# Production-Based Exchange Rates: The *q*-Theory of Multinationals*

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**Abstract**

I derive a novel production-based asset pricing result that relates exchange rates to the intertemporal marginal rates of transformation of capital (i.e., returns on investment) within multinational firms with headquarters at home and foreign affiliates abroad. This *q*-theory of multinationals is analogous to a standard international consumption-based model, but uses multinational producers and production functions rather than consumers and stochastic discount factors. Its structural estimation infers exchange rates from US multinational aggregates on real investment, output, and capital stocks. Tested across 44 countries since the 1980s, it successfully prices cross-sectional carry trade spreads through the fundamentals of US foreign affiliates.

JEL codes: E13, E22, E23, E32, F31, F44, G12, G15

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## 1 Introduction

From a demand perspective, it is well established that exchange rates $e_{t+1}$ are linked to the intertemporal marginal rates of substitution—i.e., stochastic discount factors (SDFs)—of foreign and home consumer households $M_{f,t+1}$ and $M_{h,t+1}$ (Backus and Smith (1993)). This paper provides a novel, yet equally important, supply-based perspective to exchange rate determination. I am the first to show that exchange rates are analogously linked to the intertemporal marginal rates of transformation of capital (MRTs)—i.e., returns on investment—of multinational firms with *headquarters* at home and *foreign affiliates* abroad $R^I_{HQ,t+1}$ and $R^I_{A,t+1}$. In equilibrium, exchange rates are, hence, tied to both consumption through the classical equation

$$\frac{e_{t+1}}{e_t} = \frac{M_{f,t+1}}{M_{h,t+1}} \tag{1}$$

and production, through the new result derived from optimal investments at home and abroad

$$\frac{e_{t+1}}{e_t} = \frac{R^S_{t+1} - \omega_{HQ,t} R^I_{HQ,t+1}}{(1-\omega_{HQ,t})R^I_{A,t+1}} \tag{2}$$

where $R^S_{t+1}$ is the stock return of the home multinational firm, $\omega_{HQ,t}$ is the weight of the firm's market value attributed to its headquarter, while $(1-\omega_{HQ,t})$ is the weight on its foreign affiliate. When the exchange rate increases $\uparrow e_{t+1}$ the foreign currency appreciates, in real terms, against the home currency.[1]

Ceteris paribus, from the consumption side of the economy in (1) the value of a foreign currency is high when the marginal rate of substitution of the home agent is low $\downarrow M_{h,t+1}$ (i.e., good relative times for the home investor), while from the production side its value is high in (2) when the marginal rate of transformation of capital at home is low $\downarrow R^I_{HQ,t+1}$ (i.e., low relative returns for investing at home). Hence, exchange rate determination takes place in the typical sense of risk sharing between representative consumers, but also internally within representative multinationals that diversify foreign and domestic risk through real investment. In essence, the consumption-based model might say "a foreign currency is expected to appreciate because home consumption growth is relatively high," while the production-based asset pricing model can make statements like "the foreign currency is expected to appreciate because *within-firm* returns on investment at home are relatively low." Both equations are a result of financial market completeness and, in equilibrium, are analogous outcomes of two sides (supply and demand) of the same coin (exchange rate determination). This paper pursues the microfounded

[1]Formally, working with real variables, $e_{t+1}$ is the real exchange rate expressed in units of the home consumption good numéraire per unit of the foreign good numéraire. $M_{h,t+1}$ and $R^I_{HQ,t+1}$ are rates of change from $t$ to $t+1$ in the home numéraire.

cross-border $q$-theory approach of *multinational production-based exchange rates*.

