

How do tax courts affect tax compliance and tax audits? A game-theoretic analysis

Thomas Kourouxous[†] Peter Krenn[‡] Rainer Niemann[§]

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Abstract

We investigate how the possibility to appeal tax audit results in court influences the taxpayer's income declaration and the tax authority's auditing process. We employ a game-theoretic model in which the taxpayer's income declaration may be subject to an audit conducted by a designated auditor on behalf of the tax authority. The taxpayer can challenge audit errors by filing a tax court appeal. We find that in the absence of a tax court, a separating equilibrium with no tax evasion can be achieved if tax audit fees are low and penalties for tax evasion are neither too low nor excessively high. If penalties are too high, taxpayers may overstate their income to avoid arbitrary punishment as a result of a negligent tax audit. The possibility of having erroneous tax audits overturned in court incentivizes the tax auditor to increase audit intensity and avoid negligent tax audits. However, the tax authority may reduce its audit frequency in anticipation of audit results being overturned in court. The overall impact of tax courts depends crucially on the parameter setting. Generally, we can infer that tax evasion cannot be meaningfully curbed if tax audits are prohibitively costly or if audit fees are excessively high.

Keywords: Tax Declaration, Tax Audit, Court Appeal, Game Theory

[†]University of Graz, Institute of Accounting and Taxation (Corresponding Author), E-Mail: thomas.kourouxous@uni-graz.at.

[‡]University of Graz, Institute of Accounting and Taxation.

[§]University of Graz, Institute of Accounting and Taxation.

1 Introduction

Tax audits, including subsequent tax court litigation, are an integral part of the tax collection process. For instance, in 2023, the Austrian Ministry of Finance recorded 57,404 tax-related business audits, which resulted in approximately EUR 1.5 billion in recommended additional tax.¹ In the same year, the federal finance court of Austria (‘Bundesfinanzgericht’) recorded 11,665 new appeal filings, with about 40% of all material settlements being decided in the taxpayer’s favor (BFG, 2024). In 2023, the IRS completed 582,944 tax return audits, resulting in \$31.9 billion in recommended additional tax.² Historical analysis of US tax court data shows that approximately 85% of docketed cases result in pre-trial settlements, revoking previous penalties at least in part.³ Yet, strategic interactions between income tax declarations and tax audits including subsequent court litigation are not fully understood.

At first glance, the relation between tax audits and subsequent court litigation may seem disconnected, as each process is part of a different branch of government, pursuing different objectives. While tax audits are primarily conducted to deter potential tax evaders from under-declaring their taxable income, the tax court also concerned with protecting honest taxpayers from unjust penalties by the tax authority. Such a situation may for instance arise when the tax auditor comes to an erroneous conclusion during a tax audit. The tax court can overrule the tax auditor’s decision, which in turn has an ex-ante effect on the tax auditor’s audit intensity. Thus, tax courts in addition to serving justice, indirectly also influence the taxpayer’s income declaration process by altering the tax authority’s audit incentives.

While early theoretical work on tax evasion analyzes non-compliance largely as a risky portfolio optimization decision with exogenously given detection probabilities (e.g. Allingham and Sandmo (1972), Srinivasan (1973), and Yitzhaki (1974)), later contribu-

¹The additional revenues exceed the total expenses of the entire Austrian fiscal administration which amount to EUR 852 million in 2023 (BMF, 2024).

²Similar to the Austrian data, the additional revenues exceed the total expenses of the IRS which amount to \$ 16.1 billion for the fiscal year 2023 (United States Internal Revenue Services, 2024).

³See Dubroff and Hellwig (2014). Approximately 30,000 US tax court petitions are filed annually (National Taxpayer Advocate, 2018).

tions incorporate strategic interactions between taxpayers and the tax authority. Among others, Reinganum and Wilde (1985) and Graetz, Reinganum, and Wilde (1986) show that accounting for the tax authority's incentives and responses to the anticipated taxpayer's behavior leads to new and interesting predictions relative to the occurrence and determinants of tax evasion. Sansing (1993) analyzes how an additional signal that supplements the information conveyed in the taxpayer's income declaration influences the tax authority's auditing incentives. Border and Sobel (1987) analyze the characteristics of optimal audit schemes. Those schemes involve random auditing strategies and post-audit rebates. When analyzing optimal stochastic audit strategies, Melumad and Mookherjee (1989) find those strategies need to allow the taxpayer for at least some consumption in every period. Halperin and Tzur (1990) analyze the relation between audit frequency, penalty levels, and the political weight of tax evaders in an attempt to explain why taxpayers in the United States face low penalties and low audit rates. Gordon (1989) studies morality and reputation in addition to penalties as deterrants to tax evasion. Sansing and Phillips (1998) examine the effect of banning contingent fees for tax return preparation services. Contrary to conventional wisdom, such legislation would not discourage practitioners from taking aggressive tax return positions. Bayer (2006) applies a costly state verification model to analyze the relation between tax evasion and the resources required to avoid being detected. Chen and Chu (2005) and Crocker and Slemrod (2005) develop models for corporate tax evasion, the latter explicitly accounting for agency costs between shareholders and the firm's tax management. Also related is Wei (1992) who analyzes the impact of corrupt tax officials in a static model where the tax authority does not act strategically. He shows that limiting bribery by disciplining corrupt tax officials can help to reduce tax evasion. However, the joint influence of tax courts and tax auditors on the declaration of taxable income has not yet been analyzed.⁴

Our paper extends the previous literature by incorporating an honest but effort-averse tax auditor and a tax authority that behave strategically, anticipating that the taxpayer

⁴As part of their comprehensive literature reviews, Slemrod (2007), Alm (2012), and Alm and Malézieux (2021) call for more research into tax evasion. In particular in the early tax evasion literature, tax courts are notably absent. Comprehensive reviews of that literature include Cowell (1985), Andreoni, Erard, and Feinstein (1998), Slemrod and Yitzhaki (2002), and Sandmo (2005).

may file a tax court appeal if treated unjustly. More specifically, we address the following three research questions: How does the option to appeal an erroneous tax audit decision in court influence (a) the tax auditor’s audit precision, (b) the taxpayer’s incentives to file an accurate tax declaration, and (c) the tax authority’s audit policy? In doing so, we contribute to the literature on tax compliance and tax enforcement. While previous literature has primarily focused on the determinants of filing an appeal and the outcomes of such appeals e.g., Eisenberg and Heise (2015), Lederman and Hrungr (2006), Galanter (1974), and Lederman (1999), we focus on how the possibility of filing a tax court appeal influences taxpayers’ incentives to submit a truthful income declaration, the tax auditor’s audit intensity, and the tax authority’s audit frequency.

In a nutshell, our theoretical model depicts a situation where a taxpayer can strategically decide whether to file a truthful income statement. The tax authority observes the declared income and can choose to audit the taxpayer. We incorporate a rational, effort-averse tax auditor who can exert either high or low effort. The tax authority delegates the tax audit to this auditor who can come to an erroneous conclusion when performing the audit with low, rather than high, effort. After receiving the auditor’s report, the taxpayer has the possibility to challenge audit errors by filing a tax court appeal. We assume that the tax court appeal imposes additional costs to at least one of the involved parties. The tax court can detect the actual taxable income and thus protects honest taxpayers from erroneous tax audits.⁵

Our results indicate that in the absence of a tax court, the tax auditor has no incentive to conduct diligent audits, which is detrimental to the prevention of tax evasion. Furthermore, the lack of incentives for diligent audits may lead taxpayers to voluntarily overstate their income to avoid unjustified penalties arising from erroneous audit reports. This tendency is most pronounced when penalties for tax evasion are high and auditor compensation is low. On the other hand, if taxpayers can appeal the auditor’s verdict in court, incentives for over-declaration diminish. The presence of a tax court countervails tax auditor negligence in particular if the cost of filing a tax court appeal is sufficiently

⁵We are aware that courts come to erroneous conclusions and that the legal appeals process therefore comprises multiple levels. We simplify this process and assume that the final result of the appeal is uncovering the truth.

low. At the same time, the tax authority scales down the number of tax audits, since the results of the audit may now be overturned. The overall impact of tax courts on income declaration depends crucially on the parameter setting. Generally, we can infer that tax evasion cannot be curbed meaningfully if tax audits are costly and the fee for conducting such audits is prohibitively high.

The possibility to appeal the outcome of a tax audit enables taxpayers to defend themselves against incorrect administrative action. This is a crucial element of legal protection, which is pivotal for the rule of law. From a policy perspective, legislators should be aware of the conditions under which legal protection is effective. Our results offer insights into the conditions under which taxpayers opt for legal protection and when society can benefit from reduced tax evasion.

The paper proceeds as follows. In section 2 we derive the benchmark model that analyses tax declarations and tax audits in the absence of a tax court. In section 3 we expand the benchmark model and introduce a tax court to subsequently derive results for the extended model and compare them to the benchmark results. Section 4 summarizes and concludes.

2 Benchmark Case – No Tax Court

2.1 Model Setup

We employ a game-theoretical approach to analyze interactions between the taxpayer’s income declaration, a possible tax audit and potential litigation after tax court appeal. For the sake of simplicity, we assume that taxable income is drawn from a discrete set of states with two possible outcomes. This approach is comparable to Graetz et al. (1986) and Erard and Feinstein (1994). However, our model includes a rational, risk-neutral tax auditor (TA), in addition to the risk-neutral taxpayer (TP) and the risk-neutral tax authority (FA).⁶ Moreover, in contrast to the aforementioned contributions, we refrain

⁶For the sake of simplicity, we abstract from incorporating potential agency conflicts between the three parties. The inclusion would unnecessarily complicate the analysis without meaningfully adding to our results. For a review of theoretical literature analyzing interactions between agency conflicts and taxation see Bauer, Kourouxous, and Krenn (2018).

from including a class of taxpayers who always file truthful tax statements (‘habitual taxpayers’) as our main focus lies with the tax auditor’s incentives and the impact of a potential tax court appeal.⁷ The game is comparable to a signaling game where the sender (the taxpayer) privately learns his type (the actual income) and has to send a signal (the income statement) to the receivers (the tax authority and the tax auditor). The sequence of events is depicted in the subsequent game tree in Figure 1.⁸

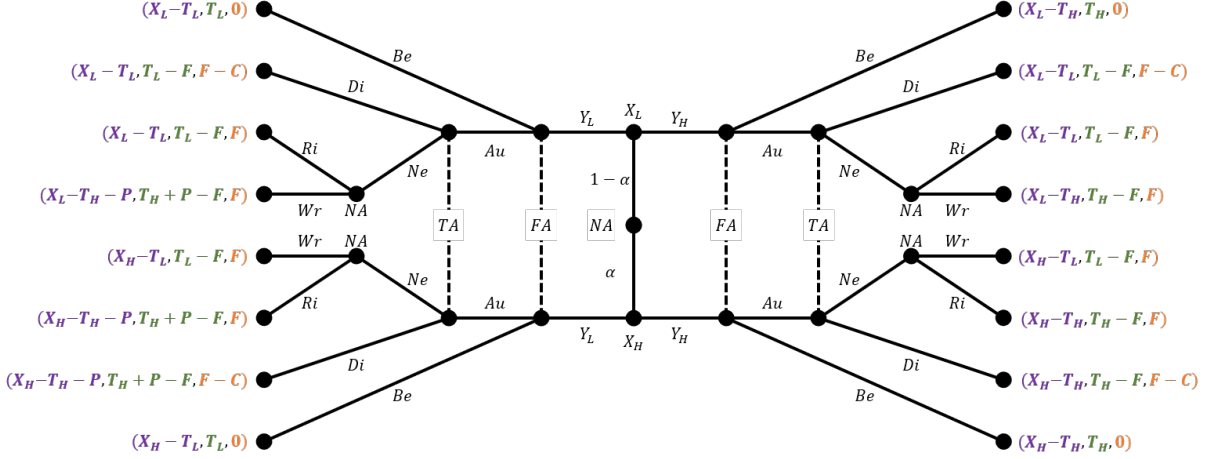


Figure 1: Game tree for the benchmark case

The realized utility levels of all three players can be seen in the row vectors (u_{TP}, u_{FA}, u_{TA}) at the end nodes of the tree where u_{TP} represents the utility of the taxpayer, u_{FA} represents the utility of the tax authority, and u_{TA} represents the utility of the tax auditor. Without the possibility to appeal the tax auditor’s report findings in the benchmark case, the time-line comprises up to five stages.

At stage 0, nature (NA) chooses the taxpayer’s type X_i from a set of two possible states, $X_i \in \{X_L, X_H\}$ with $0 \leq X_L < X_H$. Type X_i represents the actual income of TP . The ex-ante probability of X_H is equal to $Pr(X_H) = \alpha$ and therefore, $Pr(X_L) = 1 - \alpha$.

At stage 1, the taxpayer privately learns his actual taxable income, X_i , and files an income statement where $Y_i \in \{Y_L, Y_H\}$ denotes the level of declared income. The tax authority uses this signal to calculate taxes due $T(Y_i)$. The set of possible income statements corresponds with the set of possible actual income levels, that is, $Y_L = X_L$ and

⁷Adding habitual taxpayers would complicate the analysis without adding to the main results.

⁸Please note that the starting point is in the center of the game tree. Dashed lines are used to indicate decision nodes within the same information set.

$Y_H = X_H$. Further, for ease of exposition we define $T(Y_L) = T_L$ and $T(Y_H) = T_H$ with $0 \leq T_L < T_H$. Each taxpayer type individually decides about the income level Y_i he wants to declare. We allow for mixed strategies and hence the taxpayer's equilibrium actions are characterized by the probability of declaring low income Y_L . More precisely, type X_L chooses Y_L (files a conforming income statement) with probability $Pr(Y_L|X_L) = c$ and type X_H chooses Y_L (evades taxes) with probability $Pr(Y_L|X_H) = e$, where $c, e \in [0, 1]$.⁹

At stage 2, the tax authority receives the income statement Y_i without being able to observe the actual taxable income X_i . It can choose to either audit (Au) or believe (Be) the taxpayer's income statement dependent on the observed income statement. The tax authority's equilibrium action is characterized by the probability of an audit. More precisely, the tax authority does an audit after observing Y_L with probability $Pr(Au|Y_L) = a_L$ and after observing Y_H with $Pr(Au|Y_H) = a_H$, where $a_L, a_H \in [0, 1]$.

The events at stage 3 depend on the tax authority's decision at stage 2. If the tax authority decides to believe the taxpayer's income statement the game ends at stage 2 and no action from the tax auditor is required. In this case, the taxpayer's payoff u_{TP} equals $X_i - T(Y_i)$, the tax authority's payoff u_{FA} equals $T(Y_i)$ and the tax auditor's payoff u_{TA} equals 0. If the tax authority decides to audit the taxpayer's income statement, it delegates this task to the tax auditor who has the capacity of performing the audit. The tax auditor receives a fixed lump sum payment $F > 0$ for the audit activity which can neither be negotiated nor modified by the FA and TA .¹⁰ Conditionally on being assigned to the audit task, the tax auditor has the same information as the tax authority, i.e. income statement Y_i . The tax auditor can choose between two different levels of audit intensity which are equivalent to two different levels of audit quality. That is, the

⁹Consequently, type X_L overstates with probability $1 - c$ and type X_H files a truthful statement with probability $1 - e$.

