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Fisher College of Business
Working Paper Series

Charles A. Dice Center for Research in Financial Economics

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Dice Center WP 2023-14
Fisher College of Business WP 2023-03-014

The Subjective Risk and Return Expectations of Institutional Investors

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This Version: December 2024§

Abstract

We use the long-term Capital Market Assumptions of major asset managers and institutional investor consultants from 1987 to 2022 to study their subjective risk and return expectations across 19 asset classes. We find a strong and positive subjective risk-return tradeoff, with the vast majority of the variability in subjective expected returns coming from subjective risk premia (compensation for market beta) rather than subjective alphas. Belief variation and the positive risk-return tradeoff are both stronger across asset classes than across institutions, underscoring the need to study subjective beliefs across multiple asset classes. We also show that subjective expected returns aggregated across institutions predict future realized returns across asset classes and over time, with most of this predictability driven by subjective risk premia, not alphas. Overall, our findings suggest it is important to include a strong risk premia component when modeling the subjective return expectations of institutional investors.

JEL Classification: G11, G12, G23

Keywords: Institutional Investors, Subjective Beliefs, Subjective Expected Returns, Subjective Risk, Subjective Risk Premia.

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§We are very grateful to the institutions that shared data on Capital Market Assumptions with us. This paper benefited a lot from conversations with members of these institutions as well as from helpful comments from Federico Bastianello, Francesca Bastianello, David Brown, Scott Cederburg, Magnus Dahlquist, Ricardo De la O, Anisha Ghosh, Ahmed Gueciouer, Markus Ibert, Alberto Manconi, Sean Myers, Stefan Nagel, Michael O'Doherty, Cameron Peng, Xiao Yin, Paul Whelan, and seminar participants at the Arizona State University, Norwegian School of Economics, Penn State University, Purdue University, Rutgers University, The Ohio State University, University of Notre Dame, University of Southern California, Vienna Graduate School of Finance, 2024 Adam Smith Workshop, 2024 AFA Meeting, 2024 Alpine Finance Summit, 2024 EFA Meeting, 2024 Finance Conference at WashU, 2024 FIRS Meeting, 2024 Helsinki Finance Summit, 2024 NFA Meeting, 2024 UW Summer Finance Conference, 2023 EUROFIDAI-ESSEC Paris December Meeting, 2023 INSEAD Finance Symposium, 2023 Annual Valuation Workshop at USC, and 2023 Brazilian Finance Society Meeting.

Introduction

The link between risk and return is a cornerstone of asset pricing. The literature typically explores this topic by linking measures of risk to the subsequent realized returns of financial assets (e.g., Fama and MacBeth (1973)). This approach recovers the risk-return tradeoff investors perceive if they have rational expectations (i.e., their expectations are objective, or reflect the true return generating process). However, if investors' subjective risk and return expectations deviate from rational expectations, then there is a disconnect between the risk-return tradeoff they perceive and the one implied by realized returns. This is similar to the disconnect between subjective and objective sources of price variation recently documented in the literature (e.g., De La O and Myers (2021) and De La O, Han, and Myers (2024)).

The literature on subjective beliefs (reviewed by Adam and Nagel (2023)) documents important deviations from rational expectations based on the subjective expected returns of different economic agents. In contrast, large asset managers have subjective equity premia that reasonably reflect the cyclical properties of the objective equity premium (Dahlquist and Ibert (2024a)). However, there is little work on the connection between subjective risk and return expectations, particularly for institutional investors and across asset classes. As Adam and Nagel (2023) put it, "*We need more work that explores how investors...risk perceptions are linked to the subjective risk premia that they demand to hold risky assets*."

In this paper, we fill this important gap in the literature by studying the long-term Capital Market Assumptions (CMAs) of major asset managers and institutional investor consultants. In doing so, we evaluate their subjective risk and return expectations across 19 asset classes from 1987 to 2022. We uncover two important stylized facts. First, most of the variability in subjective expected returns arises from subjective risk premia (compensation for market beta) rather than subjective alphas. Figure 1(a) shows a strong and positive subjective risk-return tradeoff, with a pooled regression of subjective expected returns on subjective market betas yielding an R^2 of 69%. This positive risk-return relation (and overall belief variation) is stronger across asset classes than across institutions, highlighting the importance

(a) Subjective Expected Returns vs Risk **(b) Expected Returns: Data vs Beliefs**

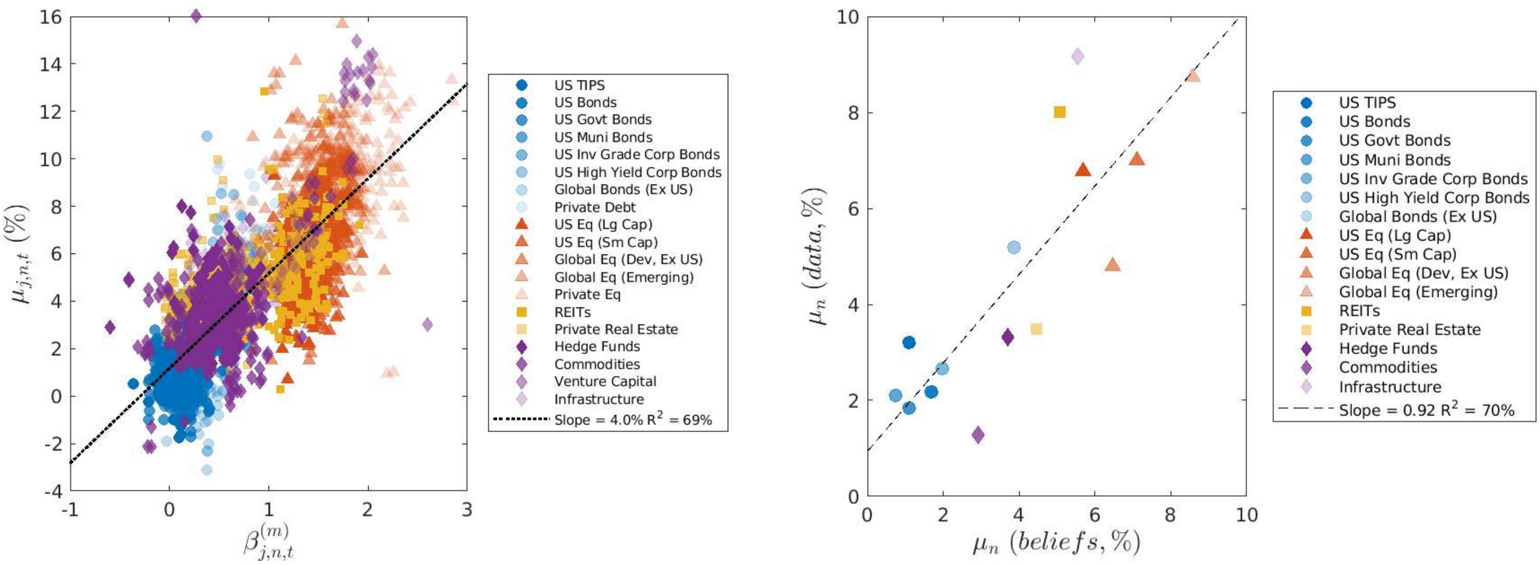

Figure 1
Summary of Main Results

Panel (a) plots subjective expected excess returns ($\mu_{j,n,t}$) against subjective risk (market betas, $\beta^{(m)}_{j,n,t}$), with the market proxied by the aggregate wealth portfolio of US pension funds. Subjective beliefs are obtained from our CMAs and vary across investors (j), asset classes (n), and years (t). Panel (b) plots subjective μ_n (aggregated over time and across investors) against the respective μ_n from average realized excess returns. See Section 1 for details on our subjective beliefs data and Section 3 for the matching of beliefs to realized returns.

of studying beliefs across multiple asset classes. Second, institutional investors' subjective beliefs largely (but not fully) align with objective beliefs. Figure 1(b) shows a regression of realized returns (averaged over the longest available sample for each asset class) on average subjective expected returns. It has a slope near 1 and an R^2 of 70%, with similar results for data moments related to risk (volatilities and betas). Furthermore, subjective expected returns predict future realized returns across asset classes and over time. Importantly, most of this predictability is driven by subjective risk premia rather than alphas, as subjective alphas do not predict future realized alphas. Overall, our findings highlight the importance of incorporating a strong subjective risk premia component when modeling institutional investors' subjective return expectations.

We begin by describing our dataset on the long-term CMAs of institutional investors. It represents an unbalanced panel covering cash and 19 other asset classes from 1987 to 2022 (see Figure 1(a)). For each institution-year, the CMAs data include expected returns, volatilities, and return correlations across asset classes. The subjective volatilities and correlations reflect a key innovation in our paper as they jointly allow us to compute subjective variances and covariances. We use these subjective return moments to calculate their excess of cash counterparts. This provides us with subjective expected excess returns (μ) and subjective market betas ($\beta^{(m)}$) for each institution (j), asset class (n), and year (t). The betas are based on two market proxies: the US equities large cap asset class (leading to an "Equity CAPM") and the aggregate portfolio of US pension funds (leading to a "Pension CAPM").

Regressing μ on $\beta^{(m)}$ under the Pension CAPM leads to Figure 1(a), with a slope coefficient of $\lambda^{(m)} = 4.0\%$. However, risk-based asset pricing models allow the slope coefficient (or market risk premium) to vary across institutions and over time. So, we also show that $\lambda^{(m)} > 0$ for all institution-years in regressions of μ_n onto $\beta_n^{(m)}$ across asset classes (omitting the j, t subscript for simplicity). Doing so allows us to study CAPM restrictions since the model implies $\lambda^{(m)} = \mu^{(m)} - \lambda^{(0)}$, where $\lambda^{(0)}$ reflects the intercept of the regression with $0 \leq \lambda^{(0)} < \mu^{(m)}$ (Black (1972)). We find that these restrictions describe our data reasonably well, but that we can statistically reject the slope restriction. As such, subjective risk and return expectations are largely (but not entirely) consistent with our CAPM implementation.

To understand the relative importance of the risk-return tradeoff, we decompose subjective expected returns (μ_n) into subjective risk premia ($rp_n^{(m)} = \lambda^{(0)} + \lambda^{(m)} \cdot \beta_n^{(m)}$) and subjective alphas ($\alpha_n^{(m)} = \mu_n - rp_n^{(m)}$) for each institution-year. We analyze a restricted CAPM version ($\lambda^{(0)} = 0$ and $\lambda^{(m)} = \mu^{(m)}$) as well as an unrestricted version that uses regression estimates for $\lambda^{(0)}$ and $\lambda^{(m)}$. We find that $\alpha_n^{(m)}$ tends to be economically small and only weakly related to μ_n for our asset classes. Even for maximum Sharpe ratio portfolios, subjective alphas remain small unless short positions are allowed, which is impractical at the asset class level.

We further quantify the risk-return tradeoff by decomposing μ variation into $rp^{(m)}$ and $\alpha^{(m)}$ components. We find that the vast majority of μ variability (76% to 91% across CAPM specifications) arises from $rp^{(m)}$, with the remaining variability driven by $\alpha^{(m)}$. In contrast, $\alpha^{(m)}$ plays a quantitatively larger role in explaining institutional disagreement, accounting for 30% to 50% of μ variability across institutions depending on the CAPM specification.

The prior paragraph highlights how belief heterogeneity across asset classes (risk-return tradeoff) and institutions (disagreement) play distinct roles in shaping overall beliefs. To explore this further, we decompose within-year expected return variability using institution and asset class fixed effects. Asset class fixed effects drive over 80% of this variability, which explains why expected return variation is largely driven by subjective risk premia, and not alphas. Specifically, alphas are important for explaining expected return disagreement across institutions. However, even though disagreement is non-trivial, it is minor in comparison to the expected return variation across asset classes (which is mostly explained by risk premia).

Having established a tight link between subjective risk and return expectations, we turn to studying the connection between subjective beliefs and realized returns. To start, we estimate regressions across asset classes of average expected returns, volatilities, and betas on their realized return counterparts averaged over the longest available sample for each asset class. We find slope coefficients close to 1 and high R^2 values (e.g., see Figure 1(a)). In contrast, the connection between average subjective and realized alphas across asset classes is weak.

The link between subjective beliefs and realized return moments is not only present across asset classes on average, but also when we compare subjective beliefs with future realized return moments. For instance, a pooled regression of $r_{n,t+1}$ on $\mu_{n,t}$ (aggregated across institutions and annualized) has a near-zero intercept, a slope close to one, and an R^2 above 6%. We also calculate out-of-sample R^2_{OOS} values to contrast the predictive ability of μ with that of historical average returns. We find R^2_{OOS} values that exceed 12% for both predictability across asset classes and over time. The R^2 and R^2_{OOS} values are mainly driven by risk premia, not alphas, showing that $rp^{(m)}$ determines the μ predictive ability. Similar results (with stronger R^2) appear in pooled panel regressions of year $t+1$ realized risk on year t subjective risk. However, subjective risk only predicts future realized risk across asset classes, not over time. Moreover, alphas fail to predict future realized alphas across asset classes or over time.

Contribution to the Literature

We contribute to the subjective beliefs literature in asset pricing.[1] The review by Adam and Nagel (2023) outlines two important directions for future research: (i) studying the relationship between subjective risk and return expectations and (ii) investigating the beliefs of less-studied agents (e.g., institutional investors). Specifically, Adam and Nagel (2023) emphasize the need to understand "how investors'...risk perceptions relate to the subjective risk premia they require for holding risky assets," and they question whether "investor groups typically omitted from individual investor surveys exhibit systematically different expectations."

Our work advances the literature on both of these dimensions. For direction (i), we examine the connection between subjective risk and return expectations across asset classes. Some prior papers analyze how subjective expected returns relate to (objective or subjective) risk measures within equities. They do so through surveys of households or individual investors (e.g., Amromin and Sharpe (2014), Giglio et al. (2021), Gnan and Schleritzko (2023), and Jo, Lin, and You (2024)) or through the views of professional forecasters, sell-side analysts, and CFOs (e.g., Brav, Lehavy, and Michaely (2005), Wu (2018), Bastianello (2024), Jensen (2024), Nagel and Xu (2023), and Gormsen and Huber (2024b)).[2] We add to this literature by studying the subjective risk and return expectations of *institutional investors for many asset classes over multiple decades.* A key innovation is that we are able to measure subjective risk

[1]The literature is too large to summarize. Nevertheless, some recent (including some contemporaneous and subsequent) papers that study subjective risk and/or return expectations are Chen, Da, and Zhao (2013), Greenwood and Shleifer (2014), Piazzesi, Salomao, and Schneider (2015), Adam, Marcet, and Beutel (2017), Cieslak (2018), Wu (2018), Adam, Matveev, and Nagel (2021), De La O and Myers (2021), Giglio et al. (2021), Nagel and Xu (2023), Wang (2021), Andonov and Rauh (2022), Lochstoer and Muir (2022), Beckmeyer and Gueciouer (2023), Begenau, Liang, and Siriwardane (2023), Beutel and Weber (2023), Gnan and Schleritzko (2023), Boons, Ottonello, and Valkanov (2023), Gandhi, Gormsen, and Lazarus (2023), Bastianello (2024), Bastianello and Fontanier (2024a,b), Bastianello and Peng (2024), Balloch and Peng (2024), Dahlquist, Ibert, and Wilke (2024), Dahlquist and Ibert (2024a,b), Décaire and Graham (2024), Décaire, Sosyura, and Wittry (2024), De La O, Han, and Myers (2024), Egan, MacKay, and Yang (2024), Fukui, Gormsen, and Huber (2024), Gormsen and Huber (2024a,b), Gormsen, Huber, and Oh (2024), Jensen (2024), Jo, Lin, and You (2024), Kremens, Martin, and Varela (2024), and Loudis (2024).

[2]The risk-return analysis by Jo, Lin, and You (2024) covers four asset classes beyond cash and equities. Notably, they find that household survey responses indicate a negative subjective risk-return tradeoff across asset classes, contrasting with the positive tradeoff we find in institutional investors' CMAs.

in the form of market betas alongside subjective expected returns. This allows us to quantify the roles of risk premia and alphas in subjective expected returns. For instance, we find that more than 75% of the variation in subjective expected returns is driven by subjective risk premia. Finally, we show that subjective expected returns predict future realized returns across asset classes and over time, which contrasts the common finding that households display extrapolative return expectations (e.g., Greenwood and Shleifer (2014)).

In terms of direction (ii), we study the CMAs of asset managers and institutional investor consultants, which differ drastically from the commonly used surveys of households, individual investors, and CFOs as well as from reports of professional forecasters and sell-side analysts. In this context, the closest paper to ours in the prior literature is Dahlquist and Ibert (2024a). They explore the CMAs of asset managers (with a brief analysis of consultants in their appendix) and find that their subjective equity premia are countercyclical, with the degree of countercyclicality roughly matching that of the objective equity premium. They also demonstrate that these subjective equity premia affect portfolio allocations.

We add to Dahlquist and Ibert (2024a) in three important ways. First, we study 19 asset classes simultaneously instead of exploring only equities. Second, we focus on the link between subjective risk and return expectations across asset classes, which they do not study. And third, we provide a comprehensive analysis of the link between subjective risk and return expectations with their respective realized return moments. It is also important to point out that we substantially expand their sample in the time-series dimension (as documented later). Using our longer time series, we confirm and extend one of their main findings, namely that the subjective equity premium is countercyclical (i.e., it moves together with the S&P 500 earnings yield).

The rest of this paper is organized as follows. Section 1 details our subjective beliefs data, Section 2 explores the link between subjective risk and return expectations, and Section 3 studies the connection between subjective beliefs and realized returns. Section 4 concludes, with the Internet Appendix providing data details and supplementary empirical results.

1 Subjective Risk and Return Expectations: Motivation and Data

We start by motivating our analysis of subjective risk and return expectations in Subsection 1.1. Then, Subsection 1.2 describes Capital Market Assumptions (CMAs), Subsection 1.3 details the dataset of CMAs we use, and Subsection 1.4 evaluates the comprehensiveness of our CMAs dataset by comparing it to the dataset in Dahlquist and Ibert (2024a).

Throughout this paper, t indexes time (i.e., year), n indexes asset (i.e., asset class), and j indexes investor (i.e., institution). Moreover, R reflects gross returns while $r = R - R_f$ reflects excess returns relative to a benchmark asset, which we refer to as the risk-free asset (and proxy for by using the US Cash asset class). Finally, to simplify notation, we suppress time indexes inside moments when convenient (e.g., $\mathbb{E}_{j,t}[r_n] \equiv \mathbb{E}_{j,t}[r_{n,t \to t+h_j}]$).

1.1 Motivation for Studying Subjective Risk and Return Expectations

In traditional asset pricing models, investors have preferences that lead to a tradeoff between risk and return. Loosely speaking (and suppressing t for simplicity), these models lead to first order conditions of the form $\mathbb{E}_j[r_n] = \lambda_j \cdot \beta_{j,n}$, where $\beta_{j,n}$ reflects investor j perception of asset n risk exposure and $\lambda_j > 0$ reflects the compensation per unit of risk required by investor j. So, traditional asset pricing models imply a positive link between subjective risk and return expectations. Historically, these models have been tested under the added assumption that investors have full-information rational expectations (FIRE). In this case, investor's first order conditions imply $r_{n,t+1} = \lambda \cdot \beta_n + \varepsilon_{n,t+1}$, which can be tested using realized returns.

However, as we formally show in Internet Appendix A, if investors do not have FIRE, then tests based entirely on realized returns can "reject" valid asset pricing models or "accept" invalid asset pricing models. As such, it is essential to study the link between risk and return implied by asset pricing models using data on subjective risk and return expectations. Subjective beliefs can also be combined with realized returns to help us understand what aspects of belief dynamics are consistent with FIRE.

Given the importance of studying subjective risk and return expectations, the rest of this section details the dataset we build for this purpose. It focuses on institutional investors since much of the work on subjective beliefs covers other agents (Adam and Nagel (2023)). The two subsequent sections then study the link between the subjective risk and return expectations of institutional investors and relate their subjective beliefs to realized returns.

1.2 Capital Market Assumptions (CMAs)

Investment institutions often create long-term outlooks on the risk and return profiles of various asset classes. These outlooks are generally called Capital Market Assumptions (CMAs).[3] As Envestnet PMC states in their 2023 CMAs methodology report,

"Capital markets assumptions are the expected returns, standard deviations, and correlation estimates that represent the long-term risk/return forecasts for various asset classes. We use these values to score portfolio risk, assist advisors in portfolio construction, construct our own asset allocation models and create Monte Carlo simulation inputs for portfolio wealth forecasts."

Relatedly, JP Morgan states in their 2022 CMAs report,

"We formulate our Long-Term Capital Market Assumptions (LTCMAs) as part of a deeply researched proprietary process that draws on quantitative and qualitative inputs as well as insights from experts across J.P. Morgan Asset Management. Our own multi-asset investment approach relies heavily on our LTCMAs: The assumptions form a critical foundation of our framework for designing, building and analyzing solutions aligned with our clients' specific investment needs."

It is clear from these statements that CMAs are created at least in part to guide asset allocations and to advise clients on their asset allocations. In fact, these institutions often employ teams of economists, researchers, and financial analysts to construct and refine their CMAs. Institutions also often analyze quantitative data as part of this process. Doing so enables them to evaluate how they expect various factors to affect long-term asset returns.

[3]Hereafter, we sometimes use the term "CMA" to refer to a single CMAs report and the term "CMAs" to refer to multiple CMAs reports (the terminology usage should be clear from the context).

A corollary of the above paragraph is that CMAs are not responses to questions designed by a third party research team (as is the case with much of the literature on subjective return expectations). Instead, CMAs are fully developed documents produced systematically and organically by institutions. Moreover, CMAs tend to rely heavily on quantitative financial research. As such, the subjective beliefs implied from CMAs are fundamentally different from the subjective beliefs implied from typical surveys. For instance, disagreement across two institutions more plausibly reflects differences in quantitative modeling techniques, inputs, and assumptions, rather than differences in psychological processes of belief formation.

An important consequence of the above discussion is that CMAs plausibly reflect beliefs that are sophisticated (relative to beliefs derived from surveys of individual investors). We provide evidence that largely supports this view. In particular, CMA-implied subjective expected returns are deeply linked to subjective risk (measured by subjective market betas) and predict future realized returns (mainly through their subjective risk premia component).

As anecdotal evidence for the sophistication of CMAs (and for the quantitative results we later provide), CMAs sometimes directly discuss concepts such as market beta and time-varying risk premia. For instance, the Envestnet PMC 2023 CMAs methodology report explicitly states that they use the assumptions that, "The global capital markets are largely efficient in the long run, where the efficiency of the markets is measured by the Capital Asset Pricing Model (CAPM)" and that, "Risk premia are time-varying". It even points out that the CAPM is quantitatively used through the Black and Litterman (1991, 1992) methodology: "To obtain the long-term expected return estimates as implied by the CAPM...we use Black-Litterman (Black & Litterman, 1991) methodology."

Importantly, it is likely that CMAs reflect concepts of risk and time-varying risk premia because the agents producing these CMAs are sophisticated and read academic papers (many even have PhDs in quantitative fields). For instance, in talking with those (teams) responsible for producing CMAs, it is clear that they are familiar with asset pricing research, especially research on risk factors and on the predictability of future returns by macroeconomic variables, yields, and valuation ratios. While understanding how research affects the belief formation underlying CMAs is interesting, our paper focuses on the belief properties of CMAs (i.e., how subjective risk and return expectations are connected to each other and to future returns). The reason is that in traditional asset pricing models belief properties (however formed) affect portfolio allocations, which determine asset prices in equilibrium. So, we study the properties of CMA-implied beliefs in this paper while exploring belief determinants and the connection between beliefs and portfolio allocations in our subsequent work (Couts et al. (2024) and Andonov et al. (2024)).

1.3 Our CMAs Dataset

Our beliefs dataset goes from 1987 to 2022 and is based on the CMAs of asset managers (or "managers" for short) and investment consultants (or "consultants" for short). The details about our data collection and belief measurement are provided in Internet Appendix B.1. Our goal is to understand the risk and return expectations of institutional investors. As such, the inclusion of managers is important. We also include consultants as we argue that their views provide an indirect way to learn about the beliefs of institutional investors.[4] This argument is valid if (i) the beliefs of institutional investors and investment consultants are simultaneously based on common signals and/or (ii) the beliefs of investment consultants have a causal impact on the beliefs of institutional investors. While we do not attempt to explore these two channels in our paper, Internet Appendix C.3 shows that our main results are similar if we use only managers or only consultants.

Our sample is based on the long-term CMAs of 45 institutions in total (22 managers and 23 consultants). The forecasting horizons vary from 4 years to 30 years, with the modal horizon being 10 years (which comprises 45.2% of the institution-year observations for which the horizon is available).[5] The bulk of the data (83.4% of the institution-year observations)

[4] Begenau, Liang, and Siriwardane (2023) show that consultant fixed effects explain a large fraction of the variation in the portfolio allocation of pension funds and Andonov et al. (2024) show that the beliefs and portfolio allocations of pension funds are connected to the CMAs of their consultants.

[5] In our baseline analysis, we combine data from all institution-year observations regardless of forecasting horizon (keeping the 10-year horizon when multiple horizons are available). This empirical decision does not create any conceptual issue with our analysis of the subjective risk-return tradeoff in Section 2 since a version

comes directly from the CMAs of the institutions we cover through direct data requests and/or online searches for their CMAs. However, we supplement the direct CMAs of these institutions with indirect CMAs obtained from pension funds through their internal reports.[6] In Internet Appendix C.3, we show that our results are similar whether we rely only on the direct CMAs of these institutions or only on the indirect CMAs we obtain from pension funds, alleviating potential concerns with either data collection approach.

Table 1 shows the list of managers and consultants that have at least one CMA in our sample.[7] Our sample covers many of the major asset managers and investment consultants. These managers have Assets Under Management (AUM) totaling more than $37 Trillion at the end of 2021. This is equivalent to more than 42% of the total AUM among the top 50 asset managers in the world (by AUM). Our consultants encompass the primary consultant's for more than 50% of the US public pension funds in a typical year over the sample period. This represents more than 70% of the AUM in this class of funds.[8]

The first five rows in Panel A of Table 2 provide the number of institutions in our sample by year as well as the split between managers and consultants.[9] To conserve space, we show

of the CAPM holds for any given investors' horizon (see Levhari and Levy (1977)) and also when investors have heterogeneous horizons (see Lee, Wu, and Wei (1990) and Brennan and Zhang (2020)). However, in Internet Appendix C.3, we show that our main results are similar if we use only CMAs with a 10 year horizon.

[6]Many pension funds list the long-term CMAs of their consultants or relevant asset managers in their internal reports (such as their capital allocation reports and/or in their CMAs reports).

[7]Two comments are in order. First, our definition of "investment consultants" is broad enough to encompass (i) institutions that focus on consulting for institutional investors (e.g., Callan), (ii) institutions that provide broad investment advising services that reach asset managers, institutional investors, and retail investors (e.g., Research Affiliates), and (iii) institutions (often called "wealth advisers") that tend to focus more on investment advising for wealthy individuals (e.g., CWO). Second, many institutions (e.g., BNY Mellon, Cliffwater, and Envestnet) have both an asset management business as well as a consulting or wealth advising business. The classification in Table 1 is based on our (somewhat subjective) assessment of whether the asset management side of a given institution is large enough to justify classifying it as an asset manager (e.g., by analysing their AUM and website self-descriptions). However, our classification has no impact on our main analysis since all results presented in the main text are based on a sample that combines asset managers and investment consultants.

[8]The underlying data on the primary consultants of US public pension funds come from Center for Retirement Research at Boston College (which starts in 2001). Some of our consultants are included in the "All Others" category in this dataset, which means we cannot identify how many pension funds have them as primary consultants in a typical year. As such, the total coverage of US pension funds and their AUM provided in Table 1 is a lower bound on the true coverage of the consultants included in our dataset.

[9]Years are defined based on the approximate timing of the institution's information set. For instance, if

years 1987, 1996, 1997, and 1998 to 2022 in steps of two years (but Internet Appendix Table IA.1 reports all years). From 1987 to 1996, our dataset covers a single institution. However, in 1997 two new institutions enter the dataset, which then grows over time, for a total of 361 institution-year observations. Note that the maximum number of institutions in any given year is 30 even though we have 45 unique institutions in our dataset. The reason is that the data for some institutions come from CMAs we obtain online or from pension fund reports, nether of which ensures continuous coverage of a given institution over time. For all data sent to us directly by the underlying institutions, our coverage has no gap years.

Each institution-year CMA covers a range of asset classes. Our final sample contains a risk-free asset class proxy (*US Cash*) as well as 19 risky asset classes. We narrow down our study to these 19 risky asset classes by considering three criteria.. First, whether the asset class is a major asset class for institutional investors. Second, whether the asset class is covered by a reasonable number of institutions in our sample. And third, whether the asset class is covered over a reasonably long time period in the sample. We include all asset classes that perform well along these dimensions. Because the asset class names and corresponding indexes can differ both across institutions and over time, we use our judgment when mapping asset classes within and across institutions to the asset classes included in our final sample. Internet Appendix B.1 provides further details.

The last two rows in Panel A of Table 2 show the number of unique asset classes (including *US Cash*) as well as the average number of asset classes per institution. The sample starts with 4 asset classes in 1987, growing to 20 asset classes by 2011 and thereafter. There is some variation in the coverage of asset classes across institutions and over time, but since 2004 the average institution covers at least 11 asset classes.

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a CMA contains $\mathbb{E}_t[R_{t\to t+10}]$, then our year variable is t. Figure IA.3 in the Internet Appendix shows the distribution of CMA production months. While most CMAs associated with year t have a production month of December of year t, some CMAs have a production month from a few months before or a few months after December of year t. The most common alternative to December of year t is January of year $t+1$, which reflects the fact that some institutions form their CMAs with information as of December of year t and build their CMA file in January of year $t+1$. We return to this issue when discussing return forecasting regressions in Subsection 3.4.

Panel B of Table 2 shows the name of each asset class and the number of institutions covering it in a given year. We require *US Cash* to be covered for each institution-year so that it is always available for use as a base asset when computing excess expected returns. Our data cover four risky asset class categories: Fixed Income, Equities, Real Estate, and Alternatives. For Fixed Income, we have eight asset classes covering different parts of the debt market, with *US Bonds*, *Global Bonds (Ex US)*, and *Private Debt* being three broad ones and *US TIPS*, *US Govt Bonds*, *US Muni Bonds, US Inv Grade Corp Bonds*, and *US High Yield Corp Bonds* being more specialized ones. For Equities, we have five asset classes, with *US Equities (Large Cap)* and *US Equities (Small Cap)* covering the US public market, *Global Equities (Dev, Ex US)* and *Global Equities (Emerging)* covering the international public market, and *Private Equity* covering the private market. For Real Estate, we have two asset classes, with *REITs* covering the public market and *Private Real Estate* covering the private market. Finally, we have *Hedge Funds*, *Commodities*, *Venture Capital*, and *Infrastructure* as the four asset classes under our Alternatives asset class category.

Like *US Cash*, the *US Equities (Large Cap)* asset class is covered by all institutions present in any given year, with a total of 361 institution-year observations. However, there is variation in the level of coverage among other asset classes. For instance, *US Bonds*, *Global Equities (Dev, Ex US)*, and *Private Real Estate* have excellent coverage (each with more than 300 institution-year observations), while *Private Debt*, *Venture Capital*, and *Infrastructure* have more modest coverage (each with less than 100 institution-year observations).

Our sample (as reflected in Table 2) only includes CMAs that contain expected returns, volatilities, and correlations data since all three variables are needed for our analysis.[10] From these belief quantities, we construct analogous belief quantities for returns on our 19 risky

[10]CMAs can differ in their definition of expected returns. In particular, 77.8% of our CMAs report expected arithmetic returns while 58.4% of our CMAs report expected geometric returns (with 36.3% of our CMAs reporting both). To ensure the conceptual definition underlying our expected return measure is the same for all institution-year observations, we always use expected arithmetic returns in our baseline analysis (since they are the most commonly reported). For the 22.2% of CMAs that do not report expected arithmetic returns, we obtain them from the reported beliefs under the properties of a log-Normal distribution (see Internet Appendix B.1 for details). Internet Appendix C.3 provides results (similar to our baseline results) using expected geometric returns as the baseline conceptual definition underlying our expected return measure.

asset classes (indexed by n) in excess of our risk-free asset class (indexed by f). Specifically, letting $\mathbb{E}_{j,t}[R]$ and $\Sigma^R_{j,t}$ represent the subjective expected return vector and covariance matrix for institution j at time t (with *US Cash* as the first asset class), we construct expected excess returns ($\mu_{j,n,t} \equiv \mathbb{E}_{j,t}[r_n] = \mathbb{E}_{j,t}[R_n] - \mathbb{E}_{j,t}[R_f]$) and covariance matrices of excess returns ($\Sigma_{j,t} = \Omega\Sigma^R_{j,t}\Omega'$, where $\Omega = [-1, \mathrm{I}]$, with 1 representing a column vector of ones and I an identity matrix). Our analysis is based on these $\mu_{j,n,t}$ and $\Sigma_{j,t}$ values (for simplicity, the rest of the text refers to μ as expected returns instead of expected excess returns). As an example of the data we observe, Internet Appendix Table IA.2 shows the average values of $\mathbb{E}_{j,t}[R]$ and $\Sigma^R_{j,t}$ as well as $\mu_{j,n,t}$ and $\Sigma_{j,t}$ at the end of 2022 pooled across institutions.

1.4 The Comprehensiveness of our CMAs Dataset

Dahlquist and Ibert (2024a) assemble a comprehensive dataset of subjective equity expected returns from CMAs. Since a substantial portion of their paper focuses on the portfolio allocations of mutual funds, their main text studies asset managers, with an analysis of investment consultants provided in their internet appendix. Given our goal of studying subjective risk and return expectations implied by CMAs, we keep both managers and consultants in our main analysis. Furthermore, we require our CMA data to include expected volatilities and correlations (beyond expected returns), which allows us to compute our risk measures. While this extra requirement could lead to a less comprehensive CMAs dataset, this is not the case empirically. For instance, we have 22 managers and 23 consultants in our main analysis whereas they have 22 managers in their main analysis with 25 consultants in their internet appendix. Moreover, our dataset covers 361 institution-year observations whereas theirs covers 240 institution-year observations (with 135 institution-year observations studied in their main text).[11] As Internet Appendix Figure IA.2 shows, our dataset has better time-

[11]Note that we focus on the risk and return expectations of multiple asset classes for annual observations of managers and consultants (leading to our 361 institution-year CMA observations, with an average of 12 risky asset classes per institution-year). In contrast, Dahlquist and Ibert (2024a) focus on US equity expected returns for asset managers that include multiple forecasting horizons and multiple CMAs for a given manager-year (leading to the 383 US equity expected return observations from managers they cite in their main text). So, the total number of expected return observations are not comparable across the two papers. This is why

series coverage whereas their complete dataset (that includes the consultants in their internet appendix) covers more institutions in some of the most recent years.

Dahlquist and Ibert (2024a) show that the subjective equity premium from CMAs is coutercyclical. Figure 2 plots the subjective equity premium in our dataset aggregated across all institutions each year (based on *US Equities (Large Cap)*) together with the S&P 500 earnings yield (i.e., the log of the inverted CAPE ratio from Shiller), which Dahlquist and Ibert (2024a) use to identify the equity premium cyclicality in their data.[12] As it is clear from the figure, the subjective equity premium is also highly countercyclical in our dataset. More specifically, its correlation with the S&P 500 earnings yield is around 0.59. This finding further demonstrates the comprehensiveness of our dataset since it can be used to extend one of the main findings in Dahlquist and Ibert (2024a) on the time-series dimension, which is important as the equity premium countercyclicality is inherently a time-series property.

2 The Link Between Subjective Risk and Return Expectations

To study the link between subjective risk and return expectations, we need a measure of subjective risk. For this purpose, we use the market risk exposure (beta) of each asset class as perceived by each institution:

$$\beta^{(m)}_{j,n,t} = \frac{\mathbb{C}ov_{j,t}[r_n, r^{(m)}]}{\mathbb{V}ar_{j,t}[r^{(m)}]} = \frac{1_n' \Sigma_{j,t} w_{m,t}}{w_{m,t}' \Sigma_{j,t} w_{m,t}} \tag{1}$$

where the superscript (m) refers to the market portfolio, with weight vector $w_{m,t}$. Our subjective risk measure is motivated by the Capital Asset Pricing Model (CAPM), but our goal is to quantify the strength of the link between subjective expected returns (μ) and subjective risk ($\beta^{(m)}$) rather than to formally test the CAPM. Nevertheless, in the process of studying the connection between μ and $\beta^{(m)}$, we explore CAPM restrictions as they provide a useful

we evaluate the comprehensiveness of our dataset by comparing our number of institution-year observations relative to theirs from the RFS dataverse (which includes managers and consultants).

[12] Our aggregate subjective equity premium accounts for differences in sample composition over the years. The aggregation methodology details are provided in the header of Figure 2 and in Subsection 3.1.

benchmark for the link between μ and $\beta^{(m)}$.[13]

One challenge is that, as pointed out by Roll (1977), it is infeasible to obtain a perfect proxy for the market portfolio weights, $w_{m,t}$. With this issue in mind, we consider two different market proxies. The first leads to the Equity CAPM ($m = e$), with $w_{m,t} = 1_e$ based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with $w_{m,t}$ based on the aggregate allocations of US Public Pension Funds.[14]

The main advantages of the Equity CAPM are that (i) its market proxy is common in much of the literature that tests the CAPM using realized returns and (ii) we observe beliefs for *US Equities (Large Cap)* for all our institution-year observations. The downside of the Equity CAPM is that it relies on a market proxy that does not reflect the overall wealth portfolio or even the wealth portfolio of institutional investors. Our Pension CAPM addresses this issue as it considers a much broader market portfolio, with weights on several asset classes.[15] The downside of the Pension CAPM is that some institution-year observations do not have beliefs for some of the asset classes included in the aggregate portfolio of pension funds. In these cases, we adjust the market portfolio weights so that they continue to sum to one (effectively

[13]Two comments are important. First, we use the CAPM as a benchmark due to its simplicity and widespread use, not because it is the most comprehensive model to capture the risk-return tradeoff perceived by institutional investors. Second, even though the traditional CAPM derivation relies on homogeneous beliefs, the CAPM can still hold with mean-variance investors who have heterogeneous beliefs (see Lintner (1969) and Levy, Levy, and Benita (2006)), and thus there is no contradiction in studying CAPM restrictions while recognizing that beliefs are heterogeneous.

[14]Internet Appendix Section B.2 details our construction of the Pension CAPM wealth portfolio weights (see Figure IA.1 for the time-series of weights). The data come from the Center for Retirement Research at Boston College. The average weights (over time) are 2% for Cash (matched to *US Cash*), 34% for US Equities (matched to *US Equities (Large Cap)*), 20% for Int Equities (matched to *Global Equities (Dev, Ex US)* and, if missing, to *US Equities (Large Cap)*), 7% for Private Equity (matched to *Private Equity*), 24% for US Fixed Income (matched to *US Bonds* and, if missing, to *US Govt Bonds* or *US Inv Grade Corp Bonds*), 3% for Global Bonds (matched to *Global Bonds (Ex US)* and, if missing, to the asset class matched to US Fixed Income), 7% for Real Estate (matched to *REITs* and, if missing, to *Private Real Estate*), and 4% for Hedge Funds (matched to *Hedge Funds*).

