Screening Employees Using Stock Options with Early Vesting Period

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September 2025

We thank Tim Baldenius, Paul Fischer, Jonathan Glover, Ilan Guttman, Mirko Heinle, Michael Kopel, Brian Mittendorf, Shyam Sunder, and workshop participants at the Columbia University Theory Conference, Yonsei University, and 15th workshop on Accounting and Economics for helpful comments.

Abstract

We explore vesting periods of executive stock-option grants. A manager's productivity depends on effort and on how well the manager matches the firm's needs. But the firm does not directly observe the manager's effort or match, and the manager does not know the match with the firm unless the manager joins and works for the firm. We show that options with short and long vesting periods play distinct contracting roles. Options with *long* vesting provide long-term effort incentives for managers. Options with *short* vesting periods – the main focus of this paper – potentially improve screening of a newly hired manager's match with the firm. By allowing early option exercises, firms create incentives for poorly matched managers to exercise early and voluntarily depart. Conversely, well-matched managers are motivated to stay and reap the full rewards of continued employment; for these managers, options with long vesting provide incentives to exert high effort.

1 Introduction

Attracting, retaining, and motivating top executive talent are critical challenges for firms. Managers play a vital role in steering organizational strategy and driving performance. An executive team well matched to the company's strategy, technology, problems, culture and opportunities is an important predictor of organizational success (Lazear and Gibbs 2014). To make good hiring decisions, firms use screening processes to evaluate the quality of job candidates and how well each candidate "fits" with the organization (Lazear and Oyers 2012). Most research considering questions of organizational fit posits that a manager possesses private information about his or her own quality and firm fit; the company does not have access to this private information. Substantial work has gone into developing mechanisms for firms to extract managerial private information.

Prior to joining a firm, however, a potential hire lacks a clear vision of the fit with the firm. Joining the firm and experiencing its culture and business model allows the hired manager to develop an idea about whether he or she is a good match for the firm (Jovanovic 1979). We focus on this situation: the firm and the manager face symmetric initial uncertainty about the manager-firm match, and the manager, through experience, privately learns more about the match. Contrary to the conventional view that only options with long vesting periods are beneficial to firms – primarily due to their incentive-alignment and retention properties – we demonstrate that stock options with shorter vesting periods also play an important role in retaining well-matched managers and in delivering incentives to those retained.

The vesting period is a key component of executive incentive plans. A common critique of stock options is that they may induce managerial short-termism by prioritizing short-term stock performance over long-term investments that create enduring value (Jensen 2004; Jensen and Murphy 2004; Bolton et al. 2006). The proposed "conventional-wisdom" solution is to incorporate a lengthy vesting period to mitigate short-termism (Bolton et al. 2006). For example, post-2008-crisis financial regulations stipulated that senior-executive incentive pay must take the form of restricted stock that cannot vest until bailout funds were repaid (Walker

2010).

We concur that there is an important role for long vesting periods to provide long-term incentives. However, we show that the arguments for only long vesting ignore an important consideration: rapid vesting allows a firm to quickly filter out managers who are likely to be a poor long-term fit. Several prominent technology companies apparently recognize this advantage of short vesting. At Netflix, stock option grants were not subject to vesting restrictions; all options vested immediately and were exercisable as of the grant date. The management rationale was that vesting restrictions could inadvertently incentivize disengaged employees to remain with the firm solely to await the vesting of their option awards (Larcker et al. 2010). Spotify grants employees options that start vesting only three months after the grant date. Snap Inc. and Pinterest likewise offer employee options with rapid vesting. This provides incentives for poorly matched managers to voluntarily depart rather than lingering in an unsatisfactory role; firms fear that long vesting periods can lead to retention of ill-fitting employees (McCord 2014).

At the same time, early vesting creates risks: managers may leave prematurely without a full opportunity to develop firm-specific skills that enhance company value over time. To maximize long-term success, retaining the "right" managers and convincing them to invest in firm-specific human capital is essential. Early vesting to screen for fit must be optimally balanced against longer vesting periods to avoid excessive voluntary attrition and under-investment in firm-specific expertise.

To analyze these conflicting effects of early vesting, we develop a model that employs option-based compensation to guide managerial retention and to provide incentives for effort. A manager's performance is superior when the manager is well-matched to the firm, but neither the company nor the manager possesses precise information about this match at the point of hiring and contracting. Information about the match is only revealed to the manager after he or she begins employment – the manager acquires private information after the contracting stage (Holmstrom 1999; Hermalin and Weisbach 1998; Hermalin 2005). This information is informative about the manager's match with the firm, but it does not perfectly reveal it – some residual uncertainty remains even for the manager.

Delayed realization of the manager-firm match prevents firms from hiring the best-matched managers upfront. With a long vesting period, a manager who learns that he or she is likely to be a poor match with the firm will remain employed "too long" through the end of the vesting period. This is costly for the firm because of lower productivity of poor-match managers. Early vesting then plays an important role – while the manager *privately* observes the match signal after working for the firm for some time, the firm can convince the low-match-signal manager to use this private information, exercise the early-vesting options, and voluntarily depart.

Long vesting periods play an important role as well. Retained managers must be given appropriate effort incentives. Options with long vesting periods are ideally suited for this. Thus, we show distinct roles for options with various vesting schedules: early-vesting options allow firms to screen managers for fit; long-vesting options provide effort incentives to retained managers.

We consider several modeling variations to demonstrate the robustness of the screening role of early-vesting options. First, we include the possibility that managers have other employment opportunities. A manager's outside employment opportunities reduce the firm's cost to screen managers, yet early vesting remains an important screening component. Second, we include the possibility that a manager can make firm-specific investments that increase the chances of high terminal firm value; early vesting remains valuable in this setting.

There is a small theoretical literature examining option vesting periods. Brisley (2006) analyzes progressive *performance* vesting to encourage managerial risk-taking by creating convex payoffs; early vesting conditioned on realized stock prices is included to preserve payoff convexity. Laux (2012) considers vesting periods in the context of a manager's short-termism; he shows that with short vesting ousted managers retain residual claims on investments after departing the firm; this mitigates managerial short-termism.

There are several important empirical analyses of option vesting periods. Cadman et al. (2013) investigate the determinants of vesting periods in stock-based compensation. Their work suggests several important vesting-period determinants: accounting standards, retention incentives, and the CEO's bargaining power. There are many examples of short vesting periods: their sample shows that about 42% of stock options vest within the first year. Gopalan et. al. (2014) develop an empirical pay-duration measure encompassing equity vesting terms, identifying various determinants: growth opportunities, asset composition, and R&D intensity. Evans et al. (2018) examine the lengths of measurement periods prior to executive performance-based compensation. Abstracting from unobservable effort and private information, they show that under certain conditions short performance periods are preferred by a firm. Huang et al. (2022) examine how investment sentiment relates to option vesting periods, finding that shorter vesting coincides with high sentiment as shareholders seek to sustain overvaluation.

Our analysis extends prior theoretical work by considering the incentive implications of early and long option vesting. Options with long vesting periods provide effort incentives; options with early vesting can be useful in screening for firm-manager match quality. Our work suggests novel empirical predictions: our results suggest that early vesting is used more frequently by firms facing greater initial uncertainty about managerial fit (e.g., technology startups); and that firms using short vesting periods experience more frequent managerial turnover.

The remainder of this paper proceeds as follows. Section 2 presents our base model. In Section 3.1 we present "benchmark" analysis of the no-private-information situation where the firm and the manager both obtain the manager's match signal. Section 3.2 considers options with only a long vesting period. In Section 3.3 we allow early- and long-vesting options and identify situations where early-vesting options are valuable to the firm. Section 4 contains several extensions: the effect of the manager's outside employment opportunities on the use of early vesting options, and the effect of early vesting options on the manager's investment in personally-costly firm-specific capital. Section 5 concludes the analysis and discusses implications and possible future research directions. All proofs are in the appendix.

2 Setup

2.1 Model

There are two risk-neutral parties: "the firm", which refers to the firm's owner(s), and "the manager" hired by the firm to manage operations. The firm designs and commits to the manager's compensation contract. The manager is protected by limited liability – payments to the manager must be non-negative. The firm's terminal value is a function of (i) the manager's unobservable effort, and (ii) the manager's "match" with the firm – how well the manager's abilities, experiences, preferences, and style match the firm's strategy, markets, customers, and other firm characteristics (Lazear and Oyer 2012). At the time of contracting, neither the firm nor the manger knows the manager's match. While working for the firm, the manager privately receives an informative (but imperfect) signal about the match with the firm (Hermalin 2005; Lazear and Oyer 2012).

