliance on external financing. All three metrics show a consistent increase across disagreement deciles, suggesting that our measure effectively identifies financially distressed firms, which are typically characterized by greater information asymmetry, and which have been associated with disagreement effects on prices (Avramov et al., 2009). Hence, our disagreement measure appears capable of capturing such effects.

In unreported tests similar to those depicted in Table 4, we analyze the relationship between disagreement (DIS) and various additional firm characteristics. Our findings indicate that elevated levels of DIS are associated with several attributes: younger firms (AGE), increased net income idiosyncratic volatility (NI-IVOL), heightened concentration of institutional investors (as measured by the Herfindahl and Hirschman Index, HHI), intensified research and development expenditures (R&D), augmented net equity issuance (ISS), greater information asymmetry (ASC), elevated standardized unexplained volume (SUV), increased turnover (TO), higher change in market-adjusted turnover (DTO), greater opinion divergence among mutual funds (DBREADTH; see Chen, Hong, and Stein, 2002), wider bid-ask spreads (BIDASK), and reduced volumes of Google searches, both overall (SV) and specifically investment-related (SV-INV). For detailed definitions of these variables, please refer to Appendix A. Several of these characteristics correspond to one of many disagreement proxies employed in the literature, each demonstrating the anticipated relationship with DIS, thereby providing additional credibility for DIS as a valid measure of disagreement. See Appendix B for further details.

#### *4.2.4. Fama and MacBeth (1973) Cross-Sectional Regressions*

Next, we perform cross-sectional regressions on the excess return (EXRET) at month $t+1$ in the spirit of Fama and MacBeth (1973). The cross-sectional coefficients, weighted for each month, are averaged across all months to remove any year/month related effects. Table 5 presents the results of regression and reports $t$-statistics adjusted by the method proposed by Newey and West (1987).

Columns (1) to (4) of Table 5 present the coefficients from cross-sectional regressions for excess returns (EXRET). In Column (1), we find that disagreement, DIS, has a negative impact on future excess returns, evidenced by a coefficient of $-1.55$ and a statistically significant $t$-statistic of $-3.98$. This coefficient (divided by 100) times a given level of DIS indicates the predicted excess return (in percentage) under the regression model for that DIS level. For example, from Table 4, the spread between the average level of DIS in the low and high deciles, 66.67 ($= 1.00 - 67.67$), multiplied by the regression coefficient ($-1.55$), predicts a return spread of 1.03% per month (or 12.4% annualized), which is comparable to the decile-portfolio excess return spread reported in Table 3.

In Column (2) of Table 5, we observe that this negative relationship persists even after accounting for each of the major individual components of our disagreement measure, with the composite measure continuing to predict returns negatively (coefficient = $-0.97$, $t$-statistic = $-3.23$), even though the composite measure is substantially correlated with IVOL

**Table 5: Fama and MacBeth (1973) Cross-Sectional Regressions**

| | (1) EXRET$_{t+1}$ | (2) EXRET$_{t+1}$ | (3) EXRET$_{t+1}$ | (4) EXRET$_{t+1}$ |
|---|---|---|---|---|
| DIS | $-1.55^{***}$ | $-0.97^{***}$ | $-1.36^{***}$ | $-0.96^{***}$ |
| | $(-3.98)$ | $(-3.23)$ | $(-4.59)$ | $(-3.27)$ |
| SIR | | $-3.75^{***}$ | | $3.45^{**}$ |
| | | $(-2.64)$ | | $(-2.46)$ |
| IVOL | | $-3.71$ | | $-4.95$ |
| | | $(-0.56)$ | | $(-0.94)$ |
| ISKEW | | $-0.38^{**}$ | | $-0.23$ |
| | | $(-2.16)$ | | $(-1.39)$ |
| DISP | | $-0.16^{**}$ | | $-0.14^{**}$ |
| | | $(-2.22)$ | | $(-2.16)$ |
| LnSIZE | | | $-0.04$ | $-0.09^{**}$ |
| | | | $(-1.00)$ | $(-2.08)$ |
| BTM | | | 0.09 | 0.06 |
| | | | (0.58) | (0.45) |
| MOM | | | 0.34 | 0.30 |
| | | | (1.62) | (1.41) |
| BETA | | | $-0.07$ | $-0.03$ |
| | | | $(-0.55)$ | $(-0.26)$ |
| ILLIQ | | | $-0.47$ | $-0.02$ |
| | | | $(-1.33)$ | $(-0.07)$ |
| Constant | $0.82^{***}$ | $1.29^{***}$ | $0.98^{**}$ | $1.71^{***}$ |
| | (2.94) | (4.91) | (2.07) | (3.26) |
| Observations | 719,940 | 719,940 | 701,159 | 701,159 |

**Notes:** This table displays the regression coefficients and $t$-statistics from a monthly Fama and MacBeth (1973) regression analysis conducted on excess returns, adhering to the methodologies in Fama and French (1993) and Carhart (1997). These regressions are performed against a lagged measure of disagreement and various lagged stock and firm characteristics. The sample ranges from January 1994 to December 2022. The coefficients on DIS, IVOL, DISP, and MOM are multiplied by 100 for readability. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively. For detailed definitions of the variables, please refer to Appendix A.

and ISKEW. It appears to absorb most of the negative variation associated with idiosyncratic volatility (IVOL), while skewness (ISKEW), DISP, and SIR remain negative predictors of future returns. In Table 6, we include regression results isolated to DIS and IVOL each alone and together as predictors. With DIS included, the significance of IVOL drops from being significant at the 5% level to insignificance. Based on our model, these results suggest a disagreement-based explanation for the return predictability of IVOL as DIS largely absorbs its predictability.

In Column (3) of Table 5, we incorporate well-established return-predicting characteristics from asset pricing literature, illiquidity (ILLIQ), log of market size (LOG-SIZE), book-to-market ratio (BTM), momentum (MOM), and beta (BETA). Even with these control

**Table 6: Fama and MacBeth (1973) Cross-Sectional Regressions with DIS and IVOL**

| | (1) $\text{EXRET}_{t+1}$ | (2) $\text{EXRET}_{t+1}$ | (3) $\text{EXRET}_{t+1}$ |
|---|---|---|---|
| DIS | $-1.55^{***}$ | | $-1.26^{***}$ |
| | $(-3.98)$ | | $(-3.64)$ |
| IVOL | | $-15.98^{**}$ | $-7.63$ |
| | | $(-2.14)$ | $(-1.00)$ |
| Constant | $0.82^{***}$ | $1.01^{***}$ | $0.92^{***}$ |
| | $(2.94)$ | $(3.93)$ | $(3.58)$ |
| Observations | $719,940$ | $719,940$ | $719,940$ |

**Notes:** This table displays the regression coefficients and $t$-statistics from a monthly Fama and MacBeth (1973) regression analysis conducted on excess returns, adhering to the methodologies in Fama and French (1993) and Carhart (1997). These regressions are performed against a lagged measure of disagreement and lagged IVOL. The sample ranges from January 1994 to December 2022. The coefficients on DIS an IVOL are multiplied by 100 for readability. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively. For detailed definitions of the variables, please refer to Appendix A.

variables, our disagreement measure maintains its robustness, continuing to negatively predict returns (coefficient $= -1.36$, $t$-statistic $= -4.59$). The final model in Column (4) of Table 5, which regresses EXRET on the composite disagreement measure, the individual disagreement components, and control variables, shows that the composite measure's predictive power is unaffected by the inclusion of these variables (coefficient $= -0.96$, $t$-statistic $= -3.27$). The model predicts that the marginal contribution of DIS to a low-minus-high decile spread portfolio is 7.7% annualized, after accounting for the predicted return impact of the average levels of the other cross-sectional determinants including disagreement proxies.

Notably, SIR and DISP consistently emerge as negative predictors of future returns. Given that SIR and DISP increase with disagreement deciles, it is postulated that SIR and DISP each reflect predictability other than that linked to disagreement, which is captured by the composite measure. In unreported tests, we also control for the bid-ask spread. The return predictability of bid-ask spreads—which is also negative and, therefore, consistent with it being a disagreement measure in the literature—is absorbed by DIS: it is no longer significant at the 5% level.

Our model does not rely on short-sales frictions; whereas, the leading explanation for the disagreement–expected return effect appeals to such frictions. We predict, therefore, that the effect can manifest even for stocks with little to no short-sales frictions. To test this hypothesis, following the literature, we use institutional ownership (IO) as an inverse proxy of short-sales constraints—higher IO implies less frictions (Nagel, 2005). Table 7 reports excess returns regressed on DIS for five quintiles of IO. As IO increases across IO quintiles, the significance of DIS decreases, indicating that short-sales frictions likely play a role in the disagreement–expected return effect.[25] Nevertheless, DIS remains significant at the 5% level in all IO quintiles, even the highest IO quintile, which suggests that short-sale frictions are not the sole determinant of disagreement–expected return predictability and, therefore, the effect can arise outside of such frictions, consistent with our model.

**Table 7: Fama and MacBeth (1973) Cross-Sectional Regressions by Institutional Ownership Quintile**

| IO | (Low) $\text{EXRET}_{t+1}$ | (2) $\text{EXRET}_{t+1}$ | (3) $\text{EXRET}_{t+1}$ | (4) $\text{EXRET}_{t+1}$ | (High) $\text{EXRET}_{t+1}$ |
|---|---|---|---|---|---|
| DIS | $-1.66^{***}$ | $-2.02^{***}$ | $-2.07^{***}$ | $-2.45^{**}$ | $-2.60^{**}$ |
| | $(-4.35)$ | $(-3.16)$ | $(-3.06)$ | $(-2.44)$ | $(-2.32)$ |
| Constant | $0.74^{***}$ | $0.90^{***}$ | $0.93^{**}$ | $0.87^{***}$ | $0.74^{***}$ |
| | (2.71) | (3.44) | (3.38) | (3.11) | (2.48) |
| Observations | 144,132 | 143,991 | 143,986 | 143,991 | 143,840 |

**Notes:** This table displays the regression coefficients and $t$-statistics from a monthly Fama and MacBeth (1973) regression analysis conducted on excess returns, adhering to the methodologies in Fama and French (1993) and Carhart (1997). These regressions are performed against a lagged measure of disagreement for stocks within one of five institutional ownership quintiles within the month. The sample ranges from January 1994 to December 2022. The coefficients on DIS are multiplied by 100 for readability. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively. For detailed definitions of the variables, please refer to Appendix A.

Our disagreement measure, DIS, is a nonlinear combination of other disagreement measures. How relevant is this nonlinear structure? Note that from Table 5, even when including all of its major disagreement components as controls, DIS remains a significant negative predictor. In addition to this evidence, to verify that our proposed measure outperforms linear combination approaches, we employ two distinct methods for deriving alternative disagreement measures. First, we dissect the individual components of DIS—which include

[25] The magnitude of the coefficients on DIS increase with increasing IO quintile because the level of DIS decreases, on average, as IO increases. Nevertheless, the magnitude of the $t$-statistics decrease with increasing IO.

analyst dispersion (DISP), idiosyncratic volatility (IVOL), and short interest ratio (SIR)—by sorting each into deciles monthly. We then calculate a straightforward, equally weighted disagreement score, DEW, ranging from 1 (indicating the lowest level of disagreement) to 10 (representing the highest). In a more sophisticated approach, we leverage principal component analysis (PCA) to identify underlying patterns across the same disagreement proxies. By reducing dimensionality, while retaining most of the variance in the data, PCA allows us to extract common factors that summarize the variations within these proxies. For our analysis, we focus particularly on the first principal component, DPCA, which accounts for 40% of the variance in the disagreement proxies. This component should serve as a compact representation of the primary underlying factor that drives the observed variations in disagreement levels.

