Common Ownership, Competition, and Corporate Governance

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Keywords: Common ownership, corporate governance, antitrust

JEL Classifications: G34, L13

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Common Ownership, Competition, and Corporate Governance*

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Abstract

This paper presents a theoretical framework for determining the ownership stakes held by financial investors in companies competing in the same product market, or, in other words, the level of common ownership. In our model, the primary motivation for these investors is the anticipation of capital gains resulting from the impact of common ownership on product market competition, which leads to increased profitability for the firms involved. On the other hand, common ownership undermines effective corporate governance by reducing monitoring, increasing extraction of private benefits by the manager, and inhibiting investments that contribute to firm value. These negative effects on corporate governance act as limiting factors, ultimately determining the equilibrium level of common ownership.

Keywords: Antitrust; Common Ownership; Corporate Governance.

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1 Introduction

This paper presents a new theory of the drivers of common ownership, which is consistent with the emerging consensus regarding its anticompetitive effects. Common ownership occurs when financial investors hold stakes in companies that compete in the same product market. The traditional view used to be that these investors aim to simply diversify risk and that they are passive, not seeking to influence the strategies of their portfolio companies. However, a new consensus is emerging in the field of industrial organization, which asserts that companies with overlapping ownership engage in less intense competition compared to traditional profit-maximizing entities. The reduced competition leads to higher prices and profits, thereby enhancing the value of the companies involved. If this is indeed the case, the potential to capture this additional value must be one of the reasons behind the financial investments that lead to common ownership. In fact, these motives might easily outweigh the incentives for risk diversification, as institutional investors already maintain well-diversified portfolios, limiting the marginal benefits from further diversification.

Considering the positive impact of common ownership on firms' profits and the permissive policy environment,[1] the question that arises for these companies is what factors constrain the extent of common ownership. We argue that the downside of common ownership is its exacerbation of agency problems within companies. For example, consider a scenario where each firm competing in a market has a manager who can appropriate private benefits of control or avoid making efforts to reduce costs.

We consider the case where firms have blockholders that have the incentives to closely monitor the managers in order to limit these opportunistic behaviors. When external financial investors acquire shares from these shareholders with the aim of softening the intensity of competition, they reduce the residual ownership stake of blockholders, thereby diminishing their incentives to engage in value-enhancing behaviors. Financial investors, on the other hand, may lack either the ability or the incentive to take actions that mitigate the agency problems.[2] In this case, a trade-off emerges: the transfer of shares from the monitoring blockholder to the financial investor reduces the intensity of competition but simultaneously facilitates the appropriation of private benefits from the manager.

The equilibrium level of common ownership strikes a balance between these conflicting effects. We show that the optimal balance depends on several factors, with the intensity of product market competition and the quality of corporate governance rules and institutions being among the most significant. We demonstrate that as competition becomes more intense, the degree of common ownership increases. Additionally, the greater the need for monitoring managers, the lower the degree of common ownership. This finding implies that improvements in corporate governance,

[1] Proposals to limit the degree of overlapping ownership have been put forward in the scholarly debate (see e.g. Elhauge 2016, Posner 2017 and Rock and Rubinfield, 2018) but have had little impact on antitrust policy so far.

[2] The financial investor may have less incentive to monitor the manager, because a reduction in the cost of a firm would lead to lower profits for its rivals, of which it owns a share.

which reduce the necessity for manager monitoring, are beneficial for shareholders but have adverse effects on consumers, as they result in greater common ownership and hence eventually in higher prices.

The trade-off between softer competition and better corporate governance would not arise if the financial investor acquired its shares from dispersed shareholders. However, as explained in Grossman and Hart (1980), far-sighted dispersed shareholders would demand a price for their shares that reflects their ex-post value, thus incorporating all the benefits from common ownership. Large blockholders, on the the other hand, internalize the acquisition externality. That is, they might be willing to tender their shares for a price lower than the ex-post value, understanding that without doing so, the acquisition would not take place. Therefore, our theory applies to firms that have shareholders with substantial ownership stakes, alongside a multitude of dispersed shareholders.

Despite the free-rider problem, the financial investor can profitably acquire some shares from the dispersed shareholders if it simultaneously acquires a stake from the controlling blockholders. This is because once it holds a positive stake in a company, the investor can leverage this position to capture some of the advantages resulting from reduced competition, even if dispersed shareholders cannot be exploited. Indeed, we demonstrate that it is always optimal to complement the acquisition of shares from large shareholders with the acquisition from dispersed shareholders. However, even for the latter acquisition, there is a limiting force: as the stake of the financial investor increases, the value of the shares held by the blockholders increases, and the financial investor must pay a higher price to acquire them.

While our baseline model focuses on the case where the common owner is an external investor, another possibility is that a company's blockholders can directly acquire a stake in a rival company. We refer to this case as cross-ownership, with a slight abuse of terminology. As long as each blockholder has an advantage in monitoring the company it controls, the trade-off between softer competition and less effective governance also arises under cross-ownership. However, there are notable differences between cross-ownership and common ownership. Firstly, cross-ownership can be profitable even when it is unilateral, meaning that only one firm acquires a stake in a rival company. In contrast, an external investor must acquire stakes in at least two competing companies to achieve a reduction in competition. Secondly, cross-ownership may be advantageous even if the acquisition is solely from dispersed shareholders. In fact, the benefits arising from reduced competition are captured by the acquirer through the initial stake it already owns in one of the firms.

Our baseline model focuses on the role of large blockholders in monitoring managers who may extract private benefits of control. However, we also explore two additional variants that introduce different types of agency problems. In the first variant, the controlling blockholders themselves have the ability to appropriate private benefits at the expense of smaller shareholders. If such appropriation is inefficient, as in Burkart et al. (1998), the extent of overlapping ownership will be limited because smaller blockholders have less incentive to internalize the deadweight losses resulting from rent extraction. In the second variant, overlapping ownership diminishes the large shareholders' incentive to make investments that increase firm profits but come with private costs, as in Anton et al. (2022). The trade-off between softer product market competition and more pronounced agency problems emerges in these alternative frameworks as well.

The remainder of the paper is organized as follows. Section 2 provides a review of the relevant literature. In Section 3, we introduce the baseline model of common ownership. Section 4 offers a comprehensive analysis of the equilibrium ownership structure, focusing on the scenario where financial investors exclusively acquire shares from large shareholders. Expanding on this, Section 5 extends the analysis to the more general case of acquiring shares from both large and dispersed shareholders. In Section 6, we turn to the case of cross-ownership. Section 7 investigates the alternative agency problems mentioned earlier. Finally, Section 8 concludes the paper by discussing various model extensions and the potential policy implications. All proofs are included in Appendix A, while Appendix B presents specific examples.

2 Relation to the literature

In this paper, we borrow from two strands of the literature, one on common ownership and the other one on ownership structure and corporate governance.

One of our basic assumption is that common ownership leads to softer competition. The notion that overlapping ownership mitigates the intensity of product market competition was first put forward by Rotemberg (1984) and O'Brien and Salop (2000). Rotemberg (1984) assumes that companies act in the interest of their shareholders, and that any heterogeneity in shareholders' interests is accounted for by forming a weighted average of their payoffs, with weights given by their respective ownership shares. As a result, under common ownership each firm maximizes a linear combination of own and rivals' profits. The higher the relative weight given to rivals' profits, which is commonly referred to as the "lambda," the less aggressively firms compete in product markets, and hence the higher prices and profits.

The literature has proposed various mechanisms that may lead firms' managers to internalize the interests of minority shareholders. For example, Azar (2017) develops a theory where a company's management proposes a strategic plan to its shareholders and dislikes their disapproval or opposition.[3] As another example, Anton et al. (2023) study a mechanism based on managerial incentives. They argue that firms with common owners tolerate managerial slack to a higher degree in order to keep prices and profits high. Schmalz (2021) reviews these and other possible governance mechanisms whereby common ownership may affect competitive outcomes. Shekita (2021) analyzes the channels through which common ownership influences firm behavior empirically. Studying 30 cases of common ownership, he documents three main corporate governance mechanisms – voice and engagement, executive compensation, and voting – that affect firm decision-making.

Some scholars, on the other hand, continue to adhere to the traditional view that

[3] Azar (2020) further argues the anticompetitive effects of common ownership are mitigated when managers are entrenched. Yet, they only disappear in the extreme case where managers are fully insulated from shareholders dissent.

index funds are passive investors that do not intervene in their portfolio companies and thus cannot facilitate anticompetitive behavior. See e.g. Bebchuk et al. (2017) for a recent articulation of this view.

The last few years have witnessed a blooming of empirical studies on common ownership, motivated by the observation that overlapping ownership, and in particular common ownership, has been on the rise in the last decades. For example, Backus at al. (2021b) calculate the weight that S&P 500 companies would place on rivals' profits in their objective function under the proportional-control assumption and show that the average weight tripled in the last decades, from 0.2 in the 1980s to almost 0.7 in 2017.

Like the theoretical literature, the empirical literature on the effects of common ownership is also unsettled. In a pioneering contribution, Azar, Schmalz and Tecu (2018) show that common ownership increases prices in the U.S. airline industry. Their findings are confirmed in the analysis carried out by Park and Seo (2019). However, Kennedy et al. (2017) and Dennis et al. (2022), using a different structural model of the US airline industry or different measures of investor control of airlines operating in bankruptcy, do not find evidence that common ownership raises airline prices.[4] Azar, Schmalz and Tecu (2021) address these critiques and argue that in fact they do not invalidate their main finding. He and Huang (2017), using a sample of U.S. public firms from 1980 to 2014, find evidence suggesting that institutional cross-ownership facilitates explicit forms of product market collaboration (e.g., within-industry joint ventures, strategic alliances, or within-industry acquisitions) and improves innovation productivity and operating profitability.[5] On the other hand, Backus et al. (2021a) find little support for markup effects of common ownership in the ready-to-eat cereal industry.

In spite of the ongoing controversies, a consensus seems to be emerging that common ownership does affect product market competition.[6] Some authors have ventured to quantify the welfare effects of common ownership. For example, Ederer and Pellegrino (2022) estimate that the welfare cost of common ownership, measured as the ratio of deadweight loss to total surplus, has increased more than tenfold in

[4] In a similar vein, Lewellen and Lowry (2021) contend that the effects that are commonly attributed to common ownership are caused by other factors, such as differential responses of firms (or industries) to the 2008 financial crisis. Controlling for thes factors, they find little robust evidence that common ownership affects firm behavior.

[5] Theory indeed shows that overlapping ownerships may affect not only prices but also other strategic choices of the firms, such as for instance investment in R&D: see e.g. Lopez and Vives (2019).

[6] While most of the literature focuses on a single industry, Azar and Vives (2021) analyze common ownership in a general equilibrium oligopoly model.

They argue that when common ownerships extends from a specific sector to the whole economy, it will reduce markups and prices. This follows from the fact that when an industry expands, it creates positive externalities for other industries. Inter-industry common ownership allows firms to better internalize these externalities, creating incentives for firms to expand output and reduce prices. In an attempt to empirically test this prediction,

Azar and Vives (2022) reconsider the US airline industry and find that common ownership by the Big Three (BlackRock, Vanguard and State Street), taken as a proxy for economy-wide common ownership, is associated with lower airline prices, whereas common ownership by other investors, taken as a proxy for industry-specific common ownership, is associated with higher prices.

about 20 years, from 0.3% in 1994 to over 4% in 2018.

The second strand of the literature that is relevant for our paper deals with the impact of the ownership structure on corporate governance and firm value. Jensen and Meckling (1976) argue that the ownership structure that maximizes total firm value is the one where the entrepreneur or the manager is the sole owner as it reduces agency costs. One component of the agency cost are monitoring costs. Manne (1976), on the other hand, presents an opposing viewpoint, suggesting that even firms with dispersed ownership can maximize share value in the presence of an active market for corporate control. Deviations from value maximization trigger a disciplinary takeover that ousts the less competent or opportunistic manager. However, Grossman and Hart (1980) demonstrate the flaws in this argument, revealing that atomistic shareholders tend to free-ride on each other, resulting in the failure of value-increasing tender offers. Shleifer and Vishny (1986) show that the large shareholders can mitigate the free-rider problem highlighted by Grossman and and Hart. In fact, a large shareholder, while not profiting from shares acquired in a tender offer, can benefit from the toehold owned prior to the takeover. A common thread in these papers is the notion that a more concentrated ownership can generate higher firm value. However, the extent of ownership concentration is limited by various factors. One evident constraint is the lack of diversification (see Demsetz and Lehn (1985)). Additionally, Burkart et al. (1997), building on Aghion and Tirole (1997), point out that increased monitoring by a large shareholder can stifle manager's initiative. The optimal ownership structure strikes a balance between the manager's incentives to exert effort and the large shareholder's monitoring. A survey on corporate governance and its impact of firm ownership and firm value is contained in Shleifer and Vishny (1997).

3 The model and preliminary results

In this section, we outline the assumptions of our baseline model of common ownership and derive some preliminary results.

3.1 Model assumptions

As mentioned in the introduction, we concentrate on financial investors that are already well diversified. Their acquisition of stakes in competing firms is solely motivated by the expectation of capital gains, resulting from the anticipated increase in firm value due to reduced competition.

