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Constructing a Positive Shock

Growth Through the Lens of Option Pricing

Mehmet Cangul

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WORKING PAPER

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**Constructing a Positive Shock:
Growth Through the Lens of Option Pricing
Prepared by Mehmet Cangul***

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ABSTRACT: Low-income economies face negative shocks whose frequency and disproportionate impact overcome growth trajectories, producing a negative drift. COVID-19 was the latest such episode. To escape this negative drift, and build a durable recovery, there is a need for a counter-balancing force: to construct a positive shock. Growth is realized through decisions that fall under two categories, routine and non-linear. While routine decisions modify existing economic behavior along the same path, non-linear decisions describe riskier options that involve transformation. Option pricing theory can be useful to describe the latter, and construct the positive shock required to escape the negative drift.

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WORKING PAPERS

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Prepared by Mehmet Cangul¹

¹ The author(s) would like to thank Papa N'Diaye, Mokhtar Benlamine, Akito Matsumoto, Marcos Poplawski Ribeiro, Charalambos G. Tsangarides, Jaroslaw Wieczorek, and Jiaxiong Yao for valuable comments. We are also grateful to Heesun Kiem, Vicky Pilouzoue, Cecilia Prado, and Weronika Synak for excellent research and editorial assistance with special gratitude to Li Wan for generously giving his time with the use of Mathematica.

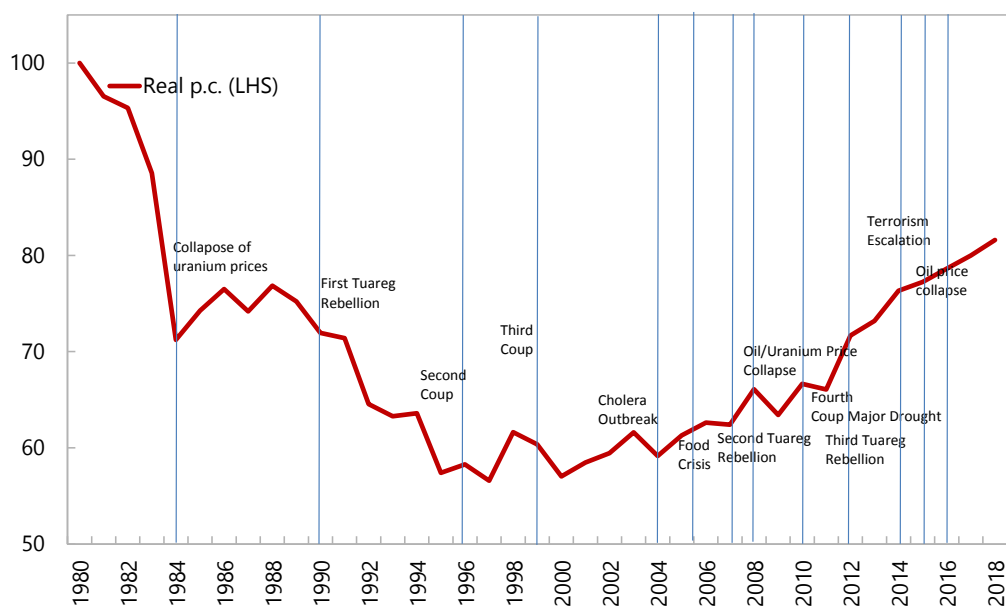
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I. Introduction

LIC economies face an unfavorable speed differential. On the one hand, they are subject to routine negative shocks that they are poorly equipped to deal with, while their average growth rate is insufficient to build sustainable gains over the cycle of these shocks, ranging from rapidly changing climatic conditions to security threats, latest being the COVID-19 crisis. As Dabla-Norris and Balgunduz point out (2012), LICs are subject to more frequent negative shocks than emerging or advanced economies (IMF, 2011). Insufficient development of institutions, policy frameworks, buffers and infrastructures make LICs more vulnerable to these shocks.² We claim, the speed of reforms to precisely address these weaknesses does not match the speed of the negative shocks.³ This speed differential, in turn, forces these economies to play constant catch-up, producing a downward drift over time. Furthermore, to respond to these shocks, countries are often confronted with difficult trade-offs that force them to adopt procyclical fiscal and debt stabilization policies in the short run, which can unavoidably further widen the speed differential, and reinforce the downward drift. (Guerguil et al., 2014; and Melina and Poplawski-Ribeiro, 2021).

Figure 1 Niger: Per Capita GDP, 1980-2018 and Timeline of Negative Shocks
(1980 = 100)



Source: IMF, WEO.

² Acemoglu et al, 2003.

³ Dabla-Norris and Balgunduz (2011) show that these shocks often cause structural breaks in the growth paths of LICs rather than cyclical disruptions.

Figure 1 above is an illustrative example highlighting Niger. Between 1980 until present, Niger faced 19 negative shocks most of which were unique to Niger with an average frequency of 1.5 years between two negative events, narrowing further with time.⁴

The implication for Niger and other fragile LICs is that much like a ship that must maintain a certain speed to escape the drift of the waves in an unpredictable ocean, these economies have to achieve a minimum rate of transformation over shorter spans of time between two negative shocks to achieve durable gains.

As an indication of this drift, in the section below, we will more formally propose a statistical measure that calculates the ratio of positive to negative standard deviations per a given time frame. We hypothesize that this ratio is less than one for most LICs. In the case of Niger for example, the positive to negative standard deviation ratio was significantly less than 1 at 0.5 between 1980-2018, implying negative deviations from the mean well overtook the positive ones. Per capita real GDP has still not caught up to its level in 1980; what this implies is that on average, recoveries following negative shocks to growth are not enough to make up for lost ground, exacerbated by subsequent shocks that grow this difference over time; this is the sense in which there is a negative drift. For example, like Niger, the real per capita GDP of fragile economies in SSA is still not at par with the level in 1979. Similarly, the LIC economies in SSA have taken almost 30 years just to recover to their real per capita GDP between 1980-2008.

We will also propose there exists a minimum economic growth rate necessary to escape this speed differential between two shocks, what we call the “escape speed” or ES. One objective of this paper will be to propose a solution, using option pricing theory, as to what the ES should be for a given economy and what would be its potential cost since it would require some transformation to add an additional growth premium above the average trend for existing potential growth. To achieve ES, policymakers should construct what we call a positive shock that can perpetuate a virtuous cycle with progressive steady gains over the cycle of routine negative shocks.

The most recent negative shock in the cycle, COVID-19, highlights the pertinence of this approach. Options will thrive in a more volatile post-COVID setting. Thus, in an unpredictable world with a higher frequency of large shocks and technological changes, there must be a more careful reflection of strategic risk taking that mirrors an options strategy to not only protect against negative shocks, but also take advantage of positive shocks. Can such an option be engineered?⁵ This paper will attempt to answer this question in the context of solving for ES.

We will propose that just like options on publicly traded stocks or real options in the economy, there is also an implicit option on the overall economy, which represents a risk premium toward transformation. Conceptually, this represents a degree of flexibility in the economy toward transformation for marginal risk. We propose that all economies have an inherent option or a speculative risk premium. Flexible economies with potential for transformation have high speculative risk premiums. Economies that are “too stable” and rigid (as an extreme, the former USSR) have a low speculative risk premium that results in weak potential for transformation.