**Table 8: Fama and MacBeth (1973) Cross-Sectional Regressions by DIS and Linear Composite Disagreement Alternatives**

| | (1)<br>$\text{EXRET}_{t+1}$ | (2)<br>$\text{EXRET}_{t+1}$ | (3)<br>$\text{EXRET}_{t+1}$ | (4)<br>$\text{EXRET}_{t+1}$ | (5)<br>$\text{EXRET}_{t+1}$ |
|---|---|---|---|---|---|
| DIS | $-1.55^{***}$<br>$(-3.98)$ | $-1.20^{***}$<br>$(-4.31)$ | $-1.54^{***}$<br>$(-4.59)$ | $-1.01^{***}$<br>$(-3.68)$ | $-1.70^{***}$<br>$(-4.83)$ |
| DEW | | $-0.08$<br>$(-1.39)$ | | | |
| DEW+ | | | $-0.04$<br>$(-0.79)$ | | |
| DPCA | | | | $-0.20^{*}$<br>$(-1.80)$ | |
| DPCA+ | | | | | $-0.02$<br>$(-0.30)$ |
| Constant | $0.82^{***}$<br>$(2.94)$ | $1.25^{***}$<br>$(4.55)$ | $1.05^{***}$<br>$(3.64)$ | $0.74^{***}$<br>$(2.59)$ | $0.80^{***}$<br>$(2.84)$ |
| Observations | 719,940 | 719,940 | 717,344 | 719,940 | 717,344 |

**Notes:** This table displays the regression coefficients and $t$-statistics from a monthly Fama and MacBeth (1973) regression analysis conducted on excess returns, adhering to the methodologies in Fama and French (1993) and Carhart (1997). These regressions are performed against a lagged measure of disagreement and lagged alternative linear disagreement measures. The sample ranges from January 1994 to December 2022. DEW is a linear disagreement score constructed as follows: in each month, assign each stock to a decile based on each of DISP, IVOL, and SIR, and then take a simple average of these decile ranks. DEW+ is constructed similarly, except deciles are formed on DISP, IVOL, and ASIR, as well as four disagreement variables BIDASK, SUV, DTO, and DBREADTH. DPCA is the first component of a principal component analysis based on DISP, IVOL, and SIR. DPCA+ is the first component of a principal component analysis based on DISP, IVOL, ASIR, BIDASK, SUV, DTO, and DBREADTH. The coefficients on DIS are multiplied by 100 for readability. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively. For detailed definitions of the variables, please refer to Appendix A.

To address potential biases related to variable selection, we conduct a separate analysis incorporating a broader array of disagreement proxies previously proposed by the literature. Beyond DISP and IVOL, this set includes abnormal short interest (ASIR; see Chen, Da, and Huang, 2019), the high-low bid-ask spread (BIDASK; see Handa, Schwartz, and Tiwari, 2003), changes in mutual fund ownership (DBREADTH; see Chen, Hong, and Stein, 2002), market-adjusted turnover changes (DTO), and standardized unexplained volume (SUV; see Garfinkel, 2009 for an overview on those measures). Similar to our earlier method, we compute an equally weighted measure, DEW+, and also perform PCA on this expanded set of disagreement proxies. The resultant first principal component, DPCA+, explains 26% of the total variance in these proxies, thus offering another perspective on the dynamics influencing disagreement.

Table 8 reports the predictive regression results for DIS controlling for each of the linear disagreement measures described above. DIS remains a robust negative predictor of expected returns in all cases. From this evidence, we conclude that the nonlinear structure implied by the model is relevant for capturing disagreement.

## 5. Conclusion

Our paper makes several contributions. Though our model is simple, we are able to gain a number of insights from it. First, for each disagreement proxy considered in isolation, our model provides new theoretical support for that proxy's connection to investor disagreement. Second, because our model encapsulates a collection of these disparate measures in a unified framework, we can provide theoretical guidance for the relationships among proxies, such as how one variable can amplify or diminish the connection of another variable to underlying investor disagreement. Third, we use these connections to construct a composite measure of disagreement, which is a nonlinear combination of several proxies into a single score. One could potentially complexify the model to make it richer, e.g., add considerations for real-world frictions, but we leave that for further research.

Fourth, we provide empirical support for the importance of the nonlinear structure of the composite measure. This empirical evidence is statistically and economically meaningful. For example, a value-weighted decile spread portfolio sorted on our composite measure of disagreement generates over 12% annualized excess returns and over 21% annualized CAPM alpha with similarly large multifactor alphas. These remarkably large alphas arise because stocks with higher disagreement tend not only to have lower expected returns but also a confluence of characteristics that would predict higher expected returns under traditional models: higher betas, less liquidity, and higher measures of financial distress. Fifth, although it is well-documented that the disagreement–expected return effect is concentrated in smaller stocks, we extend our model (in Appendix F) to provide a novel explanation for this concentration.

Finally, we establish a disagreement-based rationale for the IVOL effect and provide evidence that our disagreement proxy absorbs the return predictability of IVOL, even after controlling for other standard cross-sectional determinants from the literature. Of the major disagreement proxies incorporated into our composite measure, IVOL is also the most correlated with it. This evidence suggests that IVOL is the most reliable individual disagreement proxy among the major proxy categories we study.

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# Disagreement of Disagreement
## Internet Appendix

### Appendix A  Empirical Variable Descriptions

AGE is the age of the firm in years following the methodology of Jiang, Lee, and Zhang (2005).

ANALYST is the number of analysts that issued a valid fiscal year forecast in month $t$ divided by the average number of analysts per firm with valid forecasts in month $t$.

ASC is the adverse selection component of the bid-ask spread in month $t$ derived from Armstrong et al. (2011) using intraday data from TAQ. Data is available until 2020.

ASIR is the difference between a firm's actual and predicted SIR in month $t$. Predicted SIR is computed as the moving average SIR over the past twelve months.

BETA is the coefficient of the regression of daily returns on daily market returns for month $t$.

BIDASK is the high-low bid ask estimator for month $t$ following Corwin and Schultz (2012).

BTM is the ratio of book value of equity (following Davis, Fama, and French (2000)) from the most recent quarter to the market value of equity which is defined as shares outstanding times the share price in month $t$.

CAPM-$\alpha$ is the intercept from the regression of monthly excess portfolio returns on a constant and excess market return for month $t+1$.

DBREADTH is the change in the ratio of mutual funds with long positions in stock $i$ in quarter $q$ to all mutual funds, multiplied by minus one.

DEW is the equally weighted average disagreement decile in month $t$ based on same-month deciles of DISP, IVOL, and SIR.

DEW+ is the equally weighted average disagreement decile in month $t$ based on same-month deciles of ASIR, BIDASK, DBREADTH, DISP, DTO, IVOL, and SUV.

DIS is one plus the natural log of our composite measure of disagreement in equation (15).

DISP is the standard deviation of fiscal year end analyst EPS forecasts valid in month $t$ across analysts for stock $i$ scaled by the average absolute EPS forecast, multiplied by 100.

DPCA is the first component of a principal component analysis on DISP, IVOL, and SIR in month $t$.

DPCA+ is the first component of a principal component analysis on ASIR, BIDASK, DBREADTH, DISP, DTO, IVOL, and SUV in month $t$.

DTO is the average daily market-adjusted turnover in month $t$ less the average daily market-adjusted turnover over the past three months.

EXRET$_{t+1}$ is the difference between monthly raw returns and the risk-free rate for stock $i$ in month $t+1$.

FF3-$\alpha$ is the intercept from the regression of monthly excess portfolio returns on a constant, the excess market return, a size factor, and a book-to-market factor in month $t+1$ following Fama and French (1993).

FFC-$\alpha$ is the intercept from the regression of monthly excess portfolio returns on a constant, the excess market return, a size factor, a book-to-market factor, and a momentum factor in month $t+1$ following Fama and French (1993) and Carhart (1997).

HHI is the Herfindahl-Hirschman Index defined as the summed, squared market shares of individual stock $i$ in quarter $q$ for the $j$th mutual fund.

ILLIQ is the ratio of the absolute monthly stock return to its dollar trading volume, multiplied by $10^6$ as in Amihud (2002).

IO is the ratio of shares held by institutional investors to total shares outstanding as in Chen, Hong, and Stein (2002).

IVOL is the standard deviation of the daily residuals estimated from the regression of excess portfolio returns on a constant, the excess market return, a size factor, and a book-to-market factor at a monthly level following Ang et al. (2006), multiplied by 100.

ISKEW is the expected idiosyncratic skewness based as defined in Boyer, Mitton, and Vorkink (2010).

ISS is the difference between a firm's equity issuance and its buybacks, scaled by total assets following Bradshaw, Richardson, and Sloan (2006).

KZ-INDEX is the Kaplan and Zingales (1997) Index as a measure of reliance on external financing which is based on a linear combination of cash flows, PP&E, Tobin's Q, Total Debt, Dividends, and Cash.

NI-IVOL is the standard deviation of the residual of a regression of net income scaled by assets on the same measure lagged by a year.

R&D is research and development expenses scaled by assets.

RMAX is the average of the five highest daily returns for stock $i$ in month $t$, multiplied by 100.

RVOL is the one-month ex-post return volatility based on daily data for stock $i$ in month $t$, multiplied by 100.

MOM is the product of monthly gross returns for month $t-12$ until $t-1$, minus 1, multiplied by 100.

O-SCORE is the Ohlson O-Score, which is a measure of the probability of bankruptcy as defined in Ohlson (1980).

$\text{RET}_{t+1}$ is the raw monthly return for month $t+1$, multiplied by 100.

SIR is the ratio of shares sold short to shares outstanding for stock $i$ in month $t$ as by Chen, Da, and Huang (2019).

SIZE is the product of shares outstanding and the share price of stock $i$ in month $t$.

SUV is the standardized residual of a regression of trading volume on positive and negative returns over the past three months following Garfinkel (2009).

SV is the value of the Google search volume index for a specific ticker as provided by trends.google.com. Data is available until 2018.

SV-INV is the value of the modified Google search volume index for a specific ticker and investment-related searches as provided by trends.google.com and following the methodology of deHaan, Lawrence, and Litjens (2023). Data is available until 2018.

TO is the average ratio of daily trading volume to shares outstanding in month $t$.

Z-SCORE is the Altman Z-Score, which is a measure of bankruptcy risk as a linear combination of working capital, retained earnings, EBIT, sales (all scaled by total assets), and the proportion of market value of equity to total liabilities as defined in Altman (1968).

## Appendix B Measures of Disagreement

This section summarizes the most commonly used proxies for investor disagreement and gives examples of papers using the respective measure published in top tier journals across accounting, economics, finance, and management studies.