[15]Note that an alternative interpretation for our Pension CAPM is that it reflects an asset pricing model based on the wealth portfolio of institutional investors. If institutional investors rely on a mean-variance framework, then we have an SDF that is linear in their wealth portfolio even if there is no SDF that is linear in the market portfolio (which combines the wealth portfolio of institutional investors with the wealth portfolio of other investors). This alternative interpretation is in line with Bretscher, Lewis, and Santosh (2024), who study the pricing of investor-specific betas using data on institutional investors' holdings.

allocating the weights for the missing asset class to the other asset classes in the portfolio).[16] While we generally view the Pension CAPM as a better model to capture the risk-return tradeoff perceived by institutional investors allocating capital to multiple asset classes, we present results for both the Equity CAPM and Pension CAPM throughout the paper.

The rest of this section is organized as follows. Subsection 2.1 focuses on the risk-return tradeoff, Subsection 2.2 studies subjective alphas, Subsection 2.3 decomposes the variation in subjective expected returns into risk premia and alphas, and Subsection 2.4 demonstrates that most of the variation in subjective expected returns reflects variation across asset classes.

2.1 The Positive Risk-Return Tradeoff

The first four columns of Table 3 provide, for each asset class, average values of subjective expected returns (μ) and subjective risk measures (σ, $\beta^{(e)}$, and $\beta^{(p)}$).[17] The key observation from these columns is that asset classes with higher μ also tend to have higher subjective risk, suggesting a positive relation between subjective risk and return. This positive subjective risk-return tradeoff is observed not only in average values, but also more broadly. In particular, Figure 1(a) in the introduction considers the pooled regression

$$\mu_{j,n,t} = \lambda^{(0)} + \lambda^{(m)} \cdot \beta^{(m)}_{j,n,t} + \alpha^{(m)}_{j,n,t}, \tag{2}$$

with the estimation leading to a strong and positive subjective risk-return tradeoff, summarized by the $R^2 = 69\%$ and $\lambda^{(m)} = 4.0\%$ ($t_{stat} = 12.5$ with clustering by j, n, and t). That figure uses $m = p$, but the positive subjective risk-return tradeoff is very similar if we use

[16]Empirically, this adjustment has only a small effect. Specifically, on average (across institution-year observations), only 3.7% of the wealth portfolio weights gets reassigned to other asset classes when we compute the $\beta^{(p)}_{j,t}$ vector associated with a given institution-year observation. The reason is that the asset classes that have the most weight on the wealth portfolio of pension funds are the ones for which our beliefs data is most complete.

[17]We report average asset class values to highlight variation across asset classes (which is the main focus of our paper), but there is also non-trivial variation over time and across institutions. For instance, Internet Appendix Table IA.4 presents an analogous table that summarizes variation across institutions (i.e., disagreement). Our subsequent work provides a much more detailed analysis of time variation and disagreement in subjective beliefs (see Couts et al. (2024)).

$m = e$ instead (in that case, $R^2 = 68\%$ and $\lambda^{(m)} = 5.4\%$, with $t_{stat} = 10.9$).[18]

Nevertheless, risk-based models (like the conditional CAPM) do not impose the restriction that $\lambda^{(0)}$ and $\lambda^{(m)}$ are constant over time. These models also do not impose the restriction that $\lambda^{(0)}$ and $\lambda^{(m)}$ are the same across institutions (unless one imposes rational expectations, which we do not). Risk-based models only impose the restriction that $\lambda^{(0)}$ and $\lambda^{(m)}$ do not vary across asset classes. As such, a more general way to study the subjective risk-return tradeoff is to consider the equation

$$\mu_{j,n,t} = \lambda_{j,t}^{(0)} + \lambda_{j,t}^{(m)} \cdot \beta_{j,n,t}^{(m)} + \alpha_{j,n,t}^{(m)}. \tag{3}$$

We estimate Equation 3 separately for each institution-year observation (i.e., each CMA) from a regression of μ onto $\beta^{(m)}$ across asset classes. For both the Equity CAPM and Pension CAPM, we find that $\lambda_{j,t}^{(m)} > 0$ for all institution-year observations, indicating that the positive risk-return tradeoff holds much more generally.

Our objective is not to provide a formal test of the CAPM, but it is instructive to study the extent to which the estimated $\lambda^{(0)}$ and $\lambda^{(m)}$ values line up with CAPM restrictions. Under the CAPM, we have $\lambda_{j,t}^{(m)} = \mu_{j,t}^{(m)} - \lambda_{j,t}^{(0)}$, with the restriction on $\lambda_{j,t}^{(0)}$ depending on the version of the model. The original version of the CAPM (from Sharpe (1964) and Lintner (1965)) allows investors to borrow at the risk-free rate, which leads to $\lambda_{j,t}^{(0)} = 0$. In contrast, the Black (1972) version of the CAPM prevents investors from borrowing at the risk-free rate, leading to $0 < \lambda_{j,t}^{(0)} < \mu_{j,t}^{(m)}$. The results from our analysis of the CAPM restrictions are provided in Figure 3 and Table 4, with a discussion below.

We start by investigating the $\lambda^{(0)}$ restriction. Table 4 Panel A shows the average estimated $\lambda^{(0)}$ is positive and statistically significant for both models (1.45% in the Equity CAPM and

[18] One might conjecture that this link between subjective risk and return expectations arises only through σ since beta can be written as $\beta_{j,n,t} = \mathbb{C}or[r_n, r^{(m)}] \cdot \sigma_{j,n,t}/\sigma_{j,t}^{(m)}$. However, such a conjecture is incorrect. If we we control for $\sigma_{j,n,t}/\sigma_{j,t}^{(m)}$, the R^2 increases only slightly (to 72% in both versions of the CAPM) and $\beta^{(m)}$ remains highly statistically significant despite the strong collinearity between $\beta_{j,n,t}$ and $\sigma_{j,n,t}/\sigma_{j,t}^{(m)}$. Nevertheless, there is some truth to the idea that volatility matters beyond beta. In particular, $\sigma_{j,n,t}/\sigma_{j,t}^{(m)}$ is statistically significant in these regressions. Moreover, as we later discuss, idiosyncratic volatility has a significant relation to the alphas we study (see Internet Appendix C.1.1).

1.14% in the Pension CAPM). Figures 3(a) and 3(b) further show that almost all $\lambda^{(0)}$ values are positive. These results reject the original CAPM restriction, $\lambda^{(0)}_{j,t} = 0$, in our subjective beliefs dataset. In contrast, the Black CAPM restriction, $0 < \lambda^{(0)}_{j,t} < \mu^{(m)}_{j,t}$, seems to reasonably describe the subjective beliefs of the institutions we study.

Moving to the market risk premium restriction, Figures 3(c) and 3(d) provide scatterplots of $\lambda^{(m)}$ against $\mu^{(m)}$ with a 45 degree line added. It is visually clear these two variables are highly related. To test this link more formally, Table 4 Panel B considers regressions of the form $\lambda^{(m)}_{j,t} = a + b \cdot \mu^{(m)}_{j,t} + \varepsilon_{j,t}$, with the original CAPM restriction implying $a = 0$ and $b = 1$. The results suggest that $\lambda^{(m)}$ is significantly related to $\mu^{(m)}$. In particular, we have $b = 0.742$ ($t_{b=0} = 16.8$) with $R^2 = 41.9\%$ in the Equity CAPM and $b = 0.698$ ($t_{b=0} = 12.8$) with $R^2 = 44.1\%$ in the Pension CAPM. Yet, we can statistically reject the $b = 1$ hypothesis in both models ($t_{b=1} = -5.86$ in the Equity CAPM and $t_{b=1} = -5.53$ in the Pension CAPM), indicating the relation is not fully in line with the original CAPM.

However, the market risk premium under the Black CAPM is given by $\lambda^{(m)}_{j,t} = \mu^{(m)}_{j,t} - \lambda^{(0)}_{j,t}$, which implies the regression $\lambda^{(0)}_{j,t} + \lambda^{(m)}_{j,t} = a + b \cdot \mu^{(m)}_{j,t} + \varepsilon_{j,t}$, with restrictions $a = 0$ and $b = 1$.[19] Table 4 Panel B shows that, in this case, we have $b = 0.979$ ($t_{b=1} = -0.56$) for the Equity CAPM and $b = 1.016$ ($t_{b=1} = 0.66$) for the Pension CAPM, indicating we can no longer reject the hypothesis that $b = 1$. Moreover, the R^2 values substantially increase (to 71.0% and 85.4%, respectively). One mismatch, however, is that in both models the a coefficient is statistically significant. Specifically, a is is 1.6% ($t_{a=0} = 7.36$) in the Equity CAPM and 0.4% ($t_{a=0} = 3.49$) in the Pension CAPM. The average $\mu^{(m)}$ in the Equity CAPM is 5.4%, and thus the 1.6% deviation is economically significant (it represents almost 30% of the average $\mu^{(m)}$). In contrast, the average $\mu^{(m)}$ in the Pension CAPM is 4.6% so that the 0.4% deviation seems less relevant from an economic standpoint (it represents less than 10% of the average $\mu^{(m)}$). So, our overall conclusion is that the estimated market risk premium is broadly (but not perfectly) in line with the Black CAPM restriction, with the mismatch being economically

[19]In theory, we could use the regression $\lambda^{(m)}_{j,t} = a + b \cdot (\mu^{(m)}_{j,t} - \lambda^{(0)}_{j,t}) + \varepsilon_{j,t}$ for the Black CAPM. Empirically, however, this regression would suffer from an error-in-variables problem that would bias b toward zero (due to the fact that $\lambda^{(0)}_{j,t}$ contains estimation error).

more relevant for the Equity CAPM than the Pension CAPM.

2.2 Subjective Alphas

The prior subsection shows that the subjective risk-return tradeoff is positive and largely (but not fully) consistent with the CAPM restrictions on $\lambda^{(0)}$ and $\lambda^{(m)}$. To assess the strength of this tradeoff, we also examine the $\alpha^{(m)}$ term in Equation 3. If $\beta^{(m)}$ fully captures subjective risk (as implied by the CAPM), then we have $\alpha^{(m)}_{j,n,t} = 0$ under risk-based models. More realistically, we expect $\alpha^{(m)}_{j,n,t} \neq 0$ due to two factors. First, $\beta^{(m)}$ does not fully capture subjective risk (i.e., the CAPM is not correct), which means $\alpha^{(m)}$ includes model mispecification. Second, investors may have beliefs that are not consistent with risk-based models, in which case $\alpha^{(m)}$ captures other factors (beyond risk exposure) that affect subjective expected returns. So, this subsection focuses on quantifying the magnitude of $\alpha^{(m)}$.

To start, we rewrite Equation 3 as

$$\mu_{j,n,t} \;=\; \alpha^{(m)}_{j,n,t} \;+\; rp^{(m)}_{j,n,t} \tag{4}$$

where $rp^{(m)}_{j,n,t} = \lambda^{(0)}_{j,t} + \lambda^{(m)}_{j,t} \cdot \beta^{(m)}_{j,n,t}$ is the asset subjective risk premium and $\alpha^{(m)}_{j,n,t} = \mu_{j,n,t} - rp^{(m)}_{j,n,t}$ is the asset class subjective alpha. We consider two specifications for $\lambda^{(0)}$ and $\lambda^{(m)}$ (and consequently for $rp^{(m)}$ and $\alpha^{(m)}$). The first specification (which we refer to "unrestricted") uses the $\lambda^{(0)}$ and $\lambda^{(m)}$ values from the regression we estimate in the prior subsection for each institution-year (based on Equation 3). The second specification (which we refer to as "restricted") imposes the original CAPM restrictions so that $rp^{(m)}_{j,t} = \mu^{(m)}_{j,t} \cdot \beta^{(m)}_{j,n,t}$ and $\alpha^{(m)}_{j,n,t} = \mu_{j,n,t} - rp^{(m)}_{j,t}$ are obtained directly from beliefs without any estimation.[20]

Table 3 provides the average values of $rp^{(m)}$ and $\alpha^{(m)}$ for each asset class. Alphas tend to be smaller in magnitude than risk premia, especially for the unrestricted CAPM, which does not force $\lambda^{(0)} = 0$. Moreover, we see a strong connection between μ and $rp^{(m)}$, which is a

[20] We impose the original CAPM restrictions instead of the Black CAPM restrictions to have an alpha measure that requires no estimation, addressing the criticism that alphas are small because the parameters in Equation 3 are estimated to minimize the sum of squared alphas. Under the Black CAPM restrictions, we would still have one parameter to estimate, $\lambda^{(0)}$, even though the other parameter, $\lambda^{(m)}$, would be restricted. So, the Black CAPM is an intermediate case between our unrestricted and restricted specifications.

consequence of the positive relation between μ and $\beta^{(m)}$ highlighted in the prior subsection. In contrast, the link between μ and $\alpha^{(m)}$ is weak. Figure 4 provides a visual representation of these result by plotting average μ against average $\alpha^{(m)}$ across asset classes. Overall, average alphas tend to be small in magnitude for each asset class. For instance, even though there is large variation in average subjective expected returns across asset classes, there is little variation in average alphas, and such variation has little to no relation to the variation in average expected returns.

While asset class alphas averaged across institution-years are relatively small, it might be possible to create portfolios that have large alphas at the institution-year level. To explore this, we compute alphas for maximum Sharpe ratio (tangency) portfolios using $\alpha^{(m)}_{max,j,t} = w'_{max,j,t}\alpha^{(m)}_{j,t}$, where $\alpha^{(m)}_{j,t}$ is the vector of asset class alphas and $w_{max,j,t} = \Sigma^{-1}_{j,t}\mu_{j,t}/|\mathbf{1}'\Sigma^{-1}_{j,t}\mu_{j,t}|$.[21] Figure 5 plots the distribution of annualized $\alpha^{(m)}_{max}$ across all institution-years in our sample (the histogram is truncated at a maximum 10% alpha for visual clarity). Two results emerge. First, the typical tangency portfolio alpha is relatively small, with the median $\alpha^{(m)}_{max}$ being 2.8% and 2.5% for the restricted versions of the CAPM, and even lower for the unrestricted versions of the CAPM (1.2% and 1.4%). Second, in nearly 10% of institution-years, $\alpha^{(m)}_{max} \geq$ 10%, creating skewness that raises the average $\alpha^{(m)}_{max}$ well above its median.

Despite the non-trivial fraction of large $\alpha^{(m)}_{max}$ values, they often concentrate in economically less relevant portfolios with negative positions in some risky asset classes. To demonstrate this aspect, Figure 6 plots the distribution of annualized $\alpha^{(m)}_{max}$ with a no short-sales constraint (i.e., $w_{max} \geq 0$). Here, $\alpha^{(m)}_{max}$ generally declines compared to the short-sales-allowed case. For instance, the median $\alpha^{(m)}_{max}$ is 2.0% and 1.5% for the restricted versions of the CAPM and 0.5% and 0.6% for the respective unrestricted versions. Moreover, the fraction of large $\alpha^{(m)}_{max}$

[21]The results in Gibbons, Ross, and Shanken (1989) imply that $\alpha_{k,j,t}/i\sigma_{k,j,t} = \sqrt{\mathbb{SR}[r_k^{(max)}] - \mathbb{SR}[r^{(m)}]}$, where $i\sigma$ reflects idiosyncratic volatility and k reflects an arbitrary positive alpha portfolio of our 19 risky asset classes. Since $r_k^{(max)}$ is the maximum Sharpe ratio portfolio that can be formed with r_k and $r^{(m)}$, our $\alpha^{(m)}_{max}$ reflects the maximum alpha available for the level of idiosyncratic volatility of the maximum Sharpe ratio portfolio. The same Gibbons, Ross, and Shanken (1989) expression implies that $\alpha^{(m)}_{max}$ also measures (in alpha units) how far the market portfolio is from the maximum Sharpe ratio portfolio, with the original CAPM implying the two should coincide.

declines so that the average $\alpha_{max}^{(m)}$ values are only slightly higher than the respective median values. Thus, under a reasonable no short-sales restriction, we cannot reliably combine asset classes to form portfolios with economically large alphas relative to our market proxies.

2.3 Explaining Variation in Subjective Expected Returns: Risk Premia vs Alphas

While the prior subsections focus on the risk-return tradeoff and the magnitude of alphas, this subsection formally quantifies the role of subjective risk premia and alphas in explaining variation in subjective expected returns. For this purpose, we take variance on both sides of Equation 4 to obtain the variance decomposition

$$\begin{aligned} \mathbb{V}ar\left[\mu_{j,n,t}\right] &= \mathbb{C}ov\left[\mu_{j,n,t}\ ,\ \alpha_{j,n,t}^{(m)}\right] + \mathbb{C}ov\left[\mu_{j,n,t}\ ,\ rp_{j,n,t}^{(m)}\right] \\ &\Downarrow \\ 1 &= \underbrace{\frac{\mathbb{C}ov\left[\mu_{j,n,t}\ ,\ \alpha_{j,n,t}^{(m)}\right]}{\mathbb{V}ar\left[\mu_{j,n,t}\right]}}_{\mathbb{V}_{\alpha}=\%\text{ of }\mu\text{ Variation from Alphas}} + \underbrace{\frac{\mathbb{C}ov\left[\mu_{j,n,t}\ ,\ rp_{j,n,t}^{(m)}\right]}{\mathbb{V}ar\left[\mu_{j,n,t}\right]}}_{\mathbb{V}_{rp}=\%\text{ of }\mu\text{ Variation from Risk Premia}} \end{aligned} \tag{5}$$

where, as before, we have $rp_{j,n,t}^{(m)} = \lambda_{j,t}^{(0)} + \lambda_{j,t}^{(m)} \cdot \beta_{j,n,t}^{(m)}$. To simplify exposition in this subsection, we only present the results from the restricted CAPM specifications in the main text. In this case, no estimation is required to measure $\alpha^{(m)}$ and $rp^{(m)}$ since we have $\lambda_{j,t}^{(0)} = 0$ and $\lambda_{j,t}^{(m)} = \mu_{j,t}^{(m)}$. However, we provide analogous (and qualitatively similar) results for the unrestricted CAPM specifications in Internet Appendix C.1 (see Figure IA.5 and Table IA.5).

To start, Figure 7 visually represents of the variance decomposition in Equation 5, plotting μ against $rp^{(m)}$ and $\alpha^{(m)}$. The results show μ varies much more with $rp^{(m)}$ than with $\alpha^{(m)}$ in both the Equity CAPM and the Pension CAPM. We confirm this visual observation by estimating $\mathbb{V}_{rp}$ and $\mathbb{V}_{\alpha}$ (from Equation 5) as slopes on regressions of $rp^{(m)}$ and $\alpha^{(m)}$ onto μ, with the $\mathbb{V}_{rp}$ and $\mathbb{V}_{\alpha}$ estimates written in the graphs. For the Equity CAPM, more than 75% of the μ variation is explained by $rp^{(m)}$. For the Pension CAPM, the risk premia effect is even stronger as more than 90% of the μ variation is explained by $rp^{(m)}$.

Table 5, Panel A, provides the $\mathbb{V}_{rp}$ and $\mathbb{V}_{\alpha}$ values for different specifications of the regressions of $rp^{(m)}$ and $\alpha^{(m)}$ onto μ. Column [1] repeats the overall variance decomposition described in the prior paragraph. Columns [2] and [3] focus on variation across asset classes by adding either year and institution fixed effects or year-by-institution fixed effects. Strikingly, the decompositon of variation across asset classes is almost identical to the overall variance decomposition in Column [1]. The subsequent columns provide the same variance decompositions but separately within each of our asset class categories (Fixed Income, Equities, Real Estate, and Alternatives). We see that the explanatory power of $rp^{(m)}$ declines when we focus on a single asset class category (except for Real Estate, which only has two asset classes). This pattern holds for both the Equity CAPM and the Pension CAPM. It also holds whether we look at overall variation or only variation across asset classes. In all cases, subjective risk premia remain a very important determinant of subjective expected returns, but for some specifications alphas have a higher explanatory power than risk premia (e.g., within Equities under the Equity CAPM). These results indicate that the importance of $rp^{(m)}$ in explaining variation in μ is stronger across asset class categories than within them. That is, when we analyse multiple asset classes that display large beta heterogeneity, $rp^{(m)}$ is much more important than $\alpha^{(m)}$ in explaining subjective expected return variation. In contrast, $\alpha^{(m)}$ plays a large role when we analyze asset classes that are relatively homogeneous on beta (by focusing on a single asset class category). In some of these cases $\alpha^{(m)}$ even plays a larger role than $rp^{(m)}$ in explaining μ variation.

Table 5, Panel B, reports $\mathbb{V}_{rp}$ and $\mathbb{V}_{\alpha}$ for specifications that explore variation across institutions and over time. Column [1] uses asset class fixed effects to focus on variation across institutions and over time jointly. In this case, we have that $rp^{(m)}$ explains 51% of the μ variation under the Equity CAPM and 62% under the Pension CAPM. Columns [2] and [3] focus on μ variation across institutions or over time, with comparable numbers. These results display non-trivial differences as we focus on particular asset class categories. In particular, the explanatory power of $rp^{(m)}$ for μ variation across institutions and over time tends to be stronger within Equities and Real Estate and weaker within Fixed Income and Alternatives.

Altogether, the results in Table 5 show that risk premia explain the vast majority of the overall μ variation as well as the μ variation across asset classes. Risk premia also explain much of the μ variation across institutions and over time, but in these cases alphas play a significant quantitative role. It is important to highlight an economic distinction between these different sources of variation. All risk premia variation across asset classes (within an institution-year) comes from risk quantity (through $\beta^{(m)}_{j,n,t}$) since $\lambda^{(0)}_{j,t}$ and $\lambda^{(m)}_{j,t}$ do not vary across asset classes (n). As such, the explanatory power of risk premia for μ variation across asset classes highlights the subjective risk-return tradeoff. In contrast, risk premia variation over time and across institutions has both risk quantity and risk price effects because $\beta^{(m)}_{j,n,t}$, $\lambda^{(0)}_{j,t}$, and $\lambda^{(m)}_{j,t}$ all vary across j and over t. Whether the connection between subjective expected returns and risk quantity across institutions and over time is stronger or weaker than the connection between μ and risk premia depends on the correlation between risk quantities and risk prices. Exploring this issue is out of the scope of this paper, but the subsequent work by Couts et al. (2024) studies μ variation over time and across institutions while separately identifying the effects of risk quantity and risk price.

2.4 Belief Heterogeneity Across Asset Classes and Institutions

The alpha component shows up strongly on μ disagreement but not on overall μ variation. To explore why, this subsection decomposes within-year variation in μ into variation originating from institutions and asset classes. To start, we estimate the fixed-effects model

$$\mu_{j,n,t} = \overline{\mu}_{j,t} + \overline{\mu}_{n,t} + \eta_{j,n,t}, \tag{6}$$

where $\eta_{j,n,t}$ are residuals and $\overline{\mu}_{j,t}$ and $\overline{\mu}_{n,t}$ are (year t) institution and asset class fixed effects.

Figures 8(a) and 8(b) plot $\mu_{j,n,t}$ against $\overline{\mu}_{j,t}$ and $\overline{\mu}_{n,t}$, respectively. Figure 8(a) shows there is only a weak positive relation between $\overline{\mu}_{j,t}$ and $\mu_{j,n,t}$. Moreover, we have a high degree of $\mu_{j,n,t}$ heterogeneity for any given level of $\overline{\mu}_{j,t}$, indicating only a small fraction of the variability in $\mu_{j,n,t}$ is due to heterogeneity across institutions (captured by $\overline{\mu}_{j,t}$). In contrast, Figure 8(b) shows a strong positive relation between $\overline{\mu}_{n,t}$ and $\mu_{j,n,t}$, with a more moderate degree of $\mu_{j,n,t}$ heterogeneity at each level of $\overline{\mu}_{n,t}$. Consequently, a large fraction of the variability in $\mu_{j,n,t}$ is due to heterogeneity across asset classes (captured by $\overline{\mu}_{n,t}$).

To more precisely quantify the patterns observed in Figures 8(a) and 8(b), we take variance on both sides of Equation 6 to obtain the following decomposition:

$$1 \quad = \quad \underbrace{\frac{\mathbb{C}ov\left(\mu_{j,n,t}\ ,\ \overline{\mu}_{j,t}\right)}{\mathbb{V}ar(\mu_{j,n,t})}}_{\%\text{ of }\mu_{j,n,t}\text{ Variation from }\overline{\mu}_{j,t}} \quad + \quad \underbrace{\frac{\mathbb{C}ov\left(\mu_{j,n,t}\ ,\ \overline{\mu}_{n,t}\right)}{\mathbb{V}ar(\mu_{j,n,t})}}_{\%\text{ of }\mu_{j,n,t}\text{ Variation from }\overline{\mu}_{n,t}} \quad + \quad \underbrace{\frac{\mathbb{C}ov\left(\mu_{j,n,t}\ ,\ \eta_{j,n,t}\right)}{\mathbb{V}ar(\mu_{j,n,t})}}_{\%\text{ of }\mu_{j,n,t}\text{ Variation from }\eta_{j,n,t}} \tag{7}$$

Figure 9(a) plots decomposition results based on Equation 7 for each year in which we have at least four institutions. The results indicate that the vast majority of within-year expected return variability is driven by variability across asset classes ($\overline{\mu}_{n,t}$), with only a small portion driven by variability across institutions ($\overline{\mu}_{j,t}$).[22]

One limitation of the baseline econometric model in Equation 6 (and its respective variance decomposition in Equation 7) is that it does not allow institutions to have expected return heterogeneity in fixed effects that differs across asset classes. For example, if the fixed effect of institution j indicates relative optimism for *US Bonds* at time t, then it also indicates relative optimism for all other asset classes at time t (i.e., $\overline{\mu}_{j,t}$ is common across asset classes). To address this issue, we also consider the alternative fixed-effects model

$$\mu_{j,n,t} \ = \overline{\mu}_{n,j} \ + \ \overline{\mu}_{n,t} \ + \ \eta_{j,n,t} \tag{8}$$

which leads to a variance decomposition analogous to the one in Equation 7.

Figures 8(c) and 8(d) show that the results from Equation 8 are similar to the results from Equation 6. That is, $\mu_{j,n,t}$ has a strong relation with $\overline{\mu}_{n,t}$ and a relatively weak relation with $\overline{\mu}_{n,j}$. Moreover, Figure 9(b) implements the variance decomposition associated with Equation 8. While the institution effect (through $\overline{\mu}_{n,j}$) explains more of the expected return variability than it does in Equation 7 (through $\overline{\mu}_{j,t}$), it remains true that the majority of the expected return variability is driven by the asset class effect ($\overline{\mu}_{n,t}$).

Overall, the results in this subsection explain why expected return variation is largely

[22]Internet Appendix C.1.3 provides results for a decomposition analogous to Equation 7, but applied to βs and αs (using the restricted version of the CAPM for simplicity). In a nutshell, the results indicate that almost all the variability in βs is driven by variability across asset classes while disagreement is roughly as important as variation across asset classes in explaining overall alpha variation (see Figure IA.6).

driven by risk premia and not alphas. Specifically, subjective alphas are important in explaining subjective expected return variation across institutions (for a given asset class). However this variation in subjective expected returns across institutions (driven by disagreement) is small relative to the variation in subjective expected returns across asset classes (driven by the positive risk-return tradeoff). That said, it is important to not interpret this finding as indicating that there is little disagreement across institutions, which is not the case (as we detail in Internet Appendix C.1.4). So, while our dataset reflects significant disagreement, the variation in subjective expected returns across asset classes is substantially larger than the variation in expected returns across institutions.

3 The Link Between Subjective Beliefs and Realized Returns

The prior section studies the link between subjective risk and return expectations, effectively connecting two different dimensions of subjective beliefs. In this section, we explore the link between subjective beliefs and realized returns. Defining $\mathbb{E}_{o,t}[\cdot]$ to be the rational or objective expectation operator, we have that $r_{n,t+1} = \mathbb{E}_{o,t}[r_n] + \epsilon_{n,t+1}$, with $\mathbb{E}_{o,t}[\epsilon_n] = 0$, and a similar equation for other return moments. As such, the link between subjective beliefs and future realized returns reflects a connection between subjective and objective expectations.

Classical asset pricing models assume Full Information Rational Expectations (FIRE), implying $\mathbb{E}_{j,t}[r_n] = \mathbb{E}_{o,t}[r_n] \;\; \forall\, j$. However, this benchmark is less informative for our analysis since the beliefs implied from CMAs largely vary across institutions (as shown in Subsection 2.4). Belief disagreement between two institutions requires at least one of them to deviate from FIRE. Therefore, we focus on aggregated beliefs. If institutions have expectations that are rational given their information sets (and the union of all information sets spans the full information set), then $\mathbb{E}_{j,t}[r_n] = \mathbb{E}_{o,t}[r_n] + \xi_{j,n,t}$, with $\mathbb{E}_{o,t}[\xi_{j,n,t}] = 0$ across institutions. In this case, expected returns aggregated across institutions satisfy FIRE even though no individual institution has FIRE.[23] While our goal is not necessarily to formally test for FIRE, we guide

[23]For instance, if $\mathbb{E}_{j,t}[r_n] = \mathbb{E}_{o,t}[r_n] + \xi_{j,n,t}$, then a panel regression of $r_{n,t+1}$ onto $\mathbb{E}_{j,t}[r_n]$ would lead to a slope coefficient below one (i.e., it would reject FIRE) even though a time-series regression of $r_{n,t+1}$ onto

our empirical analysis using FIRE for aggregated beliefs as a benchmark.[24]

The rest of this section is organized as follows. Subsection 3.1 describes how we aggregate the beliefs data across institutions and provides information about the realized return data we rely on. Subsection 3.2 contrasts the risk-return tradeoff implied from the subjective beliefs of our institutions with the risk-return tradeoff observed in the realized return data. Subsection 3.3 shows that, on average, beliefs are largely consistent with their realized return counterparts, but points out important exceptions. Subsection 3.4 provides an analysis of the predictability of realized return moments using subjective beliefs.

3.1 Belief Aggregation and Realized Return Data

Given the above discussion, we aggregate beliefs across institutions to study the link between subjective beliefs and realized returns. Specifically, we estimate the fixed effects regression

$$\theta_{j,n,t} \; = \overline{\theta}_{n,j} \; + \; \overline{\theta}_{n,t} \; + \; \eta_{j,e,t} \tag{9}$$

and then obtain the aggregate belief quantity

$$\theta_{n,t} \; = \; \left[\frac{1}{T} \cdot \sum_{t=1}^{T} \left(\frac{1}{J_t} \sum_{j=1}^{J_t} \theta_{j,n,t} \right) \right] \; + \; \left(\overline{\theta}_{n,t} \; - \; \frac{1}{T} \cdot \sum_{t=1}^{T} \overline{\theta}_{n,t} \right) \tag{10}$$

where $\theta_{j,n,t}$ represents a generic belief quantity (μ, σ, $\mu^{(m)}$, $\beta^{(m)}$).[25] In turn, we obtain aggregate alphas using $\alpha^{(m)}_{n,t} = \mu_{n,t} - \beta^{(m)}_{n,t} \cdot \mu^{(m)}_{t}$.[26]

$\mathbb{E}_{j,t}[r_n]$ aggregated across institutions would lead to $b = 1$ (i.e., it would not reject FIRE). The intuition is that, in this case, $\mathbb{E}_{j,t}[r_n]$ serves as a noisy proxy for the FIRE of $r_{n,t+1}$.

[24]It is important to understand that whether FIRE for all investors and FIRE for aggregated beliefs lead to the same asset pricing implications depends on the economic environment. For instance, if mean-variance investors disagree on expected returns but have rational expectations on average and face no constraints, then asset prices are consistent with the CAPM that obtains under FIRE for all investors (Levy, Levy, and Benita (2006)). In contrast, if investors disagree on expected returns and face short selling restrictions, then asset prices can differ from those obtained under the CAPM with FIRE for all investors (Miller (1977)).

[25]For $\beta^{(p)}_{j,n,t}$, asset classes in the market portfolio vary across j (since some institutions do not cover all asset classes present in the pension market portfolio). So, instead of aggregating $\beta^{(p)}_{j,n,t}$ each year, we aggregate the covariance for each pair of asset classes and combine aggregate variances and covariances with the pension market portfolio weights to obtain $\beta^{(p)}_{n,t}$. However, we obtain similar results directly aggregating $\beta^{(p)}_{j,n,t}$.

[26]Directly aggregating $\alpha^{(m)}_{j,n,t}$ yields similar $\alpha^{(m)}_{n,t}$ values, but we opt for using $\alpha^{(m)}_{n,t} = \mu_{n,t} - \beta^{(m)}_{n,t} \cdot \mu^{(m)}_{t}$ so that we still have the equation $\mu_{n,t} = \alpha^{(m)}_{n,t} + \beta^{(m)}_{n,t} \cdot \mu^{(m)}_{t}$, with the CAPM implying $\alpha^{(m)}_{n,t} = 0$.

Intuitively, $\overline{\theta}_{n,t}$ captures time variation in beliefs controlling for time variation in the composition of institutions (through the $\overline{\theta}_{n,j}$ fixed effects). However, the time series average of $\overline{\theta}_{n,t}$ cannot be interpreted as the unconditional mean for the belief quantity associated with asset class n because it ignores components of beliefs about asset class n that do not vary over time. As such, our aggregation takes the time demeaned $\overline{\theta}_{n,t}$ and adds to it the time-series average of the cross-section averages of $\theta_{j,n,t}$, which is an estimate for $\mathbb{E}[\theta_n]$.

Note that the time-series average of $\theta_{n,t}$ matches the time-series average of the alternative aggregated series $\frac{1}{J_t}\sum_{j=1}^{J_t}\theta_{j,n,t}$. However, our aggregation method underlying $\theta_{n,t}$ is designed to produce time variation that accounts for sample composition (whereas $\frac{1}{J_t}\sum_{j=1}^{J_t}\theta_{j,n,t}$ is not). The reason is that aggregating without accounting for sample composition would induce time variation in beliefs that is purely due to variation in the set of institutions providing the given belief quantity (i.e., it would exist even if beliefs were constant over time within each institution).

Also note that our aggregation method partially accounts for the fact that institutions have heterogeneous forecasting horizons and CMA production months.[27] Forecasting horizon and CMA production months play a role when we attempt to forecast returns. We explain how we deal with these issues in Subsection 3.4.

Given aggregate beliefs, we also need realized returns for each of the asset classes underlying our beliefs data. Some institutions provide tradable indices as references for the different asset classes in their CMAs while other institutions do not. Moreover, for the ones that do provide tradable indices, different institutions may use different indices for the same asset class. So, the matching between beliefs and return data is bound to be imperfect. We obtain

[27] Specifically, the year-t CMA of institution j is meant to represent the institution j forecast for the period from $t+1$ to $t+h_j$ (where h_j is the institution forecasting horizon). Moreover, while most CMAs have a production month as of December of year t, some CMAs have a production month from a few months before or a few months after December of year t (see Figure IA.3 in the Internet Appendix). To the extent that h_j and the CMA production month gap (relative to December) is fixed over time for each institution, our $\overline{\theta}_{n,j}$ fixed effects account for that. Time variation in horizons and CMA production month gaps are not controlled for in our aggregation method, but should be of second-order importance (and would add noise to our belief data, which would serve to weaken our finding of a strong relation between subjective beliefs and future realized returns).

return data for 17 out of the 20 asset classes we study. This is because of the difficulties in obtaining quality return data over our sample period for the asset classes removed (*Private Equity*, *Private Debt*, and *Venture Capital*). Table 6 provides the details for the return data we use, including the names of the indices, the data sources, and the sample period over which we observe returns for each index.

We obtain monthly returns for each index (quarterly returns for *Private Real Estate*) and compound these returns within each year to create time series of annual returns. The realized return unconditional moments for μ_n, σ_n^2, $\beta_n^{(m)}$, and $\mu^{(m)}$ are the respective sample estimates using the full time-series of annual returns for each asset class. Moreover, $rp_n^{(m)} = \beta_n^{(m)} \cdot \mu^{(m)}$ and $\alpha_n^{(m)} = \mu_n - rp_n^{(m)}$ are analogous to their belief counterparts. Hereafter, we use hats to denote data moments. For instance, $\widehat{\mu}_n$ is the average return for asset class n.

3.2 Risk-Return Tradeoff: Subjective Beliefs vs Realized Returns

This subsection contrasts the risk-return tradeoff implied by the subjective beliefs of our institutions with the risk-return tradeoff observed in the realized return data.

Figure 10 plots expected returns (μ_n) against risk premia ($rp_n^{(m)}$) for our asset classes using both beliefs and realized returns. Figures 10(a) and 10(b) are analogous to Figures 7(a) and 7(b), except that they rely on μ_n and $rp_n^{(m)}$ values aggregated across institutions and over time (and thus only vary across asset classes). Not surprisingly, these figures uncover the strong subjective risk-return tradeoff we highlight in Section 2. Figures 10(c) and 10(d) repeat this analysis but using $\widehat{\mu}_n$ and $\widehat{rp}_n^{(m)}$. These figures demonstrate that the risk-return tradeoff observed in the realized return data is almost as strong as the subjective risk-return tradeoff of our institutions.[28] It is important to point out that we expect the realized risk-return tradeoff to be weaker than the subjective risk-return tradeoff because average returns

[28]The strong realized risk-return tradeoff in Figures 10(c) and 10(d) seems to contradict the fact that the CAPM has limited success in explaining variation in average realized returns across many test assets (see Fama and French (2004) for a broad discussion of this issue). However, there is no contradiction. The crux of the matter is that, due to the nature of CMAs, our analysis focuses on expected return variation across asset classes. This focus differs from much of the asset pricing literature, which largely focuses on expected return variation across test assets within a given asset class (often equities).

provide noisy estimates for unconditional expected returns (Fama and French (2002)). As such, it is not surprising that Figures 10(c) and 10(d) display somewhat weaker R^2 values relative to Figures 10(a) and 10(b).

Another way to examine the risk-return tradeoff is by testing whether betas predict alphas across asset classes. Within asset classes, this is known as the "low-beta anomaly" (see Frazzini and Pedersen (2014)). Internet Appendix C.2.1 shows that there is no low-beta anomaly in subjective beliefs across asset classes whereas there is a (very weak) low-beta anomaly in realized returns across asset classes (see Figure IA.7). This pattern indicates that beliefs are not fully consistent with realized returns. However, the discrepancy is so small that it has little impact on the overall message that the subjective risk-return tradeoff is consistent with the risk-return tradeoff observed in the realized return data.

3.3 Average Subjective Beliefs vs Unconditional Realized Return Moments

This subsection shows that, on average, beliefs are largely consistent with their unconditional realized return moment counterparts.