¹⁰This is the case, for example, when tax auditors are civil servants, who are remunerated with a fixed monthly salary, receive overtime payment for additional audit tasks or when tax auditors are hired to complete additional audit jobs. In our setting, the tax authority can require the auditor to undertake a certain number of audits within any time period. This results in an average 'fee' F the tax auditor receives per audit that cannot be negotiated individually. This assumption deviates from the literature on statutory audits where the auditor's compensation is determined individually by a contract that optimizes the audit client's expected utility.

tax auditor can either audit diligently (Di) or negligently (Ne). The actual level of due diligence is not observable to any of the other players. It impacts the outcome of the audit process and therefore also the tax auditor's conclusions and payoffs at the final stage. Each audit intensity causes the auditor some costs $C(Di)$ and $C(Ne)$ that can be interpreted as his personal loss of utility from exerting effort. We assume that $C(Di) = C > 0$ and $C(Ne) = 0$. Further, we assume that the fixed payment to the tax auditor always exceeds his incurred costs, $F > C$. The tax auditor's equilibrium actions are characterized by the probability of a diligent audit conditionally on the observed income statement. More precisely, the tax auditor chooses $Pr(Di|Y_L) = d_L$ and $Pr(Di|Y_H) = d_H$ where $d_L, d_H \in [0, 1]$.

At the final stage, nature determines the outcome of the audit. The resulting tax payment $T(M_i)$ relies on the auditor's report uncovering the actual taxable income ($M_i \in \{M_L, M_H\}$ with $M_L = X_L$ and $M_H = X_H$) instead of the taxpayer's reported income Y_i . Additionally, if the auditor reports M_H and the taxpayer filed an income statement of Y_L the tax authority punishes the taxpayer for committing tax evasion and the taxpayer has to pay an additional penalty fee, $P > 0$, to the tax authority.

Following a diligent audit, the tax auditor always reveals the actual taxable income X_i correctly and hence, M_i always equals X_i . Following a negligent audit, the tax auditor makes an error with some positive probability and comes to an erroneous conclusion relative to the actual income ($M_i \neq X_i$ for some cases). This results in a tax payment $T(M_i)$ that does not reflect the true tax burden as it is not based on the actual taxable income $T(X_i)$. This also means that a taxpayer of type X_L who truthfully reports a conforming income statement Y_L might be unjustly penalized for tax evasion. In anticipation, the taxpayer might overstate his true income to avoid possible litigation and arbitrary penalties.¹¹ This model feature is important for deriving the taxpayers' incentives to file a tax

¹¹There are many instances where taxpayers fail to deduct legitimate expenses on their tax returns because of fear of an audit (Stancil, 2008). A recent survey of U.S. taxpayers finds that 42% prefer to overpay their taxes, because they are afraid of getting a bill from the IRS (Mercado, 2019). Gandhi and Kuehlwein (2016) examine whether significant income tax overwithholding in the United States can be explained as rational risk-neutral taxpayers try to avoid underwithholding penalties. In Ireland, according to the Irish Tax and Customs, over 27% of individual tax returns are overpaid (Irish Tax and Customs, 2024).

court appeal in the extended model setting in section 3. We assume that the ex-ante probability of an error by a negligent auditor $Pr(M_i \neq X_i \cup Y_j)$ depends on the actual income and the reported income. To keep the notation simple, we define error probabilities $Pr(M_i \neq X_i \cup Y_j) \equiv \omega_{ij}$ where $i, j \in \{L, H\}$. Further, we assume that $0 < \omega_{ij} < \frac{1}{2} \quad \forall i, j$. Otherwise, flipping a coin would result in more correct ‘audit’ outcomes than the engagement of a negligent auditor and hence, the tax authority would have no incentives to hire such an auditor. Consequently, the ex-ante probability of a correct audit opinion (Ri) following a negligent audit equals $1 - \omega_{ij} > \frac{1}{2}$.

The game either ends at stage 2 (if no audit is conducted) or after the auditor has issued its audit opinion in stage 5. The payments to all players (u_{TP}, u_{FA}, u_{TA}) are made according to the assumptions above and are illustrated within the game tree in Figure 1.

2.2 Preliminary Analysis

In this section, we present the essential elements of the benchmark game (strategies, beliefs, utilities, possible equilibria, etc.). We apply a ‘weak perfect Bayesian equilibrium’ concept. In a first step, we define strategy spaces for all players in the game. The taxpayer’s strategy s_{TP} consists of the probabilities to file a low income statement Y_L after observing either type X_L or X_H . Let us remind that type X_L chooses Y_L with probability $Pr(Y_L|X_L) = c$ and type X_H chooses Y_L with probability $Pr(Y_L|X_H) = e$, where $c, e \in [0, 1]$. Hence, $s_{TP} = (c, e)$. The tax authority’s strategy s_{FA} is equal to the probabilities of conducting an audit following each possible income statement (Y_L and Y_H). Hence, $s_{FA} = (a_L, a_H)$. Finally, the tax auditor’s strategy s_{TA} consists of the probabilities to conduct a diligent audit for every possible income statement, $s_{TA} = (d_L, d_H)$.

Next, we define the beliefs of the players FA and TA when reaching their decision nodes. The tax authority and the tax auditor have beliefs μ_{FA} and μ_{TA} , respectively that a taxpayer with income report Y_L has an actual income of X_H . Hence, $\mu_k = Pr(X_H|Y_L)$ represent beliefs of encountering a taxpayer that is a tax evader. Similarly, beliefs κ_{FA} and κ_{TA} are the respective tax authority’s and the tax auditor’s beliefs that a taxpayer with income report Y_H has an actual income of X_H . Thus, $\kappa_k = Pr(X_H|Y_H)$ represent beliefs of encountering an honest taxpayer with high income.

In the following, we define each player’s expected utility function. If the tax auditor

decides to conduct a negligent audit (Ne), the probability of a wrong audit outcome equals ω_{ij} . Hence, the utility levels of all players following a negligent audit can be expressed in expected terms as follows: $u_k^{ij,Ne} = (1 - \omega_{ij})u_k^{ij,Ri} + \omega_{ij}u_k^{ij,Wr}$ where $k \in \{TP, FA, TA\}$ and i and j represent the actual income and the reported income respectively. Table 1 illustrates the corresponding utility levels $u_k^{ij,Ne}$ for each combination of k , i , and j .

ij	$k = TP$	$k = FA$	$k = TA$
LL	$X_L - T_L - \omega_{LL}(T_H - T_L + P)$	$T_L - F + \omega_{LL}(T_H - T_L + P)$	F
LH	$X_L - T_L - \omega_{LH}(T_H - T_L)$	$T_L - F + \omega_{LH}(T_H - T_L)$	F
HL	$X_H - T_L - (1 - \omega_{HL})(T_H - T_L + P)$	$T_L - F + (1 - \omega_{HL})(T_H - T_L + P)$	F
HH	$X_H - T_L - (1 - \omega_{HH})(T_H - T_L)$	$T_L - F + (1 - \omega_{HH})(T_H - T_L)$	F

Table 1: Expected utilities $u_k^{ij,Ne}$ following a negligent audit

The player's conditional expected utilities given their beliefs are as follows:¹²

$$U_{TP}(s_{TP}|X = X_L) = X_L - T_L - c\hat{a}_L(1 - \hat{d}_L)\omega_{LL}(T_H - T_L + P) - (1 - c)[1 - \hat{a}_H + \hat{a}_H\omega_{LH}(1 - \hat{d}_H)](T_H - T_L) \quad (1)$$

$$U_{TP}(s_{TP}|X = X_H) = X_H - T_L - e\hat{a}_L[1 - \omega_{HL}(1 - \hat{d}_L)](T_H - T_L + P) - (1 - e)[1 - \hat{a}_H(1 - \hat{d}_H)\omega_{HH}](T_H - T_L) \quad (2)$$

$$U_{FA}(s_{FA}, \mu_{FA}|Y = Y_L) = T_L - a_L F + a_L[(1 - \mu_{FA})(1 - \hat{d}_L)\omega_{LL} + \mu_{FA}(1 - \omega_{HL} + \hat{d}_L\omega_{HL})](T_H - T_L + P) \quad (3)$$

$$U_{FA}(s_{FA}, \kappa_{FA}|Y = Y_H) = T_H - a_H F - a_H[\kappa_{FA}(1 - \hat{d}_H)\omega_{HH} + (1 - \kappa_{FA})(1 - \omega_{LH} + \hat{d}_H\omega_{LH})](T_H - T_L) \quad (4)$$

$$U_{TA}(s_{TA}, \mu_{TA}|Y = Y_L) = \hat{a}_L[F - d_L C] \quad (5)$$

$$U_{TA}(s_{TA}, \kappa_{TA}|Y = Y_H) = \hat{a}_H[F - d_H C] \quad (6)$$

The weak perfect Bayesian equilibrium concept requires that all player's strategies are sequentially rational given their beliefs. Therefore, it is possible to exclude certain strategies before deriving and analyzing possible equilibria of this game. First, we observe

¹²Variables with a hat symbol (^) represent conjectured values.

that the tax auditor's expected utility does not depend on his beliefs regarding the actual type of the taxpayer. As a diligent audit incurs the tax auditor with a cost of C without providing an extra benefit, it is sequentially rational for him to always perform a negligent audit conditionally on being assigned to the audit task. Therefore, an equilibrium can only survive with the tax auditor's optimal strategy $(d_L, d_H) = (0, 0)$. Second, it is not optimal for the tax authority to audit an income statement of Y_H . This can be seen by inspection of Equation (4). Independent from belief κ_{FA} , any $a_H > 0$ reduces the tax authority's expected utility. Therefore, $a_H = 0$ has to be part of the tax authority's optimal strategy in equilibrium.

2.3 Results

In Lemma 1, we characterize the possible equilibria in this game.

Lemma 1 (Equilibrium in the benchmark setting). *In the benchmark setting, the constituting equilibrium depends on the size of the audit fee F and the penalty P . Define threshold values \underline{F} , \overline{F} , $\overline{\overline{F}}$, \underline{P} and \overline{P} as follows:*

$$\begin{aligned}\underline{F} &= \omega_{LL}(T_H - T_L + P), & \underline{P} &= \frac{\omega_{HL}}{1 - \omega_{HL}}(T_H - T_L), \\ \overline{F} &= [\alpha(1 - \omega_{HL}) + (1 - \alpha)\omega_{LL}](T_H - T_L + P), & \overline{P} &= \frac{1 - \omega_{LL}}{\omega_{LL}}(T_H - T_L), \\ \overline{\overline{F}} &= (1 - \omega_{HL})(T_H - T_L + P).\end{aligned}$$

1. *If $P < \underline{P}$ and $F < \overline{F}$, a pooling equilibrium in pure strategies as follows constitutes:*

$$\begin{aligned}s_{TP} &= (1, 1) & s_{FA} &= (1, 0) & s_{TA} &= (0, 0) \\ \mu_{FA} &= \mu_{TA} = \alpha & \kappa_{FA}, \kappa_{TA} &\in [0, 1]\end{aligned}$$

2. *If $F > \overline{F}$, a pooling equilibrium in pure strategies as follows constitutes:*

$$\begin{aligned}s_{TP} &= (1, 1) & s_{FA} &= (0, 0) & s_{TA} &= (0, 0) \\ \mu_{FA} &= \mu_{TA} = \alpha & \kappa_{FA}, \kappa_{TA} &\in [0, 1]\end{aligned}$$

3. *If $P > \overline{P}$ and $F < \overline{\overline{F}}$, a pooling equilibrium in pure strategies as follows constitutes:*

$$\begin{aligned}s_{TP} &= (0, 0) & s_{FA} &= (1, 0) & s_{TA} &= (0, 0) \\ \mu_{FA} &\in]\mu^*, 1] & \mu_{TA} &\in [0, 1] & \kappa_{FA} &= \kappa_{TA} = \alpha\end{aligned}$$

$$\text{with } \mu^* = \frac{F - \omega_{LL}(T_H - T_L + P)}{(1 - \omega_{HL} - \omega_{LL})(T_H - T_L + P)}$$

4. If $\underline{P} < P < \overline{P}$ and $F < \underline{F}$, a separating equilibrium in pure strategies as follows constitutes:

$$\begin{array}{lll} s_{TP} = (1, 0) & s_{FA} = (1, 0) & s_{TA} = (0, 0) \\ \mu_{FA} = \mu_{TA} = 0 & \kappa_{FA} = \kappa_{TA} = 1 & \end{array}$$

5. If $P > \underline{P}$ and $\underline{F} < F < \overline{F}$, a partially pooling equilibrium in mixed strategies as follows constitutes:

$$\begin{array}{lll} s_{TP} = (1, e^*) & s_{FA} = (a_L^*, 0) & s_{TA} = (0, 0) \\ \mu_{FA} = \mu_{TA} = \mu^* & \kappa_{FA} = \kappa_{TA} = 1 & \end{array}$$

$$\begin{aligned} \text{with } \mu^* &= \frac{F - \omega_{LL}(T_H - T_L + P)}{(1 - \omega_{HL} - \omega_{LL})(T_H - T_L + P)} \\ e^* &= \frac{1 - \alpha}{\alpha} \left[\frac{F - \omega_{LL}(T_H - T_L + P)}{(1 - \omega_{HL})(T_H - T_L + P) - F} \right] \\ a_L^* &= \frac{T_H - T_L}{(1 - \omega_{HL})(T_H - T_L + P)} \end{aligned}$$

Proof. See Appendix. □

Figure 2 illustrates graphically the different Equilibria 1-5 of Lemma 1.

Lemma 1 and Figure 2 provide a few notable results with regard to the benchmark case. First, due to missing incentives the tax auditor acts 'passively' and never conducts a diligent audit in the absence of an appeals court. This result is independent of the size of penalties and audit fees. Second, the prevailing equilibrium depends on the severity of penalties P and on the size of the audit fee F . Due to $\omega_{HL} < 1/2$ and $\omega_{LL} < 1/2$ the relation between the different threshold values is in general $\underline{P} < \overline{P}$ and $\underline{F} < \overline{F} < \overline{\overline{F}}$. Most interestingly, the equilibrium declaration behavior of the taxpayer does not only include (partial) under-declaration and honest declaration but also over-declaration. In the following, we provide some intuition relative to the possible equilibria of the game and draw first conclusions.

