

# **Asset Pricing with Dividend Surprises\***

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## **Abstract**

In this paper, we derive an intertemporal dividend-surprise-augmented asset-pricing model and show that the expected risk premium compensates for stock returns' exposure to (i) the market-wide dividend-surprise hedge portfolio based on dividend yield surprise and volatilities, in addition to (ii) the excess market return without dividend yield (as in the conventional CAPM) and (iii) the market-wide dividend yield factor without uncertainty. Our model implies that the uncertainty on dividend-surprise yield is attributable to systematic risk and should be priced at the cross-section, thereby theoretically supporting the existing empirical studies on the relation between dividend surprise and cross-sectional stock returns.

**JEL Classification:** G11, G12, G35

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## 1. Introduction

The well-known dividend-ratio model (e.g., Williams, 1938; Gordon and Shapiro, 1956; Gordon, 1962) opens important new avenues for predicting equity prices. Dividend yields have long been used to evaluate the expected equity returns (e.g., Campbell and Shiller, 1988; Goetzmann and Jorion, 1993; Kellard et al., 2010; Choi et al., 2017). Traditionally, the dividend-ratio model assumes a constant dividend growth rate in the absence of uncertainty which is ideal for companies with steady growth rates. However, a growing number of empirical studies find that stock returns react to the shocks to expected dividends (e.g., Lee, 1995; Grullon et al., 2002; Charitou et al., 2011; Andres et al., 2013; Von Eije et al., 2014; Henry et al., 2017; Kumar, 2017; Kreidl and Scholz, 2021; Sun et al., 2021), highlighting the importance of dividend surprises to investors. These studies suggest that firms' exposure to systematic risk increases (drops) when firms' newly announced dividends fail (exceed) investors' expected dividends. For instance, from a cross-sectional perspective, Jacoby et al. (2022) show that stocks with negative (positive) dividend surprises generate higher (lower) cross-sectional stock returns, implying a risk-based explanation such that investors demand a greater (fewer) risk premium for firms with negative (positive) dividend surprises. While the negative association between firms' dividend surprises and their risk premium is supported empirically, to the best of our knowledge, there is a lack of theoretical models elaborating the mechanism through which dividend surprise systematically affects asset returns. In this paper, we aim to fill the gap by analyzing the equilibrium implications of dividend surprise for both the aggregate market and the cross-section.

In the spirit of Merton (1973), we formulate the impact of the unexpected portion of dividend (i.e., dividend surprise) on asset prices based on an intertemporal dividend-surprise-augmented asset-pricing model. Specifically, the dividend yield follows an Ornstein-Uhlenbeck mean-

reverting process. In this sense, the stochastic instantaneous return on an asset observed by the market participants can be decomposed into three components, including capital gains yield, long-term dividend yield, and the unexpected portion of the dividend surprise yield. Our model shows that the expected risk premium for an asset arises through the following channels: (i) the return sensitivity of an asset to the excess market return without dividend yield (as in the beta measuring exposure to market risk in the conventional CAPM); (ii) the return sensitivity of an asset to a market-wide dividend factor in the absence of dividend surprise; and (iii) the return sensitivity of an asset to a hedge portfolio based on dividend surprise and volatility. In addition, we find that the additional risk originating from dividend surprise alters not only factor risk premiums but also factor sensitivities in equilibrium. Therefore, our model implies that the uncertainty in dividend-surprise yield serves as an additional channel of systematic risk and should be priced at the cross-section of stock returns.

Our paper contributes to the literature by providing the existing studies with a theoretical foundation for explaining the impacts of firms' dividend surprise on their stock returns (e.g., Grullon et al., 2002; Charitou et al., 2011; Von Eije et al., 2014; Henry et al., 2017; Sun et al., 2021; Jacoby et al., 2022). We find that in equilibrium, dividend surprise enters into the stochastic discount factor, thereby augmenting the systematic risk in the presence of dividend uncertainty. We further show that the presence of dividend surprise could affect investors' optimal portfolio demand and the risk-return trade-off by adding an additional risk premium induced by dividend uncertainty. Moreover, our work provides implications for firm dividend and financial policies. The higher uncertainty in dividend yield, the greater the expected returns required to compensate for the additional source of uncertainty.

The rest of the paper is organized as follows. Section 2 presents our model setup and assumptions. Section 3 illustrates the investor's optimal portfolio choice and derivation of the equilibrium pricing equation with dividend surprises. Section 4 concludes.