In models of higher order beliefs, a key assumption for prices to move is that investors are uncertain about the opinions of other investors (Allen, Morris, and Shin (2006); Banerjee, Kaniel, and Kremer (2009)). Houge et al. (2001) demonstrate how investors' opinions on an IPO vary depending on how they perceive the distribution of returns. Both Kandel and Pearson (1995) and Hsiao (2022) illustrate how disagreement can arise when traders have different interpretations of a public signal. Hsiao (2022) further argues that, due to traders' uncertainty regarding the information quality of the interpretation, they perceive other investors' interpretations of a signal as ambiguous. However, disagreement caused by ambiguity can also simply refer to the signal's ambiguous information quality (Illeditsch (2011)). Additionally, differences of opinion can occur when investors do not condition their decisions on prices (Banerjee (2011)). In other words, unlike a rational expectation model where investors agree on the interpretation of signals and condition on prices effectively to infer the private information of others, investors may not agree on the return distribution and, as a result, may not use prices to update their beliefs. Hong and Stein (1999) also make a claim about information diffusion, suggesting that information diffuses only gradually into prices among diversely informed investors. Consequently, investors may disagree because they perceive the same information differently, receive different information, or view the information received from other investors as ambiguous. Finding a comprehensive measure of disagreement is challenging due to these numerous sources. Nevertheless, the literature has created a number of measures to capture disagreement, each with its own set of advantages and disadvantages, which are discussed in this section.

### *B.A Measures Based on Analyst Forecasts*

Banerjee, Kaniel, and Kremer (2009) as well as Verardo (2009) make the case that professional analysts' forecasts are the result of their underlying assumptions and information-processing models. Additionally, forecast dispersion is also likely to be free of any liquidity, trading costs, and/or size-related bias (Alexandridis, Antoniou, and Petmezas (2007)). Analyst forecasts are also frequently released, creating a near-constant flow of information (Ajinkya, Atiase, and Gift (1991)). As such, dispersion can be a suitable proxy for disagreement, provided that analysts express their unbiased opinion and that investors base their opinions on those of analysts (Garfinkel (2009)). Diether, Malloy, and Scherbina (2002) propose a disagreement measure based on the dispersion of analysts' forecasts, computed as the standard deviation across analysts divided by the mean absolute value of EPS forecasts (e.g., Ali et al. (2019); Barinov (2013); Daniel, Klos, and Rottke (2023)). This measure has been used with various scaling factors, including the stock price (e.g., Banerjee (2011); Cheong and Thomas (2011)), the book value of equity (e.g., Cen, Wei, and Yang (2017); Dittmar and Thakor (2007)) or assets (e.g., Golez and Goyenko (2022); Johnson (2004)), and actual earnings (Park (2005)). According to Cen, Wei, and Yang (2017), this measure is relatively robust to the selection of the scaling factor, suggesting that the standard deviation in the numerator reflects the degree of disagreement.

However, the metric has a drawback in that it can be only computed for reasonably large companies with at least two analysts following them, leaving out roughly one-third of tradeable equity securities (Danielsen and Sorescu (2001)). Depending on the normalization term, dispersion may not only capture disagreement but also other information (Banerjee (2011); Cen, Wei, and Yang (2017)). Additionally, it is influenced by an IBES data bias (Dechow, Hutton, and Sloan (2000); Garfinkel (2009)), stale forecasts (Garfinkel (2009)), conflicts of interest and career concerns (Johnson (2004)), overoptimism (Hong and Stein (2003)), underreaction to public information (Erturk (2006)), overconfidence in private information (Abarbanell and Bernard (1992)), and herding (Bernhardt, Campello, and Kutsoati (2006); Hong, Lim, and Stein (2000)). While Johnson (2004) argues that analyst forecast disagreement is a nonsystematic risk, the opposing literature contends that dispersion only serves as a proxy for disagreement of opinion caused by asymmetric information (e.g., Berkman et al. (2009); Diether, Malloy, and Scherbina (2002); Park (2005)). However, later studies suggest that dispersion is more closely related to uncertainty than to information asymmetry. Barron, Stanford, and Yu (2009) show that fluctuations in analysts' forecast dispersion reflect changes in information asymmetry around earnings announcements, whereas levels of dispersion reflect levels of uncertainty before earnings announcements. Furthermore, information asymmetry issues seem to be infrequent and unrelated to divergent investor opinions when analysts monitor a company, whereas concerns about information asymmetry are much greater and are strongly correlated with diverging investor opinions when there is no analyst coverage (Garfinkel (2009)).

*Measures Based on Analyst Forecast Dispersion for Individual Firms*

a) Standard deviation of EPS estimates for the currently unreported fiscal year scaled by the absolute value of the mean EPS forecast for the same fiscal year. The same measure can be used on a monthly, quarterly, or semi-annually basis: e.g., Ajinkya and Gift (1985); Ajinkya, Atiase, and Gift (1991); Ali et al. (2019), Barinov (2013); Boehme, Danielsen, and Sorescu (2006); Cen, Wei, and Yang (2017); Daniel, Klos, and Rottke (2023); Diether, Malloy, and Scherbina (2002); Dittmar and Thakor (2007) (as measure of overvaluation); Doukas, Kim, and Pantzalis (2006); Erturk (2006); Garfinkel (2009); Goetzmann and Massa (2005); Hibbert et al. (2020); Hong and Sraer (2016); Jiang and Sun (2014); Johnson (2004); Park (2005); Sadka and Scherbina (2007); Verardo (2009)

b) Unadjusted analyst forecast dispersion measured by the standard deviation of EPS estimates. Similar to a) where forecast period can be chosen: e.g., Alexandridis, Antoniou, and Petmezas (2007); Banerjee (2011); Hong and Sraer (2016); Moeller, Schlingemann, and Stulz (2007); Sheng and Thevenot (2012); Yu (2011)

c) Standard deviation of EPS forecasts scaled by absolute median forecast: e.g., Banerjee (2011); Sheng and Thevenot (2012)

d) Standard deviation of EPS forecasts scaled by lagged price for annual estimates or by price at estimation window end or by the beginning of estimation window: e.g., Banerjee (2011); Chatterjee, John, and Yan (2012); Cheong and Thomas (2011); Danielsen and Sorescu (2001); Doukas and McKnight (2005); Garfinkel (2009); Ge, Lin, and Pearson (2016); Rees and Thomas (2010);Zhang (2006a); Zhang (2006)

e) Variance of EPS forecasts scaled by stock price: e.g., Barron, Stanford, and Yu (2009)

f) Difference between high and low forecasts standardized by average EPS forecast: e.g., Goetzmann and Massa (2005)

g) Standard deviation of EPS forecasts scaled by the book value of equity per share: e.g., Cen, Wei, and Yang (2017); Dittmar and Thakor (2007)

h) Standard deviation of EPS scaled by value of total assets per share: e.g., Cen, Wei, and Yang (2017); Daniel, Klos, and Rottke (2023); Golez and Goyenko (2022); Johnson (2004)

i) Difference between high and low forecasts standardized by the time-series standard-deviation of EPS: e.g., De Bondt and Forbes (1999)

j) Standard deviation of EPS forecasts scaled by time-series standard deviation of EPS: e.g., De Bondt and Forbes (1999)

k) Difference between a firm's EPS from the quarter prior to the actual EPS disclosure divided by the actual EPS: e.g., Dittmar and Thakor (2007)

l) Absolute value of the difference between the actual earnings and the mean of the forecasts for the current fiscal year end, and scaled by the absolute value of the actual earnings (Forecast error): e.g., Hibbert et al. (2020)

m) Standard deviation of EPS forecasts normalized by actual earnings: e.g., Park (2005)

n) Change in standard deviation (in log) of analysts' earnings forecasts: e.g., Cen, Wei, and Yang (2017)

o) Diversity (disagreement) of opinion measures: 1 minus the consensus measured by the correlation in forecast errors across analysts (known as the diversity part of the BKLS 1998 measure which defines dispersion as the sum of diversity and uncertainty): e.g., Barron et al. (1998); Barron, Stanford, and Yu (2009); Doukas, Kim, and Pantzalis (2006)

*Measures Based on Analyst Forecast Dispersion for Portfolios/Market*

a) Standard deviation of EPS forecasts value-weighted by market cap (focus on long-term forecasts: 3-5 years); this measure is on the portfolio/market level: e.g., Beddock (2021); Huang, O'Hara, and Zhong (2021); Yu (2011)

b) Standard deviation of long-term forecasts (3-5 years) weighting each stock by pre-ranking beta; this measure is at the portfolio/market level: e.g., Hong and Sraer (2016); Huang, O'Hara, and Zhong (2021)

*Other Measures Based on Analyst Forecasts*

a) The magnitude of the estimated regression coefficient's deviation from one of an analyst's forecast on the previous forecast issued by another analyst mapped to $-1$, 1, or 0 based on whether the coefficient—scaled by its standard error—is statistically below one, above one, or statistically insignificant, respectively, at the 1% level in a two-tailed test: Fischer, Kim, and Zhou (2022)

### *B.B Measures Based on Volatility*

Other studies propose idiosyncratic volatility as a measure of differences in opinion which is frequently used at the same time to quantify information asymmetry (e.g., Berkman et al. (2009); Boehme, Danielsen, and Sorescu (2006); Dierkens (1991)). Theoretical evidence suggests that return volatility and divergent investor opinions are positively correlated (Shalen (1993); Wang (1998)). Hence, measures of volatility should act as a proxy for dispersion in opinions. One of the most frequently used methods to derive idiosyncratic volatility is calculating the standard deviation of the residuals from a market model, for instance, Fama and MacBeth (1973), Fama and French (1993), or Brown and Warner (1985) (e.g., Ang et al. (2006); Danielsen and Sorescu (2001); Jiang and Sun (2014)). The use of stock market data enables the inclusion of even small firms in the sample, providing an accurate representation of market volatility. While most studies use a firm-level setting to proxy for disagreement, some propose a value-weighted approach to proxy for market disagreement (e.g., Boehme, Danielsen, and Sorescu (2006); Huang, Li, and Wang (2021)).

*Measures Based on Volatility*

a) Standard deviation of daily market adjusted residuals (e.g., from a regression w/ FF3 factors): e.g., Alexandridis, Antoniou, and Petmezas (2007); Boehme, Danielsen, and Sorescu (2006); Chatterjee, John, and Yan (2012); Danielsen and Sorescu (2001); Ge, Lin, and Pearson (2016); Huang, O'Hara, and Zhong (2021); Jiang and Sun (2014); Moeller, Schlingemann, and Stulz (2007); Zhang (2006)

b) Stock return volatility using TAQ as sample standard deviation or stock return volatility over a period: e.g., Chang, Cheng, and Yu (2007); Garfinkel (2009)

### *B.C Measures Based on Trading Volume*

Numerous studies demonstrate that trading volume is indicative of the degree of disagreement between investors regarding the value of a security, based on either different information or divergent opinions (Bamber (1987); Bamber, Barron, and Stober (1999); Lee and Swaminathan (2000); Banerjee (2011); Boehme, Danielsen, and Sorescu (2006); Varian (1986)). Banerjee (2011) documents a positive correlation between disagreement and expected trading volume, whereas there is a negative correlation between analyst forecast dispersion and returns. This implies that the two measures of disagreement reflect two distinct types of divergence of opinion. For this study, volume is measured as monthly average turnover, TO, which is the sum of trading volume over the average shares outstanding for stock $i$ during month $t$ (Boehme, Danielsen, and Sorescu (2006); Garfinkel (2009)).