Figure 11 plots realized return moments against time series averages of their aggregated beliefs counterparts. Figures 11(a) and 11(b) show that $\widehat{\mu}_n$ and $\widehat{\sigma}_n$ are largely consistent with μ_n and σ_n. For instance, the slope coefficient from regressing $\widehat{\mu}_n$ on μ_n is 0.92 (close to one), with $R^2 = 70\%$ (considering that $\widehat{\mu}_n$ contains substantial estimation noise, this R^2 value is very high). The link between $\widehat{\sigma}_n$ and σ_n is even stronger, with a slope coefficient of 1.00 and $R^2 = 91\%$. We observe similar results for betas. However, alphas have a much weaker link. For instance, a regression of $\widehat{\alpha}^{(p)}$ on $\alpha^{(p)}$ results in $R^2 = 31\%$ (although the slope coefficient is close to one at 0.92).

Our objective in Figure 11 is to understand whether average beliefs are consistent with unconditional return moments. So, as in the prior subsection, Figure 11 uses the entire return time-series for each asset class (i.e., returns start at the "First Return Date" column of Table 6). While this approach minimizes the large noise associated with estimating return moments, it has a key drawback. Namely, if institutions obtain their beliefs by simply estimating past return moments, the relation between average beliefs and unconditional return moments reflects a "consistency with historical data" effect. Asymptotically, this consistency with historical data ensures that beliefs are unconditionally rational (i.e., unconditional beliefs match unconditional return moments). However, beliefs can be unconditionally rational while displaying extrapolative behavior (which would lead them to not be rational conditional on time t information set). The next section addresses this issue by testing whether beliefs predict subsequent realized returns and related moments.[29]

3.4 Subjective Beliefs Predicting Future Realized Return Moments

This subsection shows that subjective expected returns predict subsequent realized returns across asset classes and over time. Moreover, the quantitative link between subjective expected returns and subsequent realized returns is consistent with aggregate beliefs satisfying FIRE. We also find that subjective risk predicts realized risk across asset classes but not over time, and that subjective alphas do not predict subsequent realized alphas.

If our institutions have FIRE on average, then $\mathbb{E}_{o,t}[r_{n,t+1}] = \mathbb{E}_t[r_{n,t+1}] = \mu_{n,t}^{(1y)}$, which yields

$$r_{n,t+1} = a + b \cdot \mu_{n,t}^{(1y)} + \epsilon_{n,t+1} \tag{11}$$

with the restrictions that $a = 0$ and $b = 1$.

We do not observe $\mu_{n,t}^{(1y)}$ (the 1-year subjective expected return), but we do observe $\mu_{n,t}$ (the subjective expected average annual return over the next H years). If $\mu_{n,t}^{(1y)}$ follows an AR(1), then $\mu_{n,t}$ and $\mu_{n,t}^{(1y)}$ are perfectly positively correlated, and thus we can replace $\mu_{n,t}^{(1y)}$ with $\mu_{n,t}$ and Equation 11 continues to hold (except for the $a = 0$ and $b = 1$ implications).

Table 7 (Panel A, first block) estimates Equation 11 using annual data with $\mu_{n,t}$ replacing

[29] An alternative approach to address the "consistency with historical data" effect would be to replicate Figure 11 using data moments estimated from returns that are subsequent to their respective beliefs. Internet Appendix C.2.2 does exactly that (see Figure Figure IA.8). The graphs for μ, σ, $\beta^{(e)}$, and $\beta^{(p)}$ are similar to (albeit noisier than) the respective graphs in Figure 11, indicating that they reflect more than a "consistency with historical data" effect. In stark contrast, the alpha graphs show that $\widehat{\alpha}$ has effectively no connection to α, suggesting subjective alphas are consistent with historical alphas, but not with future alphas. Since the results from Subsection 3.4 allow us to reach similar inferences (and in fact more refined inferences on the time-series dimension), we keep Figure IA.8 in the Internet Appendix to conserve space.

$\mu_{n,t}^{(1y)}$. The results show $\mu_{n,t}$ strongly predicts $r_{n,t+1}$, with $b = 1.14$ ($t_{b=0} = 2.63$) and a moderate $R^2 = 3.4\%$. Focusing on variation across asset classes (by adding year fixed effects) or only on variation over time (by adding asset class fixed effects), there is a slight increases in (within) R^2 values (to 4.4% and 4.6% respectively). Interestingly, when exploiting variation across asset classes, we have $b = 1.05$ ($t_{b=0} = 1.98$), which is statistically indistinguishable from 1 ($t_{b=1} = 0.09$). In this case, the $b = 1$ restriction is approximately valid because a regression with time fixed effects is similar to a cross-sectional regression of average $r_{n,t+1}$ onto average $\mu_{n,t}$, and $\mu_{n,t}$ and $\mu_{n,t}^{(1y)}$ have the same unconditional expectation. In contrast, when exploiting variation over time, we have $b = 4.70$ ($t_{b=0} = 2.00$), which is far from 1. This is expected: when exploring variation over time, the $b = 1$ restriction is far from valid. The reason is that, due to mean reversion, the time-variation in long-run expected returns ($\mu_{n,t}$) is much smaller than the respective time variation in 1-year expected returns ($\mu_{n,t}^{(1y)}$), so that replacing $\mu_{n,t}^{(1y)}$ with $\mu_{n,t}$ in Equation 11 induces $b > 1$.

To check whether the predictability we observe in the time series is consistent with the $b = 1$ implication from Equation 11, we construct a proxy for $\mu_{n,t}^{(1y)}$. Specifically, we note that if $\mu_{n,t}^{(1y)}$ follows an AR(1) process, then we have

$$\mu_{n,t}^{(1y)} = \overline{\mu}_n + H \cdot \frac{1-\phi_n}{1-\phi_n^H} \cdot (\mu_{n,t} - \overline{\mu}_n) \tag{12}$$

where $\overline{\mu}_n$ is the unconditional mean of $\mu_{n,t}$, H is the $\mu_{n,t}$ forecasting horizon, and ϕ_n is the $\mu_{n,t}$ AR(1) persistence.[30] We then set $H = 10$ years (the modal horizon for our CMAs), estimate $\overline{\mu}_n$ from $1/T_n \cdot \sum_{t=1}^{T_n} \mu_{n,t}$, and estimate ϕ_n from the sample persistence of $\mu_{n,t}$.

Table 7 (Panel A, second block) estimates Equation 11 using $\mu_{n,t}^{(1y)}$ from Equation 12. In the pooled estimation, we find $a = 0.00$ ($t_{a=0} = 0.11$) and $b = 1.02$ ($t_{b=0} = 2.84$), with the

[30]Specifically, if we have $(\mu_{n,t+1}^{(1y)} - \overline{\mu}_n) = \phi_n \cdot (\mu_{n,t}^{(1y)} - \overline{\mu}_n) + \epsilon_{t+1}$, then:

$$\mu_{n,t} - \overline{\mu}_n = \frac{1}{H} \cdot \sum_{h=0}^{H-1} \mathbb{E}_t[\mu_{n,t+h}^{(1y)}] = \frac{1}{H} \cdot \sum_{h=0}^{H-1} \phi_n^h \cdot (\mu_{n,t}^{(1y)} - \overline{\mu}_n) = \frac{1}{H} \cdot \frac{1-\phi_n^H}{1-\phi_n} \cdot (\mu_{n,t}^{(1y)} - \overline{\mu}_n)$$

which yields Equation 12. Note that the unconditional expectation of $\mu_{n,t}$ and the persistence of $\mu_{n,t} - \overline{\mu}_n$ are the same as the unconditional expectation of $\mu_{n,t}^{(1y)}$ and persistency of $\mu_{n,t}^{(1y)} - \overline{\mu}_n^{(1y)}$, which allows us to estimate $\overline{\mu}_n$ and ϕ_n directly from $\mu_{n,t}$ data (as described below Equation 12).

joint hypothesis that $a = 0$ and $b = 1$ far from being rejected (p-value is 0.99). Moreover, the b coefficient remains statistically indistinguishable from 1 whether we focus on variation across asset classes or over time. The R^2 values are also non-trivial. For instance, the pooled R^2 is 6.7% and the within R^2 values are higher than 5%. Following, Goyal and Welch (2008) and Campbell and Thompson (2008), we also consider an out-of-sample R^2 metric (R^2_{OOS}), which evaluates the predictability of $\mu^{(1y)}_{n,t}$ against benchmark models based on historical average returns (and can be negative).[31] R^2_{OOS} is high both across asset classes (12.2%) and over time (12.7%).[32]

Table 7 (Panel A, third block) considers a regression analogous to Equation 11, but that uses 3-year subjective expected returns ($\mu^{(3y)}_{n,t}$) to predict the average excess return over the next three years ($\overline{r}^{(3y)}_{n,t\rightarrow t+3}$). Specifically, we have $\overline{r}^{(3y)}_{n,t\rightarrow t+3} = \frac{1}{3}\cdot\sum_{h=1}^{3} r_{t+h}$ and $\mu^{(3y)}_{n,t} = \overline{\mu}_n + \frac{1}{3}\cdot\frac{1-\phi_n^H}{1-\phi_n}\cdot(\mu^{(1y)}_{n,t} - \overline{\mu}_n)$. The idea is to evaluate the predictability of longer-term returns given that beliefs have multi-year horizons. However, our sample is short on the time dimension so that it is not meaningful to evaluate predictability at horizons that are too long (e.g., with a 3-year horizon, the longest time series in our sample has less than 12 independent 3-year return observations). The results indicate that $\mu^{(3y)}_{n,t}$ predicts $\overline{r}^{(3y)}_{n,t\rightarrow t+3}$ with similar strength

[31] Let $\overline{r}_{n,t}$ be the historical (expanding window) average return for asset class n up to time t. We define

$$R^2_{OOS} = 1 - \frac{\sum_{n=1}^{N}\sum_{t=1}^{T_n}(r_{n,t+1} - \mu^{(1y)}_{n,t})^2}{\sum_{n=1}^{N}\sum_{t=1}^{T_n}(r_{n,t+1} - \widehat{\mu}_{n,t})^2},$$

where $\widehat{\mu}_{n,t} = \overline{r}_{n,t}$ for variation over time and $\widehat{\mu}_{n,t} = \widehat{\mu}_t = \sum_{n=1}^{N_t}\overline{r}_{n,t}$ for variation across assets. R^2_{OOS} is defined analogously in columns that use alternative independent and/or dependent variables in the regression. Importantly, to ensure reasonable historical averages, we require at least 20 years in the expanding window, removing from the R^2_{OOS} calculation any asset class that does not satisfy this criterion.

[32] Internet Appendix C.2.3 addresses two potential concerns with these return predictability results (and explains why they are not that concerning conceptually). First, some CMAs used to construct $\mu_{n,t}$ are produced after December of year t (see Figure IA.3 in the Internet Appendix). Most of these "late CMAs" likely still use information at of December of year t since the most common production month among them is January of year $t+1$. However, as a robustness check, we use only CMAs produced by December of year t or earlier, yielding results consistent with our main findings (see Panel A of Internet Appendix Table IA.6). Second, our aggregation method is not "out-of-sample" from the perspective of the econometrician as it requires full sample estimates of institution fixed effects. To address this issue, we apply an out-of-sample aggregation method that implements the steps in Subsection 3.1 on an expanding window. The results are also similar to our main results (see Panel B of Internet Appendix Table IA.6).

as $\mu_{n,t}^{(1y)}$ predicts $\overline{r}_{n,t+1}^{(1y)}$ (somewhat better in the cross-section and somewhat worse in the time-series). Importantly, we continue to not reject the $a = 0$ and $b = 1$ FIRE restrictions.

Section 2 emphasizes the importance of subjective risk premia as a determinant of subjective expected returns. Table 7, Panels B and C, show that risk premia are also an important determinant of the ability of μ to predict future returns. Specifically, for each R^2 number reported in Panel A, we provide a Shapley decomposition to identify the portion of the R^2 value that originates from $rp^{(m)}$ and $\alpha^{(m)}$.[33] The main determinant of R^2 values is $rp^{(m)}$, with $\alpha^{(m)}$ even displaying a negative contribution in some instances. Interestingly, $\alpha^{(m)}$ tends to contribute positively to R^2 values associated with 1-year return predictions, but negatively to R^2 values associated with 3-year predictions, highlighting that $rp^{(m)}$ is particularly important for long-term forecasts.

To further explore return predictability over time, Table 8 shows results for Equation 11 estimated separately for each asset class using $\mu_{n,t}^{(1y)}$. Two key findings emerge. First, we can only reject the $b = 1$ hypothesis for *US High Yield Corp Bonds* (even at 10% significance), with *US REITs* close to rejection. That said, statistical power is generally lower for single asset classes, resulting in non-rejection of $b = 0$ at 10% significance for 5 of the 19 asset classes we study. This underscores the importance of combining asset classes to study return predictability (as in Table 7), where statistical power is notably higher. Second, R^2_{OOS} values are generally positive, except for *Hedge Funds* ($R^2_{OOS} = -11.2\%$), likely due to diverse hedge fund strategies creating a poor match between beliefs and realized returns. Overall, μ tends to predict future returns within individual asset classes.

[33]Let R^2_μ reflect an R^2 number reported in Panel A of Table 7. Then, Panels B and C report

$$\text{Shapley}(rp) = 0.5 \cdot (R^2_\mu - R^2_\alpha) + 0.5 \cdot (R^2_{rp} - 0) \quad \text{and} \quad \text{Shapley}(\alpha) = 0.5 \cdot (R^2_\mu - R^2_{rp}) + 0.5 \cdot (R^2_\alpha - 0),$$

where R^2_{rp} and R^2_α (with $R^2_\mu = R^2_{rp} + R^2_\alpha$) reflect the respective R^2 numbers obtained by replacing $\mu_{n,t}$ with $\mu^{rp}_{n,t} = \overline{\alpha} + rp_{n,t}$ and $\mu^{\alpha}_{n,t} = \alpha_{n,t} + \overline{rp}$, respectively (with α and rp from the restricted CAPM). Intuitively, the first and second terms of Shapley(rp) capture the marginal R^2 improvement of allowing for rp variation relative to only allowing for α variation and allowing for no variation, respectively (and similarly for Shapley(α)). The Shapley R^2 decomposition builds on game theoretical principles developed by Shapley (1953), with econometric decomposition applications dating back to Shorrocks (1982). See Chapter 9 of Molnar (2022) for a recent textbook treatment.

Table 9 (Panels A and B, first two blocks) repeats the predictability exercise for risk measures (but without any adjustment for horizon).[34] For both σ^2 and $\beta^{(m)}$, we observe strong predictability across asset classes (i.e., when including year fixed effects), with b coefficients that are not that far from 1 (albeit we can statistically reject the $b = 1$ restriction for σ^2). However, we observe no predictability over time (i.e., when including asset class fixed effects), with b coefficients that are negative (and statistically insignificant) for both σ^2 and $\beta^{(m)}$. So, the institutions in our sample seem to reasonably predict risk variation across assets. However, they are unable to predict risk variation over time. One limitation of this analysis is that if short-term risk and long-term risk are disconnected, then it is possible that time variation in subjective risk predicts time variation in long-term realized risk. We cannot meaningfully test this hypothesis given the short time series dimension of our sample.

Table 9 (Panels A and B, third block) repeats the predictability test for $\alpha^{(m)}$, adjusting it for horizon (and calling it $\alpha^{(m,1y)}$) similarly to μ. There is little connection between $\alpha^{(m,1y)}$ and subsequent alphas. Although the b coefficient is positive in all specifications, it is always below 0.50 and statistically insignificant. Also, the R^2 values are all under 1%, indicating that subjective alphas explain almost no variation in future alphas. Positive and non-trivial R^2_{OOS} values appear, but the near 0% in-sample R^2 values suggest this result merely reflects historical alphas being even less predictive of future alphas than subjective alphas are.

4 Conclusion

In this paper, we analyze the long-term Capital Market Assumptions (CMAs) of major asset managers and institutional investor consultants from 1987 to 2022. We focus on their risk and return expectations for 19 asset classes. Our findings reveal a strong positive risk-return tradeoff, with the bulk of variability in subjective expected returns being attributed to risk premia rather than alphas. Notably, belief variability and the positive risk-return relationship are more pronounced across asset classes than across institutions, highlighting the

[34] Adjusting for horizon in expected risk is more challenging because the H-year subjective risk is not the subjective expectation of the average of 1-year realized risks over the next H-years.

importance of examining beliefs across multiple asset classes. Additionally, we demonstrate that subjective expected returns aggregated across institutions predict subsequent realized returns across asset classes and over time. Lastly, we find that the predictive power of return expectations is primarily driven by risk premia (not alphas) similar to our findings on expected return variation.

Our paper provides an important contribution to the literature on subjective beliefs. In particular, we present the first analysis of the subjective risk and return expectations of institutional investors as implied by their CMAs. Importantly, our findings indicate that incorporating a robust subjective risk premia component is crucial for accurately modeling the return expectations of institutional investors.

Our results also open the door to many new questions, some of which we explore in subsequent work. For instance, Couts et al. (2024) study the drivers of time variation and disagreement in the return expectations of institutional investors. Relatedly, Andonov et al. (2024) study the pass through of the overall beliefs implied by CMAs to the portfolio allocations of institutional investors. Overall, these are important steps towards shedding more light on the beliefs of institutional investors, which represent an important set of economic agents that has been understudied in the beliefs literature.

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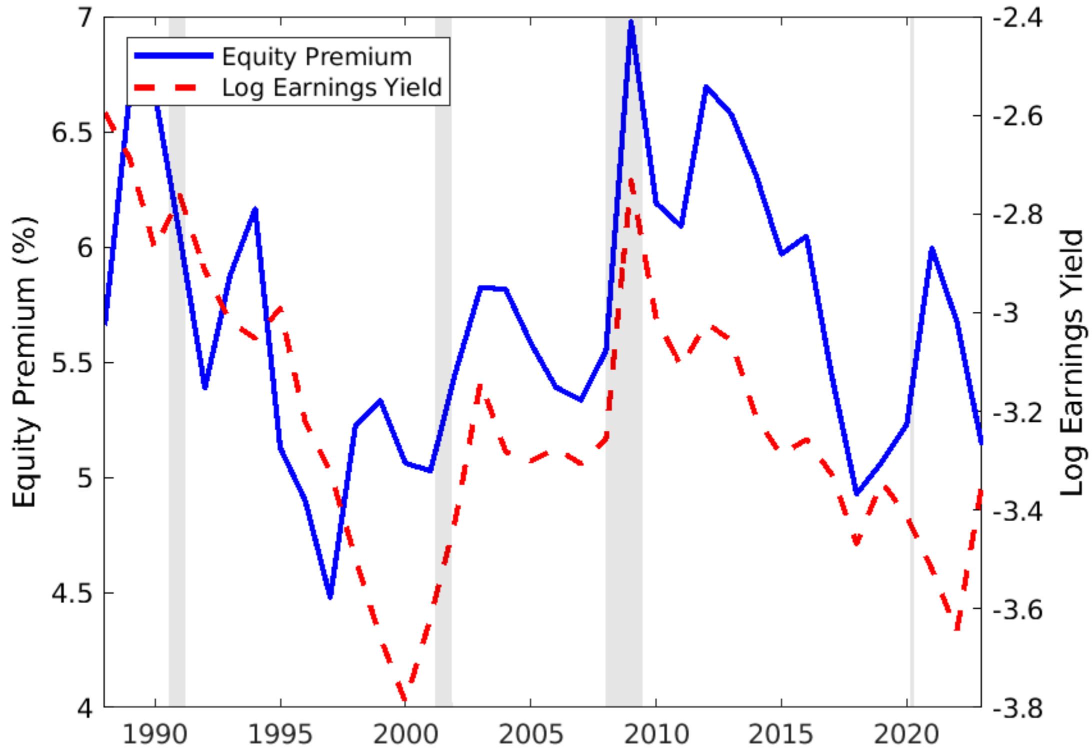


Figure 2
The Aggregate Subjective Equity Premium from Institutions

This figure plots the subjective equity premium aggregated across our institutions together with the S&P 500 earnings yield, proxied with $log(1/CAPE)$, where CAPE refers to the Cyclically Adjusted PE Ratio from Robert Shiller (available under `http://www.econ.yale.edu/~shiller/data.htm`). The shaded regions reflect US economic recessions as defined by the National Bureau of Economic Research (NBER). The subjective equity premium of institution j at time t is given by $\mu_{j,e,t} = \mathbb{E}_{j,t}[R_e] - \mathbb{E}_{j,t}[R_f]$ with R_e reflecting returns from *US Equities (Large Cap)* and R_f reflecting returns from *US Cash*. Section 1 provides more details about our subjective beliefs data and the analysis in this figure. Our aggregation method accounts for differences in sample composition over the years (see Subsection 3.1 for details). Specifically, we estimate a panel regression of $\mu_{j,e,t}$ onto institution fixed effects and year fixed effects (with no intercept and one manager dummy suppressed),

$$\mu_{j,e,t} = \overline{\mu}_{e,j} + \overline{\mu}_{e,t} + \eta_{j,e,t}$$

and then plot the following quantity:

$$\mu_{e,t} = \left[\frac{1}{T} \cdot \sum_{t=1}^{T} \left(\frac{1}{J_t} \sum_{j=1}^{J_t} \mu_{j,e,t}\right)\right] + \left(\overline{\mu}_{e,t} - \frac{1}{T} \cdot \sum_{t=1}^{T} \overline{\mu}_{e,t}\right)$$

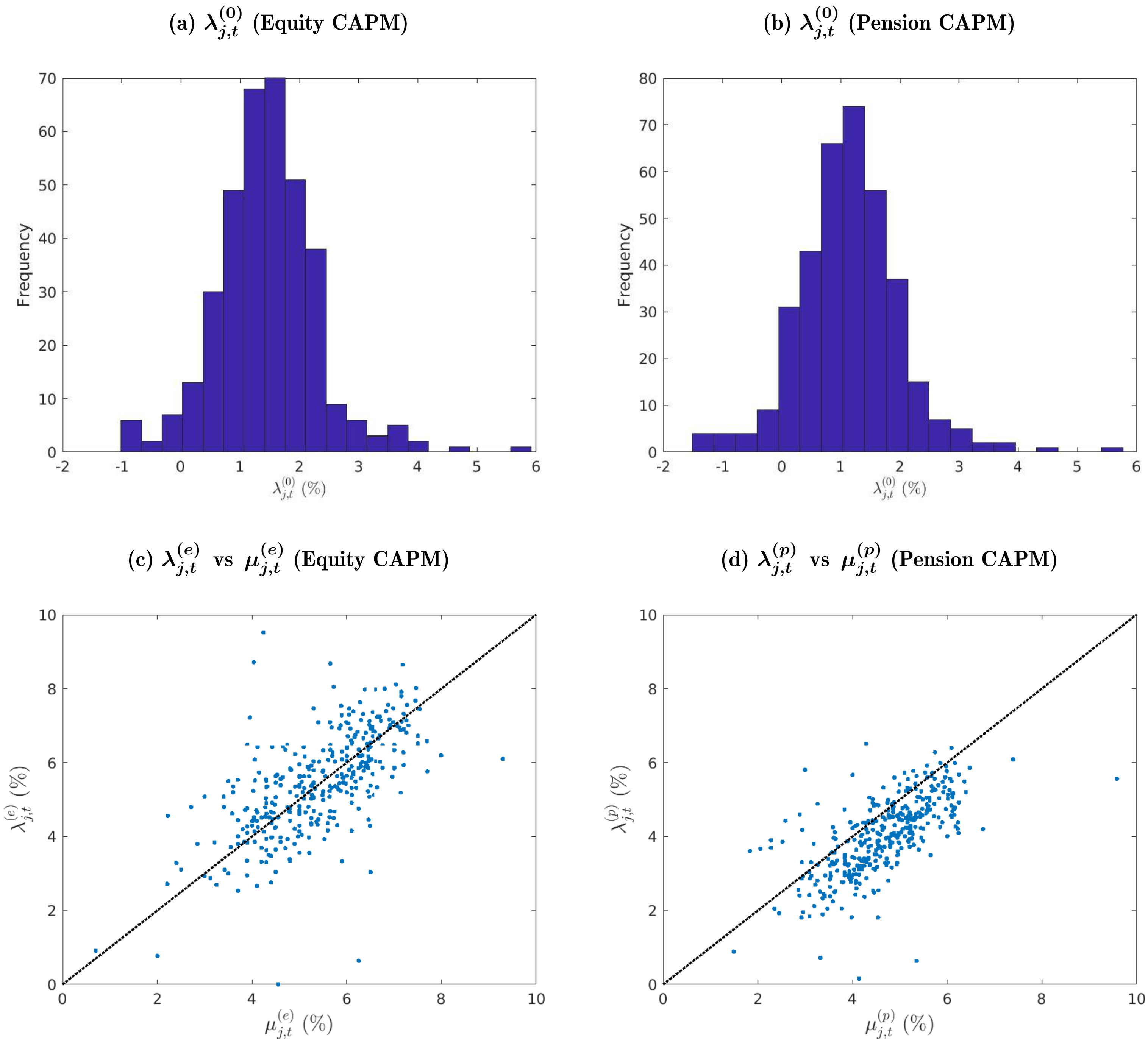


Figure 3

Parameters from Regressing Subjective Expected Returns on Subjective Market Betas

The pricing of subjective betas is based on the institution-year regression $\mu_{j,n,t} = \lambda_{j,t}^{(0)} + \lambda_{j,t}^{(m)} \cdot \beta_{j,n,t}^{(m)} + \alpha_{j,n,t}^{(m)}$, where $\mu_{j,n,t}$ and $\beta_{j,n,t}^{(m)}$ are obtained directly from the CMA of institution j at time t. Panels (a) and (b) plot histograms for the distribution of $\lambda_{j,t}^{(0)}$ estimates across institution-year observations. Panels (c) and (d) provide scatterplots of $\lambda_{j,t}^{(m)}$ estimates against $\mu_{j,t}^{(m)}$, which is the expected excess return on the market portfolio. We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Section 1.3 provides more details about our subjective beliefs data and Section 2.2 provides more details about the analysis reported in this figure.

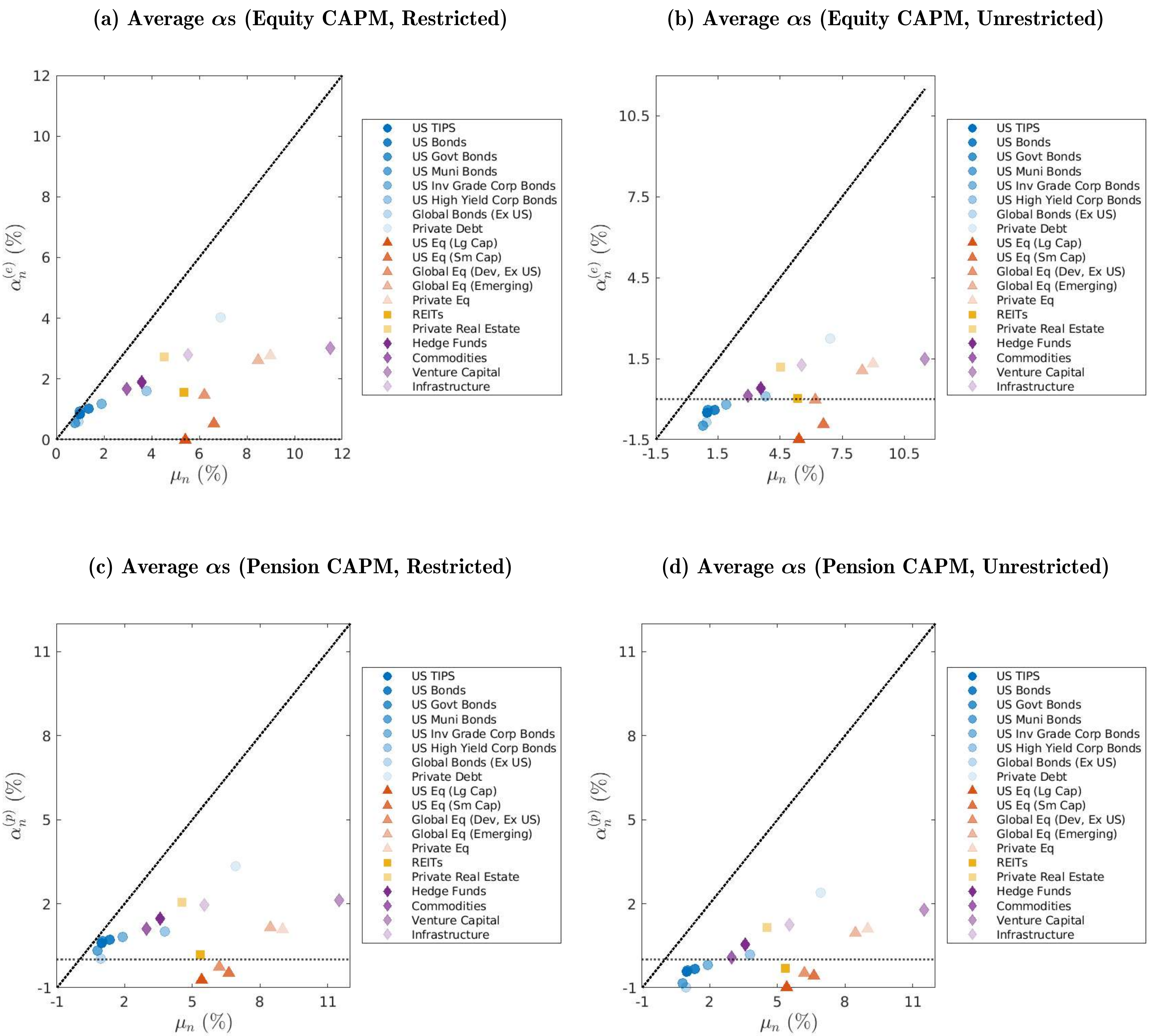


Figure 4
Average Subjective Alphas of Asset Classes

This figure plots the subjective expected return of each asset class (averaged across institutions and years) against its subjective alpha (averaged across institutions and years). Alphas are based on Equation 3, with Panels (a) and (c) imposing CAPM restrictions ($\lambda_{j,t}^{(0)} = 0$ and $\lambda_{j,t}^{(m)} = \mu_{j,t}^{(m)}$) and Panels (b) and (d) using the $\lambda_{j,t}^{(0)}$ and $\lambda_{j,t}^{(m)}$ estimated from the regression in Equation 3. We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Section 1.3 provides more details about our subjective beliefs data and Section 2.2 provides more details about the analysis reported in this figure.

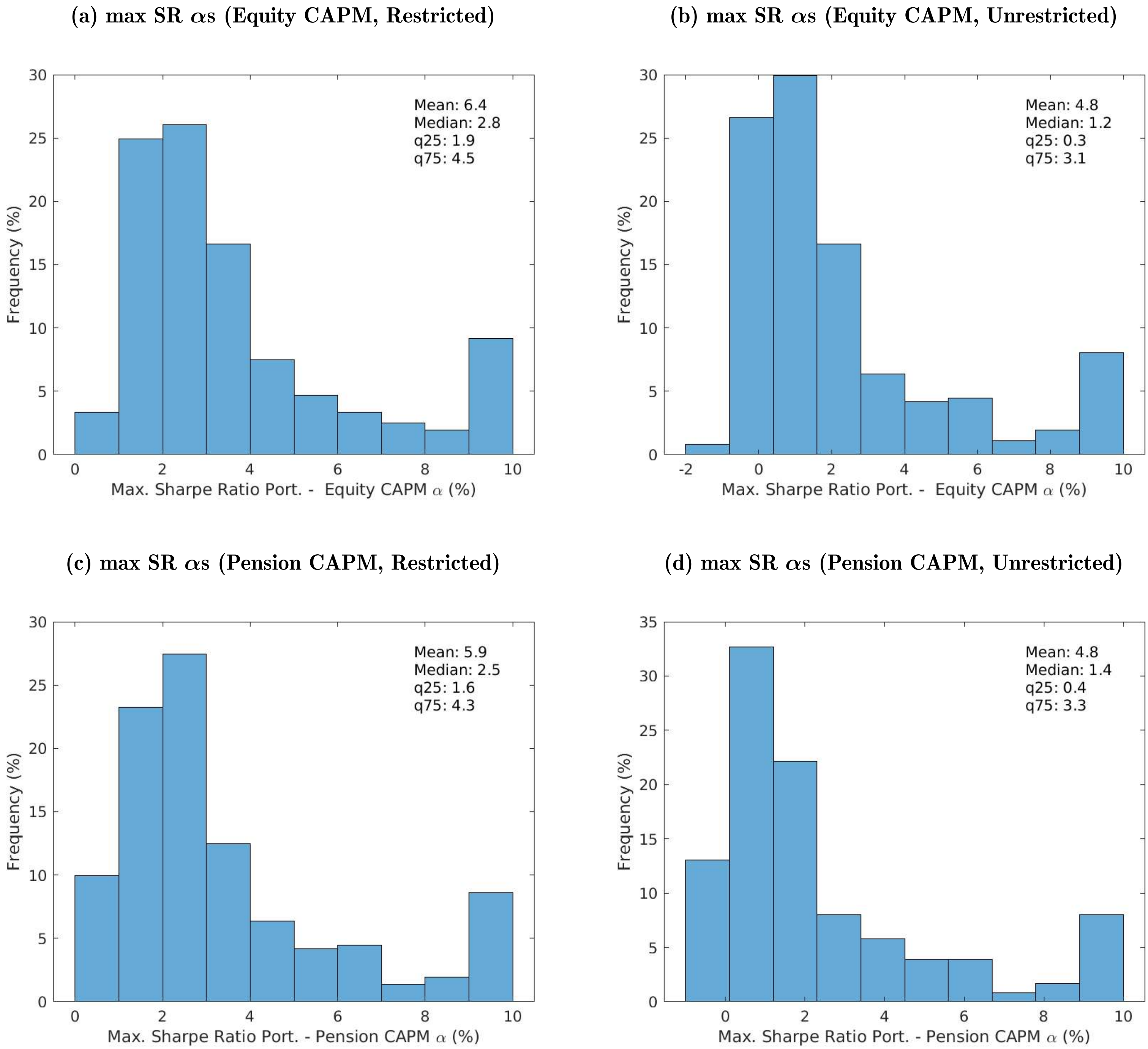


Figure 5

Subjective Alphas of Maximum Sharpe Ratio Portfolios

This figure plots the distribution of annualized alphas of maximum Sharpe ratio (or tangency) portfolios across all institution-year observations in our sample. Specifically, for each institution-year, we obtain $\alpha^{(m)}_{max,j,t} = w'_{max,j,t}\alpha^{(m)}_{j,t}$, where $\alpha^{(m)}_{j,t}$ is the vector of asset class alphas and $w_{max,j,t} = \Sigma^{-1}_{j,t}\mu_{j,t}/|\mathbf{1}'\Sigma^{-1}_{j,t}\mu_{j,t}|$. Asset class alphas are based on Equation 3. Panels (a) and (c) imposing CAPM restrictions ($\lambda^{(0)}_{j,t} = 0$ and $\lambda^{(m)}_{j,t} = \mu^{(m)}_{j,t}$). Panels (b) and (d) using the $\lambda^{(0)}_{j,t}$ and $\lambda^{(m)}_{j,t}$ estimated from the regression in Equation 3. We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Section 1.3 provides more details about our subjective beliefs data and Section 2.2 provides more details about the analysis reported in this figure.