## 2. Model Setup

**Assumption 1.** The stochastic instantaneous return observed by the market participants at time  $t$  is expressed as follows:

$$r_{i,t} = c_{i,t} + d_{i,t} + y_{i,t}$$

where  $c_{i,t}$  is the capital gains yield on asset  $i$  over a short time period between time  $t$  and time  $t+dt$ , such that:  $c_{i,t} = \ln\left(\frac{P_{i,t+dt}}{P_{i,t}}\right)$ ;  $d_{i,t}$  is the long-term dividend yield on asset  $i$  over  $dt$  calculated based on the long-term target dollar dividend  $(D_{i,t+dt}^*)$ , such that:  $d_{i,t} = \ln\left(\frac{D_{i,t+dt}^*}{P_{i,t}}\right)$ ; and  $y_i$  is the unexpected portion of the dividend yield over time  $dt$ , such that:  $y_{i,t} = \ln\left(\frac{D_{i,t+dt} - D_{i,t+dt}^*}{P_{i,t}}\right)$ , where  $D_{i,t+dt}$  is the stochastic realized dollar dividend. Note that the long-term target dividend yield is deterministic as the long-term (expected) dividend policy is constant. For notational simplicity, we omit time notations henceforth.

**Assumption 2.** The capital gains yield for every asset  $i$  ( $i = 1, 2, \dots, n$ ) follows a Gaussian process, as presented below:

$$dc_i = \mu_{c_i} dt + \sigma_{c_i} d\omega_i, \quad dd_i = \mu_{d_i} dt,$$

where  $\mu_{c_i}$ ,  $\sigma_{c_i}$ , and  $\mu_{d_i}$  are constants;  $\omega_i$  is the standard Wiener processes.

**Assumption 3.** The unexpected dividend yield for every asset  $i$  ( $i = 1, 2, \dots, n$ ) follows an Ornstein-Uhlenbeck mean-reverting process, as presented below:

$$dy_i = k_i(\mu_{y_i} - y_i)dt + \sigma_{y_i} dz_i,$$

$$E[d\omega_i, dz_i] = \rho_{c_i, y_i} dt$$

where  $\mu_{y_i}$ ,  $\sigma_{y_i}$ , and  $k_i$  are constants;  $z_i$  follows the standard Wiener process.

We assume that the dividend-surprise yield follows a mean-reverting process to account for reversals in market price or unexpected dividend growth. The drift term  $k_i(\mu_{y_i} - y_i)$  gives the long-term level of changes in dividend-surprise yield. With a speed of mean reversion, the level of dividend-surprise yield fluctuates around a long-term state mean,  $\mu_{y_i}$ , which is constant for security  $i$ . The parameter  $\sigma_{y_i}$  measures the magnitude of the innovation in dividend-surprise yield.

By applying *Itô's* lemma, the mean of the instantaneous return could be written as  $\mu_{r_i} = \mu_{c_i} + \mu_{d_i} + \mu_{y_i}$ , which is the sum of the three components: mean instantaneous capital gain yield, long-term mean of the dividend yield, and long-term mean of dividend-surprise yield. The variance of instantaneous return is given by:  $\sigma_{r_i}^2 = \sigma_{c_i}^2 + 2\sigma_{c_i, y_i} + \sigma_{y_i}^2$ , where  $\sigma_{c_i, y_i}$  is the instantaneous covariance ( $\sigma_{c_i, y_i} = \rho_{c_i, y_i} \sigma_{c_i} \sigma_{y_i}$ ), and the term  $\rho_{c_i, y_i}$  denotes the instantaneous correlation between the capital yield and the dividend-surprise yield on stock  $i$ .

The above assumptions imply that the instantaneous return on the asset  $i$  is given as follows:

$$dr_i = (\mu_{c_i} + \mu_{d_i} + k_i(\mu_{y_i} - y_i))dt + \sqrt{\sigma_{c_i}^2 + 2\sigma_{c_i, y_i} + \sigma_{y_i}^2} dv_i, \quad (1)$$

where  $v_i$  follows a standard Wiener process:  $dv_i = \frac{\sigma_{c_i}}{\sigma_{r_i}} dw_i + \frac{\sigma_{y_i}}{\sigma_{r_i}} dz_i$ .<sup>1</sup>