There are three dates: t_0, t_1, and t_2. At the *initial* date t_0, the firm's value is exogenous $X_0 \geq 0$. The firm hires a manager to manage the firm's assets to maximize the firm's terminal value; at this point, neither the firm nor the manager knows the manager's match with the firm $\theta \in \{\theta_L, \theta_H\}$ (Holmstrom 1999; Jongjaroenkamol and Laux 2017). The probability of a good match $Pr(\theta_H) = \mu$ is common knowledge with $\mu \in (0, 1)$; the probability of a bad match is then $Pr(\theta_L) = 1 - \mu$. To focus on the screening effects of option vesting schedules, we restrict attention to option-based compensation contracts: the manager is offered options at the strike price X_0. The firm offers $\beta_2 \geq 0$ *long-vesting* options; these vest at the end of t_2. The firm can also offer $\beta_1 \geq 0$ *early-vesting* options; the early-vesting options vest and expire at the end of t_1.

At the *interim* date t_1, the firm's value has evolved to $V_1 > X_0$ independent of the manager's match. By now, the manager has been with the firm for one time period and during the period, the manager has learned about the firm's operations and its economic environment; this allows the manager to update beliefs about how well he and the firm match. We capture the manager's updated beliefs with the private signal $\sigma \in \{\sigma_L, \sigma_H\}$ that the manager receives at t_1. This signal is informative about the manager's match with the firm; the precision of the signal is $\lambda \in \left(\frac{1}{2}, 1\right)$:

$$\Pr\left(\sigma_H | \theta_H\right) = \lambda; \, \Pr\left(\sigma_L | \theta_H\right) = 1 - \lambda;$$
$$\Pr\left(\sigma_L | \theta_L\right) = \lambda; \, \Pr\left(\sigma_H | \theta_L\right) = 1 - \lambda.$$

After the manager privately learns the match signal σ at t_1, conditional probabilities on $\theta \in \{\theta_L, \theta_H\}$ given the realization of σ are:

$$\Pr(\theta_H|\sigma_H) = \frac{\mu\lambda}{\mu\lambda + (1-\mu)(1-\lambda)}; \; \Pr(\theta_L|\sigma_H) = \frac{(1-\mu)(1-\lambda)}{\mu\lambda + (1-\mu)(1-\lambda)}; \tag{1}$$
$$\Pr(\theta_L|\sigma_L) = \frac{(1-\mu)\lambda}{(1-\mu)\lambda + \mu(1-\lambda)}; \; \Pr(\theta_H|\sigma_L) = \frac{\mu(1-\lambda)}{(1-\mu)\lambda + \mu(1-\lambda)}. \tag{2}$$

After learning the realization of σ, the manager can quit. If the manager's contract includes early-vesting options $\beta_1 > 0$, the manager can exercise these options when leaving the firm; the unvested long-dated options β_2 are forfeited. If the manager quits, the firm hires a new manager; the firm's terminal value following the manager's replacement is V_N (net of the new manager's compensation).

The manager who remains with the firm chooses the level of effort $e \in \{0, 1\}$ at a personal cost $C\left(e\right)$ with $C\left(0\right) = 0$ and $C\left(1\right) = c > 0$. (Recall that the manager does not learn the realization of the match θ; the manager knows only the interim signal σ about the match.) Neither the effort nor the personal cost are observed by the firm. At the terminal date t_2, the firm's terminal value is $V_2 \in \{V_{2L}, V_{2H}\}$, with $V_{2L} < X_0 < V_{2H}$ and $V_{2L} < V_N < V_{2H}$.

The manager's match θ and effort e determine the terminal-value probability structure: for $\theta \in \{\theta_L, \theta_H\}$ and $e \in \{0, 1\}$,

$$\Pr(V_{2H}|\theta, e) = \begin{cases} pe + q(1-e) & \text{if } \theta = \theta_H \\ 0 & \text{if } \theta = \theta_L \end{cases}$$
$$\Pr(V_{2L}|\theta, e) = 1 - \Pr(V_{2H}|\theta, e),$$

where $p \in \left(\frac{1}{2}, 1\right)$ and $p > q > 0$. With this probability structure, a low-match manager provides the firm with the low terminal value V_{2L} regardless of effort. For a high-match manager, high effort results in the high terminal value V_{2H} with probability p; low effort results in the high terminal value with probability $q < p$. The firm would thus prefer to hire a manager if and only if the manager is a good match. We focus on the situation where the firm faces non-trivial effort incentives: the firm wants the manager who is retained to select the high effort level $e = 1$ at t_1 if and only if $\sigma = \sigma_H$.

We assume that the discount factor across periods equals to one. The manager's utility u depends only on the total option-based compensation s and the manager's effort e: $u(\cdot) = s - C(e)$. We normalize the manager's reservation utility to zero. The firm's objective is to maximize the firm's value net of the manager's compensation.

3 Analysis

3.1 Benchmark: No Private Information

To explore the trade-offs faced by the firm when designing the manager's compensation, initially consider the simplified "benchmark" situation without private information: both the manager and the firm observe the realization of the manager's signal σ at t_1. Observing the signal allows the firm to decide whether the manager is replaced when the signal is σ_L. There is thus no reason for the firm to grant any early-vesting options. When the manager is retained, the firm uses long-vesting options $\beta_2 \geq 0$ to provide period-t_2 effort incentives for the manager. Recall that μ is the probability that the manager is a good match for the firm, and that λ describes the information content of the period-t_1 signal σ: $\Pr(\sigma_H|\theta_H) = \Pr(\sigma_L|\theta_L) = \lambda > \frac{1}{2}$. Two possibilities must be considered to characterize the firm's maximum value in the benchmark setting:

1. When the ex-ante probability that the manager is a good match μ is low, or when λ – the information content of the period-t_1 signal σ – is high , the firm will use the signal σ

to guide its replacement decision: replace the manager if an only if $\sigma = \sigma_L$. The manager with the signal σ_H remains with the firm; long-vesting options β_2 provide the manager's effort incentives. We compute the maximum expected value of the firm that uses the realization of σ to guide its replacement decision in Program 1.

2. When μ is high and/or λ is sufficiently low, the downside of a type-I error – replacing a high-match manager who received the "wrong" signal σ_L – is greater than the value of using the realization of σ for the manager-replacement decision; in this case, the firm prefers not to use the period-t_1 signal σ to screen for high match. The firm instead retains the manager regardless of the signal. The firm uses long-vesting options β_2 to provide the retained manager with period-t_2 high-effort incentives if only if the the period-t_1 signal $\sigma = \sigma_H$. We compute the maximum expected value of the firm that retains the manager regardless of the realization of σ in Program 2.

Program 1 is the firm's value-maximization problem when the firm replaces the manager if and only if $\sigma = \sigma_L$:

$$\underset{\beta_2 \geq 0}{Max} \Bigg[\mu \left[\lambda \left(p \left(V_{2H} - \beta_2 \left(V_{2H} - X_0 \right) \right) + (1-p) V_{2L} \right) + (1-\lambda) V_N \right] + (1-\mu) \left[\lambda V_N + (1-\lambda) V_{2L} \right] \Bigg] \tag{3}$$

subject to:

$$\mu \left[\lambda \left(p \beta_2 (V_{2H} - X_0) - c \right) \right] - (1-\mu)(1-\lambda) c \geq 0 \tag{4}$$

$$\Pr \left(\theta_H | \sigma_H \right) p \beta_2 (V_{2H} - X_0) - c \geq 0 \tag{5}$$

$$\Pr \left(\theta_H | \sigma_H \right) p \beta_2 (V_{2H} - X_0) - c \geq \Pr \left(\theta_H | \sigma_H \right) q \beta_2 (V_{2H} - X_0) \tag{6}$$

The first constraint (4) is the ex-ante *individual-rationality* (IR) condition; the second constraint (5) is the interim (IR) for the manager with the signal σ_H; and the third constraint (6) is the *incentive- compatibility* (IC) condition providing high-effort incentives for the manager with the signal σ_H. The manager's limited liability constraint is satisfied by $\beta_2 \geq 0$.

Lemma 1. *When the firm retains the manager if and only if the period-t_1 signal is σ_H, the firm offers the manager β_2^R long-vesting options:*

$$\beta_2^R = \frac{c}{\Pr(\theta_H|\sigma_H)(p-q)(V_{2H}-X_0)}. \tag{7}$$

The firm's expected value from Program 1 is:

$$\Pi^R = \mu\lambda p\left(V_{2H} - \frac{c}{Pr(\theta_H|\sigma_H)(p-q)}\right) + \Pr(\sigma_L)V_N + (\Pr(\sigma_H) - \mu\lambda p)V_{2L}. \tag{8}$$

Program 2 is the firm's value-maximization problem when the firm does not "screen" using the realization of σ: the manager is retained regardless of the realization of σ, and the firm provides the manager with period-t_2 effort incentives if and only if the period-t_1 signal $\sigma = \sigma_H$:

$$\underset{\beta_2\geq 0}{Max}\Big[\mu\Big[\lambda\left(p\left(V_{2H}-\beta_2\left(V_{2H}-X_0\right)\right)+(1-p)V_{2L}\right)$$
$$+(1-\lambda)\left(q\left(V_{2H}-\beta_2\left(V_{2H}-X_0\right)\right)+(1-q)V_{2L}\right)\Big]+(1-\mu)V_{2L}\Big] \tag{9}$$

subject to:

$$\mu\left[\lambda\left(p\beta_2(V_{2H}-X_0)-c\right)+(1-\lambda)q\beta_2(V_{2H}-X_0)\right]-(1-\mu)(1-\lambda)c\geq 0 \tag{10}$$

$$\Pr(\theta_H|\sigma_H)p\beta_2(V_{2H}-X_0)-c\geq 0 \tag{11}$$

$$\Pr(\theta_H|\sigma_H)p\beta_2(V_{2H}-X_0)-c\geq \Pr(\theta_H|\sigma_H)q\beta_2(V_{2H}-X_0) \tag{12}$$

As in Program 1, the three constraints are: the ex-ante (IR) constraint (10), the interim (IR) constraint (11) for the manager with the signal σ_H, and and the (IC) constraint (12) for the manager with the signal σ_H.[1]

Lemma 2. *When the firm does not screen the manager, the firm offers the manager β_2^{NR} long-vesting options:*

$$\beta_2^{NR} = \frac{c}{\Pr(\theta_H|\sigma_H)(p-q)(V_{2H}-X_0)}. \tag{13}$$

[1]With long-vesting options set at β_2^{NR} in (13), the manager with the signal σ_L will remain with the firm (in the hope of a type-I error) and select the low effort level.

