Investors' Information Acquisition and the Manager's Value-Risk Tradeoff

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Abstract

This paper studies a model in which investors' information acquisition and the manager's investment choice (hence the moments of the firm's cash flow) are jointly determined. I show that a lower information acquisition cost alters the information environment in a way that motivates the manager to prioritize reducing the variance of cash flow over improving its mean. I present conditions under which a decrease in the cost of information acquisition reduces stock valuations and investors' welfare. The analysis highlights the importance of considering the joint determination of firm risk in studying investors' information acquisition. The model's predictions are relevant to the growing literature that studies technological advancements and regulatory requirements that lower the cost for investors to acquire and process information.

Keywords: Endogenous Firm Risk; Information Acquisition Cost; Value-risk Tradeoff; Welfare

JEL Classifications: D82; G14; M41

1 Introduction

Stock price disciplines managerial decisions, and its disciplinary role depends on price informativeness (e.g., Fishman and Hagerty, 1989; Holmström and Tirole, 1993; Kanodia and Lee, 1998). To the extent that price aggregates investors' private information, a key determinant of price informativeness, and, hence its disciplinary role, is the cost for investors to acquire information. Recent technological and computational advancements have lowered the cost of information acquisition. A growing empirical literature investigates implications of technological advancements (e.g., social media and big data) and regulatory requirements (e.g., EDGAR) that lower the cost for investors to acquire and process information. The evidence suggests that the adoption of these technologies or regulatory changes motivates more information production from market participants, improves price informativeness and market liquidity, and deters managers' opportunistic behaviors (e.g., Blankespoor et al., 2014; Gao and Huang, 2020; Zhu, 2019).[1]

This paper studies investors' information acquisition in a production economy in which their information acquisition and a manager's investment choice (hence the moments of the firm's cash flow) are jointly determined. Consistent with empirical evidence, a decrease in the cost of information acquisition is shown to be associated with more private information production, a more informative price, higher liquidity, and a lower cost of capital. However, I show that these "benefits" can be observed when the firm's underlying cash flow, stock valuation, and the investors' welfare are decreasing in the background. The "benefit" and "cost" are linked to the same underlying cause that managers are pressured by the market to pursue safer investments. That is, a decrease in the cost of acquiring information changes the market information environment in a way that incentivizes the manager to choose investments with low risk and low return.

To elaborate on the model, it consists of a continuum of risk-averse investors (she) and a risk-averse manager (he) who operates a firm. The distribution of the firm's future cash flow is

[1]See Blankespoor et al. (2020) and Goldstein et al. (2019) for a review of the literature.

influenced by the investment that the manager chooses privately. The manager subsequently sells his shareholding in a competitive market. As in Grossman and Stiglitz (1980), each of the investors chooses whether to acquire information at a cost before submitting her trade, and a price is formed to clear the market. In equilibrium, the fraction of investors who choose to become informed and the manager's investment choice are jointly determined. The main friction the model aims to capture is the value-risk tradeoff managers face in operating a firm. That is, projects with a higher expected value are often riskier. In the main model, the value-risk tradeoff is captured by the fact that higher investments increase both the expected cash flow and its variance (as in, say, Kanodia and Lee, 1998 and Kurlat and Veldkamp, 2015).[2] In the Extension Section, I examine an alternative approach to model the value-risk tradeoff, in which the manager allocates his limited attention between increasing the mean and decreasing the variance of the cash flow.

Regardless of which of the two approaches is taken to model the value-risk tradeoff, I show that a lower information acquisition cost incentivizes the manager to prioritize reducing the variance of the cash flow over improving its mean. To understand the intuition, note that the manager controls the cash flow and his payoff is tied to the stock price. When the manager's decision is not directly observed by investors, it is the *sensitivity* of price to the underlying cash flow that determines his perceived benefit to moving the distribution of the cash flow. Specifically, the sensitivity between the first (and second) moments of price and the cash flow is linked to the manager's marginal benefit of increasing the mean of the cash flow (and decreasing the variance of the cash flow, respectively).[3] Both sensitivities will be higher when price is more responsive to its fundamental. In addition, a more responsive stock price increases the second-moment sensitivity faster than the first-moment sensitivity. As a result, the anticipation of a more responsive price (due to a lower information acquisition cost) motivates the manager to prioritize decreasing the variance of the cash flow over improving its mean.

[2] Tsui (2018) documents evidence that firms face a value-risk tradeoff in investment choices.

[3] The sensitivity between the first and second moments of the price p and the cash flow θ is $\frac{dE[p]}{dE[\theta]}$ and $\frac{d\mathrm{var}[p]}{d\mathrm{var}[\theta]}$.

The analysis suggests that investors' information acquisition is prone to a prisoner's dilemma type of coordination failure when the manager faces a tradeoff between improving the mean and decreasing the variance of the firm's cash flow. Each investor behaves optimally. As a group, however, investors fail to internalize the externalities their collective actions impose on the manager, leading to excessive risk avoidance in making investment choices.

I also analyze how a decrease in information acquisition cost affects investors' welfare, which can be decomposed into a speculative component and an endowment-related component. If the distribution of the cash flow were given exogenously, as in Diamond (1985), a decrease in the cost of information acquisition would reduce investors' welfare unless all investors had already become informed.[4] The result suggests that the potential for the investors to benefit from a lower information acquisition cost comes from its influence on the production side, i.e., changing the firm's underlying cash flow distribution. Recall that a lower cost results in a safer cash flow in the model (at the cost of a lower mean). A safer cash flow reduces the speculative payoff because investors earn surplus from bearing risks (e.g., Guttman et al., 2006), and a safer cash flow lowers the risks for which the investors are rewarded. In contrast, a safer cash flow can benefit investors who have a large share endowment to begin with. This is because investors with a large share endowment are expected to be net sellers in the market, and a safer project can raise stock price by lowering the cost of capital. Moreover, investors with a large share endowment are particularly concerned about the price volatility they face when selling those shares. Such an investor benefits from a decrease in information acquisition cost because it motivates the manager to reduce firm risk, which then decreases the price volatility the investor faces ex ante.

Results in the paper speak to the empirical literature examining the effect of technological advancements and regulatory requirements that reduce the cost for investors to acquire and process information. The paper highlights the importance of considering the joint determination of firm risk in studying investors' information acquisition. The conventional wisdom

[4] This result applies to both informed and uninformed investors. As a caveat, the welfare analysis in this paper does not consider noise traders who provide liquidity shocks for exogenous reasons.

that lowering information acquisition cost is beneficial is likely to be intact if the manager can only affect the mean of the cash flow but not its variance. In contrast, caution is called for if researchers believe that managers can also reduce the variance of the cash flow and faces a tradeoff between increasing the mean and decreasing the variance of the cash flow. In this case, observing an increase in price informativeness or liquidity and a lower cost of capital may be evidence that managers have been pressured to undertake low-risk and low-return investments. The concern is particularly relevant for innovation-related investments because these investments, especially at early stages, involve an inherent risk-and-return tradeoff and are often not observed by external parties due to proprietary cost concerns. To the extent that basic research is riskier than applied applications in terms of their cash flow implications, the model provides a rationale for Arora et al. (2021), who document a shift in corporate R&D composition towards less "R" and more "D" over time.

This paper relates to the broad literature that studies the role of market price in providing managerial incentives. Prior studies have examined how the disciplinary or detrimental role of price depends on *observable* factors, such as the number of blockholders (e.g., Edmans and Manso, 2011), frequency of disclosure (e.g., Gigler et al., 2014), and the quality of disclosure (e.g., Fishman and Hagerty, 1989). In contrast, the incentive effect of price in this paper is determined jointly with the manager's risk choice, which is unobservable to the market. The unobservable nature enables me to capture managers' behind-the-scenes project selection and internal R&D. Heinle and Smith (2017) examine a model in which investors are uncertain about the variance of the cash flow. In the current paper, the equilibrium is in pure strategies and, therefore, rational expectations imply that investors correctly conjecture the manager's choice of firm risk in equilibrium.

The fact that managerial incentives are tied to short-term price reminds readers of studies on managerial myopia. Stein (1989) shows that, when the market does not observe a manager's actions, price pressure can cause the manager to behave myopically even though the market correctly conjectures the myopic behavior. Gigler et al. (2014) show more frequent reporting requirement increases the price pressure and exacerbates managerial myopia. Paul (1992) argues that the manager will allocate more resources to improving short-term performance if the market has superior information about the short-term performance than the long-term performance. In this paper, the tradeoff is not about short- versus long-term cash flow but between improving the mean and decreasing the variance of the cash flow. A more subtle difference is that capital market in prior studies is passive: it does not take any actions other than pricing the firm rationally. Hence, inefficiencies in myopia models are typically caused by the *manager's* incentive to "fool" investors by secretly changing actions (even though no one is fooled in equilibrium).[5] This paper brings investors to the center of the analysis, models their information acquisition choices, and shows their collective actions directly contribute to the inefficient value-risk tradeoff in equilibrium.

The paper's focus on endogenous firm risk relates to studies on induced managerial risk avoidance. Prior studies show that career concerns from the labor market can cause the manager to be more averse to risky investment than the principal (e.g., Hirshleifer and Suh, 1992; Hirshleifer and Thakor, 1992; Holmstrom and Costa, 1986). Another source of managerial risk avoidance stems from motivating costly effort in principal-agent models: managers must carry risks to be motivated, which makes the managers value risk differently than the risk-neutral principal does. Lambert (1986) studies such a model in which a manager exerts unobservable effort to generate information about a risky project and, conditional on the information he generates, chooses between the risky project and a safe fallback project. In these agency models, a manager's risk preference is determined by a combination of his (exogenous) risk aversion and the (endogenous) curvature of the incentive contract designed by the principal. There is no career concern or compensation design in this model. The manager's risk preference arises from a combination of his (exogenous) risk aversion and the investors' (endogenous) information acquisition and the pricing of their information. Compared to the incentive contract

[5]Stein (1989) makes this point and writes: "The situation is analogous to the prisoner's dilemma. The preferred cooperative equilibrium would involve no myopia on the part of managers, and no conjecture of myopia by the stock market. Unfortunately, this cannot be sustained as a Nash equilibrium. If the market conjectures no myopia, managers will have an incentive to fool it by boosting current earnings."

that the principal chooses publicly before the manager exerts effort, investors' decisions in the paper are decentralized and chosen simultaneously as the manager makes the investment choice. The focus is to examine the joint determination of investors' information acquisition and the manager's investment choice.

The paper proceeds as follows. Section 2 presents the model. Section 3 solves the equilibrium and presents its properties. Section 4 analyzes investors' welfare, and Section 5 discusses the model's empirical implications. Section 6 examines an alternative modeling choice, along with an extension on the role of public disclosure. Section 7 concludes.

2 Model Setup

The model consists of a firm, a manager, and a continuum of investors. The manager operates the firm and influences the distribution of the firm value

$$v = \theta + e. \tag{1}$$

The cash flow $\theta \sim N(\mu_\theta, \sigma_\theta^2)$ in (1) is normally distributed, and its distribution is affected by the manager's operational decisions (to be discussed below). The shock $e \sim N(0, \sigma_e^2)$ is normally distributed with an exogenous variance σ_e^2.

The main friction the paper aims to capture is the value-risk tradeoff managers face in operating a firm. That is, projects with a higher expected value are often riskier in nature. To capture the tradeoff simply, I assume in the main model that the manager chooses an investment k, and the value-risk tradeoff is captured by the fact that higher investments increase both the mean and variance of the firm's future cash flow (e.g., Kanodia and Lee, 1998; Kurlat and Veldkamp, 2015). In particular, the expected cash inflow from investing $k \geq 0$ is $k\mu$ and the variance of the cash flow is $k^2\Sigma$, where μ and Σ are positive parameters.

The cost of the investment is $\frac{k^2}{2}$. Therefore, the distribution of the future cash flow is

$$\theta \sim N(k\mu - \frac{k^2}{2},\ k^2\Sigma). \tag{2}$$

Examples of such investments include companies investing in basic research and developing a new class of drugs as opposed to working on variations of current drugs. These investments, if successful, improve firm value, but their experimental nature means an uncertain outcome. Assuming a quadratic $\text{var}(\theta) = k^2\Sigma$ in (2) is not crucial. The analysis can be extended to other specifications of $\text{var}(\theta)$ as long as it is increasing in k and its second derivative is not too negative, as will be shown in Proposition 2.

The value-risk tradeoff the paper aims to capture is behind the scenes, meaning that the investors do not directly observe firm's investment choice. This assumption does not hold, of course, for all investments. Building a physical plant is often observed by investors either by literally seeing the building or by it being capitalized as an asset in financial reports. Akcigit et al. (2022) argue that many innovation-related investments are unobservable. Examples include the utilization rate of equipment, managerial input, process improvements, and internal R&D activities. For example, consider investments in basic research and drug development discussed in the previous paragraph. Such R&D investments are reported as expenses under the U.S. GAAP prior to technological feasibility, and expenditures across different projects are often pooled together (see SFAS No. 2 and ASC Topic 730), making it difficult for investors to identify investments related to a specific project. Koh and Reeb (2015) further document evidence suggesting that many firms with material R&D activities lump R&D expenses with other operating expenses in financial reports and, as a result, "fail to provide any information regarding their corporate R&D efforts." The lack of quality information on innovation-related activities is at least in part due to proprietary cost concerns.

The firm is traded in the market, and its stock price p is determined in a competitive market à la Grossman and Stiglitz (1980). There is a continuum of risk-averse investors, indexed by $i \in [0, 1]$, and a risk-free asset that serves as the numeraire. Investors have constant absolute risk averse utility functions with a common risk-aversion parameter $\rho > 0$. Investor i is endowed with s_i shares, where s_i is drawn independently from a uniform distribution on the interval $[0, 2S]$ for a finite $S \geq 0$. I follow the convention that a law of large numbers holds for the continuum economy, so that $\int_0^1 s_i d_i = S$ is a constant.[6] Each investor i chooses whether to pay an information acquisition/processing cost F to learn the realization of θ prior to trading. I call those who choose to learn θ *informed* investors and those who choose not to learn θ *uninformed* investors. Denote by λ the fraction of informed investors in equilibrium. Uninformed investors only observe price p, and informed investors learn θ in addition to p. Let $d^I(p)$ and $d^U(p)$ be the demand of shares from the informed and uninformed investors, respectively. Price p is determined so that the market clears. That is,

$$\lambda d^I(p) + (1-\lambda) d^U(p) = S + \epsilon, \tag{3}$$

where $\epsilon \sim N(0, \sigma_\epsilon^2)$ is the asset supply from noise traders who trade for exogenous reasons. The supply shock ϵ prevents the price from being fully revealing.

At the beginning of the game, the manager chooses an investment $k \geq 0$ to maximize his expected payoff

$$U^M = E[p] - \rho_M \text{var}(p), \tag{4}$$

where $\rho_M \geq 0$ in (4) is the manager's risk aversion parameter. One can think of the preference above as assuming that the manager has a negative exponential utility $-\exp(-\rho_M \times w)$ and his wealth, w, is the proceeds from selling his shareholding (normalized to one) at price p.[7] The model takes the manager's shareholding as exogenous. In particular, the manager's investment choice affects the firm's cash flow (hence, its stock price) but not the number of shares the manager owns. Empirically, this assumption is likely to be descriptive for long-term executives who have already earned significant shares and for executives who anticipate receiving a fixed

[6]See the technical appendix in Vives, 2010, p.383-385. The argument is not unique to uniform distributions and holds for a continuum of independent random variables with uniformly bounded variances.