The volume measure may not be the only way to measure disagreement. Benston and Hagerman (1974) as well as Petersen and Fialkowski (1994) suggest that volume serves as a proxy for liquidity if a stock consistently trades at a high volume. Garfinkel (2009) argues that volume only includes executed market orders, making it a poor substitute for all investors' private valuations. Holthausen and Verrecchia (1994) and Karpoff (1987) posit that the news that investors trade on may have a higher information content, implying that volume reflects informedness. Finally, Tkac (1999) shows that stock volume is positively correlated with market volume, indicating that market-wide factors influence volume. To address this, Garfinkel and Sokobin (2006) develop the change in market-adjusted turnover measure, $\Delta$TO, which considers both volume attributable to liquidity trading and market-wide factors. $\Delta$TO in this paper is defined as the average daily market-adjusted turnover in month $t$ less an earlier measure similarly calculated for stock $i$ over the past three months (Chen et al. (2022); Garfinkel (2009)):

However, because $\Delta$TO assumes similar price movements during the measurement and control window, it does not completely rule out a potential component in volume related to the market's reaction to the information content of news. On the other hand, standardized unexplained volume, SUV, controls for all components of volume that are not attributable to differences in opinion and are not completely captured by $\Delta$TO (Chen et al. (2022); Garfinkel (2009); Garfinkel and Sokobin (2006)).

However, an attenuation bias may impede the capacity of volume-based measures and bid-ask spreads to serve as a proxy for differences in opinions since both measures do not account for non-executed limit orders, which provide a more complete picture of the market participants' willingness to buy or sell a security. Hollifield et al. (2006) and Garfinkel (2009) argue that execution prices might not accurately reflect all investors' private valuations. To this end, Garfinkel (2009) proposes a new proxy to measure differences in opinion by utilizing investors' expressions of interest in stocks through their market and limit orders in order to capture additional information on investors' private information.

*Measures Based on Volume*

a) Average turnover: e.g., Banerjee (2011); Boehme, Danielsen, and Sorescu (2006) (scaled by shares outstanding); Danielsen and Sorescu (2001); Dittmar and Thakor (2007) (as measure of overvaluation); Lee and Swaminathan (2000); Ge, Lin, and Pearson (2016); Golez and Goyenko (2022)

b) Unexplained volume: average daily market-adjusted turnover over a control window minus a similarly calculated measure before the control window: e.g., Chen (2023); Garfinkel (2009); Garfinkel and Sokobin (2006)

c) Standard unexplained volume obtained from subtracting a predicted turnover measure derived from a regression (turnover = positive return + negative return; for each company) from the real turnover measure and dividing the result by the standard deviation of the regression residuals: e.g., Chen et al. (2022); Garfinkel (2009); Garfinkel and Sokobin (2006); Huang, O'Hara, and Zhong (2021)

d) Daily Short Interest: proposed by, e.g., Bessembinder, Chan, and Seguin (1996) but daily data was not available at that time

e) Abnormal Short Interest: e.g. Chang et al. (2022); Chen, Da, and Huang (2019); Karpoff and Lou (2010)

f) Contemporaneous correlation coefficient (at the end of a given month) of daily trading volume and absolute price change over the past two months, multiplied by -1: Hsiao (2022)

*B.D Other Disagreement Measures*

*B.D.1 Measures Based on Mutual Funds*

a) Breadth: Number of mutual funds that hold a long position in the target's stock scaled by the total number of mutual funds (on a quarterly level): e.g., Jiang and Sun (2014); Moeller, Schlingemann, and Stulz (2007)

b) Change in breadth of ownership: the change in the number of funds divided by the total number of funds (on a quarterly level): e.g., Chatterjee, John, and Yan (2012); Chen, Hong, and Stein (2002); Dittmar and Thakor (2007) (as measure of overvaluation)

c) Standard deviation of the funds' active holdings of a particular stock: the difference between the weight of the stock in each fund manager's portfolio and that in the manager's benchmark index: e.g., Golez and Goyenko (2022); Jiang and Sun (2014)

d) Change in mutual fund dispersion: Standard deviation of the funds' active holdings of a particular stock: the difference between the weight of the stock in each fund manager's portfolio and that in the manager's benchmark index: e.g., Jiang and Sun (2014)

*B.D.2 Measures Based on Bid-Ask Spreads*

a) Spreads and relative spreads: e.g., Garfinkel (2009); Handa, Schwartz, and Tiwari (2003); Houge et al. (2001)

*B.D.3 Measures Based on Intraday Distances*

a) Standard deviation of the distance between each order's requested price and the most recent trade price preceding that order using TAQ: e.g., Garfinkel (2009)

*B.D.4 Other Disagreement Measures*

a) Dispersion measure using investor accounts data: e.g., Goetzmann and Massa (2005)

b) Disagreement measure from options trades: e.g., Ge, Lin, and Pearson (2016); Golez and Goyenko (2022); Huang, O'Hara, and Zhong (2021)

c) Dispersion measure from household survey data: e.g., Huang, O'Hara, and Zhong (2021); Li and Li (2021)

d) Disagreement as uncertainty which is the sum of the projected variance of mean forecast errors (obtained from a GARCH model) and the observed dispersion: e.g., Sheng and Thevenot (2012)

e) Disagreement based on macroeconomic indicators: e.g., Anderson, Ghysels, and Juergens (2009); Berkman et al. (2009); Huang, O'Hara, and Zhong (2021)

f) Standard deviation of stock returns using all transactions during normal trading hours (with TAQ data): e.g., Garfinkel (2009)

g) Historical income volatility: e.g., Berkman et al. (2009)

h) Disagreement from social media posts (such as Twitter, Stocktwits, etc.): e.g., Cookson et al. (2023); Cookson and Niessner (2020)

i) Standard deviation of weekly raw returns: e.g., Danielsen and Sorescu (2001)

j) Estimated analyst dispersion from an OLS model for firms with no analyst data by running a regression of dispersion (when it is available) on idiosyncratic volatility and turnover: e.g., Boehme, Danielsen, and Sorescu (2006)

k) Geographic dispersion of a firm's investors as the variation in the locations of requests for the firms' filings to EDGAR: e.g., Chen (2023)

l) Distribution of the trades of institutional investors: e.g., Hibbert et al. (2020)

m) Machine-learning based forecasts: Bali et al. (2024)

## Appendix C Proofs

*Proof of Proposition 1.* Denote the absolute risk aversion at initial wealth by $A = -\frac{u''(w_0)}{u'(w_0)}$ and the absolute risk prudence at initial wealth by $P = -\frac{u'''(w_0)}{u''(w_0)}$. Denote date–1 wealth for investor type $i$ as a function of price $p$ by

$$w_i(p) = w_0 + x_i(p)(\widetilde{y} - p) \tag{C1}$$

so that the first-order condition of type–$i$ investor's expected utility maximization is

$$\mathbf{E}_i[u'(w(p))(\widetilde{y} - p)] = 0. \tag{C2}$$

We will use (C1) and (C2) and differentiation applied twice to (C1) and (C2) to derive exact expressions for the coefficients $x_i(\mu_i)$, $x_i'(\mu_i)$, and $x_i''(\mu_i)$ of the Taylor expansion of $x_i(p)$ around $\mu_i = \mu + D\Delta_i$, namely,

$$x_i(p) = x_i(\mu_i) + x_i'(\mu_i)(p - \mu_i) + \frac{x_i''(\mu_i)}{2}(p - \mu_i)^2 + h_i(p)(p - \mu_i)^2, \tag{C3}$$

where $h_i(p)$ is an unknown function that converges to 0 as $p \to \mu_i$ by Taylor's theorem.

First, because $w_i(\mu_i) = w_0$ for $x_i(\mu_i) = 0$, it is clear that

$$x_i(\mu_i) = 0 \tag{C4}$$

satisfies (C2) and this solution is unique because $u$ is strictly concave.

Second, we differentiate both sides of (C2) with respect to $p$ to obtain:

$$\mathbf{E}_i[u''(w_i(p))w_i'(p)(\widetilde{y} - p) - u'(w_i(p))] = 0. \tag{C5}$$

Note that the first derivative of wealth is $w_i'(p) = x_i'(p)(\widetilde{y} - p) - x_i(p)$, which evaluates to $w_i'(\mu_i) = x_i'(\mu_i)(\widetilde{y} - \mu_i)$ because of (C4). From (C5) evaluated at $p = \mu_i$, we derive that $u''(w_0)x_i'(\mu_i)\mathbf{E}_i[(\widetilde{y} - \mu_i)^2] - u'(w_0) = 0$, and therefore that

$$x_i'(\mu_i) = -\frac{1}{A}\frac{1}{\sigma^2}, \tag{C6}$$

where we have used the identity $\sigma^2 = \mathbf{E}_i[(\widetilde{y} - \mu_i)^2]$ and the definition of absolute risk aversion at initial wealth, $A = -\frac{u''(w_0)}{u'(w_0)}$. Note that $x_i(p)$ is locally strictly decreasing because $x_i'(\mu_i) < 0$, which follows by (C6) and the facts that $A > 0$ and $\sigma^2 > 0$.

Third, we differentiate both sides of (C5) with respect to $p$ to obtain:

$$\mathbf{E}_i[u'''(w_i(p))(w_i'(p))^2(\widetilde{y} - p) + u''(w_i(p))(w_i''(p)(\widetilde{y} - p) - 2w_i'(p))] = 0. \tag{C7}$$

Note that the second derivative of wealth is $w_i''(p) = x_i''(p)(\widetilde{y} - p) - 2x_i'(p)$, which evaluates to $w_i''(\mu_i) = x_i''(\mu_i)(\widetilde{y} - \mu_i) - 2x_i'(\mu_i)$. From (C7) evaluated at $p = \mu_i$, we derive that $u'''(w_0)(x_i'(\mu_i))^2\mathbf{E}_i[(\widetilde{y} - \mu_i)^3] + u''(w_0)x_i''(\mu_i)\mathbf{E}_i[(\widetilde{y} - \mu_i)^2] - 4u''(w_0)u'(w_0)\mathbf{E}_i[\widetilde{y} - \mu_i] = 0$, and therefore that

$$x_i''(\mu_i) = \frac{P}{A^2}\frac{s}{\sigma^3}, \tag{C8}$$

where we have used the identities (C6), $s\sigma^3 = \mathbf{E}_i[(\widetilde{y} - \mu_i)^3]$, $\sigma^2 = \mathbf{E}_i[(\widetilde{y} - \mu_i)^2]$, and $0 = \mathbf{E}_i[\widetilde{y} - \mu_i]$; the definition of absolute risk aversion at initial wealth, $A = -\frac{u''(w_0)}{u'(w_0)}$; and the definition of absolute risk prudence at initial wealth, $P = -\frac{u'''(w_0)}{u''(w_0)}$. For $s > 0$,we have $x_i''(\mu_i) > 0$, which follows by (C8) and the facts that $P > 0$ and $\sigma^3 > 0$.

Finally, we plug in (C4), (C6), and (C8) into (C3) to obtain:

$$x_i(p) = -\frac{1}{A}\frac{1}{\sigma^2}(p - \mu_i) + \frac{1}{2}\frac{P}{A^2}\frac{s}{\sigma^3}(p - \mu_i)^2 + h_i(p)(p - \mu_i)^2, \tag{C9}$$

where $h_i(p)$ is an unknown function that converges to 0 as $p \to \mu_i$ by Taylor's theorem.

For $u(w)$ in the CRRA family as in (1), we have $A = \frac{\gamma}{w_0}$ and $P = \frac{\gamma+1}{w_0}$. Therefore, (C9) simplifies to

$$x_i(p) = -\frac{w_0}{\gamma}\frac{1}{\sigma^2}(p - \mu_i) + \frac{1}{2}\frac{w_0}{\gamma}\left(1 + \frac{1}{\gamma}\right)\frac{s}{\sigma^3}(p - \mu_i)^2 + h_i(p)(p - \mu_i)^2, \tag{C10}$$

which matches (3) and completes the proof.