The firm's expected value from Program 2 is:

$$\Pi^{NR} = \mu\left(\lambda(pV_{2H} + (1-p)V_{2L}) + (1-\lambda)(qV_{2H} + (1-q)V_{2L})\right) + (1-\mu)V_{2L} \\ - \frac{c(1-\mu+\lambda(2\mu-1))(\lambda p + (1-\lambda)q)}{\lambda(p-q)}. \tag{14}$$

The ranking of the maximum expected values from Program 1 and Program 2 depends on the values of the various parameters: firm values V_N, V_{2L}, and V_{2H}; the manager's cost of high effort c; the probabilities μ, q, and p; and the information content of the period-t_1 signal (captured by λ). The ranking depends unambiguously on the ex-ante probability that the manager is a good match: $\left(\Pi^R - \Pi^{NR}\right)$ strictly decreases in this probability μ. As a consequence, the decision whether or not to use the realization of the signal σ to guide the manager's replacement decision takes the form of a threshold $\mu^B > 0$. Π^R (the expected firm value with low-signal manager replaced) is higher than Π^{NR} (the expected firm value of "no-replacement") if and only if μ is below this threshold.

Lemma 3. *There exists a threshold value $\mu^B > 0$ such that $\Pi^R > \Pi^{NR}$ if and only if $\mu < \mu^B$.*

3.2 Private Information and Long-Vesting Options

For the remainder of the paper, we return to the main information-asymmetry setting: the manager privately learns the realization of the signal σ at t_1, and the firm does not observe the manager's signal. In this subsection, we examine the firm's problem with long-vesting options only, i.e., $\beta_1 = 0$ (our main results, with both long- and early-vesting options, are below in section 3.3). Without early-vesting options, the firm is not able to screen on the manager's signal – the manager with the low signal σ_L stays with the firm, selects the low effort level, and earns strictly positive expected utility because, with probability $\Pr(\theta_H|\sigma_L) > 0$, the low signal is a type-I error.

From the firm's perspective, not knowing the realization of σ and using long-vesting options only is identical to learning the realization of σ and ignoring it as in section 3.1. The firm's value-maximization problem is then identical to Program 2 in section 3.1. The optimal level of long-vesting options β_2^{NE} and the firm's maximum expected value Π^{NE} are thus equal to the "no-replacement" ones from section 3.1, Lemma 2: $\beta_2^{NE} = \beta_2^{NR}$ and $\Pi^{NE} = \Pi^{NR}$. We document this in Proposition 1.

Proposition 1.

When the manager privately observes the signal σ, and the firm does not offer early-vesting options:

1. *The optimal level of long-vesting options β_2^{NE} is:*

$$\beta_2^{NE} = \frac{c}{\Pr(\theta_H|\sigma_H)(p-q)(V_{2H}-X_0)}. \tag{15}$$

2. *The manager remains with the firm regardless of the realization of the period-$t_1$signal σ.*

3. *The firm's expected value i*s:

$$\begin{aligned}\Pi^{NE} = {} & \mu\left(\lambda\left(pV_{2H} + (1-p)V_{2L}\right) + (1-\lambda)\left(qV_{2H} + (1-q)V_{2L}\right)\right) + (1-\mu)V_{2L} \\ & - \frac{c(1-\mu+\lambda(2\mu-1))(\lambda p + (1-\lambda)q)}{\lambda(p-q)}.\end{aligned} \tag{16}$$

4. *$\beta_2^{NE} = \beta_2^{NR}$ and $\Pi^{NE} = \Pi^{NR}$.*

3.3 Private Information with Early- and Long-Vesting Options

We next turn to the main question of the paper: why would the firm offer the manager both long-vesting options $\beta_2 > 0$ and early-vesting options $\beta_1 > 0$? Benchmark analysis in section 3.1 suggests the answer: long-vesting options β_2 are adequate to provide period-t_2 effort incentives. But long-vesting options alone cannot provide period-t_1 screening incentives – the manager who receives the "poor-match" signal σ_L cannot be prevented from staying with the firm, selecting the low level of effort, and waiting for the realization of the firm's terminal value in the hopes that the "poor-match" signal was a type-I error. As we show below, early-vesting options can help the firm screen – they can be designed to convince the manager with the "poor-match" signal to leave the firm.

There are, however, two disadvantages to using early-vesting options to screen. First, as already described in section 3.1, there is the chance that the manager is replaced because of a type-I error; this is greatest when μ is high and λ is low. Second, and new here, to convince the manager with the "poor-match" signal σ_L to leave the firm at t_1, the manager, via $\beta_1 > 0$, receives strictly positive informational rent. To decide whether or not to use early-vesting options $\beta_1 > 0$, the firm compares its expected value with long vesting only (from Proposition 1) with the maximum expected value with early- and long-vesting options, which we compute with Program 3 below:

$$\underset{\beta_1,\beta_2\geq 0}{Max}\left[\mu\left[\lambda\left(p\left(V_{2H}-\beta_2\left(V_{2H}-X_0\right)\right)+(1-p)V_{2L}\right)+(1-\lambda)\left(V_N-\beta_1(V_1-X_0)\right)\right]\right.$$
$$\left.+(1-\mu)\left[\lambda\left(V_N-\beta_1(V_1-X_0)\right)+(1-\lambda)V_{2L}\right]\right] \tag{17}$$

subject to:

$$\mu\left[\lambda\left(p\beta_2(V_{2H}-X_0)-c\right)+(1-\lambda)\beta_1\left(V_1-X_0\right)\right]$$
$$+(1-\mu)\left[\lambda\beta_1(V_1-X_0)-(1-\lambda)c\right]\geq 0 \tag{18}$$

$$\Pr\left(\theta_H|\sigma_H\right)p\beta_2\left(V_{2H}-X_0\right)-c\geq \Pr\left(\theta_H|\sigma_H\right)q\beta_2\left(V_{2H}-X_0\right) \tag{19}$$

$$\Pr\left(\theta_H|\sigma_H\right)p\beta_2\left(V_{2H}-X_0\right)-c\geq \beta_1\left(V_1-X_0\right) \tag{20}$$

$$\beta_1\left(V_1-X_0\right)\geq \Pr\left(\theta_H|\sigma_L\right)q\beta_2\left(V_{2H}-X_0\right) \tag{21}$$

$$\beta_1(V_1-X_0)\geq \Pr\left(\theta_H|\sigma_L\right)p\beta_2\left(V_{2H}-X_0\right)-c \tag{22}$$

The ex-ante (IR) constraint (18) and the effort (IC) constraint (19) are familiar from Programs 1 and 2. The interim (IR) constraint from Programs 1 and 2 is transformed into the "no-exit" condition (20) to reflect the fact that the manager with the high signal σ_H can earn strictly positive utility $\beta_1\left(V_1-X_0\right)$ by leaving the firm and exercising early-vesting options. The two "exit" constraints (21)-(22) provide the manager with the low-match signal σ_L with the incentive to exit and exercise early-vesting options: (21) guarantees that this manager is better off leaving than staying and selecting the low effort level; (22) assures that this manager is better off leaving than staying and selecting the high effort level.