Equation (1) implies that  $\left\{(\mu_{c_i} + \mu_{d_i} + k_i(\mu_{y_i} - y_i), \sqrt{\sigma_{c_i}^2 + 2\sigma_{c_i, y_i} + \sigma_{y_i}^2}, \rho_{c_i, y_i})\right\}$  is a sufficient set of statistics for the investment opportunity set at any given point in time. Asset allocation

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<sup>1</sup> Application of *Itô's* Lemma implies that:  $dv_i = \frac{\sigma_{c_i}}{\sigma_{r_i}} dw_i + \frac{\sigma_{y_i}}{\sigma_{r_i}} dz_i$ , and  $E(dv_i dz_i) = \frac{\sigma_{c_i} \rho_{c_i, y_i} + \sigma_{y_i}}{\sigma_{r_i}} dt$ .

decisions depend on the knowledge of the state variables governing the stochastic process of generating asset returns.

We further denote the instantaneous correlation coefficient between  $d\omega_i$  and  $d\omega_j$  for two different assets  $i$  and  $j$  with  $\rho_{c_i, c_j}$  and the instantaneous correlation coefficient between  $d\omega_i$  and  $dz_j$  with  $\rho_{c_i, y_j}$ . That is,  $E[dw_i dw_j] = \rho_{c_i, c_j} dt$ , and  $E[dw_i dz_j] = \rho_{c_i, y_j} dt$ , and  $E[dz_i dz_j] = \rho_{y_i, y_j} dt$ . Hence, the covariance between instantaneous returns on any two assets  $i$  and  $j$  is given by:  $\sigma_{r_i r_j} = \sigma_{c_i, c_j} + \sigma_{c_i, y_j} + \sigma_{c_j, y_i} + \sigma_{y_i, y_j}$ .

We assume that there are  $L$  investors with preferences as described by Merton (1973), and the objective of the  $l^{\text{th}}$  investor is to maximize his/her expected lifetime utility of wealth by solving the following portfolio selection problem:

$$J[W(t), y(t), t] = \max E_t \left[ \int_t^{T^l} U^l[C^l(s), s] ds + B^l [W^l(T^l), y(T^l), T^l] \right], \quad \forall l = 1, 2, \dots, L, \quad (2)$$

where  $E_t$  is the time- $t$  expectation operator, conditional on the current value of investor  $l$ 's wealth. The state variables of the investment opportunity set are given by:  $W^l(t) = W^l$ . The vector of dividend-surprise yield is denoted by  $y(t)$ .  $U^l$  is the von Neumann-Morgenstern utility function of consumption for the  $l^{\text{th}}$  investor, which is strictly concave. The initial value of the  $l^{\text{th}}$  investor's wealth is given by  $W^l(t) = W^l$ .  $T^l$  is the  $l^{\text{th}}$  investor's investment horizon and  $C^l(s)$  is the instantaneous consumption flow at time  $t$ . Finally,  $B^l$  denotes a strictly concave utility function of terminal wealth. The terminal value of lifetime utility in equation (2) is given by:

$$J[W(T), y(T), T] = B[W(T), y(T), T].$$

We assume that there are  $n$  risky assets in the market and a single instantaneously riskless asset issued by the U.S. Treasury, which pays a certain rate of return  $r_f$  over the next instant of time  $dt$ .

Given the  $It\hat{o}$  processes for the dividend-yield-adjusted return on security  $i$ , the wealth accumulation equation for an investor is given by equation (3):

$$dW = \sum_{i=1}^n q_i W (dr_i - r_f dt) + r_f W dt - C dt \quad (3)$$

where  $q_i$  is the proportion of the investor's wealth invested in the  $i^{\text{th}}$  asset, and it satisfies the summation of one requirement:  $q_f + \sum_{i=1}^n q_i = 1$ . Besides,  $q_f$  is the proportion of the investor's wealth invested in the US-based riskless asset. By substituting for  $dr_i$  from equation (1), equation (3) could be written as follows:

$$dW = [\sum_{i=1}^n q_i (\mu_{c_i} + \mu_{d_i} + k_i (\mu_{y_i} - y_i) - r_f) + r_f] W dt + \sum_{i=1}^n q_i W \sigma_{r_i} dv_i - C dt \quad (3.1)$$

where  $\sigma_{r_i} = \sqrt{\sigma_{c_i}^2 + 2\sigma_{c_i,y_i} + \sigma_{y_i}^2}$ .