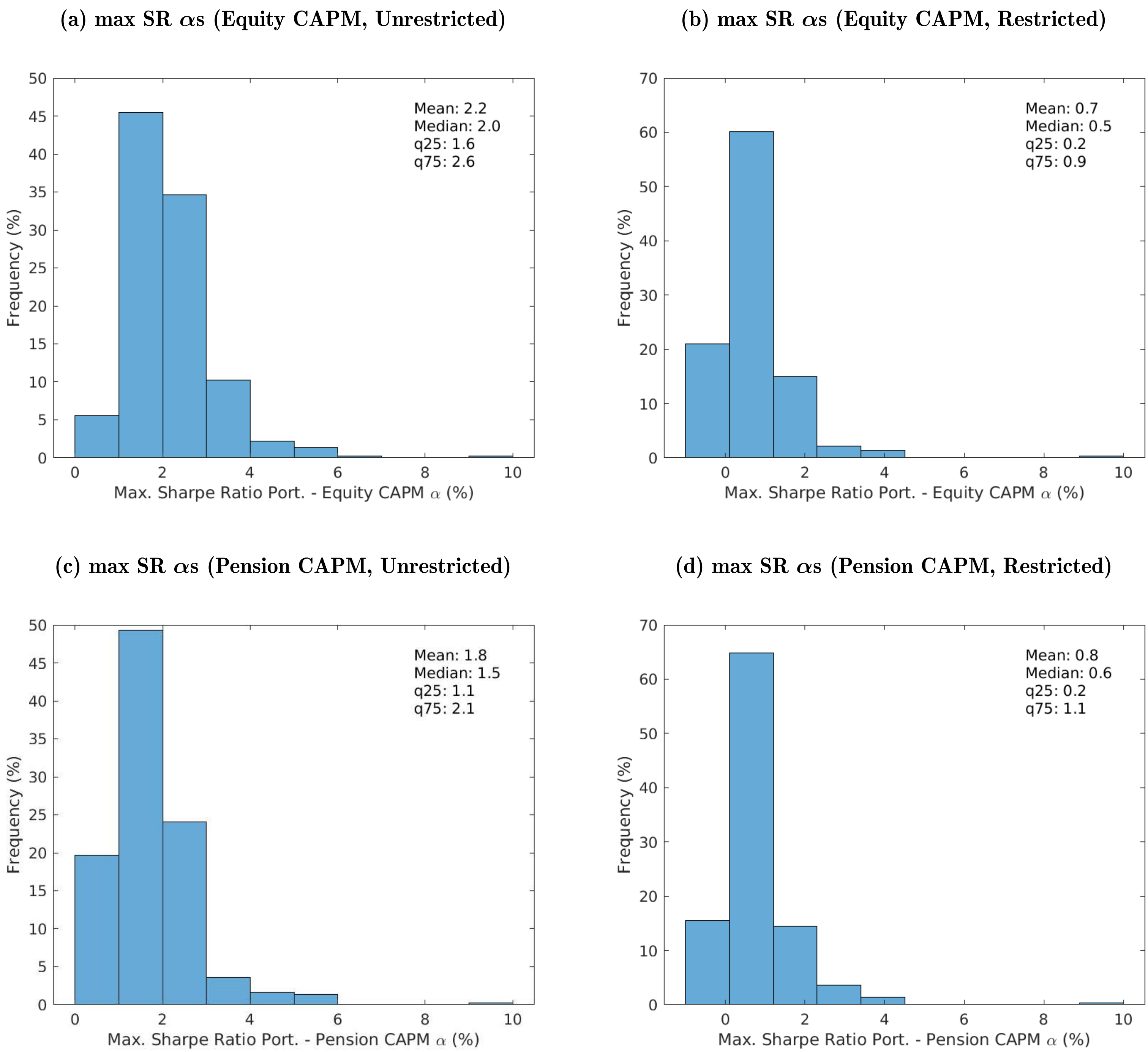
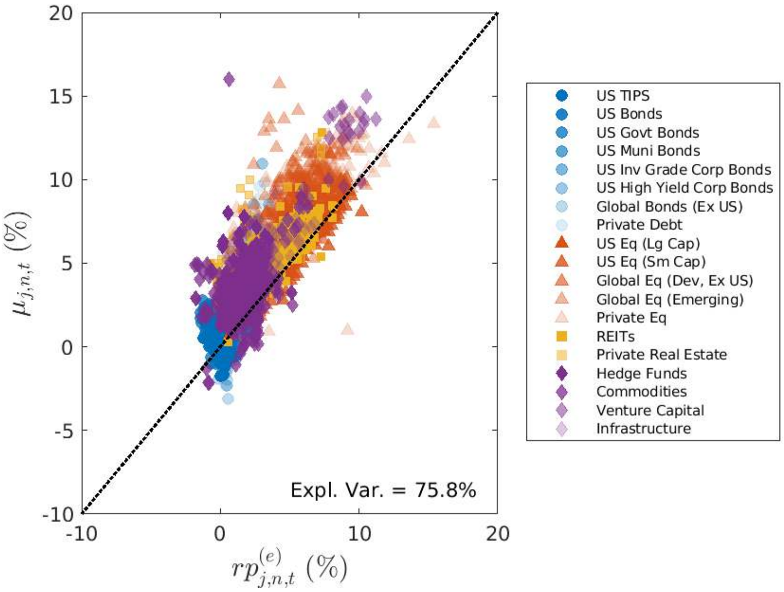


Figure 6

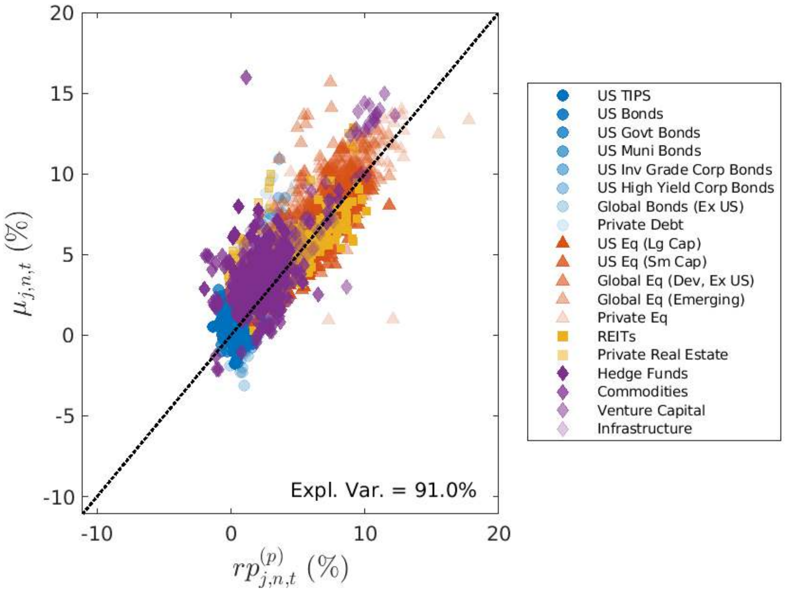
Subjective Alphas of Maximum Sharpe Ratio Portfolios (no Short Sales)

This figure plots the distribution of annualized alphas of no short sale maximum Sharpe ratio portfolios across all institution-year observations in our sample. Specifically, for each institution-year, we obtain $\alpha^{(m)}_{max,j,t} = w'_{max,j,t}\alpha^{(m)}_{j,t}$, where $\alpha^{(m)}_{j,t}$ is the vector of asset class alphas and $w_{max,j,t}$ is the weight vector that maximizes the subjective Sharpe ratio under $w_{max,j,t} \geq 0$. Asset class alphas are based on Equation 3. Panels (a) and (c) imposing CAPM restrictions ($\lambda^{(0)}_{j,t} = 0$ and $\lambda^{(m)}_{j,t} = \mu^{(m)}_{j,t}$). Panels (b) and (d) using the $\lambda^{(0)}_{j,t}$ and $\lambda^{(m)}_{j,t}$ estimated from the regression in Equation 3. We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Section 1.3 provides more details about our subjective beliefs data and Section 2.2 provides more details about the analysis reported in this figure.

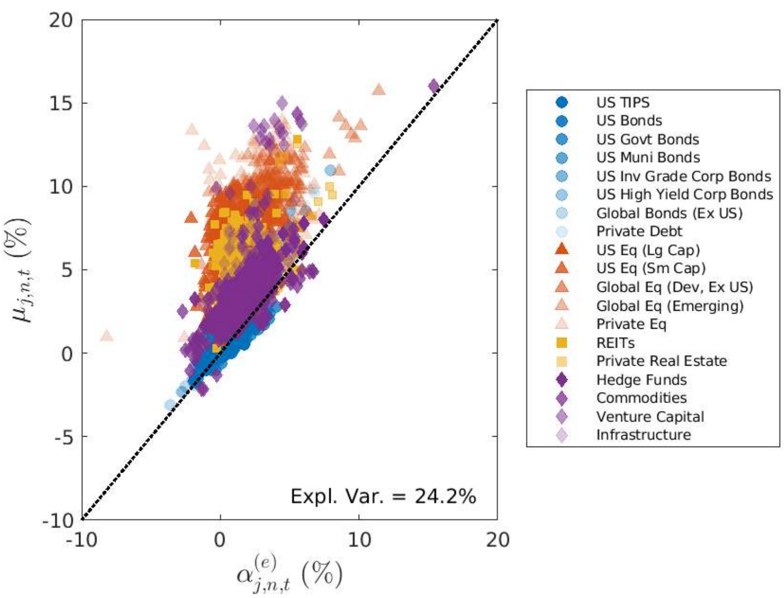
(a) $\mu_{j,n,t}$ vs $rp^{(e)}_{j,n,t}$ (Equity CAPM)



(b) $\mu_{j,n,t}$ vs $rp^{(p)}_{j,n,t}$ (Pension CAPM)



(c) $\mu_{j,n,t}$ vs $\alpha^{(e)}_{j,n,t}$ (Equity CAPM)



(d) $\mu_{j,n,t}$ vs $\alpha^{(p)}_{j,n,t}$ (Pension CAPM)

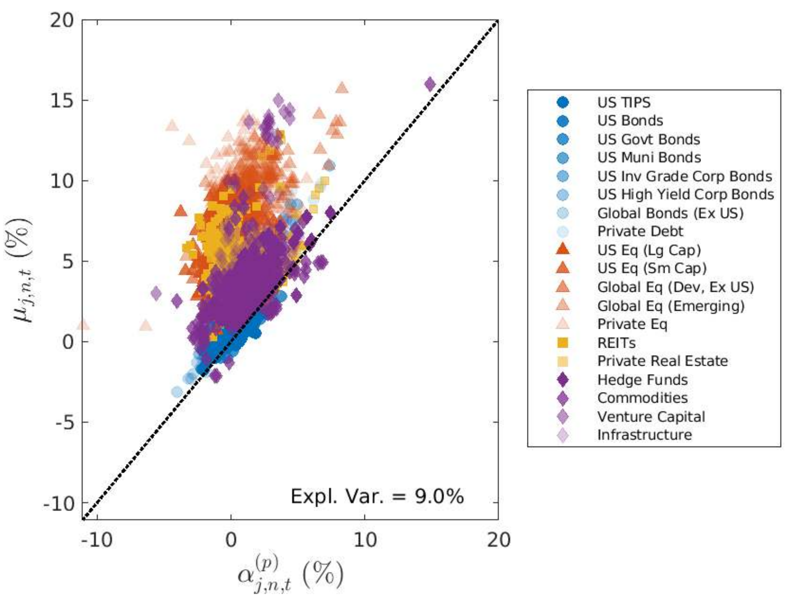


Figure 7

Subjective Expected Return Variation: Risk Premia vs Alphas

This figure plots the subjective expected return of each observation ($\mu_{j,n,t}$) against the respective subjective risk premium ($rp^{(m)}_{j,n,t} = \lambda^{(0)}_{j,t} + \lambda^{(m)}_{j,t} \cdot \beta^{(m)}_{j,n,t}$) or the respective subjective alpha ($\alpha^{(m)}_{j,n,t} = \mu^{(m)}_{j,n,t} - rp^{(m)}_{j,n,t}$). All four panels impose CAPM restrictions ($\lambda^{(0)}_{j,t} = 0$ and $\lambda^{(m)}_{j,t} = \mu^{(m)}_{j,t}$), with Internet Appendix Figure IA.5 providing analogous plots that use the $\lambda^{(0)}_{j,t}$ and $\lambda^{(m)}_{j,t}$ estimated from the regressions in Equation 3. We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Section 1.3 provides more details about our subjective beliefs data and Section 2.3 provides more details about the analysis reported in this figure.

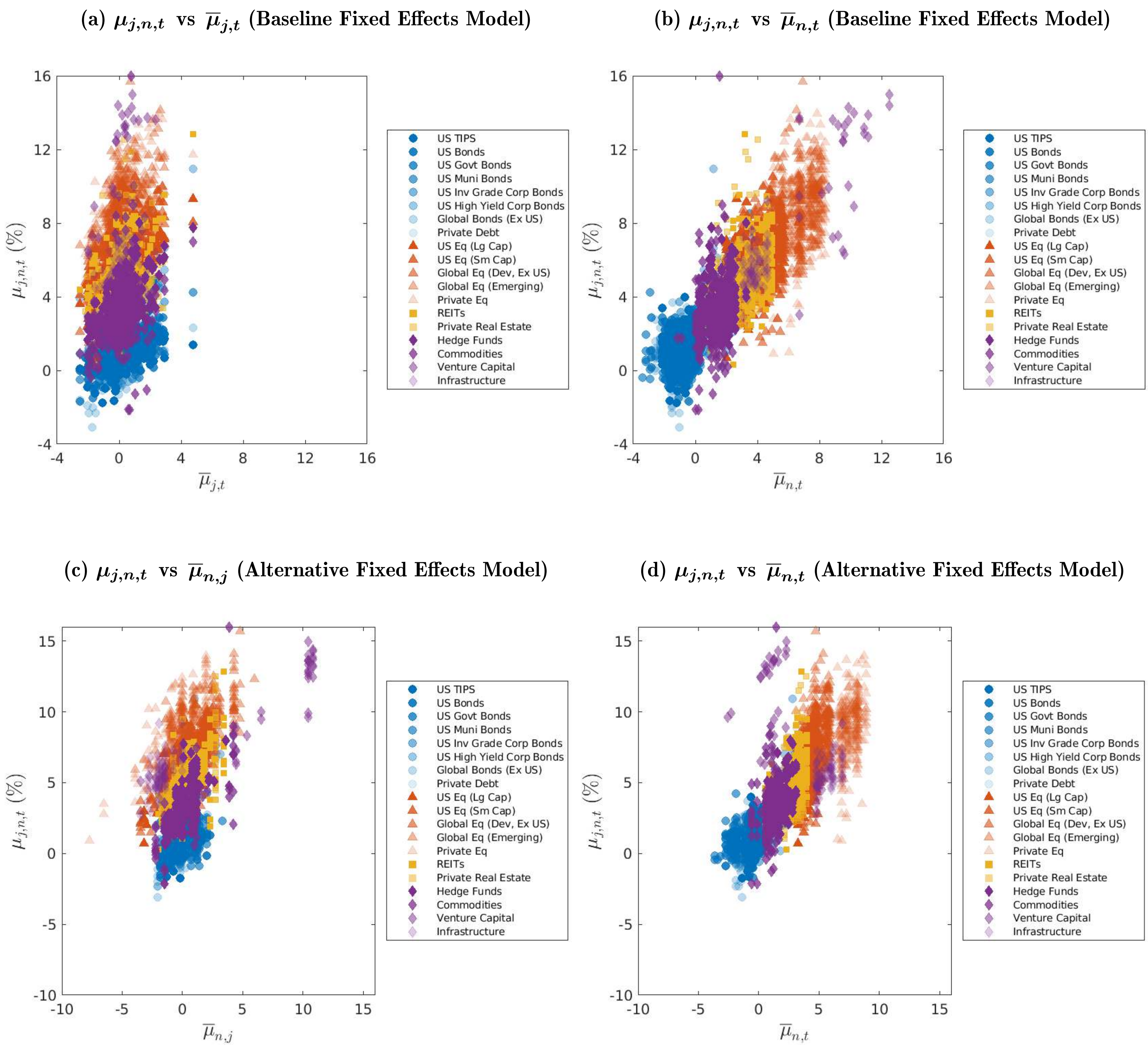
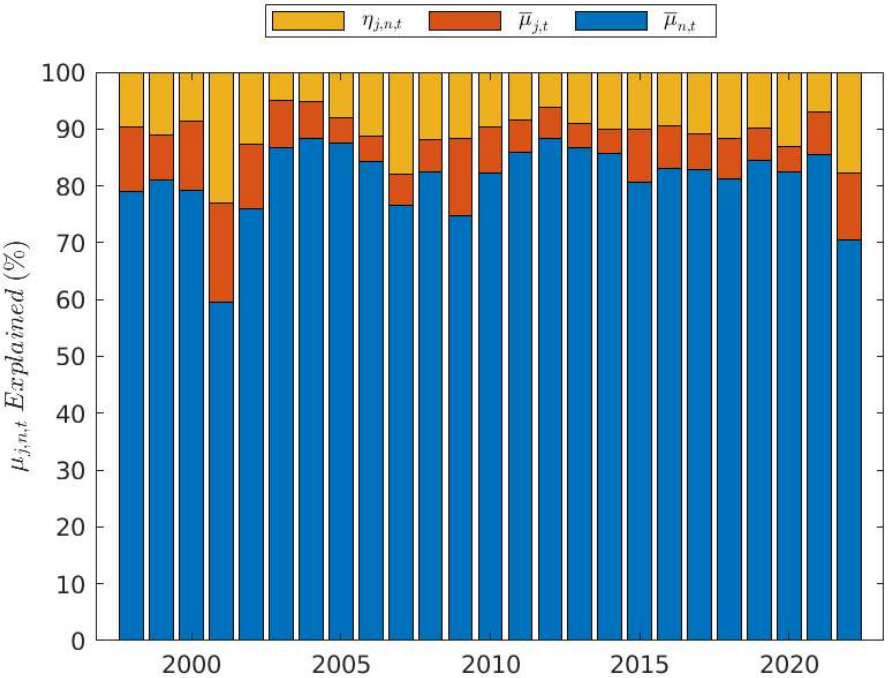


Figure 8

Heterogeneity in Subjective Expected Returns: Institution vs Asset Class Effects

This figure plots subjective expected returns ($\mu_{j,n,t}$) against institution or asset class fixed effects. The institution fixed effect is given by $\overline{\mu}_{j,t}$ or $\overline{\mu}_{n,j}$ depending on whether we are in the baseline model (Equation 6) or in the alternative model (Equation 8). The asset class fixed effect is always given by $\overline{\mu}_{n,t}$. Section 1.3 provides more details about our subjective beliefs data and Section 2.4 provides more details about the analysis reported in this figure.

(a) Baseline Fixed Effects Model: $\boldsymbol{\mu_{j,n,t} = \overline{\mu}_{n,t} + \overline{\mu}_{j,t} + \eta_{j,n,t}}$



(b) Alternative Fixed Effects Model: $\boldsymbol{\mu_{j,n,t} = \overline{\mu}_{n,t} + \overline{\mu}_{n,j} + \eta_{j,n,t}}$

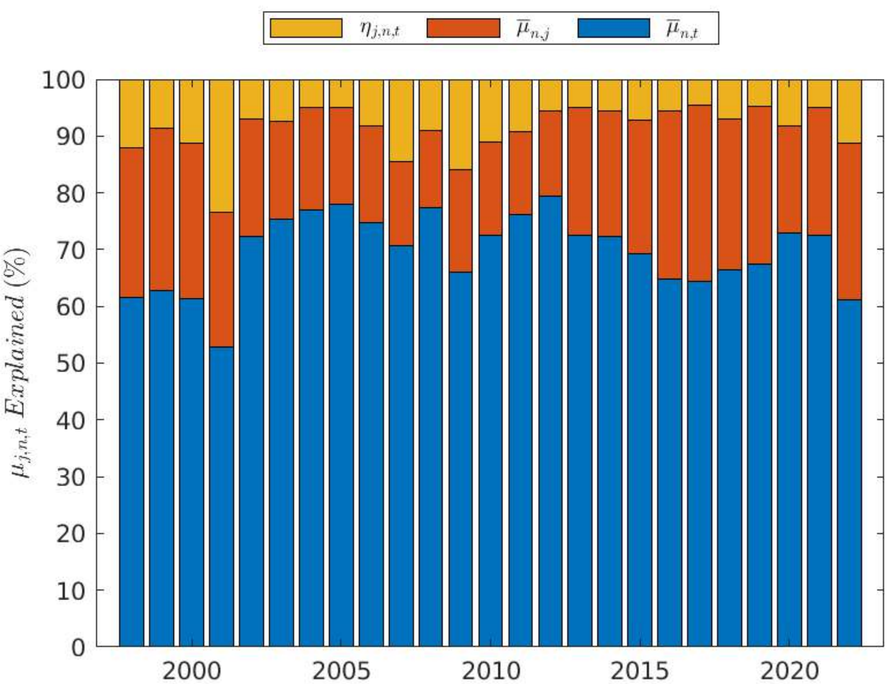


Figure 9

Decomposing Variability in Subjective Expected Returns: Institution vs Asset Class Effects

This figure plots the decomposition of within year expected return variability into the effect of asset class heterogeneity (in blue), institution heterogeneity (in red), and residuals (in orange). The institution fixed effect is given by $\overline{\mu}_{j,t}$ or $\overline{\mu}_{n,j}$ depending on whether we are in the baseline model (Equation 6) or in the alternative model (Equation 8). The asset class fixed effect is always given by $\overline{\mu}_{n,t}$. The variance decomposition is based on Equation 7 (or its analogue for the alternative fixed effects model). Section 1.3 provides more details about our subjective beliefs data and Section 2.4 provides more details about the analysis reported in this figure.

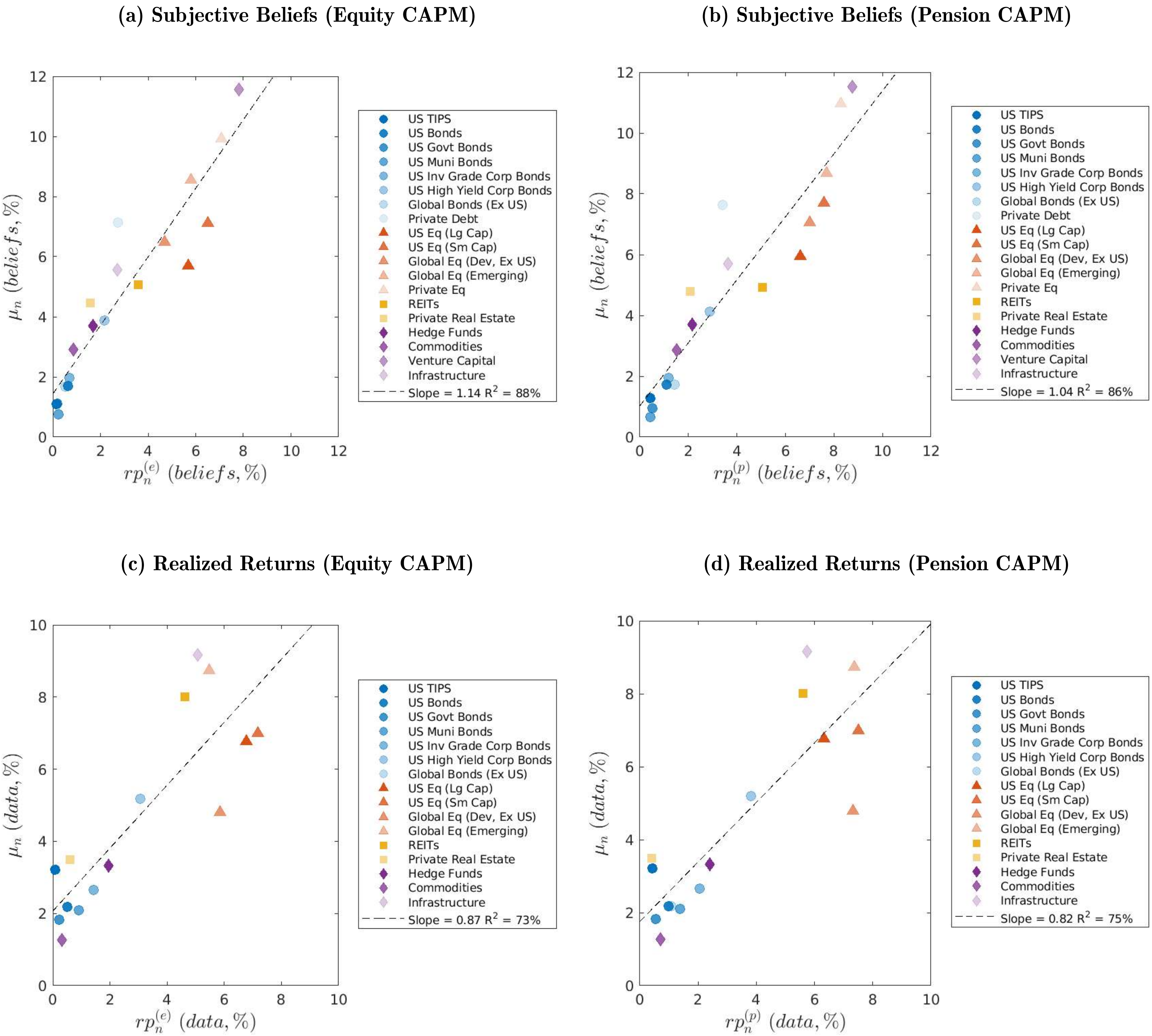


Figure 10

Risk-Return Tradeoff: Subjective Beliefs vs Realized Returns

This figure plots expected returns ($\mu_n = \mathbb{E}[r_n]$) against risk premia ($rp_n^{(m)} = \beta_n^{(m)} \cdot \mu^{(m)}$) across asset classes. Panels (a) and (b) use beliefs data (averaged across institutions and years), and thus reflect the subjective risk-return tradeoff. Panels (c) and (d) use unconditional data moments (μ_n is based on average excess returns and $rp_n^{(m)}$ is based on estimated beta and market average excess return), and thus reflect the objective risk-return tradeoff. We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Sections 1.3 and 3.1 provide more details about the data and Section 3.2 provides more details about the analysis reported in this figure.

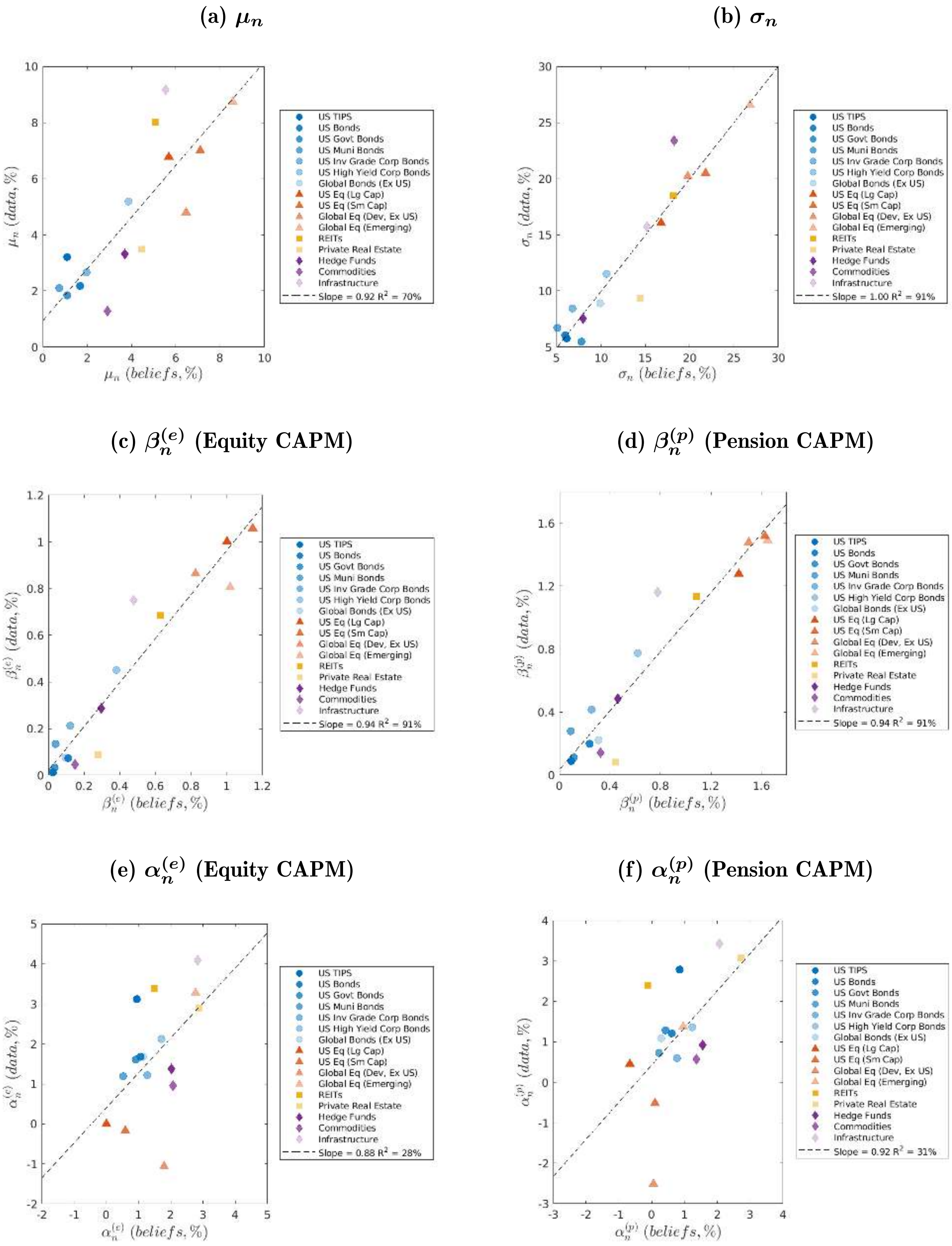


Figure 11
Average Subjective Beliefs vs Unconditional Realized Return Moments

This figure plots unconditional data moments against average values for their respective beliefs (averaged across institutions and years). We use returns over the full sample for each asset class (see dates in Table 6). For Panels (c) to (f), we consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Sections 1.3 and 3.1 provide more details about the data and Section 3.3 provides more details about the analysis reported in this figure.

Table 1

Asset Managers and Investment Consultants in our Sample

The table reports information on the institutions that have at least one CMA in our sample. Panel A details the asset managers (or simply "managers"), with information on their Assets Under Manager (AUM) ranking, dollar value (in trillion of dollars), and fraction relative to the total AUM of the top 50 AUM managers in the world. The data is from the October 2022 report on the world's largest 500 asset managers by the Thinking Ahead Institute and the Pensions & Investments company (TAI-P&I). Three of the managers in our dataset were not covered in the TAI-P&I reports, and thus we obtained their AUM from alternative sources and provided the rank they would have if present in the 2022 TAI-P&I report (this is why Merrill Lynch and Morgan Stanley both are ranked #18). BGI (or Barclays Global Investors) was acquired by Blackrock in 2009, and thus we do not report BGI AUM information. One institution has not provided authorization to release their name, so we refer to it as "Manager #22" (and do not report their AUM information). Panel B details the investment consultants (or simply "consultants"), with information on their role as primary consultants for US public pension funds (note that each US public pension fund has at most one primary consultant). Specifically, each row in Panel B reports information about the pension funds for which the given consultant is the primary consultant of on average from 2001 to 2021. The data is from the Center for Retirement Research at Boston College. Section 1.3 provides more details about our subjective beliefs data.

PANEL A - Asset Managers				PANEL B - Investment Consultants			
	AUM Information				US Pension Fund Consultant		
Manager Name	**Rank**	**$ AUM**	**% Top 50**	**Consultant Name**	**# Funds**	**% Funds**	**% AUM**
BGI	-	-	-	**Angeles**	1	0.3%	0.4%
BNY Mellon	9	$2.43 T	2.8%	**Aon**	19	8.8%	14.8
Capital Group	7	$2.72 T	3.1%	**Buckingham**	-	-	-
Envestnet	77	$0.36 T	0.4%	**Callan**	24	11.2%	11.9%
Franklin Templeton	17	$1.58 T	1.8%	**CAPTRUST**	-	-	-
Goldman Sachs	8	$2.47 T	2.9%	**Cliffwater**	1	0.5%	0.6%
Invesco	15	$1.61 T	1.9%	**CSG**	-	-	-
JP Morgan	5	$3.11 T	3.6%	**CWO**	-	-	-
Merrill Lynch	18	$1.57 T	1.8%	**Inspire**	-	-	-
Morgan Stanley	18	$1.49 T	1.7%	**MacroAnalytics**	-	-	-
Northern Trust	16	$1.61 T	1.9%	**Meketa**	6	2.6%	2.4%
Nuveen	22	$1.26 T	1.5%	**Mercer**	6	2.6%	2.5%
PFM	156	$0.13 T	0.2%	**Milliman**	1	0.7%	0.2%
PGIM	13	$1.74 T	2.0%	**NEPC**	19	8.7%	4.5%
PIMCO	12	$2.00 T	2.3%	**PCA**	9	4.0%	7.4%
Russell Inv	81	$0.34 T	0.4%	**ResearchAffiliates**	-	-	-
SEI	85	$0.31 T	0.4%	**RVK**	8	3.8%	6.2%
T. Rowe Price	14	$1.69 T	2.0%	**Segal**	2	0.7%	0.1%
Vanguard	2	$8.47 T	9.8%	**Sellwood**	-	-	-
Voya	71	$0.41 T	0.5%	**Strategic Inv Sol**	8	3.5%	6.5%
Wells Fargo	16	$1.61 T	1.9%	**Verus**	4	1.7%	1.2%
Manager #22	-	-	-	**Wilshire**	11	5.3	13.1
				WTW	2	0.7%	0.5%
Total	-	> $37 T	> 42.7%	**Total**	≥ 121	≥ 55.1%	≥ 72.3%

Table 2

Sample Coverage of CMAs

This table reports information on our sample of Capital Market Assumptions (CMAs) over time. Panel A details, for each year, the number of asset managers (or simply "managers"), institutional investor consultants (or simply "consultants"), and CMAs (managers+consultants) in our sample. We have data obtained directly from the CMAs of the underlying institutions (under "# Direct CMAs") and data obtained indirectly from the reports of pension funds (which is given by "# CMAs" - "# of Direct CMAs"). Panel A also reports, for each year, the number of unique asset classes in our sample as well as the average number of asset classes covered by an institution in a given year. Panel B reports the number of CMAs covering each of the asset classes in our sample. Section 1.3 provides more details about our subjective beliefs data.

PANEL A - Number of Managers, Consultants, and Asset Classes in our Sample (by Year)

	1987	1996	1997	1998	2000	2002	2004	2006	2008	2010	2012	2014	2016	2018	2020	2022	Total
# CMAs	1	1	3	4	5	8	8	10	12	15	18	15	15	23	25	30	361
# Direct CMAs	0	1	3	4	5	5	6	8	9	11	13	13	14	19	22	28	301
# Manager CMAs	0	0	1	1	1	1	1	1	4	5	6	5	6	12	11	18	128
# Consultant CMAs	1	1	2	3	4	7	7	9	8	10	12	10	9	11	14	12	233
# Asset Classes	4	8	13	13	13	16	17	18	18	19	20	20	20	20	20	20	-
# Asset Classes per CMA	4	8	9	9	9	9	11	12	12	12	13	14	14	13	14	14	-

PANEL B - Number of Institutions Covering Each Asset Class in our Sample (by Year)

Asset Class	1987	1996	1997	1998	2000	2002	2004	2006	2008	2010	2012	2014	2016	2018	2020	2022	Total
US Cash	1	1	3	4	5	8	8	10	12	15	18	15	15	23	25	30	361
US TIPS	0	0	0	1	1	5	7	10	11	15	17	14	14	19	20	25	299
US Bonds	1	1	3	4	5	8	8	10	12	13	17	14	15	21	22	26	342
US Govt Bonds	0	0	1	0	1	2	3	3	4	4	7	9	10	15	14	23	176
US Muni Bonds	0	0	0	0	0	1	1	1	3	4	6	6	6	9	11	12	107
US Inv Grade Corp Bonds	0	0	0	0	0	1	2	1	2	3	7	5	5	7	11	17	108
US High Yield Corp Bonds	0	0	2	3	5	5	6	8	10	12	14	12	13	18	21	26	282
Global Bonds (Ex US)	0	1	3	4	4	4	5	7	9	11	14	12	12	15	18	25	271
Private Debt	0	0	0	0	0	0	0	0	0	1	2	2	3	4	9	13	56
US Equities (Large Cap)	1	1	3	4	5	8	8	10	12	15	18	15	15	23	25	30	361
US Equities (Small Cap)	0	1	2	2	2	4	3	6	7	9	13	11	12	14	15	20	233
Global Equities (Dev, Ex US)	0	1	3	4	5	8	8	10	11	13	17	15	15	22	25	29	351
Global Equities (Emerging)	0	0	2	3	5	5	6	8	10	12	15	13	14	20	22	28	293
Private Equity	0	1	1	1	3	6	8	10	10	13	16	13	12	18	19	23	298
REITs	0	0	1	2	2	3	3	6	7	8	11	11	12	16	19	22	226
Private Real Estate	1	1	2	2	3	4	7	10	10	12	17	14	13	17	20	23	302
Hedge Funds	0	0	0	0	0	3	4	7	7	10	13	11	11	16	20	21	230
Commodities	0	0	1	1	0	0	1	6	8	11	14	12	13	18	21	24	242
Venture Capital	0	0	0	0	0	0	0	0	0	0	1	2	3	2	2	5	28
Infrastructure	0	0	0	0	0	0	0	1	2	2	5	5	4	7	12	12	88
Total	4	8	27	35	46	75	88	124	147	183	242	211	217	304	351	434	4,654

Table 3

Subjective Belief Components: Average Values

This table reports the average values (by asset class) for the belief quantities we study. Specifically, for each belief quantity, we calculate the average belief value each year and report the time-series mean of these average values. We consider expected excess returns ($\mu_{j,n,t}$), excess return volatilities ($\sigma_{j,n,t}$), market betas ($\beta^{(m)}_{j,n,t}$), risk premia ($rp_{j,n,t} = \lambda^{(0)}_{j,t} + \lambda^{(m)}_{j,t} \cdot \beta^{(m)}_{j,n,t}$), and alphas ($\alpha_{j,n,t} = \mu_{j,n,t} - rp_{j,n,t}$). For risk risk premia and alphas, the restricted CAPM imposes $\lambda^{(0)}_{j,t} = 0$ and $\lambda^{(m)}_{j,t} = \mu^{(m)}_{j,t}$ whereas the unrestricted CAPM uses $\lambda^{(0)}_{j,t}$ and $\lambda^{(m)}_{j,t}$ estimated from the regressions in Equation 3. We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Section 1.3 provides more details about our subjective beliefs data.

	Subjective Risk and Return				**Restricted CAPM**				**Unrestricted CAPM**			
Asset Class	μ	σ	$\beta^{(e)}$	$\beta^{(p)}$	$rp^{(e)}$	$rp^{(p)}$	$\alpha^{(e)}$	$\alpha^{(p)}$	$rp^{(e)}$	$rp^{(p)}$	$\alpha^{(e)}$	$\alpha^{(p)}$
US TIPS	1.1%	6.1%	0.03	0.08	0.1%	0.4%	1.0%	0.7%	1.6%	1.5%	-0.5%	-0.4%
US Bonds	1.7%	6.0%	0.11	0.22	0.7%	1.1%	1.0%	0.6%	2.3%	2.1%	-0.6%	-0.4%
US Govt Bonds	1.1%	7.8%	0.03	0.11	0.2%	0.5%	0.9%	0.6%	1.6%	1.6%	-0.5%	-0.5%
US Muni Bonds	0.7%	5.1%	0.04	0.10	0.3%	0.5%	0.4%	0.2%	1.9%	1.7%	-1.2%	-1.0%
US Inv Grade Corp Bonds	2.0%	6.8%	0.12	0.22	0.7%	1.1%	1.3%	0.9%	2.2%	2.2%	-0.2%	-0.2%
US High Yield Corp Bonds	3.9%	10.6%	0.38	0.57	2.1%	2.7%	1.8%	1.2%	3.8%	3.7%	0.1%	0.2%
Global Bonds (Ex US)	1.7%	9.9%	0.09	0.27	0.6%	1.3%	1.1%	0.3%	2.2%	2.4%	-0.6%	-0.7%
Private Debt	7.1%	14.5%	0.48	0.69	3.0%	3.7%	4.2%	3.5%	4.8%	4.7%	2.3%	2.5%
US Equities (Large Cap)	5.7%	16.8%	1.00	1.33	5.7%	6.4%	0.0%	-0.7%	7.2%	6.7%	-1.5%	-1.0%
US Equities (Small Cap)	7.1%	21.8%	1.15	1.54	6.4%	7.4%	0.7%	-0.3%	7.9%	7.6%	-0.8%	-0.5%
Global Equities (Developed, Ex US)	6.5%	19.8%	0.82	1.38	4.7%	6.6%	1.8%	-0.1%	6.3%	6.9%	0.2%	-0.5%
Global Equities (Emerging)	8.6%	26.8%	1.02	1.50	5.6%	7.0%	2.9%	1.6%	7.3%	7.3%	1.3%	1.2%
Private Equity	9.9%	28.5%	1.25	1.81	7.1%	8.8%	2.9%	1.2%	8.6%	8.8%	1.4%	1.1%
REITs	5.1%	18.1%	0.63	1.00	3.4%	4.6%	1.7%	0.5%	5.0%	5.2%	0.1%	-0.2%
Private Real Estate	4.5%	14.4%	0.28	0.47	1.6%	2.3%	2.8%	2.1%	3.3%	3.3%	1.1%	1.2%
Hedge Funds	3.7%	8.0%	0.30	0.43	1.6%	2.1%	2.1%	1.6%	3.2%	3.1%	0.5%	0.6%
Commodities	2.9%	18.3%	0.15	0.26	0.8%	1.3%	2.1%	1.6%	2.6%	2.6%	0.4%	0.3%
Venture Capital	11.3%	31.5%	1.38	1.78	8.5%	9.2%	2.9%	2.1%	9.9%	9.6%	1.5%	1.8%
Infrastructure	5.6%	15.2%	0.48	0.72	2.6%	3.4%	2.9%	2.1%	4.2%	4.2%	1.4%	1.3%

Table 4

CAPM Restrictions: Parameters from Regressing Subjective μ on Subjective β

This table reports results associated with the $\lambda^{(0)}_{j,t}$ and $\lambda^{(m)}_{j,t}$ coefficients from the regression $\mu_{j,n,t} = \lambda^{(0)}_{j,t} + \lambda^{(m)}_{j,t} \cdot \beta^{(m)}_{j,n,t} + \alpha^{(m)}_{j,n,t}$, estimated separately for each institution-year. The CAPM implies $0 \leq \lambda^{(0)}_{j,t} < \mu^{(m)}_{j,t}$, with the stronger $\lambda^{(0)}_{j,t} = 0$ restriction if investor can borrow at the risk-free rate. As such, Panel A reports average values of $\lambda^{(0)}$ across institution-year observations (and Figure 3 displays the full distribution of $\lambda^{(0)}$ values). The CAPM also implies the restriction $\lambda^{(m)}_{j,t} = \mu^{(m)}_{j,t} - \lambda^{(0)}_{j,t}$. As such, Panel B reports results from the regression $(\lambda^{(0)}_{j,t} + \lambda^{(m)}_{j,t}) = a + b \cdot \mu^{(m)}_{j,t} + \varepsilon_{j,t}$ under both $\lambda^{(0)}_{j,t} = 0$ and $\lambda^{(0)}_{j,t} \neq 0$, with the CAPM implying $a = 0$ and $b = 1$. We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. To account for residual correlation across institutions and over time, standard errors are based on Driscoll and Kraay (1998), with the number of lags selected according to Newey and West (1994). Section 1.3 provides more details about our subjective beliefs data and Section 2.1 provides more details about the analysis reported in this table.