$\square$

*Proof of Proposition 2.* Denote the absolute risk aversion at initial wealth by $A = -\frac{u''(w_0)}{u'(w_0)}$ and the absolute risk prudence at initial wealth by $P = -\frac{u'''(w_0)}{u''(w_0)}$. In the Taylor expansion for $x_i(p)$,

$$x_i(p) = -\frac{1}{A}\frac{1}{\sigma^2}(p-\mu_i) + \frac{1}{2}\frac{P}{A^2}\frac{s}{\sigma^3}(p-\mu_i)^2 + h_i(p)(p-\mu_i)^2, \tag{C11}$$

note that each of the expressions $x_i'(\mu_i) = -\frac{1}{A}\frac{1}{\sigma^2}$ and $x_i''(\mu_i) = \frac{P}{A^2}\frac{s}{\sigma^3}$ do not depend on $i$. Thus, for simplicity we drop the explicit reference to argument $\mu_i$ and write $X_1$ and $X_2$ to denote $x_i'(\mu_i)$ and $x_i''(\mu_i)$, respectively, to express (C11) equivalently as

$$x_i(p) = X_1(p-\mu_i) + \frac{1}{2}X_2(p-\mu_i)^2 + h_i(p)(p-\mu_i)^2. \tag{C12}$$

The market-clearing condition states

$$\sum_{i=1}^{n} \pi_i x_i(p) = 0. \tag{C13}$$

Substituting $x_i(p)$ from (C12) into (C13) yields

$$X_1 \sum_{i=1}^{n} \pi_i(p-\mu_i) + \frac{1}{2}X_2 \sum_{i=1}^{n} \pi_i(p-\mu_i)^2 + H(p) = 0, \tag{C14}$$

where we define

$$H(p) = \sum_{i=1}^{n} \pi_i h_i(p)(p-\mu_i)^2. \tag{C15}$$

First, note that $\sum_{i=1}^{n} \pi_i(p-\mu_i) = p-\mu$ because $\widetilde{\Delta}$ has zero mean: $\sum_{i=1}^{n} \pi_i\Delta_i = 0$. Second, note that $\sum_{i=1}^{n} \pi_i(p-\mu_i)^2 = \sum_{i=1}^{n} \pi_i(p-\mu-D\Delta_i)^2 = \sum_{i=1}^{n} \pi_i[(p-\mu)^2 - 2D\Delta_i(p-\mu) + D^2\Delta_i^2] = (p-\mu)^2 + D^2$ because $\widetilde{\Delta}$ has zero mean and unit variance: $\sum_{i=1}^{n} \pi_i\Delta_i^2 = 1$. So, (C14) becomes

$$X_1(p-\mu) + \frac{1}{2}X_2[(p-\mu)^2 + D^2] + H(p) = 0. \tag{C16}$$

We now treat price as a function of $D$ and use (C16) to compute the Taylor expansion of price around $D = 0$:

$$p(D) = p(0) + p'(0)D + \frac{1}{2}p''(0)D^2 + h(D)D^2, \tag{C17}$$

where $h(D)$ is an unknown function that converges to 0 as $D \to 0$ by Taylor's theorem.

First, at $D = 0$ it is clear that

$$p(0) = \mu \tag{C18}$$

satisfies (C14) because $\mu_i = \mu$ at $D = 0$.

Second, we differentiate both sides of (C16) with respect to $D$ to obtain:

$$X_1 p' + \frac{1}{2} X_2 \left[2(p-\mu)p' + 2D\right] + \frac{\partial H(p)}{\partial D} = 0. \tag{C19}$$

Note that $\frac{\partial H(p)}{\partial D}$ goes to zero as $D \to 0$ because it is the weighted sum of terms of the form $\frac{\partial h_i(p)(p-\mu_i)^2}{\partial D}$, which each go to zero as $D \to 0$: $\frac{\partial h_i(p)(p-\mu_i)^2}{\partial D} = h_i'(p)p'(p-\mu_i)^2 + 2h_i(p)(p-\mu_i)p'$, which goes to zero as $D \to 0$. Therefore, evaluating (C19) at $D = 0$ yields

$$p'(D) = 0. \tag{C20}$$

Third, we differentiate both sides of (C19) with respect to $D$ to obtain:

$$X_1 p'' + \frac{1}{2} X_2 \left[2(p')^2 + 2(p-\mu)p'' + 2\right] + \frac{\partial^2 H(p)}{\partial D^2} = 0. \tag{C21}$$

Note that $\frac{\partial^2 H(p)}{\partial D^2}$ goes to zero as $D \to 0$ because it is the weighted sum of terms of the form $\frac{\partial^2 h_i(p)(p-\mu_i)^2}{\partial D^2}$, which each go to zero as $D \to 0$: $\frac{\partial^2 h_i(p)(p-\mu_i)^2}{\partial D^2} = h_i''(p)(p')^2(p-\mu_i)^2 + h_i'(p)[p''(p-\mu_i)^2 + 2(p')^2(p-\mu_i)] + 2h_i'(p)(p')^2(p-\mu_i) + 2h_i(p)[(p')^2 + (p-\mu_i)^2 p'']$, which goes to zero as $D \to 0$. Therefore, evaluating (C21) at $D = 0$ yields

$$p''(0) = \frac{P}{A}\frac{s}{\sigma}. \tag{C22}$$

Therefore, by (C24), (C18), (C20), and (C22), the Taylor expansion of price around $D = 0$ is given by:

$$p(D) = \mu + \frac{1}{2}\frac{P}{A}\frac{s}{\sigma}D^2 + h(D)D^2, \tag{C23}$$

where $h(D)$ is an unknown function that converges to 0 as $D \to 0$ by Taylor's theorem.

For $u(w)$ in the CRRA family as in (1), we have $A = \frac{\gamma}{w_0}$ and $P = \frac{\gamma+1}{w_0}$. Therefore, (C23) simplifies to

$$p(D) = \mu + \frac{1}{2}\left(1 + \frac{1}{\gamma}\right)\frac{s}{\sigma}D^2 + h(D)D^2, \tag{C24}$$

which matches (6) and completes the proof.

□

*Proof of Lemma 5.* Note that $\frac{\partial x_i(D)}{\partial \Delta_i} > 0 \Leftrightarrow 1 - \frac{P}{A}\frac{s}{\sigma}\left(\frac{1}{2}\frac{P}{A}\frac{s}{\sigma}D^2 - D\Delta_i\right) > 0$, where $\frac{P}{A} = \frac{1+\gamma}{\gamma}$. For $s = 0$, this condition is satisfied. For $s > 0$, we have $\frac{\partial x_i(D)}{\partial \Delta_i} > 0 \Leftrightarrow \Delta_i > \frac{1}{2}\frac{P}{A}\frac{s}{\sigma}D - \frac{1}{\frac{P}{A}\frac{s}{\sigma}D}$, which is satisified for $D$ not too large. For $s < 0$, we have $\frac{\partial x_i(D)}{\partial \Delta_i} > 0 \Leftrightarrow \Delta_i < \frac{1}{2}\frac{P}{A}\frac{s}{\sigma}D - \frac{1}{\frac{P}{A}\frac{s}{\sigma}D}$, which, again, is satisified for $D$ not too large. □

*Proof of Lemma 6.* For $D$ not too large, the equilibrium demand of a type–$i$ investor satisifies: $\Delta_i \frac{\partial x_i}{\partial D} > 0$. To see this, note the following. If $\Delta_i < 0$, then

$$x_i' < 0 \Leftrightarrow \begin{cases} D < \frac{A}{P}\frac{\sigma}{|s|}(\Delta_i + \sqrt{2+\Delta_i^2}), & s > 0, \\ D < \frac{A}{P}\frac{\sigma}{|s|}|\Delta_i|, & s < 0, \\ s = 0, \end{cases} \tag{C25}$$

If $\Delta_i > 0$, then

$$x_i' > 0 \Leftrightarrow \begin{cases} D < \frac{A}{P}\frac{\sigma}{|s|}(\Delta_i + \sqrt{2+\Delta_i^2}), & s < 0, \\ D < \frac{A}{P}\frac{\sigma}{|s|}|\Delta_i|, & s > 0, \\ s = 0, \end{cases} \tag{C26}$$

Thus, if $s = 0$ or $D < \frac{A}{P}\frac{\sigma}{|s|}\min\{||\Delta_i|, \sqrt{2+\Delta_i^2} - |\Delta_i|\}$, then $\Delta_i x_i'(D) > 0$. This establishes the result for $D$ not too large. □

*Proof of Proposition 3.* Denote $x_i(D)$ as $x_i$ for simplicity of notation. Without loss of generality, we assume that $\Delta_i$ is distinct for each $i$, and relabel investor types $i = 1, \ldots, n$ so that $\Delta_1 < \Delta_2 < \cdots < \Delta_n$. We assume that all $\Delta_i$ are nonzero. By Lemma 5, $x_1 < x_2 < \cdots < x_n$, which means that the equilibrium demands of different type investors line up in the same order as their $\Delta$'s.

Because $\sum_{i=1}^n \pi_i x_i = 0$ by market clearing, there exists some investor type $i_x < n$ such that $x_i < 0$ for all $i \le i_x$ and $x_i > 0$ for all $i > i_x$. Because $\mathbf{E}[\widetilde{\Delta}] = 0$, there exists some investor type $i_\Delta < n$ such that $\Delta_i < 0$ for all $i \le i_\Delta$ and $\Delta_i > 0$ for all $i > i_\Delta$. If $i_x \le i_\Delta$, then $x_i < 0$ and $\Delta_i < 0$ for all $i \le i_x$. If $i_x > i_\Delta$, then $x_i > 0$ and $\Delta_i > 0$ for all $i > i_x$. In either case, $-\sum_{i \le i_x} \pi_i x_i = \sum_{i > i_x} \pi_i x_i$. In the first case, $\sum_{i \le i_x} \pi_i |x_i| = -\sum_{i \le i_x} \pi_i x_i$, which is increasing in $D$ for $D$ not too large because $\Delta_i < 0$ for all $i \le i_x$ by Lemma 6. In the second case, $\sum_{i > i_x} \pi_i |x_i| = \sum_{i > i_x} \pi_i x_i$, which is increasing in $D$ for $D$ not too large because $\Delta_i > 0$ for all $i > i_x$ by Lemma 6. Either case equals exactly half of the trading volume because of the market clearing condition. Therefore, trading volume is increasing in $D$ for $D$ not too large. □

*Proof of Proposition 7.* Denote the absolute risk aversion at initial wealth by $A = -\frac{u''(w_0)}{u'(w_0)}$ and the absolute risk prudence at initial wealth by $P = -\frac{u'''(w_0)}{u''(w_0)}$. In the second-order demand polynomial of investor $i$,

$$\widehat{x}_i(p) = -\frac{1}{A}\frac{1}{\sigma^2}(p - \mu_i) + \frac{1}{2}\frac{P}{A^2}\frac{s}{\sigma^3}(p - \mu_i)^2, \tag{C27}$$

note that each of the expressions $x_i'(\mu_i) = -\frac{1}{A}\frac{1}{\sigma^2}$ and $x_i''(\mu_i) = \frac{P}{A^2}\frac{s}{\sigma^3}$ do not depend on $i$. Thus, for simplicity we drop the explicit reference to argument $\mu_i$ and write $X_1$ and $X_2$ to denote $x_i'(\mu_i)$ and $x_i''(\mu_i)$, respectively, to express (C27) equivalently as

$$\widehat{x}_i(p) = X_1(p-\mu_i) + \frac{1}{2}X_2(p-\mu_i)^2. \tag{C28}$$

The condition $\sum_{i=1}^n \pi_i \widehat{x}_i(p) = X$ is equivalent to

$$X_1 \sum_{i=1}^n \pi_i(p-\mu_i) + \frac{1}{2}X_2 \sum_{i=1}^n \pi_i(p-\mu_i)^2 = X. \tag{C29}$$

First, note that $\sum_{i=1}^n \pi_i(p-\mu_i) = p-\mu$ because $\widetilde{\Delta}$ has zero mean: $\sum_{i=1}^n \pi_i\Delta_i = 0$. Second, note that $\sum_{i=1}^n \pi_i(p-\mu_i)^2 = \sum_{i=1}^n \pi_i(p-\mu-D\Delta_i)^2 = \sum_{i=1}^n \pi_i[(p-\mu)^2-2D\Delta_i(p-\mu)+D^2\Delta_i^2] = (p-\mu)^2+D^2$ because $\widetilde{\Delta}$ has zero mean and unit variance: $\sum_{i=1}^n \pi_i\Delta_i^2 = 1$. So (C29) becomes

$$X_1(p-\mu) + \frac{1}{2}X_2[(p-\mu)^2+D^2] = X. \tag{C30}$$

We now treat price as a function of $D$ and use (C30) to compute the Taylor expansion of price around $D=0$:

$$p(D) = p(0) + p'(0)D + \frac{1}{2}p''(0)D^2 + h(D)D^2, \tag{C31}$$

where $h(D)$ is an unknown function that converges to 0 as $D \to 0$ by Taylor's theorem.