Based on the above assumptions and wealth-accumulation process, we solve for an investor's consumption-investment optimal choice, which results in the following Hamilton-Jacobi-Bellman (HJB) equation:

$$0 = \max_{C,q} \left[ U[C(t), t] dt + J_t dt + J_W E_t(dW) + \frac{1}{2} J_{WW} E_t(dW)^2 + \sum_{i=1}^n J_{y_i} E_t(d(y_i)) + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n J_{y_i y_j} E_t(dy_i dy_j) + \sum_{i=1}^n J_{W y_i} E_t(dW dy_i) + o(dt) \right] \quad (4)$$

Equation (4) implies a Gaussian process of wealth accumulation, and it can be illustrated as below:

$$0 = \max_{C,q} \left[ U[C(t), t] + J_t + J_W [(\sum_{i=1}^n q_i (\mu_{c_i} + \mu_{d_i} + \mu_{y_i} - r_f) + r_f) W - C] + \frac{1}{2} J_{WW} \sum_{i=1}^n \sum_{j=1}^n q_i q_j \sigma_{r_i, r_j} W^2 + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n J_{y_i y_j} \sigma_{y_i, y_j} + \sum_{i=1}^n \sum_{j=1}^n q_i W J_{W y_j} \sigma_{r_i, y_j} \right] \quad (5)$$

The  $n+1$  first-order conditions for each investor derived from (5) are given by:

$$0 = U_C(C(t), t) - J_W(W, t, y), \quad (5.1)$$

$$0 = J_W(\mu_{c_i} + \mu_{d_i} + \mu_{y_i} - r_f) W + J_{WW} \sum_{j=1}^n q_j W^2 \sigma_{r_i, r_j} + \sum_{j=1}^n J_{W y_j} W \sigma_{r_i, y_j}, \quad (5.2)$$

$\forall i = 1, 2, \dots, n$ , and  $\forall j = 1, 2, \dots, n$ , where  $C^* = C(W, t, y)$  and  $q_i^* = q_i(W, t, y)$  represent the optimal level of consumption and the optimal weights for assets in portfolio.

### 3. Derivation of the Intertemporal Dividend-augmented Asset-pricing Model

#### 3.1 The Investor's Optimal Portfolio Choice

Using matrix notation, we rewrite equation (5.2) for the  $n$  risky assets as follows:

$$0 = J_W \tilde{\mu}_r + J_{WW} W \Sigma_{rr} q + \Sigma_{ry} J_{WY}, \quad (6)$$

where  $\tilde{\mu}_r$  is the vector of the long-term mean of excess returns ( $\mu_{c_i} + \mu_{d_i} + \mu_{y_i} - r_f$ ) on the  $n$  risky securities,  $\Sigma_{rr}$  represents the  $n \times n$  variance-covariance matrix of instantaneous returns with elements  $\sigma_{r_i, r_j} = \sigma_{c_i, c_j} + \sigma_{c_i, y_j} + \sigma_{c_j, y_i} + \sigma_{y_i, y_j}$  for security  $i$  and security  $j$ . The optimal weight on each asset is denoted by  $q$  vector. The notation  $\Sigma_{ry}$  represents the  $n \times n$  matrix of covariance terms between the instantaneous security return on security  $i$  and the dividend-surprise yield on security  $j$ , in which components of the matrix are given by  $\sigma_{r_i, y_j} = \sigma_{c_i, y_j} + \sigma_{y_i, y_j}$ .

Equation (6) can be written in the following form of a vector of required excess return on securities:

$$\tilde{\mu}_r = \gamma \Sigma_{rr} q + \Sigma_{ry} \gamma_y \quad (7)$$

where  $\gamma = -J_{WW} W / J_W$  denotes the parameter of the investor's relative risk aversion; and  $\gamma_y = -J_{WY} / J_W$  with  $J_{WY}$  being a second-order cross-derivative relative to wealth and the state variable, and  $J_W$  being the marginal value with respect to wealth. Solving equation (7), we obtain the vector of optimal global portfolio weights for the  $n$  risky securities as given below:

$$q^* = \frac{1}{\gamma} \Sigma_{rr}^{-1} (\tilde{\mu}_r - \Sigma_{ry} \gamma_y) \quad (8)$$