[7]The manager's certainty equivalent, $E[p] - \frac{\rho_M}{2}\text{var}(p)$, is qualitatively similar to (4).

number of shares in the near future (e.g., under time-based stock awards).[8] The focus of the paper is how a change in the cost of investors' information acquisition would jointly affect their information choice and the manager's investment decision. I discuss how incentive contracts might evolve in the Conclusion Section.

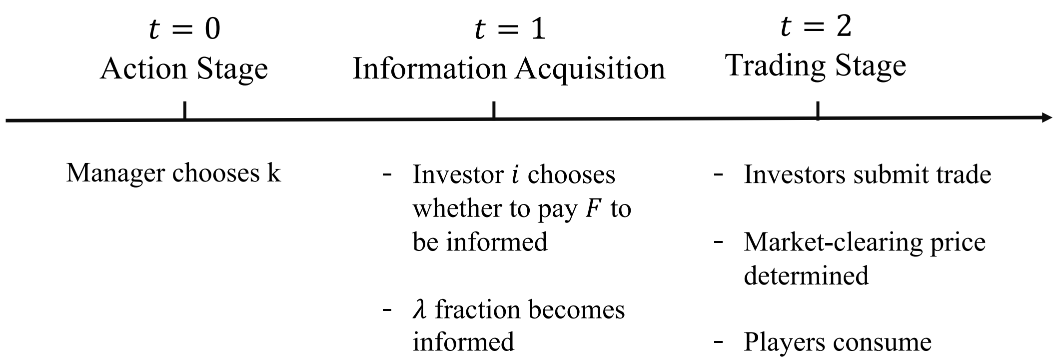

Figure 1: Sequence of Events

Figure 1 summarizes the timeline of the main model. In the Extension Section, I study an alternative approach to model the value-risk tradeoff, in which the manager allocates limited attention between increasing the mean and decreasing the variance of the firm's cash flow. The alternative setup comports with the literature showing how the allocation of managerial attention can explain differences in firm behaviors (e.g., Dessein and Santos, 2021). I find qualitatively similar results regardless of which of the two approaches is taken to model the value-risk tradeoff the manager faces.

3 Equilibrium Analysis

The joint determination of the manager's risk choice and investors' information acquisition is at the center of the analysis. Because investors do not directly observe the investment choice,

[8]Time-based stock awards are based on the passage of time. Bettis et al. (2018) find that more firms granted time-based awards than performance-based awards in each of their sample periods from 1998 to 2012.

they form a conjecture $\hat{k}$ and use the conjectured prior precision $\hat{\tau}_\theta = (\hat{k}^2\Sigma)^{-1}$ in deciding whether to acquire information and how to trade. The investors' information acquisition decisions affect how price is formed and, in turn, the value-risk tradeoff perceived by the manager in making investments in the first place. Therefore, we cannot analyze investors' information choices and the manager's investment separately, but determine their actions jointly from a rational expectations equilibrium.

The equilibrium is solved backwards. I first take the investors' conjectured $\hat{k}$ as given and solve for the trading subgame, and then endogenize the manager's choice of k at $t = 0$. Note that the distribution of firm value is "known" to investors once the conjectured $\hat{k}$ is given, and solving the subsequent trading game is a standard exercise (e.g., Grossman and Stiglitz, 1980). In particular, for any given conjecture $\hat{k}$, there exists a unique linear pricing function as follows that clears the market

$$p = \hat{A} \times \mu_\theta(\hat{k}) + \hat{B} \times \theta - \hat{C} \times \epsilon - \hat{D} \times S. \tag{5}$$

The nonnegative price coefficients are $\hat{A} = \frac{(1-\lambda)\hat{\tau}_\theta}{\lambda\tau_e+\hat{\tau}_\theta+\tau_p}$, $\hat{B} = \frac{\tau_p+\lambda(\tau_e+\hat{\tau}_\theta)}{\lambda\tau_e+\hat{\tau}_\theta+\tau_p}$, $\hat{C} = \frac{\rho(\tau_e+\hat{\tau}_\theta+\tau_p/\lambda)}{\tau_e(\lambda\tau_e+\hat{\tau}_\theta+\tau_p)}$, and $\hat{D} = \frac{\rho(\tau_e+\hat{\tau}_\theta+\tau_p)}{\tau_e(\lambda\tau_e+\hat{\tau}_\theta+\tau_p)}$, where $\hat{\tau}_\theta$ is the conjectured prior precision and $\tau_p = (\lambda\tau_e/\rho)^2\tau_\epsilon$ is the precision of price used as a signal of θ. The "hat" notations emphasize their dependence on the conjectured $\hat{k}$ as opposed to the actual k, which is unobservable to the investors. The last term in (5) is the price discount investors demand for clearing the average supply $S \geq 0$. It is a known result that all investors will choose to be informed if the cost F is sufficiently small and that no one will be informed if F is sufficiently large. That is, $\lambda = 1$ for $F < \underline{F}$ and $\lambda = 0$ for $F > \overline{F}$. (The thresholds $\underline{F}$ and $\bar{F}$ are specified later.) For intermediate cost $F \in [\underline{F}, \overline{F}]$, λ is determined by the indifference condition

$$\exp(\rho * F) = \sqrt{1 + \frac{\tau_e}{(\lambda\tau_e/\rho)^2\tau_\epsilon + \hat{\tau}_\theta}}. \tag{6}$$

The manager takes the investors' conjecture $\hat{k}$ and, hence, the price function (5) as given and chooses the best-response k^{BR} at $t = 0$ to maximize his payoff. That is,

$$k^{BR} = \arg\max_{k} \; E[p|\hat{k}, k] - \rho_M \text{var}(p|\hat{k}, k). \tag{7}$$

Let $\mu_\theta(k) = E[\theta|k]$ and $\sigma^2_\theta(k) = \text{var}(\theta|k)$. One can use the price function (5) to obtain $E[p|\hat{k}, k] = \hat{A}\,\mu_\theta(\hat{k}) + \hat{B}\mu_\theta(k) - \hat{D}S$ and $\text{var}(p|\hat{k}, k) = \hat{B}^2\sigma^2_\theta(k) + \hat{C}^2\sigma^2_\epsilon$. The first-order condition of the investment choice is

$$\hat{B} \times \frac{d\mu_\theta}{dk} = \rho_M \hat{B}^2 \times \frac{d\sigma^2_\theta}{dk}. \tag{8}$$

The left-hand side of (8) is the manager's marginal benefit of investment in increasing expected market price. The marginal benefit depends both on how a bigger investment changes the mean cash flow (i.e., $\frac{d\mu_\theta}{dk}$) and on how fast the mean cash flow is picked up by the stock price (i.e., $\frac{dE[p]}{d\mu_\theta} = \hat{B}$). To understand the marginal benefit, note that the investors always place some positive weight on the conjectured prior mean $\mu_\theta(\hat{k})$ in pricing the firm. Because the manager takes the investors' conjecture $\hat{k}$ as given and cannot change it, the price is formed in a way that is only partially responsive to the manager's investment k. This can be seen by noting that a higher k increases the expected price at a lower rate than it increases the expected cash flow, i.e., $\frac{dE[p]}{dk} = \hat{B}\frac{d\mu_\theta}{dk} < \frac{d\mu_\theta}{dk}$. The price coefficient $\hat{B}$ is the sensitivity of price to the underlying cash flow, and it captures the extent to which the manager internalizes the effect of investment k on the mean cash flow.[9]

The right-hand side of (8) is the manager's perceived marginal cost of a higher investment in scaling up price volatility. The marginal cost depends on how fast higher investments scale up variance of the cash flow (i.e., $\frac{d\sigma^2_\theta}{dk}$) as well as how fast the variance of cash flow is transmitted into price volatility (i.e., $\frac{d\text{var}[p]}{d\sigma^2_\theta} = \hat{B}^2$). Note that $\hat{B}^2$ is the second-moment sensitivity of price to fundamental. A higher pricing coefficient $\hat{B}$ means that the price is synchronized more closely to its fundamental value θ, making the price more volatile from the ex ante point of view. The finding is consistent with the well-known result that better information reduces

[9]See Kanodia and Lee (1998) and Xue and Zheng (2021) for a similar argument.

uncertainty ex post but increases the uncertainty ex ante (e.g., Christensen et al., 2010; Gao, 2010; Holthausen and Verrecchia, 1988).

A novel feature of the model is the endogeneity of firm risk, σ_θ^2. To see its significance, consider a benchmark in which the variance of the cash flow, σ_θ^2, is exogenous and the investment only changes the mean cash flow. When σ_θ^2 is exogenous, the first-order condition (8) becomes $\hat{B} \times (\mu - k) = 0$, and the manager chooses $k = \mu$ regardless of the investors' information acquisition (given $\hat{B} > 0$). The takeaway here is that the manager's investment k and investors' information choice λ are determined *separately* when σ_θ^2 is exogenous.[10] With endogenous firm risk, however, investment k and investors' information acquisition λ are intertwined and must be determined jointly. Proposition 1 below summarizes the equilibrium.

Proposition 1 *Investment k and the fraction λ of informed investor are jointly determined.*

i. for $F \leq \underline{F} \equiv \frac{1}{\rho} \log(\sqrt{1 + \frac{\mu^2 \tau_e \Sigma}{(1+2\rho_M \Sigma)^2 + (\mu \tau_e / \rho)^2 \tau_\epsilon \Sigma}})$, $\lambda = 1$ and $k = \frac{\mu}{1+2\rho_M \Sigma}$.

ii. for $F \in (\underline{F}, \overline{F})$, $\lambda \in (0,1)$ and $k \leq \mu$ are uniquely characterized from the system

$$\begin{aligned} \mu &= \left[1 + 2\rho_M \Sigma \frac{\tau_p + \lambda\left(\tau_e + \sigma_\theta^{-2}(k)\right)}{\lambda \tau_e + \tau_p + \sigma_\theta^{-2}(k)}\right] k, \\ \exp(\rho F) &= \sqrt{1 + \frac{\tau_e}{(\lambda \tau_e / \rho)^2 \tau_\epsilon + \sigma_\theta^{-2}(k)}}. \end{aligned} \tag{9}$$

iii. for $F \geq \overline{F} \equiv \frac{1}{\rho} \log(\sqrt{1 + \mu^2 \tau_e \Sigma})$, $\lambda = 0$. Any $k \leq \mu$ can be sustained as an equilibrium.

The equilibrium is unique whenever the price coefficient $B > 0$, which holds if and only if $F < \overline{F}$. For $F \geq \overline{F}$, the equilibrium features $B = 0$ and can be thought of as a "babbling equilibrium" in which stock price does *not* respond to different realizations of firm value. This is because, when $B = 0$, the price function (5) depends on the conjectured $\hat{k}$ but not the actual k. For $F \geq \overline{F}$ (i.e., Part iii of the proposition), any conjecture $\hat{k} \leq \mu$ will result in $\lambda = 0$

[10] This is because the benefit of becoming an informed investor (i.e., the right-hand side of (6)) is fixed when σ_θ^2 is exogenous. In this case, the investors' information acquisition λ is determined solely by the cost F and is independent of k.

(hence $B = 0$) and can be sustained as an equilibrium.[11] The discussion in the remainder of the paper confines attention to $F < \overline{F} = \frac{1}{\rho}\log(\sqrt{1+\mu^2\tau_e\Sigma})$, which ensures the fraction of informed investors $\lambda > 0$ and the uniqueness of the equilibrium.

Figure 2 illustrates the equilibrium (λ, k) using a numerical example in which $\mu = 4, \rho = \rho_M = \Sigma = \sigma_\epsilon = \sigma_e = 1$ and $S = 0.5$. It is intuitive to see that more investors choose to become informed when their information acquisition cost decreases: λ increases from 0 to 1 when F is reduced from $\overline{F} = 1.417$ to $\underline{F} = 0.248$. The interesting result in Figure 2 is that a lower F incentivizes the firm to invest less, reducing both the mean and variance of the cash flow. That is, a decrease in the investors' information acquisition cost changes the firm's information environment in a way that incentivizes the manager to prioritize reducing the variance of future cash flow over improving its mean.

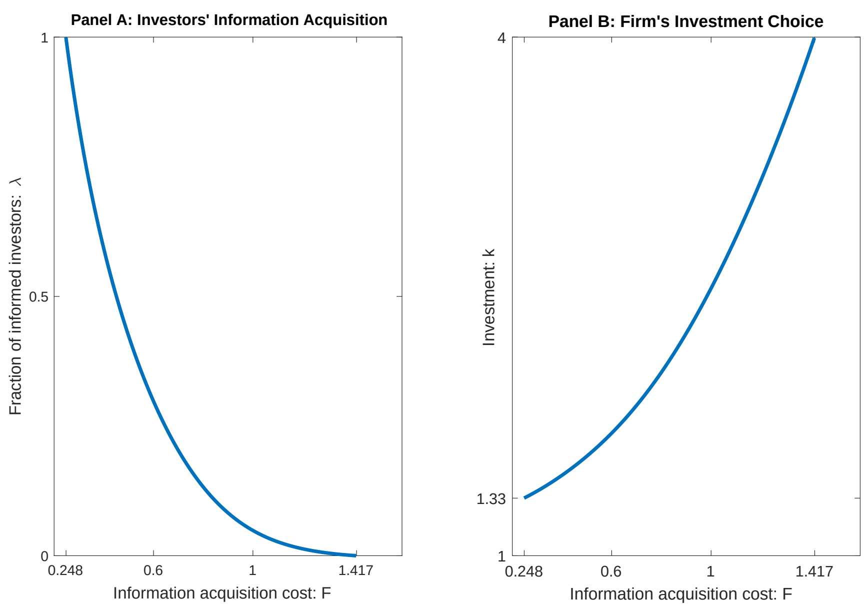

Figure 2: Illustration of Equilibrium

Why would a decrease in the investors' information acquisition/processing cost reduce investment k in equilibrium? I illustrate the intuition using a heuristic belief updating process. Suppose the investors naively believe that the manager's investment (hence the variance of

[11] If one introduces a public disclosure about the firm value, as in Section 6.1, the equilibrium k will be unique even if $\lambda = 0$ because the price coefficient $B > 0$ is guaranteed.

cash flow σ_θ^2) is unaffected by a lower information acquisition cost F. This means the value of information (i.e., the right-hand side of Equation 6) does not change, and a lower cost F would unambiguously motivate more investors to become informed. Holding σ_θ^2 constant, having more informed investors results in a more responsive price, i.e., a higher $\hat{B}$ in (5). The investors then realize that their initial belief of facing the same distribution of cash flow is incorrect because a higher $\hat{B}$ affects the marginal benefit and cost of investment differently. Specifically, Equation (8) shows that a more responsive price scales up the marginal benefit linearly and the marginal cost quadratically, as captured by $\frac{dE[p]}{d\mu_\theta} = \hat{B}$ and $\frac{d\text{var}[p]}{d\sigma_\theta^2} = \hat{B}^2$. Because of the differential sensitivities, the anticipation of a more responsive price incentivizes the manager to prioritize reducing the variance of the cash flow over improving its mean. This relative advantage can also be seen by dividing both sides of the first-order condition (8) by $\hat{B}$ and obtaining $\mu - k = \rho_M \hat{B} \frac{d\sigma_\theta^2}{dk}$, from which we know the following:

$$\frac{dk}{d\hat{B}} = -\frac{\rho_M \frac{d\text{var}(\theta)}{dk}}{1 + \rho_M \hat{B} \frac{d^2\text{var}(\theta)}{dk^2}} < 0. \tag{10}$$

It remains to verify the manager's belief that a lower F will indeed result in a more responsive price in equilibrium, i.e., a higher B. This claim is not as intuitive as it may appear because of the firm's endogenous risk. Note that $B = \frac{\tau_p + \lambda(\tau_e + \tau_\theta)}{\lambda\tau_e + \tau_\theta + \tau_p}$ is determined both by the fraction of informed investors λ and by the endogenous investment k via its impact on the firm risk. Recall that a lower cost F results in more informed investors and a safer cash flow, i.e., a higher λ and $\tau_\theta = 1/\sigma_\theta^2$. It shows below that a decrease in F has two countervailing effects on price responsiveness, B:

$$\frac{dB}{dF} = \underbrace{\frac{\partial B}{\partial \lambda}\frac{d\lambda}{dF}}_{-} + \underbrace{\frac{\partial B}{\partial \tau_\theta}\frac{\partial \tau_\theta}{\partial k}\frac{dk}{dF}}_{+}. \tag{11}$$

Price will become more responsive if more investors are informed (i.e., $\frac{\partial B}{\partial \lambda} > 0$) but less responsive if investors are facing a safer cash flow (i.e., $\frac{\partial B}{\partial \tau_\theta} < 0$). To understand the second effect, recall that investors always attach some weight to the conjectured prior cash flow, $E[\theta|\hat{k}]$, in pricing the firm. Having a safer cash flow means that the investors' prior beliefs are more precise. The investors therefore place more weight on their prior beliefs and less weight on signals about the actual cash flow. This increased reliance on prior beliefs lowers price responsiveness. I show in the appendix that the negative λ-effect in (11) dominates the other risk effect, meaning that a lower F results in a higher B in equilibrium.