Additionally, we assume the presence of shareholders who initially are large enough to internalize the acquisition-price externality and to monitor the managers. We simplify the model by assuming that each company initially has a single blockholder, along with a mass of dispersed shareholders.

After developing this baseline model, we shall consider several variants in the subsequent sections.

3.1.1 Agents

Consider an industry comprising two initially symmetric firms, denoted as $i = 1, 2$, that compete in the same product market. Initially, firm i is owned by blockholder $\mathcal{B}_i$, who possesses a fraction β of the firm's shares, and a multitude of dispersed shareholders that collectively hold the remaining fraction $1 - \beta$.[7]

A financial investor, referred to as $\mathcal{I}$, may acquire a stake s_i of firm i from its initial owners. It is assumed that $\mathcal{I}$'s portfolio is already well diversified.[8] Therefore, $\mathcal{I}$ will proceed with the acquisition to maximize its capital gains. Such capital gains may arise due to the impact of common ownership on the intensity of competition, which is a crucial aspect of the analysis. We allow the investor to acquire shares from both the blockholder and dispersed shareholders. Denote the stake acquired from the blockholder as s_i^B and from the dispersed shareholders as s_i^D. Thus, $s_i = s_i^B + s_i^D$, and after the acquisition, $\mathcal{B}_i$ will retain a remaining ownership share of $\beta - s_i^B$.

Firm i is managed by $\mathcal{M}_i$, who, if not supervised, diverts a fraction ξ of the firm's profits for her personal benefit. (To avoid confusion, we use feminine pronouns for managers, masculine pronouns for blockholders, and neutral pronouns for financial investors).

The extraction of private benefits by managers is limited by the monitoring activities of shareholders. It is assumed that monitoring efforts are solely undertaken by blockholders. Dispersed shareholders free ride on the blockholder's monitoring efforts and do not contribute any efforts of their own. The financial investor, on the other hand, is assumed to have a lower capacity for monitoring the managers compared to the blockholders and thus does not attempt to duplicate their activity.

Blockholder $\mathcal{B}_i$'s monitoring, denoted as m_i, reduces the manager's private benefits from $\xi\pi_i$ to $\xi(1-m_i)\pi_i$. The private cost of monitoring, $C(m_i)\pi_i$, is assumed to be proportional to the firm's profit. This facilitates the analysis by making $\mathcal{B}_i$'s optimal choice of m_i independent of product market competition. The function $C(m_i)$ is assumed increasing and convex, with $C(0) = 0$. To ensure the existence of an interior solution, we assume $C'(0) = 0$ and $C'(1) > \xi$.

3.1.2 Payoffs

Under these assumptions, blockholder $\mathcal{B}_i$'s payoff can be expressed as:

$$B_i = \left(\beta - s_i^B\right) [1 - \xi(1 - m_i)]\, \pi_i - C(m_i)\pi_i + P_i^B(s_i^B). \tag{1}$$

The first term on the right-hand side denotes the value of $\mathcal{B}_i$'s remaining stake $(\beta - s_i^B)$, the second term represents the cost of monitoring, and $P_i^B(s_i)$ represents the revenue obtained from the sale of the stake s_i^B to the investor. Similarly, the

[7]As a rule, we use latin letters to denote endogenous variables, calligraphic letters for agents, and greek letters for exogenous parameters. The one exception is profits, which are denoted by π.

[8]See Shy and Stenbacka (2020) for a model that instead emphasizes the diversification motive for common ownership.

investor's payoff is given by:

$$I = \sum_{i=1}^{2} \left\{ s_i \left[1 - \xi(1 - m_i)\right] \pi_i - P_i^B(s_i^B) - P_i^D(s_i^D) \right\}, \tag{2}$$

where $P_i^D(s_i^D)$ is the total payment to dispersed shareholders for the block s_i^D.

Regarding the managers, it is assumed that they appropriate whatever private benefits they can while being subject to monitoring by blockholders. Additionally, managers are responsible for making choices related to product market competition. Building upon the recent literature on common ownership discussed in section 2, it is assumed that when managers make these strategic decisions, they aim to maximize a linear combination of the shareholders' payoffs, where the weights are proportional to their respective ownership shares:

$$O_i = \left(\beta - s_i^B\right) B_i + \theta s_i I. \tag{3}$$

In equation (3), the level of influence of the financial investor is parametrized by θ, with $\theta \leq 1$ to account for the possibility that it may be lower than that of other large shareholders. As discussed in Section 2, various mechanisms have been proposed why managers may consider, at least to some extent, the interests of financial investors. Our findings do not rely on a specific mechanism and remain applicable as long as $\theta > 0$.[9] The payoff of dispersed shareholders is not included in equation (3) since the term representing a generic dispersed shareholder, $\mathcal{D}_h$, who holds a share ε_{hi} of firm i, is proportional to ε_{hi}^2. Therefore, when $\varepsilon_{hi} \approx 0$, this term becomes negligible. As a result, the interests of dispersed shareholders do not influence managerial decisions.

In the subsequent analysis, we will simplify expression (3) by substituting B_i with $\left(\beta - s_i^B\right) \pi_i$ and I with $\sum_{j=1}^{2} s_j \pi_j$. In other words, we assume that acquisition prices do not impact the company's objective function, and managers do not assign a negative weight to the profits they appropriate[10] or to the monitoring costs. As a result, the firm's objective function becomes:[11]

$$\tilde{O}_i = \left(\beta - s_i^B\right)^2 \pi_i + \theta s_i \sum_{j=1}^{2} s_j \pi_j. \tag{4}$$

The objective function (4) may be rewritten as:

$$\tilde{O}_i = \pi_i + \lambda_i \pi_j, \tag{5}$$

[9] It is worth noting that the assumption of $\theta > 0$ is not necessary in the model with cross-ownership, nor in the model with privately costly investment. In these models analyzed later in the paper, the effects discussed still arise even if $\theta = 0$.

[10] If anything, they should assign a higher weight to the diverted profits than to those retained by the shareholders.

[11] Our main results hold true for both specifications, (3) and (4). However, in what follows we will focus on the latter, which is simpler and more intuitive.

where:

$$\lambda_i = \frac{\theta s_i s_j}{\left(\beta - s_i^B\right)^2 + \theta s_i^2}. \tag{6}$$

Therefore, when $\theta > 0$, firms aim to maximize a weighted average of their own profit and the profit of their rivals. The weight assigned to the rival's profit, λ_i, is zero if either s_i, s_j, or both, vanish.

3.1.3 Product market competition

For the sake of generality, we adopt a reduced-form model for product market competition. Each firm i selects a strategic variable x_i (such as price or quantity), and these choices determine the firms' profits $\pi_i(x_i, x_j)$. (For ease of notation, we treat x_i as a scalar, but the analysis remains the same if it were a vector). As the firms are ex-ante symmetric, the functions $\pi_i(x_i, x_j)$ are assumed to be symmetric as well. Furthermore, we assume that these functions are quasi-concave and twice continuously differentiable within the relevant range. In Appendix B, we provide specific models of product market competition that satisfy these assumptions.

3.1.4 Bargaining

Due to the free-riding among dispersed shareholders, the payment to them, $P_i^D(s_i^D)$, must be equal to the ex-post value of the s_i^D shares. Instead, the acquisition prices $P_i^B(s_i^B)$ are established through a bargaining process between the investor and the blockholders. We will examine both the scenario where the terms of the agreement between $\mathcal{I}$ and $\mathcal{B}_i$ may depend on the agreement reached with $\mathcal{B}_j$, and that where such conditioning is not possible.

3.1.5 Timing

The game proceeds in three stages. In the first stage, investor $\mathcal{I}$ chooses the stakes s_1^D and s_2^D to be acquired from dispersed shareholders and engages in negotiations with blockholders $\mathcal{B}_1$ and $\mathcal{B}_2$ regarding the stakes to be acquired, s_1^B and s_2^B, and the acquisition prices, $P_1^B(s_1^B)$ and $P_2^B(s_2^B)$. In the second stage, firms engage in product market competition, which determines the equilibrium profits π_i. Lastly, in the final stage of the game, blockholders select their monitoring efforts m_i, and the payoffs are realized.

3.2 Preliminary results

We are interested in the subgame perfect equilibrium of the game, and thus we solve the model in reverse order. In this subsection, we present the equilibrium in the last two stages of the game. The analysis of acquisition prices and the equilibrium ownership structure will be addressed in the subsequent sections.

3.2.1 Monitoring

In the final stage of the game, blockholder $\mathcal{B}_i$ selects m_i to maximize his payoff, B_i. In this stage, the values of s_i^B, P_i^B and profits π_i are pre-determined, so the blockholder's objective function reduces to:

$$\left\{ \left(\beta - s_i^B\right) \left[1 - \xi_i(1 - m_i)\right] - C_i(m_i) \right\} \pi_i.$$

Since our assumptions ensure an interior solution, the equilibrium level of monitoring is determined by the first-order condition:

$$C'(m_i) = \left(\beta - s_i^B\right) \xi. \tag{7}$$

Note that this level of monitoring is inefficiently low from the shareholders' aggregate perspective. From this viewpoint, the optimal monitoring would be determined by the condition $C'(m) = \xi$.

As mentioned earlier, our specification of monitoring costs implies that the optimal level of monitoring, m_i^*, does not depend on π_i.The convexity of $C(m_i)$ implies that it increases with the blockholder's residual ownership share, $\beta - s_i^B$, and the manager's ability to steal, ξ. To highlight the dependence of m_i^* on $\beta - s_i^B$, we will write $m_i^* = m^*(\beta - s_i^B)$.

3.2.2 Product market equilibrium

Manager $\mathcal{M}_i$ chooses x_i to maximize $\tilde{O}_i = \pi_i + \lambda_i \pi_j$. To keep things simple, we assume the existence of a unique interior Nash equilibrium, which is characterized by the following first- and second-order conditions:[12]

$$\frac{\partial \pi_i}{\partial x_i} + \lambda_i \frac{\partial \pi_j}{\partial x_i} = 0 \tag{8}$$

$$\left(\frac{\partial^2 \pi_i}{\partial x_i^2} + \lambda_i \frac{\partial^2 \pi_j}{\partial x_i^2} \right) < 0. \tag{9}$$

Equilibrium profits are denoted as $\pi_i^*(\lambda_i, \lambda_j)$.

4 Equilibrium ownership structure

We now characterize the ownership structure of the firms and examine how it is influenced by the underlying economic parameters.

In general, the investor has the option to acquire shares from dispersed shareholders, the blockholders, or both. To gradually develop an understanding of the drivers of common ownership in our framework, in this section, we focus on the scenario where $\mathcal{I}$ exclusively deals with the blockholders $\mathcal{B}_1$ and $\mathcal{B}_2$. In the following section, we will analyze the acquisition from dispersed shareholders.

[12] The analysis could be extended to the case of multiple equilibria by using monotone comparative statics techniques.

4.1 Bargaining

To proceed, we need to analyze the bargaining between the investor and the blockholders regarding the acquisition of stakes and their corresponding prices. Given the restriction $s_i^D = 0$, we have $s_i^B = s_i$, allowing us to simplify the notation by omitting the superscripts. Furthermore, since the weights λ_i now depend only on the stakes s_i, we will denote equilibrium profits $\pi_i^*(\lambda_i, \lambda_j)$ as $\pi_i^*(s_i, s_j)$.

We assume that either the buyer or the sellers of the shares make a take-it-or-leave-it offer. Let us denote α as the probability that the investor makes the offers and the blockholders receive them, while with a probability of $1-\alpha$, these roles are reversed. Therefore, α represents the share of the bargaining surplus that the investor obtains on average, which serves as a measure of its bargaining power.[13]

At this point, we can distinguish between two bargaining protocols based on whether the offers from $\mathcal{I}$ to $\mathcal{B}_i$, or from $\mathcal{B}_i$ to $\mathcal{I}$, can be conditional on the agreement between $\mathcal{I}$ and $\mathcal{B}_j$ or not. In theory, this could impact the players' reservation payoffs. However, in the specific framework we are considering, these payoffs are not influenced by whether offers can be conditional or not.

To see this, let us first examine the reservation payoffs of the blockholders. In the case of conditional offers, $\mathcal{I}$ would not purchase the target stake in firm i from $\mathcal{B}_i$ unless an agreement with $\mathcal{B}_j$ is also reached.[14] The outside option for blockholder $\mathcal{B}_i$ in this scenario is his equilibrium payoff when $s_i = s_j = 0$, which can be expressed as:

$$\bar{B}_i = \beta\left[1 - \xi(1 - m^*(\beta))\right]\pi_i^*(0,0) - C(m^*(\beta))\pi_i^*(0,0). \tag{10}$$

If instead offers cannot be conditioned on common acceptance, $\mathcal{I}$ must purchase from $\mathcal{B}_i$ even without an agreement with $\mathcal{B}_j$. In this case, the outside option for blockholder $\mathcal{B}_i$ is his equilibrium payoff when $s_i = 0$, resulting in a payoff of:

$$\overline{\overline{B}}_i(s_j) = \beta\left[1 - \xi(1 - m^*(\beta))\right]\pi_i^*(0,s_j) - C(m^*(\beta))\pi_i^*(0,s_j). \tag{11}$$

However, it can be observed from (6) that both weights λ_i and λ_j become zero as soon as either stake, s_i or s_j, becomes zero. This implies that $\pi_i^*(0,s_j) = \pi_i^*(0,0)$. Consequently, the two reservation payoffs actually coincide: $\overline{\overline{B}}_i(s_j) \equiv \bar{B}_i$.