Specifically, the optimal weight in equation (8) can be expressed for every asset  $i$  as follows:

$$q_i^* = \frac{1}{\gamma} \sum_{j=1}^n \left( \delta_{r_i, r_j} (\mu_{c_j} + \mu_{d_j} + \mu_{y_j} - r_f) - \sum_{j=1}^n (\sigma_{r_i, y_j} \gamma_y) \right) \quad (9)$$

$\forall i = 1, 2, \dots, n$ . The notation  $\delta_{r_i, r_j}$  denotes an element in the inverse of  $n \times n$  variance-covariance matrix  $\Sigma_{rr}$ . The term  $\sigma_{r_i, y_j}$  is the element of  $\Sigma_{ry}$ , where  $\sigma_{r_i, y_j} = \rho_{c_i, y_j} \sigma_{c_i} \sigma_{y_j} + \rho_{y_i, y_j} \sigma_{y_i} \sigma_{y_j}$ . Assuming a special case in which the occurrence of dividend-yield surprise on any asset  $i$  is independent of the occurrence of dividend-yield surprise or change in capital yield on another asset  $j$ , then equation (9) can be reduced as follows:

$$q_i^* = \frac{1}{\gamma} \sum_{j=1}^n \left( \delta_{r_i, r_j} (\mu_{c_j} + \mu_{d_j} + \mu_{y_j} - r_f) \right) - (\rho_{c_i, y_i} \sigma_{c_i} \sigma_{y_i} + \sigma_{y_i}^2) \frac{\gamma_y}{\gamma}. \quad (10)$$

Equation (10) implies that the optimal portfolio weight  $q_i^*$  on security  $i$  contains two components: one is a universal weight, which is related to the investor's general risk tolerance level and variance-covariance scaled mean excess return (a tangency portfolio); the other one is a personalized portion, which hedges against adverse shock to the dividend-yield on itself.

### **3.2 An Equilibrium Pricing Equation with Unexpected Shocks to Dividends**

In this section, we derive an intertemporal equilibrium pricing equation to explore the impact of dividend-surprise yield on security's risk premium (see the details in the Appendix). Finally, our result shows that expected asset risk premiums arise through the following channels: (i) the sensitivity of asset return to the excess market return without dividend yield (as in the standard CAPM); (ii) the return sensitivity to a market-wide dividend factor; and (iii) the return sensitivity to hedge portfolios related to dividend surprise and volatility. Specifically, the expected excess return on security  $i$  takes the following form:

$$\mu_{r_i} - r_f = \beta_i^m (\mu_{c_m} + \mu_{d_m} - r_f) + \beta_i^m \mu_{y_m} + H_i, \quad \forall i = 1, 2, \dots, n, \quad (11)$$

where  $\mu_{r_i} = \mu_{c_i} + \mu_{d_i} + \mu_{y_i}$ , which is the expected return on security  $i$  with dividend yield,  $\mu_{c_m}$  is the weighted average of market return without dividend yield,  $\mu_{d_m}$  is the weighted average of long-term dividend yield for market portfolio, and  $\mu_{y_m}$  is the weighted average of dividend-yield surprises for market portfolio. The market beta measures the sensitivity of security  $i$ 's return to the market excess return with dividend yield, where  $\beta_i^m = \frac{\sigma_{r_i, r_m}}{\sigma_{r_m}^2} = \frac{\sigma_{r_i, c_m} + \sigma_{r_i, y_m}}{\sigma_{r_m}^2}$ .

The last term,  $H_i$ , in Equation (11) is expressed as:  $H_i = \sum_{k=1}^n \sum_{j=1}^n (\beta_i^m \sigma_{r_m, y_j} - \sigma_{r_i, y_j}) b_{r_h, j, r_h, k} (\pi_k^h - \beta_k^h \pi^m)$ , where  $\pi^m = \mu_{c_m} + \mu_{d_m} + \mu_{y_m} - r_f$  represents the expected dividend-yield-adjusted market risk premium, and  $\pi_k^h = \mu_{c_h} + \mu_{d_h} + \mu_{y_h} - r_f$  is the risk premium with dividend for the  $k^{th}$  hedge portfolio. The term  $(\beta_i^m \sigma_{r_m, y_j} - \sigma_{r_i, y_j})$  is an element of the  $n \times n$  matrix  $\Gamma_r$ ;  $b_{r_h, j, r_h, k}$  is an element of the inverse matrix  $\Gamma_r^{-1}$ ; and  $(\pi_k^h - \beta_k^h \pi^m)$  measures the unpredicted portion of  $k^{th}$  hedge portfolio with respect to market portfolio,  $\forall k = 1, 2, \dots, n$  (see details in Appendix Equation 6.7).