Formally, one can apply the implicit function theorem to the system of equations used to determine the equilibrium investment k and the fraction λ of informed investors. Denote by $[g_1(k, \lambda; F) = 0, g_2(k, \lambda; F) = 0]$ the two equations in (9). The marginal effects of the investors' information acquisition cost F on the equilibrium k and λ are obtained from

$$\begin{bmatrix} \frac{dk}{dF} \\ \frac{d\lambda}{dF} \end{bmatrix} = - \begin{bmatrix} \frac{\partial g_1}{\partial k} & \frac{\partial g_1}{\partial \lambda} \\ \frac{\partial g_2}{\partial k} & \frac{\partial g_2}{\partial \lambda} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial g_1}{\partial F} \\ \frac{\partial g_2}{\partial F} \end{bmatrix}. \tag{12}$$

While the paper assumes $\sigma_\theta^2 = k^2\Sigma$, the proposition below shows that qualitatively similar comparative statics $\frac{dk}{dF} > 0$ and $\frac{d\lambda}{dF} < 0$ hold over a broad range of specifications of the variance of the cash flow σ_θ^2.

Proposition 2 *A lower F stimulates more informed investors but decreases investment k as long as the variance of the cash flow satisfies $\frac{d\mathrm{var}(\theta)}{dk} > 0$ and $\frac{d^2\mathrm{var}(\theta)}{dk^2} \geq -\frac{1}{\rho_M}$.*

Given the mean cash flow $\mu_\theta = \mu k$, one can extend the analysis and allow k to increase the variance of cash flow in a convex, linear, or even mildly concave manner. The paper follows Kanodia and Lee (1998) and adopts a quadratic specification, $\sigma_\theta^2 = k^2\Sigma$. A specific functional form is needed to obtain the closed-form expressions of $\frac{dk}{dF}$ and $\frac{d\lambda}{dF}$ (according to Equation 12), which will be used to study implications of investors' welfare in the next section.

4 Investors' Welfare

Ranking different investments is not straightforward in the paper because a higher investment increases both the mean and variance of the cash flow. I follow Verrecchia (2019) and study implications on investors' welfare in this section. In particular, I examine how a decrease in the cost of information acquisition, F, would affect the payoffs of the continuum of investors.

It is without loss of generality to analyze the welfare from an *uninformed* investor's point of view because, in any equilibrium with $\lambda \in (0,1)$, investors are indifferent between paying the cost to become informed and staying uninformed. An uninformed investor i's demand given a market price p is $d = \frac{E(v|p)-p}{\rho\,\mathrm{var}(v|p)}$. Therefore, investor i's wealth for a given price is $\frac{E(v|p)-p}{\rho\,\mathrm{var}(v|p)} \times (v-p) + p\,s_i$, where s_i is the investor's endowment of the firm's shares. We can then express i's *interim* welfare $E(U|p)$ after observing price p as

$$E(U|p) = -\exp\{-(ps_i + \frac{(E(v|p)-p)^2}{2\,\mathrm{var}(v|p)})\}. \tag{13}$$

Ex ante, price p is a random variable. We integrate (13) over p and apply the moment generating function a chi-squared distribution to obtain the investors' ex ante payoff $E[U] = E[E[U|p]]$. The next result summarizes the closed-form expression of an investor i's ex ante payoff.

Proposition 3 *The expected payoff of an investor endowed with s_i shares is*

$$E[U] = -\left(\sqrt{1 + \frac{\mathrm{var}[E(v|p)-p]}{\mathrm{var}(v|p)}}\right)^{-1} \exp[-\left(s_i\Psi + S^2D^2\,\frac{\Phi}{2} - s_i^2\Delta\right)], \tag{14}$$

where S is the average supply, $\Psi = \rho\mu_\theta - SD^2\Phi$, $\Phi = \frac{1}{\mathrm{var}(v|p)+\mathrm{var}[E(v|p)-p]}$, and $\Delta = \rho^2B^2\sigma_\theta^2\frac{(\tau_e+\tau_\theta+\tau_p)\Phi}{2\tau_e\tau_p}$.

The payoff $E[U]$ measures the welfare of both the informed and uninformed investors in the model. Note, however, $E[U]$ is not a social-welfare measure because it does not incorporate the welfare of the unmodeled noise traders, who supply $\epsilon \sim N(0, \sigma_\epsilon^2)$ for exogenous reasons outside the model.

To understand how a decrease in the information acquisition cost affects investors' welfare, it is helpful to express $E[U]$ in terms of a *speculative component* $-\left(\sqrt{1+\frac{\mathrm{var}[E(v|p)-p]}{\mathrm{var}(v|p)}}\right)^{-1}$ and an *endowment-related component* $\exp[-\left(s_i\Psi + S^2D^2\frac{\Phi}{2} - s_i^2\Delta\right)]$, as in (14). The *speculative component* is the payoff that an investor expects to earn by clearing the supply shock ϵ and its expression is independent of the investor's endowment s_i or the average supply S. The *endowment-related component* relates to an investor selling her share endowment s_i in the market when the average asset supply is $S \geq 0$. Below, I analyze how a lower F affects the two components of the welfare separately.

The *speculative component* $-\left(\sqrt{1+\frac{\mathrm{var}[E(v|p)-p]}{\mathrm{var}(v|p)}}\right)^{-1}$ in (14) has a one-to-one increasing relation to $\mathbf{R} \doteq \frac{\mathrm{var}[E(v|p)-p]}{var(v|p)}$. The ratio $\mathbf{R}$ makes it clear that investors prefer a lower residual uncertainty $\mathrm{var}(v|p)$ ex post, but benefit from having a higher *ex ante* volatility $\mathrm{var}[E(v|p)-p]$. One can use the price function derived in (5) and show $\mathbf{R} \doteq \frac{\mathrm{var}[E(v|p)-p]}{var(v|p)} = \frac{\rho^2\tau_\theta(\tau_\theta+\tau_p+\tau_e)}{\tau_e\tau_\epsilon(\tau_\theta+\tau_p+\lambda\tau_e)^2}$. It is straightforward to verify the following after substituting $\frac{d\lambda}{dF} < 0$ and $\frac{dk}{dF} > 0$ from (12):

$$\frac{d\mathbf{R}}{dF} = \frac{\partial\mathbf{R}}{\partial\lambda}\frac{d\lambda}{dF} + \frac{\partial\mathbf{R}}{\partial\tau_\theta}\frac{\partial\tau_\theta}{\partial k}\frac{dk}{dF} > 0. \tag{15}$$

The fact $\frac{d\mathbf{R}}{dF} > 0$ means that a lower cost F decreases the speculative component of the welfare (14). To understand the result, note that a lower cost results in more informed traders and hence a more informative price, which serves as a public signal for investors. It follows from prior theoretical work that the presence of a more precise public signal can make investors worse off by reducing uncertainty prior to trading and, hence, creates an adverse effect on risk sharing (e.g., Diamond, 1985; Indjejikian, 1991; Hirshleifer, 1971). Secondly, the investors earn surplus from bearing risks (e.g., Bertomeu and Cheynel, 2016; Kurlat and Veldkamp, 2015). It is therefore not surprising that a lower cost can hurt investors when it pressures the manager to reduce firm risk for which the investors are rewarded.

Having shown that a lower F reduces the *speculative component* in (14), we know that any potential for investors to benefit from a lower cost F must arise from the *endowment-related* *component* $\exp[-\big(s_i\Psi + S^2D^2\,\frac{\Phi}{2} - s_i^2\Delta\big)]$, which can be rewritten as

$$\exp\{-\underbrace{\left[s_i\rho\mu_\theta + SD^2(\frac{S}{2} - s_i)\Phi - s_i^2\rho^2B^2\sigma_\theta^2\frac{(\tau_e+\tau_\theta+\tau_p)}{2\tau_e\tau_p}\Phi\right]}_{\Gamma}\}.$$

Investor welfare (14) is increasing in Γ, which consists of the terms inside the square bracket above. We see below that a decrease in F affects Γ (and, hence, investor welfare) via three channels:

$$\frac{d\Gamma}{dF} = \underbrace{\frac{\partial\Gamma}{\partial\mu_\theta}\frac{d\mu_\theta}{dk}\frac{dk}{dF}}_{\text{Mean Effect }(+)} + \underbrace{\frac{\partial\Gamma}{\partial\lambda}\frac{d\lambda}{dF}}_{\text{Information environment Effect }(+)} + \underbrace{\frac{\partial\Gamma}{\partial\tau_\theta}\frac{d\tau_\theta}{dk}\frac{dk}{dF}}_{\text{Firm Risk Effect }(+/-)}. \tag{16}$$

An increase in the mean cash flow μ_θ makes the investor's share endowment $s_i \geq 0$ worth more. It is therefore easy to see $\frac{\partial\Gamma}{\partial\mu_\theta} \geq 0$. Because a decrease in F is shown in Proposition 2 to lower the mean cash flow μ_θ (by decreasing k), it lowers the endowment-related welfare via the Mean Effect in (16).

A decrease in F also lowers investor welfare via the "Information environment Effect" in (16) by motivating more investors to become informed. This result is less intuitive and follows $\frac{\partial\Gamma}{\partial\lambda} \leq 0$. To see why a higher λ would decrease the endowment-related payoff, note that a higher λ has two effects on price: (i) it increases price (by lowing cost of capital $E[v-p] = D \times S$), and (ii) it increases ex ante price volatility (by making price more revealing). Both effects hurt investor i if she is expected to be a net buyer of the stock (i.e., $s_i < S$): she buys at a higher price and faces greater price volatility. A net seller (i.e., $s_i > S$) benefits from the lower cost of capital but suffers from an elevated ex ante price volatility. One can verify that the higher price volatility is the dominant effect given the linear pricing function. This explains $\frac{\partial\Gamma}{\partial\lambda} \leq 0$, with strict inequality whenever $s_i \neq S$.[12]

Recall that a smaller F motivates the manager to reduce firm risk, causing a higher τ_θ. We know from (16) that $\frac{\partial\Gamma}{\partial\tau_\theta} > 0$ in the "Firm Risk Effect" is a *necessary* condition for a

[12] $\frac{\partial\Gamma}{\partial\lambda} = 0$ for $s_i = S$ because neither the level nor the volatility of price matters when no trade is expected.

lower cost F to increase an investor's endowment-related payoff. All else equal, having a safer cash flow not only lowers the cost of capital but also reduces price volatility. This differs from having more informed investors, which increases price volatility ex ante. As argued above, a lower cost of capital (hence, higher price) benefits an investor only if she is a net seller, i.e., $s_i > S$. The reduced price volatility benefits all investors, and the benefit is more pronounced for investors with a larger share endowment s_i. It is therefore not surprising to see that $\frac{\partial \Gamma}{\partial \tau_\theta} > 0$ if and only if s_i is sufficiently large, i.e., when $s_i > S \times \Omega$ for some positive constant Ω specified in the Appendix. The next result summarizes the discussion.

Proposition 4 *A decrease in the cost of information acquisition F reduces investors' welfare* (14) *for those whose share endowment $s_i \leq S \times \Omega$. Hence, having a sufficiently large share endowment is a necessary condition for an investor to benefit from a lower F.*

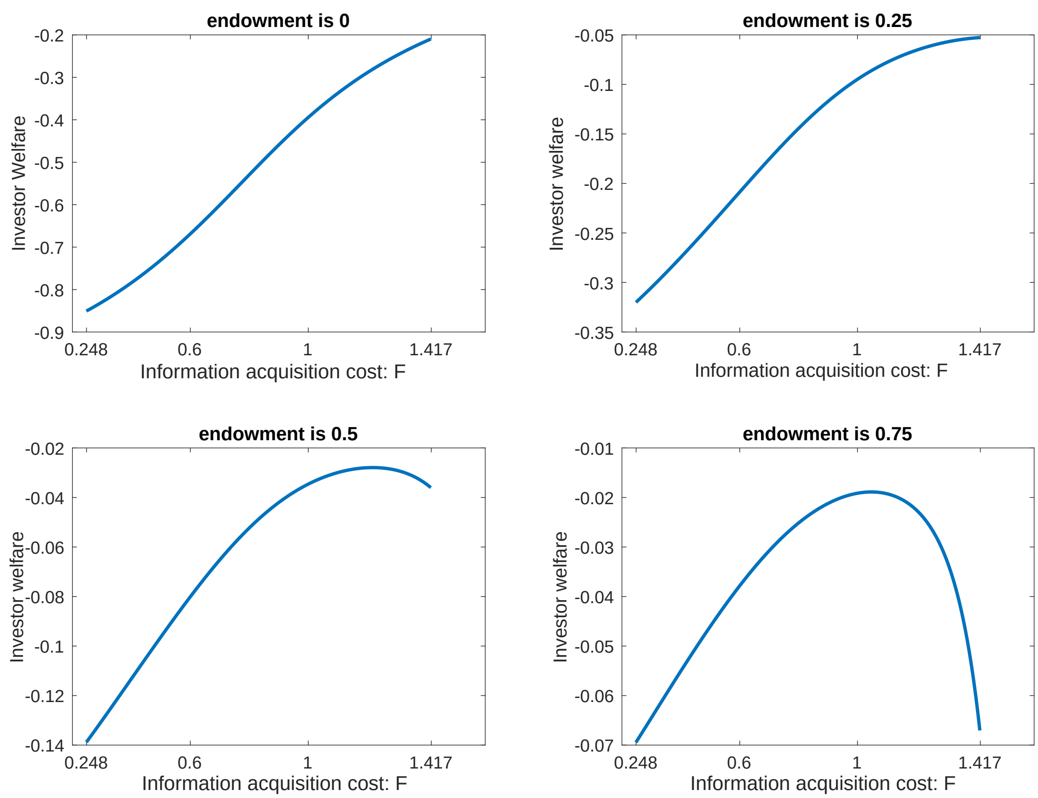

Figure 3: Investor welfare under different levels of share endowment s_i (fixing $S = 0.5$)

Figure 3 plots the investor welfare $E[U]$ as a function of information acquisition cost F. The four panels in the figure correspond to different levels of share endowment, s_i. One can see that having a large endowment is a necessary condition for an investor to benefit from a decrease in F. Even for investors with a large share endowment (i.e., the bottom two panels in Figure 3), they only benefit from a lower cost F when it is sufficiently high to begin with. This is because a high F in the model is associated with large investment and, hence, a sufficiently risky cash flow. Proposition 4 and Figure 3 together illustrate two necessary conditions for an investor to benefit from a lower cost: (i) the investor is endowed with many shares and (ii) the cash flow is highly risky to begin with. Both conditions point to a scenario where the investor is particularly concerned about the price volatility she is facing ex ante. In this case, the investor benefits from a decrease in F because it motivates the manager to lower the variance of the cash flow, which, in turn, reduces price volatility.