PANEL A - Average $\boldsymbol{\lambda^{(0)}_{j,t}}$

	Equity CAPM	**Pension CAPM**
$\mathbb{E}[\boldsymbol{\lambda}^{(0)}]$	1.45%	1.14%
$(\boldsymbol{t}_{\mathbb{E}[\boldsymbol{\lambda}^{(0)}]=0})$	(9.96)	(7.90)

PANEL B - Results from Regression $(\boldsymbol{\lambda}^{(0)} + \boldsymbol{\lambda}^{(m)}) = \boldsymbol{a} + \boldsymbol{b} \cdot \boldsymbol{\mu}^{(m)}_{j,t} + \boldsymbol{\varepsilon}_{j,t}$

	Equity CAPM		**Pension CAPM**	
	$\boldsymbol{\lambda}^{(0)} = 0$	$\boldsymbol{\lambda}^{(0)} \neq 0$	$\boldsymbol{\lambda}^{(0)} = 0$	$\boldsymbol{\lambda}^{(0)} \neq 0$
$\boldsymbol{a}$	0.014	0.016	0.008	0.004
$(\boldsymbol{t}_{a=0})$	(5.49)	(7.36)	(2.89)	(3.49)
$\boldsymbol{b}$	0.742	0.979	0.698	1.016
$(\boldsymbol{t}_{b=0})$	(16.8)	(28.8)	(12.8)	(42.4)
$[\boldsymbol{t}_{b=1}]$	[-5.86]	[-0.56]	[-5.53]	[0.66]
$\boldsymbol{R^2}$	41.9%	71.0%	44.1%	85.4%

Table 5
Fraction of Expected Return Variation Explained by Risk Premia vs Alphas

This table reports the fraction of variability in subjective expected excess returns ($\mu_{j,n,t}$) that is explained by subjective (restricted) risk premia ($rp^{(m)}_{j,n,t} = \mu^{(m)}_{j,t} \cdot \beta^{(m)}_{j,n,t}$) and subjective alphas ($\alpha^{(m)}_{j,n,t} = \mu_{j,n,t} - rp^{(m)}_{j,n,t}$) based on the decomposition in Equation 5. Panel A focuses on overall variation in μ as well as μ variation across asset classes. Panel B focuses on μ variation across institutions and over time. Different blocks of each panel define the asset class category included in the analysis. Different columns within each block consider different fixed effects (to focus on different sources of variation). We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Section 1.3 provides more details about our subjective beliefs data and Section 2.3 provides more details about the analysis reported in this table.

PANEL A - Overall Variation and Variation Across Asset Classes

	All Asset Classes			Fixed Income			Equities			Real Estate			Alternatives		
	[1]	[2]	[3]	[1a]	[2a]	[3a]	[1b]	[2b]	[3b]	[1c]	[2c]	[3c]	[1d]	[2d]	[3d]
Equity CAPM															
% of $\mathbb{V}ar[\mu]$ due to rp	76%	76%	76%	47%	49%	52%	50%	34%	24%	81%	75%	105%	56%	52%	54%
% of $\mathbb{V}ar[\mu]$ due to α	24%	24%	24%	53%	51%	48%	50%	66%	76%	19%	25%	-5%	44%	48%	46%
Pension CAPM															
% of $\mathbb{V}ar[\mu]$ due to rp	91%	91%	92%	55%	57%	60%	64%	49%	39%	104%	98%	135%	62%	58%	60%
% of $\mathbb{V}ar[\mu]$ due to α	9%	9%	8%	45%	43%	40%	36%	51%	61%	-4%	2%	-35%	38%	42%	40%
Year FE		x			x			x			x			x	
Institution FE		x			x			x			x			x	
Year×Institution FE			x			x			x			x			x

PANEL B - Variation Across Institutions and Over Time

	All Asset Classes			Fixed Income			Equities			Real Estate			Alternatives		
	[1]	[2]	[3]	[1a]	[2a]	[3a]	[1b]	[2b]	[3b]	[1c]	[2c]	[3c]	[1d]	[2d]	[3d]
Equity CAPM															
% of $\mathbb{V}ar[\mu]$ due to rp	51%	49%	46%	20%	18%	18%	67%	60%	56%	68%	65%	59%	28%	30%	26%
% of $\mathbb{V}ar[\mu]$ due to α	49%	51%	54%	80%	82%	82%	33%	40%	44%	32%	35%	41%	72%	70%	74%
Pension CAPM															
% of $\mathbb{V}ar[\mu]$ due to rp	62%	61%	55%	27%	24%	25%	77%	72%	65%	87%	83%	74%	34%	38%	31%
% of $\mathbb{V}ar[\mu]$ due to α	38%	39%	45%	73%	76%	75%	23%	28%	35%	13%	17%	26%	66%	62%	69%
Year FE		x			x			x			x			x	
Institution FE			x			x			x			x			x
Asset Class FE	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x

Table 6
Realized Return Data: Index Names, Data Sources, and Sample Period

This table provides a list of the indices used for the realized returns matched to each asset class present in our beliefs data (i.e., the CMAs of our institutions).

CMA Asset Class	Index Name	Data Source	First Return Date	Last Return Date
US Cash	3-Month Treasury Bill Return	CRSP	04/1957	12/2022
US TIPS	Bloomberg Barclays US Treasury Inflation Notes	Bloomberg	03/1997	12/2022
US Bonds	Bloomberg Barclays US Aggregate Bond Index	Bloomberg	01/1976	12/2022
US Govt Bonds	Bloomberg Barclays US Treasury Index	Bloomberg	01/1973	12/2022
US Muni Bonds	ICE Bank of America US Municipal Securities Index	Bloomberg	01/1980	12/2022
US Inv Grade Corp Bonds	Bloomberg Barclays US Investment Grade Index	Bloomberg	01/1973	12/2022
US High Yield Corp Bonds	Bloomberg Barclays US High Yield Index	Bloomberg	07/1983	12/2022
Global Bonds (Ex US)	Bloomberg Barclays Global Aggregate- Ex US	Bloomberg	01/1990	12/2022
Private Debt	-	-	-	-
US Equities (Large Cap)	S&P 500 Index	CRSP & Bloomberg[a]	04/1957	12/2022
US Equities (Small Cap)	Russell 2000 Index	Bloomberg	01/1979	12/2022
Global Equities (Dev, Ex US)	MSCI World ex US Index	Bloomberg	02/1970	12/2022
Global Equities (Emerging)	MSCI Emerging Markets	Bloomberg	01/1988	12/2022
Private Equity	-	-	-	-
REITs	FTSE NAREIT US Real Estate Index (All Equity)	NAREIT	01/1972	12/2022
Private Real Estate	NCREIF Value Weighted Index	NCREIF	Q1/1978	Q4/2022
Hedge Funds	HFRI FOF: Diversified Index	Hedge Fund Research	01/1990	12/2022
Commodities	S&P GSCI or Bloomberg Commodity Index	Bloomberg	02/1970	12/2022
Venture Capital	-	-	-	-
Infrastructure	Dow Jones Brookfield Global Infrastructure Index	Bloomberg	01/2003	12/2022

[a]The S&P 500 inception date is March 4, 1957 (so, the first complete monthly return is 04/1957). However, Bloomberg only has total return data for the S&P500 starting in 02/1970. So, we use CRSP for the S&P 500 monthly returns from 04/1957 to 01/1970. CRSP only reports ex-dividend returns for the S&P 500 ($R^{ex}_{SP,t}$). To obtain the S&P 500 total return from CRSP, we use $R_{SP,t} = R^{(ex)}_{SP,t} + (R_{vw,t} - R^{(ex)}_{vw,t})$, where $R_{vw,t}$ and $R^{(ex)}_{vw,t}$ are the cum- and ex-dividend returns for the CRSP value-weighted index. Over the period we directly observe $R_{SP,t}$ from Bloomberg, the correlation between the two $R_{SP,t}$ values (from CRSP and Bloomberg) is 99.95%. Moreover, the difference in average annualized returns is only 0.04%. As such, the $R_{SP,t}$ imputed from CRSP data seems to be a reliable proxy for S&P 500 returns. Note that series commonly used in the literature for the period before 1957 (e.g., Robert Shiller's S&P 500 returns) do not actually refer to the S&P 500 before March 4, 1957 (the inception date).

Table 7
Predicting Realized Returns using Subjective Expected Returns

This table reports results from the estimation of equations analogous to Equation 11. $\mu_{n,t}$ is the subjective expected return aggregated across institutions as described in Subsection 3.1. $\mu^{(hy)}_{n,t}$ is a linear transformation of $\mu_{n,t}$ to reflect h-year subjective expected returns (e.g., see Equation 12). We consider pooled regressions (Variation=All), regressions with year fixed effects (Variation=Across AC), and regressions with asset class fixed effects (Variation=Over Time). Standard errors rely on double clustering (by year and asset class). $p_{a=0,b=1}$ reflects the p-value for a Wald test of the joint hypothesis that $a = 0$ and $b = 1$. R^2 reflects the pooled R^2 in the Variation=All case and within R^2 values in the other two cases. R^2_{OOS} reflects an out-of-sample R^2 metric, which evaluates the predictability of the given predictor against the predictability of a historical returns benchmark (see Footnote 31). The R^2 decompositions in Panels B and C rely on the Shapley decomposition (see Footnote 33). Sections 1.3 and 3.1 provide more details about the data and Section 3.4 provides more details about the analysis reported in this table.

PANEL A - Predicting Realized Returns

	$r_{n,t+1} = a + b \cdot \mu_{n,t} + \epsilon_t$			$r_{n,t+1} = a + b \cdot \mu^{(1y)}_{n,t} + \epsilon_t$			$\overline{r}_{n,t\to t+3} = a + b \cdot \mu^{(3y)}_{n,t} + \epsilon_t$		
Variation =	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**
$\boldsymbol{a}$	0.00			0.00			0.01		
$(\boldsymbol{t_{a=0}})$	(-0.25)			(0.11)			(1.58)		
$\boldsymbol{b}$	1.14	1.05	4.70	1.02	0.80	1.13	0.91	0.90	0.77
$(\boldsymbol{t_{b=0}})$	(2.63)	(1.98)	(2.00)	(2.84)	(2.48)	(1.97)	(5.21)	(6.08)	(1.53)
$[\boldsymbol{t_{b=1}}]$	[0.32]	[0.09]	[1.58]	[0.04]	[-0.62]	[0.23]	[-0.55]	[-0.67]	[-0.46]
$\{\boldsymbol{p_{a=0,b=1}}\}$	{0.94}			{0.99}			{0.28}		
$\boldsymbol{R^2}$	3.4%	4.4%	4.6%	6.7%	5.4%	5.4%	11.3%	13.3%	3.5%
$\boldsymbol{R^2_{OOS}}$		5.3%	5.9%		12.2%	12.7%		14.0%	11.2%

PANEL B - Decomposing Predictive R^2 Values (Equity CAPM)

	$r_{n,t+1} = a + b \cdot \mu_{n,t} + \epsilon_t$			$r_{n,t+1} = a + b \cdot \mu^{(1y)}_{n,t} + \epsilon_t$			$\overline{r}_{n,t\to t+3} = a + b \cdot \mu^{(3y)}_{n,t} + \epsilon_t$		
Variation =	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**
$\boldsymbol{R^2}$ **due to** $\boldsymbol{rp}$	3.4%	4.7%	3.6%	5.2%	5.6%	3.6%	11.8%	14.9%	3.1%
$\boldsymbol{R^2}$ **due to** $\boldsymbol{\alpha}$	-0.1%	-0.3%	1.0%	1.5%	-0.2%	1.8%	-0.5%	-1.6%	0.4%
$\boldsymbol{R^2_{OOS}}$ **due to** $\boldsymbol{rp}$		4.8%	5.0%		8.1%	8.4%		15.0%	13.9%
$\boldsymbol{R^2_{OOS}}$ **due to** $\boldsymbol{\alpha}$		0.5%	0.8%		4.0%	4.3%		-1.0%	-2.6%

PANEL C - Decomposing Predictive R^2 Values (Pension CAPM)

	$r_{n,t+1} = a + b \cdot \mu_{n,t} + \epsilon_t$			$r_{n,t+1} = a + b \cdot \mu^{(1y)}_{n,t} + \epsilon_t$			$\overline{r}_{n,t\to t+3} = a + b \cdot \mu^{(3y)}_{n,t} + \epsilon_t$		
Variation =	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**
$\boldsymbol{R^2}$ **due to** $\boldsymbol{rp}$	3.2%	4.2%	3.5%	5.5%	5.2%	4.0%	11.2%	13.4%	3.1%
$\boldsymbol{R^2}$ **due to** $\boldsymbol{\alpha}$	0.1%	0.1%	1.1%	1.2%	0.2%	1.4%	0.1%	-0.1%	0.4%
$\boldsymbol{R^2_{OOS}}$ **due to** $\boldsymbol{rp}$		4.9%	5.2%		8.8%	9.0%		15.9%	14.8%
$\boldsymbol{R^2_{OOS}}$ **due to** $\boldsymbol{\alpha}$		0.4%	0.7%		3.4%	3.6%		-1.9%	-3.5%

Table 8
Predicting Realized Returns using Subjective Expected Returns (by Asset Class)

This table reports results from the estimation of Equation 11 by asset class. $\mu_{n,t}$ is the subjective expected return aggregated across institutions as described in Subsection 3.1. The predictor in these regressions, $\mu_{n,t}^{(1y)}$, is a linear transformations of $\mu_{n,t}$ to reflect 1-year expected returns (as per Equation 12). Standard errors rely on Newey and West (1987, 1994). R^2_{OOS} reflects an out-of-sample R^2 metric, which evaluates the predictability of the given predictor against the predictability of the historical average return for the given asset class, with a minimum of 20 years of data for the historical average return (see Footnote 31). N_{R^2} reflects the number of years of forecasting errors we have to compute R^2_{OOS}. We require $N_{R^2} \geq 5$ to compute the R^2_{OOS} value for a given asset class (this is why it is not available for Infrastructure). Sections 1.3 and 3.1 provide more details about the data and Section 3.4 provides more details about the analysis reported in this table.

	a	$(t_{a=0})$	b	$(t_{b=0})$	$[t_{b=1}]$	R^2	R^2_{OOS}	N_{R^2}
US TIPS	0.03	(1.61)	0.32	(0.61)	[-1.27]	2.9%	3.4%	5
US Bonds	0.01	(0.68)	0.89	(1.83)	[-0.23]	5.8%	2.3%	27
US Govt Bonds	0.02	(1.46)	0.62	(1.94)	[-1.20]	13.1%	7.7%	24
US Muni Bonds	0.01	(1.47)	1.09	(3.98)	[0.31]	14.5%	9.8%	19
US Inv Grade Corp Bonds	0.01	(0.50)	1.07	(1.77)	[0.12]	18.9%	18.9%	20
US High Yield Corp Bonds	-0.05	(-1.67)	2.55	(5.37)	[3.26]	59.7%	45.2%	19
Global Bonds (Ex US)	0.00	(0.04)	1.15	(1.69)	[0.22]	4.8%	15.4%	13
US Equities (Large Cap)	0.03	(0.27)	1.02	(0.69)	[0.01]	1.8%	3.3%	35
US Equities (Small Cap)	0.05	(0.59)	0.27	(0.24)	[-0.66]	0.2%	11.7%	24
Global Equities (Developed, Ex US)	-0.12	(-1.52)	2.46	(2.34)	[1.39]	11.9%	10.0%	32
Global Equities (Emerging)	0.15	(0.91)	-0.75	(-0.42)	[-0.98]	1.2%	15.1%	15
REITs	-0.06	(-0.68)	2.78	(2.40)	[1.54]	17.2%	10.4%	25
Private Real Estate	0.00	(-0.05)	0.89	(2.11)	[-0.27]	10.0%	16.0%	25
Hedge Funds	0.00	(-0.01)	0.68	(0.77)	[-0.37]	2.5%	-11.2%	13
Commodities	-0.08	(-2.31)	2.25	(1.75)	[0.97]	12.8%	7.2%	21
Infrastructure	-0.02	(-0.33)	1.45	(1.86)	[0.58]	15.7%	-	-

Table 9
Predicting Realized Risk and Alphas using Subjective Risk and Alphas

This table reports results from the estimation of equations analogous to Equation 11. Realized risk and alpha metrics are calculated from monthly returns within year $t+1$ (with alphas multiplied by 12). $\sigma^2_{n,t}$, $\beta^{(m)}_{n,t}$, and $\alpha^{(m)}_{n,t}$ are the belief quantities aggregated across institutions as described in Subsection 3.1. $\alpha^{(m,1y)}_{n,t}$ is a linear transformation of $\alpha^{(m)}_{n,t}$ to reflect 1-year subjective alphas (similar to Equation 12). We consider pooled regressions (Variation=All), regressions with year fixed effects (Variation=Across AC), and regressions with asset class fixed effects (Variation=Over Time). Standard errors rely on double clustering (by year and asset class). $p_{a=0,b=1}$ reflects the p-value for a Wald test of the joint hypothesis that $a=0$ and $b=1$. R^2 reflects the pooled R^2 in the Variation=All case and within R^2 values in the other two cases. R^2_{OOS} reflects an out-of-sample R^2 metric, which evaluates the predictability of the given predictor against the predictability of a historical risk or alpha benchmarks (see Footnote 31). Sections 1.3 and 3.1 provide more details about the data and Section 3.4 provides more details about the analysis reported in this table.

PANEL A - Equity CAPM

	$\widehat{\sigma}^2_{n,t+1} = a + b \cdot \sigma^2_{n,t} + \epsilon_t$			$\widehat{\beta}^{(e)}_{n,t+1} = a + b \cdot \beta^{(e)}_{n,t} + \epsilon_t$			$\widehat{\alpha}^{(e)}_{n,t+1} = a + b \cdot \alpha^{(e,1y)}_{n,t} + \epsilon_t$		
Variation =	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**
a	0.00			0.03			0.00		
$(t_{a=0})$	(-0.09)			(0.56)			(0.15)		
b	0.65	0.67	-0.04	0.95	0.98	-0.11	0.37	0.35	0.44
$(t_{b=0})$	(10.53)	(7.20)	(-0.09)	(16.73)	(17.09)	(-0.56)	(0.76)	(0.74)	(0.84)
$[t_{b=1}]$	[-5.65]	[-3.54]	[-2.24]	[-0.89]	[-0.30]	[-5.77]	[-1.30]	[-1.39]	[-1.06]
$\{p_{a=0,b=1}\}$	{0.00}			{0.67}			{0.40}		
R^2	29.0%	38.2%	0.0%	60.7%	67.0%	0.3%	0.7%	0.6%	0.9%
R^2_{OOS}		8.0%	-28.8%		60.2%	-14.8%		4.1%	4.6%

PANEL B - Pension CAPM

	$\widehat{\sigma}^2_{n,t+1} = a + b \cdot \sigma^2_{n,t} + \epsilon_t$			$\widehat{\beta}^{(p)}_{n,t+1} = a + b \cdot \beta^{(p)}_{n,t} + \epsilon_t$			$\widehat{\alpha}^{(p)}_{n,t+1} = a + b \cdot \alpha^{(p,1y)}_{n,t} + \epsilon_t$		
Variation =	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**
a	0.00			0.04			0.00		
$(t_{a=0})$	(-0.09)			(0.48)			(0.17)		
b	0.65	0.67	-0.04	0.96	0.99	-0.07	0.24	0.33	0.19
$(t_{b=0})$	(10.53)	(7.20)	(-0.09)	(14.01)	15.44	-0.49	(0.44)	(0.62)	(0.31)
$[t_{b=1}]$	[-5.65]	[-3.54]	[-2.24]	[-0.62]	-0.21	-7.97	[-1.39]	[-1.27]	[-1.31]
$\{p_{a=0,b=1}\}$	{0.00}			{0.82}			{0.37}		
R^2	29.0%	38.2%	0.0%	61.2%	64.8%	0.1%	0.3%	0.5%	0.2%
R^2_{OOS}		8.0%	-28.8%		61.0%	-12.7%		2.7%	2.5%

Internet Appendix

"The Subjective Risk and Return Expectations of Institutional Investors"

By Spencer J. Couts, Andrei S. Gonçalves, and Johnathan A. Loudis

Contents

A Theoretical Motivation

In this section, we suppress the time index, t, to simplify exposition. However, our analysis in this section can be thought of as reflecting time t beliefs about time $t+h$ outcomes for an arbitrary h.

Suppose the Stochastic Discount Factor (SDF) for a set of $j = 1, 2, ..., J$ investors is given by M. If these investors have access to $n = 1, 2, ..., N$ assets (or asset classes) over time, then we have

$$1 = \mathbb{E}_j[M \cdot R_n] \qquad \Rightarrow \qquad \mathbb{E}_j[r_n] = \beta_{j,n} \cdot \lambda_j \tag{IA.1}$$

where $r_n = R_n - R_f$, $\beta_{j,n} = \frac{\mathbb{C}ov_j[-M, r_n]}{\mathbb{V}ar_j[M]}$, and $\lambda_j = \frac{\mathbb{V}ar_j[M]}{\mathbb{E}_j[M]} > 0$.

Equation IA.1 shows that investor j perceives a positive risk-return tradeoff. That is, investor j believes that assets that negatively comove with M deliver high expected returns. In typical models, M reflects marginal utility so that risky assets are the ones that investors think will perform poorly in states of nature where they have high marginal utility.

It is important to note that $\mathbb{E}_j[\cdot]$ represents investor j subjective expectation operator, which can differ from the objective expectation operator, $\mathbb{E}_o[\cdot]$. As such, assets that investor j perceives to have high beta may not actually have high beta. Similarly, assets that investor j perceives to have high expected returns may not actually have high expected returns. Taken in isolation, the only theoretical restriction Equation IA.1 provides is that subjective expected returns are positively linked to subjective expected risk (i.e., subjective betas).

From Equation IA.1, we can recover the objective risk-return tradeoff. To start, let $\pi_{j,s}$ reflect investor j subjective probability that state of nature s will materialize next period. Then, aggregating Equation IA.1 across investors yields

$$1 = \frac{1}{J} \cdot \sum_{j=1}^{J} \mathbb{E}_j[M \cdot R_n] = \frac{1}{J} \cdot \sum_{j=1}^{J} \sum_{s=1}^{S} \pi_{j,s} \cdot M_s \cdot R_{n,s} = \sum_{s=1}^{S} (M_s \cdot R_{n,s} \cdot \underbrace{\frac{1}{J} \cdot \sum_{j=1}^{J} \pi_{j,s}}_{\pi_s}) \tag{IA.2}$$

which can be written as

$$1 = \sum_{s=1}^{S} \pi_s \cdot M_s \cdot R_{n,s} = \sum_{s=1}^{S} \pi_{o,s} \cdot \Pi_s \cdot M_s \cdot R_{n,s} \;=\; \mathbb{E}_o[\Pi \cdot M \cdot R_n] \tag{IA.3}$$

where $\Pi_s = \pi_s/\pi_{o,s}$ reflects the average probability distortion for state of nature s (i.e., "investor sentiment"). As such, the distorted SDF, $\Pi \cdot M$, captures the risk-return tradeoff in the realized return data (i.e., under the objective measure):

$$1 = \mathbb{E}_o[\Pi \cdot M \cdot R_n] \qquad \Rightarrow \qquad \mathbb{E}_o[r_n] = \beta_{o,n} \cdot \lambda_o, \tag{IA.4}$$

where $\beta_{o,n} = \frac{\mathbb{C}ov_o[-\Pi \cdot M, r_n]}{\mathbb{V}ar_o[\Pi \cdot M]}$, and $\lambda_o = \frac{\mathbb{V}ar_o[\Pi \cdot M]}{\mathbb{E}_o[\Pi \cdot M]} > 0$

Since we rarely observe the beliefs of investors, we typically assume investors have rational/objective beliefs on average (i.e., $\Pi_s = 1$). As such, we regularly test an SDF model ($\widehat{M}$) using the moment condition $\mathbb{E}_o[\widehat{M} \cdot R_n] = 1$. Such a test can lead to two important mistakes:

1. We may "reject" a valid SDF. Suppose $\widehat{M} = M$ so that our SDF model is valid. If Π comoves positively with $1/M$, then the risk-return tradeoff based on $\widehat{M}$ will look weaker than investors perceive it to be. For instance (at the extreme), if $\Pi \propto 1/M$ then there is no risk-return tradeoff under the objective measure even though investors perceive a positive risk-return tradeoff. We would conclude that exposure to $\widehat{M}$ is not priced by investors even though it is. This extreme scenario could arise if beliefs are distorted optimistically, for instance. Specifically, $\Pi_s = \pi_s/\pi_{o,s} \propto 1/M_s$ implies investors assign relatively high probabilities to states of nature with low M (good states with low marginal utility) and relatively low probabilities to states of nature with high M (bad states with high marginal utility). $\mathbb{C}or_o(\Pi, 1/M) > 0$ leads to a similar qualitative result.

2. We may "accept" an invalid SDF. Suppose $\widehat{M} \neq M$ so that our SDF model is invalid. If Π comoves positively with $\widehat{M}/M$, then the risk-return tradeoff based on $\widehat{M}$ will look more positive than investors perceive it to be. For instance (at the extreme), if $M = 1$ and $\Pi \propto \widehat{M}$ then there is a positive risk-return tradeoff under the objective measure even though investors perceive no risk-return tradeoff. We would conclude

that exposure to $\widehat{M}$ is priced by investors even though it is not. Such an extreme scenario could arise if investors are risk-neutral but beliefs are distorted pessimistically. Specifically, $\Pi_s = \pi_s/\pi_{o,s} = \widehat{M}_s$ implies investors assign relatively high probabilities to states of nature with high $\widehat{M}$ (states with high model-based marginal utility) and relatively low probabilities to states of nature with low $\widehat{M}$ (states with low model-based marginal utility). $\mathbb{C}or_o(\Pi, \widehat{M}/M) > 0$ leads to a similar qualitative result.

This analysis illustrates that it is important to understand the subjective risk-return tradeoff when there is concern that it may deviate from the objective risk-return tradeoff. Doing so helps to avoid incorrectly accepting or rejecting a particular model for the SDF. In this context, Section 2 in the main text explores the link between subjective risk and return based on the subjective beliefs of institutional investors about different asset classes in the context of the CAPM. In turn, Section 3 in the main text studies the link between subjective and objective beliefs (through realized return moments).

B Data and Measurement Details

This section provides details on the data used in our analysis.

B.1 Subjective Beliefs Data and Measurement

This section provides details on our Capital Market Assumptions (CMAs) dataset.

B.1.1 Data Collection Process

We collected the long-term CMAs of 45 institutions in total: 22 asset managers and 23 investment consultants. We relied in three complementary data collection approaches:

(i) The bulk of the data comes directly from the institutions covered in our CMAs. We identify individuals within each institution that are connected to the production of CMAs and contact them directly to request access to their data. In all successful cases, the institution sent us continuous (at least annual) data covering from some initial year until their most recent CMA, and thus these data are free of issues related to attrition rates.

(ii) To complement the data sent directly by institutions, we also obtain data from online sources. As in Dahlquist and Ibert (2024), our approach is simple: we obtain the most recent CMA of each institution directly from their website and complement these CMAs with prior CMAs obtained through google searches and `archive.org`.

(iii) The third data collection approach is based on indirect data obtained from pension funds. Specifically, we contacted various pension funds to request the CMAs they rely on when deciding their portfolio allocations (sometimes through a freedom of information act request). The pension funds sent us their portfolio allocation reports and/or their CMA reports. In turn, these reports include the numbers (expected returns, volatilities, and correlations) associated with the CMAs of third party institutions that the pension fund rely on (including consultants and sometimes asset managers).

Whenever an institution-year observation is available through more than one of the three methods above, we rely on a pecking order for data selection (we use (i) when available, (ii) when (i) is not available, and (iii) when nether (i) nor (ii) is available). Given this pecking order, 83.4% of the institution-year observations in our baseline sample is based on data collected from methods (i)+(ii), which we refer to as "direct CMAs". Some direct CMAs are based on quarterly reports, in which case we keep only the Q4 report for each year.

As we explain in Section C.3, while the results we report in the main text combine consultants with managers and rely on this pecking order, we explore different subsets of the data in our robustness checks to demonstrate that our results are not due to particular biases that may be present in the data. For example, we show that our results are qualitatively similar if we rely only on consultants or only on managers, indicating that our results are not due to any particular incentive that is specific to either of these types of institutions. Analogously, we show that our results are qualitatively similar if we rely only on data collection process (i)+(ii) or only on data collection process (iii), indicating that our results are not due to potential biases that may arise from collecting data directly from the institution or collecting data indirectly through pension funds.

Figure IA.2 compares the number of CMAs in our dataset relative to the number of CMAs in the complete dataset of Dahlquist and Ibert (2024) (that includes the consultants from their internet appendix). Their dataset has more CMAs than ours from 2015 to 2020 whereas ours has more CMAs than theirs every year from 1987 to 2013 as well as in 2021 (with the same number of institutions in 2014). So, our dataset has better time-series coverage than theirs whereas their dataset covers more institutions than ours in six of the most recent ten years.

Our final sample contains a risk-free asset class proxy (*US Cash*) as well as 19 risky asset classes. To decide on these 19 risky asset classes, we consider three aspects. First, whether the asset class is a major asset class for institutional investors. Second, whether the asset class is covered by a reasonable number of institutions in our sample. And third, whether the asset class is covered over a reasonable time period within the institutions that cover it.

We include all asset classes that perform well in these dimensions, with the final list of asset classes covered in our study available in the first column of Table IA.3.

It is important to note that each institution-year CMA covers a range of asset classes with names that do not necessarily match the exact names of the 20 asset classes we study (which we refer to as the "broad asset classes"). Moreover, the asset classes covered in the CMAs vary across institutions and over time. So, we develop a detailed procedure to match the asset classes covered in the CMAs to our broad asset classes. First, we identify the asset classes in each CMA based on the asset class name used in the CMA report and/or the actual portfolio index stated in the CMA report. Second, we manually map each asset class in each institution-year CMA to an institution-specific asset class name (fixed over time) that reflects the underlying asset class well. Third, we map each institution-specific asset class name to a slightly more general asset class name (which we refer to as the master asset class) that reflects the institution-specific asset class name reasonably well while allowing for small mismatches to accommodate asset classes from different institution under the same master asset class. Fourth, for each CMA, we match each of our broad asset classes to the most closely related master asset class available (with the possibility of no match).

Table IA.3 provides the results from our asset class matching procedure. The first column shows the asset classes we cover in the paper. The other columns provide the list of master asset classes that we match to these asset classes. They also provide the fraction of institution-year CMAs that have the respective match (within the institution-year CMAs that have some match for the given asset class). For all asset classes, the option #1 master asset class is responsible for at least 50% of the matches. Section C.3 provides a robustness analysis that focuses on institution-year observations for which the primary asset class name matches the name of the underlying master asset class used (the results are very similar to the ones reported in the main text).

B.1.2 Extracting Beliefs from the CMAs

As JP Morgan details in their 2015 report, their expected returns (μ), expected volatilities (σ), and expected correlations (ρ) are all obtained based on their views on log returns through a log-Normal transformation.[IA.1] Specifically, they first form their beliefs in log return space and then translate them to the raw return space space using the assumption that log returns are normally distributed. In mathematical terms, they report (for each asset class n)

$$\begin{pmatrix} \mu_n \\ \sigma_n^2 \\ \rho_{n,k} \end{pmatrix} = \begin{pmatrix} e^{\widetilde{\mu}_n + \frac{1}{2}\cdot\widetilde{\sigma}_n^2} - 1 \\ e^{2\cdot(\widetilde{\mu}_n + \frac{1}{2}\cdot\widetilde{\sigma}_n^2)} \cdot (e^{\widetilde{\sigma}_n^2} - 1) \\ \left(e^{\widetilde{\rho}_{n,k}\cdot\widetilde{\sigma}_n\cdot\widetilde{\sigma}_k} - 1\right) / \sqrt{(e^{\widetilde{\sigma}_n^2} - 1)\cdot(e^{\widetilde{\sigma}_k^2} - 1)} \end{pmatrix} \tag{IA.5}$$

which imply

$$\begin{pmatrix} \widetilde{\sigma}_n^2 \\ \widetilde{\mu}_n \\ \widetilde{\rho}_{n,k} \end{pmatrix} = \begin{pmatrix} log\left(1 + \sigma_n^2/(\mu_n + 1)^2\right) \\ log(\mu_n + 1) - \frac{1}{2}\cdot\widetilde{\sigma}_n^2 \\ log\left(1 + \rho_{n,k}\cdot\sqrt{(e^{\widetilde{\sigma}_n^2} - 1)\cdot(e^{\widetilde{\sigma}_k^2} - 1)}\right) / \left(\widetilde{\sigma}_n\cdot\widetilde{\sigma}_k\right) \end{pmatrix} \tag{IA.6}$$

where $\widetilde{\mu}$, $\widetilde{\sigma}$, and $\widetilde{\rho}$ are the expected log returns, expected log volatilities, and expected log correlations. The JP Morgan 2015 report also provides $e^{\widetilde{\mu}_n} - 1$ as the "expected compound return" or "expected geometric return".

All institution-year CMAs in the sample used in our main analysis contain σ_n and $\rho_{n,k}$. However, they vary on whether they contain only expected arithmetic returns (which is μ in our notation), only expected geometric returns (which is $e^{\widetilde{\mu}_n} - 1$ in our notation), or both. In particular, 77.8% of our CMAs report expected arithmetic returns while 58.4% of our CMAs report expected geometric returns (with 36.3% of our CMAs reporting both). To ensure the conceptual definition underlying our expected return measure is the same for all our institution-year observations, we always use expected arithmetic returns in our baseline analysis (since they are the most commonly reported). To avoid loosing observations, we use

[IA.1] The use of μ and σ in this section embeds a slight abuse of notation since μ and σ reflect expected excess return and excess return volatility in the main text while they reflect expected return and return volatility in this section (so, they are not in excess of US Cash here).

the transformation $\mu_n = e^{\widetilde{\mu}_n + \frac{1}{2}\cdot\widetilde{\sigma}_n^2}$ for the 22.2% of CMAs without μ_n.[IA.2] Internet Appendix C.3 provides results (similar to our baseline results) that use expected geometric returns instead, with the transformation based on the first two equations of the Equation System IA.6 (applied to the 41.6% of CMAs without $\widetilde{\mu}_n$).

B.2 Wealth Portfolio Weights Data and Measurement

In the Equity CAPM, we assign 100% of the weight to *US Equities (Large Cap)*, which is available for all CMAs. However, in the Pension CAPM, we proxy for the wealth portfolio weights using the aggregated portfolio weights of US public pension funds obtained from the Center for Retirement Research (CRR) at Boston College. Specifically, we collect information on the dollar allocations across asset classes for each pension fund (from the "Detailed Investment Data" dataset). Then, we aggregate allocations across pension funds each year covered in the dataset (i.e., from 2001 to 2021) and use 2001 weights for earlier years and 2021 weights for later years.

CRR provides information on allocations for a range of asset classes, many of which have zero weights for most pension funds. To create the aggregate allocations on our asset classes, we start by summing the allocations on their underlying asset classes as follows (with asset class names as recorded in the CRR dataset):

1. **Cash**: Cash + FICash
2. **US Fixed Income**: FIUS + ω_{FIUS}·FIOther
3. **Int Fixed Income**: FIExUS + $(1 - \omega_{\text{FIUS}})$·FIOther

[IA.2] Specifically, we observe σ_n and $\widetilde{\mu}_n$ and solve the system of equations

$$\begin{pmatrix} \mu_n \\ \widetilde{\sigma}_n^2 \end{pmatrix} = \begin{pmatrix} e^{\widetilde{\mu}_n + \frac{1}{2}\cdot\widetilde{\sigma}_n^2} - 1 \\ log\left(1 + \sigma_n^2/(\mu_n + 1)^2\right) \end{pmatrix}$$

for μ_n and $\widetilde{\sigma}_n^2$ numerically using a root-solving algorithm.

4. **US Equities**: EQUS + ω_{EQUS}·EQOther

5. **Int Equities**: EQExUS + $(1 - \omega_{\mathrm{EQUS}})$·EQOther

6. **Private Equity**: EQPrivate + PrivatePlacement + MLP

7. **Real Estate**: RECore + REMisc + RENonCore + PrivRealEstate + REIT + RealAssets + RETriple

8. **Hedge Funds**: AbsRtrn + RelativeRtrn + RiskParity + CoveredCall + CreditOpp + DistrssedDebt + DistrssedLend + HedgeEQ + AltInflation + Hedge + MultiClass + OppDebt + OppEQ + Opp + GTAA

9. **Commodities**: Commod+Farm+NatResources+Timber

10. **Infrastructure**: Infrast

FIUS and EQUS reflect asset classes that are based on US investments, FIExUS and EQExUS reflect asset classes that are based on international investments, and FIOther and EQOther reflect asset classes that can potentially combine US and Ex US investments. We construct these six asset classes as follows:

1. **FIUS**: FIDomestic + FITIPS + FITreasury

2. **EQUS**: EQDomesticLarge + EQDomesticMisc + EQDomesticMid + EQDomesticSmall

3. **FIExUS**: FIEmerg + FIGlobal + FIIntl

4. **EQExUS**: EQIntlActv + EQIntlDev + EQGlobal + EQIntlMisc + EQIntlPass + EQIntlEmerg + EQGlobalGrowth

5. **FIOther**: FICore + FINonCore + FICorpBonds + FIValue + FIConv + FIAlt + FIFundsFunds + FIMisc + FIETI + FINominal + FIInvestGrd + FIBelowInvestGrd +

FIHighYield + FILoans + PrivateDebt + FIMortgage + FIGIPS + FIOpp + FIStructured + ω_{FI}·MiscEQFI

6. **EQOther**: EQCore+EQLarge+EQMisc+EQMicro+EQSmall+EQOpportunistic+EQSocialResp + $(1-\omega_{\text{FI}})$·MiscEQFI

The ω_{FIUS}, ω_{EQUS}, and ω_{FI} weights are constructed as follows:

1. ω_{FI} equals the allocation on FIUS+FIExUS+FIOther* dividend by the allocation on FIUS+FIExUS+FIOther*+EQUS+EQExUS+EQOther*, where FIOther* and EQOther* are the FIOther and EQOther asset classes with MiscEQFI set to zero (so that we do not need ω_{FI} to compute them).