⁴ As a negative shock, we considered all coups, civil conflicts, wars, epidemics, major food shortages, commodity price collapses, and droughts.

⁵ There is ongoing research by Cohen et al. (IMF, 2020) and others that discuss design of debt instruments with embedded optionality in them, linking payouts to GDP thresholds. Although the question this paper poses is for an options integration into growth strategy generally, the development of such instruments can play a role in incentivizing agents, somewhat analogous to employee stock options.

We will argue that in a typical LIC economy, due to various reasons, including weak financial intermediation and resource misallocation, agents with the most resources to add risk premium toward transformation are complacent with the average growth path. Those willing to take marginal risk on the other hand lack the resources and an enabling environment to compensate this marginal risk.

This is the context in which we will introduce a binary decision framework. Agents face two fundamental choices at any given time in the allocation of their resources, including time, net of consumption and leisure: either they continue to vary parameters on the same economic activity, or they undertake significant risk toward transformation. The first decision type D1 can be a farmer deciding to produce a different amount of the same product based on rain patterns whereas the second decision type D2 can be the same farmer deciding to take training on micro irrigation for a more efficient production or investing in equipment to transform her product for higher value addition. We will use option pricing to model this second decision type. Sufficient number of agents undertaking the second decision type in relatively close time periods will produce a critical mass with a shock effect at the aggregate level toward transformation over a shorter span of time. We propose that such a “synchronization” can be catalyzed by pursuing a positive shock strategy.

We argue that absent this “synchronization” of the second decision type D2, it is, on average taken randomly and rarely with little aggregate impact. This in turn reverts growth and transformation to a trajectory that is insufficient to escape the speed differential. Indeed, we will demonstrate that for a type of transformation described, ES needs to be at a level to compensate the additional risk the agent would take by deciding to invest in these riskier decisions. This makes intuitive sense. Higher growth would give more agents in the economy the buffer they need to take more risk. With more agents purchasing options, the likelihood they will be exercised at a profit, increases, which in turn leads other agents to purchase options to partake in the positive shock, increasing the overall probability of success of the strategy, leading to higher growth, which leads to a higher option premium.

An aggregate estimate for the premium of these riskier options also represents the potential cost of the positive shock. Finally, real options theory will shed some light on how the positive shock could be constructed toward engaging the private sector in specific ways since it is a bottom to top approach to economic development.

We organize the rest of the paper as follows. Section II discusses the motivation for why option pricing is an appropriate framework to answer the relevant questions for the paper. Section III surveys the literature, and sheds light on a similar development approach that was proposed previously, and resuscitated in different occasions, the so called “Big Push Theory”. Section IV introduces a statistical representation to demonstrate the concept of the negative drift more robustly, the so-called ratio of positive to negative standard deviations of growth. Section V discusses the model, including the assumptions, the quantitative results, and their interpretation vis-à-vis the conceptual framework. Section VI discusses the policy implications from these results and sheds light on the design of the positive shock, using principles from the real options theory. Section VII concludes.

II. Motivation for an Option Pricing Approach

In order to both motivate and reach some basic conclusions about the design of the positive shock, we propose an options pricing framework. An option is a derivative on an underlying asset which, in return for a premium, gives its holder the right, but not the obligation to buy or sell (call or put option) the underlying asset at a fixed price (strike) before a given date (expiry). The underlying asset in this case, is broadly the economy, or the set of aggregate assets, but later we will specify it to be the GDP. An option trader makes two bets on the underlying asset. The first bet is on the direction of change, the second is on the speed of that change. As argued above, GDP is a function of continuous economic decisions that are inherently based on these two bets on the GDP itself: the direction of GDP, (whether the economy is expanding or contracting) and how fast that direction is taking shape, the growth rate. This also holds true for economic transformation. There are certain types of decisions that make an inherent bet on the direction and speed of this transformation.

If private agents believe in the credibility of the positive shock, they will want to hold “real” options that they could exercise for large gains if this perceived shock is successful. More agents holding these options relatively close to each other in time, will increase the likelihood of the positive shock succeeding, feeding back into more agents holding more options, producing a self-sustaining virtuous cycle, and ultimately a shock effect.

An example of a real option in this case is acquiring a new skill. The time and resources the agent invests to acquire it comprise the premium on this “option”. In case there are new private sector jobs that emerge (as expected from the outcome of the positive shock), the agent would like to have an option she could exercise to increase her income by being eligible to fill these jobs. The expected gain from this exercise needs to at least equal the premium she will pay.

Beyond this general motivation, there are three reasons why an options pricing mechanism is an appropriate framework to re-think economic growth and transformation.

1. *Fractional price and synergy*

Options give investors an opportunity to invest a fraction of the price of the underlying asset to protect against downward shifts or partake in upward swings of the underlying asset without a fundamental re-orientation of their portfolio. In the real economy, agents will also not shift their resources over-night to new endeavors, but would rather test a perceived direction, while taking limited risk by purchasing real options that would enable them to mimic taking this limited risk to potentially take advantage of a gain from a large directional change. Moreover, by design, the type of options available would be such that even if this risk does not pay off, the “purchase of the option” would still provide a lesson value. For example, even if the agent does not end up obtaining one of these new jobs, the process of acquiring the requisite skills would make her gain transferable skills she could apply elsewhere.

2. *Speculative premium: micro re-orientation feeding macro transformation*

An options pricing framework is also appropriate to describe the decision of the agent in so far as it involves a risk for a re-orientation of the individual economic behavior; this involves taking marginal risk relative to economic behaviors that perpetuate the same state of the world. Options thrive in environments

where there is elevated risk and or rapid change.⁶ An options pricing framework implicitly describes a situation where the agent either protects against risk or takes the risk to partake in the rapid change. As argued above, decisions that result in economic production fall under: (i) D1, routine decisions; and (ii) D2, non-routine, riskier decisions. An options pricing framework is a natural candidate to model these second set of rarer D2s.

The value of an option has two components, inherent value, and speculative premium. An out of the money option has an inherent value of zero (since it is below the strike price), therefore its only value comes from the speculative premium. In this paper, we will assume agents consider only out of the money options on the real economy as the underlying asset. They are betting that the economy will do “sufficiently” better when they purchase options they hope to exercise at a future time. Thus, these options’ value derives from their speculative premium, which reflects the overall speculative premium agents place in the economy. A low speculative premium reflects a low-risk appetite. A high speculative premium reflects the degree of confidence that transformation of the economy translating into sizeable gains for private enterprise is more likely. D2 therefore is more likely to take place if there is a higher speculative premium the agent places on the prospects of the economy.

3. *Volatility*

Finally, if options thrive when underlying assets are expected to make large movements in shorter horizons with significant volatility, then this framework is also appropriate to describe the positive shock as we define it, with a large impact over a short horizon. This type of framework would be especially pertinent now as the world economy is facing higher uncertainty even as it emerges from the pandemic. New challenges and emergence of technologies both in response to these, but also as a function of forces outside policy makers’ control contribute to an inherently more volatile environment where higher risk taking will not only be desirable, but to an extent, necessary to survive.