Compared to welfare analysis in exchange economies (e.g., Diamond, 1985; Verrecchia, 2019), the new insight in this section relates to the endogenous production. To see the difference, the corollary below summarizes how a lower F would affect investors' welfare in an exchange economy where the distribution of the cash flow is given exogenously.[13]

Corollary 1 *If the distribution of the cash flow is exogenous and the fraction of informed investor* $\lambda \in (0, 1)$*, a decrease in information acquisition cost reduces investor welfare* (14)*.*

The only consequence of a lower cost F in exchange economies is that more investors will become informed. We argued following Equations (15) and (16) that, all else equal, having more informed investors (i.e., a higher λ) reduces both the speculative- and endowment-related components of the welfare (14). Comparing Corollary 1 to previous discussions highlights

[13] A noteworthy technical difference is the source of supply noise, ϵ. Diamond (1985) and Verrecchia (2019) assume that ϵ arises from investors' random endowments, whereas ϵ is attributed to noise traders here. Their models assume that the variance of individual endowments goes to infinity, which prevents investors from learning about the aggregate supply from their private endowments (e.g., Diamond, 1985, p.1075). Ganguli and Yang (2009) show that, under a more realistic assumption that endowments have a finite variance, assuming ϵ coming from random endowments often results in multiple equilibria. Assuming ϵ coming from noise traders, as in this paper, eliminates the concern of multiple equilibria while maintaining a finite variance for investors' share endowments. Admittedly, introducing unmodeled noise traders limits the scope of welfare analysis.

the importance of considering feedback effects on the firm risk when we study investors' information production. The significance of considering the joint determination of firm risk will resurface as we discuss the model's empirical implications in the next section.

5 Empirical Implications

A growing literature investigates the consequences of technological advancements (e.g., social media and big data) and regulatory requirements (e.g., EDGAR) that reduce the cost for investors to acquire or process information. It is documented that a decrease in information acquisition cost is associated with more private informative production, an increase in liquidity, and better disciplined managerial behaviors (e.g., Blankespoor et al., 2014; Gao and Huang, 2020; Zhu, 2019).

Building on the same numerical example seen before, Figure 4 shows that the model is consistent with the empirical evidence. A decrease in the information acquisition cost makes the price more responsive to the firm value, improves price informativeness, increases market depth (a liquidity measure in this model), and reduces the cost of capital. While these observations are often interpreted as evidence that lowering the cost of information acquisition is beneficial, the next result shows that these "benefits" can be observed at the same time as the firm value and its stock valuation are decreasing.

Proposition 5 *As the information acquisition cost decreases,*

(i) cost of capital decreases, as measured by a smaller price discount $E[v-p]$;

(ii) the expected firm value $E[v]$ *and investment* k *decrease;*

(iii) the expected price $E[p]$ *decreases if and only if the mean supply* S *is low (i.e.,* $S < \overline{S}$*) and the manager's risk aversion is high (i.e.,* $\rho_M > \underline{\rho}$*).*

The cost of capital $E[v-p] = D \times S$ is the price discount investors demand for clearing the average supply $S \geq 0$, and $D = \frac{\rho(\tau_e+\tau_\theta+\tau_p)}{\tau_e(\lambda\tau_e+\tau_\theta+\tau_p)}$ is specified in the price function. Recall that

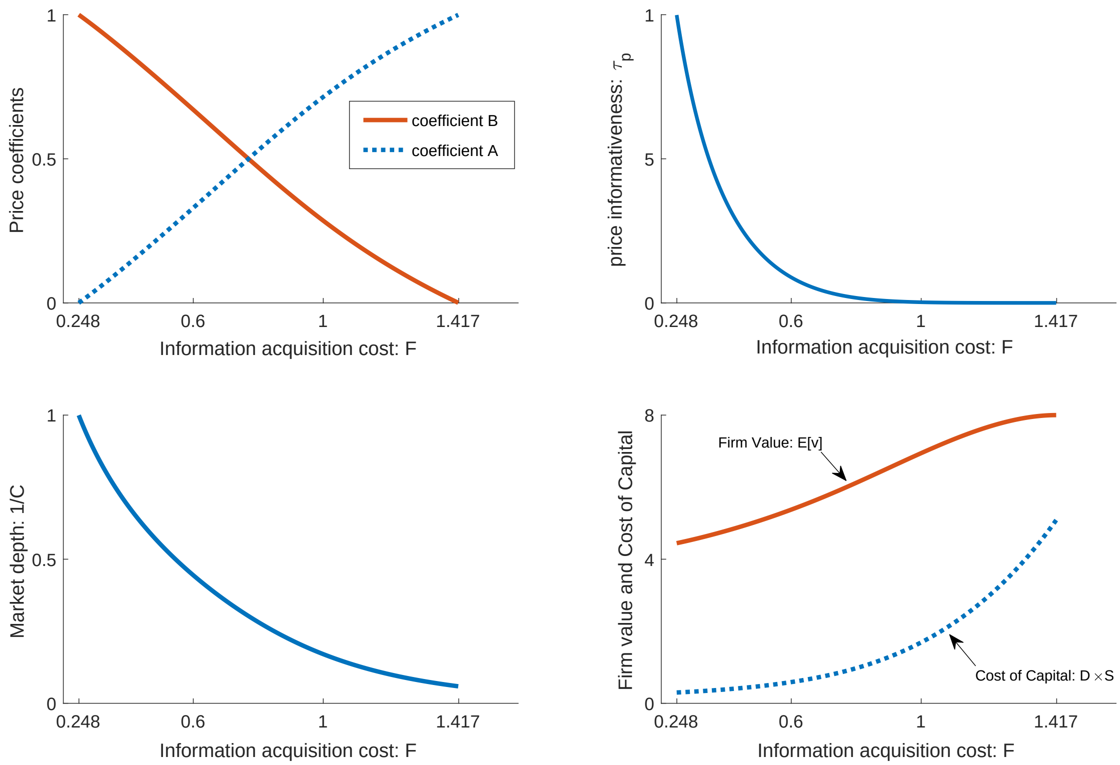

Figure 4: An illustration of empirical implications

a lower information acquisition cost results in more informed investors and a safer cash flow. Both changes reduce the risk investors bear and, hence, lower the firm's cost of capital. In particular, a safer cash flow reduces the risk level to begin with, and having more informed investors makes price more informative (and, hence, investors can learn more from it).

Part (iii) of the proposition is about stock valuation $E[p] = E[v] - E[v-p]$, which jointly accounts for the cost of capital effect $E[v-p]$ discussed above and the cash-flow effect $E[v]$. It shows that a lower information acquisition cost can decrease stock valuation, which occurs when the lower cost of capital is dominated by the decrease in the expected cash flow. To understand the conditions on S and ρ_M for a lower price, note that the average supply S does *not* affect equilibrium actions (λ, k) or the price coefficients.[14] The implication is that a higher

[14]This can be verified by noting that S does not enter the system of equations (9) characterizing the equilibrium. Intuitively, S is irrelevant to the investors' information acquisition choice because the value of information depends on the reduction in uncertainties, and different S only affects the expected price but not its volatility. From the manager's point of view, he takes the investors' conjectures $\hat{k}$, and, hence, the price

S does not change the expected cash flow $E[v] = \mu k - \frac{k^2}{2}$ but scales up the cost of capital $E[v-p] = D \times S$. It is therefore not surprising that the cost of capital effect dominates for sufficiently large S, i.e., for $S \geq \overline{S}$. For $S < \overline{S}$, the cash flow effect dominates if and only if the manager is sufficiently averse to risk, i.e., $\rho_M > \underline{\rho}$. All else equal, a lower cost F is more likely to reduce stock valuation $E[p]$ when the manager is more risk-averse.

Investigating the gap between the mean cash flow $E[v]$ and cost of capital in the last panel of Figure 4, one can see that lowering F first increases and then decreases stock valuation $E[p]$. To understand the non-monotonic relationship, note that the thresholds $\overline{S}$ and $\underline{\rho}$ in the conditions $S < \overline{S}$ and $\rho_M > \underline{\rho}$ are functions of the endogenous actions λ and k, which, in turn, depend on F. It is shown in the appendix that both conditions are more easily satisfied as F decreases.[15] The result suggests that lowering investors' information acquisition cost from a high level is associated with an increase in stock price at first, thanks to the decreasing cost of capital. The mean cash-flow effect eventually becomes the dominant effect, in which case lowering the cost F will hurt stock valuation.

The discussion in this section highlights the importance of accounting for the joint determination of firm risk when studying investors' information production. The conventional wisdom that lowering the cost of information acquisition is beneficial is likely to be intact if the manager can only affect the mean cash flow but not its variance (say, as in Edmans and Manso, 2011). In contrast, caution should be given if researchers believe that managers can reduce the variance of the cash flow and there is a tradeoff between improving the mean and decreasing the variance of the future cash flow. In this case, observing an increase in price informativeness and liquidity and a decrease in cost of capital may be evidence that managers are pressured to undertake inefficiently low-risk investments. This concern is particularly relevant for innovation-related investments because these investments, especially at early stages, involve an inherent risk-and-return tradeoff and are often not observed by external parties due to proprietary cost concerns. To my best knowledge, these cost considerations are under-

discount $\hat{D}S$ in (5) as given and cannot change it. Therefore, $\hat{D}S$ drops out of the manager's decision problem.

[15] That is, a lower F decreases $\underline{\rho}$ and increases $\overline{S}$, making it easier to satisfy $S < \overline{S}$ and $\rho_M > \underline{\rho}$.

explored in the literature studying the investors' information acquisition or processing cost because most empirical studies take firm riskiness as given.

6 Extensions

6.1 Value of a public disclosure

A main takeaway of the paper is that a decrease in investors' information acquisition cost could pressure the manager to pursue less risky investments, which can reduce the expected firm value and investors' welfare. This section examines the role of public disclosure in mitigating the problem. The timeline is the same as that in Figure 1, with the addition that all investors observe a public disclosure $y = \theta + \phi$ prior to trading. The common noise term ϕ in the disclosure is normally distributed with mean zero and precision $\tau_y = 1/\sigma_y^2$, and the precision τ_y is known to all investors.

Given a precision $\tau_y \geq 0$ of the public disclosure, one can follow the similar steps in Proposition 1 and show that there exists a unique linear price equilibrium with the price function

$$p = \alpha_0 + \alpha_\theta \times \theta + \alpha_y \times \phi - \alpha_\epsilon \times \epsilon - \alpha_S \times S, \tag{17}$$

where ϕ is the common noise in the disclosure y. I solve the equilibrium for all $\tau_y \geq 0$ and obtain the next result by comparing equilibria across different τ_y.

Proposition 6 *A finite disclosure precision τ_y^* maximizes the expected firm value $E[v]$. The precision τ_y^* also maximizes the welfare of the investors without share endowment.*

Investors without any share endowment (i.e., $s_i = 0$) can be thought of as "young" traders who have yet to enter the market. The disclosure quality τ_y^* (characterized in the Appendix) maximizes the welfare of these "young" investors. Generally, one cannot find a single disclosure quality τ_y that maximizes the welfare of investors with different share endowments.[16] This is

[16]This is the case even in exchange economies. For example, Indjejikian (1991) writes: "The significance

clear from the discussions following Proposition 3: how information would affect an investor's welfare (14) depends on the investor's initial share endowment, s_i.

The result about firm value $\mathbf{E}[v]$ in Proposition 6 follows from the fact that the equilibrium investment k is increasing in the quality of disclosure for $\tau_y \leq \tau_y^*$ and decreasing for $\tau_y > \tau_y^*$. For $\tau_y \leq \tau_y^*$, the fraction of informed investors λ and investment k are jointly determined from the investors' indifference condition $\exp(\rho * F) = \sqrt{1 + \frac{\tau_e}{\tau_y + (\lambda \tau_e/\rho)^2 \tau_\epsilon + \tau_\theta}}$ and the manager's first-order condition $\mu - k = \rho_M \alpha_\theta \frac{d\,\mathrm{var}(\theta)}{dk}$, both of which extend (9) to incorporate the public disclosure. Applying the implicit function theorem to the two equations yields:

$$\frac{dk}{d\tau_y} > 0 \text{ and } \frac{d\lambda}{d\tau_y} < 0. \tag{18}$$

$\frac{d\lambda}{d\tau_y} < 0$ means more precise disclosure crowds out investors' private information acquisition. This is intuitive because disclosure lowers the uncertainty investors face, reducing the advantage that informed investors have over uninformed ones (e.g., Demski and Feltham, 1994; Fischer and Stocken, 2010). The fraction of informed investors, λ, is reduced to zero at $\tau_y = \tau_y^*$. For $\tau_y > \tau_y^*$, λ stays at zero, and the equilibrium investment k is the unique fixed-point to $\mu - k = 2\rho_M \frac{\tau_y}{(k^2\Sigma)^{-1} + \tau_y} k\Sigma$, with

$$\frac{dk}{d\tau_y} < 0. \tag{19}$$

To understand the opposite effects that disclosure quality τ_y has on firm investment seen in (18) and (19), it is instructive to analyze how a more precise disclosure changes price responsiveness $\alpha_\theta = \frac{\lambda(\tau_e + \tau_\theta) + \tau_y + \tau_p}{\lambda \tau_e + \tau_\theta + \tau_y + \tau_p}$ in (17). For $\tau_y > \tau_y^*$, we use $\lambda = 0$ to express $\alpha_\theta = \frac{\tau_y}{\tau_\theta + \tau_y}$. Suppose the investors start with a belief that an increase in τ_y does not affect equilibrium investment k and, hence, $\tau_\theta = 1/(k^2\Sigma)$. A more precise disclosure would increase $\alpha_\theta = \frac{\tau_y}{\tau_\theta + \tau_y}$. As argued following Proposition 2, the anticipation of a more responsive price incentivizes the manager to prioritize reducing the variance of the cash flow over improving its mean, resulting a lower k. This explains $\frac{dk}{d\tau_y} < 0$ in (19).

of the endowment effect depends on the magnitude of the risky asset endowment and hence cannot be easily combined with or compared to the risk sharing and information cost effects."