First, suppose $D=0$. Consider

$$p(0) = \mu + p_X, \tag{C32}$$

where $p_X$ satisfies $X_1p_X + \frac{1}{2}X_2p_X^2 = X$. If $s=0$, then $p_X = -A\sigma^2X$ because $X_2 = 0$. Otherwise, let $p_X$ be the negative solution to the quadratic equation, namely, $p_X = \frac{A\sigma(1-\sqrt{1+2Ps\sigma X})}{Ps}$. Note that by L'Hospital's rule that $\lim_{s\to 0}\frac{A\sigma(1-\sqrt{1+2Ps\sigma X})}{Ps} = -A\sigma^2X$, so that

$$p_X = \begin{cases} \frac{A\sigma(1-\sqrt{1+2Ps\sigma X})}{Ps} & s \neq 0, \\ -A\sigma^2X & s = 0, \end{cases} \tag{C33}$$

is continuous in $s$ and $p(0) = \mu + p_X$ satisfies (C30) at $D=0$.

Second, we differentiate both sides of (C30) with respect to $D$ to obtain:

$$X_1p' + \frac{1}{2}X_2\left[2(p-\mu)p' + 2D\right] = 0. \tag{C34}$$

Evaluating (C34) at $D=0$ yields

$$p'(D) = 0. \tag{C35}$$

Third, we differentiate both sides of (C34) with respect to $D$ to obtain:

$$X_1p'' + \frac{1}{2}X_2\left[2(p')^2 + 2(p-\mu)p'' + 2\right] = 0. \tag{C36}$$

Evaluating (C36) at $D = 0$ yields

$$p''(0) = \frac{P}{A}\frac{s}{\sigma}\frac{1}{\sqrt{1+2Ps\sigma X}}. \tag{C37}$$

Therefore, by (C31), (C32), (C33), (C35), and (C37), the Taylor expansion of price around $D = 0$ is given by:

$$p(D) = \mu + \left\{ \begin{array}{ll} \frac{A\sigma(1-\sqrt{1+2Ps\sigma X})}{Ps} & s \neq 0, \\ -A\sigma^2 X & s = 0. \end{array} \right\} + \frac{1}{2}\frac{P}{A}\frac{s}{\sigma}\frac{1}{\sqrt{1+2Ps\sigma X}}D^2 + h(D)D^2, \tag{C38}$$

where $h(D)$ is an unknown function that converges to 0 as $D \to 0$ by Taylor's theorem. For $u(w)$ in the CRRA family as in (1), we have $A = \frac{\gamma}{w_0}$ and $P = \frac{\gamma+1}{w_0}$. Therefore, (C38) simplifies to

□

## Appendix D Equilibrium Demands

In this section, we develop the argument and intuition behind the relationship between equilibrium volume and the scale of disagreement. First, we analyze the sensitivity of each investor's equilibrium demand to the investor's underlying source of heterogenity in belief, i.e., how $x_i$ varies with $\Delta_i$:

$$\frac{\partial x_i}{\partial \Delta_i} = \frac{1}{\gamma}\frac{1}{\sigma^2}D - \frac{1}{\gamma}\left(1+\frac{1}{\gamma}\right)\frac{s}{\sigma^3}D^2\left(\frac{1}{2}\left(1+\frac{1}{\gamma}\right)\frac{s}{\sigma}D - \Delta_i\right). \tag{D39}$$

Lemma 5 indicates that (D39) is positive, and hence equilibrium demand $x_i$ is increasing in $\Delta_i$, for $D$ not too large.

**Lemma 5.** *Equilibrium demand $x_i$ is increasing in $\Delta_i$ for $D$ not too large.*

*Proof of Lemma 5.* See Appendix C. □

Figure D.1 shows how $x_i$ increases with $\Delta_i$ for $D$ not too large. Because equilibrium price $p$ is higher than $\mu$ for $s > 0$ and small $D > 0$, equilibrium demand is negative at $\Delta_i = 0$, which corresponds to belief $\mu_i = \mu < p$. At $\Delta_i = \frac{p-\mu}{D} > 0$, we have $\mu_i = p$ so that $x_i = 0$.

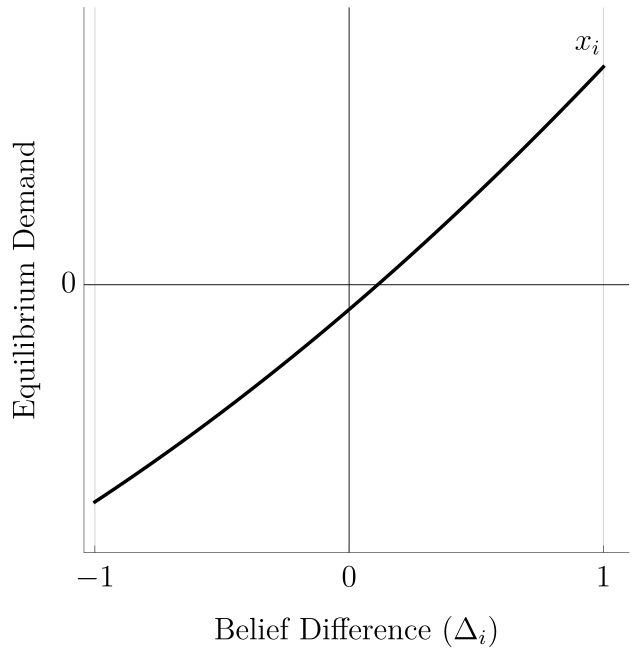
The intuition that Lemma 5 supports is that equilibrium demands line up in the same order as underlying beliefs, $\{\Delta_i\}$. Without loss of generality, we relabel the beliefs in order from lowest to highest belief difference:

$$\Delta_1 < \Delta_2 < \cdots < \Delta_n. \tag{D40}$$

Without loss, we also assume each of the $\Delta_i$'s are unique. Lemma 5 tells us that the same ordering will apply to equilibrium demands:

$$x_1 < x_2 < \cdots < x_n. \tag{D41}$$

**Figure D.1:** Equilibrium demand $x_i$ as a function of underlying belief heterogenity $\Delta_i$.




**Notes:** This figure plots the 2nd-order Taylor polynomial of the equilibrium demand $x_i$ as a function of $\Delta_i$ over the domain $\Delta_i \in [-1, 1]$ for the following parameter values: $D = 0.05$, $\mu = 1$, $\sigma = 0.2$, $s = 0.6$, $\gamma = 2$, $w_0 = 1$.

This intuition is also evident in Figure 1b, which has $\Delta_1 < \Delta_2$ and shows that demands at the vertical dotted line, which corresponds to the market-clearing price, satisfy $x_1 < x_2$.

Second, we analyze the sensitivity of equilibrium demand to the scale of disagreement, i.e., how $x_i$ varies with $D$:

$$\frac{\partial x_i}{\partial D} = \left(\frac{P}{A}\frac{s}{\sigma}D - \Delta_i\right)\left[-\frac{1}{A}\frac{1}{\sigma^2} + \frac{P}{A^2}\frac{s}{\sigma^3}D\left(\frac{1}{2}\frac{P}{A}\frac{s}{\sigma}D - \Delta_i\right)\right]. \tag{D42}$$

Lemma 6 indicates that (D42) has the same sign as $\Delta_i$, and hence the product of $\Delta_i$ with equilibrium demand $x_i$ is increasing in $D$, for $D$ not too large.

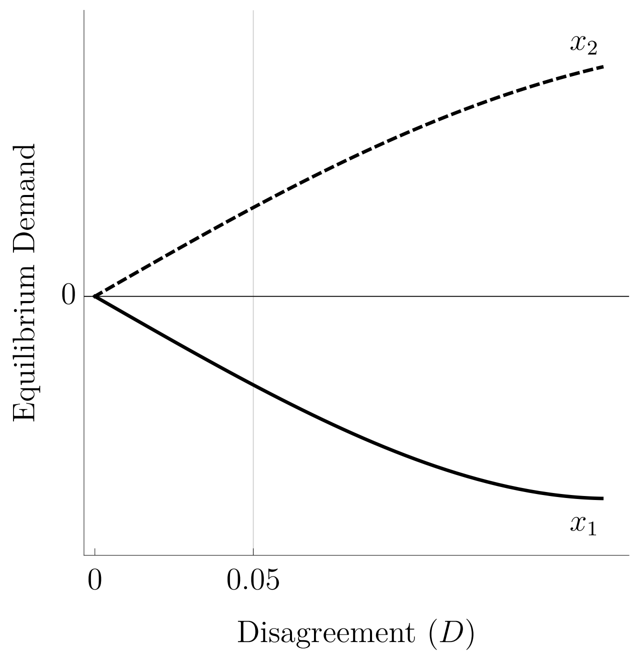
**Lemma 6.** *The product of $\Delta_i$ with equilibrium demand $x_i$, $\Delta_i x_i$, is increasing in $D$ for $D$ not too large.*

$$\Delta_i \frac{\partial x_i(D)}{\partial D} > 0. \tag{D43}$$

*Proof of Lemma 6.* See Appendix C. □

Figure D.2 shows how $x_i$ increases with $D$ for $\Delta_i > 0$ and decreases with $D$ for $\Delta_i < 0$, for $D$ not too large. The intuition is as follows. Because demand is strictly decreasing in price in a neighborhood of $\mu_i$ and crosses the price axis at $p = \mu_i = \mu + D\Delta_i$ (see Figure 1b), then as $D$ increases, demand will cross the price axis at a higher price if $\Delta_i > 0$, and hence be higher at all prices less than this new price. Likewise if $\Delta_i < 0$, then demand will cross the price axis at a lower price and demand will be lower at all prices higher than this new price. Thus, demands expand away from the price axis as $D$ increases. For $D$ not too large, the corresponding increase in the equilibrium price does not overcome this expansion effect of demands away from the price axis around the equilibrium price.