III. Literature Review

The concept of the negative drift is similar to hysteresis, especially as elaborated in a larger sense by Cerra et al, (2020). The distinction of this paper is the extension of a pull beyond business cycles and negative shocks. These events reach a defining point once combined with a stable equilibrium in the absence of negative shocks. This combination renders the plateau between two negative events not strong enough to resist the pull from the previous negative shock unless there is a counter-veiling positive shock partially because of eroded savings during the negative shock.

This notion is partially related to path dependence. A fundamental constraint in achieving ES is that economic agents have a short-term horizon and cannot internalize the drift produced by the speed differential over a longer period. There is a path dependence that reinforces existing patterns of economic behavior and relations (D1), and risk aversion toward transformation (D2). In general, path dependence is a well explored phenomenon in economics initially popularized by Paul David in 1985, and extended to other social sciences, (P. David, 2007). In the context of this paper, we argue that influential agents who have the most potential to effectuate economic transformation are content with the status-quo and are thus averse to marginal risk. Therefore, despite the long-term drift and the potential to do better, the overall risk

⁶ This is the case for a world on the precipice of the great re-set as it tries to emerge from the COVID-19 pandemic.

appetite for higher gains is low partly due to this horizon mismatch and lack of internalization because of an inherent free-rider problem. The idea of path dependence is defined more formally below following the idea of growth as a function of continuous decisions and their interactions.

This constraint adds a speed element to economic development to escape the negative drift, the so-called escape speed (ES) from the inertia inherent in the path dependence, requiring in turn a concerted push.

In the early 1940s, the concept of the “big push model” was developed by Paul Rosenstein-Rodan. Murphy, Shleifer and Vishny (1989) resuscitated this approach using game theory. They argued that it was unprofitable for a given agent to invest in industrialization unless others also industrialized at the same time, leading to demand for industrialized products, higher income, and more industrialization in turn. They used game theory to make the point that there existed equilibria that could push the system toward industrialization even if it was unprofitable for a given agent to industrialize. In some equilibria, an initial government intervention through several policy options could be the catalyst for this push.

More recently in 2005, Jeffrey Sachs, as argued by William Easterly (2006) also advocated a “Big Push” idea in his book: *The End of Poverty: Economic Possibilities of Our Time* where he makes the case that scaling of financial resources to LICs toward complementary investments, could make a significant difference to end poverty. Easterly (2006) criticizes this approach based on “information” and “incentive” problems faced by a “large planning exercise.” He proposes an alternative of piecemeal interventions.

This paper uses the options framework to contribute yet to a new resuscitation of the “big push” hypothesis using a novel analytical tool to argue for a new approach to economic growth. The argument in the paper goes beyond the traditional determinants of growth to instead discuss the urgency of time and how to beat it with a sustained, concerted push over shorter periods. The options framework makes this case more robustly and modifies the traditional big push theory by introducing a hybrid between planning and randomness. The planning comes in the way of choosing strategies most likely to yield complementarity, and implementing them relatively close in time, for synergy and shock. Randomness is introduced conceptually through optionality where, as elaborated later, sufficient flexibility is retained to modify a given course based on continuous outcomes. Even if there is initial failure, repeated lessons from experimentation yields lesson values, for example, through development of skills, to eventually produce successful outcomes that blossom and develop without deliberate planning.

Using a completely different angle, a real options framework is used by Dihle (2015) who looks at the macro implications of microeconomic irreversibility and idiosyncratic uncertainty in a simple growth model. This study provides an important insight into what real options approach in micro decisions, for example project selection in capital spending, imply for growth. In a related context, the principles of abandon and modification in the options framework that mitigate irreversibility, are discussed in the policy section.

IV. Statistical Ratio to Measure Drift

Volatility of economic growth can be measured by standard deviation:

(1)

$$V(g) = \sqrt{\frac{\sum_{t=1}^n (g_t - g_m)^2}{n-1}}$$

where $V(g)$ is volatility of economic growth and where g_m is mean growth. This definition of volatility lumps positive and negative deviations from the mean. In this paper, we would like to isolate them, and hence propose a ratio of positive to negative deviations where:

$$V(g_+) / V(g_-) = VR$$

$$VR = \frac{\sqrt{\frac{\sum_{t=1}^n (g_{(t+)} - g_m)^2}{n-1}}}{\sqrt{\frac{\sum_{t=1}^n (g_{(t-)} - g_m)^2}{n-1}}}$$

We hypothesize that over long stretches of time, VR is less than one for LICs, meaning that over time, negative deviations from the mean overtake the positive ones, producing a negative drift. There are two important considerations however to consider. The first is balance. One could imagine a scenario in which a few large negative deviations by themselves would produce large negative volatility overtaking smaller but more numerous positive deviations. Therefore, to put weight on this statistic, we have to ensure that in number of observations, there is sufficient balance between the numerator and the denominator. The second consideration is that by itself, this measure does not tell us whether the growth path is sustainable, but rather in combination with population growth, it is an indication of whether the growth path over the selected period is avoiding the drift. So not only does overall growth rate have to exceed population growth rate by a sufficient margin, but that it also must have large enough positive deviations to overcome the pull of negative ones. Therefore, we will consider per capita GDP growth when measuring this statistic for various groups in the table below:

Table 1. Volatility Ratios for Selected Groups 1/
(1980-2018)

PPP Real GDP Growth

	Ratio	Balance*
<i>SSA</i>	0.72	0.95
<i>SSA LIC</i>	0.93	0.86
<i>SSA Fragile Countries</i>	0.77	0.70
<i>SSA Middle Income</i>	0.60	0.70
<i>WAEMU</i>	1.01	0.95
<i>Emerging LICs</i>	2.75	1.17

Source: WEO and author's calculations.

1/ Country groups are obtained from IMF SSA REO.

*Balance represents the ratio of number of positive to number of negative deviations regardless of the size of the deviation.

It is interesting to note that even for the middle-income countries in SSA, we notice a ratio significantly below 1, consistent with middle income trap.

V. The Model

A. Growth as Function of Decisions

We posit that the economy is comprised of two types of agents, the private sector agent p_i , and the government, G .

Furthermore, economic production is a result of the accumulation of a series of decisions and interactions among them, dp_i , dG , $dp_i dp_{ii}$ and $dG dp_i$, that lead to a continuous output at any given time:

$$Y_t = f(dp_i, dG, dp_i dp_{ii}, dG dp_i) \quad (2)$$

As explained above, there are two types of fundamental decision types that drive economic production. The first decision type, what we denote to be “routine”, D_1 , sustains the same production structure for a given agent. The second type of decision, what we call, structural and non-linear, D_2 , is a decision that changes the structure of production for a given agent and involves considerably more risk.

Similarly, just as there are two decision types that drive economic production, and hence growth, there are also two parts to growth itself. One part of growth is higher production of the same set of products, the second part of growth is by new types of production, new products and the transformation they bring. Note that this is a larger concept than the productivity coefficient in the Solow Growth Model. The latter is about efficiency, but here, this part of growth encompasses not only efficiency but also higher value products, (Hidalgo et al, 2007) and new ways of producing wealth. In the context of the model through different types of decisions that are riskier. To this end, we will assume growth has a linear component driven by D_1 and a transformative component, the speculative premium in the options context, driven by D_2 . *The set of D_2 that results in the speculative premium value part of growth involves purchase and exercise of options.*

Time is defined continuously. Although decisions are made over discrete periods, with a multitude of economic agents in constant interaction, over time, there is a continuity that emerges as the number of agents and the interaction of their decisions tend to infinity.