For the same reason, the investor's reservation payoff is zero in both cases of conditional and unconditional offers. Therefore, in the baseline model, it does not matter whether the bargaining is bilateral or multilateral, as both protocols lead to the same outcome.

This outcome is *constrained-efficient*, meaning it is efficient from the perspective of the large shareholders. (Hereafter, the term "large shareholders" refers to the institutional investor $\mathcal{I}$ and the blockholders $\mathcal{B}_1$ and $\mathcal{B}_2$.)

Proposition 1 *The equilibrium ownership structure* (s_1^*, s_2^*) *maximizes the joint*

[13] In our framework, the same expected outcome would actually be produced by many different bargaining solutions.

[14] This is proved formally below.

payoff of the investor and the blockholders:

$$S = I + B_1 + B_2 = \sum_{i=1}^{2} \left\{ \beta \left[1 - \xi(1 - m^*(\beta - s_i)) \right] - C \left(m^*(\beta - s_i) \right) \right\} \pi_i^* \tag{12}$$

This result is based on the fact that $\mathcal{B}_i$'s disagreement payoff does not depend on s_j. If it were dependent, the investor could manipulate the offer to $\mathcal{B}_j$ to enhance its bargaining position with respect to $\mathcal{B}_i$.[15] Without such strategic motives, the bargaining solution maximizes the overall surplus of the large shareholders, S. The acquisition prices then distribute this surplus among the players.[16]

4.2 The monitoring-competition trade-off

Due to the symmetry of firms and the concavity of the joint payoff (12), Proposition 1 implies that the equilibrium ownership structure is symmetric: $s_1^* = s_2^* = s^*$.[17] Proposition 1 then implies that s^* maximizes:

[15] That is the reason why, with three or more firms, Proposition 1 would hold only if the investor could make conditional offers. With unconditional offers, the offer made to a certain blockholder could be distorted in order to affect the outside option of the other blockholders.

[16] These prices are determined as follows. When the investor makes the offers, which occurs with a probability of α, the prices P_i are set in a way that satisfies the blockholders' participation constraints, $B_i \geq \bar{B}_i$; that is,

$$\begin{aligned} P_i^L &= \left\{ \beta \left[1 - \xi(1 - m^*(\beta)) \right] - C(m^*(\beta)) \right\} \pi_i^*(0,0) + \\ &\quad - \left\{ (\beta - s_i) \left[1 - \xi(1 - m^*(\beta - s_i)) \right] - C(m^*(\beta - s_i)) \right\} \pi_i^*(s_i, s_j). \end{aligned}$$

On the other hand, when the blockholders make the offers, which occurs with a probability of $1-\alpha$, the investor's payoff is set to its reservation value, which is zero. Therefore:

$$P_i^H = s_i \left[1 - \xi(1 - m^*(\beta - s_i)) \right] \pi_i^*(s_i, s_j).$$

Consequently, the average acquisition prices are:

$$\begin{aligned} P_i^B(s_i, s_j) &= \alpha P_i^L + (1 - \alpha) P_i^H \\ &= \alpha \left\{ \beta \left[1 - \xi(1 - m^*(\beta)) \right] - C(m^*(\beta)) \right\} \pi_i^*(0,0) + \\ &\quad - \alpha \left\{ \beta \left[1 - \xi(1 - m^*(\beta - s_i)) \right] - C(m^*(\beta - s_i)) \right\} \pi_i^*(s_i, s_j) + \\ &\quad + s_i \left[1 - \xi(1 - m^*(\beta - s_i)) \right] \pi_i^*(s_i, s_j). \end{aligned}$$

[17] Even without symmetry, Proposition 1 implies that the investor acquires a stake in one firm only if it can also acquire a stake in the other firm. To see this, consider the case where $s_i > 0$ and $s_j = 0$, assuming the most favorable scenario for the investor ($\alpha = 1$). Using the fact that $\pi_i^*(s_i, 0) = \pi_i^*(0,0)$, the investor's net payoff is:

$$\begin{aligned} & s_i \left[1 - \xi(1 - m^*(\beta - s_i)) \right] \pi_i^*(0,0) - P_i^*(s_i, 0) \\ = \; & \left\{ \beta \left[1 - \xi(1 - m^*(\beta - s_i)) \right] - C(m^*(\beta - s_i)) \right\} \pi_i^*(0,0) + \\ & - \left\{ \beta \left[1 - \xi(1 - m^*(\beta)) \right] - C(m^*(\beta)) \right\} \pi_i^*(0,0) < 0, \end{aligned}$$

where the equality follows from the expression for the equilibrium prices given in the previous footnote, and the inequality from the fact that $m^*(\beta)$ maximizes $\beta \left[1 - \xi(1 - m_i) \right] - C(m_i)$. Intuitively, when $s_j = 0$, the acquisition of a stake in firm i by the investor does not impact the product market

$$S = \nu^*(s)\Pi^*(s). \tag{13}$$

where

$$\nu^*(s) = \beta \left\{1 - \xi \left[1 - m^*(\beta - s)\right]\right\} - C\left[m^*(\beta - s)\right] \tag{14}$$

and

$$\Pi^*(s) = \pi_1^*(s,s) + \pi_2^*(s,s). \tag{15}$$

The first factor in expression (13), ν^*, represents the large shareholders' aggregate payoff per unit of profit, taking into account managers' appropriations and monitoring costs. The second factor, Π^*, corresponds to industry profits.

We will now demonstrate that a change in s affects the two factors in opposite directions. Therefore, the choice of the ownership structure entails a trade-off between reduced competition in the product market and diminished monitoring. To identify the trade-off, let us consider the marginal effects of a change in the level of common ownership. From equation (13), we obtain:

$$\frac{dS}{ds} = \nu^*\Pi^* \left(\frac{\partial \nu^*}{\partial s}\frac{1}{\nu^*} + \frac{\partial \Pi^*}{\partial s}\frac{1}{\Pi^*}\right) \tag{16}$$

The first term in brackets represents the impact a change in the degree of common ownership s on the large shareholders' payoff per unit of profit, the second represents the impact on industry profits. We now analyze each effect separately.

4.2.1 The softening-of-competition effect

A crucial aspect of the model is that an increase in the degree of common ownership leads to a rise in industry profits Π^* by reducing competition in the product market.

Lemma 1 *If $\theta > 0$, industry profits monotonically increase with the degree of common ownership s:*

$$\frac{\partial \Pi^*}{\partial s} \geq 0.$$

The derivative is strictly positive for $0 < s < \beta$ and becomes zero at $s = 0$ and $s = \beta$.

This effect has received significant attention in recent literature on overlapping ownership. The increase in profits is due to the fact that as s increases, each firm assigns greater importance to the rival's profits and adopts a less aggressive stance.

While this effect is well known, it is important to highlight that it vanishes when common ownership is very low ($s = 0$) or very high ($s = \beta$). At $s = 0$, the effect vanishes because the weight

$$\lambda = \frac{\theta s^2}{(\beta - s)^2 + \theta s^2} \tag{17}$$

equilibrium but reduces the blockholder's incentive to monitor. Since there is already insufficient monitoring, the acquisition destroys value and does not occur in equilibrium.

depends on the product of the two stakes. Hence, the impact of an increase in s on λ is second order at $s = 0$.

At $s = \beta$, the effect disappears for two reasons. First, as s approaches β, the impact of s on λ vanishes. This can be seen from the derivative:

$$\frac{\partial \lambda}{\partial s} = \frac{2\theta s \beta \left(\beta - s\right)}{\left[\left(\beta - s\right)^2 + \theta s^2\right]^2}. \tag{18}$$

Second, when $s = \beta$, the weight λ equals one, indicating perfect collusion between firms. Consequently, near this point, industry profits are close to their maximum, suggesting that a slight change in s has a second-order effect.

Although we cannot exclude the possibility of more intricate patterns, the derivative $\frac{\partial \Pi^*}{\partial s}$ typically exhibits an inverted-U shape. This implies that the impact of increasing the level of common ownership on industry profits is most significant for intermediate levels of common ownership. Appendix B demonstrates that the inverted-U shape arises under various commonly employed models of the product market.

4.2.2 The corporate governance effect

The second crucial element of the model is the reduction in monitoring resulting from an increase in the degree of common ownership, as the blockholder retains a smaller residual stake. This reduction has an impact on the aggregate payoff per unit of profit for the large shareholders, ν^*. Formally:

$$\begin{aligned} \frac{\partial v^*}{\partial s} &= \left[\xi\beta - C'(m^*)\right] \frac{\partial m^*}{\partial s} \\ &= -\frac{\xi \left[\xi\beta - C'(m^*)\right]}{C''(m^*)} < 0. \end{aligned} \tag{19}$$

When $s = 0$, monitoring is at the efficient level from the perspective of the blockholders (and therefore the large shareholders). Therefore, by the envelope theorem, the monitoring effect is second order. However, when $s > 0$, monitoring becomes inefficiently low, leading to a first-order decrease in the net payoff per unit of profit, ν^*. The magnitude of this effect tends to increase with s. A sufficient condition for this is $C'''(m) \geq 0$.

4.3 The limits to common ownership

From the above, it appears that the first term of the derivative (16) represents the marginal cost of common ownership in terms of reduced monitoring, while the second term (i.e., the semi-elasticity of industry profits) represents the marginal benefit.

As noted, under mild regularity conditions, the marginal cost increases with s, while the marginal benefit follows an inverted-U shape. At $s = 0$, both the marginal cost and the marginal benefit vanish, whereas at $s = \beta$ the marginal benefit vanishes again while the marginal cost reaches its highest point. This implies that the negative effect on monitoring must eventually outweigh the positive effect on profits. Beyond that point, any further increase in common ownership leads to a decrease in the aggregate payoff of the large shareholders. Threfore, common ownership is invariably limited.

Proposition 2 *Common ownership is always partial:* $0 \leq s^* < \beta$.

4.4 The emergence of common ownership

Starting from a scenario where there is no common ownership in equilibrium, suppose the underlying parameters gradually change, resulting in increased benefits of common ownership and/or decreased costs. An interesting feature of our model is that as this process unfolds, there comes a point where common ownership emerges with a discrete jump rather than infinitesimal increments. This implies that even a small change in the underlying conditions can lead to a significant change in the level of common ownership.

To understand why this is the case, let us examine the function $S(s) = \nu^*(s)\Pi^*(s)$. At $s = 0$, this function is always flat, as both the marginal cost and marginal benefit of common ownership vanish. However, the point $s = 0$ can be a global maximum, a local maximum (but not global), or a local minimum. This is because, considering that the marginal cost of common ownership always exceeds the marginal benefit at $s = \beta$, only three possibilities can arise except for degenerate cases. First, the marginal cost curve may lie entirely above the marginal benefit curve, implying that $s = 0$ is a global maximum, as shown in Figure 1. Second, the marginal cost curve may intersect the marginal benefit curve an even number of times, starting from above. In this case, $s = 0$ is a local maximum, but not necessarily a global one, as illustrated in Figure 2. Third, the marginal cost curve may intersect the marginal benefit curve an odd number of times, starting from below, implying that $s = 0$ is a minimum, as depicted in Figure 3.

Depending on the specific combination of parameter values, any of these scenarios can occur. As the model parameters vary, the shape of the function $S(s)$ transitions from one extreme to the other, passing through the intermediate case.[18] The transition to common ownership occurs in the intermediate case (Figure 2), when both local maxima become global maxima. At this point, any perturbation in the underlying parameters will cause a jump in s^* from 0 to a strictly positive level, or vice versa.

4.5 Comparative statics

Now let us analyze the factors that determine the equilibrium level of common ownership. Some of these factors affect only the residual income per unit of profit, ν^*, others only industry profits, Π^*, and still others both. Generally, the first set of factors relates to corporate governance, while the second set relates to product market

[18] The direction of change for each of the model's parameters will be analyzed separately in the following subsection.

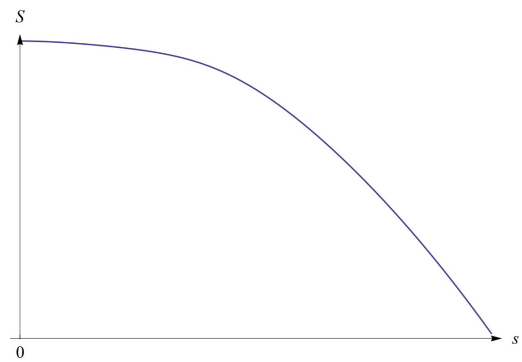

Figure 1: The case where $s = 0$ is a global maximum. The picture is drawn for Example 1 in Appendix B, with $\gamma = 2$, $\beta = \frac{4}{5}$, $\xi = \frac{3}{4}$, $\theta = \frac{4}{5}$ and $\delta = \frac{3}{5}$.