For empirical simplicity and applicability, we assume a special case in which the occurrence of dividend-yield surprise on any asset  $i$  can be hedged by one market-wide dividend-surprise hedge

portfolio instead of  $k$  hedge portfolios. Therefore, the specification in the last term,  $H_i$ , boils down to the following:  $H_i = \sum_{j=1}^n (\beta_i^m \sigma_{r_m, y_j} - \sigma_{r_i, y_j}) b_{r_h, j, r_h} (\pi^h - \beta^h \pi^m)$ . We rewrite Equation (11) as follows:

$$\mu_{r_i} - r_f = \beta_i^m (\mu_{c_m} + \mu_{d_m} + \mu_{y_m} - r_f) + \beta_i^h (\pi^h - \beta^h \pi^m), \quad (12)$$

$\forall i = 1, 2, \dots, n$ , the market beta  $\beta_i^m$  consists of two components:  $\frac{\sigma_{r_i, c_m}}{\sigma_{r_m}^2}$  and  $\frac{\sigma_{r_i, y_m}}{\sigma_{r_m}^2}$ , and  $\beta_i^h = \sum_{j=1}^n (\beta_i^m \sigma_{r_m, y_j} - \sigma_{r_i, y_j}) b_{r_h, j, r_h}$ .

Equation (12) provides a simple equilibrium relation between the asset risk premium and two sources of risk: the systematic risk from security market and the systematic risk from adverse shocks to dividend yield in the market. It demonstrates that how the asset systematic risk (beta) is related to risk associated with the *dividend-surprise yield*,  $\sigma_{r_i, y_m}$ , in addition to the standard CAPM-type term  $\sigma_{r_i, c_m}$ . Recall that in a CAPM world, all investors hold the market portfolio. Hence, when the weighted dividend surprise at the market level ( $y_m$ ) is low, investors would prefer security  $i$  to pay a higher return ( $r_i$ ) to offset the loss from the unexpected low dividend. In other words, investors prefer  $\sigma_{r_i, y_m}$  for security  $i$  to be negative so that it hedges against adverse shocks to realized dividends at the market level. This means that investors demand a risk premium for securities with a higher  $\sigma_{r_i, y_m}$ .

The variance of market return is also enhanced by additional variance and covariance terms related to dividend-surprise yield, as:  $\sigma_{r_m}^2 = \sigma_{c_m}^2 + 2\sigma_{c_m, y_m} + \sigma_{y_m}^2$ . The intuition is that the presence of  $\sigma_{y_m}^2$  and  $\sigma_{c_m, y_m}$  shows explicitly that uncertainty in dividend yield cannot be diversified away, even in the context of a very large portfolio such as the market portfolio. The last term  $\beta_i^h (\pi^h - \beta^h \pi^m)$  in Equation (12) contributes to the explanation of asset return with dividend yield. Intuitively,  $(\pi^h - \beta^h \pi^m)$  measures the orthogonal portion of hedge portfolio with respect to market portfolio. And  $\beta_i^h$  captures the security  $i$ 's sensitivity to the orthogonal portion of hedge portfolio to hedge against the risk originating from dividend-surprise yield. Overall, our model shows that risk related to dividend-surprise not only affects factor risk premiums, but also factor sensitivities. As a result, the uncertainty on dividend-surprise yield is systematic and should be priced at the cross-section.

#### 4. Conclusion

We release the assumption of constant dividend growth rate and derive an intertemporal dividend-surprise-augmented asset-pricing model in the spirit of Merton (1973). We show that the uncertainty on dividend-surprise yield is systematic and should be priced at the cross-section, thereby implying a negative association between firms' risk premium and their dividend surprises. Our paper provides a theoretical foundation for the existing empirical studies on the relation between dividend surprises and stock returns.