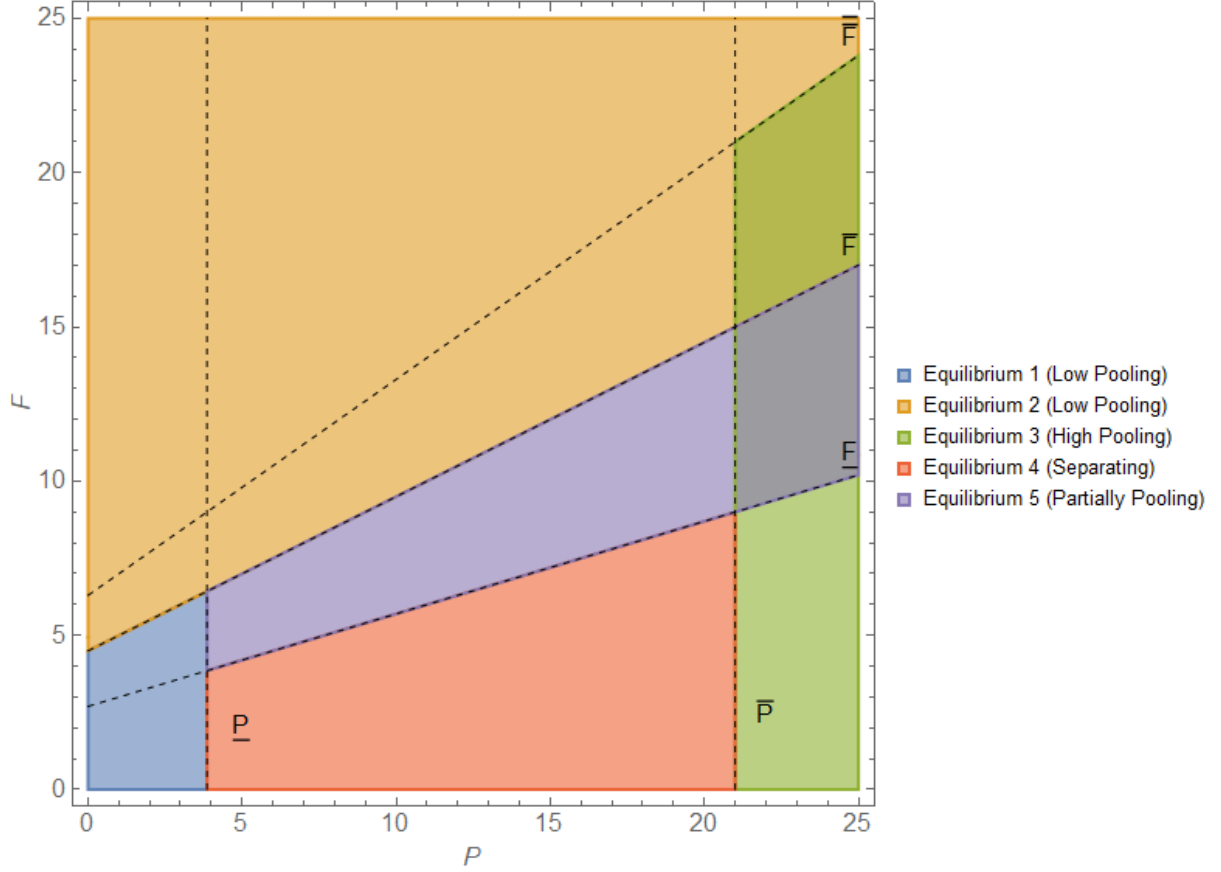


Figure 2: Graphical Illustration of Equilibria with $\alpha = 0.5$, $\omega_{ij} = 0.3 \forall i, j \in \{L, H\}$, $T_H = 10$ and $T_L = 1$

In Equilibrium 1, both types of taxpayers pool on low income which means that the high income earner will always under-declare his income and hence commit tax evasion. Due to very low audit costs, the tax authority decides to audit all income statements with low income. However, if penalties are too small ($P < \underline{P}$), even an audit rate of 100% cannot prevent tax evasion entirely. This is because the tax auditor is prone to mistakes. Therefore, a tax evader will only be detected with probability $1 - \omega_{HL}$ and if penalties are sufficiently low, they do not outweigh the potential benefit from evading taxes.

Pooling on low income (under-declaration) may also occur in Equilibrium 2. If the audit fee is too high ($F > \bar{F}$), the tax authority will decide against auditing the taxpayer's income declaration. This, in turn, removes any incentive for taxpayers to file an income statement with high income and leads to under-declaration.

In Equilibrium 3, both types of taxpayers pool on high income. This means that the low income earner overstates his income. This can occur if penalties are very high and

audit fees are low enough such that the tax authority would like to always trigger an audit following a low income statement ($F < \bar{F}$). In this case, the penalty does not only deter the high income earner from tax evasion but also provides the low income earner with incentives to pay voluntarily more taxes than he actually owes. This counter-intuitive result can be explained by the tax auditor's limited audit accuracy in case of negligent audits. If the low income earner has to fear being convicted unjustifiably of tax evasion and penalties are very high, he would prefer to pay a higher amount of taxes instead of risking these high penalties. Note that Equilibrium 3 may intersect with some of the other equilibria if the audit fee is sufficiently high. This is mainly due to the fact that this equilibrium is dependent on the tax authority's belief μ_{FA} if a deviation from the equilibrium path occurs.

Equilibrium 4 is a separating equilibrium where all taxpayers declare their income truthfully. Such a 'truth-telling' situation can occur if penalties are neither too high nor too low to trigger over-declaration and under-declaration. Further, the audit fee has to be sufficiently low such that the tax authority finds it optimal to audit all low income statements.

Finally, in Equilibrium 5 the low income earner always files a low income statement whereas the high income earner randomizes between tax evasion and declaring honestly high income with some positive probability. This equilibrium materializes if penalties are sufficiently high to prevent complete under-declaration but requires sufficiently high audit fees such that the tax authority does not find it optimal to audit each low income statement.

To summarize, the benchmark case illustrates that if there exists no tax court of appeals and tax auditors are prone to audit errors we might observe scenarios of under-declaration, over-declaration as well as honest declaration. The prevailing scenario actually depends on the (relative) size of penalties and of audit fees.

3 Presence of a Tax Court

3.1 Extended Model Setup

In the following, we extend the benchmark model of the previous section 2.1 with an additional stage, in which the taxpayer can decide whether to accept the tax bill he receives (*Ac*) or object and file an appeal to the tax court (*Ob*). The extended structure of the game and adjusted payoffs are depicted in Figure 3.

When deciding on whether to file a tax court appeal, the taxpayer has perfect information about his type and the previously filed income statement (perfect recall). Further, he knows if an audit has been conducted and what the outcome of this audit was. However, he cannot distinguish between cases where the auditor has conducted a diligent audit and cases where the auditor discovered the actual income ‘by chance’ following a negligent audit.¹³ The court of appeals is able to reveal the taxpayer’s type with certainty and to correct the tax bill for any mistakes due to an audit failure and/or wrong income statements.¹⁴ This means that an honest taxpayer of type X_L can avoid the additional tax payment and penalty following a wrong audit opinion by lodging an appeal. However, a tax evader will have to pay higher taxes and an additional penalty if he triggers a court trial. Thus, a tax evader would never have an incentive to make an appeal and hence risking a detection by the court.

An appeal can result in additional costs for some of the players. First, the taxpayer bears some legal costs ($L_{TP} > 0$) that are independent from the court decision. Those legal costs represent, for instance, costs from hiring an attorney or investing time and effort in

¹³Relaxing this assumption does impact the solution of the game, because the taxpayer’s payoffs following the same subsequent action are identical in each case.

¹⁴In accordance with common law theory, an appeals court has the purpose to correct serious errors in prior judgment. We abstract from having multiple court levels or erroneous court judgments and assume that ultimately the truth is revealed through the court process.

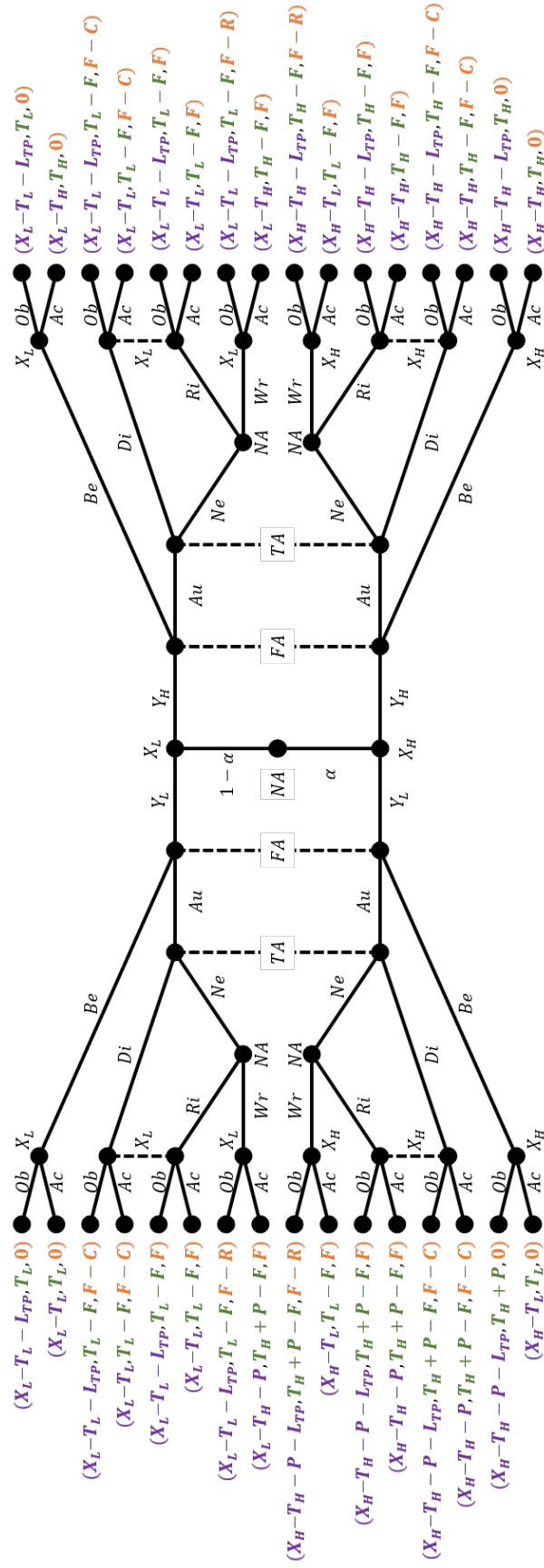


Figure 3: Game tree for the case with a court of appeals

making an appeal.¹⁵ Second, as the tax court is able to determine the actual income with certainty, it reveals the case in which the tax auditor has performed a negligent audit and came to an erroneous conclusion ($M_i \neq X_i$). In case of revelation, the tax auditor is subject to some exogenous cost $R > 0$ in order to redeem for this error. This might involve having to work extra hours, a temporary demotion, or the obligation to take a course on how to better conduct future audits.

3.2 Preliminary Analysis

The presence of a tax court does not alter strategy spaces or the ex-ante beliefs of either the tax authority or the tax auditor, thus they are equal to the corresponding expressions in the previous section. The taxpayer, however, is now able to lodge an appeal at the tax court following tax auditor's report.

Therefore, his strategy is extended by an additional component O :

$$s_{TP} = (c, e, O) \tag{7}$$

We model the strategy component O as a 2x2x3 matrix that captures the taxpayer's decision to object the audit report if he is of type X_i with an income statement of Y_j and an audit status of l where $i, j \in \{L, H\}$ and $l \in \{Be, AuRiDi, AuWr\}$. For each combination of i and j , the tax authority can come to the following conclusions:

1. *Be*: The tax authority approves the income statement.
2. *AuRiDi*: The tax authority triggers an audit and the tax auditor comes to the right conclusion (either due to a diligent audit or by 'luck').
3. *AuWr*: The tax authority triggers an audit and the tax auditor comes to an erroneous conclusion.

¹⁵We do not explicitly model legal costs for the tax authority. Although it is very likely that tax authority incurs legal costs as a result of a court trial, we believe that these costs are relatively low or simply part of the tax authorities overhead and therefore insignificant for decision making. Further, we explicitly abstract from considering the operational costs of the court itself. This is mainly because those 'overhead' costs are borne by the public and cannot be attributed to any of the players.

Element o_{ijl} of matrix O equals 1 if the taxpayer objects the audit report and 0 otherwise. The information sets at the final stage are either singleton (cases with Be and $AuWr$) or contain decision nodes that imply identical payoffs for each decision (cases with $AuRiDi$). Under perfect recall this means that the taxpayer's beliefs are irrelevant at this stage and it is possible to derive matrix O^* that contains for each case the taxpayer's optimal response o_{ijl}^* .

We restrict our attention to cases where the introduction of a tax court can have an effect within the specified model. Therefore, we make the following two assumptions. First, we assume that a taxpayer of type X_L who has unjustifiably been convicted of tax evasion due to a wrong conclusion of the tax auditor will have an incentive to make an appeal in front of the tax court. That is, $L_{TP} < T_H - T_L + P$. Second, we rule out cases where a taxpayer of type X_L intentionally makes an over-declaration with Y_H and would make an appeal after the tax authority confirms this over-declaration (Be).¹⁶ This requires that $L_{TP} > T_H - T_L$.

Lemma 2 presents the taxpayer's optimal objection response O^* .

Lemma 2 (Taxpayer's optimal objection response). *The taxpayer's optimal objection response O^* is defined as follows:*

$$o_{ijl}^* = \begin{cases} 1 & \text{if } i = j = L \text{ and } l = AuWr, \\ 0 & \text{otherwise,} \end{cases}$$

with $i, j \in \{L, H\}$ and $l \in \{Be, AuRiDi, AuWr\}$.

Proof. The Lemma follows from a direct comparison between the taxpayer's payoffs $u_{TP}(o_{ijl} = 1)$ and $u_{TP}(o_{ijl} = 0)$ at each end node for all combinations of i, j and l . \square

In equilibrium, all player's strategies are sequentially rational. Therefore, O^* is part of the taxpayer's equilibrium strategy s_{TP} . As this is anticipated by both the tax authority and the tax auditor, we can restrict our further analysis to those cases that include O^* .

¹⁶The second assumption is also related to the general question, whether taxpayers have an opportunity to make an appeal against a tax payment that is consistent with their initial income statement. From a legal perspective this is possible. We do not exclude this possibility in our model ex-ante but make an assumption that rules out such behavior.

Under consideration of O^* , we can express the utility levels of each player following a negligent audit in expected terms (see Table 2). A comparison between Table 1 and

ij	$k = TP$	$k = FA$	$k = TA$
LL	$X_L - T_L - \omega_{LL}L_{TP}$	$T_L - F$	$F - \omega_{LL}R$
LH	$X_L - T_L - \omega_{LH}(T_H - T_L)$	$T_L - F + \omega_{LH}(T_H - T_L)$	F
HL	$X_H - T_L - (1 - \omega_{HL})(T_H - T_L + P)$	$T_L - F + (1 - \omega_{HL})(T_H - T_L + P)$	F
HH	$X_H - T_L - (1 - \omega_{HH})(T_H - T_L)$	$T_L - F + (1 - \omega_{HH})(T_H - T_L)$	F

Table 2: Expected utilities $u_k^{ij, Ne}$ following a negligent audit with a tax court and O^*

Table 2 reveals that as a direct consequence of O^* the payoffs following Ne change only in the case where type X_L has filed an income statement of Y_L . The expected transfer of additional taxes and a penalty $\omega_{LL}(T_H - T_L + P)$ from the taxpayer to the tax authority in the benchmark setting is replaced by the respective expected legal costs of a court trial and the tax auditor's expected cost $\omega_{LL}R$ to redeem for any errors occurred is included.