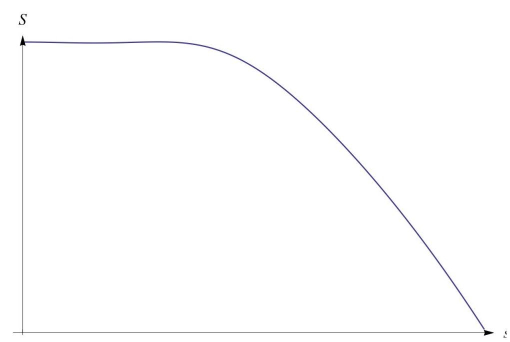

Figure 2: The case where $s = 0$ is a local but not global maximum. The picture is drawn for Example 1 in Appendix B, with $\gamma = 2$, $\beta = \frac{4}{5}$, $\xi = \frac{19}{30}$, $\theta = \frac{4}{5}$ and $\delta = \frac{3}{5}$.

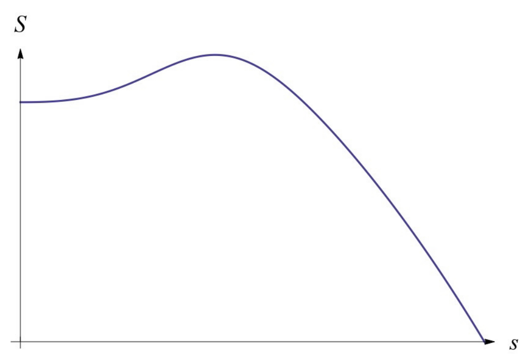

Figure 3: The case where $s = 0$ is a local minimum. The picture is drawn for Example 1 in Appendix B, with $\gamma = 2$, $\beta = \frac{4}{5}$, $\xi = \frac{1}{2}$, $\theta = \frac{4}{5}$ and $\delta = \frac{3}{5}$.

competition. We will explore each category in detail before moving on to the factors that simultaneously affect both ν^* and Π^*.

4.5.1 Corporate governance

Generally speaking, corporate governance rules and institutions play a crucial role in determining the managers' ability to appropriate a portion of the firm's profits and the cost associated with monitoring their actions. In legal systems that offer greater protection to shareholders, managers typically have fewer opportunities to extract private rents (resulting in a lower value of ξ), and monitoring becomes more feasible and cost-effective (reflected in a smaller cost $C(m)$). The analysis presented in this paper suggests that shifts in the effectiveness of corporate governance institutions may have been a contributing factor to the substantial increase in the level of common ownership observed in recent decades.

To delve deeper into this matter, it is convenient to use a quadratic specification of the monitoring-cost function:

$$C(m) = \frac{1}{2}\gamma m^2. \tag{20}$$

Under this specification, the parameter γ quantifies the magnitude of monitoring costs.[19] Hence, both parameters ξ and γ exhibit an inverse relationship with the "quality" of corporate institutions. Specifically, for any given monitoring level m, the residual payoff per unit of profit, $\nu = \beta\left[1 - \xi\left(1-m\right)\right] - \frac{1}{2}\gamma m^2$, decreases as either ξ or γ, or both, increase. (However, neither ξ nor γ has an effect on industry profits Π^*.)

Surprisingly, however, the impact of ξ and γ on the equilibrium level of common ownership differs. A lower ξ, indicating fewer opportunities for managers to divert revenues into private benefits, leads to an increase in common ownership, but a lower γ, indicating easier monitoring of managers, results in a decrease in common ownership.

Proposition 3 *The equilibrium level of common ownership s^* is monotonically decreasing in ξ and monotonically increasing in γ.*

To understand why ξ and γ have opposite effects on the equilibrium level of common ownership, it is helpful to compare the monitoring level chosen in equilibrium by the blockholder with the level that would be optimal from the perspective of the large shareholders as a whole. Using the quadratic specification (20), the former is given by:

$$m^* = \frac{\xi(\beta - s)}{\gamma}, \tag{21}$$

while the latter is $\frac{\xi\beta}{\gamma}$. The difference between the two, $\frac{\xi s}{\gamma}$, increases with ξ but decreases with γ. Consequently, if ξ decreases, the cost of common ownership in

[19] With this specification, condition $C'(1) > \xi$, which guarantees an interior solution for m^*, becomes $\gamma > \xi$.

terms of reduced monitoring diminishes, leading to an equilibrium with more common ownership. Conversely, if γ decreases, the cost of common ownership in terms of reduced monitoring increases. In this case, improved corporate governance results in less common ownership.

The above observations raise an interesting policy question: assuming that improving corporate governance is possible, is it always desirable to do so? Let us consider a scenario where the policymaker aims to maximize the welfare of a representative agent who acts as both a dispersed shareholder and a consumer of the products offered by the two firms. In their role as an investor, the representative agent receives a payoff proportional to $[1-\xi(1-m^*)]\,\Pi^*$. As ξ decreases, the representative agent benefits directly from an increase in the term inside the square brackets and indirectly from the higher equilibrium level of common ownership, which positively affects industry profits. However, as a consumer, the representative agent experiences a decrease in their consumer surplus as the degree of common ownership rises. Consequently, if the weight assigned to the consumer surplus in the representative agent's payoff is significant enough, the policymaker may choose not to lower ξ even if it were feasible to do so.[20]

4.5.2 Product market competition

Let us now explore the factors that influence how common ownership affects industry profits Π^*. One such factor is θ, which influences $\frac{d\lambda}{ds}$ and consequently $\frac{\partial\Pi^*}{\partial s}$. Another factor is the level of product market competition. Although this variable has not been explicitly defined, it undoubtedly plays a role in determining how Π^* depends on s. (On the other hand, neither of these factors affects ν^*.)

To proceed, let us introduce a parameter σ that quantifies the intensity of product market competition. Although there are various ways to parameterize it, we posit that σ influences the semi-elasticity $\frac{\partial\Pi^*}{\partial\lambda}\frac{1}{\Pi^*}$:

$$\frac{\partial}{\partial\sigma}\left(\frac{\partial\Pi^*}{\partial\lambda}\frac{1}{\Pi^*}\right) > 0. \tag{22}$$

Intuitively, a higher level of competition results in lower profits Π^* for firms when $\lambda = 0$ (i.e., when each firm solely maximizes its own profits). When instead $\lambda = 1$, firms do not compete, and they always achieve monopoly profits. Hence, the stronger the competition, the greater the increase in industry profits when transitioning from $\lambda = 0$ to $\lambda = 1$. Condition (22) asserts that this holds not only for the transition from $\lambda = 0$ to $\lambda = 1$ but also for any small increment in λ: it is essentially as a monotonicity condition. Appendix B demonstrates that in standard models, commonly utilized measures of competition intensity, such as an increase in the degree of product substitutability or a switch from Cournot to Bertrand, align with our definition (22).

[20] On the other hand, the policymaker will always strive to minimize γ since reducing γ benefits the representative agent in both their roles as a consumer and an investor. This is because, as product market competition becomes less intense, consumer surplus experiences a greater decline compared to the increase in industry profits.

Proposition 4 *The equilibrium level of common ownership s^* monotonically increases with θ and with the intensity of product market competition σ.*

The underlying intuition is straightforward. With a higher θ, indicating greater influence of the financial investor in strategic decisions, the softening-of-competition effect resulting from common ownership becomes more pronounced. Consequently, investors have a stronger incentive to acquire stakes in competing firms. Similarly, the impact of common ownership on competition reduction becomes more significant in the presence of intense competition. For instance, when the two companies operate in separate markets, there is no competition even at $\lambda = 0$, rendering common ownership devoid of any benefits.

4.5.3 Other factors

There are two remaining parameters in our model: α and β. The parameter α represents the investor's bargaining power during negotiations with blockholders. As per Proposition 1, it has no impact on the equilibrium level of common ownership. On the other hand, the parameter β represents the initial stake owned by the blockholder, and it affects both ν^* and Π^*. This complicates the comparative statics analysis; however, we can establish the following proposition:

Proposition 5 *The equilibrium level of common ownership s^* monotonically increases with β.*

The proof of this proposition uncovers various subtle effects resulting from changes in β, which complement the purely mechanical effect of the blockholder having fewer shares to sell as β decreases, without altering the qualitative conclusion that would hold if solely the mechanical effect were at work.

5 Dispersed shareholders

We now return to the general case, where the financial investor has the possibility to purchase shares not only from the blockholders but also from dispersed shareholders.

This case is more complex as Proposition 1 no longer holds. Moreover, the distinction between conditional and unconditional offers becomes important.[21] For the sake of simplicity, we assume that the investor possesses all the bargaining power ($\alpha = 1$) and we concentrate on the case of conditional offers, where the offers made by the investor to the two blockholders are valid only if both parties accept them. The acquisition from dispersed shareholders, on the other hand, is unconditional.

[21] The reason for this is that when the investor acquires a stake in both firms from dispersed shareholders, their objective functions assign a positive weight to the profit of the other firm, even if its blockholder does not sell a stake to the investor. In the case of non-conditional offers, if only one blockholder sells his shares, the shares held by the other blockholder become more valuable due to reduced competition. As a result, the blockholders' outside options improve, and the investor needs to pay a higher price to each blockholder. With conditional offers, this effect does not arise.

Before delving into the analysis, it may be helpful to provide an intuitive explanation for why it could be optimal to acquire shares from dispersed shareholders. At first glance, it might appear that this cannot be a profitable strategy, because dispersed shareholders demand a price that fully reflects the ex-post value of their shares, making it impossible to directly benefit from such acquisitions. However, acquiring shares from dispersed shareholders can yield an indirect gain to the financial investor, as the value of the shares acquired from the blockholders may increase. Naturally, this means that the investor will never acquire shares solely from dispersed shareholders.

5.1 The investor's payoff

Let us consider the bargaining between the investor and the blockholders in this new scenario.

The investor chooses the amount of shares it intends to purchase, denoted as s_1^B, s_1^D, s_2^B and s_2^D. The value of the shares acquired from dispersed shareholders must be equal to the corresponding acquisition price. Therefore, the investor's payoff (2) becomes:

$$I = \sum_{i=1}^{2} s_i^B \left[1 - \xi(1 - m^*(b - s_i^B))\right] \pi_i^* - \sum_{i=1}^{2} P_i^B(s_i^B). \tag{23}$$

Regarding the acquisition price $P_i^B(s_i^B)$, remember that the investor possesses all the bargaining power *vis-a-vis* the blockholders. As a result, the acquistion prices must leave to the blockholders exactly their reservation payoff. Since the offers to the blockholders are conditional upon mutual acceptance, the reservation payoff for blockholder $\mathcal{B}_i$ now is:

$$\bar{B}_i = \beta \left[1 - \xi(1 - m^*(\beta))\right] \pi_i^*(0,0) - C(m^*(\beta))\pi_i^*(0, s_1^D, 0, s_2^D), \tag{24}$$

where equilibrium profits are denoted as $\pi_i^*(s_1^B, s_1^D, s_2^B, s_2^D)$. Therefore, prices are:

$$\begin{aligned} P_i^B(s_i^B) &= \{\beta\left[1 - \xi(1 - m^*(\beta))\right] - C(m^*(\beta))\}\, \pi_i^*(0, s_1^D, 0, s_2^D) + \\ &\quad - \left\{(\beta - s_i)\left[1 - \xi(1 - m^*(\beta - s_i^B))\right] - C(m^*\left(\beta - s_i^B\right))\right\} \pi_i^*(s_1^B, s_1^D, s_2^B, s_2^D). \end{aligned} \tag{25}$$

Using (25), the investor's payoff rewrites as:

$$\begin{aligned} I &= \sum_{i=1}^{2} \{\beta[1 - \xi(1 - m^*(\beta - s_i^B))] - C(m^*(\beta - s_i^B))\}\pi_i^*(s_1^B, s_1^D, s_2^B, s_2^D) \\ &\quad - \sum_{i=1}^{2} \{\beta[1 - \xi(1 - m^*(\beta))] - C(m^*(\beta))\}\pi_i^*(0, s_1^D, 0, s_2^D). \end{aligned} \tag{26}$$

The first line of this expression represents the joint payoff of the large shareholders, S. In the case examined in the previous section, the equilibrium profit in the second line would be $\pi_i^*(0,0,0,0)$, rendering the second line a constant. Consequently, the equilibrium ownership structure would maximize S (Proposition 1). However, in this case, the second line depends on the investor's acquisition strategy, implying that maximizing I is not equivalent to maximizing S.

Like S, the function I is symmetric and concave, implying that in equilibrium $s_1^B = s_2^B = s^B$ and $s_1^D = s_2^D = s^D$. The investor's payoff therefore becomes, with self-explaining notation:

$$I = \nu^*(s^B)\Pi^*(s^B, s^D) - \nu^*(0)\Pi^*(0, s^D). \tag{27}$$

5.2 Acquisition from dispersed shareholders

Upon inspection of (27), it becomes apparent that the choice of s^B involves the same trade-off as in the previous section, with the only difference being that equilibrium profits are evaluated at $s^D \geq 0$. Consequently, all the qualitative results obtained in the previous section extend to the present, more comprehensive scenario.