Optimal β_1 and β_2 are derived from constraints. The effort (IC) constraint (19) is identical to the (IC) constraint in Program 1 in (6). The (IC) constraint (19) provides the necessary and sufficient minimum level of long-vesting options:

$$\beta_2 \geq \frac{c}{\Pr(\theta_H|\sigma_H)(p-q)(V_{2H}-X_0)}. \tag{23}$$

In the proof of Proposition 2, we show that maximizing the objective in (17) requires that β_1 and β_2 be set as low as possible (subject to the constraints). Thus, the necessary and sufficient condition (23) holds with equality. Substituting $\beta_2 = \frac{c}{\Pr(\theta_H|\sigma_H)(p-q)(V_{2H}-X_0)}$ into the exit constraint (21) and rearranging provide the lower bound on early-vesting options required to convince the manager with the low-match signal σ_L to exit:

$$\beta_1 \geq \underline{\beta}_1 = \frac{\Pr(\theta_H|\sigma_L)}{\Pr(\theta_H|\sigma_H)} \frac{q\,c}{(p-q)(V_1-X_0)}. \tag{24}$$

In the proof of Proposition 2, we show that both the second exit constraint (22) and the ex-ante (IR) constraint (18) are satisfied if early- and long-vesting options satisfy (23)-(24). The "no-exit" constraint (20) requires that the manager with the high-match signal σ_H prefers to stay with the firm instead of leaving and exercising early-vesting options; this provides the upper bound on early-vesting options β_1:

$$\beta_1 \leq \overline{\beta}_1 = \frac{q\,c}{(p-q)(V_1-X_0)}. \tag{25}$$

This upper bound on β_1 is greater than the lower bound in (24):

$$\underline{\beta}_1 = \frac{\Pr(\theta_H|\sigma_L)}{\Pr(\theta_H|\sigma_H)} \frac{q\,c}{(p-q)(V_1-X_0)} = \frac{\Pr(\theta_H|\sigma_L)}{\Pr(\theta_H|\sigma_H)}\overline{\beta}_1 < \overline{\beta}_1;$$

this guarantees the existence of infinitely many β_1's satisfying (24)-(25). As discussed above, maximizing the objective in (17) requires that β_1 be set as low as possible, subject to the program constraints; this means that the optimal β_1 satisfies (24) with equality. We formalize the discussion above and compute the maximum expected firm value in Proposition 2.

Proposition 2.

When the manager privately observes the signal σ, and the firm offers early- and long-vesting options:

1. *The optimal level of early-vesting options β_1^E is:*

$$\beta_1^E = \frac{\Pr(\theta_H|\sigma_L)}{\Pr(\theta_H|\sigma_H)} \frac{q\,c}{(p-q)(V_1 - X_0)}. \tag{26}$$

2. *The optimal level of long-vesting options β_2^E is:*

$$\beta_2^E = \frac{c}{\Pr(\theta_H|\sigma_H)(p-q)(V_{2H} - X_0)}. \tag{27}$$

3. *The manager with the high-match signal σ_H does not exercise early-vesting options and remains with the firm by exerting the high effort. The manager with the low-match signal σ_L exercises early-vesting options and leaves the firm.*

4. *The firm's expected value is*

$$\begin{aligned}\Pi^E = {} & \mu\left(\lambda\left(pV_{2H} + (1-p)V_{2L}\right) + (1-\lambda)V_N\right) + (1-\mu)\left(\lambda V_N + (1-\lambda)V_{2L}\right) \\ & - \frac{c(1-\mu+\lambda(2\mu-1))(\lambda p + (1-\lambda)q)}{\lambda(p-q)}.\end{aligned} \tag{28}$$

Proposition 2 describes the optimal option levels when the firm uses early-vesting options $\beta_1^E > 0$ to screen – to convince the manager with the low-fit signal σ_L to exercise these options and leave the firm. As described in the "benchmark" no-private-information section 3.1, the firm may instead forego screening: even with information symmetry between the firm and the manager, the cost of screening – the risk that the low-fit signal is a type-I error – may exceed the benefit. Here, with the manager privately observing the signal σ, the cost of screening is higher than the one in section 3.1: screening requires the firm to pay the manager informational rent. We next compare the firm's values in two scenarios: (i) with screening using early- and long-vesting options from Proposition 2, and (ii) without screening and using long-vesting options only from Proposition 1. As in the "benchmark" section 3.1, the comparison of the firm's values with screening Π^E and without screening Π^{NE} depends on the values of various parameters: terminal values V_N, V_{2L}, and V_{2H}; the manager's cost of high effort c; the probabilities μ, q, and p; and the information content of the period-t_1 signal λ. We show below in Proposition 3 that the difference in values $\left(\Pi^E - \Pi^{NE}\right)$ strictly decreases in the ex-ante probability μ that the manager is a good match.

Proposition 3.

1. *There exists a threshold value $\mu^\star > 0$ such that $\Pi^E > \Pi^{NE}$ if and only if $\mu < \mu^\star$, where*

$$\mu^\star = \frac{\lambda\left(V_N - V_{2L}\right)}{(1-\lambda)q\left(V_{2H} - V_{2L}\right) + (2\lambda - 1)\left(V_N - V_{2L}\right)}.$$

2. *The difference in firm values with and without early vesting $\left(\Pi^E - \Pi^{NE}\right)$ strictly decreases in μ.*

Proposition 3 shows that early-vesting options increase the firm's profit when the probability of the manager being a good match μ is small – when screening for managerial match problem is more important than avoiding a type-I error. There are two possibilities that we illustrate with numerical examples and graphs below.

1. The "typical" scenario.

We illustrate this in Figure 1: the firm's expected value with screening Π^E is higher than the value without screening Π^{NE} if and only if μ is below a threshold $\mu^\star \in (0, 1)$. The firm uses long-vesting options only if $\mu > \mu^\star$; the firm uses both early- and long-vesting options if $\mu \leq \mu^\star$.

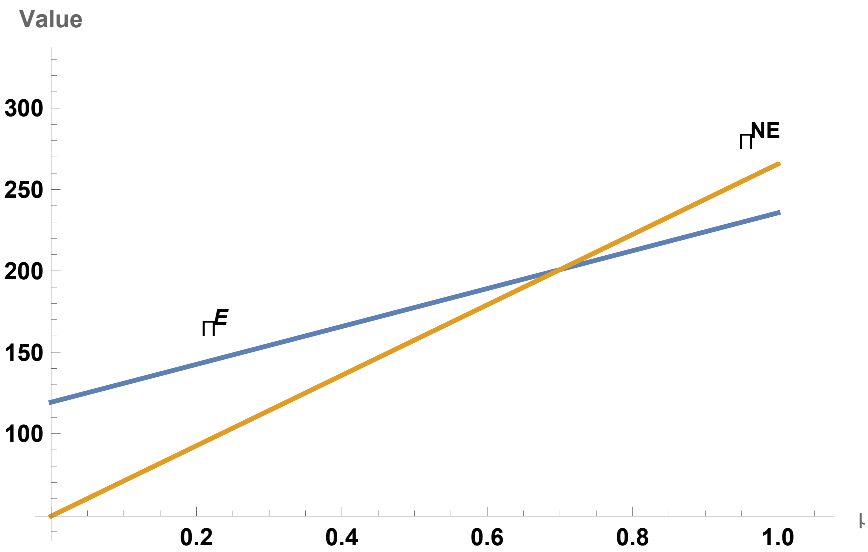

Figure 1: $\Pi^E$and Π^{NE} for $\mu \in (0,1)$ with $\lambda = 0.7; p = 0.8; q = 0.5; c = 50; V_{2H} = 500; V_{2L} = 100; V_N = 200$.

2. Very high benefit of screening

When the benefit of screening is very high with high $\lambda, (p-q)$ and $(V_N - V_{2L})$, the firm's expected value with early- and long-vesting options exceeds the value with only long-vesting options for all values of $\mu \in (0,1)$, and it is always optimal for the firm to use both early- and long-vesting options. We illustrate this in Figure 2.

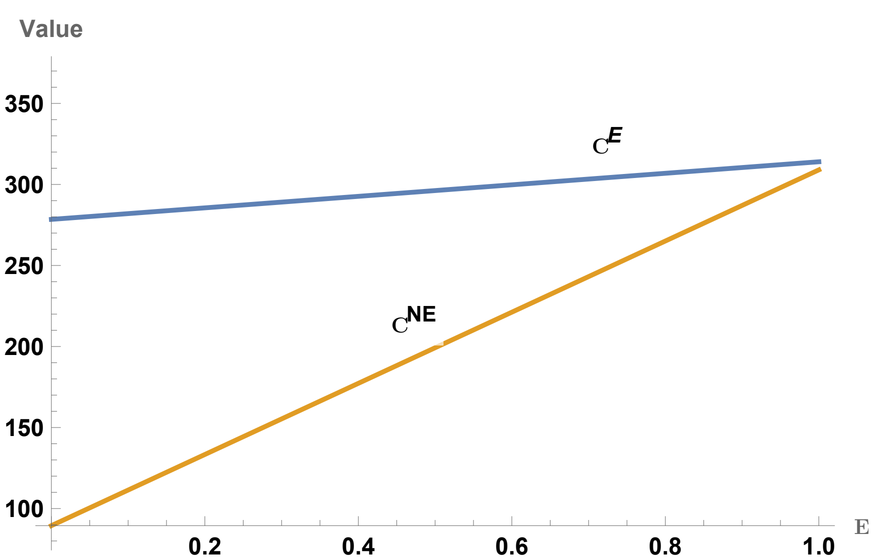

Figure 2. $\Pi^E$and Π^{NE} for various values of $\mu \in (0,1)$, with $\lambda = 0.9; p = 0.8; q = 0.4; c = 50; V_{2H} = 500; V_{2L} = 100; V_N = 310$.