**Figure D.2:** Equilibrium demand $x_i$ as a function of disagreement $D$.




**Notes:** This figure plots the 2nd-order Taylor polynomial of the equilibrium demands $x_1$ (solid curve) and $x_2$ (dashed curve) as functions of $D$ over the domain $D \in [0, 0.16]$ for the following parameter values: $\Delta_1 = -1$, $\Delta_2 = 1$, $\mu = 1$, $\sigma = 0.2$, $s = 0.6$, $\gamma = 2$, $w_0 = 1$.

The intuition for the relationship of equilibrium volume to the scale of disagreement is related to the intuition for Lemma 6. Because demands expand away from the price axis as $D$ increases, negative demands become more negative and positive demands become more positive, and their absolute values increase, which increases equilibrium volume $V$.

## Appendix E Higher-Order Sensitivities

### *E.A Sensitivity of Trading-Volume–Volatility Relationship to Disagreement in Equilibrium*

Differentiating $V$ with respect to $\sigma$ in (11), we obtain the following relationship between trading volume and payoff volatility:

$$\frac{\partial V}{\partial \sigma} = \frac{2}{\gamma} \frac{D}{\sigma^3} \left[ -1 + \left(1 + \frac{1}{\gamma}\right)^2 \frac{s^2}{\sigma^2} D^2 \right]. \tag{E44}$$

For $D$ not too large, (E44) characterizes the nature of comovement between trading volume and volatility, holding all else equal. This relationship, multiplied by $-1$, increases in the scale of disagreement:

$$\frac{\partial}{\partial D} \left( -\frac{\partial V}{\partial \sigma} \right) = \frac{2}{\gamma} \frac{1}{\sigma^3} \left[ 1 - 3 \left(1 + \frac{1}{\gamma}\right)^2 \frac{s^2}{\sigma^2} D^2 \right] > 0, \tag{E45}$$

for $D$ not too large. This result indicates that the correlation between trading volume and volatility, multipled by $-1$, is a proxy for disagreement.

Compare this result with Hsiao (2022), who proposes correlation between absolute returns and dollar volume, multiplied by $-1$, as an investor disagreement proxy. The behavior of absolute returns is tied to volatility, establishing a link to our results, but arrived at under distinct assumptions.

### *E.B Sensitivity of Trading-Volume–Skewness Relationship to Disagreement in Equilibrium*

Differentiating $V$ with respect to $s$ in (11), we obtain the following relationship between trading volume and skewness:

$$\frac{\partial V}{\partial s} = -\frac{1}{\gamma} \frac{D^3}{\sigma^4} \left(1 + \frac{1}{\gamma}\right)^2 s. \tag{E46}$$

For $D$ not too large, (E46) characterizes the nature of comovement between trading volume and skewness, holding all else equal. This relationship, multiplied by $-1$, increases in the scale of disagreement:

$$\frac{\partial}{\partial D} \left( -\frac{\partial V}{\partial s} \right) = \frac{3}{\gamma} \frac{D^2}{\sigma^4} \left(1 + \frac{1}{\gamma}\right)^2 s > 0, \tag{E47}$$

for $s > 0$. This result indicates a novel proxy for investor disagreement, namely, the correlation between trading volume and skewness, multipled by $-1$. Likewise, the correlation between trading volume and cubed returns, multiplied by $-1$, is suggested as an investor disagreement proxy.

## Appendix F Role of Market Size

It is widely known that many pricing anomalies are concentrated in small stocks, including the disagreement–expected return anomaly (Diether, Malloy, and Scherbina (2002)). Extant models of the disagreement effect, however, do not explain this small-stock concentration effect. Our model can be extended to show that the negative disagreement–expected return effect is stronger for smaller stocks.

To this point, we have assumed net zero supply for tractability and simplicity. In this section, we explore the role of positive supply for equilibrium prices and derive a new price function. We use approximate demand (second-order Taylor polynomial) in the market-clearing condition to solve for the corresponding price and expected return functions. Let $X > 0$ denote the supply, or shares outstanding, of the risky asset. Denote the second-order polynomial of type–$i$ demand in (3) of Proposition 1 by $\widehat{x}_i(p) = -\frac{w_0}{\gamma}\frac{1}{\sigma^2}(p-\mu_i) + \frac{1}{2}\frac{w_0}{\gamma}\left(1+\frac{1}{\gamma}\right)\frac{s}{\sigma^3}(p-\mu_i)^2$. Applying the analogous market-clearing condition that equates demand and supply, $\sum_{i=1}^{n}\pi_i\widehat{x}_i(p) = X$, we can derive the associated second-order Taylor polynomial of price as a function of disagreement, $D$, using a similar approach as in the proof of Proposition 2:

**Proposition 7** (Price under positive supply)**.** *The second-order Taylor polynomial of the price as a function of the scale of disagreement, $D$, around $D = 0$, corresponding to the condition $\sum_{i=1}^{n}\pi_i\widehat{x}_i(p) = X$, is given by:*[26]

$$p(D) = \mu - \left\{ \begin{array}{ll} \frac{\sqrt{1+2\frac{\gamma+1}{w_0}s\sigma X}-1}{\left(1+\frac{1}{\gamma}\right)\frac{s}{\sigma}} & s \neq 0, \\ \frac{\gamma}{w_0}\sigma^2 X & s = 0. \end{array} \right\} + \frac{\frac{1}{2}\left(1+\frac{1}{\gamma}\right)\frac{s}{\sigma}}{\sqrt{1+2\frac{\gamma+1}{w_0}s\sigma X}}D^2. \tag{F48}$$

*Proof of Proposition 7.* See Appendix C. □

When there is no skewness (i.e., $s = 0$), the price is shifted down relative to the mean payoff ($\mu$) by $-\frac{\gamma}{w_0}\sigma^2 X$, which reflects the traditional risk premium necessary to induce investors to hold $X$ units of the risky asset in equilibrium. For positive skewness ($s > 0$) and no disagreement ($D = 0$), the price is shifted down relative to the mean payoff by the first term in curly braces in (F48). This term reflects the risk premium necessary to induce investors to hold $X$ units of the risky asset in equilibrium while accounting for the complex impact of the third moment of the payoff distribution on risk, given that our investors

[26] Again, this 2nd-order Taylor polynomial approximation is reasonable for $D$ not too large. Note that as shares outstanding approach zero—i.e., $X \to 0$—the price equation (F48) of Proposition 7 reduces to the second-order Taylor polynomial of the price equation (6), which was derived under the assumption of zero supply in Proposition 2—linking the two analyses.

have utility with a positive third derivative (demand for precautionary savings). When disagreement is positive, this risk premium is further offset by the last term of (F48), which increases in the scale of disagreement, $D > 0$.

Using the positive-supply-based market price of Proposition 7 and the identities $\sigma = \sigma_r p$ and $d = D/p$, the corresponding expected return is

$$\mathbf{E}[\widetilde{r}] = \begin{Bmatrix} \frac{\sqrt{1+2(\gamma+1)s\sigma_r \frac{pX}{w_0}}-1}{\left(1+\frac{1}{\gamma}\right)\frac{s}{\sigma_r}} & s \neq 0, \\ \gamma\sigma_r^2 \frac{pX}{w_0} & s = 0. \end{Bmatrix} - \frac{\frac{1}{2}\left(1+\frac{1}{\gamma}\right)\frac{s}{\sigma_r}}{\sqrt{1+2(\gamma+1)s\sigma_r \frac{pX}{w_0}}} d^2. \tag{F49}$$

Holding all else equal, (F49) predicts that expected returns are decreasing in disagreement for positively skewed returns, just as predicted in (7) following Proposition 2: $\frac{\partial \mathbf{E}[\widetilde{r}]}{\partial d} = -\frac{\left(1+\frac{1}{\gamma}\right)\frac{s}{\sigma_r}}{\sqrt{1+2(\gamma+1)s\sigma_r \frac{pX}{w_0}}} d < 0$. Moreover, (F49) reveals how expected returns respond to model parameters as well as to the market size of the asset, $pX$, which is price per share times shares outstanding. In particular, the disagreement effect is weaker the larger the market size of the firm. In sum, we should expect returns to be lower for higher disagreement and this effect to be stronger among smaller stocks.

### *F.A Empirical Analysis*

**Table F.1: Fama-MacBeth Cross-Sectional Regressions by Size Quintile**

| SIZE | (Small) $\text{EXRET}_{t+1}$ | (2) $\text{EXRET}_{t+1}$ | (3) $\text{EXRET}_{t+1}$ | (4) $\text{EXRET}_{t+1}$ | (Big) $\text{EXRET}_{t+1}$ |
|---|---|---|---|---|---|
| DIS | $-1.58^{***}$ | $-1.45^{**}$ | $-2.66^{*}$ | $-3.63$ | 1.04 |
| | $(-4.93)$ | $(-2.19)$ | $(-1.65)$ | $(-1.55)$ | (0.32) |
| Constant | $0.92^{***}$ | $0.86^{***}$ | $0.85^{***}$ | $0.86^{***}$ | $0.77^{***}$ |
| | (2.71) | (2.72) | (3.11) | (3.32) | (3.68) |
| Observations | 144,132 | 143,991 | 143,986 | 143,991 | 143,840 |

**Notes:** This table displays the regression coefficients and $t$-statistics from a monthly Fama and MacBeth (1973) regression analysis conducted on excess returns, adhering to the methodologies in Fama and French (1993) and Carhart (1997). These regressions are performed against a lagged measure of disagreement for stocks within one of five size quintiles within the month. The sample ranges from January 1994 to December 2022. The coefficients on DIS are multiplied by 100 for readability. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively.

In Table F.1, we test this hypothesis of a strengthening of disagreement–expected return predictability for smaller stocks. We separate our sample into five quintiles sorted by firm size each month. We find that the coefficient on DIS is negative and more significant the smaller the size quintile, consistent with our hypothesis. Moreover, we find that DIS is insignificant in the quintile of the largest firms, but significant in all other quintiles.[27]

[27] The level of DIS is lower as size increases, which leads to coefficient estimates increasing in size despite

**Table F.2: CAPM-$\alpha$ of Portfolio Sorts on Various Disagreement Proxies for the Smallest Quintile of Stocks**

| | DIS | DISP | IVOL | SIR |
|---|---|---|---|---|
| Low | 0.59 | 0.20 | 0.45 | 0.53 |
| 2 | 0.33 | 0.34 | 0.24 | 0.51 |
| 3 | 0.27 | 0.14 | 0.18 | 0.26 |
| 4 | 0.25 | 0.18 | 0.18 | 0.11 |
| 5 | −0.01 | −0.13 | −0.13 | 0.26 |
| 6 | −0.34 | −0.25 | −0.37 | −0.29 |
| 7 | −0.61 | −0.41 | −0.15 | −0.15 |
| 8 | −0.51 | −0.49 | −0.46 | −0.52 |
| 9 | −0.62 | −0.61 | −1.00 | −0.94 |
| High | −1.66 | −0.83 | −0.86 | −0.94 |
| Low-High | 2.25 | 1.03 | 1.30 | 1.47 |
| $t$-stat. | (5.87) | (4.07) | (3.27) | (3.64) |

**Notes:** This table reports the CAPM-$\alpha$ estimated as the intercept from a time-series regression of value-weighted monthly excess returns at month $t+1$ against the market excess return at month $t$ for univariate portfolios sorted according to various disagreement proxies at date $t$, restricted to the smallest quintile of stocks each month. Each column represents a different sorting variable listed at the top of the column: disagreement (DIS), analyst forecast dispersion (DISP), IVOL, or short interest (SIR). The sample ranges from January 1994 to December 2022. The row labeled "Low-High" presents the differences in monthly returns between the 10th decile (representing high values of the sorting variable) and the 1st decile (representing low values of the sorting variable). $t$-statistics, adjusted according to Newey and West (1987), are presented in parentheses along with their corresponding $p$ values.