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## Appendix: Derivation of Equilibrium Pricing Equation

We assume that there are  $L$  investors world-wide from different countries, Equation (6)

$0 = J_W \tilde{\mu}_r + J_{WW} W \sum_{rr} q + \sum_{ry} J_{WY}$  can be rearranged as follows:

$$a^l \tilde{\mu}_r = -W^l \sum_{rr} q^l - \sum_{ry} b^l, \quad (6.1)$$

where vector  $a^l = \left( \frac{J_W}{J_{WW}} \right)^L$  and vector  $b^l = \left( \frac{J_{WY}}{J_{WW}} \right)^L$ , for investor  $l$ ,  $l = 1, 2, \dots, L$ . Summing across the  $L$  investors and dividing by  $\sum_1^L a^l$ , we obtain:

$$\tilde{\mu}_r = -A \sum_{rr} \frac{\sum_1^L W^l q^l}{\sum_1^L a^l} - \sum_{ry} B = -A \sum_{rr} x_m - \sum_{ry} B, \quad (6.2)$$

where  $A = \frac{\sum_1^L W^l}{\sum_1^L a^l}$  is a scalar,  $B = \frac{\sum_1^L b^l}{\sum_1^L a^l}$  is a  $n \times 1$  vector, and  $x_m = \frac{\sum_1^L W^l q^l}{\sum_1^L a^l}$  is a  $n \times 1$  vector of security weights in the market equilibrium. This notation allows us to write:

$$\sum_{rr} x_m = \sigma_{r,r_m} = \sigma_{c,c_m} + \sigma_{y,c_m} + \sigma_{c,y_m} + \sigma_{y,y_m}, \text{ which is a } n \times 1 \text{ vector.}$$

$$x_m^T \sum_{ry} = \sigma_{y,r_m} = \sigma_{y,c_m} + \sigma_{y,y_m}, \text{ which is a } 1 \times n \text{ vector, and } x_m^T \sum_{rr} x_m = \sigma_{r_m}^2.$$

To compute  $A$  and  $B$  in terms of first and second moments of the tangency and hedge portfolios as well as covariance terms of individual returns with these portfolios, we further define  $h^k$  as an  $n \times 1$  vector of security weights in the  $k^{th}$  hedge portfolio,  $\forall k = 1, 2, \dots, n$ . Then we pre-multiply equation (6.2) by  $1 \times n$  transposed weight vectors,  $x_m^T$ , for the market portfolio, and the transpose vector of  $h^k$  for the  $k^{th}$  hedge portfolio  $k = 1, 2, \dots, n$ , respectively, to yield the following  $n+1$  equations:

$$\mu_{c_m} + \mu_{d_m} + \mu_{y_m} - r_f = -A \sigma_{r_m}^2 - \sigma_{r_m,y} B, \quad (6.3)$$

$$\mu_{c_h}^k + \mu_{d_h}^k + \mu_{y_h}^k - r_f = -A \sigma_{r_h,r_m}^k - \sigma_{r_h,y}^k B, \quad \forall k = 1, 2, \dots, n, \quad (6.4)$$

where  $\mu_{c_m} = \sum_{i=1}^n x_i \mu_{c_i}$  is the weighted expected capital yield for the market portfolio,  $\mu_{d_m}$  is the weighted average of long-term dividend-yield for the market portfolio. The term  $\mu_{y_m} = \sum_{i=1}^n x_i y_i$  represents the weighted average of dividend-yield surprise for the market portfolio. Similarly, to hedge against the adverse impact of dividend-yield surprise, we define  $\mu_{c_h}^k = \sum_{i=1}^n h_i^k \mu_{c_i}$  as the weighted average of expected capital yield for the  $k^{th}$  hedge portfolio. The term  $\mu_{y_h}^k = \sum_{i=1}^n h_i^k \mu_{y_i}$  is the weighted average of dividend-yield surprise for the  $k^{th}$  hedge portfolio. The term  $y_k = \sum_{i=1}^n h_i^k y_i$  represents the weighted average of dividend-surprise yield for the  $k^{th}$  hedge portfolio. Solving for  $A$  from equation (6.3), we have:

$$-A = \frac{\mu_{c_m} + \mu_{d_m} + \mu_{y_m} - r_f + \sigma_{r_m,y} B}{\sigma_{r_m}^2}. \quad (6.5)$$

Next, we further define  $\mu_{c_h}$  as a  $n \times 1$  vector with elements  $\mu_{c_h}^k$ ,  $\mu_{d_h}$  as a  $n \times 1$  vector with elements  $\mu_{d_h}^k$ , and  $\mu_{y_h}$  as a  $n \times 1$  vector with elements  $\mu_{y_h}^k$ . Substituting the above solution to  $A$  into equation (6.4) and simplifying leads to the following equation:

$$\mu_{c_h} + \mu_{d_h} + \mu_{y_h} - r_f 1 = \frac{\sigma_{r_h r_m}}{\sigma_{r_m}^2} (\mu_{c_m} + \mu_{d_m} + \mu_{y_m} - r_f) + \left( \frac{\sigma_{r_h r_m}}{\sigma_{r_m}^2} \sigma_{r_m,y} - \sigma_{r_h,y} \right) B.$$