The firm's maximum value is $\Pi^{\star} = \max\left\{\Pi^E, \Pi^{NE}\right\}$. For parameter values in Figure 2,

$\Pi^{\star} = \Pi^{E}$. Figure 3 shows this maximum value for the parameter values from Figure 1.

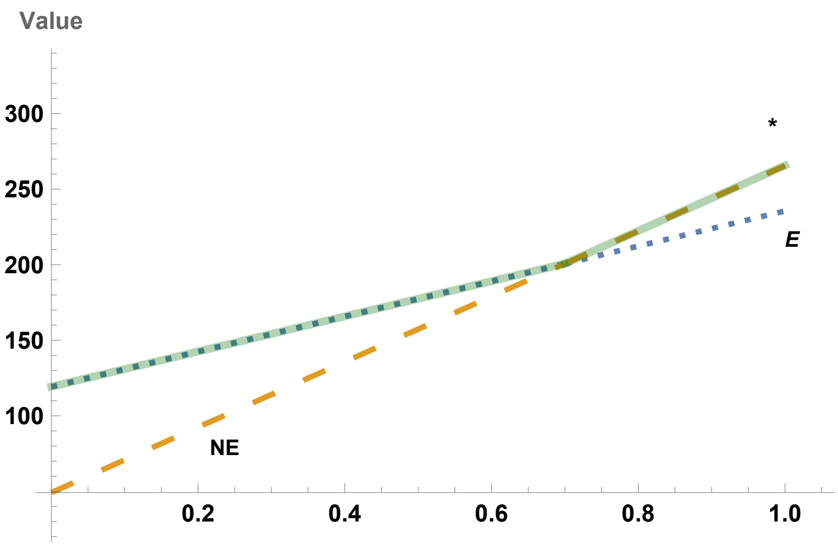

Figure 3. $\Pi^{\star} = \max\left\{\Pi^{E}, \Pi^{NE}\right\}$ for various values of $\mu \in (0,1)$, with $\lambda = 0.7; p = 0.8; q = 0.5; c = 50; V_{2H} = 500; V_{2L} = 100; V_{N} = 200$.

Figures 1 and 2 suggest two facts. First, when the probability that the manager is a good match μ is low, the firm strictly prefers using early-vesting options in addition to long-vesting ones. Second, the difference in firm values with and without early vesting $\left(\Pi^{E} - \Pi^{NE}\right)$ strictly decreases in μ (these observations were formalized in Proposition 3). Corollary 1 summarizes some comparative statics of the threshold μ^{*} with respect to key components of the model.

Corollary 1. *1.* μ^{*} decreases in q.

2. μ^{*} increases in λ.

3. μ^{*} increases in $(V_N - V_{2L})$.

Corollary 1 documents that the benefit of using early vesting increases as the information precision λ and the incremental value from replacing a poor-fit manager $(V_N - V_{2L})$ increase. On the other hand, as q increases, the threshold μ^{*} decreases and the benefit of early-vesting options decreases – it becomes more costly to convince a manager with a low match signal to leave the firm, as the payoff to the manager of waiting until the end of period t_2 and selecting the low effort level increase in q.

4 Extensions

4.1 Manager's outside employment opportunities

In the main setting, a manager's outside employment option (i.e., reservation utility) is normalized to zero for simplicity. This subsection incorporates the manager's reservation utility from outside employment $\bar{U} > 0$; we examine how this affects the manager's incentive to leave, and the firm's design of the option-based compensation. The reservation utility lowers the firm's cost of motivating a manager to leave voluntarily. We investigate whether early-vesting options are still useful.

The effect on the contract-design program is in the (IR) constraint (29): the left-hand side of the constraint must now exceed $\bar{U} > 0$:

$$\begin{aligned}
&\underset{\beta_1,\beta_2 \geq 0}{Max}\ \mu \left[\lambda \left(p \left(V_{2H} - \beta_2 \left(V_{2H} - X_0\right)\right) + (1-p)V_{2L}\right) + (1-\lambda)\left(V_N - \beta_1(V_1 - X_0)\right)\right] \\
&\quad + (1-\mu)\left[\lambda \left(V_N - \beta_1(V_1 - X_0)\right) + (1-\lambda)V_{2L}\right] \\
&\text{subject to:} \\
&\mu \left[\lambda \left(p\beta_2(V_{2H} - X_0) - c\right) + (1-\lambda)\beta_1 \left(V_1 - X_0\right)\right] \\
&\quad + (1-\mu)\left[\lambda\beta_1(V_1 - X_0) - (1-\lambda)c\right] \geq \bar{U} && (29) \\
&\beta_2 \Pr(\theta_H |\ \sigma_H)p(V_{2H} - X_0) - c \geq \beta_2 \Pr(\theta_H |\ \sigma_H)q(V_{2H} - X_0) && (30) \\
&\beta_2 \Pr(\theta_H |\ \sigma_H)p(V_{2H} - X_0) - c \geq \beta_1(V_1 - X_0) + \bar{U} && (31) \\
&\beta_1(V_1 - X_0) + \bar{U} \geq \beta_2(1 - \Pr(\theta_L |\ \sigma_L))q(V_{2H} - X_0) && (32)
\end{aligned}$$

As before, the (IC) constraint (30) will be satisfied at the minimum cost to the firm when

$$\beta_2 = \frac{c}{\Pr(\theta_H|\sigma_H)(p-q)(V_{2H} - X_0)}. \tag{33}$$

The (Exit-Low) constraint (32) is affected by the outside employment opportunity – the lower bound on early-vesting options is:[2]

$$\beta_1 \geq \underline{\beta}_1 = \max\left\{0, \frac{\beta_2 \Pr(\theta_H|\sigma_L)\, q(V_{2H} - X_0) - \bar{U}}{(V_1 - X_0)}\right\}.$$

From the (No Exit-High) constraint (31), to convince a manager with the good-match signal to stay, the upper bound on the early vesting options is

$$\beta_1 \leq \overline{\beta}_1 = \frac{\beta_2 \Pr(\theta_H|\sigma_H)\, p(V_{2H} - X_0) - c - \bar{U}}{(V_1 - X_0)}.$$

Substituting β_2 from (33) into the lower bound:

$$\begin{aligned}
\beta_1 \geq \underline{\beta}_1 &= \max\left\{0, \frac{\beta_2 \Pr(\theta_H|\sigma_L)\, q(V_{2H} - X_0) - \bar{U}}{(V_1 - X_0)}\right\} \\
&= \max\left\{0, \frac{c \Pr(\theta_H|\sigma_L)\, q}{\Pr(\theta_H|\sigma_H)\,(V_1 - X_0)(p - q)} - \frac{\bar{U}}{V_1 - X_0}\right\}.
\end{aligned}$$

The upper bound $\overline{\beta}_1$ exceeds the lower bound $\underline{\beta}_1$:

$$\begin{aligned}
\overline{\beta}_1 - \underline{\beta}_1 &= \frac{\beta_2 \Pr(\theta_H|\sigma_H) p(V_{2H} - X_0) - c - \bar{U}}{(V_1 - X_0)} - \frac{\beta_2 \Pr(\theta_H|\sigma_L)\, q(V_{2H} - X_0) - \bar{U}}{(V_1 - X_0)} \\
&= \frac{cq(\Pr(\theta_H|\sigma_H) + \Pr(\theta_L|\ \sigma_L) - 1)}{\Pr(\theta_H|\sigma_H)(p - q)(V_1 - X_0)} > 0,
\end{aligned}$$

Therefore, $\beta_1 = \max\left\{0, \frac{c \Pr(\theta_H|\sigma_L) q}{\Pr(\theta_H|\sigma_H)(V_1 - X_0)(p-q)} - \frac{\bar{U}}{V_1 - X_0}\right\}$ and $\beta_2 = \frac{c}{\Pr(\theta_H|\sigma_H)(p-q)(V_{2H} - X_0)}$.

These results are summarized in Proposition 4.

Proposition 4. *When the manager has outside employment opportunities equivalent to $\bar{U} > 0$ in utility terms:*

1. The firm grants the manager $\beta_1 = \max\left\{0, \frac{c \Pr(\theta_H|\sigma_L) q}{\Pr(\theta_H|\sigma_H)(V_1 - X_0)(p-q)} - \frac{\bar{U}}{V_1 - X_0}\right\}$ early-vesting options and $\beta_2 = \frac{c}{\Pr(\theta_H|\sigma_H)(p-q)(V_{2H} - X_0)}$ long-vesting options.

[2] Here, because of the reservation utility $\overline{U} > 0$, the lower bound on early-vesting options $\underline{\beta}_1$ may be strictly negative; in that case, the $\beta_1 \geq 0$ requirement will bind.