There are probabilities associated with each decision type.

We assume that D_1 is governed by the normal distribution. The agent will do more, less or the same of an activity with different quantities whose probabilities will fall under a standard normal distribution with zero mean and 1 variance:

$$D_1 \sim N(0,1)$$

The second decision type has two embedded probabilities. The first is the perceived probability of success by the agent from the decision if taken, $P(D_{2s})$ or if the option is exercised with profit, and the second is the probability that the decision is taken in the first place, $P(D_{2i})$ or the probability that the agent decides to purchase the option. The latter probability is conditional on the formal. Based on Bayes Theorem:

$$P(D_{2l} / D_{2s}) = P(D_{2l} \text{ and } D_{2s}) / P(D_{2s}) \quad (3)$$

$P(D_{2s})$ is a function of many factors linked to the success of the positive shock itself. However, an interesting argument in this function is also the number of agents deciding to act on the second decision type. The more agents decide to purchase these options, the more likely that the positive shock will be successful, feeding back to the profitable exercise of these options at any given time t for a given agent k .

$$\text{Therefore } P(D_{2s})_{tk} = f_t(PS, \sum_{k=1}^n D_{2k}) \quad (4)$$

where D_2 for a given agent k can either take the value of 0 or 1 and where PS stands for factors linked to the success of the positive shock, which will be further elaborated below.

and where PD_{2l} is governed by the binomial distribution.

We must also add that absent PS , as the number of agents increases for a given time t , the joint probability of these agents to purchase options for a significant impact on $P(D_{2s})$ tends to zero.

$$\prod_{k=1}^n P_t(D_{2l})_k \rightarrow 0 \text{ as } n \rightarrow a \quad (5)$$

where a represents a sufficiently large number such that it has a significant impact on $P(D_{2s})$ for the success of the positive shock.

This is a significant inference which tells us that absent an exogenous intervention, the probability of a meaningful impact from these decisions tends to zero, and hence reverts the growth path to its equilibrium, defined by the negative drift. This is a more formal definition of the concept of path dependence as it relates to the hypothesis of the positive shock theory.

The second decision type is governed by the binomial distribution. But there are two embedded probabilities that link the decision to the actual outcome. First is the probability of the decision itself, and the second is the probability of its success. The first probability is conditional on the second. Therefore, perceived probability of success of the second decision type will reinforce the expected outcome in either direction. One implication of this is that there is no middle ground. As elaborated with more detail below, there will either be a virtuous cycle or reversion to the negative drift, but a neutral path will not be stable.

Expected economic production from a given agent at any given time can thus be defined as a function of the sum of the two types of decisions:

$$EP(t_i) = P_{\text{density}} * qt(dil) + P * dsnl * P_{\text{success}} * qt(dsnl)$$

Where $qt(dil)$ is a function of the first decision type, comprising linear decisions and $qt(dsnl)$ is a function of the second decision type, comprising non-linear decisions.

Overall expected economic production can thus be defined as the sum of the expected productions of all economic agents at a given time where nag represents the number of agents:

$$E(\text{GDP}) = \sum_{n=1}^{nag} (EP(\text{tn})) \quad (6)$$

All agents in the economy face decisions whose continuous executions and interactions (based on direction and volatility) result in an overall production for the economy, what we call GDP. The sum of the value of the options embedded in the second type decisions agents face over time must also aggregate to an overall option value on the total production, GDP, which then becomes the underlying instrument of that overall option:

$$\sum_{n=1}^k (Cn) = C(\text{gdp}) \quad (7)$$

Where Cn is a call option available to a given agent inherent in any given D2 decision based on the direction and perceived speed of that direction. $C(\text{gdp})$ is the sum of all call options available to agents in the economy, representing the overall call option value on the economy. D2 is an implicit argument of the function $C(\text{gdp})$ where gdp itself is a function of both D1 and D2, but since we are calculating the option value associated with a given gdp , we are isolating D2 to calculate it.

Underlying instruments in finance must be tradeable. In this case, the overall production, by definition, is also tradeable, tradeable both internally and externally. The real options embedded in that instrument are also tradeable. If the agent “buys” the option inherent in the second decision type, she will trade goods and services associated with that decision, hence the options of the underlying economy, at the minimum, involve trading in the underlying economy. The implicit direction and volatility bet the agent makes on the second type decision also aggregates to the overall economy to result in an aggregate directional and volatility bet all agents make on the economy, and ultimately resulting in the actual direction and volatility of the economy going forward and determining growth.

Therefore, there is an aggregate option value to any economy $C(\text{gdp})$, the economy itself being the underlying instrument traded constantly by rational agents, constantly making these two types of decisions. If this is true, then we can easily value this aggregate option using a standard option valuation method.

B. Model Objectives

The first purpose of the model will be to shed light on the speed differential. More precisely, what would be the minimum expected overall economic growth rate, ES , at which agents would find it profitable to purchase real options available to them? It would be this rate, which would enable more risk taking toward economic transformation that would ultimately beat the speed trap.

The second purpose of the model will be to propose a framework to value the aggregation of the individual options available to the agent at the expected economic growth rate. This will represent a hypothetical overall option on the economy and represent the speculative premium. Estimation of this greater option value will serve two purposes. The first will be an estimate of the speculative premium, which estimated for all countries, would yield a relative rank.

The second purpose will be a minimum threshold for the overall size of the positive shock. If the value of the hypothetical option is the sum of the value of the individual options available to the agents or the premium, then the aggregate option on the economy will represent the sum of these premiums.

A representation of the cost of the positive shock can be thought of as a generic subsidy for the purchase of these options by devoting the additional amount the agents need to cross into the threshold of more risk taking. An example for an instrument of optionality can be thought of as a pool of funds saved in good times to take advantage of opportunities to engineer a positive shock. Of course, this does not mean that planners of the positive shock identify each option available to the private agent and purchase it for her. Rather, it means that the difference between the aggregate premium and what the premium should be for the option strategy to be profitable, represents what the planner could view as the additional cost of the positive shock to the economy. Thus, estimation can also serve the purpose of determining a limit.

Next, we discuss the assumptions we make to apply the Black-Scholes formula to the set of specific arguments needed, and their mapping to the relevant economic variables.

C. General Assumptions

Our baseline in this specification to find a closed-form solution to estimate the ES will be the Black-Scholes model. The first departure point will be the value of a given option available to an agent. For this exercise, we will make the following assumptions:

1. The agent faces two decision types at any given time t . First decision type is routine in nature that continues the same economic behavior, varying its parameters according to a normal distribution, involving the return to the mean over the cycles of economic boom and bust. The second decision type involves the purchase of “real” options in more risky projects that involve some level of transformation of the projects underlying the routine decision type.
2. The strike price of the option, K , is the current per capita GDP, g , factored by a propensity, c (that takes out consumption), that the agent is willing to pay for the project underlying the option at the time of exercise.
3. The current value of the project underlying the option, S_t is K (defined as above) that is expected to grow at an economic growth rate, g , discounted to present value by the prevailing real interest rate.
4. The agent splits the net resources available to her (after consumption) between linear activity (represented by investment in the prevailing economy) and the purchase of the option, involving more risk, betting on a higher growth rate.
5. Furthermore, given the third assumption, unless the implied return of the option with respect to her income now outweighs the return on the prevailing growth rate, she will expend the entirety of resources available after consumption on behavior underlying the routine decision type.
6. If the return on the value of the option is greater than the return on the prevailing economy, she would invest a fraction of the net income, f , after consumption, on the option. This fraction reaches an asymptotic maximum with higher returns on the option.
7. For simplicity’s sake, we will assume no taxes in this model.