The opposite result $\frac{dk}{d\tau_y} > 0$ for $\tau_y \leq \tau_y^*$ can be understood by showing that a higher τ_y *decreases* price responsiveness α_θ. The counterintuitive result rests on the interactions between the public disclosure and private information acquisition $\lambda > 0$. Intuitively, α_θ captures how much information about the fundamental is impounded into the price via trading. α_θ is increasing in (i) the fraction λ of *informed* investors who observe θ and (ii) the quality of information that *uninformed* investors have about θ, captured by $\text{var}^{-1}(\theta|p, y) = \tau_\theta + \tau_y + \tau_p$. As the precision of disclosure τ_y increases, it is important to note that λ will reduce in a way that maintains the total information available to the uninformed investors unchanged. This can be seen by analyzing the condition that determines λ (recall $\tau_p = (\lambda\tau_e/\rho)^2\tau_\epsilon$):

$$\exp(\rho * F) = \sqrt{1 + \frac{\tau_e}{\tau_\theta + \tau_y + \tau_p}}. \tag{20}$$

Start with a belief that a higher τ_y does not affect k (hence τ_θ). The fact λ is chosen endogenously from (20) means that a more precise disclosure does *not* affect total information available to the uninformed investors because the higher τ_y will be offset by a decrease in τ_p. Therefore, the impact of a higher τ_y on $\alpha_\theta = \frac{\lambda(\tau_e+\tau_\theta)+\tau_y+\tau_p}{\lambda\tau_e+\tau_\theta+\tau_y+\tau_p}$ is limited to its indirect effect in reducing the fraction λ of informed investors, resulting in a smaller α_θ.

The welfare result in Proposition 6 is related to a well-known result in Diamond (1985) that public disclosure improves investor welfare by crowding out investors' private information production. The difference is that Diamond (1985) studies a pure exchange economy and Proposition 6 extends the argument to a production economy in which the distribution of the cash flow is endogenous. The analysis suggests that investors face a coordination failure in their private information production when the manager faces a behind-the-scenes value-risk tradeoff in operating the firm. Each investor's information choice is optimal from an individual point of view. Collectively, however, the investors acquire too much information in that the manager is pressured to pick projects with a low risk and a low return. Public disclosure mitigates the problem by helping the investors coordinate information production, which, in turn, "corrects" the manager's distorted value-risk tradeoff in operating the firm.

6.2 Alternative way to model the value-risk tradeoff

This section examines an alternative approach to model the tradeoff managers face between increasing the mean and decreasing the variance of the cash flow. Consider a setting in which a manager allocates attention between mean-improving a_1 and variance-reducing a_2. Let $\mu_\theta(a_1)$ and $\sigma^2_\theta(a_2)$ be the first and second moments of the cash flow, with $\frac{d\mu_\theta}{da_1} > 0$ and $\frac{d\sigma^2_\theta}{da_2} < 0$. The value-risk tradeoff in this setup arises because the manager is subject to the following attention constraint (the total attention is normalized to one without loss of generality):

$$a_1 + a_2 \leq 1. \tag{21}$$

Alternatively, $a_1, a_2 \geq 0$ can be interpreted as the time the manager devotes to improving the mean and decreasing the variance, respectively. Examples of the attention a_2 to lower risk include ex ante risk screening and ex post risk management.[17] I allow for arbitrary functions μ_θ and σ^2_θ, provided that they satisfy $\frac{d\mu_\theta}{da_1} > 0$ and $\frac{d\sigma^2_\theta}{da_2} < 0$, with weakly decreasing marginal effects $\frac{d\mu_\theta}{da_1}$ and $|\frac{d\sigma^2_\theta}{da_2}|$.

The manager's allocation of attention is not observed by external investors and, therefore, must be determined jointly with the investors' information acquisition choice λ. It has been shown that all investors will be informed (i.e., $\lambda = 1$) if the information acquisition cost F is sufficiently small, and no one will be informed (i.e., $\lambda = 0$) for sufficiently larger F. To avoid such corner solutions, the discussion below confines attention to intermediate F to ensure that the fraction of informed investors $\lambda \in (0, 1)$ in equilibrium.[18]

Proposition 7 *There exists $\underline{\rho}_M$ and $\overline{\rho}_M$ so that $a_2 = 0$ for $\rho_M \leq \underline{\rho}_M$, and $a_1 = 0$ for $\rho_M \geq \overline{\rho}_M$. For $\rho_M \in (\underline{\rho}_M, \bar{\rho}_M)$, $a_1, a_2 > 0$, and a lower F increases the attention to risk-reducing a_2.*

It is not surprising that the manager will devote all attention to reducing firm risk if he is

[17] May (1995) documents evidence that managers reduce firm risk by choosing the diversification level in a given acquisition. Amihud and Lev (1981) argue that managers can lower firm risk by diversifying the product line or by adjusting the cost structure to obtain low operating leverage.

[18] That is, the discussion confines attention to $F \in (\underline{F}, \overline{F})$, where $\underline{F}$ and $\overline{F}$ are solved by substituting $(a_2 = 0, \lambda = 1)$ and $(a_2 = 1, \lambda = 0)$ to the investors' indifference condition (6), respectively.

sufficiently risk averse (i.e., $\rho_M > \overline{\rho}_M$) and will focus entirely on the mean cash flow if he is close to being risk neutral (i.e., $\rho_M < \underline{\rho}_M$), where $\overline{\rho}_M$ and $\underline{\rho}_M$ are specified in the appendix. The equilibrium is interior (i.e., $a_1, a_2 > 0$) otherwise. Related to the main message of the paper, we continue to see that a decrease in the investors' information acquisition cost motivates the manager to prioritize lowering firm risk over improving its expected value. The intuition can be illustrated by examining the relative marginal benefit of the attention that the manager devotes to reducing risk (a_2) over improving the mean (a_1):

$$\begin{aligned}\frac{\text{MB of } a_2}{\text{MB of } a_1} &= \rho_M \frac{\frac{d\text{var}[p]}{d\sigma_\theta^2}}{\frac{dE[p]}{d\mu_\theta}} \times \frac{\left|\frac{d\sigma_\theta^2}{da_2}\right|}{\frac{d\mu_\theta}{da_1}}, \\ &= \rho_M \times \hat{B} \times \frac{\left|\frac{d\sigma_\theta^2}{da_2}\right|}{\frac{d\mu_\theta}{da_1}}. \end{aligned} \tag{22}$$

As in the main model, the manager correctly conjectures that a lower cost F tends to improve price responsiveness in equilibrium (i.e., a higher $\hat{B}$).[19] A higher $\hat{B}$ increases the second-moment sensitivity of price to fundamental quadratically and the first-moment sensitivity linearly, as captured by $\frac{d\text{var}[p]}{d\sigma_\theta^2} = \hat{B}^2$ and $\frac{dE[p]}{d\mu_\theta} = \hat{B}$. I argued following (8) that the two sensitivities are related to managerial incentives to lower firm risk and improve its mean, respectively. Equation (22) shows that the anticipation of a higher $\hat{B}$ incentivizes the manager to shift attention from improving the mean to reducing the variance. Further, the relative sensitivity $\hat{B}$ discussed above is multiplicatively separable from how the moments of the fundamental are modeled, i.e., $\left|\frac{d\sigma_\theta^2}{da_2}\right| / \frac{d\mu_\theta}{da_1}$ in (22). Because of the multiplicatively separable structure, one can obtain qualitatively similar results under general specifications of $\mu_\theta(a_1)$ and $\sigma_\theta^2(a_2)$ in Proposition 7.

[19] I show in the appendix that a lower F results in a higher B in equilibrium. The only exception is when both $\mu_\theta(a_1)$ and $\sigma_\theta^2(a_2)$ are linear functions, in which case B is a constant.

7 Conclusion

Technological and computing advancements have lowered the cost for investors to acquire information. For example, recent studies document that decreasing information acquisition costs have made it cost efficient for asset managers to collect real-time satellite images and consumer transaction data for valuation. Relatedly, regulatory requirements such as EDGAR are believed to help reduce investors' information acquisition/processing cost.

This paper demonstrates a potential downside to decreasing the cost for investors to acquire private information in a model where their information acquisition and the manager's investment decision are jointly determined. Consistent with empirical evidence, I show that a lower information acquisition cost is associated with an increase in information acquisition, price informativeness, and liquidity and a decrease in cost of capital. However, these perceived benefits are observed in part because the manager is pressured by the pricing of the investors' information to prioritize reducing firm risk over improving its expected value, i.e., to pursue investments with low risk and low return. In addition to reducing the expected firm value, the distorted value-risk tradeoff can also lower stock valuation and the investors' welfare.

The analysis highlights the significance of accounting for feedback effects when we study technological and regulatory changes that facilitate investors' information production. The conventional wisdom that lowering the cost of information acquisition is beneficial is likely to hold if the manager can only affect the mean cash flow. In contrast, caution should be exercised if managers can also reduce firm risk and there is a tradeoff between improving the mean and decreasing the variance of future cash flow. In such cases, observing an increase in price informativeness and a lower cost of capital may be evidence that managers are pressured by the market to undertake low-risk-and-return investments. The concern is particularly relevant for innovation-related investments because these investments, especially at early stages, involve an inherent risk-and-return tradeoff and are not observed by investors due to concerns about proprietary cost. To the extent that basic research is riskier than applied applications in terms of cash flow implications, the result is consistent with Arora et al. (2021), who show a shift in corporate R&D composition towards less "R" and more "D" over time. Identifying the feedback effects associated with facilitating investors' information acquisition seems an interesting avenue for future research.

In the model, the manager's investment choice affects the firm's cash flow (hence its stock price) but not the number of shares the manager owns. Empirically, this assumption is likely to be descriptive for long-term executives who have already accumulated significant shares and for executives who anticipate receiving a fixed number of shares in the near future (e.g., under time-based stock awards). These managers view their shareholdings as fixed when making investment choices and are susceptible to excessive risk avoidance induced by changes to the investors' information acquisition. Incentive schemes might evolve over time in response to the distorted value-risk tradeoff. For example, the board can make the number of shares awarded to executives to be increasing in the firm's stock price, which introduces convexity to their payoffs and mitigates risk avoidance. This adjustment is consistent with "performance equity," an incentive scheme designed to give executives more shares if the company meets performance targets set by the board. A common criticism of performance equity is that it is an intricate means of boosting a CEO's pay.[20] The model provides a rationale for its use. Because public disclosure can also be used to mitigate the manager's risk avoidance induced by investors' information production, another avenue for future research is to study the interactions between disclosure and contract design in the presence of investors' private information production.

[20]See "The New Pay Gap: What Firms Report Paying CEOs Versus What They Take Home" from *the Wall Street Journal* published on 8/25/2019.

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A Appendix

Proof of Proposition 1. Given a conjectured $\hat{k}$, the distribution of θ is known to investors. I follow the same steps as in Grossman and Stiglitz (1980) to derive the pricing function

$$p = \hat{A} \times \mu_\theta(\hat{k}) + \hat{B} \times \theta - \hat{C} \times \epsilon - \hat{D} \times S, \tag{A.1}$$

where the nonnegative coefficients are $\hat{A} = \frac{(1-\lambda)\hat{\tau}_\theta}{\lambda\tau_e+\hat{\tau}_\theta+\tau_p}$, $\hat{B} = \frac{\tau_p+\lambda(\tau_e+\hat{\tau}_\theta)}{\lambda\tau_e+\hat{\tau}_\theta+\tau_p}$, $\hat{C} = \frac{\rho(\tau_e+\hat{\tau}_\theta+\tau_p/\lambda)}{\tau_e(\lambda\tau_e+\hat{\tau}_\theta+\tau_p)}$, and $\hat{D} = \frac{\rho(\tau_e+\hat{\tau}_\theta+\tau_p)}{\tau_e(\lambda\tau_e+\hat{\tau}_\theta+\tau_p)}$, where $\hat{\tau}_\theta = \sigma_\theta^{-2}(\hat{k})$ and $\tau_p = (\lambda\tau_e/\rho)^2\tau_\epsilon$. The price coefficients depend on the investor's conjecture $\hat{k}$ via $\hat{\tau}_\theta$. The fraction of informed investors is $\lambda = 1$ for sufficiently low F (i.e., $F < \frac{1}{\rho}\log\sqrt{1+\frac{\tau_e}{(\tau_e/\rho)^2\tau_\epsilon+\hat{\tau}_\theta}}$) and is $\lambda = 0$ for sufficiently high F (i.e., $F > \frac{1}{\rho}\log\sqrt{1+\tau_e/\hat{\tau}_\theta}$.) For intermediate F, the fraction λ is solved from the indifference condition:

$$\exp(\rho * F) = \sqrt{1+\frac{\rho^2\tau_e}{(\lambda\tau_e)^2\tau_\epsilon+\rho^2\hat{\tau}_\theta}}. \tag{A.2}$$

The right hand side of (A.2) is $\sqrt{\frac{\text{var}(v|p)}{\text{var}(v|\theta)}} = \sqrt{1+\frac{\tau_e}{\hat{\tau}_\theta+\tau_p}}$, which is the extent to which learning θ reduces uncertainties, and it depends on the conjectured investment $\hat{k}$ via $\hat{\tau}_\theta = \sigma_\theta^{-2}(\hat{k})$.

Having characterized the trading subgame for a given $\hat{k}$, I solve for the equilibrium k below in 3 steps.

Step 1: Deriving the system of equations that characterize the equilibrium, while taking $0 < \lambda < 1$ as given.

The manager takes the investors' conjecture $\hat{k}$ and, hence, the price function (A.1) as given and chooses k at $t = 0$ to maximize his payoff $E[p|\hat{k}, k] - \rho_M \text{var}(p|\hat{k}, k)$, where $E[p|\hat{k}, k] = \hat{A}\,\mu_\theta(\hat{k}) + \hat{B}\mu_\theta(k) - \hat{D}S$ and $\text{var}(p|\hat{k}, k) = \hat{B}^2\sigma_\theta^2(k) + \hat{C}^2\sigma_\epsilon^2$ follow from (A.1). The first-order condition for k is $\hat{B}\mu - \rho_M\hat{B}^2\frac{d\sigma_\theta^2}{dk} = \hat{B}k$. Because B is guaranteed to be positive for $\lambda > 0$, I rewrite the first-order condition as:

$$\mu - k = \rho_M\hat{B}\frac{d\sigma_\theta^2}{dk}. \tag{FOC k}$$

The investors' indifferent condition for determining the fraction λ of informed traders is

$$\exp(\rho * F) = \sqrt{1 + \frac{\rho^2 \tau_e}{(\lambda \tau_e)^2 \tau_\epsilon + \rho^2/\sigma_\theta^2(\hat{k})}}. \tag{Indifference}$$

The equilibrium is characterized by (FOC k), (Indifference), and rational expectations $k = \hat{k}$. I prove the existence and uniqueness of the solution to the system of non-linear equations at the end of the proof given its complexity.

Step 2: Characterizing $\underline{F}$ and $\overline{F}$.

The manager takes the price coefficient $\hat{B}$ as given and cannot change it by choosing a different k. I apply the implicit function theorem to (FOC k) and obtain the following:

$$\frac{dk}{d\hat{B}} = -\frac{\rho_M \frac{d\mathrm{var}(\theta)}{dk}}{1 + \rho_M \hat{B} \frac{d^2 \mathrm{var}(\theta)}{dk^2}} < 0. \tag{A.3}$$

I use $\mathrm{var}(\theta)$ instead of σ_θ^2 in (A.3) to ease the expression of the second-order derivative.