2. The firm's expected value with early-vesting stock options is

$$\Pi^E = \mu\left(\lambda\left(pV_{2H} + (1-p)V_{2L}\right) + (1-\lambda)V_N\right) + (1-\mu)(\lambda V_N + (1-\lambda)V_{2L})$$
$$- \frac{c(1-\mu+\lambda(2\mu-1))(\lambda p + (1-\lambda)q)}{\lambda(p-q)} - (\lambda - \mu(2\lambda-1))\bar{U}.$$

From the first part of Proposition 4, the level of early-vesting options β_1 decreases as the outside option $\bar{U}$ increases. If the reservation utility is not too high (formalized in corollary 2 below), early-vesting options are still used to convince the manager with the low-match signal σ_L to leave the firm. When the manager's reservation utility is very high, early-vesting options are not needed – the manager with the low-match signal σ_L leaves the firm to enjoy the high reservation utility. These observations are formalized in the corollary below.

Corollary 2. *1. The level of early-vesting options decreases in the manager's reservation utility:* $\frac{\partial \beta_1}{\partial \bar{U}} < 0$.

2. The firm uses early-vesting options $\beta_1 > 0$ *if and only if* $\bar{U} < \dfrac{c\,\Pr\left(\theta_H|\sigma_L\right)q}{\Pr(\theta_H|\sigma_H)(p-q)}$. *Otherwise,* $\beta_1 = 0$.

4.2 Firm specific investment

This section considers the possibility that a manager can make personally costly firm-specific investment that enhances the firm's expected value. The firm-specific investment affects a manager's productivity only in the firm, leading to the well-known hold-up problem (Becker 1962; Brisley and Douglas 2015).

The manager makes the firm-specific investment after beginning to work for the firm but before receiving the match signal σ. The investment is valuable only if the manager stays with the firm until the end of the second period. With early-vesting options, the manager may leave after he receives the low-match signal σ_L; this weakens the manager's incentive to make the firm-specific investment in the first place. However, firm-specific investment increases the manager's value of staying in the firm: the investment increases the manager's productivity, which in turn increases the probability of the high terminal value V_{2H}.

To examine the net effect of early-vesting options on the manager's firm-specific investment, let h represent the increase in the manager's productivity based on the investment: it raises the chance for the high firm value V_{2H} from p to $(p+h)$ with high effort and from q to $(q+h)$ with low effort. The manager's personal cost of investment is $kh^2/2$; assume that k is large enough to guarantee $p+h^* < 1$. With any $\beta_1 \geq 0$ and $\beta_2 \geq 0$, the manager chooses h that maximizes his payoff:

$$\underset{h}{Max}\Bigg[\mu\left[\lambda\left(\beta_2\left(p+h\right)\left(V_{2H}-X_0\right)-c\right)+(1-\lambda)\beta_1\left(V_1-X_0\right)\right] \\ +(1-\mu)\left[\lambda\beta_1\left(V_1-X_0\right)-(1-\lambda)c\right]-kh^2/2\Bigg] \tag{34}$$

The first-order condition with respect to h is $\mu\lambda\beta_2\left(V_{2H}-X_0\right)-k\,h=0$. The second derivative of the manager's expected utility with respect to h is $-k<0$, so the manager's expected utility is strictly concave in h. Solving the first-order condition gives the manager's optimal investment choice as a function of the firm's previously selected β_1 and β_2:

$$h^\star\left(\beta_1,\beta_2\right)=\frac{\mu\lambda\beta_2\left(V_{2H}-X_0\right)}{k}. \tag{35}$$

Equation (35) holds for any β_1 and β_2. The firm now computes the optimal β_1 and β_2 anticipating the manager's choice in (35):

$$\underset{\beta_1,\beta_2\geq 0}{Max}\ \mu\Big[\lambda\left(\left(p+h^{\star}\left(\beta_1,\beta_2\right)\right)\left(V_{2H}-\beta_2\left(V_{2H}-X_0\right)\right)+\left(1-p-h^{\star}\left(\beta_1,\beta_2\right)\right)V_{2L}\right)$$
$$+\left(1-\lambda\right)\left(V_N-\beta_1(V_1-X_0)\right)\Big]+\left(1-\mu\right)\left[\lambda\left(V_N-\beta_1\left(V_1-X_0\right)\right)+(1-\lambda)V_{2L}\right]$$

subject to:

$$\mu\left[\lambda\left(\left(p+h^{\star}\left(\beta_1,\beta_2\right)\right)\beta_2(V_{2H}-X_0)-c\right)+(1-\lambda)\beta_1\left(V_1-X_0\right)\right]$$
$$+(1-\mu)\lambda\left[\beta_1(V_1-X_0)-(1-\lambda)c\right]-k\frac{h^{\star}\left(\beta_1,\beta_2\right)^2}{2}\geq 0$$

$$\Pr\left(\theta_H|\sigma_H\right)\left(p+h^{\star}\left(\beta_1,\beta_2\right)\right)\beta_2\left(V_{2H}-X_0\right)-c$$
$$\geq \Pr\left(\theta_H|\sigma_H\right)\left(q+h^{\star}\left(\beta_1,\beta_2\right)\right)\beta_2\left(V_{2H}-X_0\right) \tag{36}$$

$$\Pr\left(\theta_H|\sigma_H\right)\left(p+h^{\star}\left(\beta_1,\beta_2\right)\right)\beta_2\left(V_{2H}-X_0\right)-c\geq\beta_1\left(V_1-X_0\right) \tag{37}$$

$$\beta_1\left(V_1-X_0\right)\geq\Pr\left(\theta_H|\sigma_L\right)\left(q+h^{\star}\left(\beta_1,\beta_2\right)\right)\beta_2\left(V_{2H}-X_0\right) \tag{38}$$

$$\beta_1(V_1-X_0)\geq\Pr\left(\theta_H|\sigma_L\right)\left(p+h^{\star}\left(\beta_1,\beta_2\right)\right)\beta_2\left(V_{2H}-X_0\right)-c$$

As before, we get the necessary conditions on β_2 from the (IC) constraint (36). Because the investment effect additively increases the probability of the high outcome, the manager's investment choice has no effect on this constraint. Thus, the lowest β_2 that satisfies this constraint is the same as in the solution to Program 3:

$$\beta_2^E=\frac{c}{\Pr(\theta_H|\sigma_H)(p-q)(V_{2H}-X_0)}. \tag{39}$$

Early-vesting options β_1 satisfy (38) and (37):

$$\beta_1^E=\frac{\Pr\left(\theta_H\mid\sigma_L\right)}{\Pr\left(\theta_H|\sigma_H\right)}\frac{c\left(q+h^{\star}\left(\beta_1^E,\beta_2^E\right)\right)}{(p-q)(V_1-X_0)}. \tag{40}$$

Substituting the long-vesting options β_2^E from (39) into (35), the manager's firm-specific investment level satisfies:

$$h^{\star}\left(\beta_1^E,\beta_2^E\right)=\frac{c(1-\lambda+(2\lambda-1)\mu)}{k(p-q)}. \tag{41}$$

Without early vesting options, the manager determines the optimal level of firm-specific investment as follows:

$$\underset{h}{Max}\,\mu(\lambda\beta_2(p+h)(V_{2H}-X_0)-c)+(1-\lambda)\beta_2(q+h)(V_{2H}-X_0))-(1-\mu)(1-\lambda)c-kh^2/2, \quad (42)$$

and the investment level is

$$h^{\star}(\beta_1,\beta_2)=\frac{\mu\beta_2(V_{2H}-X_0)}{k}. \quad (43)$$

Substituting $\beta_2^{NE}=\frac{c}{\Pr(\theta_H|\sigma_H)(p-q)(V_{2H}-X_0)}$,

$$h^{\star}\left(\beta_1^{NE},\beta_2^{NE}\right)=\frac{c(1-\lambda+(2\lambda-1)\mu)}{k\lambda(p-q)}, \quad (44)$$

which shows that $h^{\star}\left(\beta_1^{NE},\beta_2^{NE}\right)$ is greater than $h^{\star}\left(\beta_1^{E},\beta_2^{E}\right)$. This result is summarized in Proposition 5.

Proposition 5. *1. The use of early-vesting options lowers the level of firm specific investment:*

$$h^{\star}\left(\beta_1^{NE},\beta_2^{NE}\right)=\frac{c(1-\lambda+(2\lambda-1)\mu)}{k\lambda(p-q)}>h^{\star}\left(\beta_1^{E},\beta_2^{E}\right)=\frac{c(1-\lambda+(2\lambda-1)\mu)}{k(p-q)}.$$

2. The number of early-vesting options increases in firm-specific investment: $\frac{\partial\beta_1}{\partial h}>0$.

As confirmed in Proposition 5, firm-specific investment level is lower when early-vesting options are used. The greater chance of being replaced with early-vesting options discourages a manager from making firm-specific investment up front because the benefit of the investment is realized only if the manager remains with the firm through t_2. Part 2 of the proposition documents the fact that the higher the manager's firm-specific investment h, the greater the level of early-vesting options required to convince the low-match signal manager to voluntarily leave the firm. In summary, early-vesting options are valuable in presence of the firm-specific investment, although the increased voluntary turnover with early vesting lowers the level of the manager's firm-specific investment.