Define  $\Gamma_r$  as the following  $n \times n$  matrix:

$$\Gamma_r = \beta^m \Sigma_{r_m,y} - \Sigma_{r,y}, \text{ where } \beta^m = \frac{\sigma_{r,r_m}}{\sigma_{r_m}^2} \text{ is a } n \times 1 \text{ vector, and } \Gamma_{r_h} \text{ as the following } n \times n \text{ matrix:}$$

$$\Gamma_{r_h} = \beta^h \Sigma_{r_m,y} - \Sigma_{r_h,y}, \text{ where } \beta^h = \frac{\sigma_{r_h,r_m}}{\sigma_{r_m}^2} \text{ is a } n \times 1 \text{ vector.}$$

Assuming that the inverse matrix  $\Gamma_{r_h}^{-1}$  exists, we solve for B:

$$B = \Gamma_{r_h}^{-1} [\mu_{c_h} + \mu_{d_h} + \mu_{y_h} - r_f 1 - \beta^h (\mu_{c_m} + \mu_{d_m} + \mu_{y_m} - r_f)] = \Gamma_{r_h}^{-1} \pi^h - \beta^h \pi^m, \quad (6.6)$$

where  $\pi^m = \mu_{c_m} + \mu_{d_m} + \mu_{y_m} - r_f$  represents the expected dividend-yield-adjusted market risk premium, and  $\pi^h = \mu_{c_h} + \mu_{d_h} + \mu_{y_h} - r_f 1$  is a  $n \times 1$  vector of hedge portfolio risk premiums adjusted with the dividend-yield. Substituting solutions (6.5) and (6.6) for A and B into equation (6.2), we obtain the result as follows:

$$\tilde{\mu}_r = -A \sigma_{r,r_m} - \sigma_{r,y} B = \frac{\sigma_{r,r_m}}{\sigma_{r_m}^2} \pi^m + \left( \frac{\sigma_{r,r_m}}{\sigma_{r_m}^2} \sigma_{r_m,y} - \sigma_{r,y} \right) B = \beta^m \pi^m + \Gamma_r \Gamma_{r_h}^{-1} [\pi^h - \beta^h \pi^m] \quad (6.7)$$

where the vector term  $[\pi^h - \beta^h \pi^m]$  represents the discrepancy in predicted risk premium on hedge portfolios with respect to market portfolio, and the expected hedge portfolio risk premiums are determined by the hedge portfolio returns' sensitivities to the market portfolio, that is  $\beta^h$ .

Substituting for  $\Gamma_r$  and  $\Gamma_{r_h}$ , we rewrite Equation (6.7) to obtain our dividend-yield surprise adjusted asset-pricing equation, that is:

$$\mu_{c_i} + \mu_{d_i} + \mu_{y_i} - r_f = \beta_i^m (\mu_{c_m} + \mu_{d_m} - r_f) + \beta_i^m \mu_{y_m} + H_i, \quad (6.8)$$

where  $\beta_i^m = \frac{\sigma_{r_i,r_m}}{\sigma_{r_m}^2}$  and  $H_i = \sum_{k=1}^n \sum_{j=1}^n (\beta_i^m \sigma_{r_m,y_j} - \sigma_{r_i,y_j}) b_{r_h,j,r_h,k} (\pi_k^h - \beta_k^h \pi^m)$ . The term  $(\beta_i^m \sigma_{r_m,y_j} - \sigma_{r_i,y_j})$  is an element of the  $n \times n$  matrix  $\Gamma_r$  in Equation (6.7),  $\forall i = 1, 2, \dots, n$ , and

$\forall j = 1, 2, \dots, n$ . The term  $b_{r_h,j,r_h,k}$  denotes an element of the inverse matrix  $\Gamma_{r_h}^{-1}$ ; and  $(\pi_k^h - \beta_k^h \pi^m)$  reflects the  $k^{\text{th}}$  hedge portfolio's mean pricing error under the CAPM,  $\forall k = 1, 2, \dots, n$ .