D. Solution to the model: Black-Scholes Formula Applied

Black-Scholes Formula for a given agent at time t with the assumptions below:

$$C(S_t, t) = N(d_1) S_t - N(d_2) PV(K) \quad (8)$$

Where C represents the value of the option; S_t is the current stock price; K is the strike price on the option; PV is the present value and finally N is the cumulative density function for the normal distribution, giving us the probability associated with d_1 and d_2 defined below as in the standard Black-Scholes formula.

In our framework, S_t will equal the present value of the risky project on which the agent considers purchasing the option:

$$(9) S_t = [(i c l) (1+g)^*] e^{-r(T-t)}$$

Where i equals GDP per capita; c equals a consumption factor less than 1; l is a routine behavior factor less than 1 that takes out the resources devoted to the routine decisions; g is the expected economic growth rate; r is the risk-free real interest rate; and $T-t$ is time to maturity of the real option.

$$K = i c l \quad (10)$$

where K , the strike price represents what the agent is willing to pay today to exercise the option on the project at the time of the exercise. If we re-formulate the Black-Scholes formula above with the assumptions we make, we have for the value of the option available to an agent:

$$C(S_t, t) = N(d_1) [(i c l) (1+g)^*] e^{-r(T-t)} - N(d_2) PV(i c l) \quad (11)$$

$$= N(d_1) [(i c l) (1+g)^*] e^{-r(T-t)} - N(d_2) (i c l) e^{-r(T-t)}$$

For this option to present an attractive value, it must be equal or greater than the present value on the return the agent would expect to make based on the current economic growth rate on the same resources she would otherwise invest on the risky project:

$$(12) N(d_1) [(i c l) (1+g)^*] e^{-r(T-t)} - N(d_2) (i c l) e^{-r(T-t)} \geq [(i c l) (1+g)] e^{-r(T-t)}$$

The risky option on the left represents an uncertain future that has a higher expected g whereas the routine behavior on the right represents a less risky return the agent can make today by engaging in the routine behavior that will, on average, earn the prevailing economic growth rate. The Black-Scholes formula helps us take into account the uncertainty and the risk associated with the value on the left; there is a higher return expected, but it is uncertain. How large does this higher return have to be vis-à-vis the prevailing growth rate, to compensate for the uncertainty, is the question we are trying to answer.

where, as in the standard Black-Scholes formula, d_1 and d_2 are

$$(13) d_1 = [1 / \sigma (\sqrt{T-t})] [\ln(S_t / K) + (r + \sigma^2/2) (T-t)]$$

$$(14) d_2 = d_1 - \sigma (\sqrt{T-t})$$

where σ represents volatility.

What is the minimum value g^* must take for the inequality above to hold true?

We first re-arrange the inequality above to isolate g^* , with basic algebraic steps, we arrive at the following intermediate formula:

$$(15) g^* = [((1+g) + N(d_2)) / N(d_1)] - 1$$

Inside N , we also have g^* as part of the definition for d_1 and d_2 . Therefore, to isolate g^* , we expand the cumulative normal distribution:

$$(16) N(d_1) = [(1 / (\sqrt{2\pi})) (d_1 - (1/6)d_1^3 + (1/40)d_1^5 - (1/336)d_1^7 \dots)]$$

In this model we will assume that the hypothetical option has a life of one year toward expectations of the realized growth rate at the end of the year.⁷

thus:

$$(17) d_2 = d_1 - \sigma$$

Therefore, d_1 can be re-written as:

$[1 / \sigma] [\ln(S_t / K) + (r + \sigma^2/2)]$ and further substituting for S_t and K :

$$(18) N(d_1) = [(1 / (\sqrt{2\pi})) ([(1 / \sigma) [\ln(((i c l) (1+g^*/100)) e^{-r(T-t)}) / (i c l)] + (r + \sigma^2/2)]) - ((1/6) ([1 / \sigma] [\ln(((i c l) (1+g^*/100)) e^{-r(T-t)}) / (i c l)] + (r + \sigma^2/2)])^3 + (1/40) ((1/6) ([1 / \sigma] [\ln(((i c l) (1+g^*/100)) e^{-r(T-t)}) / (i c l)] + (r + \sigma^2/2)])^5 - (1/336) ((1/6) ([1 / \sigma] [\ln(((i c l) (1+g^*/100)) e^{-r(T-t)}) / (i c l)] + (r + \sigma^2/2)])^7 \dots)]$$

$$(19) N(d_2) = [(1 / (\sqrt{2\pi})) ((d_1 - \sigma) - (1/6)(d_1 - \sigma)^3 + (1/40)(d_1 - \sigma)^5 - (1/336)(d_1 - \sigma)^7 \dots)]$$

Substituting for d_1

$$(20) N(d_2) = [(1 / (\sqrt{2\pi})) ([(1 / \sigma (\sqrt{T-t})) [\ln(S_t / K) + (r + \sigma^2/2)(T-t)] - \sigma] - (1/6) ([1 / \sigma (\sqrt{T-t})) [\ln(S_t / K) + (r + \sigma^2/2)(T-t)])^3 + (1/40) ((1/6) ([1 / \sigma (\sqrt{T-t})) [\ln(S_t / K) + (r + \sigma^2/2)(T-t)])^5 - (1/336) ((1/6) ([1 / \sigma (\sqrt{T-t})) [\ln(S_t / K) + (r + \sigma^2/2)(T-t)])^7 \dots)]$$

⁷ Partially this is so that g^* is more intuitive to interpret for a given year. However, this assumption is also consistent with investment decisions that have a shorter horizon in the LIC or the fragile context⁷ due to a number of uncertainties, including policy uncertainty. This is also consistent with the objective of the model to initiate a positive shock over a shorter-horizon to counter the negative drift. Given that projects with the potential for a growth impact would most likely span multiple years, the one-year horizon in the model can also be interpreted as representing the initial phase of a project. For example, an agricultural FDI in food processing is announced with basic infrastructure completed in the first year so that the investment is now bounded for its continuation to the next phase, while already producing positive feedback to agents who would adjust their expectations of their options ending up in the money accordingly. On the other hand, we also ran the model with much longer-term horizon to assess the impact on the g^* , which increases substantially.

$$\begin{aligned} &) - \sigma)^3) + (1/40) (([1 / \sigma (\sqrt{T-t})] [\ln (S_t / K) + (r + \sigma^2/2) (T - t)] \\ &) - \sigma)^5 - (1/336) (([1 / \sigma (\sqrt{T-t})] [\ln (S_t / K) + (r + \sigma^2/2) (T - t)] \\ &) - \sigma)^7 \dots)) \end{aligned}$$