Denote by $\underline{F}$ the cost threshold below which the fraction of informed investors is $\lambda = 1$ (hence, the price coefficient $B = 1$). Given (A.3), we know that the equilibrium investment is the smallest $\underline{k}$ under $B = 1$, and $\underline{k} = \frac{\mu}{1+2\rho_M \Sigma}$ is determined by substituting $B = 1$ to (FOC k). Substituting $k = \underline{k}$ and $\lambda = 1$ into (Indifference), we characterize $\underline{F}$ as

$$\underline{F} \equiv \frac{1}{\rho} \log \sqrt{1 + \frac{\mu^2 \tau_e \Sigma}{(1 + 2\rho_M \Sigma)^2 + (\mu \tau_e/\rho)^2 \tau_\epsilon \Sigma}}. \tag{A.4}$$

Note that the value of information (i.e., the right-hand side of (Indifference)) is increasing in k and decreasing in λ. Equation (A.4) means that, at $F = \underline{F}$, an investor is willing to acquire information even if the value of information achieves its minimum level (i.e., when $k = \underline{k}$ and $\lambda = 1$). It follows that $\lambda = 1$ for any $F \leq \underline{F}$.

Similarly, denote by $\overline{F}$ the threshold above which $\lambda = 0$ and, hence $B = 0$. $k = \overline{k} \equiv \mu$ is obtained by substituting $B \to 0$ to (FOC k). Substituting $k = \overline{k}$ and $\lambda = 0$ into (Indifference) yields $\overline{F}$:

$$\overline{F} \equiv \frac{1}{\rho} \log \sqrt{1 + \mu^2 \tau_e \Sigma}. \tag{A.5}$$

At $F = \overline{F}$, on one will acquire information even if the value of information is at its maximum level (i.e., when $k = \overline{k}$ and $\lambda = 0$). It follows that $\lambda = 0$ for any $F \geq \overline{F}$.

Step 3: Existence and uniqueness of the solution to (FOC k) **and** (Indifference)**.**

This step shows the existence and uniqueness of the solution to the two non-linear equations (after imposing $k = \hat{k}$). For a given $k \in (\underline{k}, \overline{k})$, $\sigma_\theta^2(k) = k^2\Sigma$ is fixed and, hence, a unique $\lambda(k)$ is determined by the indifference condition (Indifference). One can then substitute the given k and $\lambda(k)$ to obtain the price coefficient $B(k) \equiv B(\sigma_\theta^2(k), \lambda(k))$ shown in (A.1). That is, the system of equations allows us to express $\lambda(k)$ and $B(k)$ as implicit functions of k. Substituting $\lambda(k)$ and $B(k)$ into (FOC k), one obtains the equilibrium k as the fixed point to the equation:

$$k = \mu - \rho_M B(k) \frac{d\sigma_\theta^2}{dk}. \tag{A.6}$$

The left-hand side of (A.6) is increasing in k. I claim that its right-hand side is decreasing in k, which means that there exists at most one fixed point k to (A.6). It is sufficient to prove the claim by showing that a higher k results in a higher $B(k) \equiv B(\sigma_\theta^2(k), \lambda(k))$. Recall that the value of information $\sqrt{1 + \frac{\rho^2 \tau_e}{(\lambda \tau_e)^2 \tau_\epsilon + \rho^2 \sigma_\theta^{-2}(k)}}$ is decreasing in λ and increasing in k. If k increases (hence, σ_θ^2 is higher) in equilibrium, λ must also increase to maintain (Indifference) as an equality. A higher k and λ together result in an increase in $B(\sigma_\theta^2(k), \lambda(k))$ because $\frac{\partial B}{\partial \sigma_\theta^2} > 0$ and $\frac{\partial B}{\partial \lambda} > 0$. This verifies the claim.

The existence of the fixed-point k to (A.6) is proven by examining the extreme values of $k \to \underline{k}$ and $k \to \overline{k}$. Step 2 above shows that $k = \underline{k}$ and $k = \overline{k}$ satisfy (A.6) as an equality at $F = \underline{F}$ and $F = \overline{F}$, respectively. For $F \in (\underline{F}, \overline{F})$, one can verify that the left-hand side of (A.6) is less than its right-hand side for $k = \underline{k}$, while the opposite is true for $k = \overline{k}$. The existence of the fixed point $k \in (\underline{k}, \overline{k})$ to (A.6) follows from standard continuity arguments. ■

Proof of Proposition 2. To prove that lowering F increases the price coefficient B in equilibrium, suppose by contradiction that B weakly decreases instead. The manager takes the price coefficient B as given and (A.3) shows that $\frac{dk}{d\hat{B}} < 0$ for $\frac{d^2\text{var}(\theta)}{dk^2} \geq -\rho_M^{-1}$. That is, the anticipation of a lower B will motivate the manager to increase k and, hence, σ_θ^2. (Hereafter, I will drop the "hat" notation in $\hat{B}$ when discussing *equilibrium properties*.) This means that if a lower F were to weakly decrease B in equilibrium, σ_θ^2 would weakly increase. A weakly higher σ_θ^2 and a strictly lower cost of information acquisition F imply that more investors will acquire information, i.e., a higher λ. (Recall from (Indifference) that the value of information is increasing in σ_θ^2 and decreasing in λ.) The argument, however, leads to a contradiction. In particular, (A.7) below shows that a strictly higher λ and a weakly higher σ_θ^2 will result in a strictly higher $B = \frac{\tau_p + \lambda(\tau_e + \tau_\theta)}{\lambda\tau_e + \tau_\theta + \tau_p}$ *in equilibrium*, contradicting the assumed conjecture that B weakly decreases.

$$\frac{\partial B}{\partial \sigma_\theta^2} > 0, \quad \frac{\partial B}{\partial \lambda} > 0. \tag{A.7}$$

The contradiction proves the claim of $\frac{dB}{dF} < 0$ in equilibrium.

Note that the manager takes the pricing coefficient B as given in choosing k. Therefore, the comparative statics below follow from the result $\frac{dB}{dF} < 0$ shown above and (A.3):

$$\frac{dk}{dF} = \frac{dk}{dB}\frac{dB}{dF} > 0.$$

It shows above that a decrease in F results in a higher B and a lower k (hence, σ_θ^2) in equilibrium. If a lower F reduces σ_θ^2 while still resulting in a higher B, we know from (A.7) that the higher B must be driven by an increase in λ. This proves $\frac{d\lambda}{dF} < 0$. ■

Proof of Proposition 3. Given the market price p, an uninformed investor i's *interim* expected payoff is

$$E(U_i|p) = -\exp\{-\rho[E(W_i|p) - \frac{\rho}{2}\text{var}(W_i|p)]\}, \tag{A.8}$$

where W_i is i's final wealth. Let s_i be investor i's endowment of the firm's share and d_i be her demand of the share. We know $W_i = ps_i + d_i(v-p)$. Differentiating $E(U_i|p)$ with respect to d_i, we solve from the first-order condition $d_i = \frac{E(v|p)-p}{\rho\, var(v|p)}$ and express W_i as follows:

$$W_i = p \times s_i + \frac{E(v|p)-p}{\rho\, var(v|p)} \times (v-p). \tag{A.9}$$

Substituting (A.9) back to (A.8), one can rewrite $E(U_i|p)$ as

$$E(U_i|p) = -\exp\{-\frac{[E(v|p)-p]^2}{2\, var(v|p)} - \rho\, p\, s_i\}. \tag{A.10}$$

From an ex ante point of view, p is normally distributed because the price function is linear. To calculate the investor i's *ex ante* payoff, we take expectation of (A.10) over p as:

$$E[U] = E[-\exp\{-\frac{[E(v|p)-p]^2}{2\, var(v|p)} - \rho\, p\, s_i\}], \tag{A.11}$$

where $var(v|p) = \frac{1}{\tau_e} + \frac{1}{\tau_\theta+\tau_p}$, $E(v|p) = \frac{\tau_\theta}{\tau_\theta+\tau_p}\mu_\theta + \frac{\tau_p}{\tau_\theta+\tau_p}\frac{p-A\mu_\theta+D\,S}{B}$, and A, B, and D are the price coefficients derived in (A.1) and S is the average supply.

It is useful to define $\tilde{p} = p - E[p]$ so that $\tilde{p}$ is a zero-mean variable. Substituting $p = \tilde{p} + E[p]$ and the price coefficients A, B, D into (A.11), I simplify it as the following:

$$E[U] = E[-\exp(F \times \tilde{p}^2 + G \times \tilde{p} + H)], \tag{A.12}$$

where

$$F = -\frac{\tau_e \tau_\theta^2 \left(\tau_\theta + \tau_p + \tau_e\right)}{2\left(\tau_\theta + \tau_p\right)\left(\tau_e + \tau_\theta + \frac{\tau_p}{\lambda}\right)^2},$$

$$G = -\rho\left(s_i - \frac{S\left(\tau_e + \tau_\theta\right)}{\tau_e + \tau_\theta + \frac{\tau_p}{\lambda}} + \frac{\lambda S \tau_e}{\lambda\tau_e + \tau_\theta + \tau_p}\right),$$

and

$$H = \frac{\rho^2 S\left(\tau_e + \tau_\theta + \tau_p\right)\left(2\lambda\tau_e s_i + \tau_p\left(2s_i - S\right) + 2\tau_\theta s_i - S\tau_\theta\right)}{2\tau_e\left(\lambda\tau_e + \tau_\theta + \tau_p\right)^2} - \rho\mu_\theta s_i.$$

Computing $E[U]$ in (A.12) requires taking expectation of the exponential of a quadratic form of a zero-mean normal variable $\tilde{p} \sim N(0, \Sigma)$. Let $\mathbf{V} = var(p)$. We know $\mathbf{V} = B^2/\tau_\theta + C^2/\tau_\epsilon$ from the linear price function. Following (Veldkamp, 2011, p. 102), I calculate the closed-form expression of (A.12) as

$$E[-\exp(F \times \tilde{p}^2 + G \times \tilde{p} + H)] = -\frac{1}{\sqrt{1-2F\mathbf{V}}} \exp\left[\frac{G^2\,\mathbf{V}}{2(1-2F\mathbf{V})} + H\right]. \tag{A.13}$$

Substituting F, G, H and $\mathbf{V}$ defined above into (A.13), we can collect terms and express the ex ante payoff as

$$E[U] = -\left(\sqrt{1 + \frac{\mathrm{var}[E(v|p) - p]}{\mathrm{var}(v|p)}}\right)^{-1} \exp\{-S^2\,D^2\,\frac{\Phi}{2}\}\ \exp\{-\left(s_i\Psi - s_i^2\Delta\right)\}. \tag{A.14}$$

Here, $\frac{\mathrm{var}[E(v|p)-p]}{var(v|p)} = \frac{\rho^2\tau_\theta(\tau_\theta+\tau_p+\tau_e)}{\tau_e\tau_\epsilon(\tau_\theta+\tau_p+\lambda\tau_e)^2}$, $\Psi = \rho\mu_\theta - SD^2\Phi$, $\Delta = \rho^2 B^2 \sigma_\theta^2 \frac{(\tau_e+\tau_\theta+\tau_p)\Phi}{2\tau_e\tau_p}$, and

$$\Phi = \frac{1}{\mathrm{var}(v|p) + \mathrm{var}[E(v|p) - p]} = \tau_e\left(1 - \frac{\tau_e}{\tau_e + \tau_\theta + \tau_p} - \frac{\tau_\theta}{\tau_e + \tau_\theta + r^2\tau_e\tau_\epsilon(\tau_\theta + \tau_p) + 2\tau_p/\lambda}\right),$$

where $r = 1/\rho$ is the investors' risk tolerance and price coefficients B and D are specified under (A.1). ■

Proof of Proposition 4. We can rewrite the investors' welfare (A.14) as

$$E[U] = -\left(1 + \mathbf{R}\right)^{-\frac{1}{2}} \times \exp(-\Gamma), \tag{A.15}$$

where $\mathbf{R} \doteq \frac{\mathrm{var}[E(v|p)-p]}{var(v|p)}$ and $\Gamma = s_i\Psi + S^2D^2\,\frac{\Phi}{2} - s_i^2\Delta$. It follows

$$\frac{dE[U]}{dF} = -\frac{d\left(1+\mathbf{R}\right)^{-\frac{1}{2}}}{dF}\exp(-\Gamma) + \left(1+\mathbf{R}\right)^{-\frac{1}{2}}\exp(-\Gamma)\frac{d\Gamma}{dF}. \tag{A.16}$$

Substituting the price coefficients, we obtain $\mathbf{R} \doteq \frac{\mathrm{var}[E(v|p)-p]}{var(v|p)} = \frac{\rho^2\tau_\theta(\tau_\theta+\tau_p+\tau_e)}{\tau_e\tau_\epsilon(\tau_\theta+\tau_p+\lambda\tau_e)^2}$. Tedious algebra shows the following (using $\tau_\theta = 1/(k^2\Sigma)$, and $\frac{d\lambda}{dF} < 0$ and $\frac{dk}{dF} > 0$ obtained in (12)):

$$\frac{d\mathbf{R}}{dF} = \frac{\partial \mathbf{R}}{\partial \lambda}\frac{d\lambda}{dF} + \frac{\partial \mathbf{R}}{\partial \tau_\theta}\frac{\partial \tau_\theta}{\partial k}\frac{dk}{dF} > 0.$$

Note $-\frac{d(1+\mathbf{R})^{-\frac{1}{2}}}{dF} \propto \frac{d\mathbf{R}}{dF}$, meaning that $-\frac{d(1+\mathbf{R})^{-\frac{1}{2}}}{dF}$ shares the same sign as $\frac{d\mathbf{R}}{dF}$. Given $\frac{d\mathbf{R}}{dF} > 0$, we know from examining (A.16) that $\frac{d\Gamma}{dF} > 0$ is a sufficient condition for $\frac{dE[U]}{dF} > 0$. Below, we examine $\frac{d\Gamma}{dF}$ and derive a sufficient condition for $\frac{d\Gamma}{dF} > 0$.

$$\frac{d\Gamma}{dF} = \underbrace{\frac{\partial \Gamma}{\partial \mu_\theta}\frac{d\mu_\theta}{dk}\frac{dk}{dF}}_{\text{Mean Effect } (+)} + \underbrace{\frac{\partial \Gamma}{\partial \lambda}\frac{d\lambda}{dF}}_{\text{Information environment Effect } (+)} + \underbrace{\frac{\partial \Gamma}{\partial \tau_\theta}\frac{d\tau_\theta}{dk}\frac{dk}{dF}}_{\text{Firm Risk Effect } (+/-)}. \tag{A.17}$$

In the "Mean Effect" in (A.17), $\frac{dk}{dF} > 0$ is shown in Proposition 2. We know $\frac{d\mu_\theta}{dk} > 0$ because $\mu_\theta = \mu k - \frac{k^2}{2}$ and $k \leq \mu$ (see Propositions 1 and 2). The fact that the "Mean Effect" is positive follows by noting

$$\frac{\partial \Gamma}{\partial \mu_\theta} = \rho s_i \geq 0, \tag{A.18}$$

where $s_i \geq 0$ is investor i's initial share endowment.

Similarly, I have shown $\frac{d\lambda}{dF} < 0$ in Proposition 2 and one can verify that the "Information environment Effect" in (A.17) is nonnegative by noting: (recall $r = 1/\rho$)

$$\frac{\partial \Gamma}{\partial \lambda} = -(s_i - S)^2 \frac{(1-\lambda) r^2 \tau_e^2 \tau_\epsilon^2 \left(\lambda r^2 \tau_e^2 \tau_\epsilon + \tau_e + \tau_\theta\right)}{\left(\lambda^2 r^4 \tau_e^3 \tau_\epsilon^2 + r^2 \tau_e \tau_\epsilon (2\lambda \tau_e + \tau_\theta) + \tau_e + \tau_\theta\right)^2} \leq 0, \tag{A.19}$$

and the equality holds if and only if $s_i = S$.