Consistent with the model's prediction, Table F.2 indicates that the alpha is higher among smaller stocks. Among the smallest quintile of stocks, a value-weighted decile spread portfolio sorted on our composite measure of disagreement generates 27.0% annualized CAPM alpha. Again, this alpha is larger than the corresponding alphas generated by the major disagreement proxies for the smallest quintiles of stocks: 12.4% for analyst forecast dispersion, 15.6% for IVOL, and 17.6% for short interest.

their significance declining, until the highest size quintile, at which point the coefficient is primarily influenced by noise.

## Appendix G Robustness

**Table G.1: Time-Series Distributions of Cross-Sectional Correlations**

| Panel A: Correlations with DIS | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Variable | Mean | SD | Min | p10 | p25 | p50 | p75 | p90 | Max |
| SIR | 0.13 | 0.08 | −0.07 | 0.02 | 0.07 | 0.13 | 0.18 | 0.23 | 0.42 |
| IVOL | 0.62 | 0.07 | 0.29 | 0.53 | 0.58 | 0.63 | 0.66 | 0.70 | 0.77 |
| ISKEW | 0.48 | 0.13 | −0.11 | 0.32 | 0.41 | 0.50 | 0.56 | 0.62 | 0.76 |
| DISP | 0.15 | 0.07 | 0.02 | 0.06 | 0.09 | 0.14 | 0.20 | 0.25 | 0.37 |
| SIZE | −0.09 | 0.02 | −0.15 | −0.13 | −0.11 | −0.09 | −0.08 | −0.07 | −0.03 |
| BTM | 0.07 | 0.09 | −0.12 | −0.02 | 0.01 | 0.05 | 0.11 | 0.21 | 0.38 |
| MOM | −0.08 | 0.13 | −0.39 | −0.24 | −0.16 | −0.09 | 0.00 | 0.09 | 0.46 |
| BETA | 0.14 | 0.14 | −0.25 | −0.03 | 0.05 | 0.14 | 0.22 | 0.33 | 0.54 |
| ILLIQ | 0.12 | 0.11 | −0.01 | 0.02 | 0.04 | 0.09 | 0.16 | 0.27 | 0.59 |
| RMAX | 0.55 | 0.07 | 0.35 | 0.45 | 0.50 | 0.55 | 0.60 | 0.65 | 0.72 |

| Panel B: Correlations Among Major Disagreement Proxies | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Variables | Mean | SD | Min | p10 | p25 | p50 | p75 | p90 | Max |
| DISP, IVOL | 0.13 | 0.05 | 0.02 | 0.06 | 0.09 | 0.13 | 0.16 | 0.19 | 0.25 |
| DISP, SIR | 0.07 | 0.04 | −0.01 | 0.03 | 0.04 | 0.07 | 0.10 | 0.12 | 0.22 |
| IVOL, SIR | 0.24 | 0.08 | 0.01 | 0.13 | 0.18 | 0.24 | 0.31 | 0.35 | 0.44 |

**Notes:** Panel A presents the time-series distribution of the cross-sectional monthly correlation between DIS and various variables of interest. Panel B presents the time-series distribution of cross-sectional monthly correlations between each pair of major disagreement proxies: DISP, IVOL, and SIR. Each panel includes the following statistical measures for each variable: mean, standard deviation, minimum, maximum, and percentiles at the 10th, 25th, 50th (median), 75th, and 90th levels. The sample ranges from January 1994 to December 2022 and comprises all common stocks (identified as shrcd = 10, 11) traded on the NYSE, AMEX, or NASDAQ. For detailed definitions of the variables, please refer to Table 1 and Appendix A.

**Table G.2: Portfolio Sorts on Disagreement, DIS**

| DIS | Raw Return | Excess Return | CAPM-$\alpha$ | FF3-$\alpha$ | FFC-$\alpha$ |
|---|---|---|---|---|---|
| Low | 0.96 | 0.78 | 0.27 | 0.24 | 0.18 |
| 2 | 1.00 | 0.82 | 0.21 | 0.19 | 0.17 |
| 3 | 0.93 | 0.75 | 0.07 | 0.06 | 0.06 |
| 4 | 0.90 | 0.72 | −0.04 | −0.05 | −0.01 |
| 5 | 0.85 | 0.67 | −0.15 | −0.15 | −0.13 |
| 6 | 0.87 | 0.69 | −0.17 | −0.17 | −0.07 |
| 7 | 0.75 | 0.56 | −0.35 | −0.36 | −0.22 |
| 8 | 0.71 | 0.53 | −0.50 | −0.48 | −0.31 |
| 9 | 0.46 | 0.28 | −0.87 | −0.82 | −0.65 |
| High | −0.06 | −0.24 | −1.55 | −1.49 | −1.31 |
| Low-High | 1.01 | 1.01 | 1.82 | 1.73 | 1.49 |
| $t$-stat. | (2.02) | (2.02) | (4.90) | (5.57) | (4.65) |
| $p$-value | 0.04 | 0.04 | 0.00 | 0.00 | 0.00 |

**Notes:** This table reports the mean values of value-weighted monthly returns at month $t + 1$, based on univariate portfolios sorted according to disagreement (DIS) at date $t$. The sample ranges from January 1994 to December 2022. The row labeled "Low-High" presents the differences in monthly returns between the 10th decile (representing high disagreement) and the 1st decile. "Raw returns" refer to one-month-ahead monthly returns, whereas "Excess returns" are raw returns minus the risk-free rate. The alphas (CAPM-$\alpha$, FF3-$\alpha$, FFC-$\alpha$) represent intercepts from a time-series regression of monthly excess returns at month $t+1$ against the market return (CAPM), further adjusted for size and value factors (FF3), and additionally a momentum factor (FFC) at month $t$. $t$-statistics, adjusted according to Newey and West (1987), are presented in parentheses along with their corresponding $p$ values.

**Table G.3: CAPM-$\alpha$ of Portfolio Sorts on Various Disagreement Proxies**

| | DIS | DISP | IVOL | SIR |
|---|---|---|---|---|
| Low | 0.27 | 0.32 | 0.24 | 0.15 |
| 2 | 0.21 | 0.11 | 0.29 | 0.00 |
| 3 | 0.07 | 0.13 | 0.09 | 0.02 |
| 4 | −0.04 | −0.08 | −0.08 | −0.01 |
| 5 | −0.15 | 0.00 | −0.07 | −0.06 |
| 6 | −0.17 | 0.03 | −0.31 | −0.04 |
| 7 | −0.35 | −0.13 | −0.29 | −0.04 |
| 8 | −0.50 | −0.15 | −0.33 | −0.26 |
| 9 | −0.87 | −0.21 | −0.56 | −0.46 |
| High | −1.55 | −0.77 | −1.07 | −0.58 |
| Low-High | 1.82 | 1.08 | 1.31 | 0.73 |
| $t$-stat. | (4.90) | (4.23) | (3.94) | (2.66) |
| $p$-value | 0.00 | 0.00 | 0.00 | 0.01 |

**Notes:** This table reports the CAPM-$\alpha$ estimated as the intercept from a time-series regression of value-weighted monthly excess returns at month $t+1$ against the market excess return at month $t$ for univariate portfolios sorted according to various disagreement proxies at date $t$. Each column represents a different sorting variable listed at the top of the column: disagreement (DIS), analyst forecast dispersion (DISP), IVOL, or short interest (SIR). The sample ranges from January 1994 to December 2022. The row labeled "Low-High" presents the differences in monthly returns between the 10th decile (representing high values of the sorting variable) and the 1st decile (representing low values of the sorting variable). $t$-statistics, adjusted according to Newey and West (1987), are presented in parentheses along with their corresponding $p$ values.

**Table G.4: CAPM-$\alpha$ of Portfolio Sorts on Disagreement, DIS, for Various Levels of Relative Risk Aversion**

| DIS | $\gamma = \frac{1}{2}$ | $\gamma = 1$ | $\gamma = 2$ | $\gamma = 4$ | $\gamma = 8$ |
|---|---|---|---|---|---|
| Low | 0.28 | 0.27 | 0.27 | 0.29 | 0.28 |
| 2 | 0.18 | 0.17 | 0.21 | 0.23 | 0.23 |
| 3 | 0.08 | 0.13 | 0.07 | 0.08 | 0.10 |
| 4 | −0.08 | −0.04 | −0.04 | −0.10 | −0.11 |
| 5 | −0.15 | −0.22 | −0.15 | −0.12 | −0.13 |
| 6 | −0.09 | −0.09 | −0.17 | −0.20 | −0.18 |
| 7 | −0.29 | −0.27 | −0.35 | −0.34 | −0.33 |
| 8 | −0.54 | −0.56 | −0.50 | −0.54 | −0.54 |
| 9 | −0.72 | −0.80 | −0.87 | −0.82 | −0.86 |
| High | −1.46 | −1.57 | −1.55 | −1.56 | −1.56 |
| Low-High | 1.74 | 1.85 | 1.82 | 1.84 | 1.84 |
| $t$-stat. | (4.98) | (5.14) | (4.90) | (4.94) | (5.00) |

**Notes:** This table reports the CAPM-$\alpha$ estimated as the intercept from a time-series regression of value-weighted monthly excess returns at month $t + 1$ against the market excess return at month $t$, for univariate portfolios sorted according to disagreement (DIS) at date $t$, computed using various values of the relative risk aversion coefficient, $\gamma$, which varies by column. The sample ranges from January 1994 to December 2022. The row labeled "Low-High" presents the differences in monthly returns between the 10th decile (representing high disagreement) and the 1st decile (representing low disagreement). $t$-statistics, adjusted according to Newey and West (1987), are presented in parentheses along with their corresponding $p$ values.

**Table G.5: Fama-MacBeth Cross-Sectional Regressions for Various Levels of Relative Risk Aversion**

| $\gamma$ | ($\gamma = \frac{1}{2}$) EXRET$_{t+1}$ | ($\gamma = 1$) EXRET$_{t+1}$ | ($\gamma = 2$) EXRET$_{t+1}$ | ($\gamma = 4$) EXRET$_{t+1}$ | ($\gamma = 8$) EXRET$_{t+1}$ |
|---|---|---|---|---|---|
| DIS | $-0.58^{***}$ | $-1.05^{***}$ | $-1.55^{***}$ | $-2.18^{***}$ | $-2.62^{***}$ |
| | (−4.23) | (−4.35) | (−3.98) | (−4.04) | (−3.97) |
| Constant | $0.84^{***}$ | $0.82^{***}$ | $0.82^{***}$ | $0.85^{***}$ | $0.88^{***}$ |
| | (3.09) | (2.96) | (2.94) | (3.05) | (3.16) |
| Observations | 714, 310 | 718, 794 | 719, 940 | 720, 317 | 720, 480 |

**Notes:** This table displays the regression coefficients and $t$-statistics from a monthly Fama and MacBeth (1973) regression analysis conducted on excess returns, adhering to the methodologies in Fama and French (1993) and Carhart (1997). These regressions are performed against a lagged measure of disagreement, computed based on one of five levels of relative risk aversion, $\gamma$. The sample ranges from January 1994 to December 2022. The coefficients on DIS are multiplied by 100 for readability. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively.

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