Substituting for d_2 and d_1 in the following derivation:

$$(21) g^* = [((1+g) + N (d_2)) / N (d_1)] - 1$$

$$\begin{aligned} g^* = & [((1+g) + \{([1 / (\sqrt{2\pi})] (([1 / \sigma (\sqrt{T-t})] [\ln ((S_t) / (K)) + (r + \sigma^2/2) (T - t)] \\ &) - \sigma) - (1/6) (([1 / \sigma (\sqrt{T-t})] [\ln ((S_t) / (K)) + (r + \sigma^2/2) (T - t)] \\ &) - \sigma)^3) + (1/40) (([1 / \sigma (\sqrt{T-t})] [\ln ((S_t) / (K)) + (r + \sigma^2/2) (T - t)] \\ &) - \sigma)^5 - (1/336) (([1 / \sigma (\sqrt{T-t})] [\ln ((S_t) / (K)) + (r + \sigma^2/2) (T - t)] \\ &) - \sigma)^7 \dots)) \} / \{([1 / (\sqrt{2\pi})] (([1 / \sigma] [\ln (((i c l) (1+g^*)] e^{-r(T-t)}) / (i c l)) + (r + \sigma^2/2)]) - (1/6) ([1 / \sigma] [\ln (((i c l) (1+g^*)] e^{-r(T-t)}) / (i c l)) + (r + \sigma^2/2)])^3 + (1/40) ((1/6) ([1 / \sigma] [\ln (((i c l) (1+g^*)] e^{-r(T-t)}) / (i c l)) + (r + \sigma^2/2)])^5 - (1/336) ((1/6) ([1 / \sigma] [\ln (((i c l) (1+g^*)] e^{-r(T-t)}) / (i c l)) + (r + \sigma^2/2)])^7 \dots)) \}] - 1 \end{aligned}$$

Finally substituting for S_t and K in d_1 :

$$\begin{aligned} (22) g^* = & [((1+g) + \{([1 / (\sqrt{2\pi})] (([1 / \sigma] [\ln (((i c l) (1+g^*)] e^{-r} / (i c l)) + (r + \sigma^2/2)) - \sigma) - (1/6) ([1 / \sigma] [\ln (((i c l) (1+g^*)] e^{-r} / (i c l)) + (r + \sigma^2/2)) - \sigma)^3) + (1/40) (([1 / \sigma] [\ln (((i c l) (1+g^*)] e^{-r} / (i c l)) + (r + \sigma^2/2)) - \sigma)^5 - (1/336) (([1 / \sigma] [\ln (((i c l) (1+g^*)] e^{-r} / (i c l)) + (r + \sigma^2/2)) - \sigma)^7 \dots)) \} / \{([1 / (\sqrt{2\pi})] (([1 / \sigma] [\ln (((i c l) (1+g^*)] e^{-r} / (i c l)) + (r + \sigma^2/2)]) - (1/6) ([1 / \sigma] [\ln (((i c l) (1+g^*)] e^{-r} / (i c l)) + (r + \sigma^2/2)])^3 + (1/40) ((1/6) ([1 / \sigma] [\ln (((i c l) (1+g^*)] e^{-r} / (i c l)) + (r + \sigma^2/2)])^5 - (1/336) ((1/6) ([1 / \sigma] [\ln (((i c l) (1+g^*)] e^{-r} / (i c l)) + (r + \sigma^2/2)])^7 \dots)) \}] - 1 \end{aligned}$$

This equation boils down to a quadratic involving the following variables:

- 1) **icl**, which is the resources available to the agent net of consumption and routine decisions.
- 2) **r**, real “risk-free” interest rate on a government paper. We will take the rate for the one-year security since we assume that time to maturity is 1 year for the purposes of this model and subtract average inflation for the year.
- 3) **g**, “prevailing” economic growth rate, this is the latest realized growth rate.
- 4) **σ** , standard deviation of growth in the past 20 years to represent a realistic expectation for the agent.⁸
- 5) Finally, we have g^* , which is the variable we are trying to solve for, what the growth rate should be for the option premium to represent a high enough value to beat the prevailing growth rate, ES. $g^* - g$ will represent the growth premium needed to make risk toward transformation (using options) more attractive for the economy.
- 6) Finally, the difference in the valuation of the option using g^* and g respectively from the existing nominal GDP base will give us a clue about the potential overall cost of the positive shock in that this will represent the additional “premium” per agent.

We used simulations in Mathematica and Newton’s method to approximate the solution for g^* for a combination of sets of the above variables, representing selected countries, regions, and groupings. We

⁸ When we relax this assumption to run the model on volatility calculated for shorter horizons, the gap between ES, g^* and the prevailing growth rate increases by 13 percent for fragile countries in SSA, with decreasing positive volatility in the last 10 years; g^* has to be higher to compensate the weaker contribution to option value from the lower positive volatility. 20 years is selected as a baseline to estimate the standard deviation of the distribution of growth rates.

ran simulations with different values of icl to see if the outcome would differ. It is interesting to note that the results are orthogonal to levels of icl , in other words the resources available to the agent. This is somewhat counterintuitive as one would expect that higher nominal level of resources should allow an agent to take more risk for example, and lead to a lower g^* , so one could consider this as a weakness of the model. Another way to think about it would be to interpret icl as an additional unit of resources potentially available for the purchase of the option.

$$N(d_1) [(i c l) (1+g^*)] e^{-r(T-t)} - N(d_2) (i c l) e^{-r(T-t)} \geq (i c l) (1+g) e^{-r(T-t)}$$

Table 2. Results for selected groupings

	Inputs			Results			
	Real risk-free interest rate 1/	Positive Standard Deviation of Growth 2/	PPP Real GDP Growth 3/	Minimum Growth (ES)	Existent Option Value on the Economy 4/	Option Value consistent with ES	Premium Difference 5/
	r	σ	g	g^*	$C(g)$	$C(g^*)$	$C(g^*) - C(g)$
SSA LICs	1.4	2.0	4.1	6.9	162	173	11
SSA Fragile Countries	3.0	1.3	2.4	4.5	9.9	10.4	0.5
LICs	2.3	1.5	4.8	5.2			

Sources: Country authorities, WEO, UMOA titre, and author's results and calculations.

1/ Average 2018 observation for the 1 year maturity in percent.

2/ Considering only positive standard deviations of PPP Per Capital Real GDP Growth between 1980-2018, in percent.

3/ 2018 actual.

4/ The option value on the economy consistent with the latest actual growth in 2018, g : constant prices, PPP 2011 international dollars

5/ Per capita cost of the positive shock, constant prices, PPP 2011 international dollars. PPP Real GDP per capita for LICs not available for option value calculation.

We can see that for LICs in general, ES is nearly 8.3 percent higher than the prevailing growth rate or more than 0.4 percentage points; for LICs in SSA, this differential rises to 68 percent or 2.8 percentage points (potentially reflecting fragility); whereas for fragile economies in SSA, ES is nearly 88 percent higher than the prevailing growth rate or 2.1 percentage points, confirming the need for a “shock” effect.

To understand the impact of a longer-term option, proxying longer-term investment decisions, we also ran the model with a call option of a 9-year maturity. The results confirm the assumption that longer-term options, which are riskier, demand higher growth rates: for SSA LICs, the growth differential rises to 16.8 percentage points over the longer horizon; and for fragile economies in SSA, it is 8.5 percentage points of difference with the prevailing medium-term growth assumption. These results make intuitive sense in highlighting the growing “cost differential” over time as more uncertainties emerge, demanding a higher premium.