5 Conclusion

This paper examines the roles of vesting periods in executive option grants. We show that early-vesting options and long-vesting options play different roles. Compensating newly hired managers with options that vest quickly can allow a firm to efficiently screen for manager-firm match. A manager who learns that he is likely a poor match with the firm can exercise early-vesting options and depart: early vesting schedules motivate voluntary attrition of unlikely longer-term matches. Conversely, managers who learn that they are likely well-suited to the firm remain and are motivated to work via long-vesting options. Of course, screening benefits of early vesting must be balanced against countervailing effects, particularly the risk of replacing a manager whose "low-fit" signal is a type-I error. We find that the benefits of early-vesting options are valuable as a screening tool when the ex-ante probability of attracting a well-matched manager is relatively low.

We show that our results continue to hold when the manager has outside employment opportunities and when the manager can acquire firm-specific capital at a personal cost. Overall, our research highlights the importance of often-overlooked vesting timing as a component of compensation design to attract, screen, and motivate executive talent. It provides a framework for firms to structure vesting schedules to manage retention of managers with good manager-firm matches.

For future work, one interesting direction is to examine the ability of firms to control the precision of the signal about the manager's match. While our model treated this precision as exogenous, firms may be able to implement information systems, rotational job assignments, or other practices aimed at accelerating mutual learning about managerial fit. Faster information revelation may facilitate more efficient screening, to the benefit of both firms and managers.

6 Appendix

Proof of Lemma 1.

Rearranging the (IC) constraint (6) provides the minimum β_2 necessary and sufficient for (IC):

$$\beta_2 \geq \frac{c}{\Pr\left(\theta_H|\sigma_H\right)\left(p-q\right)\left(V_{2H}-X_0\right)}. \tag{45}$$

Substituting (45) into the left-hand-side of the interim (IR) constraint (5)

$$\begin{aligned}
&\Pr(\theta_H|\sigma_H)p\beta_2(V_{2H}-X_0)-c \\
&\geq \Pr(\theta_H|\sigma_H)p\frac{c}{\Pr\left(\theta_H|\sigma_H\right)\left(p-q\right)\left(V_{2H}-X_0\right)}(V_{2H}-X_0)-c \\
&= c\left(\frac{q}{p-q}\right) > 0.
\end{aligned}$$

Thus, the interim (IR) constraint (5) does not bind as long as β_2 satisfies (45). Substituting (45) and $\Pr(\theta_H|\sigma_H)$ from (1) into the left-hand side of the ex-ante (IR) constraint (4) yields:

$$\begin{aligned}
&\mu\left[\lambda\left(p\beta_2(V_{2H}-X_0)-c\right)\right]-(1-\mu)(1-\lambda)c \\
&\geq \mu\lambda p\frac{c}{\Pr\left(\theta_H|\sigma_H\right)\left(p-q\right)}-\mu\lambda c-(1-\mu)(1-\lambda)c \\
&= c\left[\frac{\mu\lambda p}{\Pr\left(\theta_H|\sigma_H\right)\left(p-q\right)}-\left(\mu\lambda+(1-\mu)(1-\lambda)\right)\right] \\
&= c\left[\frac{p\left(\mu\lambda+(1-\mu)(1-\lambda)\right)}{(p-q)}-\frac{(p-q)\left(\mu\lambda+(1-\mu)(1-\lambda)\right)}{(p-q)}\right] \\
&= c\frac{q\left(\mu\lambda+(1-\mu)(1-\lambda)\right)}{(p-q)} > 0.
\end{aligned} \tag{46}$$

The left-hand side of the ex-ante (IR) constraint is thus strictly positive as long as β_2 satisfies (45).

Substituting (45) into the firm's objective and rearranging provides the firm's benchmark profit in (8).

Proof of Lemma 2.

Rearranging the (IC) constraint (12) provides the minimum β_2 necessary and sufficient for

(IC):

$$\beta_2 \geq \frac{c}{\Pr\left(\theta_H|\sigma_H\right)\left(p-q\right)\left(V_{2H}-X_0\right)}. \tag{47}$$

Substituting (47) into the left-hand-side of the interim (IR) constraint (11):

$$\begin{aligned}
&\Pr(\theta_H|\sigma_H)p\beta_2(V_{2H}-X_0)-c \\
&\geq \Pr(\theta_H|\sigma_H)p\frac{c}{\Pr\left(\theta_H|\sigma_H\right)\left(p-q\right)\left(V_{2H}-X_0\right)}(V_{2H}-X_0)-c \\
&= c\left(\frac{q}{p-q}\right) > 0.
\end{aligned}$$

Thus, the interim (IR) constraint (11) does not bind as long as β_2 satisfies (47). Substituting (47) and $\Pr(\theta_H|\sigma_H)$ from (1) into the left-hand side of the ex-ante (IR) constraint (10):

$$\begin{aligned}
&\mu\left[\lambda\left(p\beta_2(V_{2H}-X_0)-c\right)+(1-\lambda)q\beta_2(V_{2H}-X_0)\right]-(1-\mu)(1-\lambda)c \\
&\geq \mu\left[\lambda\left(\frac{p\,c}{\Pr\left(\theta_H|\sigma_H\right)\left(p-q\right)}-c\right)+(1-\lambda)\frac{q\,c}{\Pr\left(\theta_H|\sigma_H\right)\left(p-q\right)}\right]-(1-\mu)(1-\lambda)c \\
&> \mu\left[\lambda\left(\frac{p\,c}{\Pr\left(\theta_H|\sigma_H\right)\left(p-q\right)}-c\right)\right]-(1-\mu)(1-\lambda)c && (48)\\
&= \mu\lambda p\frac{c}{\Pr\left(\theta_H|\sigma_H\right)\left(p-q\right)}-\mu\lambda c-(1-\mu)(1-\lambda)c > 0, && (49)
\end{aligned}$$

with $(1-\lambda)\frac{q\,c}{\Pr(\theta_H|\sigma_H)(p-q)} > 0$ implying the inequality (48), and inequality (49) already confirmed in the proof of Lemma 1 above (see (46)). The left-hand side of the ex-ante (IR) constraint is thus strictly positive as long as β_2 satisfies (47). Substituting (47) into the firm's objective and rearranging provides the firm's benchmark profit in (14).

Proof of Lemma 3.

From (8) and (14) and $\beta_2^R = \beta_2^{NR} = \frac{c}{\Pr(\theta_H|\sigma_H)(p-q)(V_{2H}-X_0)}$ in (8) and (13):

$$\begin{aligned}
\Pi^R - \Pi^{NR} \;&= \mu(1-\lambda)\Big(V_N - q\left(V_{2H}-\tfrac{c}{\Pr(\theta_H|\sigma_H)(p-q)}\right)-(1-q)V_{2L}\Big) \\
&\quad +(1-\mu)\lambda\left(V_N-V_{2L}\right) \\
&= (\lambda-1)\mu q\left(V_{2H}-V_{2L}\right)+\left(V_N-V_{2L}\right)\left(\lambda+\mu-2\lambda\mu\right) \\
&\quad -\frac{c\,q\left(\lambda-1\right)\left(1-\lambda-\mu+2\lambda\mu\right)}{\left(p-q\right)\lambda}
\end{aligned} \tag{50}$$

Partially differentiating $\left(\Pi^R - \Pi^{NR}\right)$ with respect to μ:

$$\begin{aligned}\frac{\partial\left(\Pi^R - \Pi^{NR}\right)}{\partial \mu} &= (\lambda-1)q\left(V_{2H}-V_{2L}\right) + \left(V_N - V_{2L}\right)(1-2\lambda) \\ &+ \frac{c\,q(\lambda-1)(2\lambda-1)}{(p-q)\,\lambda}\end{aligned} \tag{51}$$

From (51), it is clear that the second derivative of $\left(\Pi^R - \Pi^{NR}\right)$ with respect to μ is zero; thus, $\left(\Pi^R - \Pi^{NR}\right)$ is linear in μ. We next show that the partial derivative in (51) is negative. The derivative consists of three additive terms:

1. The first term $(\lambda-1)q\left(V_{2H}-V_{2L}\right)$ is negative because $\lambda < 1$, $q > 0$, and $V_{2H} > V_{2L}$.
2. The second term $\left(V_N - V_{2L}\right)(1-2\lambda)$ is negative because $\lambda > \frac{1}{2}$ and $V_N > V_{2L}$.
3. The third term is negative because $\lambda \in (1/2, 1)$ and $p > q$.

Next we show that $\left(\Pi^R - \Pi^{NR}\right) > 0$ when $\mu = 0$. Substituting $\mu = 0$ into the right-hand-side of (50):

$$\Pi^R - \Pi^{NR}\Big|_{\mu=0} = \frac{cq\left(\lambda-1\right)^2}{(p-q)\lambda} + \lambda\left(V_N - V_{2L}\right) > 0.$$

This strict inequality, along with the fact that $\left(\Pi^R - \Pi^{NR}\right)$ is linear in, and decreasing in, μ, guarantees that there exists a threshold $\mu^B > 0$ such that $\Pi^R > \Pi^{NR}$ if and only if $\mu < \mu^B$.