2. ω_{FIUS} is the allocation on FIUS dividend by the allocation on FIUS+FIExUS

3. ω_{EQUS} is the allocation on EQUS dividend by the allocation on EQUS+EQExUS

Given the allocations on these asset classes summed across pension funds, we construct wealth portfolio weights by dividing each asset class alloction by the sum of allocations to all asset classes in the given year. Since Commodities and Infrastructure are often missing from our beliefs data and they have very small weights on the CRR data (the combined weight of these two asset classes is never more than 1%), we remove them from the wealth portfolio, keeping only the other eight asset classes (and rescaling weights so that they continue to add to one). Figure IA.1 displays the pension wealth portfolio weights from 2001 to 2021 (recall that we use 2001 weights for earlier years and 2021 weights for later years).

Finally, we map these CRR asset classes into our CMA asset classes as follows:

1. CRR asset class "Cash" is mapped to CMA asset class *US Cash*

2. CRR asset class "US Fixed Income" is mapped to CMA asset class *US Bonds* (if missing, to *US Govt Bonds* and *US Inv Grade Corp Bonds* in this order of availability)

3. CRR asset class "Int Fixed Income" is mapped to CMA asset class *Global Bonds (Ex US)* (if missing, to the asset class matched to US Fixed Income)

4. CRR asset class “US Equities” is mapped to CMA asset class *US Equities (Large Cap)*
5. CRR asset class “Int Equities” is mapped to CMA asset class *Global Equities (Developed, Ex US)* (if missing, *US Equities (Large Cap)*)
6. CRR asset class “Private Equity” is mapped to CMA asset class *Private Equity*
7. CRR asset class “Real Estate” is mapped to CMA asset class *REITs* (if missing, *Private Real Estate*)
8. CRR asset class “Hedge Funds” is mapped to CMA asset class *Hedge Funds*

Note that the CRR data starts in 2001 and ends in 2021, and thus we use the 2001 weights for prior years and 2021 weights for later years in our main analysis of the Pension CAPM.

C Supplementary Empirical Results

This section provides results that complement the main findings in the paper.

C.1 Further Results: Link Between Subjective Risk and Return Expectations

This section provides further results that supplement our main findings on the link between subjective risk and return expectations (studied in Section 2).

C.1.1 Relation Between Subjective Alphas and Subjective Idiosyncratic Variance

In Section 2.2, we define alpha as $\alpha_{j,n,t}^{(m)} = \mu_{j,n,t} - rp_{j,n,t}^{(m)}$. So, any component of subjective expected returns (μ) not captured by our CAPM risk premia ($rp^{(m)}$) is included in alphas. This implies that alphas capture any other risk elements that exist but that are not captured by market betas (e.g., model mispecification). To explore this further, Figure IA.4 displays the results from a regression of subjective alphas onto the CAPM subjective idiosyncratic variances (with standard errors clustered by j, n, and t). As it is clear from the plots, there is a positive association between the alphas and idiosyncratic variances. In particular, the slope coefficient is 43.0 ($t_{stat} = 4.9$) under the Equity CAPM and 25.0 ($t_{stat} = 2.9$) under the Pension CAPM.

C.1.2 Explaining Variation in Subjective Expected Returns under Restricted CAPM

Section 2.3 decomposes expected return variability into the effect of subjective risk premia (rp) and alphas (α). To simplify exposition in the main text, that section discusses results only under the restricted CAPM so that $rp_{j,n,t}^{(m)} = \mu_{j,t}^{(m)} \cdot \beta_{j,n,t}^{(m)}$. Figure IA.5 and Table IA.5 replicate the results discussed in that section (from Figure 7 and Table 5), but using the unrestricted CAPM so that $rp_{j,n,t}^{(m)} = \lambda_{j,t}^{(0)} + \lambda_{j,t}^{(m)} \cdot \beta_{j,n,t}^{(m)}$. The results are similar to the ones reported in the main text. In particular, rp dominates overall μ variation as well as μ variability across asset classes. However, alpha plays a quantitative relevant role when we focus on particular asset classes (even explaining more than 50% of the μ variation in some instances). Moreover, α contributes more to μ variation across institutions and over time than it does for the overall μ variation or the μ variation across asset classes.

C.1.3 Alpha and Beta Heterogeneity Across Asset Classes and Institutions

Section 2.4 shows that variation in subjective expected returns across asset classes (risk-return tradeoff) dominates the variation in expected returns across institutions (disagreement). We can generalize that analysis to any other belief quantity (generically referred to as θ). Specifically, we can decompose the within-year variability in θ into variability originating from institutions and variability originating from asset classes through the generic fixed effects model

$$\theta_{j,n,t} = \overline{\theta}_{j,t} + \overline{\theta}_{n,t} + \eta_{j,n,t} \tag{IA.7}$$

which leads to the variance decomposition

$$1 = \underbrace{\frac{\mathbb{C}ov\left(\theta_{j,n,t}\ ,\ \overline{\theta}_{j,t}\right)}{\mathbb{V}ar(\theta_{j,n,t})}}_{\%\text{ of } \theta_{j,n,t} \text{ Variation from } \overline{\theta}_{j,t}} + \underbrace{\frac{\mathbb{C}ov\left(\theta_{j,n,t}\ ,\ \overline{\theta}_{n,t}\right)}{\mathbb{V}ar(\theta_{j,n,t})}}_{\%\text{ of } \theta_{j,n,t} \text{ Variation from } \overline{\theta}_{n,t}} + \underbrace{\frac{\mathbb{C}ov\left(\theta_{j,n,t}\ ,\ \eta_{j,n,t}\right)}{\mathbb{V}ar(\theta_{j,n,t})}}_{\%\text{ of } \theta_{j,n,t} \text{ Variation from } \eta_{j,n,t}} \tag{IA.8}$$

Figure IA.6 provides results for the decomposition in Equation IA.8 applied to βs and αs (for the Equity CAPM and Pension CAPM). In a nutshell, the results indicate that almost all the variability in βs is driven by variability across asset classes. In contrast, disagreement plays a large role in explaining alpha variability, with disagreement ($\overline{\alpha}_{j,t}+\eta_{j,n,t}$) being roughly as important as variation across asset classes ($\overline{\alpha}_{n,t}$) in explaining overall alpha variation.

C.1.4 Quantifying Disagreement

Table 3 in the main text reports, for each asset class, average values for the subjective belief quantities we study. Table IA.4 is an analogous table in which each number reflects disagreement across institutions. For instance, for the first table entry, we calculate the standard deviation of μ_{TIPS} across institutions each year (for years with at least five μ_{TIPS} observations), and then report the time series average of these standard deviation values. As it is clear from the table, there is plenty disagreement about both expected returns and risk.

For instance, subjective expected returns for *US Equities (Large Cap)* have a ($\pm 2 \cdot$ Sigma) range of 4% in a typical year. The disagreement tends to be lower for fixed income asset classes (with ranges around 2%) and higher for asset class with less historical information (e.g., the range is 8% for *Private Equity*). Ranges for betas are also large, suggesting that institutions do not simply use historical covariance matrices as their estimates (albeit they likely combine historical information with a forward-looking perspective). For instance, $\beta^{(p)}$ has a range of close to 0.32, on average, for *US Equities (Large Cap)* and of more than 1.5 for *Private Equity*.

Recall that Sections 2.4 and C.1.3 show that variation in subjective risk and return expectations is stronger across asset classes than across institutions. The above discussion clarifies that this result does not imply that there is little disagreement across institutions. Instead, it implies that the variation in subjective expected returns across asset classes is simply larger than the variation in expected returns across institutions (i.e., disagreement) present in our dataset.

C.2 Further Results: Link Between Subjective Beliefs and Realized Returns

This section provides further results that supplement our main findings on the link between subjective beliefs and realized returns (studied in Section 3).

C.2.1 Low Beta Anomaly

Section 3.2 in the main text shows that there is a positive risk-return tradeoff under both subjective beliefs (CMAs) and objective beliefs (realized returns). Another way to look at the risk-return tradeoff is to check whether betas predict alphas across asset classes. Within an asset class, this is known as the "low-beta anomaly" (see Frazzini and Pedersen (2014)). Figures IA.7(a) and IA.7(b) show that there is no low-beta anomaly across asset classes in the subjective beliefs of our institutions. Figures IA.7(c) and IA.7(d) show that, in the realized return data, there is a small low-beta anomaly across asset classes. This pattern indicates that beliefs are not fully consistent with realized returns. However, the discrepancy is so small that it has little impact on the overall message that the subjective risk-return tradeoff is consistent with the risk-return tradeoff observed in the realized return data.

C.2.2 Average Subjective Beliefs vs Unconditional Future Return Moments

Section 3.3 (through Figure 11) shows that average beliefs are consistent with unconditional return moments. However, to minimize estimation noise, Figure 11 uses the entire return time-series for each asset class (i.e., returns start at the "First Return Date" column of Table 6). This approach has the drawback that if institutions obtain their beliefs by simply estimating past return moments, the relation between average beliefs and unconditional return moments reflects a "consistency with historical data" effect. Asymptotically, this consistency with historical data ensures that beliefs are unconditionally rational (i.e., unconditional beliefs match unconditional return moments). However, beliefs can be unconditionally rational while displaying extrapolative behavior (which could lead them to be irrational conditional on the time t information set). Section 3.4 addresses this issue by formally testing whether beliefs predict subsequent realized returns. This section provides results from an alternative (less formal) way to deal with the same problem.

Specifically, Figure IA.8 considers unconditional return moments estimated using returns that are subsequent to their respective beliefs. For instance, in Figure IA.8, we obtain μ_e as the average of $\mu_{e,t}$ from 1987 to 2021 and $\widehat{\mu}_e$ as the average of $r_{e,t}$ from 1988 to 2022 (and similarly for all other quantities over their respective sample periods). As we can see from Figure IA.8(a), there is still a strong link between average returns and average expected returns, with a regression of $\widehat{\mu}_e$ on μ_e resulting in a slope coefficient of 0.80. The R^2 value declines to 47% (from 70% in Figure 11), but this result is expected given that the $\widehat{\mu}_n$ values used in Figure IA.8 have more estimation noise than the $\widehat{\mu}_n$ values used in Figure 11. We can clearly see this aspect from the fact that $\widehat{\mu}_{Commodities} < 0\%$ in Figure IA.8, which is likely a consequence of estimation noise (i.e., it reflects unexpected returns that did not average to zero over the relatively small sample period). In fact, just dropping $\widehat{\mu}_{Commodities}$ from Figure 11(a) increases the R^2 to 56.2%. Figures 11(b) to 11(d) further show that we continue to observe a tight link between subjective beliefs and realized return moments related to risk (volatilities and betas). Interestingly, in this case the R^2 values remain at similar levels. This result is likely a consequence of the fact that variances and covariances (underlying $\widehat{\sigma}_n$ and $\widehat{\beta}_n$) have much less estimation noise than average returns (see, e.g., Merton (1980)). So, we can conclude that the results in Figure 11 reflect more than a "consistency with historical data" effect (the same conclusion we obtain from Section 11).

In stark contrast to the paragraph above, Figures IA.8(e) and IA.8(f) show that $\widehat{\alpha}_n$ has effectively no connection to α_n. This indicates that the alpha connection in Figure IA.8 indeed reflects a pure "consistency with historical data" effect. That is, institutions estimate alphas based on past data. Since alphas do not tend to persist in the return data (if the CAPM is correct, alphas are zero over long samples), there is no link between α_n and the $\widehat{\alpha}_n$ calculated from subsequent returns. So, in this case, α_n reflects a temporary extrapolation. It is temporary because as historical data accumulates estimated alphas converge to zero (under the CAPM), which would induce beliefs to also converge to zero (if they indeed reflect past alphas), leading to beliefs that are unconditionally rational.

Note that we obtain the same conclusions from the two above paragraphs through the formal predictability tests in Section 11 (see Tables 7 to 9). However, those predictability tests are more complete since they also allow us to differentiate between cross-sectional and time-series predictability. While subjective risk measures predict realized risk in the cross-section of asset classes, they do not in the time-series.

C.2.3 Return Predictability Regressions: Addressing Two Potential Concerns

There are two potential concerns with the return predictability results in Section 3.4. This section describes them, explains why they are not that concerning conceptually, and shows that our return predictability results are empirically robust to both of these concerns.

The first potential concern is that our CMAs may contain a look-ahead bias. The reason is that some institutions produce their year t CMA in the first few months of year $t+1$.[IA.3]

[IA.3] In our terminology, years are defined based on the approximate timing of the institution's information

We observe the CMA production month for 67.6% of our CMAs, with the distribution of CMA production months displayed in Figure IA.3 (Y0 reflects year t and Y1 reflects year $t+1$ in our terminology). The majority of the CMAs are produced in December of year t or earlier, which induces no look-ahead bias. Among the year t CMAs produced in year $t+1$, the most common production month is January, with very few CMAs with a production month after that. The most likely reason is that these CMAs were produced using information as of December of year t, but the formal CMA report was only written in January of year $t+1$.[IA.4] If this explanation holds for all CMAs with a production month in year $t+1$, then we do not have a look-ahead bias (since the information set is still as of year t). However, if the CMAs with production months in year $t+1$ use realized returns over some months of year $t+1$ in their conditioning variables, then we have a look-ahead bias.

Conceptually, this look-ahead issue would actually bias us against detecting return predictability. The reason is that CMAs reflect long-term return forecasts, often relying on valuation ratios (or yields) as conditioning variables (see Figure 2 for evidence of that in the context of equities). So, high returns in the beginning of year $t+1$ would likely lead to high valuation ratios (i.e., low yields), and thus low CMA-implied expected returns. Nevertheless, we address the potential look-ahead bias empirically. Specifically, we construct alternative aggregate beliefs that are entirely based on CMAs with production month as of December of year t or earlier. The results are similar to our main results (see Panel A of Table IA.6). In particular, the cross-sectional and time-series predictability regressions (using one-year subjective μ) have slope coefficients that are statistically indistinguishable from one.

set. For instance, if a CMA contains $\mathbb{E}_t[R_{t\to t+10}]$, then we refer to it as the year t CMA. This differs from the industry convention. For instance, the JP Morgan CMA titled "2023 Long-Term Capital Market Assumptions" was created on 09/30/2022, and thus represents the 2022 JP Morgan CMA in our terminology (i.e., our year convention corresponds to the information set year and not the first forecast year).

[IA.4] Some CMA reports have a file date (i.e., the exact date when the file was finalized). Other CMA reports do not have a file date, but have a date for the data source. For instance, the 2022 JP Morgan CMA (which is titled "2023 Long-Term Capital Market Assumptions" as per the prior footnote) contains no file date, but states "Source: J.P. Morgan Asset Management; as of September 30, 2022" right above the capital market assumptions we use. When recording the CMA production date, we use the file date when available and the data source date when the file date is not available. This approach is conservative in the sense that the data source date is a better proxy for the institution's information set, but it comes earlier than the CMA file date, so we give priority to the CMA file date when constructing our CMA production month variable.

The out-of-sample R^2 values decrease a little relative to our main analysis (from 12.2% and 12.7% to 9.7% and 9.2%), but they remain economically large and this decline is expected since we are using only a fraction of the CMAs available in our dataset (which adds noise to aggregated beliefs).

The second potential concern is that our aggregation method is not "out-of-sample" from the perspective of the econometrician as it requires full sample estimates of institution fixed effects (to deal with variation in the composition of institutions in our sample over time). Conceptually, whether this worry is relevant depends on the objective of the empirical analysis. If the objective of the analysis is to use CMAs to create a trading signal, then this is a valid concern. However, if the objective of the analysis is to understand whether the subjective expected returns of these institutions are, on average, in line with objective expected returns (which is the economic question we tackle), then this is less of a worry. Nevertheless, we also address this potential issue empirically. Specifically, we consider an out-of-sample aggregation approach that implements the steps in Subsection 3.1 on an expanding window. The results are also similar to our main results (see Panel B of Internet Appendix Table IA.6). For instance, the cross-sectional and time-series predictability regressions (using one-year subjective μ) have slope coefficients that are statistically indistinguishable from one. The slope coefficient that reflects variation over time is only marginally significant ($t_{b=0} = 1.86$), but it is economically close to one ($b = 0.87$), indicating the marginal statistical significance is related to the fact that out-of-sample aggregation leads to noisier aggregated beliefs (due to estimation noise in aggregation parameters). As in Panel A, the out-of-sample R^2 values decrease a little relative to our main analysis (from 12.2% and 12.7% to 9.4% and 9.8%), but they remain economically large and this decline is also expected given the noisier aggregation process.

C.3 Main Results: Alternative Specifications

As detailed in Section B.1, our main analysis combines different data sources and institution types to maximize data coverage. It also relies on some measurement decisions (e.g., how to define asset classes and what types of expected returns to use). In this section, we show that our results are robust to these empirical decisions. This finding helps to alleviate/eliminate concerns related to different potential biases that may affect our data.

For each subsection in this section, the alternative main results related to the link between subjective risk and return expectations are provided in figures (see Figures IA.9 to IA.14) and the alternative main results related to the link between subjective beliefs and realized returns are provided in a table (see Table IA.7).

C.3.1 Main Results for Each Institution Type

Managers and consultants may have different approaches to their CMAs (and also face different incentives). So, we provide results separately for each group of institutions (with 128 CMAs from managers and 233 CMAs from consultants). Figures IA.9 and IA.10 show that our main results on the link between subjective risk and return expectations are similar for each group of investors. Table IA.7 (Panels A and B) show that our main results on the link between subjective beliefs and subsequent realized returns are similar for each group.

C.3.2 Main Results for Each Data Source

There are different potential issues with our two types of data collection approaches of (i) obtaining beliefs directly from the CMAs of the underlying institutions or (ii) obtaining beliefs from the CMAs indirectly recorded by pension funds. So, we provide results separately for the 301 direct CMAs we have as well as the 123 indirect CMAs we have.[IA.5] Figures IA.11 and IA.12 show that our main results on the link between subjective risk and return expectations are similar for each data collection approach group of investors. Table IA.7 (Panels C and D) show that our main results on the link between subjective beliefs and subsequent realized returns are similar for each group.

[IA.5] Note that the sum of the direct and indirect CMAs used in this analysis is higher than the total number of CMAs we have in our main analysis. The reason is the pecking order described in Section B.1.1. Specifically, there are some institution-years for which we observe both the direct CMA and the indirect CMA (with our pecking order giving priority to the direct CMA). So, the number of indirect CMAs used in our analysis that includes only indirect CMAs is higher than the number of indirect CMAs used in our main analysis.

C.3.3 Main Results Using Observations in which the Primary Asset Class is Available

As Table IA.3 highlights, to increase coverage, we build our 20 paper asset classes by combining data from different master key asset classes depending on data availability. This process creates slight mismatches between the asset classes of different institutions, although in cases where primary asset classes are unavailable, we choose substitutes to be closely related. To address this issue, we consider an alternative sample that uses only primary asset classes without substitution (i.e., "Option #1 in Table IA.3), except in the case of *US Cash* and *Equities (Large Cap)* since these two asset classes are needed to obtain μ and β. The main results from this analysis are similar to the ones provided in the main text. Figures IA.13 demonstrates this finding in the context of the analysis linking subjective risk and return expectations. Table IA.7 (Panel E) focuses on the analysis linking subjective beliefs and subsequent realized returns.

C.3.4 Main Results with $\mathbb{E}[\boldsymbol{R}]$ Based on Expected Geometric Returns

As explained in Section B.1.2, the CMAs used in our main analysis vary on whether they contain only expected arithmetic returns, only expected geometric returns, or both. In particular, 77.8% of our CMAs report expected arithmetic returns while 58.4% of our CMAs report expected geometric returns (with 36.3% of our CMAs reporting both). Our main analysis uses expected arithmetic returns when available (since they are the most commonly reported) and expected arithmetic returns implied from expected geometric returns when expected arithmetic returns are not directly available. As an alternative procedure, we also consider using expected geometric returns when available and expected geometric returns implied from expected arithmetic returns when expected geometric returns are not directly available. The main results from this analysis are similar to the ones provided in the main text. Figure IA.14 demonstrates this finding in the context of the analysis linking subjective risk and return expectations. Table IA.7 (Panel F) focuses on the analysis linking subjective beliefs and subsequent realized returns. All mathematical details on the transformations are provided in Section B.1.2.

C.3.5 Main Results with Beliefs Based on 10-Year Investment Horizon

The forecasting horizons in our CMAs vary from 4 years to 30 years, with the modal horizon being 10 years (capturing 44.2% of the CMAs for which we observe horizon). Conceptually, this is not an issue since a version of the CAPM holds with investors that have heterogeneous horizons, as we discuss in the main text. However, we also consider an alternative sample that focuses on the 102 CMAs that have a 10 year horizon. The main results from this analysis are similar to the ones provided in the main text. Figure IA.15 demonstrates this finding in the context of the analysis linking subjective risk and return expectations. Table IA.7 (Panel G) focuses on the analysis linking subjective beliefs and subsequent realized returns.

C.3.6 Main Results Only Requiring $\boldsymbol{\mu}$

Since we study subjective risk and return expectations jointly, our main analysis only uses CMAs that have both expected returns and covariance matrices (i.e., volatilities and correlation matrices). However, our return predictability results can be obtained even without requiring covariance matrices. So, Table IA.7 (Panel H) shows that our main return predictability results are similar to the ones we provide in the main text even if we use all CMAs whether they contain covariance matrices or not (this alternative analysis is based on 429 CMAs whereas our main analysis is based on 361 CMAs).

References for Internet Appendix

Dahlquist, M. and M. Ibert (2024). “Equity Return Expectations and Portfolios: Evidence from Large Asset Managers”. In: *Review of Financial Studies* 37.6, pp. 1887–1928.

Frazzini, A. and L. H. Pedersen (2014). “Betting Against Beta”. In: *Journal of Financial Economics* 111.1, pp. 1–25.

Merton, R. C. (1980). “On Estimating the Expected Return on the Market: An Exploratory Investigation”. In: *Journal of Financial Economics* 8, pp. 323–361.

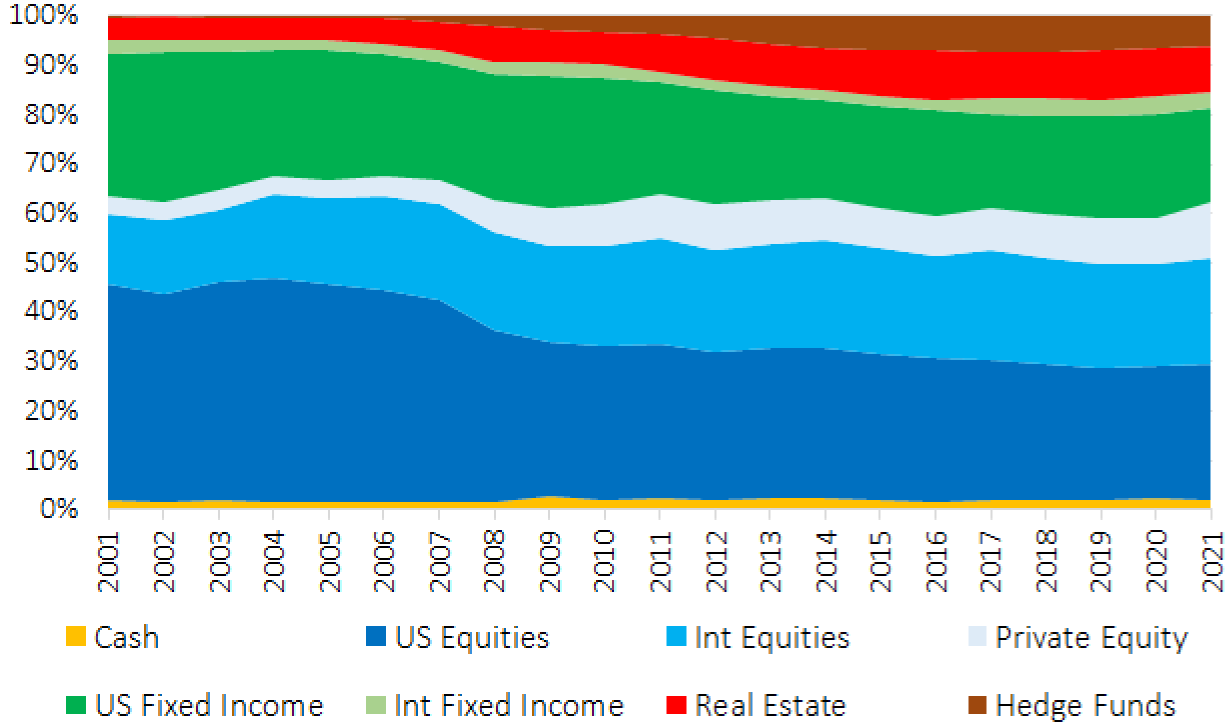


Figure IA.1
Aggregate Allocation of US Public Pension Funds

This figure plots the aggregate allocation of US Public Pension Funds from 2001 to 2021 (data from the Center for Retirement Research at Boston College). This allocation is used to form the market portfolio in the Pension CAPM ($m = p$) model we study. We use 2001 weights for earlier years and 2021 weights for later years. Section B.2 provides more details.

Number of CMAs

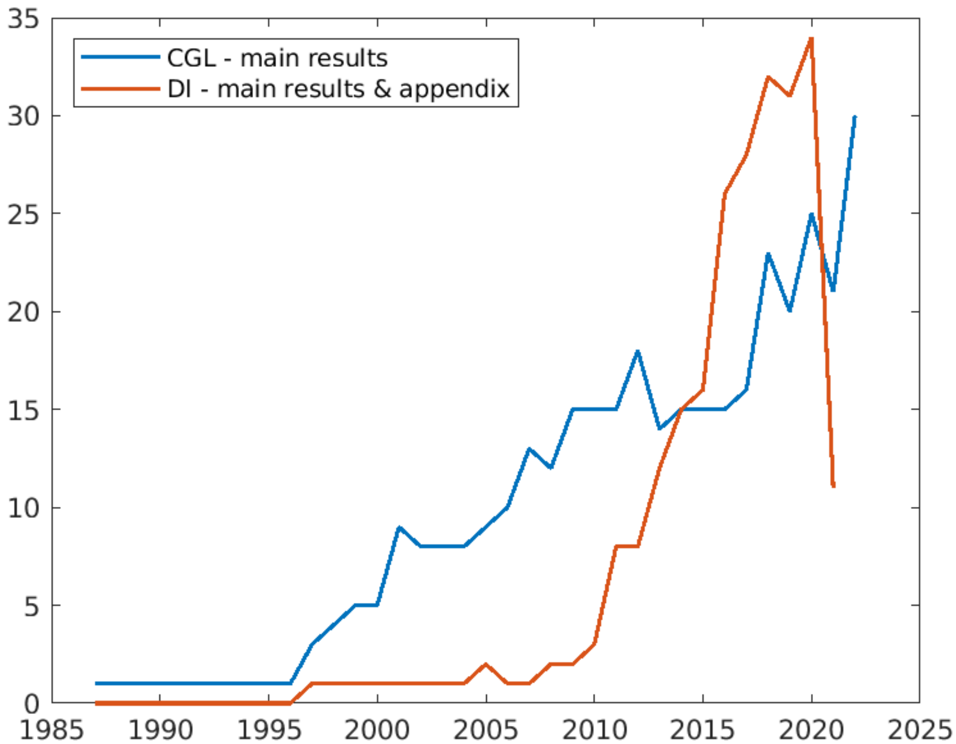


Figure IA.2
Comparing our CMAs Dataset Coverage with that of Dahlquist and Ibert (2024)

This figure plots the number of CMAs in our dataset each year (in blue) as well as the number of CMAs in the Dahlquist and Ibert (2024) dataset each year (in orange). While we study asset managers and investment consultants, Dahlquist and Ibert (2024) study only asset managers in the main text, with a short analysis of investment consultants in their internet appendix. To be conservative, the number of CMAs we report for Dahlquist and Ibert (2024) includes all CMAs studied in the main text as well as the internet appendix. Sections 1.3 and B.1 provide more details about our subjective beliefs data and Section 1.4 provides more details about the analyses reported in this figure.

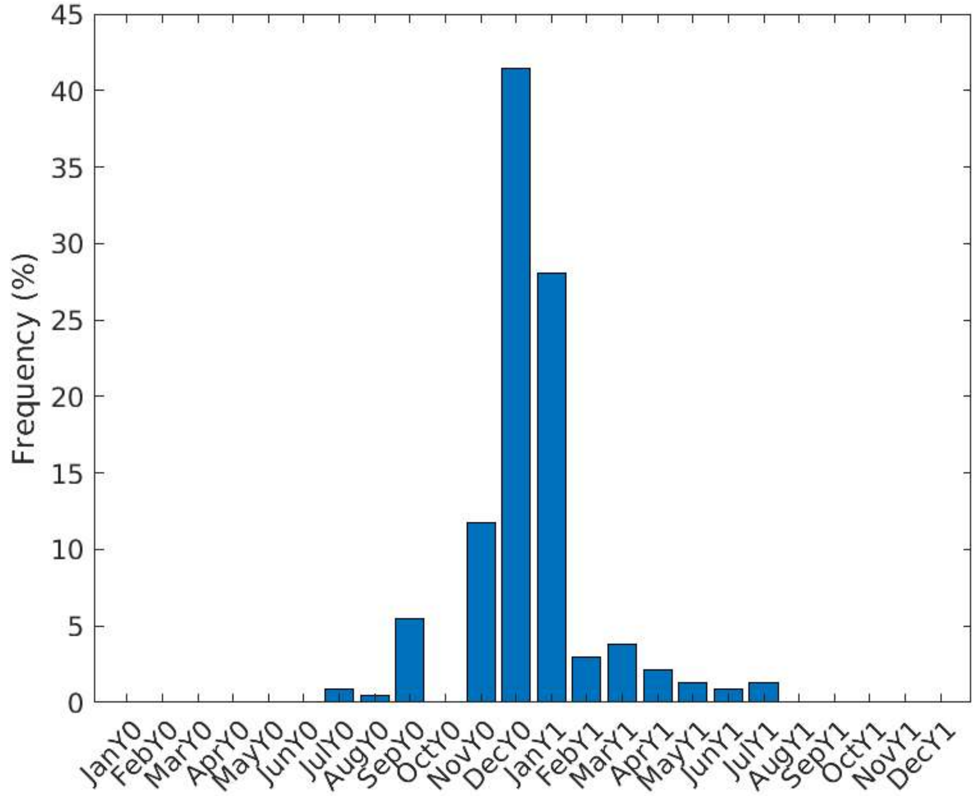
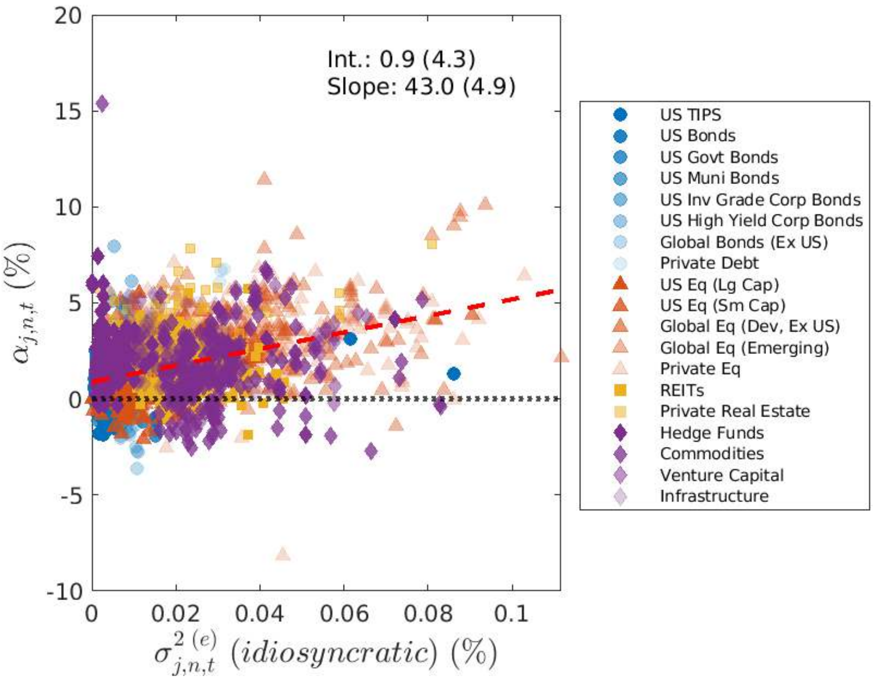


Figure IA.3

CMA Production Months: Frequency of each Month

This figure plots the distribution of CMA production months in our sample (among the 67.6% CMAs for which we observe production month). Y0 refers to year t while Y1 refers to year $t+1$. So, the bar on DecY0 reflects the fraction of CMAs with CMA production month as of December of year t. Some CMA reports have a file date (i.e., the exact date when the file was finalized). Other CMA reports do not have a file date, but have a date for the data source. When recording the CMA production date, we use the file date when available and the data source date when the file date is not available. This approach is conservative in the sense that the data source date is a better proxy for the institution's information set, but it comes earlier than the CMA file date, so we give priority to the CMA file date when constructing our CMA production month variable. Sections 1.3 and B.1 provide more details about our subjective beliefs data and Section C.2.3 provides more details about the analyses reported in this figure.

(a) Equity CAPM



(b) Pension CAPM

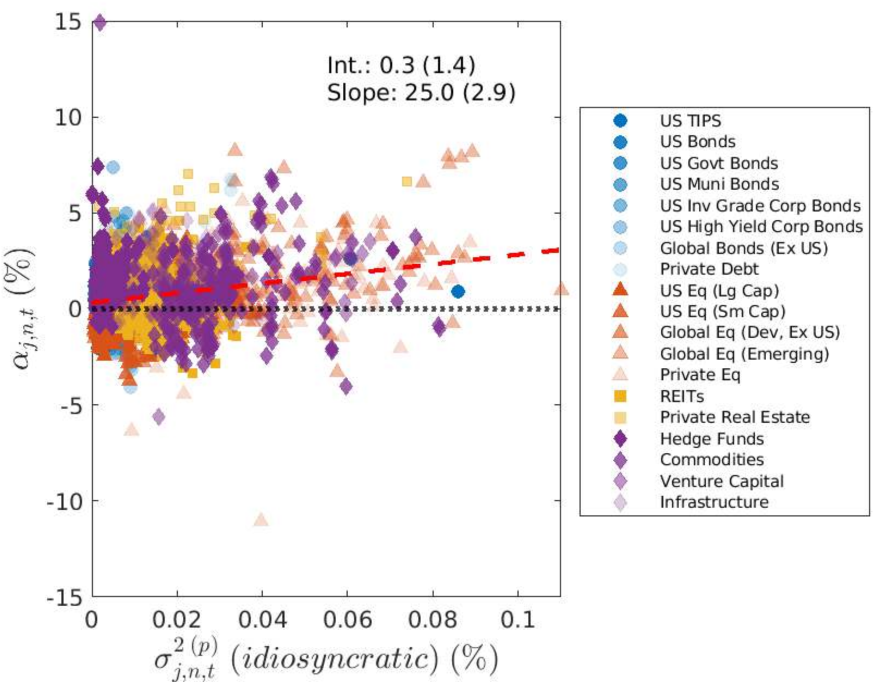


Figure IA.4

Relation Between Subjective Alphas and Subjective Idiosyncratic Variance

This figure plots the subjective alpha of each asset class (for each institution-year CMA) against its respective subjective idiosyncratic variance. Subjective alphas are obtained from $\alpha^{(m)}_{j,n,t} = \mu_{j,n,t} - \mu^{(m)}_{j,t} \cdot \beta^{(m)}_{j,n,t}$ while subjective idiosyncratic variances are obtained from $\sigma^{2(m)}_{j,n,t} = \sigma^2_{j,n,t} - (\beta^{(m)}_{j,n,t})^2 \cdot \sigma^2_{j,m,t}$. We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Sections 1.3 and B.1 provide more details about our subjective beliefs data and Section C.1.1 provides more details about the analyses reported in this figure.

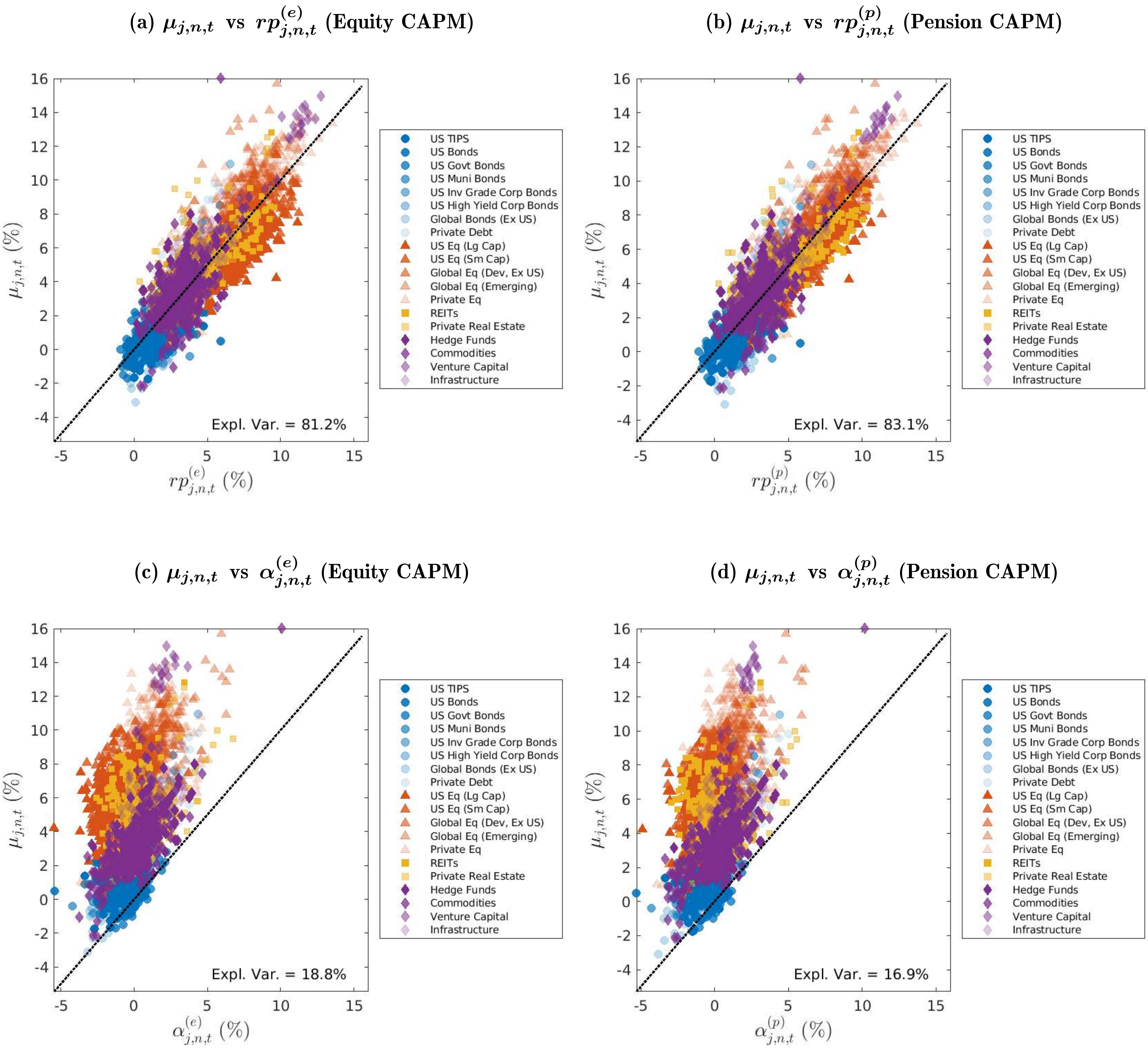


Figure IA.5

Subjective Expected Return Variation: Risk Premia vs Alphas (Unrestricted CAPM)

This figure plots the subjective expected return of each observation ($\mu_{j,n,t}$) against the respective subjective risk premium ($rp^{(m)}_{j,n,t} = \lambda^{(0)}_{j,t} + \lambda^{(m)}_{j,t} \cdot \beta^{(m)}_{j,n,t}$) or the respective subjective alpha ($\alpha^{(m)}_{j,n,t} = \mu^{(m)}_{j,n,t} - rp^{(m)}_{j,n,t}$). All four panels use the $\lambda^{(0)}_{j,t}$ and $\lambda^{(m)}_{j,t}$ estimated from the regressions in Equation 3. We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Section 1.3 provides more details about our subjective beliefs data and Section C.1.2 provides more details about the analysis reported in this figure.