VI. Policy Implications

A. Divergence or Convergence?

More options being purchased will lead to one of two outcomes. First possibility is that they converge to a critical mass that will result in being in the money and increasing the premium on the overall option and thus increasing the premium of these options in the next period. Second possibility is that there will not be a critical mass, and these options will converge to end up out of the money. There will either be a rapid succession of success stories, divergence from the existing growth path or the continuation of the same linear path, and convergence back toward the status-quo. This is also more formally explained in the model

through equation (5). A stronger initial set of coordinated measures will encourage agents and increase the probability of success; as discussed more formally above, left to its devices without an initial push, the probability of a sufficient number of transformative decisions D2 converging to a critical mass is practically zero.

In order to incentivize agents to purchase these riskier options and empower D2, there must be: to i) conditions conducive to more risk-taking for behaviors that require some transformation at the micro-level; ii) initial push at the macro level by taking strategic risks through prioritization and asymmetric concentration instead of equally and linearly spreading resources in hopes of solving all problems (the Christmas tree approach)⁹; iii) and project credibility, specificity and speed of reform in order to match the short-term horizon of the private agent who, in her options strategy, is not only betting on the direction of these reforms, but also their speed; and finally iv) simultaneity, ensuring that complimentary projects and measures are implemented relatively close in time to maximize synergy..

The next section will discuss broader principles of real option theory that can be applied to the design of the positive shock. These principles underlie the application of options theory (which was used to calculate g^*) in decisions that can push economies closer to the g^* , the so-called escape speed.

B. General Design Principles

Ultimately, it would be up to each country to decide the contents of the Positive Shock to incentivize ES. However, there are some guiding principles proposed below based on principles of real options strategy.

1. There are three axes: (i) policy tools; (ii) administrative actions, including specific ad-hoc measures, and finally, (iii) sectoral support. The first axis, policy tools can potentially comprise a range of policy incentives to governance policies and reforms in the way of lifting potential constraints on the private sector. The second axis involves actions supporting existing private sector support mechanisms, such as directly purchasing options for agents by providing training for example in needed skills to support emerging sectors, or grant competitions already experimented by some countries¹⁰ and finally the third axis involves the strategic choice to decide which sectors should in fact be emerging for which there could be targeted support such as the growth strategy that Namibia launched recently in cooperation with the Harvard Growth Lab.¹¹
2. In maximizing optionality of the overall strategy, we draw the following criteria for project/measure selection by the policy makers. A similar approach was adopted in the paper by Kim et al (2017) in using a real option to analyze renewable energy investment decisions in developing countries:
 - a. **Abandon/modification quality.** Those measures which will be easiest to modify or abandon to switch to a related strategy should be given preference relative to those that will require a significant and irreversible investment before meaningful feedback on the economy can be measured. The reason behind this feature is that if options can be exercised to abandon or modify an existing strategy based on feedback, it increases the value of the strategy significantly. The

⁹ A recent example of this is the asymmetric and strategic bet the government of Namibia is placing on developing green hydrogen in concert with the private sector. This is a project that was initially larger than the GDP of Namibia, which has since attracted more investments. This is an example of a strong and focused initial signal, which has since attracted more momentum.

¹⁰ McKenzie, D., "Identifying and Spurring High-Growth Entrepreneurship: Experimental Evidence from a Business Plan Competition", American Economic Review, 2017

¹¹ Namibia's industry targeting dashboard: <https://growthlab.app/namibia-tool>

idea here is to choose projects that even if modified or abandoned, will still yield benefits through lessons and experiences learned in the process.

- b. **Lesson value and synergy.** Those measures, that if abandoned, will still provide a lesson value in their process vis-à-vis a synergy they establish with other actions, should be given preference.
- c. **Asymmetric concentration.** The pool of resources should not be equally split across measures and projects, but rather a set of low hanging fruits with near term yields can be chosen and the rest of the resources can go to selected strategic bets., one to improve/increase production in a relatively established industry toward export capacity and the other, a new high-value sector development with existing private sector potential or initiative. One defining criterion for the strategic bet should be investing in technical skills (such as coding) that can be easily transferable. Strategic investments in technology and technology enhancing skills can provide a path toward convergence in today's highly digitalized world economy. The guiding principle here is finding an endemic advantage with an initial potential that can be exploited rather than strictly comparative advantage.
- d. **Simultaneity and speed.**¹² Reforms should have a speed element to achieve credibility: There should be a matrix of discrete actions and policies defined within a limited timeframe. The components of the matrix should be pursued simultaneously for maximum impact. If one action's timeline diverges from another component, synergy could be reduced with a sub-par overall impact and a higher overall cost. The perceived speed of reform will also induce private agents to have more confidence in the risks they are taking.¹³ The objective is to induce a positive and growing externality between transformation at the public policy level and private decisions; the assumption is that one cannot succeed without the other.

VII. Conclusion

The analysis in this paper shows that a long-term development approach alone may not be enough to escape from the cycle of negative shocks. This is evident in the significant gap the options framework identifies between the ES (growth rate necessary to escape the negative drift) and the actual prevailing

¹² Speed of structural transformation is an important factor in the strategy to strengthen the private sector. (McMillan et al. 2014). Speed is crucial in the positive shock strategy as well. A positive gain, if it cannot be achieved fast enough, will be diluted over time or stopped on its tracks eventually by the large and rapid impact of a negative shock, and hence there will be a continuous downward push in the economy. In the value of an option, this is the time value. An option's value is comprised of two elements: intrinsic value and time value. In this paper we will assume that the intrinsic value of the option we are constructing is zero since the option is not yet in the money; the strategy has not yet realized its profitability. The second element is the time value. As time progresses, there is a natural decay that eventually reduces the option's value to zero. This is how the option can become profitable, if it can beat its own expiration, and for this it needs speed, which will represent the first argument of the volatility as an input into the value of the option. If reforms under the positive shock can begin to yield useful results for the private sector in timely fashion, individual options purchased by private agents will also become profitable. If results are too slow to accrue, individual options may expire without profit or if agents do not perceive enough speed in the shock, they may not purchase these options in the first place. Individual investors face time constraints that need to be considered.

¹³ An empirical implication of this speed argument is that we should notice faster transformation for countries that are deemed to have had some success in achieving convergence as opposed to a gradual progress. A concerted shock strategy by the authorities in Ethiopia and Rwanda in the last three decades are consistent with this implication.

growth rate in LICs and SSA fragile economies, more than 88 percent for the latter,¹⁴ Options and optionality will thrive in this new setting, widening the gap further between LICs and economies with more flexibility and space for risk taking toward profitable real options. Economies subject to routine negative shocks could stagnate and drift downward unless they can also take advantage of positive shocks.

Long stretches of time may give the illusion of linearity, obscuring non-linear events and free-rider problems, requiring non-linear solutions to modify incentives. This paper argues that a synergy between private agents and governments can accelerate non-linear decisions necessary to achieve the escape speed (ES) to overcome the negative drift. Waiting for full institutional development and the perfect state of policy may not be sufficient or even necessary to achieve this outcome. A dynamic development of the private sector could eventually lead to the development of institutions dictated by the needs of the economy rather than the other way around, resulting in a longer-term virtuous cycle. However, the broader implication is that economies also need a short-term growth strategy before they can shift to this longer-term virtuous cycle.