In the presence of a tax court and optimal response O^* , the conditional expected utilities of all players are as follows:

$$U_{TP}^{O^*}(s_{TP}|X = X_L) = X_L - T_L - c\hat{a}_L(1 - \hat{d}_L)\omega_{LL}L_{TP} - (1 - c) \left[1 - \hat{a}_H + \hat{a}_H\omega_{LH}(1 - \hat{d}_H) \right] (T_H - T_L) \quad (8)$$

$$U_{TP}^{O^*}(s_{TP}|X = X_H) = X_H - T_L - e\hat{a}_L[1 - \omega_{HL}(1 - \hat{d}_L)](T_H - T_L + P) - (1 - e)[1 - \hat{a}_H(1 - \hat{d}_H)\omega_{HH}](T_H - T_L) \quad (9)$$

$$U_{FA}^{O^*}(s_{FA}, \mu_{FA}|Y = Y_L) = T_L + a_L \left[\mu_{FA}(1 - \omega_{HL} + \hat{d}_L\omega_{HL})(T_H - T_L + P) - F \right] \quad (10)$$

$$U_{FA}^{O^*}(s_{FA}, \kappa_{FA}|Y = Y_H) = T_H - a_H F - a_H [\kappa_{FA}(1 - \hat{d}_H)\omega_{HH} + (1 - \kappa_{FA})(1 - \omega_{LH} + \hat{d}_H\omega_{LH})](T_H - T_L) \quad (11)$$

$$U_{TA}^{O^*}(s_{TA}, \mu_{TA}|Y = Y_L) = \hat{a}_L [F - d_L C - (1 - d_L)(1 - \mu_{TA})\omega_{LL}R] \quad (12)$$

$$U_{TA}^{O^*}(s_{TA}, \kappa_{TA}|Y = Y_H) = \hat{a}_H [F - d_H C] \quad (13)$$

The expected utilities of the tax authority and the tax auditor following an income statement Y_H do not change in comparison to the benchmark case. This, however, implies

again that for any belief κ_{FA} (κ_{TA}) the tax authority (the tax auditor) would never find it optimal to audit Y_H (diligently). Therefore, the equilibrium includes $a_H = 0$ as well as $d_H = 0$ and conjectures of the other players are accordingly. Using this result, the expected utilities of the taxpayer from Equations (8) and (9) simplify to:

$$U_{TP}^{O*}(s_{TP}|X = X_L) = X_L - T_L - c\hat{a}_L(1 - \hat{d}_L)\omega_{LL}L_{TP} - (1 - c)(T_H - T_L) \quad (14)$$

$$U_{TP}^{O*}(s_{TP}|X = X_H) = X_H - T_L - e\hat{a}_L[1 - \omega_{HL}(1 - \hat{d}_L)](T_H - T_L + P) - (1 - e)(T_H - T_L) \quad (15)$$

In contrast to the benchmark case, the tax auditor's expected utility after observing Y_L now depends on his belief relative to the honest high income earner ($1 - \mu_{TA}$) and includes a trade-off between the direct costs of a diligent audit C and the potential indirect cost to recuperate $(1 - \mu_{TA})\omega_{LL}R$ in case of an error. For the further analysis, we distinguish between the following two basic scenarios with regard to the relation between direct and indirect costs:

1. $\omega_{LL}R < C$: In this scenario, even if the tax auditor had perfect knowledge that a taxpayer with income statement Y_L is of type X_L ($\mu_{TA} = 0$) the direct costs of a diligent audit exceed the expected cost of disciplinary measures associated with an audit failure in case of a negligent audit. As a result, the tax auditor will always prefer $d_L = 0$ over any other strategy for any belief $\mu_{TA} \in [0, 1]$. The tax auditor's optimal strategy $s_{TA} = (0, 0)$ is therefore identical to the benchmark case.
2. $\omega_{LL}R > C$: In this scenario, the tax auditor prefers $d_L = 1$ over $d_L = 0$ for sufficiently low beliefs $\mu_{TA} < \mu^\dagger$, he prefers $d_L = 0$ over $d_L = 1$ for sufficiently high beliefs $\mu_{TA} > \mu^\dagger$, and he is indifferent for belief $\mu_{TA} = \mu^\dagger$ where μ^\dagger solves the following equation:¹⁷

$$(1 - \mu^\dagger)\omega_{LL}R = C \quad (16)$$

A comparison between the benchmark case and the two scenarios analyzed above shows that a tax court can provide the tax auditor with incentives to conduct a diligent audit.

¹⁷Keep in mind that belief $\mu_{TA} = Pr(X_H|Y_L)$ captures the belief of auditing a tax evader and hence a lower belief increases incentives for a diligent audit.

This effect, however, will only exist, if the direct costs from a diligent audit do not exceed the expected cost of performing a negligent audit. For the following analysis we will restrict our attention to scenario 2 and hence make the following assumption:

Assumption 1. $\omega_{LL}R > C$.

In this case, the tax auditor's reaction function can be summarized as follows:

$$d_L(\mu_{TA}) = \begin{cases} d_L = 1, & \text{if } \mu_{TA} < \mu^\dagger \\ d_L \in]0, 1[, & \text{if } \mu_{TA} = \mu^\dagger \\ d_L = 0, & \text{otherwise,} \end{cases} \quad (17)$$

$$\text{with } \mu^\dagger = 1 - \frac{C}{\omega_{LL}R}. \quad (18)$$

Similarly, we can derive the tax authority's best response function a_L after observing Y_L for a given belief of μ_{FA} . This can be achieved by comparison between the expected utility in Equation (10) with $a_L = 0$ and $a_L = 1$. The tax authority's decision rule is as follows:

$$a_L(\mu_{FA}) = \begin{cases} a_L = 1, & \text{if } \mu_{FA} > \mu^\ddagger \\ a_L \in]0, 1[, & \text{if } \mu_{FA} = \mu^\ddagger \\ a_L = 0, & \text{otherwise} \end{cases} \quad (19)$$

$$\text{with } \mu^\ddagger = \frac{F}{(1 - \omega_{HL} + \hat{d}_L \omega_{HL})(T_H - T_L + P)}. \quad (20)$$

A comparison between μ^* and μ^\ddagger reveals that $\mu^\ddagger > \mu^*$. Thus, the tax authority will less often choose $a_L = 1$ with a court of appeals as compared to the situation without a court of appeals. The intuition behind that result is as follows. Without a court of appeals, the tax authority benefits from a wrong audit opinion by receiving additional tax payments and a penalty. However, when the taxpayer is able to make an appeal, this benefit vanishes for the tax authority. Therefore, all else equal, the tax authority will trigger an audit less often after observing Y_L .

3.3 Results

In Lemma 3, we characterize the possible equilibria of the game in the presence of a tax court.

Lemma 3 (Equilibrium with a tax court). *With a tax court, the constituting equilibrium depends on the size of the audit fee F , the penalty P , tax auditor's direct cost from a diligent audit C , the legal costs L_{TP} , and the cost of disciplinary measures R in case of an audit failure. Define in addition to threshold values from Lemma 1 the threshold values \overline{C} , $F^\dagger|_{d_L=1}$, $F^\dagger|_{d_L=0}$, F° , F^\sharp and F^\flat as follows:*

$$\begin{aligned}\overline{C} &= (1 - \alpha)\omega_{LL}R, \\ F^\dagger|_{d_L=1} &= \alpha(T_H - T_L + P), \\ F^\dagger|_{d_L=0} &= \alpha(1 - \omega_{HL})(T_H - T_L + P), \\ F^\circ &= \left(1 - \frac{C}{\omega_{LL}R}\right)(T_H - T_L + P), \\ F^\sharp &= \left(1 - \frac{C}{\omega_{LL}R}\right)(1 - \omega_{HL})(T_H - T_L + P), \\ F^\flat &= \left(1 - \frac{C}{\omega_{LL}R}\right)(T_H - T_L).\end{aligned}$$

1. *If $(C < \overline{C})$ and $(F > F^\dagger|_{d_L=1})$, a pooling equilibrium in pure strategies as follows constitutes:*

$$\begin{aligned}s_{TP} &= (1, 1, O^*) & s_{FA} &= (0, 0) & s_{TA} &= (1, 0) \\ \mu_{FA} = \mu_{TA} &= \alpha & \kappa_{FA}, \kappa_{TA} &\in [0, 1]\end{aligned}$$

2. *If $(C > \overline{C})$ and $(F < F^\dagger|_{d_L=0})$ and $(P < \underline{P})$ and $(\omega_{LL}L_{TP} < T_H - T_L)$, a pooling equilibrium in pure strategies as follows constitutes:*

$$\begin{aligned}s_{TP} &= (1, 1, O^*) & s_{FA} &= (1, 0) & s_{TA} &= (0, 0) \\ \mu_{FA} = \mu_{TA} &= \alpha & \kappa_{FA}, \kappa_{TA} &\in [0, 1]\end{aligned}$$

3. *If $(C > \overline{C})$ and $(F > F^\dagger|_{d_L=0})$, a pooling equilibrium in pure strategies as follows constitutes:*

$$\begin{aligned}s_{TP} &= (1, 1, O^*) & s_{FA} &= (0, 0) & s_{TA} &= (0, 0) \\ \mu_{FA} = \mu_{TA} &= \alpha & \kappa_{FA}, \kappa_{TA} &\in [0, 1]\end{aligned}$$

4. *If $(P > \overline{P})$ and $(F < \overline{F})$ and $(\omega_{LL}L_{TP} > T_H - T_L)$, a pooling equilibrium in pure*

strategies as follows constitutes:

$$\begin{aligned} s_{TP} &= (0, 0, O^*) & s_{FA} &= (1, 0) & s_{TA} &= (0, 0) \\ \mu_{FA} &\in]\mu^\dagger|_{d_L=0}, 1[& \mu_{TA} &\in]\mu^\dagger, 1[& \kappa_{FA} &= \kappa_{TA} = \alpha \end{aligned}$$

$$\text{with } \mu^\dagger|_{d_L=0} = \frac{F}{(1 - \omega_{HL})(T_H - T_L + P)}$$

5. If $(C > \overline{C})$ and $(F < F^\circ)$ or if $(C < \overline{C})$ and $(F < F^\dagger|_{d_L=1})$, a partially pooling equilibrium in mixed strategies as follows constitutes:

$$\begin{aligned} s_{TP} &= (1, e^*, O^*) & s_{FA} &= (a_L^*, 0) & s_{TA} &= (1, 0) \\ \mu_{FA} &= \mu_{TA} = \mu^\dagger|_{d_L=1} & \kappa_{FA} &= \kappa_{TA} = 1 \end{aligned}$$

$$\begin{aligned} \text{with } \mu^\dagger|_{d_L=1} &= \frac{F}{(T_H - T_L + P)} \\ e^* &= \frac{1 - \alpha}{\alpha} \left[\frac{F}{(T_H - T_L + P) - F} \right] \\ a_L^* &= \frac{T_H - T_L}{T_H - T_L + P} \end{aligned}$$

6. If $(C > \overline{C})$ and $(F^\sharp < F < F^\dagger|_{d_L=0})$ and $(P > \underline{P})$, a partially pooling equilibrium in mixed strategies as follows constitutes:

$$\begin{aligned} s_{TP} &= (1, e^*, O^*) & s_{FA} &= (a_L^*, 0) & s_{TA} &= (0, 0) \\ \mu_{FA} &= \mu_{TA} = \mu^\dagger|_{d_L=0} & \kappa_{FA} &= \kappa_{TA} = 1 \end{aligned}$$

$$\begin{aligned} \text{with } \mu^\dagger|_{d_L=0} &= \frac{F}{(1 - \omega_{HL})(T_H - T_L + P)} \\ e^* &= \frac{1 - \alpha}{\alpha} \left[\frac{F}{(1 - \omega_{HL})(T_H - T_L + P) - F} \right] \\ a_L^* &= \frac{T_H - T_L}{(1 - \omega_{HL})(T_H - T_L + P)} \end{aligned}$$

7. If $(C > \overline{C})$ and $(F < F^b)$ and $(P < \underline{P})$, a partially pooling equilibrium in mixed strategies as follows constitutes:

$$\begin{aligned} s_{TP} &= (1, e^*, O^*) & s_{FA} &= (1, 0) & s_{TA} &= (d_L^*, 0) \\ \mu_{FA} &= \mu_{TA} = \mu^\dagger & \kappa_{FA} &= \kappa_{TA} = 1 \end{aligned}$$

$$\begin{aligned}
\text{with } \mu^\dagger &= 1 - \frac{C}{\omega_{LL}R} \\
e^* &= \frac{1-\alpha}{\alpha} \left[\frac{\omega_{LL}R}{C} - 1 \right] \\
d_L^* &= 1 - \frac{P}{\omega_{HL}(T_H - T_L + P)}
\end{aligned}$$

8. If $(C > \overline{C})$ and $(F^\sharp < F < F^\flat)$ and $(P < \underline{P})$ and $\left(\omega_{LL}L_{TP} < \frac{\omega_{HL}F(T_H - T_L + P)}{\left(1 - \frac{C}{\omega_{LL}R}\right)(T_H - T_L + P) - F} \right)$, a partially pooling equilibrium in mixed strategies as follows constitutes:

$$\begin{aligned}
s_{TP} &= (1, e^*, O^*) & s_{FA} &= (a_L^*, 0) & s_{TA} &= (d_L^*, 0) \\
\mu_{FA} &= \mu_{TA} = \mu^\dagger & \kappa_{FA} &= \kappa_{TA} = 1
\end{aligned}$$

$$\begin{aligned}
\text{with } \mu^\dagger &= 1 - \frac{C}{\omega_{LL}R} \\
e^* &= \frac{1-\alpha}{\alpha} \left[\frac{\omega_{LL}R}{C} - 1 \right] \\
d_L^* &= 1 - \frac{(\omega_{LL}R - C)(T_H - T_L + P) - F\omega_{LL}R}{\omega_{HL}(\omega_{LL}R - C)(T_H - T_L + P)} \\
a_L^* &= \frac{T_H - T_L}{F} \left(1 - \frac{C}{\omega_{LL}R} \right).
\end{aligned}$$

Proof. See Appendix. □

In comparison to the benchmark case, the number of possible equilibria increases with a tax court of appeals. It can be observed that in addition to the size of the audit fee and penalties also the size of direct audit costs and legal costs have an impact on the prevailing equilibrium. Note that a separating equilibrium is not feasible in the presence of a tax court. Agents with different characteristics choose the same action in either pure or mixed strategies. As nearly all equilibria are specified by the relation between direct audit costs and threshold value \overline{C} , we distinguish in the following basically between the two cases $C > \overline{C}$ and $C < \overline{C}$. Figure 4 illustrates graphically the possible equilibria from Lemma 3 if $C > \overline{C}$ and Figure 5 those if $C < \overline{C}$.