Let us therefore focus on the choice of s^D. The marginal effect of s^D on the investor's payoff is:

$$\frac{\partial I}{\partial s^D} = \nu^*(s^B)\frac{\partial \Pi^*(s^B, s^D)}{\partial s^D} - \nu^*(0)\frac{\partial \Pi^*(0, s^D)}{\partial s^D}. \tag{28}$$

The first term of the derivative represents the marginal benefit of increasing s^D, which, similar to s^B, corresponds to higher profits resulting from reduced competition.[22] The second term represents the marginal cost. Unlike s^B, this cost does not pertain to a deterioration in corporate governance. Instead, it arises from the fact that when the investor purchases shares from dispersed shareholders in firm i, the value of the shares owned by the blockholders increases, and thus the investor must pay a higher price to acquire a stake from them.

We have:

Proposition 6 *In equilibrium, the investor purchases shares from dispersed shareholders, $s^D > 0$, if and only if it acquires shares from the blockholders, $s^B > 0$.*

Proposition 6 establishes two results. First, it asserts that acquiring shares from dispersed shareholders is not profitable when $s^B = 0$. This is evident from equation (27), which implies that $I \equiv 0$ whenever $s^B = 0$. The benefit from acquiring shares from dispersed shareholders is solely indirect and arises only when $s^B > 0$. Second,

[22] Note, however, that the softening-of-competition effect of the acquisition of shares from dispersed shareholders is lower compared to acquiring shares from blockholders. This can be observed by examining the expression for lambda:

$$\lambda = \frac{\theta\left(s^B + s^D\right)^2}{\left(\beta - s^B\right)^2 + \theta\left(s^B + s^D\right)^2},$$

which reveals that acquiring shares from the blockholder not only increases the weight of the common owner but also decreases the weight of the blockholder, with a more significant impact on lambda.

the proposition states that once $s^B > 0$, the financial investor will always acquire a portion of its shares from dispersed shareholders. This is because when $s^D = 0$, a small increase in s^D has no impact on the blockholder's outside option, in accordance to Lemma 1. Consequently, the cost of acquiring shares from dispersed shareholders vanishes, while the benefit remains positive if $s^B > 0$.[23]

In conclusion, it is never optimal to exclusively purchase shares from dispersed shareholders due to their atomistic nature and ability to capture all surplus resulting from reduced competition. However, the investor finds it advantageous to acquire shares from dispersed shareholders if, simultaneously, a stake is obtained from the blockholders. In this case, the investor can appropriate the increased value of the shares owned by the blockholders through a lower acquisition price.[24]

Note that the nature of the benefits from common ownership does not depend on whether shares are acquired from blockholders or dispersed shareholders: in both cases, the acquisition leads to softer competition and higher profits. However, the costs of common ownership differ: purchasing shares from a blockholder reduces monitoring, while acquiring shares from dispersed shareholders raises the price that must be paid to blockholders.

5.3 Complements or substitutes?

Proposition 6 establishes a form of complementarity between the acquisition from blockholders and dispersed shareholders: either both vanish or both are strictly positive. It is worth investigating whether this complementarity holds more generally, meaning whether an increase in s^B always leads to an increase in s^D, and vice versa.

To gain some insights, observe that the choice of s^D must satisfy the following condition:

$$\frac{\dfrac{\partial \Pi^*(s^B, s^D)}{\partial s^D}}{\dfrac{\partial \Pi^*(0, s^D)}{\partial s^D}} = \frac{\nu^*(0)}{\nu^*(s^B)}. \tag{29}$$

The right-hand side is greater than one and increases with s^B. The left-hand side can be approximated by the ratio between $\frac{\partial \Pi^*(s)}{\partial s}$ and $\frac{\partial \Pi^*(s-s^B)}{\partial s}$.[25] Remember that $\frac{\partial \Pi^*(s)}{\partial s}$ generally follows an inverted-U shape. Therefore, as long as s is sufficiently small to be on the increasing part of the curve, an increase in s^D raises the denominator of the left-hand side of (29). For the equality to hold, it must be accompanied by an increase in s^B, so that the numerator can also increase. In fact, since the derivative $\frac{\partial \Pi^*(s)}{\partial s}$ tends to be concave in s, the increase in s must be greater than the increase in s^D, indicating that s^B must also increase. This suggests that when the overall

[23] Proposition 6 further implies that when s^B experiences a discrete jump from zero to a positive level, as discussed above, so does s^D.

[24] This result is consistent with the literature on takeovers, which shows that a raider does not generate a profit by purchasing shares from dispersed shareholders, but profitability can be achieved when the raider already owns a toehold in the target company.

[25] This is an approximation because it abstracts from the slightly different impact of s^B and s^D on equilibrium profits, discussed in footnote 24.

level of common ownership is low, s^B and s^D tend to move in the same direction, confirming the complementarity.

On the other hand, when s becomes large and approaches the decreasing part of the curve, an increase in s^D must be accompanied by a decrease in s^B. Otherwise, the left-hand side of (29) would decrease, and since the right-hand side would increase, the condition would no longer hold. This suggests that when the overall level of common ownership is high, s^B and s^D tend to move in opposite directions, indicating that they become substitutes.

6 Cross ownership

In this section, we apply our theoretical framework to the case of "cross ownership." Strictly speaking, this term refers to cases where a company directly acquires a stake in a competing firm. In this section, however, we actually consider a scenario where a firm's blockholder acquires shares in a rival firm. In other words, we examine the situation where one of the two blockholders becomes the common owner. From now on, to use more precise terminology, we will refer to this scenario as "internal" common ownership, in contrast to "external" common ownership.

There are subtle differences between the cases where a company or its blockholder acquires a stake in a rival company.[26] However, for our current purposes, these differences are relatively insignificant, and we choose to adopt the blockholder formulation to facilitate comparison with the previous analysis.

6.1 Assumptions

The model in this section is similar to that in section 3, with the only difference being the absence of an external investor. Instead, a blockholder acquires a stake in the competing company.

Internal common ownership differs from external common ownership in two noticeable ways. Firstly, internal common ownership can be profitable even if only one blockholder acquires a stake in a rival company. In contrast, an external investor must acquire stakes in at least two competing companies to achieve a reduction in competition. Secondly, internal common ownership can be profitable even if $\theta = 0$, that is, even if managers exclusively prioritize the interests of their controlling stakeholder. As a result, we can simplify the analysis by assuming that only blockholder $\mathcal{B}_1$, say, acquires a stake in firm 2, and not the other way around, and by setting $\theta = 0$.

Let us denote the stake that blockholder $\mathcal{B}_1$ acquires from blockholder $\mathcal{B}_2$ as s_2^B, and the stake he acquires from dispersed shareholders as s_2^D, with $s_2 = s_2^B + s_2^D$. As before, dispersed shareholders are forward-looking and demand a price that fully reflects the ex post value of their shares. On the other hand, blockholder $\mathcal{B}_2$ may be willing to sell his shares for a lower price, anticipating that otherwise the acquisition

[26] We explore these differences in a related paper, Denicolò and Panunzi (2023).

may not take place. For simplicity, we assume that in the negotiations over the acquisition price, blockholder $\mathcal{B}_1$ has all the bargaining power.

Similar to Section 3, we maintain the assumption that each blockholder possesses a unique ability to monitor its own firm. In other words, blockholder $\mathcal{B}_1$ does not have the capacity to monitor the manager of firm 2; that responsibility solely lies with blockholder $\mathcal{B}_2$.[27]

Therefore, the payoff of blockholder $\mathcal{B}_1$ now is:

$$\begin{aligned} B_1 \quad = \quad & \beta\left[1-\xi(1-m_1)\right]\pi_1 - C(m_1)\pi_1 + \\ & + s_2\left[1-\xi(1-m_2)\right]\pi_2^* + \\ & - P_2^B(s_2^B) - P_2^D(s_2^D), \end{aligned} \tag{30}$$

while the payoff of blockholder $\mathcal{B}_2$ remains as given in (1).

Since $\theta = 0$, at the product market competition stage, managers now exclusively pursue the interests of their monitoring blockholders. Thus, firm i chooses x_i to maximize B_i. However, for simplicity, we will again focus on a simplified formulation of the objective function that disregards the "spurious" components of B_i. Hence, manager $\mathcal{M}_2$ maximizes π_2, while manager$\mathcal{M}_1$ maximizes $\beta\left(\beta\pi_1 + s_2\pi_2\right)$. This can be rewritten as:

$$\tilde{O}_1 = \pi_1 + \lambda_1\pi_2, \tag{31}$$

where

$$\lambda_1 = \frac{s_2}{\beta}. \tag{32}$$

As noted, the weight λ_1 is now positive even if $\theta = 0$. Moreover, it is directly proportional to the share s_2 instead of being dependent on the product of the shares. It is this property that implies that even unilateral acquisitions may now be profitable.

6.2 Acquisition prices

Since dispersed shareholders cannot be exploited, we can rewrite $\mathcal{B}_1$'s payoff as follows:

$$B_1 = \left\{\beta\left[1-\xi(1-m^*(\beta))\right] - C(m^*(\beta))\right\}\pi_1^* + s_2^B\left[1-\xi(1-m^*(\beta - s_2^B))\right]\pi_2^* - P_2^B(s_2^B). \tag{33}$$

The acquisition price for the block s_2^B, $P_2^B(s_2^B)$ must make $\mathcal{B}_2$ indifferent between selling or not. Therefore,

$$\begin{aligned} P_2^B(s_2^B) \quad = \quad & \left\{\beta[1-\xi(1-m^*(\beta)))] - C(m^*(\beta))\right\}\pi_2^*(0, s_2^D) + \\ & - \left\{(\beta - s_2^B)\left[1-\xi(1-m^*(\beta - s_2^B)\right] - C(m^*(\beta - s_2^B))\right\}\pi_2^*(s_2^B, s_2^D), \end{aligned} \tag{34}$$

where we omit the variables s_1^B and s_1^D in the notation $\pi_i^*(s_1^B, s_1^D, s_2^B, s_2^D)$ as they are both equal to zero in this section. Inserting this expression into (33), the payoff of

[27] In fact, all we need is that each blockholder is less efficient in monitoring the other firm and that in equilibrium there is no duplication of effort.

blockholder $\mathcal{B}_1$ can finally be expressed as

$$B_1 = \nu_1^*(0)\pi_1^*(s_2^B, s_2^D) + \nu_2^*(s_2^B)\pi_2^*(s_2^B, s_2^D) - \nu_2^*(0)\pi_2^*(0, s_2^D). \tag{35}$$

The first two terms are the value of the blocks owned by the two blockholders after the acquisition of s_2^B and s_2^D by blockholder $\mathcal{B}_1$. The third term represents $\mathcal{B}_2$'s reservation payoff. Since $\mathcal{B}_1$ has all the bargaining power, he obtains all the bilateral surplus in excess of $\mathcal{B}_2$'s reservation payoff.

6.3 Equilibrium ownership structure

The equilibrium ownership structure now maximizes (35). We will first consider the case where $\mathcal{B}_1$ acquires shares solely from $\mathcal{B}_2$, followed by the case where he acquires shares only from dispersed shareholders, and finally, the case where $\mathcal{B}_1$ acquires shares from both sources.

6.3.1 Acquisition from the blockholder

When $\mathcal{B}_1$ acquires shares solely from $\mathcal{B}_2$, the trade-off that determines the equilibrium ownership structure is similar to that in the case of external common ownership. Increasing s_2^B leads to softer competition in the product market, resulting in higher profits. However, it also reduces the incentives to monitor, worsening corporate governance. The equilibrium level of s_2^B strikes a balance between these two effects.

Proposition 7 *In equilibrium, internal common ownership always exists:* $0 < s_2^B$.

Unlike the case of external common ownership, s_2^B is always positive in this scenario. This is because even when s_2^B approaches zero, the positive impact on profits remains first order, while the negative effect on corporate governance becomes second order, because monitoring is at the efficient level when $s_2^B = 0$.

The factr that limits internal common ownership is the same as external common ownership: as s_2^B increases, $\mathcal{B}_2$ exerts less monitoring effort and this reduces the value of the shares $\mathcal{B}_1$ acquires from him. On the other hand, as s_2^B increases the increase in profits due to the softening-of-competititon effect becomes smaller and smaller. In the symmetric case where each blockholder acquires a stake in the competing company, perfect cooperation in the product market is achieved when s^B approaches $\frac{\beta}{2}$, implying that $\frac{\beta}{2}$ is an upper bound on the stakes acquired in equilibrium.

The comparative statics for internal common ownership are identical to those of external common ownership.

Proposition 8 *The equilibrium level of internal common ownership is monotonically decreasing in the managers' ability to steal ξ and monotonically increasing in the costliness of monitoring γ, the intensity of product market competition σ, and the blockholders' initial stakes β.*

The intuition behind these findings is precisely the same as in the case of external common ownership.