It follows from (A.17) that a sufficient condition for $\frac{d\Gamma}{dF} > 0$ is to have a positive "Firm Risk Effect", which, given $\frac{d\tau_\theta}{dF} < 0$ shown in Proposition 2, is equivalent to having $\frac{\partial \Gamma}{\partial \tau_\theta} < 0$. Straightforward algebra shows the following for any $S \geq 0$:

$$\frac{\partial \Gamma}{\partial \tau_\theta} < 0 \text{ if and only if } s_i \leq S \times \Omega, \tag{A.20}$$

with

$$\Omega = \frac{(1-\lambda)r^2\tau_e\tau_\theta\tau_\epsilon}{\lambda^2 r^4\tau_e^3\tau_\epsilon^2 + r^2\tau_e\tau_\epsilon(2\lambda\tau_e - \lambda\tau_\theta + 2\tau_\theta) + \tau_e + \tau_\theta}. \tag{A.21}$$

Recall from Proposition 1 that the equilibrium λ and k (hence, τ_θ) are independent of the individual endowment s_i and the average supply S. Therefore, Ω above is also independent of S and s_i. Rather, Ω is determined solely by model parameters that are common knowledge and is therefore a constant known in equilibrium. ■

Proof of Corollary 1. It is shown in (A.15) that $E[U] = -(1+\mathbf{R})^{-\frac{1}{2}} \times \exp(-\Gamma)$, where $\mathbf{R} \doteq \frac{\text{var}[E(v|p)-p]}{var(v|p)} = \frac{\rho^2\tau_\theta(\tau_\theta+\tau_p+\tau_e)}{\tau_e\tau_\epsilon(\tau_\theta+\tau_p+\lambda\tau_e)^2}$ and $\Gamma = s_i\Psi + S^2D^2\,\frac{\Phi}{2} - s_i^2\Delta$. Substituting $\tau_p = (\lambda\tau_e/\rho)^2\tau_\epsilon$, one can verify that $\frac{\partial\mathbf{R}}{\partial\lambda} < 0$. The fact that $\frac{\partial\mathbf{R}}{\partial\lambda} < 0$, together with $\frac{\partial\Gamma}{\partial\lambda} \leq 0$ shown in (A.19), suggests that $E[U]$ is decreasing in the fraction of informed investors, λ. The corollary follows by noting that a decrease in F results in a higher λ for any $\lambda \in (0,1)$. ■

Proof of Proposition 5. $E[v-p] = D \times S$ follows from the linear price function, where $D = \frac{\rho(\tau_e+\tau_\theta+\tau_p)}{\tau_e(\lambda\tau_e+\tau_\theta+\tau_p)}$ and $\tau_p = (\lambda\tau_e r)^2\tau_\epsilon$. I apply the chain rule to obtain

$$\frac{dD}{dF} = \frac{\partial D}{\partial\lambda}\frac{d\lambda}{dF} + \frac{\partial D}{\partial\tau_\theta}\frac{d\tau_\theta}{dk}\frac{dk}{dF}. \tag{A.22}$$

Tedious but straightforward algebra verifies $\frac{\partial D}{\partial\lambda} < 0$ and $\frac{\partial D}{\partial\tau_\theta} < 0$. The result $\frac{dD}{dF} > 0$ follows by subsituting $\frac{d\lambda}{dF} < 0$ and $\frac{d\tau_\theta}{dF} < 0$ shown in Proposition 2. This proves the claim that a decrease in F reduces the firm's cost of capital $E[v-p] = D \times S$.

It is easy to see that $E[v] = \mu k - \frac{k^2}{2}$ is increasing in k because I have shown $k \leq \mu$ in Proposition 1. The result $\frac{dE[v]}{dF} > 0$ then follows $\frac{dk}{dF} > 0$ shown in Proposition 2.

To prove Part (iii), we use the price function characterized in Proposition 1 and express

$$E[p] = E[v] - \frac{\rho(\tau_e+\tau_\theta+\tau_p)}{\tau_e(\lambda\tau_e+\tau_\theta+\tau_p)}S. \tag{A.23}$$

Applying the chain rule to obtain $\frac{dE[p]}{dF} = \frac{\partial E[p]}{\partial\lambda}\frac{d\lambda}{dF} + \frac{\partial E[p]}{\partial\tau_\theta}\frac{d\tau_\theta}{dk}\frac{dk}{dF}$. Substituting $\frac{d\lambda}{dF}$ and $\frac{dk}{dF}$ derived in (12), one can verify that $\frac{dE[p]}{dF} > 0$ if and only if the two conditions below are satisfied at the same time:

$$S < \bar{S} \equiv \frac{r(\mu-k)\left(k^2\lambda\Sigma\tau_e\left(\lambda r^2\tau_e\tau_\epsilon+1\right)+1\right)}{k\Sigma\left(\lambda\left(k^2\Sigma\tau_e\left(\lambda r^2\tau_e\tau_\epsilon+1\right)-1\right)+2\right)}, \tag{A.24}$$

and

$$\rho_M > \underline{\rho} \equiv -\frac{k\left(k^2\lambda\Sigma\tau_e\left(\lambda r^2\tau_e\tau_\epsilon+1\right)+1\right)S}{2\left(k^3\lambda\Sigma\tau_e(S\Sigma+r)\left(\lambda r^2\tau_e\tau_\epsilon+1\right)-k^2\lambda\mu r\Sigma\tau_e\left(\lambda r^2\tau_e\tau_\epsilon+1\right)+k(r-(\lambda-2)S\Sigma)-\mu r\right)}.$$

Straightforward algebra verifies that $\underline{\rho} \geq 0$ is guaranteed for $S < \bar{S}$ and that $\frac{d\underline{\rho}}{dS} > 0$ (with $\underline{\rho} = 0$ at $S = 0$.)

The remainder of the proof verifies that both conditions above are easier to be satisfied as F decreases. Note that $\overline{S}$ in (A.24) is independent of the mean supply S because Proposition 1 shows that the equilibrium k and λ are independent of S. It is easy to verify that $\frac{\partial\bar{S}}{\partial\lambda} > 0$ and $\frac{\partial\bar{S}}{\partial k} < 0$. Using the results $\frac{d\lambda}{dF} < 0$ and $\frac{dk}{dF} > 0$ from Proposition 2, we know

$$\frac{d\bar{S}}{dF} = \frac{\partial\bar{S}}{\partial\lambda}\frac{d\lambda}{dF} + \frac{\partial\bar{S}}{\partial k}\frac{dk}{dF} < 0.$$

Similarly, one can verify $\frac{\partial\underline{\rho}}{\partial\lambda} < 0$ and $\frac{\partial\underline{\rho}}{\partial k} > 0$, from which we know

$$\frac{d\underline{\rho}}{dF} = \frac{\partial\underline{\rho}}{\partial\lambda}\frac{d\lambda}{dF} + \frac{\partial\underline{\rho}}{\partial k}\frac{dk}{dF} > 0.$$

Collecting conditions completes the proof. ∎

Lemma 1 below is used in proving Proposition 6.

Lemma 1 *There exists a unique linear price equilibrium given a disclosure precision* $\tau_y \geq 0$.

Proof of Lemma 1. I guess and verify that the market-clearing price takes the form of

$$p = \alpha_0 \times \mu_\theta(\hat{k}) + \alpha_\theta\theta + \alpha_y\phi - \alpha_\epsilon\epsilon - \alpha_S S,$$

where ϕ is the common noise in the public disclosure y. Observing price p is informationally equivalent to observing $q = \frac{p-\alpha_0\,\mu_\theta(\hat{k})+\alpha_S S-\alpha_y\,y}{\alpha_\theta-\alpha_y} = \theta - \frac{\alpha_\epsilon}{\alpha_\theta-\alpha_y}\epsilon$, where q is normally distributed with mean θ and precision $\tau_p = (\frac{\alpha_\theta-\alpha_y}{\alpha_\epsilon})^2\tau_\epsilon$.

Denote by $\tau_I = \mathrm{var}^{-1}(v|\theta)$ and $\tau_U = \mathrm{var}^{-1}(v|p)$ the posterior precision of informed investors and uninformed investors, respectively. One can show that

$$\tau_I = \tau_e \text{ and } \tau_U = \frac{1}{\frac{1}{\tau_e} + \frac{1}{\tau_\theta+\tau_y+\tau_p}}. \tag{A.25}$$

An informed investor's demand is $d_I = \tau_I\frac{E(v|\theta)-p}{\rho}$ and an uninformed investor's demand $d_U = \tau_U\frac{E(v|p)-p}{\rho}$. The market-clearing condition is

$$\lambda d_I + (1-\lambda)d_U = S + \epsilon,$$

from which I can derive the linear pricing coefficients as follows:

$$\begin{aligned}
\alpha_0 &= \frac{(1-\lambda)\hat{\tau}_\theta}{\lambda\tau_e+\hat{\tau}_\theta+\tau_y+\tau_p},\ \alpha_\theta = \frac{\lambda(\tau_e+\hat{\tau}_\theta)+\tau_y+\tau_p}{\lambda\tau_e+\hat{\tau}_\theta+\tau_y+\tau_p},\ \alpha_y = \frac{(1-\lambda)\tau_y}{\lambda\tau_e+\hat{\tau}_\theta+\tau_y+\tau_p},\\
\alpha_\epsilon &= \frac{\rho(\tau_e+\hat{\tau}_\theta+\tau_y+\tau_p/\lambda)}{\tau_e(\lambda\tau_e+\hat{\tau}_\theta+\tau_y+\tau_p)},\quad \alpha_S = \frac{\rho(\tau_e+\hat{\tau}_\theta+\tau_y+\tau_p)}{\tau_e(\lambda\tau_e+\hat{\tau}_\theta+\tau_y+\tau_p)},
\end{aligned} \tag{A.26}$$

where $\hat{\tau}_\theta = 1/(\Sigma\hat{k}^2)$ and $\tau_p = (\lambda\tau_e/\rho)^2\tau_\epsilon$. Note that all the coefficients are functions of the conjectured $\hat{k}$ via $\hat{\tau}_\theta = (\hat{k}^2\Sigma)^{-1}$.

Having characterized the pricing function, the search for equilibrium resembles the proof of Proposition 1, and, therefore, I only point out the main differences. The first-order condition for k becomes

$$\mu - k = \rho_M\alpha_\theta\frac{d\mathrm{var}(\theta)}{dk}. \tag{A.27}$$

The investors' indifference condition that determines the equilibrium fraction of informed investors λ is

$$\exp(\rho * F) = \sqrt{1+\frac{\tau_e}{\tau_y+\tau_p+\hat{\tau}_\theta}}. \tag{A.28}$$

The interior equilibrium is fully characterized by the system of equations: (A.27), (A.28), and

$\hat{k} = k$ as in Proposition 1. Given a $\tau_y \geq 0$, the lower bound $\underline{F}(\tau_y)$ and the upper bound $\bar{F}(\tau_y)$ for $\lambda \in (0, 1)$ are characterized as in the proof of Proposition 1. ■

Proof of Proposition 6. Rewrite the two equations (A.27) and (A.28) as $f_1(k, \lambda; \tau_y) = 0$ and $f_2(k, \lambda; \tau_y) = 0$. I apply the implicit function theorem to obtain

$$\begin{bmatrix} \frac{dk}{d\tau_y} \\ \frac{d\lambda}{d\tau_y} \end{bmatrix} = - \begin{bmatrix} \frac{\partial f_1}{\partial k} & \frac{\partial f_1}{\partial \lambda} \\ \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial \lambda} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial f_1}{\partial \tau_y} \\ \frac{\partial f_2}{\partial \tau_y} \end{bmatrix}. \tag{A.29}$$

It follows from (A.29) that

$$\frac{dk}{d\tau_y} > 0 \text{ and } \frac{d\lambda}{d\tau_y} < 0. \tag{A.30}$$

Starting from $\tau_y = 0$, which is the no-disclosure case studied in Propositions 1, increasing τ_y increases k and decreases λ. Let τ_y^* be the precision level such that λ is lowered to zero for the first time. Such a τ_y^* exists and is unique. First, the value of information – the right-hand side of (A.28) – is decreasing in τ_y. Second, we know $\lambda > 0$ for $\tau_y = 0$ and $\lambda = 0$ for sufficiently large τ_y. (Recall the maintained assumption $F < \overline{F}$ in (A.5), which ensures $\lambda > 0$ at $\tau_y = 0$.)

λ will stay at zero for $\tau_y > \tau_y^*$. The price function in this case becomes

$$p = \frac{\hat{\tau}_\theta}{\hat{\tau}_\theta + \tau_y}\hat{\mu}_\theta + \frac{\tau_y}{\hat{\tau}_\theta + \tau_y} y - \rho \frac{\tau_e + \hat{\tau}_\theta + \tau_y}{\tau_e(\hat{\tau}_\theta + \tau_y)}(S + \epsilon), \tag{A.31}$$

where $\hat{\mu}_\theta = \mu\hat{k} - \frac{\hat{k}^2}{2}$ and $\hat{\tau}_\theta = (\hat{k}^2\Sigma)^{-1}$ depend on the investors' conjectured $\hat{k}$. The manager takes the price function (A.31) as given and chooses k to maximize $E[p|\hat{k}, k] - \rho_M \text{var}(p|\hat{k}, k)$. The first-order condition is $\mu - k - 2\rho_M \frac{\tau_y}{\hat{\tau}_\theta + \tau_y} k\Sigma = 0$. Imposing rational expectations, $\hat{k} = k$, one can solve the equilibrium k as the unique fixed point of the following condition:

$$\mu - k - 2\rho_M \frac{\tau_y}{(k^2\Sigma)^{-1} + \tau_y} k\Sigma = 0. \tag{A.32}$$

The existence and uniqueness of the fixed point is guaranteed by noting that the left-hand side of (A.32) is decreasing in k, is positive at $k \to 0$, and is negative for large k. It follows from the first-order condition (A.32) that

$$\frac{dk}{d\tau_y} = -\frac{2\rho_M k^3 \Sigma^2}{\tau_y k^2 \Sigma \left(k^2 \Sigma \tau_y (2\rho_M \Sigma + 1) + 6\rho_M \Sigma + 2\right) + 1} < 0. \tag{A.33}$$

(A.30) shows $\frac{dk}{d\tau_y} > 0$ for $\tau_y < \tau_y^*$ and (A.33) shows $\frac{dk}{d\tau_y} < 0$ for $\tau_y > \tau_y^*$. This verifies the claim that the expected firm value $E[v]$ achieves its maximum at τ_y^*.

The remainder of the proof verifies the claim about investors' welfare. Note that the welfare expression (14) in Proposition 3 can be easily modified to incorporate the public disclosure. If an investor does not have share endowment (i,e., $s_i = 0$), her ex ante welfare is

$$E[U] = -\frac{\exp(\mathbf{N})}{\sqrt{1+\mathbf{R}}}, \tag{A.34}$$

where $\mathbf{R} = \frac{\text{var}[E(v|p)-p]}{var(v|p)} = \frac{\rho^2(\tau_\theta+\tau_y)(\tau_\theta+\tau_y+\tau_p+\tau_e)}{\tau_e\tau_\epsilon(\tau_\theta+\tau_y+\tau_p+\lambda\tau_e)^2}$, $\mathbf{N} = -\frac{S^2\,\alpha_S^2}{2(\text{var}(v|p)+\text{var}[E(v|p)-p])}$, and the price coefficient $\alpha_S = \frac{\rho(\tau_e+\tau_\theta+\tau_y+\tau_p)}{\tau_e(\lambda\tau_e+\tau_\theta+\tau_y+\tau_p)}$ depend on the disclosure precision τ_y.