It is possible to provide a relative ranking of the various threshold values that relate to F . In general, we have $F^\sharp < F^\circ$ and $F^\dagger|_{d_L=0} < F^\dagger|_{d_L=1}$ and $F^\dagger|_{d_L=0} < \overline{F}$. Further, we have $F^\flat > F^\sharp$ if $P < \underline{P}$ and $F^\flat < F^\sharp$ otherwise.

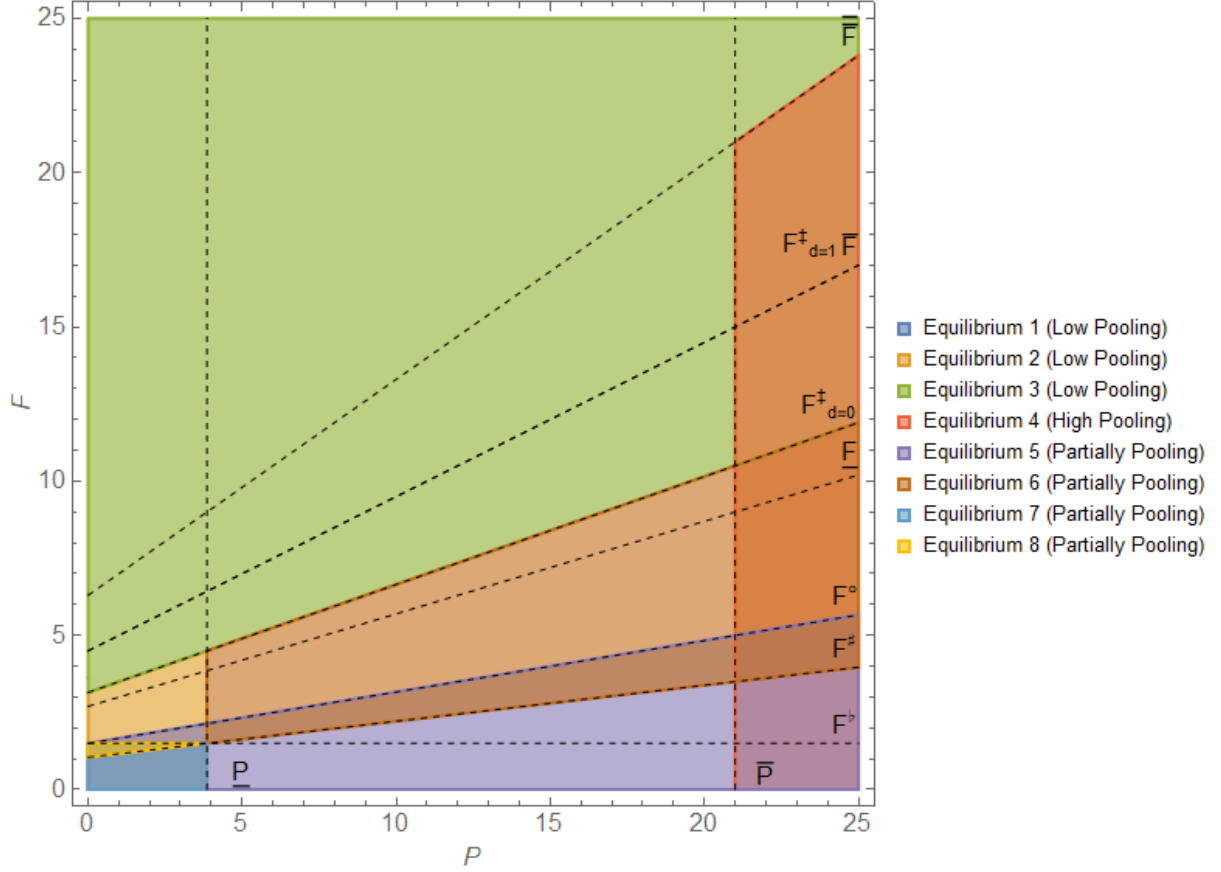


Figure 4: Graphical Illustration of Equilibria if $C > \bar{C}$ with $\alpha = 0.5$, $\omega_{ij} = 0.3 \forall i, j \in \{L, H\}$, $T_H = 10$, $T_L = 1$, $C = 1$ and $R = 4$

We abstain from discussing the various equilibria in the presence of a tax court in detail. Instead, we try to derive general conclusions relative to the impact of a tax court on audit accuracy, audit probability, tax evasion and over-declaration by comparing the benchmark case with the case that incorporates a tax court. We begin with Proposition 1 and summarize the impact of the tax court on audit accuracy.

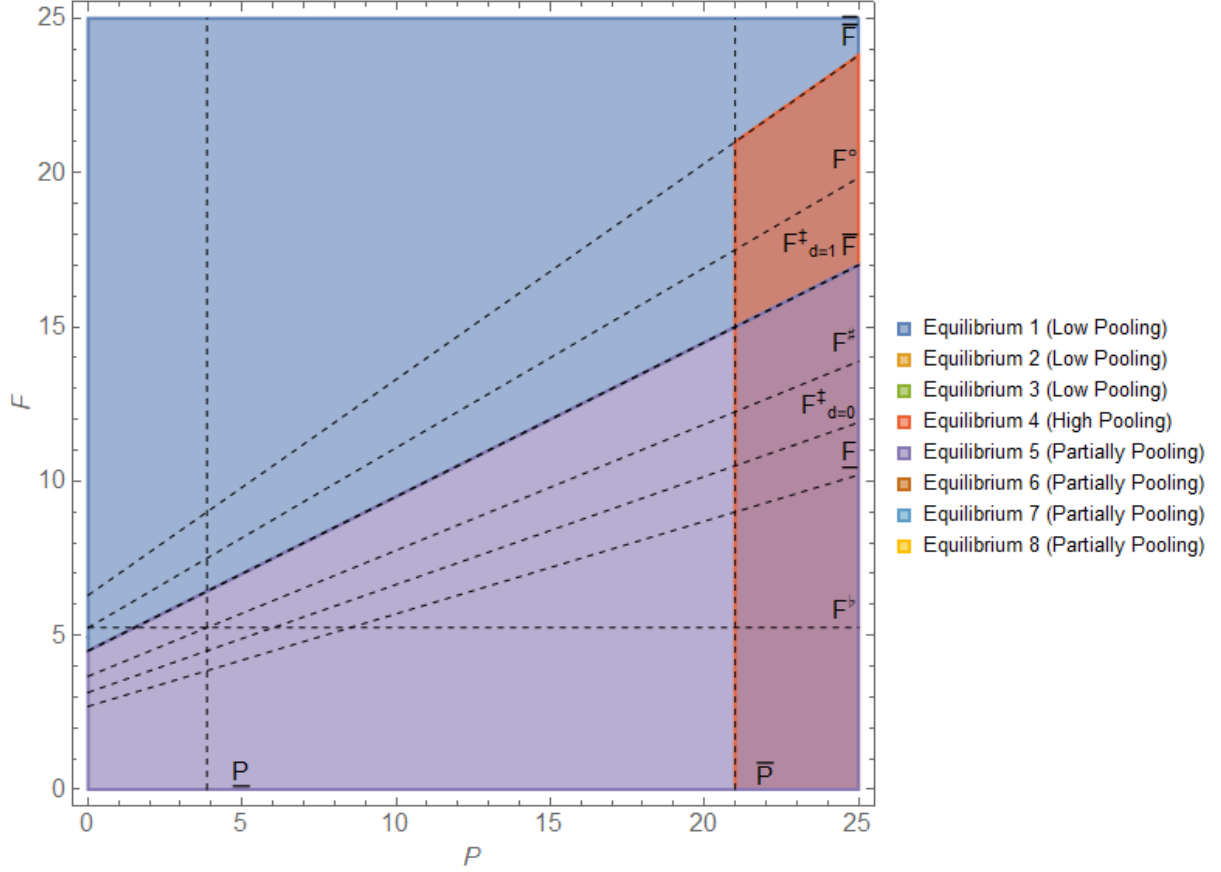


Figure 5: Graphical Illustration of Equilibria if $C < \bar{C}$ with $\alpha = 0.5$, $\omega_{ij} = 0.3 \forall i, j \in \{L, H\}$, $T_H = 10$, $T_L = 1$, $C = 1$ and $R = 8$

Proposition 1 (Effect of a Tax Court on Audit Accuracy). *A tax court improves audit accuracy. This is particularly the case if direct costs from a diligent audit are sufficiently low. More precisely, a tax court will always provide the tax auditor with incentives to increase audit accuracy if the direct audit costs in case of a diligent audit C do not exceed the expected cost of disciplinary measures to redeem for an audit failure $(1 - \alpha)\omega_{LL}R$.*

Proof. The Proposition follows from comparing the equilibria in Lemma 1 with the equilibria in Lemma 3. \square

Proposition 1 shows that a tax court can help to increase audit accuracy. The reason for this result is that a tax court is able to detect errors that are caused by negligent audits. If the expected indirect cost associated with audit failure (direct costs from a diligent audit) is sufficiently high (low) it will provide the tax auditor with incentives to audit more diligently. This behavior leads in turn to a higher audit accuracy.¹⁸ From the proposition we can also derive the following corollary:

Corollary 1. *The likelihood of a tax court's positive impact on audit accuracy increases in the probability of a wrong audit opinion following a negligent audit ω_{LL} and decreases in the proportion of high income earners α .*

A higher probability of an audit failure increases the tax auditor's expectation of incurring cost to redeem for negligence and therefore increases his incentives to audit diligently. Conversely, a larger proportion of high income earners diminishes the risk of committing an audit error that will be objected in front of a tax court. This results in less incentives to audit diligently.

In the following proposition, we would like to summarize how the tax court influences the likelihood of the tax authority conducting a tax audit.

Proposition 2 (Effect of a Tax Court on Audit Probabilities). *A tax court weakly decreases audit probabilities in all possible equilibria if the ex-ante probability of a wrong audit opinion following a low income statement of a low income earner is sufficiently high. More precisely, audit probabilities with a tax court are less or equal to audit probabilities without a tax court for all combinations of penalties, fees, audit costs and legal costs if $\omega_{LL} \geq \frac{\alpha}{1-\alpha}\omega_{HL}$. Otherwise, audit probabilities decline in almost all cases except the case where $\bar{F} < F < F^\dagger|_{d_L=1}$ and $C < \bar{C}$.*

Proof. The Proposition follows from a comparison between the equilibria in Lemma 1 and Lemma 3. All else equal, the case of $F^\dagger|_{d_L=1} > \bar{F}$ and $C < \bar{C}$ is the only case where the audit probability a_L^* increases from 0 (without a tax court) to $\frac{T_H - T_L}{T_H - T_L + P}$ (with a tax court)

¹⁸Please note that audit accuracy might also increase if C is above \bar{C} . However, in this case this effect is not universal and less pronounced.

if F is between \bar{F} and $F^\dagger|_{d_L=1}$. This solution, however, does not exist if $F^\dagger|_{d_L=1} < \bar{F}$ which is the case if $\omega_{LL} \geq \frac{\alpha}{1-\alpha}\omega_{HL}$. \square

A tax court of appeals decreases incentives for the tax authority to trigger an audit. As already indicated in the prior subsection, this may partly be attributed to the fact that a tax court hinders the tax authority to gain an extra benefit when an honest taxpayer is accused erroneously of committing tax evasion. However, the tax court also increases audit accuracy which indirectly affects the trade-offs between expected gains and costs from auditing a potential tax evader as well as the decision of a high income earner to understate his income. In total, those effects decrease audit probabilities for nearly all combinations of audit fees and penalties.

Next, we would like to summarize the impact of a tax court on the taxpayers' incentives to overstate his income.

Proposition 3 (Effect of a Tax Court on Overstatements). *A tax court prevents taxpayers from overstating their income if the legal costs associated with tax court litigation are sufficiently low. More precisely, overstatements are prevented if the expected legal costs of filing a conforming income statement ($\omega_{LL}L_{TP}$) do not exceed additional taxes as a result of an overstated tax filing ($T_H - T_L$).*

Proof. The Proposition follows from comparing the equilibria in Lemma 1 with the equilibria in Lemma 3. \square

If legal costs are not too high, a tax court reduces incentives for taxpayers to overstate their income on their tax return filings. Our result suggests that a system for legal protection can only be effective, if it is sufficiently cost efficient. An expensive device for legal protection would incur costs to the public without enhancing the rights of people who are potentially affected by wrong decisions of a (tax) administration. Intuitively, the higher the chance that an honest taxpayer will be accused erroneously of committing tax evasion, the more this taxpayer is likely to favor an over-declaration compared to risking legal costs that can occur in case of a court trial.

Next, we would like to turn the attention to the overall impact of the tax court on tax evasion, beginning with a proposition.

Proposition 4 (Effect of a Tax Court on Tax Evasion). *The occurrence and probability of tax evasion can increase or decrease due to the presence of a tax court.*

Proof. To prove this claim, it is sufficient to present one case where e^* increases and one case where e^* decreases. An example for the first case can be found by comparison between Equilibrium 4 in Lemma 1 and Equilibrium 5 in Lemma 3 for identical parameter combinations. While in the absence of a tax court a high income earner would report his income truthfully, with a tax court in place he commits tax evasion with positive probability. An example for the latter case can be found by comparison between Equilibrium 1 in Lemma 1 and Equilibrium 7 in Lemma 3. While in the absence of a tax court a high income earner would commit tax evasion with certainty, with a tax court in place he commits tax evasion only with some probability, strictly less than 1. \square

It is difficult to draw simple conclusions regarding the overall impact of a tax court on tax evasion. This is because there are several opposing forces in place that partially offset each other. On the one hand, a tax court increases audit accuracy in many cases. An increase to audit accuracy reduces incentives to under-declare income as the likelihood of being caught, conditional on getting audited, increases. Thus, tax evaders can be easier detected. On the other hand, a tax court leads to reduced audit rates in many cases which strengthen the incentive to commit tax evasion. The overall outcome depends crucially on the parameter setting. Looking at the equilibria of Lemma 3, we can infer that if tax audits are costly and the fee for conducting such tax audits is high, taxpayers will more likely pool in equilibria declaring low income. High penalties for tax evasion in combination with expensive or inefficient legal protection results in a situation where we have no tax evasion at the cost of having all taxpayers declare high income regardless of their actual income. The socially most desirable equilibria (5-8) can be separated into two categories. The first can be achieved with a combination of low penalties for tax evasion and a reasonable relation between the cost of legal protection and the tax auditor fee. In this case, it is critical to ensure that disciplinary penalties for audit failures are kept low to curb tax evasion. Alternatively, it is possible to curb tax evasion independent of the cost associated with disciplinary measures in case of an audit failure if audit fees are

sufficiently low and do not differ too much from the cost of conducting an audit while penalties for tax evasion are rather high.

4 Conclusion

Theoretical papers analyzing interactions between taxpayers and tax authorities show that enforcement influences the taxpayer's propensity to engage in tax evasion. We contribute to the literature and present novel insights relative to the impact of tax court appeals on the taxpayer's propensity of filing a truthful tax declaration, tax auditor incentives, and on the tax authority's design of an optimal audit policy. Our game-theoretic model explicitly accounts for tax auditor incentives, an imperfect audit technology, penalty levels, cost of litigation, as well as audit fees. In absence of a tax court, possible equilibria comprise understatements, overstatements, and truthful income tax statements. A tax court is able to increase audit accuracy, but this may in turn lower the tax authority's audit rates in anticipation of potentially having the audit results overturned.