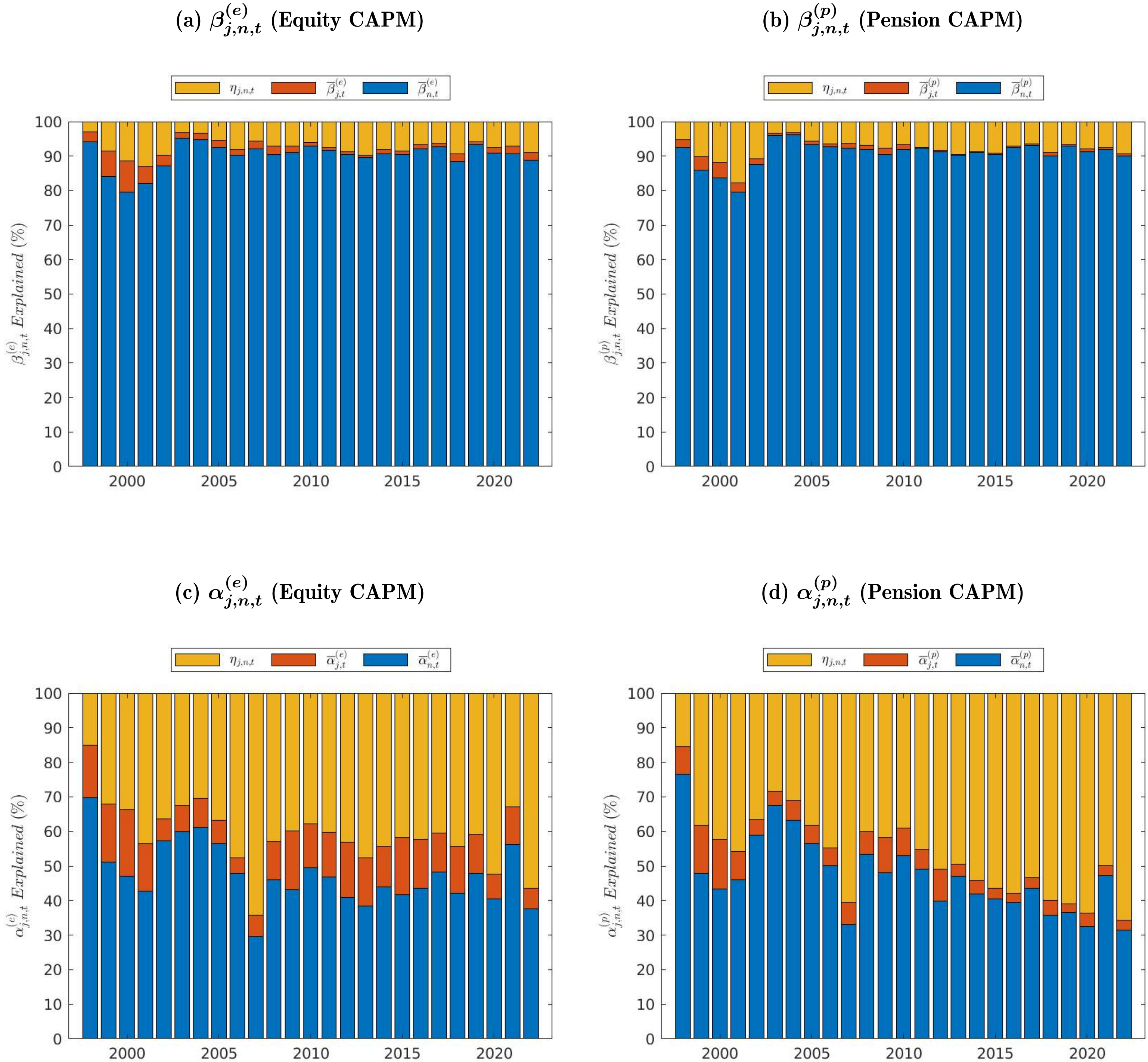


Figure IA.6

Decomposing Variability in Subjective βs and αs: Institution vs Asset Class Effects

This figure plots the decomposition of within year variability in βs and αs (based on Equation IA.8) into the effect of asset class heterogeneity (in blue), institution heterogeneity (in red), and residuals (in orange). The institution fixed effect is given by $\overline{\beta}_{j,t}$ or $\overline{\alpha}_{j,t}$ while the asset class fixed effect is given by $\overline{\beta}_{n,t}$ or $\overline{\alpha}_{n,t}$. We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Section 1.3 provides more details about our subjective beliefs data and Section C.1.3 provides more details about the analysis reported in this figure.

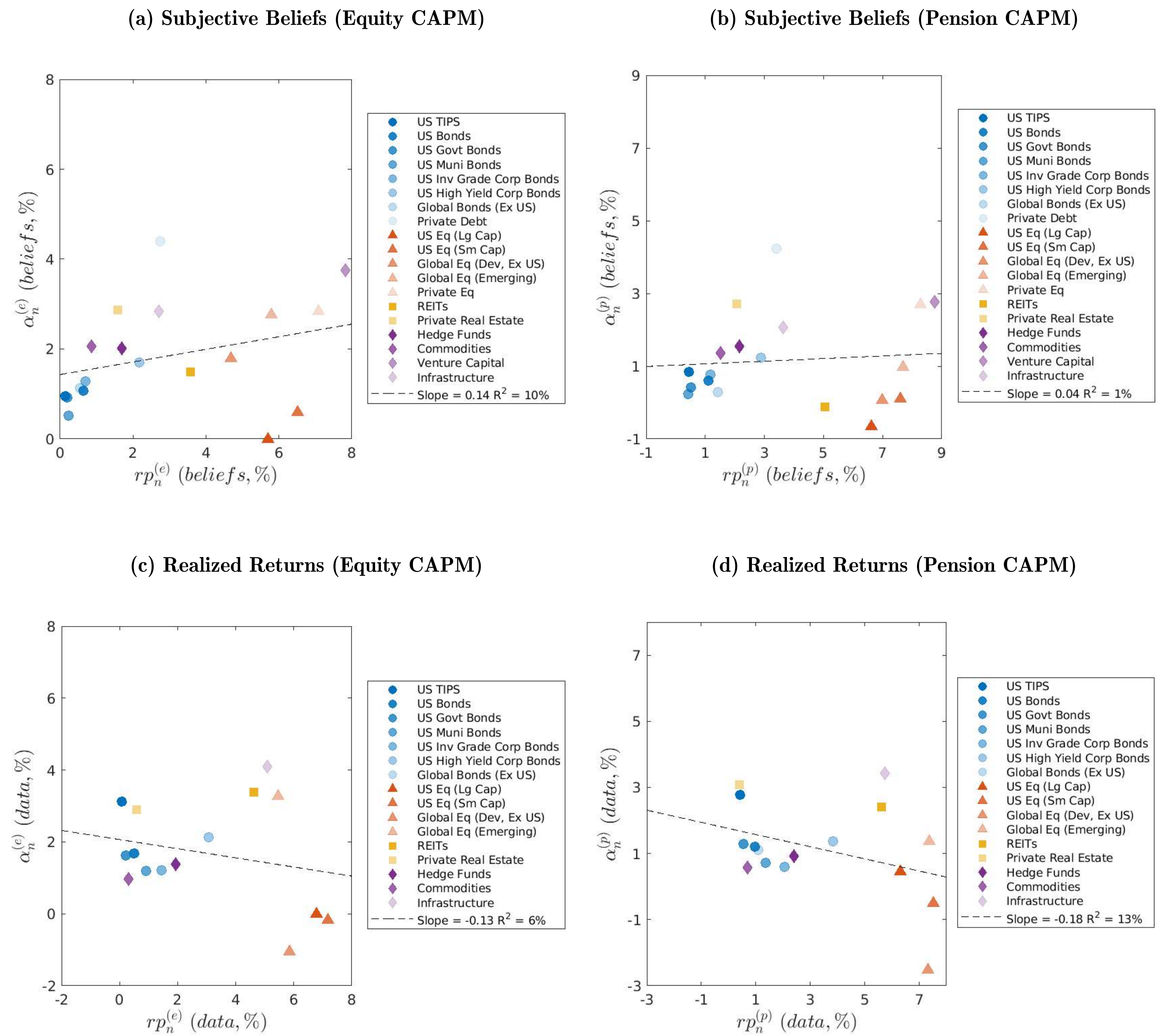


Figure IA.7

Low-Beta Anomaly: Subjective Beliefs vs Realized Returns

This figure plots alphas ($\alpha_n = \mu_n - rp_n^{(m)}$) against risk premia ($rp_n^{(m)} = \beta_n^{(m)} \cdot \mu^{(m)}$) across asset classes. Risk premia only vary with beta across asset classes (since the market risk premia is constant across asset classes). Panels (a) and (b) use beliefs (averaged across institutions and years), and thus reflect the subjective low-beta anomaly (or lack thereof). Panels (c) and (d) use unconditional data moments, and thus reflect the objective low-beta anomaly. We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Sections 1.3 and 3.1 provide more details about the data and Section C.2.1 provides more details about the analysis reported in this figure.

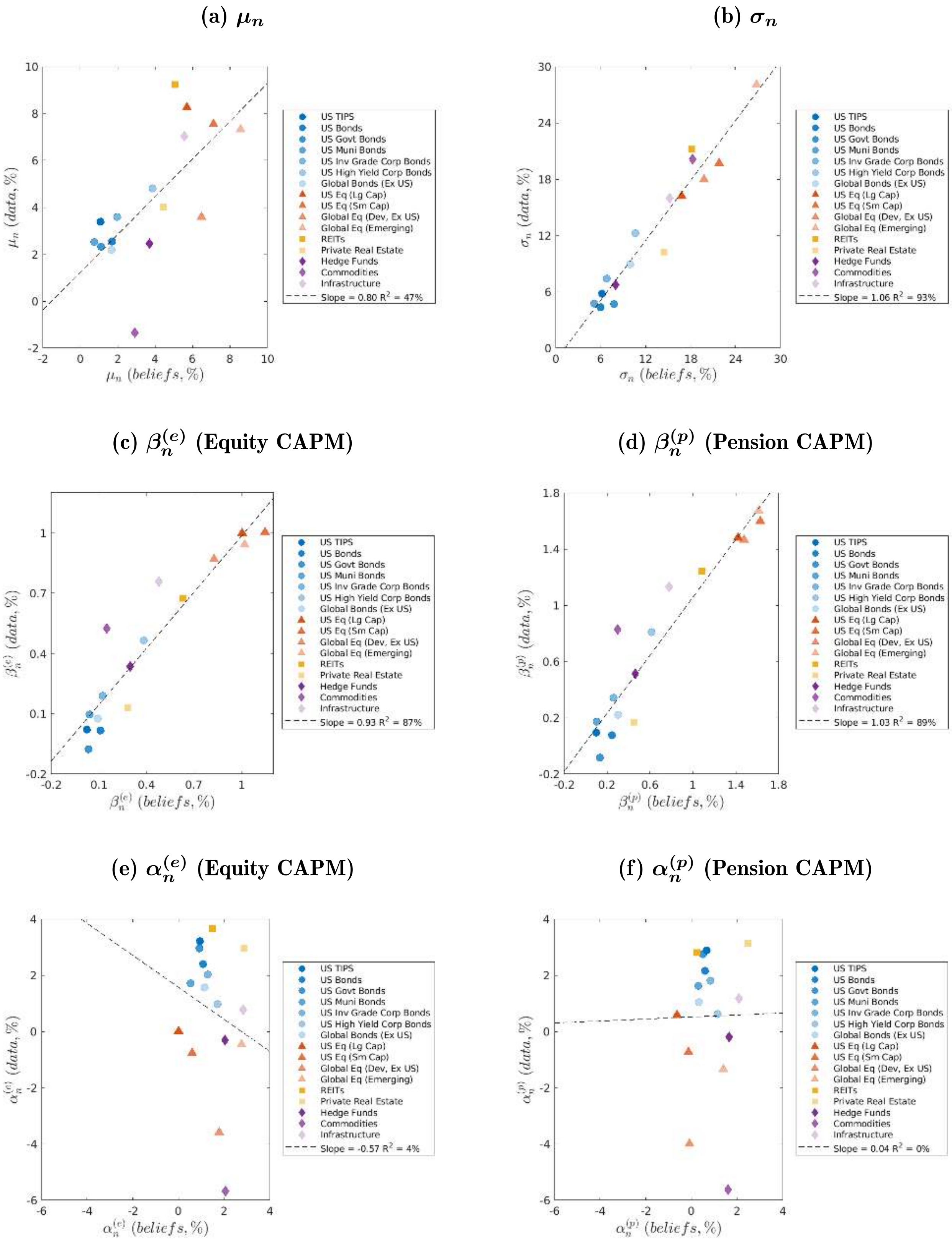


Figure IA.8

Average Subjective Beliefs vs Unconditional Future Return Moments

This figure plots unconditional data moments against average values for their respective beliefs (averaged across institutions and years). We use returns that are subsequent to the belief moments. For Panels (c) to (f), we consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Sections 1.3 and 3.1 provide more details about the data and Section C.2.2 provides more details about the analysis reported in this figure.

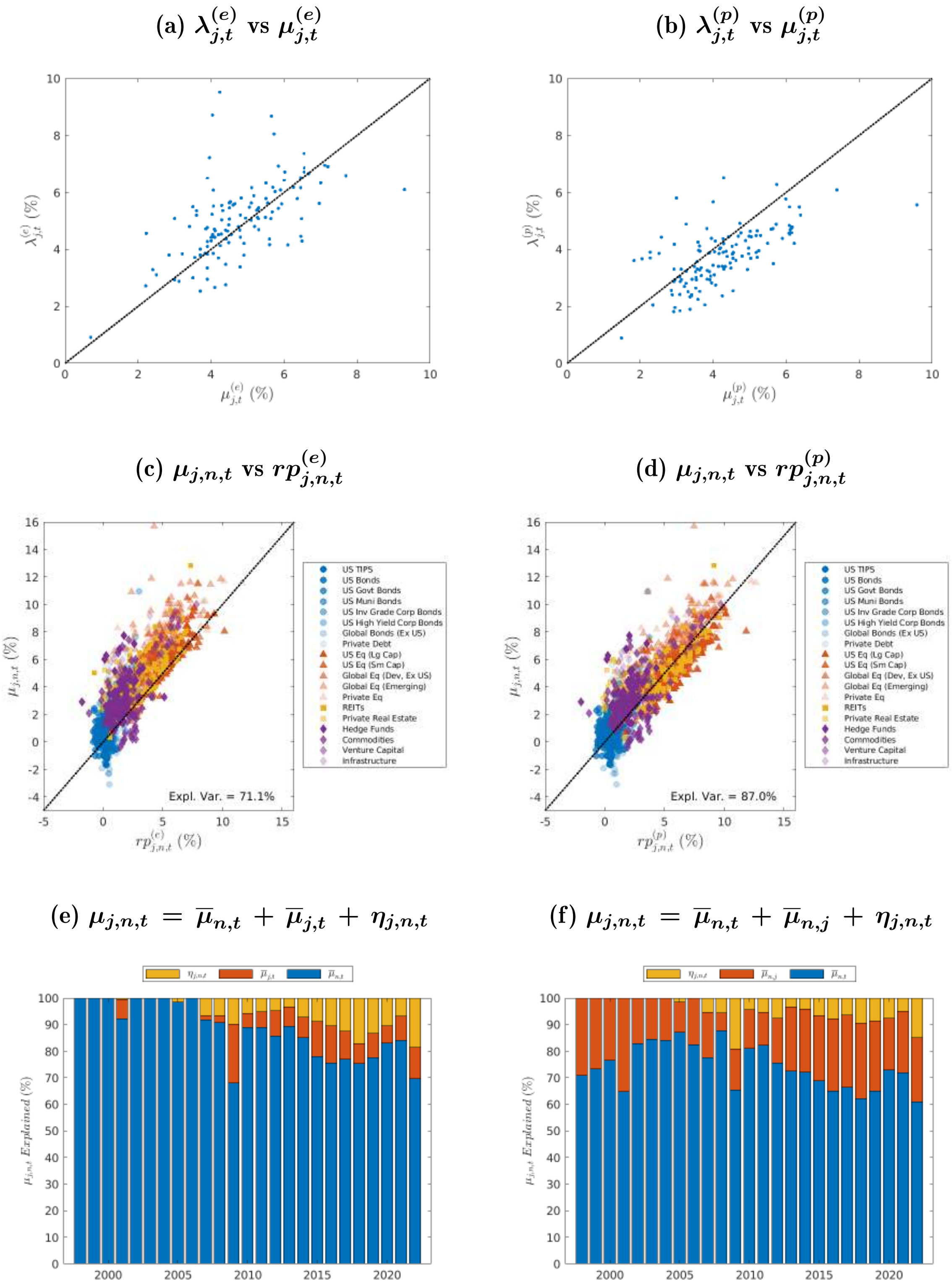


Figure IA.9

Link Between Subjective Risk and Return Expectations: CMAs of Asset Managers

This figure replicates our main results, but using only the CMAs of asset managers. Panels (a) and (b) replicate Figures 3(c) and 3(d) in the main text. Panels (c) and (d) replicates Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 9(a) and 9(b) in the main text. Sections 1 and B.1 provide more details about our subjective beliefs data and Section C.3.1 provides more details about the analysis reported in this figure.

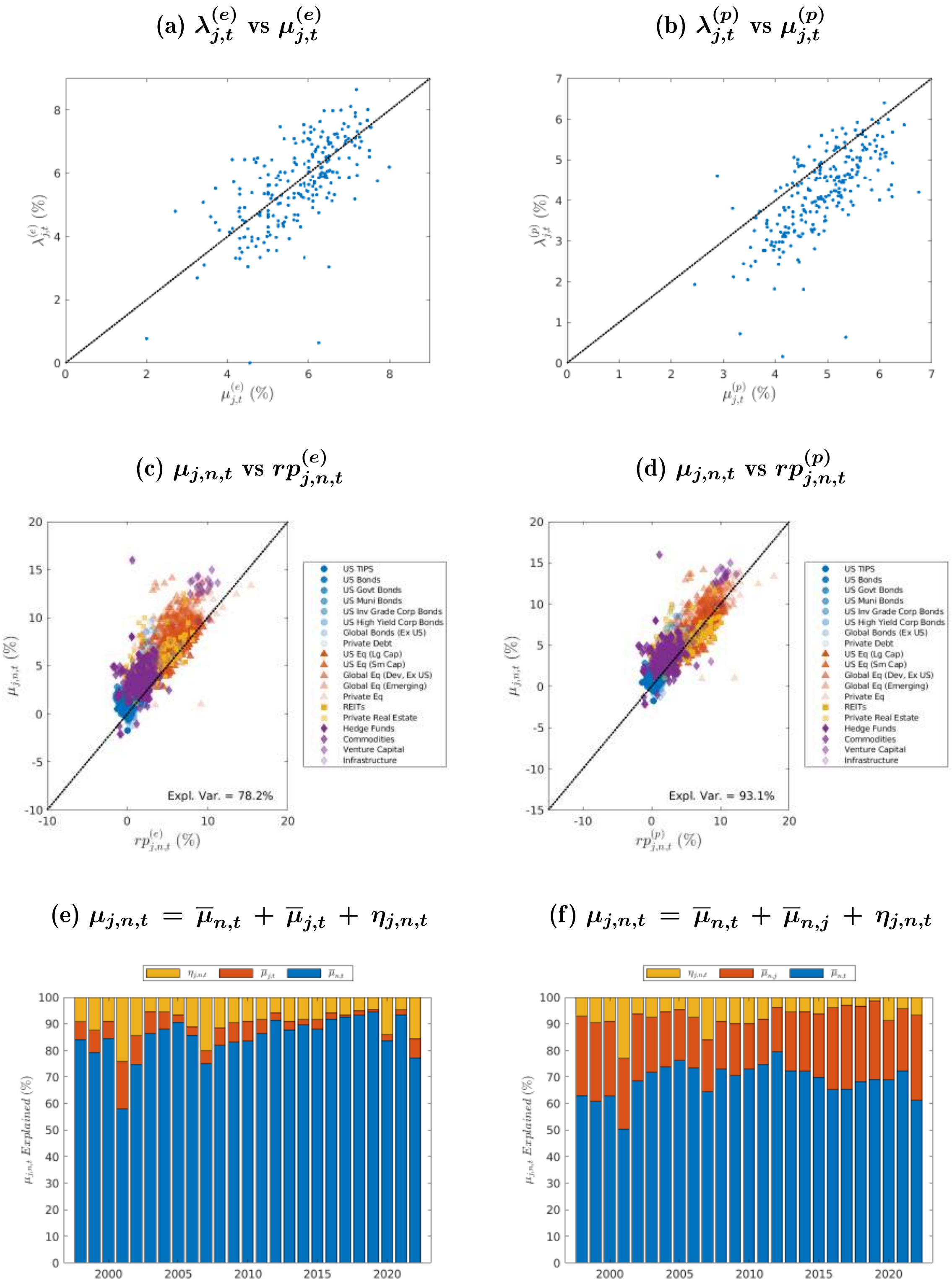


Figure IA.10
Link Between Subjective Risk and Return Expectations: CMAs of Investment Consultants

This figure replicates our main results, but using only the CMAs of institutional investor consultants. Panels (a) and (b) replicate Figures 3(c) and 3(d) in the main text. Panels (c) and (d) replicates Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 9(a) and 9(b) in the main text. Sections 1 and B.1 provide more details about our subjective beliefs data and Section C.3.1 provides more details about the analysis reported in this figure.

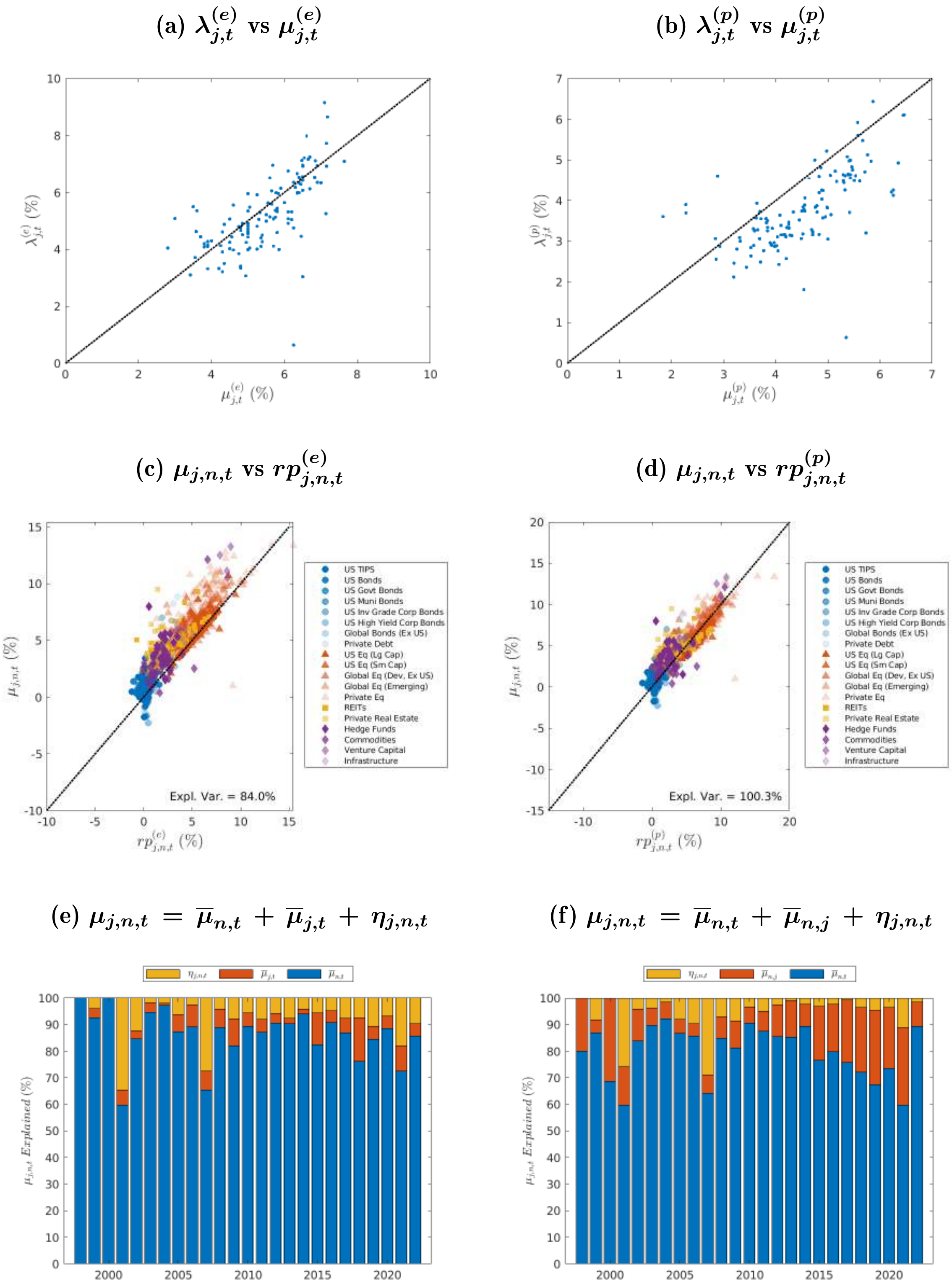


Figure IA.11

Link Between Subjective Risk and Return Expectations: CMAs Obtained Indirectly through Pension Fund Reports

This figure replicates our main results, but using only data obtained indirectly through pension fund reports. Panels (a) and (b) replicate Figures 3(c) and 3(d) in the main text. Panels (c) and (d) replicates Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 9(a) and 9(b) in the main text. Sections 1 and B.1 provide more details about our subjective beliefs data and Section C.3.2 provides more details about the analysis reported in this figure.

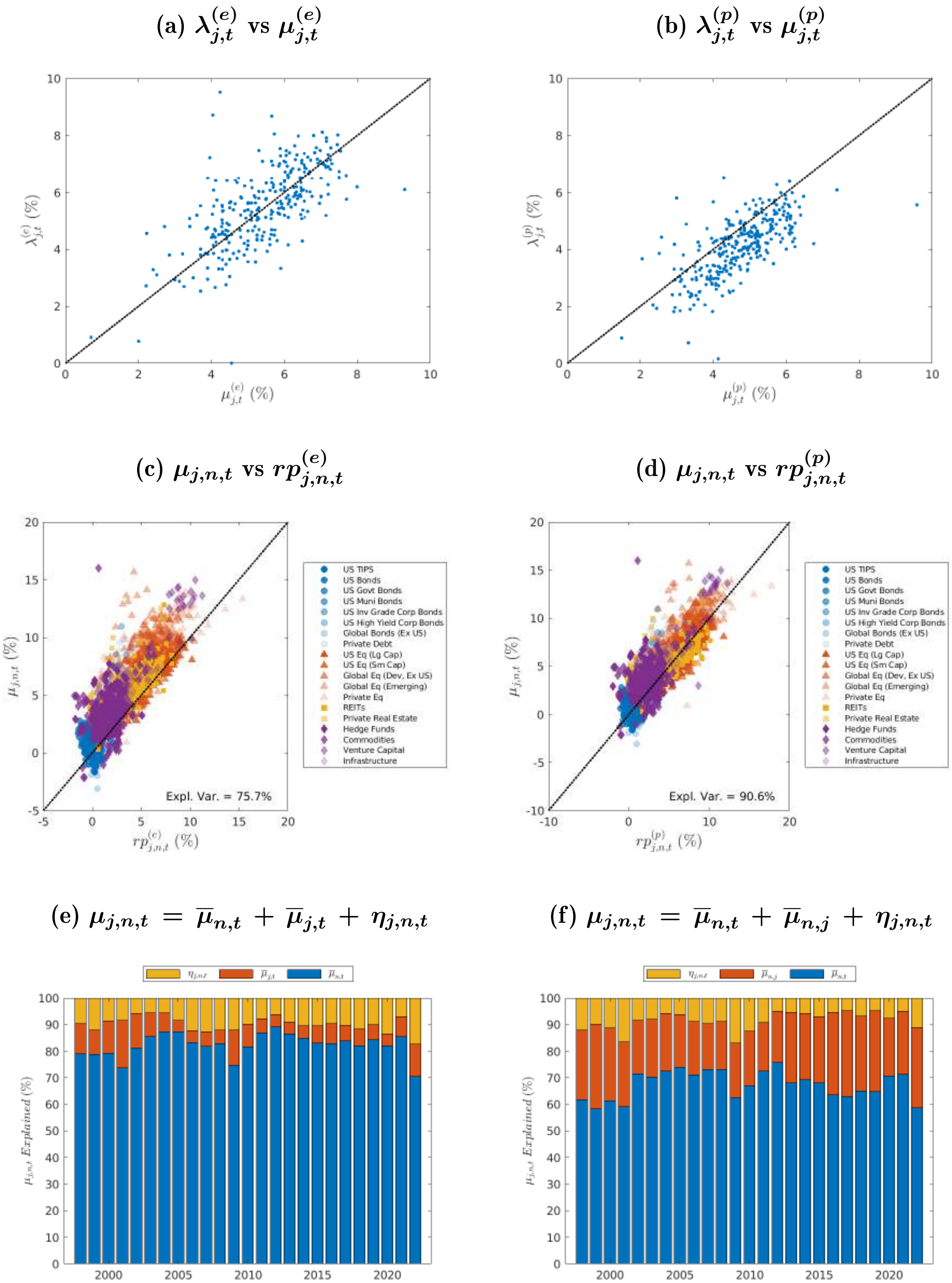


Figure IA.12

Link Between Subjective Risk and Return Expectations: CMAs Obtained Directly trough Underlying CMA Reports

This figure replicates our main results, but using only data obtained directly from the CMAs of the respective institutions. Panels (a) and (b) replicate Figures 3(c) and 3(d) in the main text. Panels (c) and (d) replicates Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 9(a) and 9(b) in the main text. Sections 1 and B.1 provide more details about our subjective beliefs data and Section C.3.2 provides more details about the analysis reported in this figure.

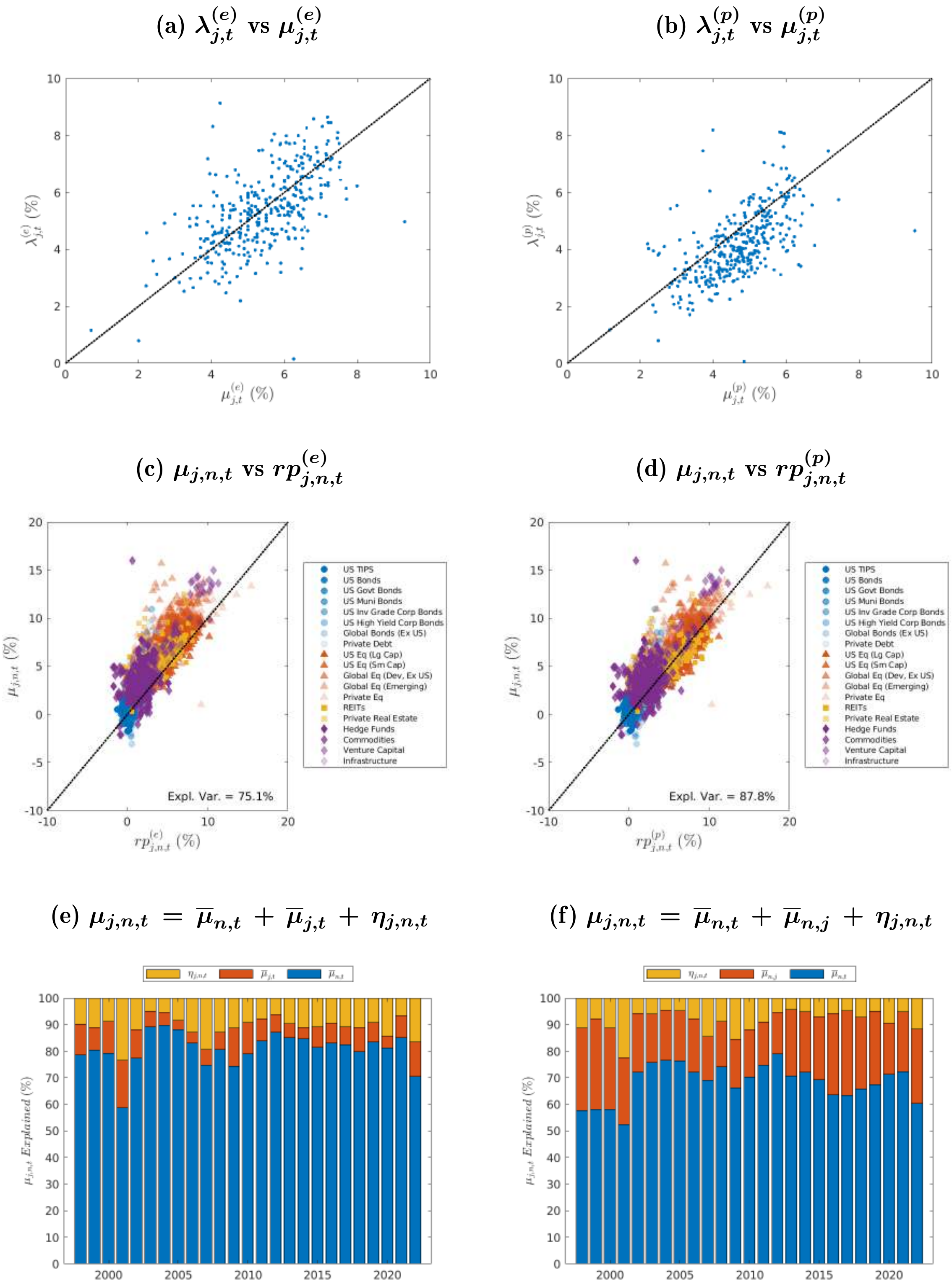


Figure IA.13

Link Between Subjective Risk and Return Expectations: CMAs with Primary Asset Class Available

This figure replicates our main results, but using only institution-year-asset class observations for which the given primary asset class used in the analysis is directly available among our master asset classes for the given institution-year observation. Panels (a) and (b) replicate Figures 3(c) and 3(d) in the main text. Panels (c) and (d) replicates Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 9(a) and 9(b) in the main text. Sections 1 and B.1 provide more details about our subjective beliefs data and Section C.3.3 provides more details about the analysis reported in this figure.

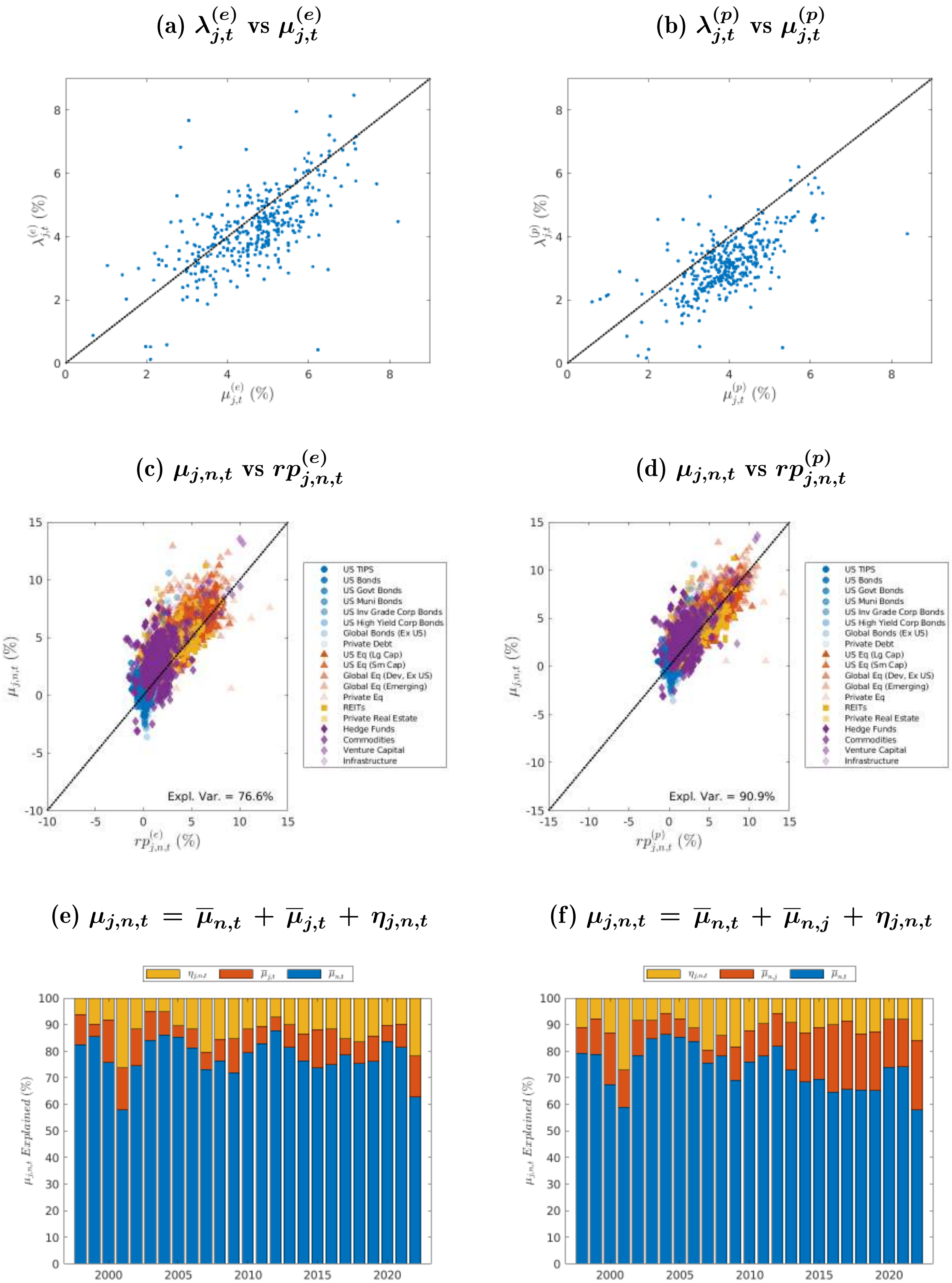


Figure IA.14
Link Between Subjective Risk and Return Expectations:
$\mathbb{E}[R]$ Based on Expected Geometric Returns

This figure replicates our main results, but measuring $\mathbb{E}[R]$ from expected geometric returns. Panels (a) and (b) replicate Figures 3(c) and 3(d) in the main text. Panels (c) and (d) replicates Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 9(a) and 9(b) in the main text. Sections 1 and B.1 provide more details about our subjective beliefs data and Section C.3.4 provides more details about the analysis reported in this figure.