¹⁴ Subsequent work aims to test the assumptions on the underlying stochastic process, which may change these baseline estimates. Although agents' actions will affect the underlying asset price and hence the value of the option in the conceptual framework, this is not modelled explicitly in the current estimates of the ES, which presents an ex-ante estimate of the needed movement in the underlying asset for the option strategy to be profitable.

Annex I. Exploring a longer option expiration¹

$$g^* = [((1+g) + N(d_2)) / N(d_1)] - 1$$

Inside N , we also have g^* as part of the definition for d_1 and d_2 . Therefore, in order to isolate g^* , we have to expand the cumulative normal distribution:

$$N(d_1) = [(1 / (\sqrt{2\pi})) (d_1 - (1/6)d_1^3 + (1/40)d_1^5 - (1/336)d_1^7 \dots)]$$

In this version we will assume that this hypothetical option has a life of 9 years toward expectations of the realized growth rate at the end of this period.

$$d_2 = d_1 - \sigma$$

Therefore, d_1 can be re-written as:

$[1 / \sigma] [\ln(S_t / K) + (r + \sigma^2/2)]$ and further substituting for S_t and K :

$$N(d_1) = [(1 / (\sqrt{2\pi})) (([1 / \sigma] [\ln((i c l) (1+g^*/100)] e^{-r(9)} / (i c l)) + (r + \sigma^2/2)]) - ((1/6) ([1 / \sigma] [\ln((i c l) (1+g^*/100)] e^{-r(9)} / (i c l)) + (r + \sigma^2/2)])^3 + (1/40) ((1/6) ([1 / \sigma] [\ln((i c l) (1+g^*/100)] e^{-r(9)} / (i c l)) + (r + \sigma^2/2)])^5 - (1/336) ((1/6) ([1 / \sigma] [\ln((i c l) (1+g^*/100)] e^{-r(9)} / (i c l)) + (r + \sigma^2/2)])^7 \dots]$$

$$N(d_2) = [(1 / (\sqrt{2\pi})) ((d_1 - \sigma) - (1/6)(d_1 - \sigma)^3 + (1/40)(d_1 - \sigma)^5 - (1/336)(d_1 - \sigma)^7 \dots)]$$

Substituting for d_1

$$N(d_2) = [(1 / (\sqrt{2\pi})) (([1 / \sigma (\sqrt{9})] [\ln(S_t / K) + (r + \sigma^2/2) (T - t)] - \sigma) - (1/6) ([1 / \sigma (\sqrt{9})] [\ln(S_t / K) + (r + \sigma^2/2) (9)] - \sigma)^3 + (1/40) ([1 / \sigma (\sqrt{9})] [\ln(S_t / K) + (r + \sigma^2/2) (9)] - \sigma)^5 - (1/336) ([1 / \sigma (\sqrt{9})] [\ln(S_t / K) + (r + \sigma^2/2) (9)] - \sigma)^7 \dots)]$$

Substituting for d_2 and d_1 in the following derivation:

$$g^* = [((1+g) + N(d_2)) / N(d_1)] - 1$$

$$g^* = [((1+g) + [(1 / (\sqrt{2\pi})) (([1 / \sigma (\sqrt{9})] [\ln((S_t) / (K)) + (r + \sigma^2/2) (9)] - \sigma) - (1/6) ([1 / \sigma (\sqrt{9})] [\ln((S_t) / (K)) + (r + \sigma^2/2) (9)] - \sigma)^3 + (1/40) ([1 / \sigma (\sqrt{9})] [\ln((S_t) / (K)) + (r + \sigma^2/2) (9)] - \sigma)^5 - (1/336) ([1 / \sigma (\sqrt{9})] [\ln((S_t) / (K)) + (r + \sigma^2/2) (9)] - \sigma)^7 \dots)]) / [(1 / (\sqrt{2\pi})) (([1 / \sigma] [\ln((i c l) (1+g^*/100)] e^{-r(9)} / (i c l)) + (r + \sigma^2/2)]) - ((1/6) ([1 / \sigma] [\ln((i c l) (1+g^*/100)] e^{-r(9)} / (i c l)) + (r + \sigma^2/2)])^3 + (1/40) ((1/6) ([1 / \sigma] [\ln((i c l) (1+g^*/100)] e^{-r(9)} / (i c l)) + (r + \sigma^2/2)])^5 - (1/336) ((1/6) ([1 / \sigma] [\ln((i c l) (1+g^*/100)] e^{-r(9)} / (i c l)) + (r + \sigma^2/2)])^7 \dots)]] - 1$$

¹ 9 years instead of 1.

$$) - \sigma)^7 \dots) \} / \{ [(1/\sqrt{2\pi}) (([1/\sigma] [\ln (([i c l] (1+g^*)) e^{-r(9)}) / (i c l)) + (r + \sigma^2/2)]) - (1/6) ([1/\sigma] [\ln (([i c l] (1+g^*)) e^{-r(9)}) / (i c l)) + (r + \sigma^2/2)])^3 + (1/40) ((1/6) ([1/\sigma] [\ln (([i c l] (1+g^*)) e^{-r(9)}) / (i c l)) + (r + \sigma^2/2)])^5 - (1/336) ((1/6) ([1/\sigma] [\ln (([i c l] (1+g^*)) e^{-r(9)}) / (i c l)) + (r + \sigma^2/2)])^7 \dots)] \} - 1$$

Finally substituting for St and K in d1:

$$g^* = [((1+g) + \{ [(1/\sqrt{2\pi}) ((([1/9\sigma] [\ln (([i c l] (1+g^*)) e^{-9r}) / (i c l)) + (r + \sigma^2/2)9]) - \sigma) - (1/6) ([1/3\sigma] [\ln (([i c l] (1+g^*)) e^{-9r}) / (i c l)) + (r + \sigma^2/2)9]) - \sigma)^3 + (1/40) ([1/3\sigma] [\ln (([i c l] (1+g^*)) e^{-9r}) / (i c l)) + (r + \sigma^2/2)9]) - \sigma)^5 - (1/336) ([1/3\sigma] [\ln (([i c l] (1+g^*)) e^{-9r}) / (i c l)) + (r + \sigma^2/2)9]) - \sigma)^7 \dots) \} / \{ [(1/\sqrt{2\pi}) (([1/3\sigma] [\ln (([i c l] (1+g^*)) e^{-9r}) / (i c l)) + (r + \sigma^2/2)9]) - (1/6) ([1/3\sigma] [\ln (([i c l] (1+g^*)) e^{-9r}) / (i c l)) + (r + \sigma^2/2)9])^3 + (1/40) ((1/6) ([1/3\sigma] [\ln (([i c l] (1+g^*)) e^{-9r}) / (i c l)) + (r + \sigma^2/2)9])^5 - (1/336) ((1/6) ([1/3\sigma] [\ln (([i c l] (1+g^*)) e^{-9r}) / (i c l)) + (r + \sigma^2/2)9])^7 \dots)] \} - 1$$

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