The analysis relies on several assumptions regarding the relative magnitude of various model parameters, particularly the cost of legal court proceedings. If the legal costs associated with tax court proceedings are sufficiently low, the court can help mitigate over-declaration. Conversely, if the tax court is less affordable, taxpayers are less likely to appeal erroneous tax audit results which weakens the court's effect on audit quality and audit rates. Overall, we illustrate how legal protection in the shape of a tax court influences incentives of both taxpayers and the tax authority.

Our study provides useful policy implications. First, it highlights that the effectiveness of any legal protection depends crucially on how costly that protection is. Protection that is not affordable will not be effective as a safeguard for honest taxpayers. Policymakers should consider this when designing a system for legal protection. Second, it shows that the overall impact of the tax court goes beyond providing legal protection for honest taxpayers. The fact that the taxpayer has the right to file a tax court appeal incentivizes the tax auditor to conduct audits more diligently, which in turn increases the ex-ante accuracy of tax audits. Thus, the mere presence of a tax court influences the tax authority's optimal audit rates as well as the economic trade-offs faced by potential tax evaders.

Our study is subject to several limitations. For analytical feasibility, we employed a binary model. Similarly, we ruled out more complex cost structures for tax audits and abstained from discussing the issue of optimal tax auditor incentives. Another complexity-reducing assumption relates to the tax court. We assume that the tax court reveals the taxpayers' true income with certainty, summarizing the real-world legal process into one stage instead of modeling a multilayered appeals system that includes error probabilities at its various stages. Although the last stage of appeals can still make mistakes, it has the final say on the taxpayers' liability, and its error probability should be sufficiently low.

Future research can build on the results of this paper by validating them within a continuous setting. Moreover, our insights may be used as a foundation for future empirical research. For instance, it would be interesting to see whether or not and to what extent the quantity and quality of tax audits, the level of disputed taxable income, and the overall collected penalties evolved differently in countries that experienced changes in the rule of law, particularly in terms of court independence (for example, Hungary, Poland, or Turkey) in comparison to countries with stable levels of court independence.

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A Appendix

A.1 Proofs

Proof of Lemma 1. In order to prove the existence of Equilibria 1-5 we start with some preliminary steps. An equilibrium can only constitute if the tax auditor chooses strategy $s_{TA} = (0, 0)$ because it strictly dominates all other strategies. Therefore, the tax auditor's behavior is not investigated further. Moreover, it is optimal for the fiscal authority to never audit a high income statement ($a_H = 0$).

The fiscal authority makes the decision to audit a low income statement Y_L conditional on his belief μ_{FA} and the conjectured strategy of the tax auditor. It will decide to audit ($a_L = 1$) if,

$$U_{FA}(a_L = 1, \mu_{FA}|Y = Y_L) > U_{FA}(a_L = 0, \mu_{FA}|Y = Y_L) \quad (\text{A.1})$$

or equivalently if

$$T_L - F + [(1 - \mu_{FA})(1 - \hat{d}_L)\omega_{LL} + \mu_{FA}(1 - \omega_{HL} + \hat{d}_L\omega_{HL})](T_H - T_L + P) > T_L \quad (\text{A.2})$$

Hence, his belief has to be:

$$\mu_{FA} > \frac{F - (1 - \hat{d}_L)\omega_{LL}(T_H - T_L + P)}{[1 - \omega_{HL} + \hat{d}_L\omega_{HL} - (1 - \hat{d}_L)\omega_{LL}](T_H - T_L + P)} \equiv \mu^* \quad (\text{A.3})$$

In equilibrium conjectured values hold and hence, $\hat{d}_L = 0$. Therefore, μ^* reduces to:

$$\frac{F - \omega_{LL}(T_H - T_L + P)}{[1 - \omega_{HL} - \omega_{LL}](T_H - T_L + P)} \equiv \mu^* \quad (\text{A.4})$$

$$\text{The fiscal authority's decision rule is: } a_L(\mu_{FA}) = \begin{cases} a_L = 1, & \text{if } \mu_{FA} > \mu^* \\ a_L \in]0, 1[, & \text{if } \mu_{FA} = \mu^* \\ a_L = 0, & \text{otherwise} \end{cases}$$

Next, we investigate different types (pooling, separating, partially pooling and fully mixed) of equilibrium candidates in order to prove the existence of Equilibria 1-5.

Pure strategy equilibria

Pooling on Y_L ($c = 1$ and $e = 1$)

We start with a pure strategy equilibrium candidate where the taxpayer pools on Y_L . This is equivalent to $c = 1$ and $e = 1$ or $s_{TP} = (1, 1)$. On the equilibrium path, the fiscal

authority's belief μ_{FA} (as well as the tax auditor's belief μ_{TA} , which is irrelevant here) has to be determined by Bayes' Rule. Hence,

$$\mu_{FA}(\hat{c}, \hat{e}) = \mu_{TA}(\hat{c}, \hat{e}) = \frac{\alpha \hat{e}}{\alpha \hat{e} + (1 - \alpha) \hat{c}}. \quad (\text{A.5})$$

Accordingly, if conjectures hold, we have $\mu_{FA} = \alpha$. From the fiscal authority's decision rule there follow two equilibrium candidates: (1) $a_L = 1$ if $\alpha > \mu^*$ and (2) $a_L = 0$ if $\alpha < \mu^*$.¹⁹ From Equation (A.4) it follows that the fiscal authority chooses $a_L = 1$ if

$$F < [\alpha(1 - \omega_{HL}) + (1 - \alpha)\omega_{LL}](T_H - T_L + P) \equiv \bar{F} \quad (\text{A.6})$$

and $a_L = 0$ if $F > \bar{F}$.

In order to prove that candidate (1) constitutes an equilibrium, we have to show that none of the taxpayers (type X_L or X_H) has an incentive to deviate. In equilibrium, the conjectured values correspond to actual values and hence the expected utility of types X_L and X_H in Equations (1) and (2) reduce to:

$$U_{TP}(s_{TP}|X = X_L) = X_L - T_L - c\omega_{LL}(T_H - T_L + P) - (1 - c)(T_H - T_L) \quad (\text{A.7})$$

$$U_{TP}(s_{TP}|X = X_H) = X_H - T_L - e(1 - \omega_{HL})(T_H - T_L + P) - (1 - e)(T_H - T_L) \quad (\text{A.8})$$

Type X_L does not deviate if

$$U_{TP}(c = 1|X = X_L) > U_{TP}(c = 0|X = X_L) \quad (\text{A.9})$$

or equivalently if

$$P < \frac{1 - \omega_{LL}}{\omega_{LL}}(T_H - T_L) \equiv \bar{P}. \quad (\text{A.10})$$

Similarly, from comparing the utility of type X_H between $e = 1$ and $e = 0$ we can derive that no deviation occurs if

$$P < \frac{\omega_{HL}}{1 - \omega_{HL}}(T_H - T_L) \equiv \underline{P}. \quad (\text{A.11})$$

From $\omega_{LL} < 1/2$ and $\omega_{HL} < 1/2$ it follows that $\underline{P} < \bar{P}$ and hence both types do not deviate, if the stricter Equation (A.11) holds. As income statement Y_H is off the equilibrium path and the fiscal authority's (the tax auditor's) strategy is independent from

¹⁹We ignore the special case of $\mu_{FA} = \mu^*$.

belief κ_{FA} (κ_{TA}) it can take any value between 0 and 1. If $P < \underline{P}$ and $F < \overline{F}$ none of the players has an incentive to deviate from their strategies $s_{TP} = (1, 1)$, $s_{FA} = (1, 0)$, and $s_{TA} = (0, 0)$ given beliefs $\mu_{FA} = \mu_{TA} = \alpha$ and $\kappa_{FA}, \kappa_{TA} \in [0, 1]$. Hence, under these conditions Equilibrium 1 prevails.

To prove that candidate (2) with $a_L = 0$ constitutes an equilibrium, it is necessary to show that neither type deviates from $c = 1$ and $e = 1$. With $\hat{a}_L = 0$, expected utilities in Equations (1) and (2) become:

$$U_{TP}(s_{TP}|X = X_L) = X_L - T_L - (1 - c)(T_H - T_L) \quad (\text{A.12})$$

$$U_{TP}(s_{TP}|X = X_H) = X_H - T_L - (1 - e)(T_H - T_L) \quad (\text{A.13})$$

As both types maximize the expected utility with $c = 1$ and $e = 1$ respectively, both types will never deviate. For the same argument as with Equilibrium 1, beliefs κ_{FA} and κ_{TA} can take any value between 0 and 1. Therefore, if $F > \overline{F}$ the strategies $s_{TP} = (1, 1)$, $s_{FA} = (0, 0)$ and $s_{TA} = (0, 0)$ are sequentially rational given the beliefs $\mu_{FA} = \mu_{TA} = \alpha$ and $\kappa_{FA}, \kappa_{TA} \in [0, 1]$. This proves that in this case Equilibrium 2 constitutes.

Pooling on Y_H ($c = 0$ and $e = 0$)

Next, we investigate equilibrium candidates with pooling at Y_H , meaning that $c = 0$ and $e = 0$. If the fiscal authority observes Y_H it would always choose not to audit ($a_H = 0$). The fiscal authority's belief κ_{FA} is determined according to Bayes' rule and equals α . The belief μ_{FA} is off the equilibrium path.

A potential deviation of either taxpayer depends on the fiscal authority's best response off the equilibrium path which is determined by the decision rule from above $a_L(\mu_{FA})$ depending on the belief μ_{FA} . We first, evaluate the situation of $\mu_{FA} > \mu^*$ and hence $a_L = 1$. For $\mu_{FA} > \mu^*$ we have to restrict μ^* to values below 1 which is equivalent to the following condition:

$$F < (1 - \omega_{HL})(T_H - T_L + P) \equiv \overline{\overline{F}} \quad (\text{A.14})$$

In the case of $a_L = 1$, type X_L 's and X_H 's expected utilities correspond with those in Equations (A.7) and (A.8). Type X_L does not deviate, if

$$U_{TP}(c = 0|X = X_L) > U_{TP}(c = 1|X = X_L) \quad (\text{A.15})$$

or equivalently, if $P > \bar{P}$. Similarly, type X_H does not deviate if $P > \underline{P}$. From $\underline{P} < \bar{P}$ it follows that the first condition is stricter than the second one. If $P > \bar{P}$ and $F < \bar{F}$, strategies $s_{TP} = (0, 0)$, $s_{FA} = (1, 0)$ and $s_{TA} = (0, 0)$ are sequentially rational given beliefs $\mu_{FA} = \mu_{TA} > \mu^*$ and $\kappa_{FA} = \kappa_{TA} = \alpha$. This proves that in this case Equilibrium 3 prevails.

If $\mu_{FA} < \mu^*$, the fiscal authority would choose $a_L = 0$. In this case, expected utility of type X_L is given by Equation (A.12). It is easy to observe that type X_L would always like to deviate from $c = 0$ as it increases his utility. Therefore, $a_L = 0$ cannot be part of a pooling equilibrium on Y_H .

Separating with $c = 1$ and $e = 0$

Next, we evaluate a candidate for a separating equilibrium with $c = 1$ and $e = 0$. In this case beliefs are $\mu_{FA} = \mu_{TA} = 0$ and $\kappa_{FA} = \kappa_{TA} = 1$. From the fiscal authority's decision rule $a_L(\mu_{FA})$ it follows that $a_L = 1$ if

$$F < \omega_{LL}(T_H - T_L + P) \equiv \underline{F} \quad (\text{A.16})$$

and $a_L = 0$ otherwise.

For $a_L = 1$ to be part of an equilibrium we have to find conditions that ensure neither type wants to deviate. Type X_L prefers $c = 1$ over $c = 0$ if the condition in Equation (A.9) holds, which is equivalent to $P < \bar{P}$. Further, type X_H prefers $e = 0$ over $e = 1$ if $P > \underline{P}$.²⁰ Hence, no deviation of both types occurs only if $\underline{P} < P < \bar{P}$. To summarize, strategies $s_{TP} = (1, 0)$, $s_{FA} = (1, 0)$, and $s_{TA} = (0, 0)$ and beliefs $\mu_{FA} = \mu_{TA} = 0$ and $\kappa_{FA} = \kappa_{TA} = 1$ form separating Equilibrium 4, if $F < \underline{F}$ and $\underline{P} < P < \bar{P}$.

For proving that $a_L = 0$ cannot be part of a separating equilibrium with $c = 1$ and $e = 0$ it is sufficient to show that type X_H has always an incentive to deviate because he could increase his expected utility by engaging in tax evasion (see Eq. (A.13)).

Separating with $c = 0$ and $e = 1$

Another possible candidate for a separating equilibrium would be $c = 0$ and $e = 1$. This would mean that type X_H always evades taxes whereas type X_L would always overstate

²⁰See Eq. (A.11) for a deviation in the reverse direction.

his income. In this case beliefs are $\mu_{FA} = \mu_{TA} = 1$ and $\kappa_{FA} = \kappa_{TA} = 0$. From the fiscal authority's decision rule $a_L(\mu_{FA})$ it follows that $a_L = 1$ if

$$F < (1 - \omega_{HL})(T_H - T_L + P) \equiv \bar{\bar{F}} \quad (\text{A.17})$$

and $a_L = 0$ otherwise.

If the above condition is met and hence, $a_L = 1$, then type X_H does not deviate from $e = 1$ if $P < \underline{P}$ (see Eq. (A.11)). However type X_L would only maintain $c = 0$ if $P > \bar{P}$ (see Eq. (A.10)). As P cannot meet both conditions at the same time, either X_L or X_H would deviate which rules out $a_L = 1$ as being part of a separating equilibrium with $c = 0$ and $e = 1$.

If $a_L = 0$, then type X_L would always like to deviate from $c = 0$ as he could increase his expected utility. Therefore, there exists no separating equilibrium with $c = 0$ and $e = 1$.

Partially pooling equilibria

As a next step, we evaluate candidates for a partially pooling equilibrium. A partially pooling equilibrium requires that the fiscal authority and the taxpayer randomize in one component of their strategy. As the fiscal authority always prefers $a_H = 0$ over $a_H = 1$, a randomization can only take place with a_L . This, however rules out all candidates where type X_L and X_H pool at Y_H , because such an equilibrium would require that $0 < a_H < 1$. Therefore, only the following two possible candidates remain:

1. Type X_L always files an income statement of Y_L ($c = 1$) and X_H randomizes between Y_L and Y_H with $e \in]0, 1[$.
2. Type X_H always files an income statement of Y_L ($e = 1$) and X_L randomizes between Y_L and Y_H with $c \in]0, 1[$.