The key novelty that results in equation (2) is the introduction of foreign subsidiaries into the maximization problem of representative firms. In contrast to standard international business cycle and international asset pricing models, which assume that representative firms in each country are standalone companies, I model firms as multinational enterprises (MNEs). By owning equity shares in the representative MNEs, the representative consumers can diversify domestic risk through the internal risk sharing that takes place within multinationals. When maximizing total firm value, multinationals maximize not only the value of the home productive business unit but also, jointly, that of foreign affiliates. In doing so, they weigh the different investment opportunities and productivities at home and abroad while diversifying foreign and domestic risk internally.[2] As such, and under mild assumptions of the Hayashi (1982) conditions, both the headquarters and foreign affiliates—henceforth business units—have within-firm values that are inferable from data and technological parameters (e.g., Tobin's $q$s) and that jointly determine the total equity value of MNEs. Managers maximize this total firm value by choosing optimal investments at home and abroad at differential capital formation rates (i.e., different investment-to-capital ratios). The intensity of these differentials is driven fundamentally by within-firm differentials in investment opportunities (i.e., Tobin's $q$ differentials), productivities (i.e., differentials in marginal products of capital (MPK)), and adjustment costs of investment between the firm's foreign affiliates abroad and its headquarters ($HQ$) at home.

These fundamental Tobin's $q$s, MPKs, and adjustment costs drive the differences in the returns on investment at home and abroad, $R^I_{HQ,t+1}$ and $R^I_{A,t+1}$, in equation (2). Therefore, exchange rate fluctuations provide, in equilibrium, the crucial *offsetting* link within multinationals through which differences in expected returns from investing at home and abroad are reconciled with the firm's total stock return $R^S_{t+1}$. Crucially, the link between within-multinational MRTs and exchange rates is enabled only when the firm is allowed to diversify domestic risk by pursuing investment opportunities in foreign markets through affiliate real investment. Otherwise, the multinational is a domestic standalone firm, and the new result in (2) collapses to the seminal production-based result in Cochrane (1991, 1996) of the equivalence between stock returns and (domestic) investment returns, $R^S_{t+1} \equiv R^I_{HQ,t+1}$.[3] The model also nests the single-country laureate result in Tobin (1969) of the equivalence of Tobin's $q$ to both marginal

[2] Canonical models in a two-country world economy have 2 aggregate capital stocks, while here each firm has 2 stocks of its own, one abroad and one at home. Without affiliates the production risk-sharing link within firms in (2) does not exist.

[3] Rearrange (2) to $\frac{e_{t+1}}{e_t}(1-\omega_{HQ,t})R^I_{A,t+1}+\omega_{HQ,t}R^I_{HQ,t+1} = R^S_{t+1}$ and set the equity stake in the affiliate to 0 by $\omega_{HQ,t} = 1$.

$q$ and to the ratio between the market value of a (domestic) asset and its replacement cost. This single-country $q$-theory literature enjoys substantial empirical support in the cross-section of US equity returns (e.g., Liu, Whited, and Zhang (2009)). However, it is only a special case of the multinational $q$-theory framework introduced in this paper, which enables the exploration of the microfoundations of exchange rates as a new asset class to this literature, and establishes a first connection to the broader literatures on international macroeconomics and finance.

Without needing to model preferences in (1), the production approach provides exchange rate changes in closed form that can be inferred from aggregate data on US production worldwide and through production functions.[4] That is, just as currencies in (1) can be inferred from data at $t$ and $t+1$ on aggregate real consumption growth and by estimating utility parameters such as risk aversion, currencies in (2) can be inferred from readily available aggregate multinational data at $t$ and $t+1$ on domestic and foreign affiliate investment- and output-to-capital ratios and by estimating technological parameters such as capital share in output and adjustment costs. This preference-free model in (2) is tested in both structural and reduced form.

Using data on US aggregate multinational production from the US Bureau of Economic Analysis (BEA), the model is tested on a broad sample of 44 countries with US foreign affiliate data starting in the 1980s. Total real sales by US-owned foreign companies worldwide grew from \$1.5 trillion in the early 1980s to \$6.5 trillion in 2019, equivalent to half of the total revenue reported by all Fortune 500 US corporations in 2015. These large figures highlight the crucial role foreign affiliates play in growth opportunities and (net present) value creation for US investors, suggesting that incorporating their role into the value maximization problem of representative firms is first-order. The $q$-theory framework explicitly integrates their overlooked role and explores a rich set of testable implications for exchange rates as arising from (2).