6.3.2 Acquisition from dispersed shareholders

One important difference between internal and external common ownership is that in the former case, it may be profitable for $\mathcal{B}_1$ to acquire a positive stake from dispersed shareholders even if $s_2^B = 0$, as $\mathcal{B}_1$ benefits from softer competition through the increased value of his shares of firm 1.

This can be seen by differentiating B_1 with respect to s_2^D while assuming $s_2^B = 0$. The resulting expression,

$$\frac{dB_1}{ds_2^D} = \nu_1^*(0)\frac{\partial \pi_1^*(0, s_2^D)}{\partial s_2^D}. \tag{36}$$

shows that $\mathcal{B}_1$ can potentially generate higher profits from his stake in firm 1 due to reduced competition in the market.

This possibility however relies on firms competing in variables that exhibit strategic complementarity. In the case of strategic substitutes, $\frac{\partial \pi_1^*(0,s_2^D)}{\partial s_2^D}$ is always negative, leading to the same conclusion as under external common ownership. However, in the case of strategic complements, $\frac{\partial \pi_1^*(0,s_2^D)}{\partial s_2^D}$ can be positive up to a certain $\bar{s}_2^D < 1$, which then becomes the equilibrium level of internal common ownership when $\mathcal{B}_1$ does not acquire shares from $\mathcal{B}_2$. In this situation, the limit for acquiring shares from dispersed shareholders is determined by factors influencing product market competition. Further details can be found in Appendix B.

6.3.3 Acquisition from both

Suppose now that $\mathcal{B}_1$ acquires shares from both $\mathcal{B}_2$ and dispersed shareholders. The trade-off involved in choosing s_2^B remains the same as when $s_2^D = 0$, so all the qualitative results obtained in subsection 6.3.1 extend to this more comprehensive scenario.

On the other hand, the choice of s_2^D changes more significantly. The derivative of B_1 with respect to s_2^D becomes:

$$\frac{dB_1}{ds_2^D} = \nu_1^*(0)\frac{\partial \pi_1^*(s_2^B, s_2^D)}{\partial s_2^D} + \nu_2^*(s_2^B)\frac{\partial \pi_2^*(s_2^B, s_2^D)}{\partial s_2^D} - \nu_2^*(0)\frac{\partial \pi_2^*(0, s_2^D)}{\partial s_2^D}. \tag{37}$$

The first two terms represent the impact of s_2^D on the profits of the two blockholders. Blockholder $\mathcal{B}_1$ now captures all of the extra profits resulting from softer competition, given his bargaining power in negotiations with $\mathcal{B}_2$. Since the second term is always positive, the sum of the first two terms is more likely to be positive than the derivative in (36).

However, the last term in expression (37) represents the impact of s_2^D on $\mathcal{B}_2$'s outside option. As in the case of external common ownership, this term is negative since a softer firm 1 increases the profits of firm 2 in the absence of an acquisition from $\mathcal{B}_2$. Thus, this term imposes an upper bound on the number of shares that $\mathcal{B}_1$ can profitably acquire from dispersed shareholders.

7 Other mechanisms

In the preceding sections, we have argued that common ownership involves a trade-off between softer competition and worse corporate governance. Thus far, our analysis has focused on a specific corporate governance mechanism: the incentives of blockholders to monitor managers and prevent the misappropriation of private benefits. However, agency problems within firms can manifest in different ways. In this section, we explore two other types of agency problems and demonstrate that a trade-off similar to what has been analyzed in the previous sections still exists.

We return to the scenario where the common owner is an external investor. To better focus on the relevant trade-off, we assume that the investor solely purchases shares from the blockholders, disregarding the possibility of acquiring shares from dispersed shareholders.

7.1 Extraction of private benefits by the blockholder

First, we consider the case where blockholders themselves can extract private benefits from the company they control, at the expense of minority shareholders. In this variant of the model, managers no longer divert cash flow to themselves; their only role is to make strategic choices at the product market competition stage.

Let a_i denote the fraction of profits privately appropriated by blockholder $\mathcal{B}_i$, with the remaining fraction $(1 - a_i)$ being distributed among all shareholders. Following Burkart, Gromb and Panunzi (1998), we assume that the diversion of profit entails a deadweight loss $D(a_i)\pi_i$, which is an increasing, convex function of a_i.

Under these assumptions, the payoff of a blockholder $\mathcal{B}_i$ who has sold a stake s_i to investor $\mathcal{I}$ is :

$$B_i = [a_i + (\beta - s_i)(1 - a_i) - D(a_i)]\,\pi_i + P_i(s_i). \tag{38}$$

The blockholder then chooses a_i to maximize B_i. The first-order condition for a maximum is:

$$D'(a_i) = 1 - (\beta - s_i)\,. \tag{39}$$

From this we get:

$$\frac{\partial a_i^*}{\partial s_i} = \frac{1}{D''(a_i)} > 0. \tag{40}$$

In other words, the lower the blockholder's residual stake $(\beta - s_i)$, the higher the fraction of the firm's profits he will privately appropriate. This represents the cost of common ownership in this setup.

The analysis then proceeds as in the baseline model. The aggregate payoff of the large shareholders is:

$$\begin{aligned} S &= I + B_1 + B_2 \\ &= \sum_{i=1}^{2} [a_i + \beta(1 - a_i) - D(a_i)]\,\pi_i. \end{aligned} \tag{41}$$

Exploiting symmetry, one sees that the joint surplus can again be written as:

$$S = \nu^{*}\Pi^{*}, \tag{42}$$

where now:

$$\nu^{*}(s) = a^{*}(s) + \beta\left[1 - a^{*}(s)\right] - D\left[a^{*}(s)\right]. \tag{43}$$

As in the baseline model, the upside of common ownership is that it increases industry profits Π^{*}, and the downside is that it decreases the net payoff per unit of profit, ν^{*}. This last point can be seen by calculating:

$$\begin{aligned}\frac{\partial \nu^{*}}{\partial s} &= \left[(1-\beta) - D'(a^{*})\right]\frac{\partial a^{*}}{\partial s} \\ &= -s\frac{\partial a^{*}}{\partial s} \leq 0\end{aligned} \tag{44}$$

Intuitively, the negative effect of common ownership now is due to the fact that as the blockholders' residual stakes decrease, their incentive to extract a portion of the companies' profits privately increases. This is inefficient since, from the perspective of the large shareholders, the level of profit extraction a^{*} is inefficiently large. Similar to the baseline model, the marginal cost of common ownership vanishes at $s = 0$.

7.1.1 Example

To get explicit solutions, let us specify the deadweight loss function as:

$$D(a_i) = \frac{1}{2}\phi a_i^2. \tag{45}$$

In this version of the model, the parameter ϕ, which measures how difficult it is for the blockholder to dilute minority shareholders, represents the quality of corporate governance. Under this specification, one obtains:

$$a_i^{*}(s_i) = \frac{1 - (\beta - s_i)}{\phi}. \tag{46}$$

and

$$\nu^{*}(s) = \beta + \frac{1}{2}\frac{(1-\beta)}{\phi}\left[(1-\beta)^2 - s^2\right]. \tag{47}$$

All the results obtained in the baseline model hold also in this version of the model. The only difference is the new corporate governance parameter, ϕ. Regarding its comparative statics, we have.

Proposition 9 *The equilibrium level of common ownership s^{*} is monotonically increasing in ϕ.*

That is, an improvement in the quality of corporate governance institutions leads to more common ownership.

7.2 Non-contractible investment

Next, we develop a model where common ownership diminishes the incentives to exert privately costly efforts that enhance the firm's profitability. These efforts can include, for instance, endeavors aimed at reducing the firm's costs or improving the quality of its products.

This model is conceptually similar to the work of Anton et al. (2023), who demonstrate that common ownership makes it optimal to offer managers less high-powered incentive schemes, consequently dampening their motivation to reduce costs. However, in our model, we assume that the efforts to improve the firm's performance are directly undertaken by the blockholders themselves. This simplification allows for a more streamlined analysis and facilitates the endogenization of the level of common ownership.

In contrast to the previous models where common ownership consistently undermines corporate governance, this new framework introduces non-monotonic effects. The non-monotonicity arises because cost reductions, or quality improvements, enhance the profits of the firm experiencing them but reduce the profits of its competitors. Consequently, in the absence of common ownership, individual firms tend to exert excessively high effort levels. Therefore, an increase in common ownership, leading to overall reduced effort levels, initially boosts aggregate profits even without considering its softening-of-competition effect. However, as common ownership surpasses a critical threshold, investment incentives decline to a point where efforts become insufficient for maximizing industry profits. At this juncture, a trade-off emerges, resembling that of the baseline model: further increases in common ownership result in inefficiently low effort levels, alongside a softening of competition. Hence, in this model variant, the equilibrium level of common ownership is constrained by the need to maintain incentives for cost reduction efforts.

7.2.1 Assumptions

To be specific, let us assume that blockholder $\mathcal{B}_i$ has the ability to lower the firm's marginal costs from c to $c - r_i$ at a private cost of $C(r_i)$, with $C(0) = 0, \quad C' > 0$, and $C'' > 0$. For simplicity, we will ignore the acquisition of shares from dispersed shareholders. Furthermore, we assume $\beta = 1$; this ensures that the non-monotonicity mentioned earlier always arises.

The sequence of events in this game is as follows: first, the common owner and the blockholders engage in negotiations to determine the stakes to be acquired, s_i. Subsequently, each blockholder $\mathcal{B}_i$ decides on their effort level and the resulting cost reduction, r_i, which is observed by the rival firm.[28] Finally, the firms compete in the product market.

Note that in this version of the model, even if the common owner had the same capability to invest in cost reduction as the blockholders, its incentive to do so would be lower. This is because a reduction in marginal costs would make the firm more

[28]The analysis can be extended to the case where a firm's cost reduction is not observed by the rival, as in Lopez and Vives (2019).

competitive and more aggressive in the product market, leading to reduced profits for rival firms, in which the common owner holds a share.

Equilibrium profits now are represented as $\pi_i^*(\lambda_i, \lambda_j, r_i, r_j)$, where π_i^* increases with r_i and decreases with r_j. The equilibrium profits depend on the stakes s_i for two reasons: firstly, the weights λ_i depend on s_i and s_j, as in the baseline model, and secondly, the investment in cost reduction and, consequently, r_i, depend on s_i.

7.2.2 Equilibrium investments

When choosing r_i, $\mathcal{B}_i$ maximizes

$$B_i = (1 - s_i)\pi_i^*(\lambda_i, \lambda_j r_i, r_j) - C(r_i). \tag{48}$$

Let us assume that the payoff B_i is a concave function of r_i. (This property holds in the examples considered in Appendix B.) Assuming an interior solution, the first-order condition with respect to r_i gives:

$$(1 - s_i)\frac{\partial \pi_i^*(\lambda_i, \lambda_j, r_i, r_j)}{\partial r_i} = C'(r_i). \tag{49}$$

The left-hand side is the increase in profits due to the cost reduction; the right-hand side is the marginal cost of effort.

Since $\frac{\partial \pi_j^*}{\partial r_i} < 0$, it is evident that at $s_i = 0$ the incentives to reduce marginal costs are excessively high from the viewpoint of the maximization of industry profits.

By the implicit function theorem, we have:

$$\frac{dr_i}{ds_i} = -\frac{-\frac{\partial \pi_i^*(\lambda_i,\lambda_j,r_i,r_j)}{\partial r_i} + (1 - s_i)\left[\frac{\partial^2 \pi_i^*(\lambda_i,\lambda_j,r_i,r_j)}{\partial \lambda_i \partial r_i}\frac{d\lambda_i}{ds_i} + \frac{\partial^2 \pi_i^*(\lambda_i,\lambda_j,r_i,r_j)}{\partial \lambda_j \partial r_i}\frac{d\lambda_j}{ds_i}\right]}{\frac{\partial^2 \pi_i^*(\lambda_i, \lambda_j, r_i, r_j)}{\partial r_i^2} - C''(r_i)}. \tag{50}$$

At $s_i = 0$, the term inside square brackets vanishes and thus r_i^* is a decreasing function of s_i. This property holds for any $s_i > 0$ provided that the marginal benefit from a cost reduction increases with the intensity of competition, as is the case in many standard models. Under this condition, the term inside the square brackets is always negative.

7.2.3 Equilibrium ownership structure

Moving backward in the analysis, let us consider the bargaining stage. To simplify the discussion, let us assume that the common owner possesses all the bargaining power and makes take-it-or-leave-it offers to the blockholders. Under conditional offers,[29] the outcome is efficient, and thus the acquisition stakes s_i and s_j will be

[29] This means that the offer made to blockholder $\mathcal{B}_i$ are valid only if the offer made to $\mathcal{B}_j$ is accepted. The reason why this conditionality is necessary for efficiency is that the acceptance of an offer by one blockholder creates a positive externality on the other blockholder, as it hinders their incentives to reduce the marginal cost. If the offers are not conditional, these externalities are not

chosen to maximize:

$$S = \sum_{i=1}^{2} \pi_i^*(\lambda_i, \lambda_j, r_i^*, r_j^*) - \sum_{i=1}^{2} C(r_i^*). \tag{51}$$

Given the symmetry of the model, we can restrict attention to the case $s_i = s_j = s$ with no loss of generality. In this case, S rewrites as

$$S = \Pi^*(\lambda, r) - 2C^*(r), \tag{52}$$

where Π^* is again industry profits and $2C^*$ is the aggregate cost of investment in cost reduction.