Partially pooling with $c = 1$ and $e \in]0, 1[$

We first evaluate candidate (1). In this case belief κ_{FA} equals 1. Randomization requires that type X_H is indifferent between $e = 0$ and $e = 1$ which occurs, if

$$U_{TP}(e = 0|X = X_H) = U_{TP}(e = 1|X = X_H). \quad (\text{A.18})$$

Rearrangement of the above equation yields the conjectured audit probability \hat{a}_L that is necessary for indifference:

$$\hat{a}_L = \frac{T_H - T_L}{(1 - \omega_{HL})(T_H - T_L + P)} \quad (\text{A.19})$$

In equilibrium the conjectured audit probability equals the optimal decision of the fiscal authority ($a_L^* = \hat{a}_L$). Randomization requires, however, that a_L^* has to be between 0 and 1 which holds if $P > \underline{P}$.

From the fiscal authority's decision rule we know that it randomizes after observing Y_L if $\mu_{FA} = \mu^*$. A reasonable belief $0 < \mu_{FA} < 1$ requires that $\underline{F} < F < \bar{F}$. The equilibrium belief of the fiscal authority μ_{FA} is determined by Bayes' Rule as indicated in Equation (A.5). Using the conjecture $\hat{c} = 1$ we can calculate the necessary conjecture of the probability of tax evasion as

$$\hat{e} = \frac{(1 - \alpha)\mu_{FA}}{\alpha(1 - \mu_{FA})} \quad (\text{A.20})$$

In equilibrium the conjectured value equals the optimal decision of the taxpayer e^* and hence

$$e^* = \hat{e} = \frac{1 - \alpha}{\alpha} \left[\frac{F - \omega_{LL}(T_H - T_L + P)}{(1 - \omega_{HL})(T_H - T_L + P) - F} \right]. \quad (\text{A.21})$$

For randomization it is required that $0 < e^* < 1$. This is the case if $\underline{F} < F < \bar{F}$ which is stricter than the condition for a reasonable μ_{FA} .

The equilibrium can only exist, if type X_L has no incentive to deviate from $c = 1$. The expected utility of type X_L can be calculated from Equation (1) by using $\hat{a}_L = a_L^*$, $\hat{a}_H = 0$ and $\hat{d}_L = \hat{d}_H = 0$:

$$U_{TP}(s_{TP}|X = X_L) = X_L - T_L - c \frac{\omega_{LL}}{1 - \omega_{HL}}(T_H - T_L) - (1 - c)(T_H - T_L) \quad (\text{A.22})$$

From $\omega_{LL} < 1/2$ and $\omega_{HL} < 1/2$ it follows that $\frac{\omega_{LL}}{1 - \omega_{HL}} < 1$ and hence the term $\frac{\omega_{LL}}{1 - \omega_{HL}}(T_H - T_L)$ is smaller than $(T_H - T_L)$. Therefore the expected utility is maximized with $c = 1$ and type X_L has no incentive to deviate. This is sufficient to show that, if $P > \underline{P}$ and $\underline{F} < F < \bar{F}$ we have Equilibrium 5 with strategies $s_{TP} = (1, e^*)$, $s_{FA} = (a_L^*, 0)$, $s_{TA} = (0, 0)$ and beliefs $\mu_{FA} = \mu_{TA} = \mu^*$, $\kappa_{FA} = \kappa_{TA} = 1$.

Partially pooling with $c \in]0, 1[$ and $e = 1$

Next, we evaluate the second candidate for a partial pooling equilibrium with $e = 1$ and $c \in]0, 1[$. In this case, belief κ_{FA} equals 0. Type X_L randomizes if

$$U_{TP}(c = 1|X = X_L) = U_{TP}(c = 0|X = X_L) \quad (\text{A.23})$$

This requires that the conjectured audit probability is as follows:

$$\hat{a}_L = \frac{T_H - T_L}{\omega_{LL}(T_H - T_L + P)} \quad (\text{A.24})$$

As in equilibrium conjectures hold, we have $a_L^* = \hat{a}_L$. The pooling equilibrium is only sustainable, if type X_H has no incentive to deviate from $e = 1$. Using the conjectured audit probabilities, the expected utility of type X_H in Equation (2) reduces to:

$$U_{TP}(s_{TP}|X = X_H) = X_H - T_L - e \frac{1 - \omega_{HL}}{\omega_{LL}}(T_H - T_L) - (1 - e)(T_H - T_L) \quad (\text{A.25})$$

From $\omega_{LL} < 1/2$ and $\omega_{HL} < 1/2$ it follows that $\frac{1 - \omega_{HL}}{\omega_{LL}} > 1$ and therefore, the expected utility is maximized with $e = 0$. This is sufficient to show that there does not exist such an equilibrium with partially pooling.

Completely mixed equilibria with $c \in]0, 1[$ and $e \in]0, 1[$

Finally, an equilibrium in completely mixed strategies would require that the fiscal authority randomizes after observing Y_H meaning that $a_H \in]0, 1[$. As this is not optimal for the fiscal authority for any belief κ_{FA} such an equilibrium cannot exist. \square

Proof of Lemma 3. In the following we derive conditions for the different equilibria that may constitute in the game. We start with pure strategies equilibria.

Pure strategy equilibria

Pooling on Y_L ($c = 1$ and $e = 1$)

First, we evaluate a candidate for a pooling equilibrium where both types file an income statement Y_L and hence, $c = 1$ and $e = 1$. According to Bayes' rule the beliefs are $\mu_{FA} = \mu_{TA} = \alpha$. Those beliefs determine the fiscal authority's and the tax auditor's optimal responses and we can distinguish the following four cases:

1. $\alpha < \mu^\dagger$ and $\alpha > \mu^\ddagger$
2. $\alpha < \mu^\dagger$ and $\alpha < \mu^\ddagger$
3. $\alpha > \mu^\dagger$ and $\alpha > \mu^\ddagger$
4. $\alpha > \mu^\dagger$ and $\alpha < \mu^\ddagger$

The fiscal authority's best response is also determined by his conjectures \hat{d}_L regarding the tax auditor's optimal response which in equilibrium equals his actual best response.

We start with case 1. Beliefs fulfill the requirements for case 1, if the following two conditions hold:

$$C < (1 - \alpha)\omega_{LL}R \equiv \bar{C} \quad (\text{A.26})$$

and

$$F < \alpha(T_H - T_L + P) \equiv F^\ddagger|_{d_L=1} \quad (\text{A.27})$$

In this case, the tax auditor plays $d_L = 1$ whereas the fiscal authority plays $a_L = 1$. Then, the expected utility of type X_H from Equation (15) is as follows:

$$U_{TP}^{O*}(s_{TP}|\hat{a}_L = 1, \hat{d}_L = 1, X = X_H) = X_H - T_L - e(T_H - T_L + P) - (1 - e)(T_H - T_L) \quad (\text{A.28})$$

As the expected utility is maximized with $e = 0$, type X_H would like to deviate and hence, under case 1 we have no pooling equilibrium.

Beliefs meet requirements in case 2 if $C < \bar{C}$ and $F > F^\ddagger|_{d_L=1}$. Then, we have the optimal responses $d_L = 1$ and $a_L = 0$. In this case, neither type X_L nor type X_H have an incentive to deviate because there will be no audit of income statement Y_L and hence both types' utilities are maximized with $c = 1$ and $e = 1$ respectively. The off-the equilibrium beliefs κ_{FA} and κ_{TA} are irrelevant for all players strategies. This is sufficient to prove existence of Equilibrium 1.

Next, we evaluate case 3. This occurs, if $C > \bar{C}$ and $F < F^\ddagger|_{d_L=0}$ where

$$\alpha(1 - \omega_{HL})(T_H - T_L + P) \equiv F^\ddagger|_{d_L=0} \quad (\text{A.29})$$

Under these conditions, the optimal responses are $d_L = 0$ and $a_L = 1$. From type X_L 's utility function in Equation (14) we see that he does not deviate if $\omega_{LL}L_{TP} < T_H - T_L$.

Similarly, type X_H will not deviate if $P < \underline{P}$. As with case 2, the off-equilibrium beliefs are irrelevant. This is sufficient to prove existence of Equilibrium 2.

Finally, case 4 will occur, if $C > \bar{C}$ and $F > F^\dagger|_{d_L=0}$ which results in optimal responses $d_L = 0$ and $a_L = 0$. As with case 2 neither type has an incentive to deviate and therefore, under these conditions Equilibrium 3 prevails.

Pooling on Y_H ($c = 0$ and $e = 0$)

We now evaluate a candidate for a pooling equilibrium where both types file an income statement Y_H and hence, $c = 0$ and $e = 0$. The beliefs on the equilibrium path are $\kappa_{FA} = \kappa_{TA} = \alpha$ and the optimal responses of the fiscal authority and the tax auditor are $a_H = 0$ and $d_H = 0$. The beliefs off the equilibrium path μ_{FA} and μ_{TA} are arbitrary as long as they support the equilibrium. We distinguish the following four cases which imply the optimal responses in brackets:

1. $\mu_{TA} < \mu^\dagger$ and $\mu_{FA} > \mu^\dagger$ ($d_L = 1, a_L = 1$)
2. $\mu_{TA} < \mu^\dagger$ and $\mu_{FA} < \mu^\dagger$ ($d_L = 1, a_L = 0$)
3. $\mu_{TA} > \mu^\dagger$ and $\mu_{FA} > \mu^\dagger$ ($d_L = 0, a_L = 1$)
4. $\mu_{TA} > \mu^\dagger$ and $\mu_{FA} < \mu^\dagger$ ($d_L = 0, a_L = 0$)

Cases 1,2 and 4 cannot be part of an equilibrium because type X_L can improve his utility strictly by deviating from $c = 0$ and playing $c = 1$ instead (see Equation (14)).

In case 3, type X_L does not deviate, if $\omega_{LL}L_{TP} > T_H - T_L$. Since we require that $L_{TP} < T_H - T_L + P$ a non-empty interval for L_{TP} exists only if

$$\frac{T_H - T_L}{\omega_{LL}} < T_H - T_L + P \quad (\text{A.30})$$

or equivalently if $P > \bar{P}$.

Similarly, it follows from Equation (15) that type X_H does not deviate from $e = 0$, if $P > \underline{P}$ which is implied by $P > \bar{P}$. Finally, a reasonable threshold $\mu^\dagger|_{d_L=0} < 1$ requires that $F < \bar{F}$. This is sufficient to prove existence of Equilibrium 4.

Separating with $c = 1$ and $e = 0$

It is straightforward to show that such an equilibrium cannot exist. The equilibrium would require beliefs $\mu_{FA} = \mu_{TA} = 0$ and $\kappa_{FA} = \kappa_{TA} = 1$. Since $\mu^\dagger > 0$ the fiscal authority's optimal response is always $a_L = 0$. This, however, provides type X_H with incentives to deviate from $e = 0$ as he can strictly increase his utility. Therefore, a separating equilibrium with $c = 1$ and $e = 0$ cannot exist.

Separating with $c = 0$ and $e = 1$

In this case, the associated beliefs are $\mu_{FA} = \mu_{TA} = 1$ and $\kappa_{FA} = \kappa_{TA} = 0$. Since $\mu^\dagger < 1$ is always true the tax auditor will play $d_L = 0$. The fiscal authority's optimal response depends on whether $\mu^\dagger|_{d_L=0}$ is larger or smaller than 1.

We have $\mu^\dagger|_{d_L=0} < 1$ if $F < \bar{\bar{F}}$. In this case, the fiscal authority would play $a_L = 1$. From the expected utility functions in Equations (14) and (15) it follows that type X_L does not deviate if $\omega_{LL}L_{TP} > T_H - T_L$ and type X_H does not deviate if $P < \underline{P}$. In the following, we show that both conditions cannot hold at the same time when we assume $L_{TP} < T_H - T_L + P$. An alternative representation of this assumption is $L_{TP} - (T_H - T_L) < P$. Since, X_H does not deviate if $P < \underline{P}$, a non-empty interval for P exists only, if $L_{TP} - (T_H - T_L) < \underline{P}$ or equivalently if

$$(1 - \omega_{HL})L_{TP} < T_H - T_L \quad (\text{A.31})$$

This, however, contradicts $\omega_{LL}L_{TP} > T_H - T_L$ because $\omega_{LL} < 1/2$ and $(1 - \omega_{HL}) > 1/2$. Therefore, an equilibrium cannot exist in this case.

If $F > \bar{\bar{F}}$, then the fiscal authority's best response is $a_L = 0$. In this case, however, type X_L could strictly improve by deviating from $c = 0$ and therefore, this cannot be part of an equilibrium.

Partially pooling equilibria

Next, we investigate equilibrium candidates that involve a partially pooling strategy of the taxpayer. For the same argument as for Lemma 1 candidates where type X_L and X_H pool at Y_H can be ruled out because this would require either a randomization of the fiscal authority or the tax auditor after Y_H is observed.

Partially pooling with $c = 1$ and $e \in]0, 1[$

Since income statement Y_H is only filed by type X_H we have beliefs $\kappa_{FA} = \kappa_{TA} = 1$. A randomization of type X_H requires that

$$U_{TP}^{O*}(e = 0|X = X_H) = U_{TP}^{O*}(e = 1|X = X_H), \quad (\text{A.32})$$

which is equivalent to the following condition on the conjectured audit probability and the probability of a diligent audit:

$$\hat{a}_L \left[\hat{d}_L + (1 - \hat{d}_L)(1 - \omega_{HL}) \right] = \frac{T_H - T_L}{T_H - T_L + P} \quad (\text{A.33})$$

For this equilibrium candidate, the fiscal authority and/or the tax auditor have to randomize when observing an income statement Y_L . In accordance with the best response functions we have therefore to consider the following five cases:

1. $\mu_{FA} = \mu_{TA} = \mu^\dagger < \mu^\ddagger$ ($d_L = 1, a_L \in]0, 1[$)
2. $\mu_{FA} = \mu_{TA} = \mu^\dagger > \mu^\ddagger$ ($d_L = 0, a_L \in]0, 1[$)
3. $\mu_{FA} = \mu_{TA} = \mu^\ddagger > \mu^\dagger$ ($d_L \in]0, 1[, a_L = 1$)
4. $\mu_{FA} = \mu_{TA} = \mu^\dagger < \mu^\ddagger$ ($d_L \in]0, 1[, a_L = 0$)
5. $\mu_{FA} = \mu_{TA} = \mu^\dagger = \mu^\ddagger$ ($d_L \in]0, 1[, a_L \in]0, 1[$)

In cases 1 and 2 the the fiscal authority randomizes whereas the tax auditor plays a pure strategy. Case 1 establishes if $\mu^\ddagger|_{d_L=1} < \mu^\dagger$ which is equivalent to:

$$\frac{F}{(T_H - T_L + P)} < 1 - \frac{C}{\omega_{LL}R}. \quad (\text{A.34})$$

Rearrangements yield the equivalent condition for F :

$$F < \left(1 - \frac{C}{\omega_{LL}R} \right) (T_H - T_L + P) \equiv F^\circ. \quad (\text{A.35})$$

In equilibrium, the players' beliefs have to be determined by using Bayes' Rule with the conjectured evasion probability \hat{e} (see Equation (A.5)). As in equilibrium conjectured values have to hold, we can infer the equilibrium strategy e^* that is necessary to make the fiscal authority indifferent as follows:

$$e^* = \hat{e} = \frac{(1 - \alpha)\mu_{FA}}{\alpha(1 - \mu_{FA})} = \frac{1 - \alpha}{\alpha} \left[\frac{F}{(T_H - T_L + P) - F} \right] \quad (\text{A.36})$$

Randomization requires that $0 < e^* < 1$ and hence, $F < \alpha(T_H - T_L + P) \equiv F^\dagger|_{d_L=1}$. By use of Equation (A.33) we can calculate the equilibrium audit probability a_L^* that is necessary to make type X_H indifferent:

$$a_L^* = \hat{a}_L = \frac{T_H - T_L}{T_H - T_L + P} \quad (\text{A.37})$$

An equilibrium is only sustainable if type X_L has no incentive to deviate from $c = 1$. Inspection of Equation (14) yields that with $\hat{d}_L = 1$ the expected utility is maximized at $c = 1$ and hence, type X_L has indeed no incentive to deviate.