Proof of Proposition 1.

With long-vesting options only, $\beta_1 = 0$, and the firm is not able to screen on the realization of the manager's signal σ. The manager remains with the firm regardless of the realization of σ; long-vesting options β_2 are used to provide high-effort incentives only to the manager with the high-signal σ_H. The firm's value-maximization program is then identical to Program 2 analyzed in Lemma 2. The results thus follow immediately from Lemma 2.

Proof of Proposition 2

Rearranging the (IC) constraint (19) provides the minimum β_2 necessary and sufficient for (IC):

$$\beta_2 \geq \frac{c}{\Pr\left(\theta_H|\sigma_H\right)(p-q)\left(V_{2H}-X_0\right)}. \tag{52}$$

Substituting (52) into the exit constraint (21) provides the necessary and sufficient condition for the manager with the low-fit signal σ_L to exercise early-vesting options and leave the firm:

$$\beta_1 \geq \frac{\Pr(\theta_H|\sigma_L)\, q\, (V_{2H} - X_0)}{V_1 - X_0} \beta_2. \tag{53}$$

The objective (17) is strictly decreasing in β_1 and β_2. Thus, β_1 and β_2 must be as low as possible subject to the constraints (18)-(21); we set them as low as allowed by (52) and (53) and check that the remaining constraints (22), (20), and (18) are satisfied:

$$\beta_1^E = \frac{\Pr(\theta_H|\sigma_L)}{\Pr(\theta_H|\sigma_H)} \frac{q\, c}{(p-q)\,(V_1 - X_0)} \tag{54}$$

$$\beta_2^E = \frac{c}{\Pr(\theta_H|\sigma_H)\,(p-q)(V_{2H} - X_0)}. \tag{55}$$

Substituting (54) into the left-hand-side of the second exit constraint (22):

$$\beta_1^E (V_1 - X_0) = \frac{\Pr(\theta_H|\sigma_L)}{\Pr(\theta_H|\sigma_H)} \frac{q\, c}{(p-q)}. \tag{56}$$

Substituting (55) into the right-hand-side of the second exit constraint (22):

$$\Pr(\theta_H|\sigma_L)\, p \beta_2^E\, (V_{2H} - X_0) - c = \frac{\Pr(\theta_H|\sigma_L)}{\Pr(\theta_H|\sigma_H)} \frac{p\, c}{(p-q)} - c. \tag{57}$$

Subtracting the right-hand-side (57) from the left-hand-side (56) and rearranging:

$$\begin{aligned} \text{LHS-RHS} &= \frac{\Pr(\theta_H|\sigma_L)}{\Pr(\theta_H|\sigma_H)} \frac{q\, c}{(p-q)} - \frac{\Pr(\theta_H|\sigma_L)}{\Pr(\theta_H|\sigma_H)} \frac{p\, c}{(p-q)} + c \\ &= c\left(1 - \frac{\Pr(\theta_H|\sigma_L)}{\Pr(\theta_H|\sigma_H)}\right) > 0, \end{aligned}$$

because $\frac{\Pr(\theta_H|\sigma_L)}{\Pr(\theta_H|\sigma_H)} < 1$. Thus, the second exit constraint (22) is satisfied when $\beta_1 = \beta_1^E$ and $\beta_2 = \beta_2^E$.

Substituting (55) into the left-hand-side of the no-exit constraint provides a necessary and sufficient condition for this constraint:

$$\beta_1 \leq \overline{\beta_1} = \frac{q\,c}{(p-q)\,(V_1 - X_0)}.$$

With $\frac{\Pr(\theta_H|\sigma_L)}{\Pr(\theta_H|\sigma_H)} < 1$, it is easy to see that $\beta_1^E = \frac{\Pr(\theta_H|\sigma_L)}{\Pr(\theta_H|\sigma_H)} \frac{q\,c}{(p-q)(V_1-X_0)} < \overline{\beta_1}$, and the no-exit constraint (20) holds when $\beta_1 = \beta_1^E$ and $\beta_2 = \beta_2^E$.

Substituting (54) and (55) into the right-hand-side of the ex-ante (IR) constraint (18):

$$\begin{aligned}
&\mu\left[\lambda\left(p\beta_2(V_{2H}-X_0)-c\right)+(1-\lambda)\beta_1\left(V_1-X_0\right)\right] \\
&\quad+(1-\mu)\left[\lambda\beta_1(V_1-X_0)-(1-\lambda)c\right] \\
&=\mu\left[\lambda\left(\frac{p\,c}{\Pr\left(\theta_H|\sigma_H\right)(p-q)}-c\right)+(1-\lambda)\frac{\Pr\left(\theta_H|\sigma_L\right)}{\Pr\left(\theta_H|\sigma_H\right)}\frac{q\,c}{(p-q)}\right] \\
&\quad+(1-\mu)\left[\lambda\frac{\Pr\left(\theta_H|\sigma_L\right)}{\Pr\left(\theta_H|\sigma_H\right)}\frac{q\,c}{(p-q)}-(1-\lambda)c\right] \qquad (58)\\
&>\mu\left[\lambda\left(\frac{p\,c}{\Pr\left(\theta_H|\sigma_H\right)(p-q)}-c\right)\right]-(1-\mu)(1-\lambda)c, \qquad (59)
\end{aligned}$$

where the inequality (59) is obtained by subtracting

$$(\mu(1-\lambda)+(1-\mu)\lambda)\,\frac{\Pr\left(\theta_H|\sigma_L\right)}{\Pr\left(\theta_H|\sigma_H\right)}\frac{q\,c}{(p-q)} > 0$$

from (58).

To complete the proof, note that the left-hand-side of (59) is greater than zero, as already shown in the proof of Lemma 1 in (46). Thus, the ex-ante (IR) constraint (18) holds when $\beta_1 = \beta_1^E$ and $\beta_2 = \beta_2^E$.

Proof of Proposition 3.

From (16) and (28):

$$\Pi^E - \Pi^{NE} \quad = \left(\lambda - \left(2\lambda - 1\right)\mu\right)\left(V_N - V_{2L}\right) - (1-\lambda)\mu q\left(V_{2H} - V_{2L}\right) \qquad (60)$$

Partially differentiating $\left(\Pi^E - \Pi^{NE}\right)$ with respect to μ:

$$\frac{\partial\left(\Pi^E - \Pi^{NE}\right)}{\partial\mu} = (\lambda-1)q\left(V_{2H} - V_{2L}\right) + (2\lambda-1)\left(V_{2L} - V_N\right) \tag{61}$$

From (61), it is clear that the second derivative of $\left(\Pi^E - \Pi^{NE}\right)$ with respect to μ is zero; thus, $\left(\Pi^E - \Pi^{NE}\right)$ is linear in μ. We next show that the partial derivative in (61) is negative. The derivative consists of two additive terms:

1. The first term $(\lambda-1)q\left(V_{2H} - V_{2L}\right)$ is negative because $\lambda < 1$.
2. The second term $(2\lambda-1)\left(V_{2L} - V_N\right)$ is negative because $\lambda > \frac{1}{2}$ and $V_N > V_{2L}$.

Next we show that $\left(\Pi^E - \Pi^{NE}\right) > 0$ when $\mu = 0$. Substituting $\mu = 0$ into the right-hand-side of (60):

$$\Pi^E - \Pi^{NE}\Big|_{\mu=0} = \lambda\left(V_N - V_{2L}\right) > 0.$$

This strict inequality, along with the fact that $\left(\Pi^E - \Pi^{NE}\right)$ is linear and decreasing in μ, guarantees that there exists a threshold $\mu^\star > 0$ such that $\Pi^E > \Pi^{NE}$ if and only if $\mu < \mu^\star$.

The threshold $\mu^\star$ is the μ that solves

$$\Pi^E - \Pi^{NE} = 0,$$

which gives $\mu^\star = \dfrac{\lambda\left(V_N - V_{2L}\right)}{(1-\lambda)q\left(V_{2H} - V_{2L}\right) + (2\lambda-1)\left(V_N - V_{2L}\right)}$.

Proof of Corollary 1.

1. $\frac{\partial \mu^*}{\partial q} = -\frac{(1-\lambda)\lambda(V_{2H}-V_{2L})(V_N-V_{2L})}{[\lambda(V_N-V_{2L})+(1-\lambda)(qV_{2H}+(1-q)V_{2L}-V_N)]^2} < 0.$
2. $\frac{\partial \mu^*}{\partial \lambda} = \frac{(V_N-V_{2L})(qV_{2H}+(1-q)V_{2L}-V_N)}{[\lambda(V_N-V_{2L})+(1-\lambda)(qV_{2H}+(1-q)V_{2L}-V_N)]^2} > 0.$
3. $\frac{\partial \mu^*}{\partial (V_N-V_{2L})} = \frac{\lambda(1-\lambda)q(V_{2H}-V_{2L})}{[\lambda(V_N-V_{2L})+(1-\lambda)(qV_{2H}+(1-q)V_{2L}-V_N)]^2} > 0.$

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