The first-order condition for surplus maximization is:

$$\frac{dS}{ds} = \frac{\partial \Pi^*}{\partial \lambda}\frac{d\lambda}{ds} + \left(\frac{\partial \Pi^*}{\partial r} - 2\frac{\partial C^*}{\partial r}\right)\frac{dr}{ds} = 0. \tag{53}$$

The case $\theta = 0$. An important feature of this variant is that the assumption $\theta > 0$ is not required, unlike in the baseline model. To emphasize this difference, let us start the analysis by considering the case where $\theta = 0$ and thus $\lambda = 0$, as in Anton et al. (2023). Consequently, each firm, at the product market competition stage, only considers its own profit.

In this case, the first term on the right-hand side of (53) vanishes, resulting in the equilibrium level of common ownership being implicitly determined by the condition:

$$\frac{\partial \Pi^*}{\partial r} - 2\frac{\partial C^*}{\partial r} = 0. \tag{54}$$

Therefore, the equilibrium level of common ownership must replicate the investment in cost reduction that would be selected if firms were able to coordinate their efforts perfectly while still competing in the product market.

It is clear from condition (54) that the equilibrium level of common ownership must be strictly positive. This is because when $s = 0$, the equilibrium efforts are excessively high in terms of maximizing industry profits. However, common ownership is partial, as setting $s = 1$ would eliminate the incentives to invest in cost reduction, and a complete absence of investment would violate condition (54).

The case $\theta > 0$. When $\theta > 0$, the term $\dfrac{\partial \Pi^*}{\partial \lambda}\dfrac{d\lambda}{ds}$ becomes positive. As a result, at equilibrium we must have:

$$\frac{\partial \Pi^*}{\partial r} - 2\frac{\partial C^*}{\partial r} > 0, \tag{55}$$

indicating that the investment in cost reduction falls below the level that maximizes industry profits. In this scenario, the equilibrium level of common ownership is determined by striking a balance between collusion and cost reduction. Increas-

internalized, which hampers the efficiency of the outcome.

ing common ownership reduces competition but comes at the cost of distorting the investment in cost reduction below the efficient level from the large shareholders' perspective.[30] However, the softening-of-competition effect of common ownership disappears when $s = 1$, as demonstrated in Section 4.1 above. Since the impact on efforts is always negative as long as condition (53) is satisfied, the equilibrium level of common ownership is always constrained by the necessity to maintain incentives to exert effort.

7.2.4 Example

To illustrate, let us consider a scenario where firms supply differentiated products, and the demand for these products is described by the equation:

$$q_i = 1 - p_i + \gamma p_j, \tag{56}$$

where $\gamma \in [0, 1]$ represents the degree of product differentiation. We also assume that firms compete on prices.[31] In this case, it can be verified that when $\theta = 0$, the equilibrium level of common ownership is given by:

$$s^* = \frac{\gamma}{2 - \gamma^2},$$

thus, the value of s^* increases as the degree of product differentiation γ increases. This observation confirms our previous findings, indicating a positive relationship between the intensity of competition and the level of common ownership.

8 Conclusion

This paper has examined the interplay between the costs and benefits of common ownership in firms. A growing body of theoretical and empirical literature suggests that common ownership reduces product market competition, leading to higher profits. This is good from the viewpoint of the shareholders. The novel contribution of this paper is the focus on the cost associated with common ownership, specifically, its impact on corporate governance effectiveness.

In our model, "active" and "passive" investors play complementary roles. Active investors, such as individuals or families holding a relatively large block of shares, have incentives to monitor managers and exert efforts to reduce costs or increase market share. On the other hand, investment funds like the Big Three, with stakes in multiple firms within the same industry, have an incentive to soften product market competition. The equilibrium ownership structure arises as the optimal response to these divergent forces. Factors that enhance the value of monitoring, such as a greater potential for diverting resources as private benefits, constrain the extent of common ownership. Conversely, more intense product market competition favors the emergence of common ownership.

[30] From the consumer's perspective, both collusion and higher costs are welfare-reducing.

[31] This corresponds to Example 2 in Appendix B.

The main trade-off in the paper does not rely on the specific modeling of the agency problem. We have explored various scenarios, including those where the manager can extract private benefits of control, a large blockholder can do so directly, or the blockholder can exert efforts to reduce marginal cost or improve product quality. Regardless of the scenario, the trade-off remains the same: softer product market competition comes at the expense of less effective corporate governance.

A noteworthy feature of our model is that common ownership does not arise as a continuous and smooth process but rather through an initial jump. Despite a sequence of enhancements in shareholder legal protection that reduce the necessity for active monitoring, there may be no immediate effect on common ownership for a period of time. However, at a certain point, there can be a sudden and discrete change in the ownership structure, resulting in a greater presence of institutional investors.

The model has intriguing implications for the political economy of corporate governance. Initially, as long as enhancements in shareholder protection do not result in the emergence of common ownership, consumers remain unaffected and thus have no reason to oppose such improvements. However, once corporate governance reaches a level where active monitoring is less necessary, leading to the equilibrium emergence of common ownership, shareholder and consumer interests begin to diverge. Voters with limited or no financial stake in stocks may prefer corporate governance structures that offer lesser protection to minority shareholders.

Another assumption that can be relaxed is the exclusive focus on profit-driven behavior for firms and their shareholders. A notable trend in recent years is the emergence of socially responsible investors who prioritize goals such as environmental preservation and human rights protection, alongside profit maximization. The literature has highlighted a significant concern known as the leakage problem. This refers to the situation where one firm reduces emissions through green technology, but the environmental benefits are partially offset by increased emissions from competitors using less sustainable technologies. Common ownership may offer a potential solution to mitigate the leakage problem and enhance the effectiveness of socially responsible investment strategies. However, if the consequence of the involvement of socially motivated investors is softer product market competition, consumers may bear the costs of social responsibility. Exploring these new trade-offs presents an exciting avenue for future research.

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Appendix A: Proofs

This appendix collects the proofs omitted in the main text.

Proof of Proposition 1. Investor $\mathcal{I}$ chooses its offers so as to maximize its net payoff I. Since the participation constraints $B_i \geq \bar{B}_i$ must bind in equilibrium, we have

$$I + B_1 + B_2 = I + \bar{B}_1 + \bar{B}_2.$$

Inspection of (10) reveals that $\bar{B}_i$ does not depend on the investor's stakes, s_1 and s_2. It follows immediately that maximization of I is equivalent to maximization of $I + \bar{B}_1 + \bar{B}_2$, and hence of $I + B_1 + B_2$. ■

Proof of Lemma 1. Clearly:

$$\frac{d\Pi^*}{ds} = \frac{\partial \Pi^*}{\partial \lambda}\frac{\partial \lambda}{\partial s}.$$

From (17), we know that $\frac{\partial \lambda}{\partial s}$ is always non-negative but vanishes at $s = 0$ and $s = \beta$. Furthermore, $\frac{\partial \lambda}{\partial s}$ is inverted-U shaped in s.

Next, consider the factor $\frac{\partial \Pi^*}{\partial \lambda}$. Using the first-order conditions (8), we obtain:

$$\frac{\partial \Pi^*}{\partial \lambda} = \left(-\lambda \frac{\partial \pi_2^*}{\partial x_1} + \frac{\partial \pi_2^*}{\partial x_1}\right)\frac{\partial x_1}{\partial \lambda} + \left(\frac{\partial \pi_1^*}{\partial x_2} - \lambda \frac{\partial \pi_1^*}{\partial x_2}\right)\frac{\partial x_2}{\partial \lambda}$$

Symmetry implies $\dfrac{\partial x_1}{\partial \lambda} = \dfrac{\partial x_2}{\partial \lambda} = \dfrac{\partial x}{\partial \lambda}$, so the above expression may be rewritten as:

$$\frac{\partial \Pi^*}{\partial \lambda} = 2\left(1 - \lambda\right)\frac{\partial \pi_i^*}{\partial x_j}\frac{\partial x}{\partial \lambda}. \tag{A1}$$

To calculate $\dfrac{\partial x}{\partial \lambda}$, we use once again the first-order conditions (8), obtaining:

$$\frac{\partial \lambda}{\partial x} = \frac{\frac{\partial^2 \pi_i^*}{\partial x_i^2} + \lambda \frac{\partial^2 \pi_j^*}{\partial x_i^2}}{\frac{\partial \pi_j^*}{\partial x_i}},$$

which implies:

$$\frac{\partial x}{\partial \lambda} = -\frac{\frac{\partial \pi_j^*}{\partial x_i}}{\left(\frac{\partial^2 \pi_i^*}{\partial x_i^2} + \lambda \frac{\partial^2 \pi_j^*}{\partial x_i^2}\right)}.$$

Plugging this expression into (A1) we eventually get:

$$\frac{\partial \Pi^*}{\partial \lambda} = -2\left(1 - \lambda\right)\frac{\left(\frac{\partial \pi_j^*}{\partial x_i}\right)^2}{\left(\frac{\partial^2 \pi_i^*}{\partial x_i^2} + \lambda \frac{\partial^2 \pi_j^*}{\partial x_i^2}\right)} \geq 0,$$

which is positive by the second-order conditions (9). The derivative is strictly positive for $\lambda < 1$, i.e., for $s < \beta$. Therefore, the sign of $\dfrac{\partial \pi^*}{\partial s}$ coincides with the sign of $\dfrac{\partial \lambda}{\partial s}$. The result then follows from the observation that $\dfrac{\partial \lambda}{\partial s}$ is always non negative and vanishes only at $s = 0$ and $s = \beta$. ■

Proof of Proposition 2. From (13) we have (omitting for notational convenience the dependence of m^* on $\beta - s$):

$$\begin{aligned}
\frac{\partial S}{\partial s} &= \Pi^* \frac{\partial m^*}{\partial s} \left. \frac{\partial \left\{ \beta \left[1 - \xi(1-m) \right] - C\left(m\right) \right\}}{\partial m} \right|_{m=m^*} + \\
&\quad + \left\{ \beta \left[1 - \xi(1-m^*) \right] - C\left(m^*\right) \right\} \frac{\partial \Pi^*}{\partial s} \\
&= -\Pi^* \frac{s\xi^2}{C''\left(m^*\right)} + \left\{ \beta \left[1 - \xi(1-m^*) \right] - C\left(m^*\right) \right\} \frac{\partial \Pi^*}{\partial s},
\end{aligned}$$

where the equality follows from condition (7), which implies $\dfrac{\partial m^*}{\partial s} = -\dfrac{\xi}{C''\left(m^*\right)}$ and $\left. \dfrac{\partial \left\{ \beta \left[1 - \xi(1-m) \right] - C\left(m^*\right) \right\}}{\partial m} \right|_{m=m^*} = \xi s$. Since $\left. \dfrac{\partial \Pi^*}{\partial s} \right|_{s=\beta} = 0$ by Lemma 1, we have

$$\left. \frac{\partial S}{\partial s} \right|_{s=\beta} < 0,$$

which implies that $s^* < \beta$. ■

Proof of Proposition 3. Monotonicity requires that $\dfrac{\partial s^*}{\partial \xi} < 0$ (resp., $\dfrac{\partial s^*}{\partial \gamma} > 0$) when $s^* > 0$, and that s^* jumps downwards (resp., upwards) as ξ (resp., γ) increases.

To show this, note first of all that with the quadratic specification (20) of the monitoring cost function, the equilibrium level of monitoring is (21). Therefore, keeping in mind that Π^* does not depend on ξ and γ, the derivative $\dfrac{\partial S}{\partial s}$ becomes:

$$\frac{\partial S}{\partial s} = \frac{\xi^2}{\gamma} H \Pi^*, \tag{A2}$$

where

$$H \equiv -s + \left[\gamma\beta \frac{1-\xi}{\xi^2} + \frac{1}{2}(\beta^2 - s^2) \right] \frac{\partial \Pi^*}{\partial s} \frac{1}{\Pi^*} \tag{A3}$$

To proceed, consider first the case in which $s^* > 0$. By Proposition 2, in this case s^* is an interior maximum of the function $S(s)$, and thus it must satisfy the first-order condition $H = 0$. By implicit differentiation, we then obtain:

$$\frac{\partial s^*}{\partial \xi} = -\frac{\frac{\partial H}{\partial \xi}}{\frac{\partial H}{\partial s}} < 0$$

and

$$\frac{\partial s^*}{\partial \gamma} = -\frac{\frac{\partial H}{\partial \gamma}}{\frac{\partial H}{\partial s}} < 0$$

where the sign follows from the fact that $\frac{\partial H}{\partial s} < 0$ by the second order condition, whereas $\frac{\partial H}{\partial \xi} < 0$ and $\frac{\partial H}{\partial \gamma} > 0$. (These latter inequalities follows immediately from (A3)).