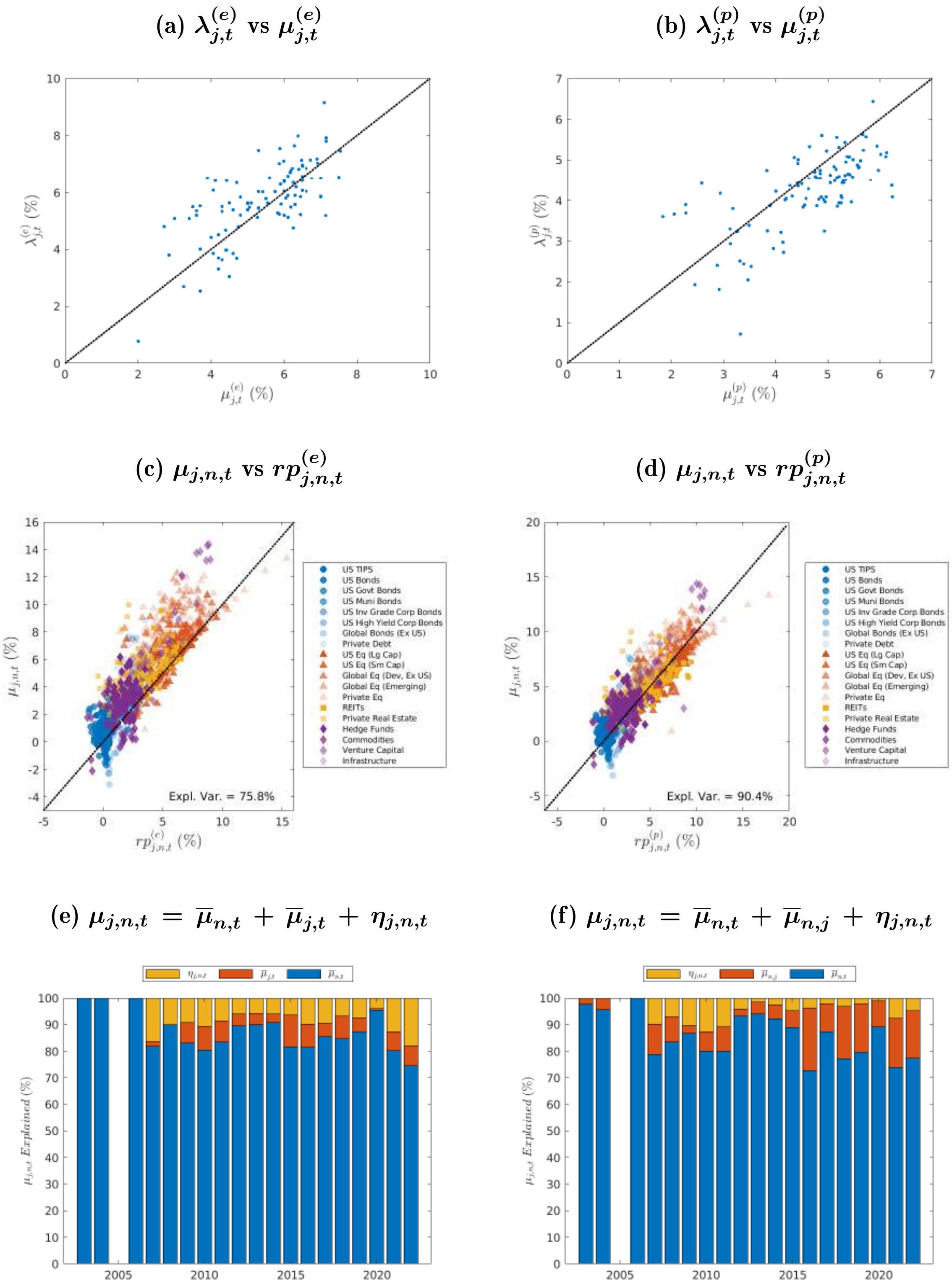


Figure IA.15

Link Between Subjective Risk and Return Expectations: CMAs with a 10-Year Investment Horizon

This figure replicates our main results, but measuring beliefs only from CMAs that state a 10 year horizon, which is the most common horizon in our dataset. Panels (a) and (b) replicate Figures 3(c) and 3(d) in the main text. Panels (c) and (d) replicates Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 9(a) and 9(b) in the main text. Sections 1 and B.1 provide more details about our subjective beliefs data and Section C.3.5 provides more details about the analysis reported in this figure.

Table IA.1
Complete Sample Coverage of CMAs (by Year)

This table reports information on our sample of Capital Market Assumptions (CMAs) over time. The left panel details, for each year, the total number of CMAs in our sample, the number of direct CMAs from the underlying institutions, the number of CMAs from asset managers (or simply "managers"), and the number of CMAs from investment consultants (or simply "consultants"). The right panel reports, for each year, the number of CMAs covering each of the asset classes in our sample (with asset class definitions below the table). Sections 1.3 and B.1 provide more details about our subjective beliefs data.

	# of CMAs in Dataset				# of CMAs Covering the Given Asset Class																			
	All CMAs	**Direct CMAs**	**Managers**	**Consultants**	**(0)**	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**	**(11)**	**(12)**	**(13)**	**(14)**	**(15)**	**(16)**	**(17)**	**(18)**	**(19)**
1987	1	0	0	1	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
1988	1	1	0	1	1	0	1	0	0	0	0	1	0	1	1	1	0	1	0	1	0	0	0	0
1989	1	1	0	1	1	0	1	0	0	0	0	1	0	1	1	1	0	1	0	1	0	0	0	0
1990	1	1	0	1	1	0	1	0	0	0	0	1	0	1	0	1	0	0	0	1	0	0	0	0
1991	1	1	0	1	1	0	1	0	0	0	0	1	0	1	1	1	0	1	0	1	0	0	0	0
1992	1	1	0	1	1	0	1	0	0	0	0	1	0	1	1	1	0	1	0	1	0	0	0	0
1993	1	1	0	1	1	0	1	0	0	0	0	1	0	1	1	1	0	1	0	1	0	0	0	0
1994	1	1	0	1	1	0	1	0	0	0	0	1	0	1	1	1	0	1	0	1	0	0	0	0
1995	1	1	0	1	1	0	1	0	0	0	0	1	0	1	1	1	0	1	0	1	0	0	0	0
1996	1	1	0	1	1	0	1	0	0	0	0	1	0	1	1	1	0	1	0	1	0	0	0	0
1997	3	3	1	2	3	0	3	1	0	0	2	3	0	3	2	3	2	1	1	2	0	1	0	0
1998	4	4	1	3	4	1	4	0	0	0	3	4	0	4	2	4	3	1	2	2	0	1	0	0
1999	5	4	1	4	5	1	5	1	0	0	3	4	0	5	2	5	3	3	2	4	0	0	0	0
2000	5	5	1	4	5	1	5	1	0	0	5	4	0	5	2	5	5	3	2	3	0	0	0	0
2001	9	5	2	7	9	3	8	1	0	0	5	5	0	9	3	9	5	7	2	6	2	0	0	0
2002	8	5	1	7	8	5	8	2	1	1	5	4	0	8	4	8	5	6	3	4	3	0	0	0
2003	8	6	1	7	8	7	8	3	1	2	6	5	0	8	3	8	6	7	3	7	3	1	0	0
2004	8	6	1	7	8	7	8	3	1	2	6	5	0	8	3	8	6	8	3	7	4	1	0	0
2005	9	7	2	7	9	8	9	3	1	1	6	7	0	9	4	9	6	9	5	9	6	6	0	0
2006	10	8	1	9	10	10	10	3	1	1	8	7	0	10	6	10	8	10	6	10	7	6	0	1
2007	13	9	3	10	13	9	13	3	2	2	9	10	0	13	8	13	8	13	7	13	9	9	0	1
2008	12	9	4	8	12	11	12	4	3	2	10	9	0	12	7	11	10	10	7	10	7	8	0	2
2009	15	11	5	10	15	13	14	4	4	3	12	10	1	15	9	13	9	13	8	12	9	11	0	2
2010	15	11	5	10	15	15	13	4	4	3	12	11	1	15	9	13	12	13	8	12	10	11	0	2
2011	15	11	4	11	15	15	14	5	4	4	12	11	1	15	10	15	12	14	9	14	11	12	1	3
2012	18	13	6	12	18	17	17	7	6	7	14	14	2	18	13	17	15	16	11	17	13	14	1	5
2013	14	11	3	11	14	14	13	7	3	6	12	11	2	14	11	14	13	13	9	14	11	12	2	5
2014	15	13	5	10	15	14	14	9	6	5	12	12	2	15	11	15	13	13	11	14	11	12	2	5
2015	15	13	6	9	15	15	14	10	7	6	13	13	2	15	12	15	14	12	12	13	11	13	2	5
2016	15	14	6	9	15	14	15	10	6	5	13	12	3	15	12	15	14	12	12	13	11	13	3	4
2017	16	15	7	9	16	16	16	11	7	6	14	13	3	16	14	16	15	12	13	14	13	14	3	5
2018	23	19	12	11	23	19	21	15	9	7	18	15	4	23	14	22	20	18	16	17	16	18	2	7
2019	20	19	10	10	20	19	20	16	9	7	17	14	4	20	16	20	20	16	15	15	16	17	2	7
2020	25	22	11	14	25	20	22	14	11	11	21	18	9	25	15	25	22	19	19	20	20	21	2	12
2021	21	21	11	10	21	20	20	16	9	10	18	16	9	21	13	20	19	18	18	17	16	17	3	10
2022	30	28	18	12	30	25	26	23	12	17	26	25	13	30	20	29	28	23	22	23	21	24	5	12
Total	361	301	128	233	361	299	342	176	107	108	282	271	56	361	233	351	293	298	226	302	230	242	28	88

(0) US Cash; (1) US TIPS; (2) US Bonds; (3) US Govt Bonds; (4) US Muni Bonds; (5) US Inv Grade Corp Bonds; (6) US High Yield Corp Bonds; (7) Global Bonds (Ex US); (8) Private Debt; (9) US Equities (Large Cap); (10) US Equities (Small Cap); (11) Global Equities (Dev, Ex US); (12) Global Equities (Emerging); (13) Private Equity; (14) REITs; (15) Private Real Estate; (16) Hedge Funds; (17) Commodities; (18) Venture Capital; (19) Infrastructure;

Table IA.2
Average Subjective Beliefs (Pooled Across Institutions in 2022)

This table reports the average values for the belief quantities we observe at the end of 2022 pooled across institutions (we observe the same information in previous years, but for a subset of the asset classes available in 2022). Panel A reports average nominal returns, $\mathbb{E}[R]$, average volatilities, $\sigma[R]$, and average correlations. Panel B reports the same quantities, but for excess returns, $r = R - R_f$, with R_f proxied by the return on *US Cash*. Sections 1.3 and B.1 provide more details about our subjective beliefs data.

PANEL A - Raw Returns (R)

Asset Class	$\mathbb{E}[R]$	$\sigma[R]$	(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
(0) US Cash	3.0%	0.8%	1																			
(1) US TIPS	4.1%	6.1%	0.06	1																		
(2) US Bonds	4.5%	5.1%	0.14	0.73	1																	
(3) US Govt Bonds	4.1%	6.5%	0.16	0.66	0.85	1																
(4) US Muni Bonds	3.7%	4.8%	0.05	0.58	0.77	0.62	1															
(5) US Inv Grade Corp Bonds	5.5%	7.2%	0.03	0.66	0.84	0.60	0.71	1														
(6) US High Yield Corp Bonds	6.8%	9.8%	-0.05	0.35	0.30	0.02	0.36	0.54	1													
(7) Global Bonds (Ex US)	4.0%	7.2%	0.11	0.59	0.68	0.65	0.55	0.66	0.30	1												
(8) Private Debt	8.7%	12.4%	-0.06	0.15	0.06	-0.21	0.11	0.36	0.69	0.14	1											
(9) US Equities (Large Cap)	7.5%	16.5%	-0.03	0.16	0.17	-0.08	0.14	0.36	0.68	0.19	0.59	1										
(10) US Equities (Small Cap)	9.1%	21.0%	-0.04	0.13	0.12	-0.16	0.13	0.34	0.68	0.14	0.58	0.89	1									
(11) Global Equities (Developed, Ex US)	8.6%	17.8%	-0.03	0.16	0.16	-0.09	0.16	0.37	0.66	0.26	0.56	0.83	0.79	1								
(12) Global Equities (Emerging)	10.5%	23.3%	-0.01	0.17	0.15	-0.11	0.15	0.34	0.63	0.20	0.53	0.72	0.70	0.80	1							
(13) Private Equity	10.6%	22.7%	-0.02	0.14	0.07	-0.15	0.07	0.28	0.61	0.16	0.60	0.76	0.73	0.70	0.62	1						
(14) REITs	8.0%	19.8%	-0.04	0.26	0.24	0.04	0.24	0.40	0.63	0.27	0.50	0.70	0.69	0.64	0.56	0.58	1					
(15) Private Real Estate	6.7%	13.4%	0.02	0.17	0.15	0.01	0.08	0.18	0.36	0.12	0.40	0.44	0.43	0.40	0.34	0.48	0.59	1				
(16) Hedge Funds	6.2%	7.8%	0.01	0.18	0.13	-0.16	0.13	0.34	0.65	0.18	0.59	0.74	0.73	0.72	0.68	0.64	0.56	0.39	1			
(17) Commodities	5.8%	17.8%	-0.01	0.17	-0.05	-0.20	-0.04	0.13	0.37	0.08	0.36	0.33	0.35	0.41	0.42	0.35	0.29	0.22	0.45	1		
(18) Venture Capital	12.7%	30.0%	-0.01	0.17	0.08	-0.10	-0.04	0.28	0.62	0.09	0.63	0.78	0.77	0.71	0.62	0.75	0.54	0.36	0.69	0.35	1	
(19) Infrastructure	7.8%	15.9%	-0.03	0.28	0.17	-0.07	0.19	0.34	0.60	0.27	0.55	0.67	0.66	0.67	0.60	0.62	0.63	0.50	0.59	0.39	0.57	1

PANEL B - Excess Returns ($r = R - R_f$)

Asset Class	$\mathbb{E}[r]$	$\sigma[r]$	(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
(0) US Cash	0.0%	0.0%	1																			
(1) US TIPS	1.0%	6.1%	0.00	1																		
(2) US Bonds	1.5%	5.0%	0.00	0.73	1																	
(3) US Govt Bonds	0.9%	6.4%	0.00	0.66	0.84	1																
(4) US Muni Bonds	0.6%	4.8%	0.00	0.59	0.77	0.61	1															
(5) US Inv Grade Corp Bonds	2.4%	7.2%	0.00	0.66	0.84	0.61	0.71	1														
(6) US High Yield Corp Bonds	3.8%	9.9%	0.00	0.36	0.31	0.03	0.37	0.55	1													
(7) Global Bonds (Ex US)	0.8%	7.2%	0.00	0.58	0.67	0.64	0.55	0.66	0.30	1												
(8) Private Debt	5.6%	12.5%	0.00	0.18	0.09	-0.19	0.13	0.37	0.69	0.15	1											
(9) US Equities (Large Cap)	4.5%	16.5%	0.00	0.17	0.18	-0.08	0.15	0.37	0.68	0.19	0.59	1										
(10) US Equities (Small Cap)	5.9%	21.1%	0.00	0.14	0.13	-0.15	0.14	0.34	0.68	0.15	0.58	0.89	1									
(11) Global Equities (Developed, Ex US)	5.6%	17.9%	0.00	0.17	0.17	-0.08	0.17	0.37	0.66	0.26	0.56	0.83	0.79	1								
(12) Global Equities (Emerging)	7.4%	23.3%	0.00	0.17	0.16	-0.11	0.16	0.34	0.63	0.20	0.53	0.72	0.70	0.80	1							
(13) Private Equity	7.5%	22.8%	0.00	0.15	0.08	-0.15	0.07	0.28	0.61	0.16	0.60	0.76	0.73	0.70	0.62	1						
(14) REITs	4.8%	19.8%	0.00	0.27	0.25	0.05	0.24	0.41	0.63	0.27	0.51	0.70	0.69	0.64	0.56	0.58	1					
(15) Private Real Estate	3.6%	13.4%	0.00	0.17	0.15	0.01	0.08	0.18	0.37	0.11	0.40	0.45	0.43	0.40	0.34	0.48	0.59	1				
(16) Hedge Funds	3.0%	7.9%	0.00	0.20	0.14	-0.15	0.13	0.35	0.66	0.18	0.60	0.74	0.73	0.72	0.68	0.64	0.57	0.39	1			
(17) Commodities	2.7%	17.8%	0.00	0.18	-0.04	-0.20	-0.04	0.14	0.37	0.08	0.36	0.34	0.35	0.41	0.42	0.35	0.29	0.22	0.45	1		
(18) Venture Capital	9.6%	30.0%	0.00	0.17	0.08	-0.10	-0.03	0.28	0.62	0.09	0.62	0.78	0.77	0.71	0.62	0.75	0.54	0.36	0.67	0.35	1	
(19) Infrastructure	4.7%	15.9%	0.00	0.29	0.19	-0.05	0.20	0.34	0.61	0.27	0.56	0.67	0.67	0.67	0.60	0.62	0.63	0.50	0.59	0.39	0.57	1

Table IA.3
Constructing our Asset Classes

This table provides the results from our procedure to match the asset classes covered in the paper to the asset classes covered in the CMAs. The procedure is as follows. First, we identify the asset classes in each CMA based on the asset class name used in the CMA report and/or the actual portfolio index stated in the CMA report. Second, we manually map each asset class in each institution-year CMA to an institution-specific asset class name (fixed over time) that reflects the underlying asset class well. Third, we map each institution-specific asset class name to a slightly more general asset class name (which we refer to as the master asset class) that reflects the institution-specific asset class name reasonably well while allowing for small mismatches to accommodate asset classes from different institution under the same master asset class. Fourth, for each CMA, we match each of our paper asset class to the most closely related master asset class available (with the possibility of no match). The first column shows the broad asset classes we cover in the paper. The other columns provide the list of master asset classes that we match to these broad asset classes. They also provide the fraction of institution-year CMAs that have the respective match (within the institution-year CMAs that have some match for the given broad asset class). Sections 1.3 and B.1 provide more details about our subjective beliefs data.

	Master Asset Classes Used (in order)				
Broad Asset Class	**Option #1**	**Option #2**	**Option #3**	**Option #4**	**Option #5**
US Cash	US Cash (95%)	3-Month Libor (4.2%)	US Treasuries ST (0.3%)	US Inflation (0.3%)	
US TIPS	US TIPS (100%)				
US Bonds	US Bonds (78%)	US Bonds Credit (1.5%)	US Core+ Fixed Income (7.9%)	US Core Fixed Income (8.8%)	US Intermediate Bonds (0.6%)
US Govt Bonds	US Treasury Bonds (55%)	US Intermediate Treasury Bonds (28%)	US Long-Term Treasury Bonds (17%)		
US Muni Bonds	US Municipal Bonds (80%)	US Intermediate Municipal Bonds (10%)	US Municipal 1 to 15 Years (9.3%)		
US Inv Grade Corp Bonds	US Inv Grade Corp Bonds (73%)	US Corporate Bonds (16%)	US Intermediate Inv Grade Corp Bonds (2.8%)	US Intermediate Corp Bonds (0.9%)	US Long-Term Corp Bonds (7.4%)
US High Yield Corp Bonds	US High Yield Corp Bonds (100%)				
Global Bonds (Ex US)	Global Ex US Bonds (50%)	Global Ex-US Corp Bonds (1.8%)	Global Ex-US Inv Grade Corp Bonds (0.7%)	Global Developed Ex-US Bonds Unhedged (5.5%)	Global Ex-US Govt Bonds (20%)
Private Debt	US Private Debt (100%)				
US Equities (Large Cap)	US Equities Large Cap (66%)	US Equities (34%)			
US Equities (Small Cap)	US Equities Small Cap (82%)	US Equities Mid & Small Cap (18%)			
Global Equities (Developed, Ex US)	Global Dev Ex US Equities (52%)	Global Ex US Equities (35%)	EAFE Index (9.7%)	Global Dev Equities (0.7%)	Global Equities (2.8%)
Global Equities (Emerging)	Emerging Market Equities (100%)				
Private Equity	US Private Equity (93%)	Global Private Equity (0.3%)	Private Equity Buyouts (5.4%)	Private Equity Venture Capital Combined (1.3%)	
REITs	REITs (92%)	Global REITs (8%)			
Private Real Estate	US Core Real Estate (83%)	Private Real Estate (13%)	Global Real Estate (2.0%)	Global Real Estate Hedged (1.7%)	
Hedge Funds	Hedge Funds (76%)	Equal-Weighted Average of Different Hedge Fund Strategies* (24%)			
Commodities	Commodities (97%)	Commodity Futures (2.9%)			
Venture Capital	Venture Capital (100%)				
Infrastructure	Global Infrastructure (75%)	Private Infrastructure (15%)	Public Infrastructure (10%)		

Broad Asset Class	**Option #6**	**Option #7**	**Option #8**	**Option #9**	**Option #10**
US Cash					
US TIPS					
US Bonds	Long-Term Bonds (3.5%)				
US Govt Bonds					
US Muni Bonds					
US Inv Grade Corp Bonds					
US High Yield Corp Bonds					
Global Bonds (Ex US)	Global Bonds (6.3%)	Global Corp Bonds (0.4%)	Global Govt Bonds (13%)	Global Ex US Govt Bonds Hedged (0.7%)	Global Bonds Hedged (2.6%)
Private Debt					
US Equities (Large Cap)					
US Equities (Small Cap)					
Global Equities (Developed, Ex US)					
Global Equities (Emerging)					
Private Equity					
REITs					
Private Real Estate					
Hedge Funds					
Commodities					
Venture Capital					
Infrastructure					

* For Hedge Funds, we take the average of all available hedge fund strategies when the primary and substitute 1 categories are missing. The hedge fund strategies are (they vary by institution-year observation): "Funds of Funds", "Multi Strategy", "Discretionary", "Open Mandate", "Directional", "Non-Directional", "Event Driven", "Market Neutral", "Relative Value", "Long-Short", "Long Bias", "Macro", "CTA", "Equity Style", "Credit Style Bonds Hedged", "Asymmetric Style", "Equity Hedged", "Convertible", "Moderate Aggregate Risk", "Absolute Returns", "Managed Futures"

Table IA.4
Subjective Belief Components: Disagreement

This table reports the average disagreement (by asset class) for the belief quantities we study. Specifically, for each belief quantity, we calculate the standard deviation across institutions each year and report the time-series mean of these standard deviation values. We consider expected excess returns ($\mu_{j,n,t}$), excess return volatilities ($\sigma_{j,n,t}$), market betas ($\beta^{(m)}_{j,n,t}$), risk premia ($rp_{j,n,t} = \lambda^{(0)}_{j,t} + \lambda^{(m)}_{j,t} \cdot \beta^{(m)}_{j,n,t}$), and alphas ($\alpha_{j,n,t} = \mu_{j,n,t} - rp_{j,n,t}$). For risk premia and alphas, the restricted CAPM imposes $\lambda^{(0)}_{j,t} = 0$ and $\lambda^{(m)}_{j,t} = \mu^{(m)}_{j,t}$ whereas the unrestricted CAPM uses $\lambda^{(0)}_{j,t}$ and $\lambda^{(m)}_{j,t}$ estimated from the regressions in Equation 3. We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Sections 1.3 and B.1 provide more details about our subjective beliefs data.

	Subjective Risk and Return				**Restricted CAPM**				**Unrestricted CAPM**			
Asset Class	μ	σ	$\beta^{(e)}$	$\beta^{(p)}$	$rp^{(e)}$	$rp^{(p)}$	$\alpha^{(e)}$	$\alpha^{(p)}$	$rp^{(e)}$	$rp^{(p)}$	$\alpha^{(e)}$	$\alpha^{(p)}$
US TIPS	0.7%	1.8%	0.05	0.08	0.3%	0.4%	0.7%	0.7%	0.7%	0.7%	0.6%	0.5%
US Bonds	0.6%	1.5%	0.05	0.09	0.3%	0.4%	0.6%	0.6%	0.8%	0.8%	0.7%	0.7%
US Govt Bonds	1.0%	4.1%	0.12	0.17	0.7%	0.9%	0.8%	0.8%	1.0%	1.0%	0.7%	0.6%
US Muni Bonds	0.6%	1.7%	0.06	0.08	0.3%	0.4%	0.6%	0.7%	0.7%	0.7%	0.5%	0.5%
US Inv Grade Corp Bonds	0.7%	1.3%	0.04	0.06	0.3%	0.3%	0.6%	0.6%	0.7%	0.6%	0.5%	0.5%
US High Yield Corp Bonds	1.0%	1.6%	0.07	0.10	0.6%	0.8%	0.8%	0.8%	1.0%	1.0%	0.7%	0.7%
Global Bonds (Ex US)	0.8%	2.3%	0.06	0.10	0.3%	0.4%	0.9%	0.8%	0.8%	0.8%	0.9%	0.9%
Private Debt	1.1%	3.5%	0.15	0.19	1.0%	1.1%	1.1%	1.2%	1.0%	1.0%	1.0%	1.0%
US Equities (Large Cap)	1.0%	1.4%	0.00	0.08	1.0%	1.1%	0.0%	0.5%	1.3%	1.3%	0.7%	0.7%
US Equities (Small Cap)	1.5%	2.2%	0.14	0.15	1.5%	1.5%	0.8%	0.9%	1.6%	1.5%	0.8%	0.8%
Global Equities (Developed, Ex US)	1.3%	1.9%	0.12	0.11	1.1%	1.3%	1.0%	0.6%	1.3%	1.3%	0.7%	0.6%
Global Equities (Emerging)	2.1%	3.3%	0.14	0.19	1.3%	1.4%	2.1%	1.9%	1.4%	1.4%	1.5%	1.4%
Private Equity	2.0%	5.2%	0.29	0.38	2.0%	2.3%	1.7%	1.6%	2.0%	2.1%	1.1%	1.1%
REITs	1.2%	2.9%	0.15	0.20	1.2%	1.4%	0.9%	0.9%	1.3%	1.3%	0.7%	0.7%
Private Real Estate	1.3%	3.7%	0.24	0.36	1.5%	1.9%	1.4%	1.6%	1.4%	1.5%	1.0%	1.1%
Hedge Funds	1.1%	2.4%	0.11	0.15	0.7%	0.8%	1.1%	1.2%	0.8%	0.8%	1.0%	1.0%
Commodities	1.8%	3.8%	0.20	0.28	1.2%	1.4%	1.8%	1.8%	1.1%	1.2%	1.5%	1.5%
Venture Capital	2.8%	5.1%	0.18	0.24	2.0%	2.1%	1.4%	1.1%	2.3%	2.3%	0.7%	0.6%
Infrastructure	0.9%	3.6%	0.22	0.32	1.2%	1.5%	1.1%	1.3%	1.2%	1.2%	0.9%	1.0%

Table IA.5
Fraction of Expected Return Variation Originating from Risk Premia vs Alphas (Unrestricted CAPM)

This table reports the fraction of variability in subjective expected excess returns ($\mu_{j,n,t}$) that is explained by subjective (unrestricted) risk premia ($rp^{(m)}_{j,n,t} = \lambda^{(0)}_{j,t} + \lambda^{(m)}_{j,t} \cdot \beta^{(m)}_{j,n,t}$) and subjective alphas ($\alpha^{(m)}_{j,n,t} = \mu_{j,n,t} - rp^{(m)}_{j,n,t}$) based on the decomposition in Equation 5. We use $\lambda^{(0)}_{j,t}$ and $\lambda^{(m)}_{j,t}$ estimated from the regressions in Equation 3. Panel A focuses on overall variation in μ as well as μ variation across asset classes. Panel B focuses on μ variation across institutions and over time. Different blocks of each panel define the set of broad asset classes included in the analysis. Different columns within each block consider different fixed effects (to focus on different sources of variation). We consider two specifications for βs (see the beginning of Section 2 for details). The first leads to the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second leads to the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocation of US Public Pension Funds. Sections 1.3 and B.1 provide more details about our subjective beliefs data and Section C.1.2 provides more details about the analysis reported in this table.

PANEL A - Overall Variation and Variation Across Asset Classes

	All Asset Classes			Fixed Income			Equities			Real Estate			Alternatives		
	[1]	[2]	[3]	[1F]	[2F]	[3F]	[1E]	[2E]	[3E]	[1R]	[2R]	[3R]	[1A]	[2A]	[3A]
Equity CAPM															
% of $\mathbb{V}ar[\mu]$ due to rp	81%	80%	79%	64%	58%	54%	57%	39%	25%	87%	82%	108%	66%	60%	61%
% of $\mathbb{V}ar[\mu]$ due to α	19%	20%	21%	36%	42%	46%	43%	61%	75%	13%	18%	-8%	34%	40%	39%
Pension CAPM															
% of $\mathbb{V}ar[\mu]$ due to rp	83%	82%	81%	63%	57%	53%	64%	48%	36%	96%	90%	120%	65%	59%	60%
% of $\mathbb{V}ar[\mu]$ due to α	17%	18%	19%	37%	43%	47%	36%	52%	64%	4%	10%	-20%	35%	41%	40%
Year FE		x			x			x			x			x	
Institution FE		x			x			x			x			x	
Year×Institution FE			x			x			x			x			x

PANEL B - Variation Across Institutions and Over Time

	All Asset Classes			Fixed Income			Equities			Real Estate			Alternatives		
	[1]	[2]	[3]	[1F]	[2F]	[3F]	[1E]	[2E]	[3E]	[1R]	[2R]	[3R]	[1A]	[2A]	[3A]
Equity CAPM															
% of $\mathbb{V}ar[\mu]$ due to rp	70%	67%	61%	62%	52%	52%	80%	76%	66%	75%	75%	63%	48%	49%	41%
% of $\mathbb{V}ar[\mu]$ due to α	30%	33%	39%	38%	48%	48%	20%	24%	34%	25%	25%	37%	52%	51%	59%
Pension CAPM															
% of $\mathbb{V}ar[\mu]$ due to rp	72%	70%	64%	62%	53%	52%	82%	78%	68%	82%	81%	68%	50%	51%	43%
% of $\mathbb{V}ar[\mu]$ due to α	28%	30%	36%	38%	47%	48%	18%	22%	32%	18%	19%	32%	50%	49%	57%
Year FE		x			x			x			x			x	
Institution FE			x			x			x			x			x
Asset Class FE	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x

Table IA.6
Predicting Realized Returns and Risk using Subjective Beliefs (December Report Date or Aggregation using a Rolling Window)

This table reports results from the estimation of equations analogous to Equation 11. $\mu_{n,t}$ and $\beta^{(e)}_{n,t}$ are the belief quantities aggregated across institutions, with the aggregation method as described in Subsection 3.1, with Panel A using only CMAs with a production month as of December of year t or earlier and Panel B implementing the aggregation on an expanding window (so that the aggregation procedure is out-of-sample). $\mu^{(1y)}_{n,t}$ is a linear transformations of $\mu_{n,t}$ to reflect 1-year expected returns (as per Equation 12). We consider pooled regressions (Variation=All), regressions with year fixed effects (Variation=Across AC), and regressions with asset class fixed effects (Variation=Over Time). Standard errors rely on double clustering (by year and asset class). $p_{a=0,b=1}$ reflects the p-value for a Wald test of the joint hypothesis that $a = 0$ and $b = 1$. R^2 reflects the pooled R^2 in the Variation=All case and within R^2 values in the other two cases. R^2_{OOS} reflects an out-of-sample R^2 metric, which evaluates the predictability of the given predictor against the predictability of a historical returns benchmark (see Footnote 31). $\widehat{\beta}^{(e)}_{n,t}$ values are calculated from monthly returns within year $t+1$. Sections 1.3 and 3.1 provide more details about the data and Section C.2.3 provides more details about the analysis reported in this table.

PANEL A - Only CMAs with Production Month as of December of year t or Earlier

	$r_{n,t+1} = a + b \cdot \mu_{n,t} + \epsilon_t$			$r_{n,t+1} = a + b \cdot \mu^{(1y)}_{n,t} + \epsilon_t$			$\widehat{\beta}^{(e)}_{n,t+1} = a + b \cdot \widehat{\beta}^{(e)}_{n,t} + \epsilon_t$		
Variation =	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**
$\boldsymbol{a}$	-0.01			0.01			0.03		
$(\boldsymbol{t_{a=0}})$	(-0.37)			(0.27)			(0.65)		
$\boldsymbol{b}$	1.23	1.08	4.22	0.93	0.72	0.93	1.01	1.02	-0.68
$(\boldsymbol{t_{b=0}})$	(2.51)	(2.14)	(1.56)	(2.80)	(2.43)	(2.05)	(15.95)	(15.73)	(-2.74)
$[\boldsymbol{t_{b=1}}]$	[0.47]	[0.16]	[1.19]	[-0.22]	[-0.96]	[-0.16]	[0.15]	[0.31]	[-6.79]
$\{\boldsymbol{p_{a=0,b=1}}\}$	{0.88}			{0.96}			{0.64}		
$\boldsymbol{R^2}$	3.9%	5.1%	4.2%	8.8%	6.3%	7.0%	61.9%	68.4%	6.9%
$\boldsymbol{R^2_{OOS}}$		4.6%	4.2%		9.7%	9.2%		62.0%	-8.2%

PANEL B - Belief Aggregation Using a Rolling Window

	$r_{n,t+1} = a + b \cdot \mu_{n,t} + \epsilon_t$			$r_{n,t+1} = a + b \cdot \mu^{(1y)}_{n,t} + \epsilon_t$			$\widehat{\beta}^{(e)}_{n,t+1} = a + b \cdot \widehat{\beta}^{(e)}_{n,t} + \epsilon_t$		
Variation =	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**
$\boldsymbol{a}$	0.00			0.01			0.04		
$(\boldsymbol{t_{a=0}})$	(-0.27)			(0.52)			(0.63)		
$\boldsymbol{b}$	1.13	1.07	4.90	0.87	0.67	0.87	0.90	0.94	-0.12
$(\boldsymbol{t_{b=0}})$	(2.66)	(2.08)	(2.25)	(3.02)	(2.34)	(1.86)	(16.43)	(16.99)	(-0.66)
$[\boldsymbol{t_{b=1}}]$	[0.30]	[0.14]	[1.79]	[-0.43]	[-1.17]	[-0.28]	[-1.77]	[-1.11]	[-6.16]
$\{\boldsymbol{p_{a=0,b=1}}\}$	{0.94}			{0.85}			{0.20}		
$\boldsymbol{R^2}$	3.7%	5.1%	5.2%	6.1%	4.5%	4.3%	58.4%	64.9%	0.3%
$\boldsymbol{R^2_{OOS}}$		5.2%	5.7%		9.4%	9.8%		57.4%	-22.6%

Table IA.7

Predicting Realized Returns using Subjective Expected Returns (Alternative Specifications)

This table reports results from the estimation of equations analogous to Equation 11 using one-year subjective expected returns, $\mu_{n,t}^{(1y)}$, which is a linear transformation of $\mu_{n,t}$ (see Equation 12). We consider regressions with year fixed effects (Variation=Across AC), and regressions with asset class fixed effects (Variation=Over Time). Standard errors rely on double clustering (by year and asset class). R^2 reflects within R^2 values. R^2_{OOS} reflects an out-of-sample R^2 metric, which evaluates the predictability of the given predictor against the predictability of a historical returns benchmark (see Footnote 31). Different panels reflect different empirical specifications studied in our robustness checks. Sections 1.3 and 3.1 provide more details about the data and Section C.3 provides more details about the analyses reported in this table.

	PANEL A		**PANEL B**		**PANEL C**		**PANEL D**	
	Only Managers		**Only Consultants**		**Only Indirect CMAs**		**Only Direct CMAs**	
Variation =	**Across AC**	**Over Time**	**Across AC**	**Over Time**	**Across AC**	**Over Time**	**Across AC**	**Over Time**
$\boldsymbol{b}$	0.74	0.96	0.58	0.77	0.82	0.87	0.87	1.23
$(\boldsymbol{t_{b=0}})$	(2.16)	(2.20)	(1.82)	(1.61)	(3.03)	(2.81)	(2.64)	(2.01)
$[\boldsymbol{t_{b=1}}]$	[-0.76]	[-0.08]	[-1.33]	[-0.49]	[-0.65]	[-0.41]	[-0.40]	[0.37]
$\boldsymbol{R^2}$	4.6%	5.2%	3.6%	3.4%	6.4%	4.2%	6.2%	6.4%
$\boldsymbol{R^2_{OOS}}$	10.7%	11.0%	11.8%	11.8%	7.7%	6.9%	13.1%	13.6%

	PANEL E		**PANEL F**		**PANEL G**		**PANEL H**	
	Primary AC Available		**Geometric Returns**		**10-Year Horizon**		**Only Requires $\boldsymbol{\mu}$**	
Variation =	**Across AC**	**Over Time**	**Across AC**	**Over Time**	**Across AC**	**Over Time**	**Across AC**	**Over Time**
$\boldsymbol{b}$	0.82	1.18	1.09	1.36	0.78	1.00	0.83	1.11
$(\boldsymbol{t_{b=0}})$	(2.62)	(2.04)	(2.78)	(2.46)	(3.59)	(2.01)	(2.79)	(2.10)
$[\boldsymbol{t_{b=1}}]$	[-0.59]	[0.32]	[0.24]	[0.65]	[-1.04]	[0.00]	[-0.57]	[0.21]
$\boldsymbol{R^2}$	5.9%	6.4%	7.3%	6.4%	9.3%	9.1%	6.6%	6.6%
$\boldsymbol{R^2_{OOS}}$	12.5%	13.0%	11.3%	11.8%	13.8%	14.3%	13.8%	14.3%