Finally, since F has to be smaller than F° as well as $F^\dagger|_{d_L=1}$, we derive a condition that determines which of both requirements is binding. F° is smaller than $F^\dagger|_{d_L=1}$ if

$$C > (1 - \alpha)\omega_{LL}R \equiv \bar{C}. \quad (\text{A.38})$$

This is sufficient to prove existence of equilibrium 5.

Case 2 establishes if $\mu^\dagger|_{d_L=0} > \mu^\dagger$ which is equivalent to:

$$\frac{F}{(1 - \omega_{HL})(T_H - T_L + P)} > 1 - \frac{C}{\omega_{LL}R} \quad (\text{A.39})$$

Rearrangements yield the equivalent condition for F :

$$F > \left(1 - \frac{C}{\omega_{LL}R}\right) (1 - \omega_{HL})(T_H - T_L + P) \equiv F^\sharp \quad (\text{A.40})$$

As in case 1 we calculate the equilibrium strategy e^* that is necessary to make the fiscal authority indifferent as follows:

$$e^* = \hat{e} = \frac{(1 - \alpha)\mu_{FA}}{\alpha(1 - \mu_{FA})} = \frac{1 - \alpha}{\alpha} \left[\frac{F}{(1 - \omega_{HL})(T_H - T_L + P) - F} \right] \quad (\text{A.41})$$

Randomization requires that $0 < e^* < 1$ and hence, $F < \alpha(1 - \omega_{HL})(T_H - T_L + P) \equiv F^\dagger|_{d_L=0}$. By use of Equation (A.33) we can calculate the equilibrium audit probability a_L^* that is necessary to make type X_H indifferent:

$$a_L^* = \hat{a}_L = \frac{T_H - T_L}{(1 - \omega_{HL})(T_H - T_L + P)} \quad (\text{A.42})$$

Since randomization requires that $0 < a_L^* < 1$, this equilibrium is only feasible if $P > \underline{P}$. From type X_L 's expected utility in Equation (14) it follows that he does not deviate from $c = 1$ if

$$a_L^* \omega_{LL} L_{TP} < T_H - T_L \quad (\text{A.43})$$

which is equivalent to

$$\omega_{LL}L_{TP} < (1 - \omega_{HL})(T_H - T_L + P). \quad (\text{A.44})$$

Since $\omega_{LL} < (1 - \omega_{HL})$ this condition is redundant to $L_{TP} < T_H - T_L + P$ and therefore, type X_L would never like to deviate.

Finally, we check when $F^\sharp < F^\dagger|_{d_L=0}$ and hence there is a non-empty interval for a feasible F . This is only the case if $C > \bar{C}$. This is sufficient to prove existence of Equilibrium 6.

In case 3, the tax auditor randomizes whereas the fiscal authority will play $a_L = 1$. The tax auditor's equilibrium strategy makes type X_H indifferent and can be determined by use of Equation (A.33) as follows:

$$d_L^* = \hat{d}_L = 1 - \frac{P}{\omega_{HL}(T_H - T_L + P)} \quad (\text{A.45})$$

Since randomization requires that $0 < \hat{d}_L < 1$ a necessary condition is that $P < \underline{P}$. Type X_H 's equilibrium strategy e^* has to be consistent with belief $\mu_{TA} = \mu^\dagger$ that makes the tax auditor indifferent:

$$e^* = \hat{e} = \frac{(1 - \alpha)\mu_{TA}}{\alpha(1 - \mu_{TA})} = \frac{1 - \alpha}{\alpha} \left[\frac{\omega_{LL}R}{C} - 1 \right] \quad (\text{A.46})$$

From that follows that randomization is only feasible if $C > \bar{C}$.

As a next step, we check under which condition $\mu^\dagger > \mu^\ddagger|_{d_L=d_L^*}$ and hence, case 3 will actually prevail. This is the case if

$$1 - \frac{C}{\omega_{LL}R} > \frac{F}{(1 - \omega_{HL} + d_L^*\omega_{HL})(T_H - T_L + P)}. \quad (\text{A.47})$$

After some rearrangements, it is possible to show that this is equivalent to:

$$F < \left(1 - \frac{C}{\omega_{LL}R}\right)(T_H - T_L) \equiv F^\flat \quad (\text{A.48})$$

Finally, an equilibrium can only sustain, if type X_L has no incentive to deviate from $c = 1$. This is the case if

$$(1 - d_L^*)\omega_{LL}L_{TP} < T_H - T_L \quad (\text{A.49})$$

or equivalently

$$L_{TP} < \frac{\omega_{HL}}{\omega_{LL}} \left(\frac{T_H - T_L}{P} \right) (T_H - T_L + P). \quad (\text{A.50})$$

We assumed that $L_{TP} < T_H - T_L + P$. Therefore, to prove that the above inequality holds, it sufficient to show that

$$\frac{\omega_{HL}}{\omega_{LL}} \left(\frac{T_H - T_L}{P} \right) > 1 \quad (\text{A.51})$$

which can be rewritten as

$$\frac{\omega_{HL}}{\omega_{LL}} (T_H - T_L) > P. \quad (\text{A.52})$$

From $\omega_{LL} < 1/2$ it follows that the left-hand side is always greater than \underline{P} and hence $\underline{P} > P$ which is also a necessary condition for this equilibrium is stricter than the above inequality. This proves that type X_L has never an incentive to deviate and Equilibrium 7 can exist.

Case 4 is not a valid equilibrium candidate due to the following reason. In this case, the fiscal authority would like to choose $a_L = 0$. This however, contradicts the necessary conjectured audit probability from Equation (A.33) that ensures that type X_H would like to randomize.

In case 5, both the fiscal authority and the tax auditor randomize after observing Y_L . As with case 3 type X_H 's equilibrium strategy has to be consistent with belief $\mu_{TA} = \mu^\dagger$ and hence

$$e^* = \hat{e} = \frac{(1 - \alpha)\mu_{TA}}{\alpha(1 - \mu_{TA})} = \frac{1 - \alpha}{\alpha} \left[\frac{\omega_{LL}R}{C} - 1 \right]. \quad (\text{A.53})$$

Randomization requires that $0 < e^* < 1$ and hence $C > \bar{C}$. The tax auditor chooses d_L^* such that $\mu^\dagger = \mu^\ddagger$ and therefore, the fiscal authority is indifferent if

$$d_L^* = \hat{d}_L = 1 - \frac{(\omega_{LL}R - C)(T_H - T_L + P) - F\omega_{LL}R}{\omega_{HL}(\omega_{LL}R - C)(T_H - T_L + P)} \quad (\text{A.54})$$

Randomization requires that $0 < d_L^* < 1$ and hence $F^\sharp < F < F^\circ$. The fiscal authority's optimal a_L^* in combination with the anticipated d_L^* is determined by Equation (A.33) that ensures indifference of type X_H . It follows that

$$a_L^* = \hat{a}_L = \frac{T_H - T_L}{F} \left(1 - \frac{C}{\omega_{LL}R} \right). \quad (\text{A.55})$$

Randomization requires that $0 < a_L^* < 1$ and hence, $F < F^\flat$. It can be observed that $F < F^\flat$ implies $F < F^\circ$. A non-empty interval for a feasible F can, however, only exist if $F^\sharp < F^\flat$. This is only the case if $P < \underline{P}$.

Finally we check when type X_L has no incentive to deviate. This is the case if the following inequality holds:

$$\omega_{LL}L_{TP} < \frac{\omega_{HL}F(T_H - T_L + P)}{\left(1 - \frac{C}{\omega_{LL}R}\right)(T_H - T_L + P) - F} \quad (\text{A.56})$$

Partially pooling with $c \in]0, 1[$ and $e = 1$

Since income statement Y_H is only filed by type X_L we have beliefs $\kappa_{FA} = \kappa_{TA} = 0$. A randomization of type X_L requires that

$$U_{TP}^{O*}(c = 0|X = X_L) = U_{TP}^{O*}(c = 1|X = X_L), \quad (\text{A.57})$$

which is equivalent to the following condition on the conjectured audit probability and the probability of a diligent audit:

$$\hat{a}_L(1 - \hat{d}_L) = \frac{T_H - T_L}{\omega_{LL}L_{TP}} \quad (\text{A.58})$$

Since the left-hand side of this equation has to be smaller than 1 (either fiscal authority or tax auditor or both randomize) this type of equilibrium can only prevail if $\omega_{LL}L_{TP} > T_H - T_L$. The different cases regarding beliefs μ_{FA} and μ_{TA} that could occur within this equilibrium candidate are identical to those that appear when type X_H does partially pooling. Any equilibrium candidate that would involve $\hat{a}_L = 0$ or $\hat{d}_L = 1$ can be ruled out because this would violate Equation (A.58). Therefore, cases 1 and 4 cannot be part of an equilibrium and we analyze only the remaining cases 2, 3, and 5.

In case 2, we have $\mu^\dagger|_{d_L=0} > \mu^\dagger$ which establishes if $F > F^\sharp$. The required equilibrium strategy of type X_L would be as follows:

$$c^* = \hat{c} = \frac{\alpha}{1 - \alpha} \left[\frac{(1 - \omega_{HL})(T_H - T_L + P) - F}{F} \right] \quad (\text{A.59})$$

Randomization requires that $0 < c^* < 1$ and hence $F < \overline{\overline{F}}$ and $F > F^\dagger|_{d_L=0}$. It can be shown that both $F > F^\dagger|_{d_L=0}$ and F^\sharp are smaller than $\overline{\overline{F}}$ and therefore, there exists a non-empty interval for F . As a lower bound $F^\dagger|_{d_L=0}$ is binding whenever $C > \overline{C}$ and F^\sharp is binding otherwise.

The fiscal authority's strategy that is consistent with randomization of type X_L follows from Equation (A.58) and equals

$$a_L^* = \hat{a}_L = \frac{T_H - T_L}{\omega_{LL}L_{TP}}. \quad (\text{A.60})$$

An equilibrium is only sustainable if type X_H has no incentive to deviate from $e = 0$. From the expected utility in Equation (15) we can derive, that in general this is the case if

$$\hat{a}_L[1 - \omega_{HL}(1 - \hat{d}_L)] < \frac{T_H - T_L}{T_H - T_L + P}. \quad (\text{A.61})$$

For case 2, this requirement translates into the following condition:

$$L_{TP} > \frac{1 - \omega_{HL}}{\omega_{LL}} (T_H - T_L + P) \quad (\text{A.62})$$

As the fraction on the right-hand side is always larger than 1 this would require a violation of the assumption that $L_{TP} < T_H - T_L + P$. Therefore, case 2 cannot be part of an equilibrium.

Case 3 requires the fiscal authority to choose $a_L = 1$ whereas the tax auditor chooses d_L^* such that type X_L is held indifferent:

$$d_L^* = \hat{d}_L = 1 - \frac{T_H - T_L}{\omega_{LL}L_{TP}} \quad (\text{A.63})$$

By use of the optimal d_L^* we can derive the condition that ensures that type X_H has no incentive to deviate from $e = 0$. No deviation occurs if

$$1 + \omega_{HL} \frac{T_H - T_L}{\omega_{LL}L_{TP}} < \frac{T_H - T_L}{T_H - T_L + P}. \quad (\text{A.64})$$

Since the left-hand side is strictly larger than 1 and the right-hand side is strictly smaller than 1 we have a contradiction. Hence, type X_H would always deviate in case 3 and this cannot be part of an equilibrium.

Finally, we evaluate case 5 where both the fiscal authority and the tax auditor randomize after observing Y_L . The tax auditor chooses d_L^* such that $\mu^\dagger = \mu^\ddagger|_{d_L=d_L^*}$ and therefore, the fiscal authority is indifferent. This is the case if

$$d_L^* = \hat{d}_L = 1 - \frac{(\omega_{LL}R - C)(T_H - T_L + P) - F\omega_{LL}R}{\omega_{HL}(\omega_{LL}R - C)(T_H - T_L + P)}. \quad (\text{A.65})$$

Randomization requires that $0 < d_L^* < 1$ and hence $F^\sharp < F < F^\circ$. The fiscal authority chooses a_L such that type X_L is indifferent and hence

$$a_L^* = \hat{a}_L = \frac{T_H - T_L}{\omega_{LL}L_{TP}} \left[\frac{\omega_{HL}(\omega_{LL}R - C)(T_H - T_L + P)}{(\omega_{LL}R - C)(T_H - T_L + P) - F\omega_{LL}R} \right] \quad (\text{A.66})$$

By inserting optimal d_L^* and a_L^* into Equation (A.61) we can derive that type X_H would not like to deviate if

$$L_{TP} > \frac{\omega_{HL}}{\omega_{LL}} \left[\frac{F}{\left(1 - \frac{C}{\omega_{LL}R}\right) (T_H - T_L + P) - F} \right] (T_H - T_L - P). \quad (\text{A.67})$$

The above condition for no deviation is only feasible, if it does not require a violation of $L_{TP} < T_H - T_L + P$. To show that the condition violates this assumption it is sufficient to prove that

$$\frac{\omega_{HL}}{\omega_{LL}} \left[\frac{F}{\left(1 - \frac{C}{\omega_{LL}R}\right) (T_H - T_L + P) - F} \right] \geq 1 \quad (\text{A.68})$$

which is equivalent to

$$F \geq \frac{\omega_{LL}}{\omega_{LL} + \omega_{HL}} \left(1 - \frac{C}{\omega_{LL}R}\right) (T_H - T_L + P). \quad (\text{A.69})$$

This condition, however, is redundant to $F > F^\#$ which is also a requirement for this candidate to be sustainable because of

$$\frac{\omega_{LL}}{\omega_{LL} + \omega_{HL}} < 1 - \omega_{HL}. \quad (\text{A.70})$$

Therefore, a violation of $L_{TP} < T_H - T_L + P$ would be required for no deviation of type X_H and consequently there cannot exist an equilibrium in case 5.

Completely mixed equilibria with $c \in]0, 1[$ and $e \in]0, 1[$

We can exclude equilibria where both types randomize because this would require either $a_H \in]0, 1[$ or $d_H \in]0, 1[$ or both. \square