Next, consider the possibility that as ξ or γ changes, s^* may jump from an interior solution where $s^* \equiv s^+ > 0$ to a corner solution where $s^* = 0$. At the switching point, we must have:

$$\Delta S \equiv S(s^+) - S(0) = \frac{\xi^2}{\gamma} K \Pi^*_+, \tag{A4}$$

where Π^*_0 is industry profits at $s = 0, \Pi^*_+$ is industry profits at $s = s^+$, and

$$K \equiv \left\{ \left[2\gamma\beta \frac{1-\xi}{\xi^2} + \beta^2 \right] \frac{\Pi^*_+ - \Pi^*_0}{\Pi^*_+} - s^{+2} \right\}. \tag{A5}$$

It follows that:

$$\left. \frac{\partial \Delta S}{\partial \xi} \right|_{\Delta S = 0} \propto \frac{\partial K}{\partial \xi} < 0,$$

where the symbol $\propto$ means "has the same sign has." This implies that when $\Delta S = 0$, an increase in ξ makes ΔS become negative, causing a downward jump of the equilibrium level of common ownership from s^+ to 0.

Likewise, we have

$$\left. \frac{\partial \Delta S}{\partial \gamma} \right|_{\Delta S = 0} \propto \frac{\partial K}{\partial \gamma} > 0,$$

implying that when $\Delta S = 0$, an increase in γ makes ΔS become positive, causing an upward jump of the equilibrium level of common ownership from 0 to s^+. ■

Proof of Proposition 4. We proceed as in the proof of Proposition 3. The function H and K defined in (A3) and (A5) clearly depend on the derivative $\frac{\partial \Pi^*}{\partial s} \frac{1}{\Pi^*}$, or its discrete analog $\frac{\Pi^*_+ - \Pi^*_0}{\Pi^*_+}$. We have:

$$\begin{aligned} \frac{\partial \Pi^*}{\partial s} \frac{1}{\Pi^*} &= \frac{\partial \Pi^*}{\partial \lambda} \frac{1}{\Pi^*} \frac{\partial \lambda}{\partial s} \\ &= \frac{2\theta s \beta (\beta - s)}{\left[(\beta - s)^2 + \theta s^2 \right]^2} \frac{\partial \Pi^*}{\partial \lambda} \frac{1}{\Pi^*}. \end{aligned}$$

It follows immediately that:

$$\frac{\partial s^*}{\partial \theta} = -\frac{\frac{\partial H}{\partial \theta}}{\frac{\partial H}{\partial s}} > 0$$

and

$$\frac{\partial s^*}{\partial \sigma} = -\frac{\frac{\partial H}{\partial \sigma}}{\frac{\partial H}{\partial s}} < 0.$$

A similar logic applies to the direction of the jump from $s^* = 0$ to $s^* = s^+$: in both cases, the jump is upwards, as $\dfrac{\Pi_+^* - \Pi_0^*}{\Pi_+^*}$ is increasing in both θ and σ. ■

Proof of Proposition 5. We proceed as in the proof of Proposition 3. Consider first the function H :

$$\begin{aligned} H &\equiv -s + \left[\gamma\beta\frac{1-\xi}{\xi^2} + \frac{1}{2}(\beta^2 - s^2)\right]\frac{\partial \Pi^*}{\partial s}\frac{1}{\Pi^*} \\ &= -s + \left[\gamma\beta\frac{1-\xi}{\xi^2} + \frac{1}{2}(\beta^2 - s^2)\right]\left\{\frac{2\theta s\beta\left(\beta - s\right)}{\left[(\beta - s)^2 + \theta s^2\right]^2}\right\}\frac{\partial \Pi^*}{\partial \lambda}\frac{1}{\Pi^*}. \end{aligned}$$

The blockholder's stake β affects H in a complex way, but tedious algebra confirms that $\dfrac{\partial H}{\partial \beta} > 0$. By a standard argument, this implies that $\dfrac{\partial s^*}{\partial \beta} > 0$, so s^* increases with β when s^* is strictly positive.

That s^* jumps upward at the discontinuity point when β increases follows from the fact that $\dfrac{\partial K}{\partial \beta} > 0$, which again is easy to confirm using (A5). ■

Proof of Proposition 6. Suppose first that $s^B = 0$. In this case, it appears that the derivative (28) vanishes, implying that the investor cannot gain by acquiring shares from the dispersed shareholders. On the other hand, suppose that $s^B > 0$. Let us evaluate the derivative

$$\frac{\partial I}{\partial s^D} = \nu^*(s^B)\frac{\partial \Pi^*(s^B, s^D)}{\partial s^D} - \nu^*(0)\frac{\partial \Pi^*(0, s^D)}{\partial s^D}$$

at $s^D = 0$. From

$$\lambda = \frac{\theta\left(s^B + s^D\right)^2}{\left(\beta - s^B\right)^2 + \theta\left(s^B + s^D\right)^2},$$

one sees that

$$\left.\frac{d\lambda}{ds^D}\right|_{s^D=0} = \frac{2\theta s^B\left(\beta - s^B\right)^2}{\left[(\beta - s^B)^2 + \theta\left(s^B\right)^2\right]^2}$$

which implies that $\left.\frac{d\lambda}{ds^D}\right|_{s^D=0} > 0$ if $s^B > 0$ and $\left.\frac{d\lambda}{ds^D}\right|_{s^D=0} = 0$ if $s^B = 0$. Therefore, $\left.\dfrac{\partial \Pi^*(s^B, s^D)}{\partial s^D}\right|_{s^D=0} > 0$ if $s^B > 0$, whereas $\left.\dfrac{\partial \Pi^*(0, s^D)}{\partial s^D}\right|_{s^D=0} = 0$. It follows that $\left.\frac{\partial I}{\partial s^D}\right|_{s^D=0} > 0$ if $s^B > 0$, which implies that at the optimum $s^D > 0$ if $s^B > 0$. ■

Proof of Proposition 7. Suppose that $s_2^D = 0$. From (35) we have:

$$B_1 = \nu_1^*(0)\pi_1^*(s_2^B, 0) + \nu_2^*(s_2^B)\pi_2^*(s_2^B, 0) - \nu_2^*(0)\pi_2^*(0, 0).$$

Differentiating we get:

$$\frac{\partial B_1}{\partial s_2^B} = \frac{\partial \nu_2^*(s_2^B)}{\partial s_2^B}\pi_2^*(s_2^B,0) + \nu_1^*(0)\frac{\partial \pi_1^*(s_2^B,0)}{\partial s_2^B} + \nu_2^*(s_2^B)\frac{\partial \pi_2^*(s_2^B,0)}{\partial s_2^B}.$$

The first term of the derivative is negative and represents the marginal cost of cross ownership, the sum of the last two terms is positive and represents the marginal benefit.

At $s_2^B = 0$ the first term vanishes, as shown above. It follows that $\left.\frac{\partial B_1}{\partial s_2^B}\right|_{s_2^B=0} > 0$, proving that cross ownership is always positive. ■

Proof of Proposition 8. The proof is identical to the proofs of Propositions 3, 4 and 5 and is therefore omitted. ■

Proof of Proposition 9. From (42) and (47) we obtain

$$\frac{\partial S}{\partial s} = \frac{(1-\beta)}{\phi} H \Pi^*,$$

where

$$H = -s + \left\{ \frac{\beta}{1-\beta}\phi + \frac{1}{2}\left[(1-\beta)^2 - s^2\right] \right\} \frac{\partial \Pi^*}{\partial s}\frac{1}{\Pi^*}$$

To proceed, consider first the case in which $s^* > 0$. In this case, s^* is an interior maximum of the function $S(s)$, and thus it must satisfy the first-order condition $H = 0$. By implicit differentiation, we then obtain:

$$\frac{\partial s^*}{\partial \phi} = -\frac{\frac{\partial H}{\partial \phi}}{\frac{\partial H}{\partial s}} > 0$$

where the sign follows from the fact that $\frac{\partial H}{\partial s} < 0$ by the second order condition, whereas

$$\frac{\partial H}{\partial \phi} = \frac{\beta}{1-\beta}\frac{\partial \Pi^*}{\partial s}\frac{1}{\Pi^*} > 0.$$

Like in the proof of Proposition 3, a similar argument applies to the case where s^* jumps from a corner solution where $s^* = 0$ to an interior solution where $s^* \equiv s^+ > 0$.

Appendix B: Examples

Here we consider various specific models of product market competition and show that standard measures of the intensity of competition accord with our condition (23).

Example 1. The firms supply differentiated products, the inverse demand for which is:

$$p_i = 1 - q_i - \delta q_j,$$

where $\delta \in [0,1]$ is a parameter that captures the degree of product differentiation: products are independent for $\delta = 0$, perfect substitutes for $\delta = 1$. Marginal costs are nil, and firms compete in quantities ($x_i = q_i$).

It is easy to verify that the profit functions are well-behaved and that the equilibrium is unique. Equilibrium prices and profits are:

$$\begin{aligned} q_i^* &= \frac{1}{2+\delta+\delta\lambda} \\ \Pi^* &= \frac{1+\delta\lambda}{(2+\delta+\delta\lambda)^2}. \end{aligned}$$

Using (17), one obtains

$$\frac{\partial \Pi^*}{\partial s} = \frac{4\theta\delta^2\beta s (\beta - s)^3}{\left[(2+\delta)(\beta - s)^2 + 2\theta(1+\delta)s^2\right]^3},$$

whence it is easy to verify that the derivative is positive and inverted-U shaped.

In this example, a natural index of the intensity of competition is the degree of product substitutability δ. Indeed, we have

$$\frac{\partial}{\partial \delta}\left[\frac{\partial \Pi^*}{\partial \lambda}\frac{1}{\Pi^*}\right] = \frac{\delta(1-\lambda)(4+\delta+3\delta\lambda)}{(1+\delta\lambda)^2(2+\delta+\delta\lambda)^2} > 0,$$

consistently with our condition (23).

Example 2. Under the same assumptions as in Example 1, suppose that firms compete in prices. The Bertrand equilibrium is:

$$\begin{aligned} p_i^* &= \frac{1-\delta}{2-\delta-\delta\lambda} \\ \Pi^* &= \frac{(1-\delta)(1-\delta\lambda)}{(1+\delta)(2-\delta-\delta\lambda)^2}. \end{aligned}$$

Using (17), one then obtains

$$\frac{\partial \Pi^*}{\partial s} = \frac{4\theta(1-\delta)\delta^2\beta s(\beta - s)^3}{(1+\delta)\left[(2-\delta)(\beta-s)^2 + 2\theta(1-\delta)s^2\right]^3}.$$

As in the case of quantity competition, the derivative is positive and inverted-U shaped.

As above, it is natural to take δ as a measure of the intensity of competition. This measure accords with our condition (23), as

$$\frac{\partial}{\partial\delta}\left[\frac{\partial\Pi^*}{\partial\lambda}\frac{1}{\Pi^*}\right]=\frac{\delta\left(1-\delta\right)\left(4-\delta-3\delta\lambda\right)}{\left(1-\delta\lambda\right)^2\left(2-\delta-\delta\lambda\right)^2}>0.$$

It is also generally recognized that competition is more intense when firms choose prices than if they choose output levels. This notion of the intensity of competition also accords with (23), as

$$\left.\frac{\partial\Pi^*}{\partial\lambda}\frac{1}{\Pi^*}\right|_{\text{Bertrand}}-\left.\frac{\partial\Pi^*}{\partial\lambda}\frac{1}{\Pi^*}\right|_{\text{Cournot}}=\frac{2\delta^3\left(1+2\lambda-3\lambda^2\right)}{\left(1-\delta\lambda\right)\left(1+\delta\lambda\right)\left(2-\delta-\delta\lambda\right)\left(2+\delta+\delta\lambda\right)}>0.$$

Internal common ownership. In the internal common-ownership model of Section 6, assuming $s_2^B=0$, the equilibrium profits are

$$\begin{aligned}\pi_1^*(s_2^D) &= \frac{\left[\beta\left(2+\delta\right)+s_2^D\delta\right]\left[\beta\left(2+\delta\right)-\left(1+\delta\right)s_2^D\delta\right]}{\left[\beta\left(4-\delta^2\right)-s_2^D\delta^2\right]^2}\\ \pi_2^*(s_2^D) &= \frac{\beta^2\left(2+\delta\right)^2}{\left[\beta\left(4-\delta^2\right)-s_2^D\delta^2\right]^2}\end{aligned}$$

in Example 1. Using these formulas, it is easy to show that B_1 is always decreasing in s_2^D.

On the other hand, in Example 2 equilibrium profits are

$$\begin{aligned}\pi_1^*(s_2^D) &= \frac{\left(1+\delta\right)\left[\beta\left(2+\delta\right)-s_2^D\delta\right]\left[\beta\left(2+\delta\right)+\left(1+\delta\right)s_2^D\delta\right]}{\left(1-\delta\right)\left[\beta\left(4-\delta^2\right)-s_2^D\delta^2\right]^2}\\ \pi_2^*(s_2^D) &= \frac{\left(1+\delta\right)\beta^2\left(2+\delta\right)^2}{\left(1-\delta\right)\left[\beta\left(4-\delta^2\right)-s_2^D\delta^2\right]^2}.\end{aligned}$$

In this case, B_1 is always increasing in s_2^D at $s_2^D=0$.

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