For $\tau_y \le \tau_y^*$, one can verify that

$$\begin{aligned}
\frac{d\mathbf{R}}{d\tau_y} &= \frac{\partial \mathbf{R}}{\partial \tau_y} + \frac{\partial \mathbf{R}}{\partial \lambda}\frac{d\lambda}{d\tau_y} + \frac{\partial \mathbf{R}}{\partial \tau_\theta}\frac{\partial \tau_\theta}{\partial k}\frac{dk}{d\tau_y} > 0, \\
\frac{d\mathbf{N}}{d\tau_y} &= \frac{\partial \mathbf{N}}{\partial \tau_y} + \frac{\partial \mathbf{N}}{\partial \lambda}\frac{d\lambda}{d\tau_y} + \frac{\partial \mathbf{N}}{\partial \tau_\theta}\frac{\partial \tau_\theta}{\partial k}\frac{dk}{d\tau_y} < 0,
\end{aligned}$$

where $\frac{dk}{d\tau_y} > 0$ and $\frac{d\lambda}{d\tau_y} < 0$ are obtained in (A.29). It follows that $E[U] = -\frac{\exp(\mathbf{N})}{\sqrt{1+\mathbf{R}}}$ is increasing in τ_y for $\tau_y \le \tau_y^*$.

For $\tau_y > \tau_y^*$, we can further simplify $\mathbf{R}$ and $\mathbf{N}$ defined above because $\lambda = 0$ in this case. One can verify the following (using $\frac{dk}{d\tau_y} < 0$ obtained in (A.33)):

$$\begin{aligned}
\frac{d\mathbf{R}}{d\tau_y} &= \frac{\partial \mathbf{R}}{\partial \tau_y} + \frac{\partial \mathbf{R}}{\partial \tau_\theta}\frac{\partial \tau_\theta}{\partial k}\frac{dk}{d\tau_y} < 0, \\
\frac{d\mathbf{N}}{d\tau_y} &= \frac{\partial \mathbf{N}}{\partial \tau_y} + \frac{\partial \mathbf{N}}{\partial \tau_\theta}\frac{\partial \tau_\theta}{\partial k}\frac{dk}{d\tau_y} \ge 0.
\end{aligned}$$

It follows that $E[U] = -\frac{\exp(\mathbf{N})}{\sqrt{1+\mathbf{R}}}$ is decreasing in τ_y for $\tau_y > \tau_y^*$, implying that $E[U]$ is uniquely maximized at $\tau_y = \tau_y^*$. ■

Proof of Proposition 7. Much of the proof is to characterize the equilibrium stated in the lemma below. Comparative statics of the equilibrium with respect to F are proved afterwards.

Lemma 2 *There exists $\underline{\rho}_M$ and $\overline{\rho}_M$ such that $a_2 = 0$ for $\rho_M < \underline{\rho}_M$ and $a_2 = 1$ for $\rho_M > \overline{\rho}_M$. For $\rho_M \in [\underline{\rho}_M, \bar{\rho}_M]$, the equilibrium $a_2 \in [0,1]$ and $\lambda \in (0,1)$ are uniquely characterized by the system of equations $\rho_M \frac{\tau_p+\lambda(\tau_e+\tau_\theta)}{\lambda\tau_e+\tau_\theta+\tau_p}|\frac{\text{dvar}[\theta]}{da_2}| = \frac{dE[\theta]}{da_1}$ and $\exp(\rho F) = \sqrt{1+\frac{\rho^2\tau_e}{(\lambda\tau_e)^2\tau_\epsilon+\rho^2/\text{var}[\theta]}}$.*

Proof of Lemma 2. Given the price coefficient $B(\hat{a}_2)$, the marginal benefit of the mean-improving a_1 perceived by the manager is $B(\hat{a}_2) \times \frac{dE[\theta]}{da_1}$ and the marginal benefit of risk-reducing a_2 is $\rho_M B^2(\hat{a}_2)|\frac{\text{dvar}[\theta]}{da_2}|$. A necessary condition for both a_1 and a_2 to be positive is that they share the same marginal benefit: $B(\hat{a}_2) \times \frac{dE[\theta]}{da_1} = \rho_M B^2(\hat{a}_2)|\frac{\text{dvar}[\theta]}{da_2}|$. Recall that the maintained assumption $\underline{F} < F < \overline{F}$ ensures $\lambda \in (0,1)$ and, hence, $B > 0$ in equilibrium. One can therefore express $B(\hat{a}_2) \times \frac{dE[\theta]}{da_1} = \rho_M B^2(\hat{a}_2)|\frac{\text{dvar}[\theta]}{da_2}|$ equivalently as:

$$\frac{dE[\theta]}{da_1} = \rho_M B(\hat{a}_2)|\frac{\text{dvar}[\theta]}{da_2}|. \tag{A.35}$$

The two equations stated in the lemma are the investors' indifferent condition (A.2) and (A.35) above (after substituting $\hat{B} = \frac{\tau_p+\lambda(\tau_e+\tau_\theta(\hat{a}_2))}{\lambda\tau_e+\tau_\theta(\hat{a}_2)+\tau_p}$ and imposing $\hat{a}_2 = a_2$).

Given a $\hat{a}_2 \in [0,1]$, a unique λ is determined from the investors' indifferent condition (A.2). Therefore, the price coefficient $B(\hat{a}_2) = \frac{\tau_p+\lambda(\tau_e+\tau_\theta(\hat{a}_2))}{\lambda\tau_e+\tau_\theta(\hat{a}_2)+\tau_p}$ is fully determined once $\hat{a}_2 \in [0,1]$ is specified. One can solve $\underline{\rho}_M$ and $\overline{\rho}_M$ from (A.35) after setting $a_2 = 0$ and $a_2 = 1$, respectively. That is,

$$\begin{aligned}\underline{\rho}_M &= \tfrac{dE[\theta]}{da_1}/(B(a_2)\,|\tfrac{\text{dvar}[\theta]}{da_2}|) \quad |_{a_2=0,a_1=1},\\ \overline{\rho}_M &= \tfrac{dE[\theta]}{da_1}/(B(a_2)\,|\tfrac{\text{dvar}[\theta]}{da_2}|) \quad |_{a_2=1,a_1=0}.\end{aligned}$$

To prove $a_2 = 0$ for $\rho_M < \underline{\rho}_M$, I differentiate $B(\hat{a}_2) = \frac{\tau_p+\lambda(\tau_e+\tau_\theta(\hat{a}_2))}{\lambda\tau_e+\tau_\theta(\hat{a}_2)+\tau_p}$ with respect to $\hat{a}_2$ and obtain

$$\frac{dB(\hat{a}_2)}{d\hat{a}_2} = \underbrace{\frac{\partial B}{\partial \lambda}\frac{d\lambda}{d\hat{a}_2}}_{-} + \underbrace{\frac{\partial B}{\partial \tau_\theta}\frac{\partial \tau_\theta}{\partial \hat{a}_2}}_{-} < 0. \tag{A.36}$$

That is, the price coefficient is decreasing in the conjectured risk-reducing attention $\hat{a}_2$. This means $B(\hat{a}_2 = 0) \geq B(\hat{a}_2)$ for any $\hat{a}_2 \in [0, 1]$. One can show the following for $\rho_M < \underline{\rho}_M$:

$$\begin{aligned} \rho_M B(\hat{a}_2)\,|\frac{d\text{var}[\theta]}{da_2}| \quad &< \quad \underline{\rho}_M\, B(\hat{a}_2)\,|\frac{d\text{var}[\theta]}{da_2}| \\ &\leq \quad \underline{\rho}_M B(\hat{a}_2 = 0)\,|\frac{d\text{var}[\theta]}{da_2}|_{a_2=0} \\ &= \quad \frac{dE[\theta]}{da_1}\,|_{a_1=1} \\ &\leq \quad \frac{dE[\theta]}{da_1} \quad \forall\, \hat{a}_1, \hat{a}_2 \in [0, 1]. \end{aligned} \tag{A.37}$$

The first inequality is by $\rho_M < \underline{\rho}_M$. The second inequality uses $B(\hat{a}_2 = 0) \geq B(\hat{a}_2)$ for any $\hat{a}_2$ (see A.36) and the assumption that var$[\theta]$ features weakly diminishing marginal returns. The equality is by the construction of $\underline{\rho}_M$, and the last inequality relies on the the assumption that $E[\theta]$ features weakly diminishing marginal returns. The inequality (A.37) shows that, for $\rho_M < \underline{\rho}$, the manager's marginal benefit of a_2 is less than his marginal benefit of a_1 no matter what conjecture $\hat{a}_2$ investors hold. Therefore, $a_2 = 0$ is the unique equilibrium for $\rho_M < \underline{\rho}_M$.

Similarly, for $\rho_M > \overline{\rho}_M$, one can show the following:

$$\begin{aligned} \rho_M B(\hat{a}_2)\,|\frac{d\text{var}[\theta]}{da_2}| \quad &> \quad \overline{\rho}_M\, B(\hat{a}_2)\,|\frac{d\text{var}[\theta]}{da_2}| \\ &\geq \quad \overline{\rho}_M B(\hat{a}_2 = 1)\,|\frac{d\text{var}[\theta]}{da_2}|_{a_2=1} \\ &= \quad \frac{dE[\theta]}{da_1}\,|_{a_1=0} \\ &\geq \quad \frac{dE[\theta]}{da_1} \quad \forall\, \hat{a}_1, \hat{a}_2 \in [0, 1]. \end{aligned} \tag{A.38}$$

The first inequality is by $\rho_M > \overline{\rho}_M$ and the second one uses $B(\hat{a}_2) \geq B(\hat{a}_2 = 1)$ for any $\hat{a}_2 \leq 1$ and var$[\theta]$ featuring weakly diminishing marginal returns. The equality is by the construction of $\overline{\rho}_M$ and the last inequality is by the weakly diminishing marginal returns. Inequality (A.38) shows that for $\rho_M > \overline{\rho}_M$, the marginal benefit of a_2 is higher than that of a_1 regardless of the investors' conjecture $\hat{a}_2$. This proves $(a_1 = 0, a_2 = 1)$ is the unique equilibrium for $\rho_M > \overline{\rho}_M$.

It remains to prove that the solution to the system of equations in Lemma 2 exists and is unique for $\rho_M \in (\underline{\rho}_M, \overline{\rho}_M)$. Note that, for a given $a_2 \in [0, 1]$, a unique $\lambda(a_2)$ is determined by the indifference condition $\exp(\rho F) = \sqrt{1 + \frac{\rho^2\tau_e}{(\lambda\tau_e)^2\tau_\epsilon + \rho^2/\text{var}[\theta]}}$. One can then substitute a_2 and $\lambda(a_2)$ to obtain the price coefficient $B(a_2) \equiv B(a_2, \lambda(a_2))$. That is, the system of equations allows us to express $\lambda(a_2)$, $B(a_2)$, and $a_1 = 1 - a_2$ all as implicit functions of a_2. Substituting $\lambda(a_2)$, $B(a_2)$, and $a_1(a_2)$ into (A.35), one can solve the equilibrium a_2^* as the fixed point to the equation below:

$$B\left(a_2, \lambda(a_2)\right) = \frac{1}{\rho_M} \frac{\frac{dE[\theta]}{da_1}}{|\frac{d\text{var}[\theta]}{da_2}|}. \tag{A.39}$$

We know from (A.36) and its discussion that the left-hand side of (A.39) decreases in a_2 and the right-hand side increases in a_2. Therefore, there exists at most one fixed point a_2 to (A.39). The existence of the fixed-point a_2 is proven by examining the boundary values of $a_2 = 0$ and $a_2 = 1$. Given how $\underline{\rho}_M$ and $\overline{\rho}_M$ are constructed above, we know that $a_2 = 0$ and $a_2 = 1$ satisfy (A.39) as an equality at $\rho_M = \underline{\rho}_M$ and at $\rho_M = \overline{\rho}_M$, respectively. For $\rho_M \in (\underline{\rho}_M, \overline{\rho}_M)$, it is easy to verify that the left-hand side of (A.39) is greater than its right-hand side at $a_2 = 0$, and the opposite is true at $a_2 = 1$. The existence of the fixed point $a_2 \in (0, 1)$ to (A.39) follows the standard fixed-point argument. ■

Having characterized the equilibrium in Lemma 2 above, the remainder proves that, for $\rho_M \in (\underline{\rho}_M, \overline{\rho}_M)$ (hence, both $a_1, a_2 > 0$), a lower F increases the attention devoted to the risk-reducing a_2. Suppose that the cost of information acquisition is reduced from F to $F' = F - \epsilon$. Denote by (a_1, a_2, B, λ) and $(a_1', a_2', B', \lambda')$ the equilibrium under F and F', respectively. I prove $a_2' > a_2, B' \geq B$, and $\lambda' > \lambda$ by showing a sequence of claims.

Claim 1: $a_2' > a_2$. Suppose by contradiction $a_2' \leq a_2$, in which case the value of information shown below is weakly higher under $a_2'(\leq a_2)$ because τ_θ is lower under a_2':

$$value = \sqrt{1 + \frac{\rho^2 \tau_e}{(\lambda \tau_e)^2 \tau_\epsilon + \rho^2 \tau_\theta}}.$$

A weakly higher value, together with a strictly lower cost (i.e., $F' < F$), results in more information acquisition, i.e., $\lambda' > \lambda$. It's easy to verify that the price coefficient $B = \frac{\tau_p + \lambda(\tau_e + \tau_\theta)}{\lambda \tau_e + \tau_\theta + \tau_p}$ satisfies

$$\frac{\partial B}{\partial \tau_\theta} < 0, \quad \frac{\partial B}{\partial \lambda} > 0. \tag{A.40}$$

We can therefore conclude the following from $\lambda' > \lambda$ and $a_2' \leq a_2$ (hence, $\tau_\theta' \leq \tau_\theta$):

$$B' > B. \tag{A.41}$$

Given the diminishing marginal returns, $a_2' \leq a_2$ (hence, $a_1' \geq a_1$) means that $|\frac{d\text{var}[\theta]}{da_2}|$ is weakly higher under a_2' and $\frac{dE[\theta]}{da_1}$ is weakly lower under a_1'. It follows from (A.39) that $B' \leq B$. However, finding $B' \leq B$ contradicts (A.41), proving that the assumption $a_2' \leq a_2$ is false.

Claim 2: $B' \geq B$. Claim 1 shows $a_2' > a_2$ and, hence, $a_1' < a_1$. It follows that $|\frac{d\text{var}[\theta]}{da_2}|$ is weakly lower under a_2' and $\frac{dE[\theta]}{da_1}$ is weakly higher under a_1'. The claim $B' \geq B$ follows from $B = \frac{dE[\theta]}{da_1} / (\rho_M |\frac{d\text{var}[\theta]}{da_2}|)$ in (A.39). Further, the equality $B' = B$ holds if and only if both $E[\theta|a_1]$ and $\text{var}[\theta|a_2]$ are linear functions, and $B' > B$ holds if at least one features strictly diminishing marginal returns.

Claim 3: $\lambda' > \lambda$. Recall from (A.40) $\frac{\partial B}{\partial \lambda} > 0$ and $\frac{\partial B}{\partial \tau_\theta} < 0$. The claim $\lambda' > \lambda$ follows from $B' \geq B$ in Claim 2 and $a_2' > a_2$ (hence, $\tau_\theta' > \tau_\theta$) in Claim 1. ■