The paper achieves three objectives. First, after introducing the model, I adopt a reduced-form approach to test the predicted link between US global production variables and the appreciation rate of foreign currencies. For example, as discussed in the model section, equation (2) offers a crucial 1-year-ahead *forecasting* hypothesis of foreign currency appreciation. Specifically, the model predicts that higher within-firm investment opportunities abroad at time $t$—captured by higher Tobin's $q$ differentials at $t$, and implicitly in the data, by higher investment-to-capital differentials between foreign affiliates and the US headquarters ($I/K_{A,t} - I/K_{HQ,t}$)—should

[4] This is feasible because (2) results from a production-based model that imposes *no* assumptions on preferences and infers exchange rates from a multinational producer's first-order conditions at home and abroad. It parallels (1), which uses no technology assumptions (i.e., valid for any production technology) and infers currencies from consumers' first-order conditions.

forecast an appreciation of foreign currencies into $t+1$. Why? to offset the higher relative marginal benefits of production abroad expected at time $t+1$.

I find strong support for this testable implication of a link between US MNE fundamentals and exchange rates. A standard deviation increase in the annual foreign affiliate $I/K$ differential is associated, on average, with foreign currencies appreciating 1-year into the future (vis-à-vis the US dollar) by 0.68% per annum.[5] Despite the considerable challenge of forecasting exchange rates from fundamentals—a disconnect regarded as one of the main puzzles in international macro-finance (Obstfeld and Rogoff (2000))—these novel findings remain robust after controlling for an extensive set of macroeconomic predictors, including differentials in output, consumption, country-wide investments in capital stocks, net exports, inflation, interest rates, and currency premia. Moreover, as part of the tradeoff between the marginal costs of investing abroad at time $t$ versus the marginal benefits realized at $t+1$, the theory also predicts a contemporaneous depreciation of foreign currencies associated with affiliate output and investment at time $t+1$. The data also support these testable implications. Importantly, taking the model to the data in a structural estimation setting, the model reproduces both the direction and size of the coefficients found in the panel regressions. That is, after deducting model-implied from realized exchange rate growth rates, there are no longer any forecastable or contemporaneous components in average foreign currency appreciation associated with US affiliate activity.

Second, I structurally estimate a parsimonious specification of the MNE $q$-theory of investment framework using the generalized method of moments (GMM). Applying established estimation techniques from the production-based literature (e.g., Cochrane (1996), Liu et al. (2009), Belo, Xue, and Zhang (2013)) to the novel pricing of currencies, I estimate the international model at the country level of US affiliates. The median estimate of capital's share across the 44 aggregate affiliates is 0.339 and is estimated with precision. This estimate is remarkably close to its typical textbook value used in real business cycle models and serves as a validity check for the estimation procedure. Moreover, formal GMM overidentification tests of model fit to the data (Hansen (1982)) fail to reject the model of exchange rate determination. Using the estimated parameters and observable data on affiliate investment- and output-to-capital, the approach delivers implied exchange rate changes and currency excess returns that are compared against realized data in formal asset pricing tests at the country and portfolio levels.

[5] In *all* tests, US foreign affiliate aggregates for investment, output, and capital $(I, Y, K)$ are measured in foreign currency real terms (e.g., in constant 2015 British pounds for affiliate investments in the UK), while HQ data are in real US dollar figures.

Third, when applied to currency premia, the framework successfully prices the cross-section of currency carry trade risk premia as first established in the seminal work of Lustig and Verdelhan (2007). That is, the model fits well the average payoffs seen in historical currency data to a US investor long in high and short in low foreign interest rate portfolios.[6] It reproduces the economically large average excess return spreads seen in the data between high minus low (H-L) carry trade portfolios, with an inferred H-L carry trade spread of 7.27% per annum, comparable to the average spread of 7.54% realized across 8 portfolios. The time-series correlation between the implied and realized series is high at 40.7%. To explain these currency patterns, the model relies on the heterogeneity in the *fundamentals* of and across US foreign affiliates. I find a link between interest rates being relatively high in foreign countries and US affiliates finding it relatively costly to expand their real productive capital stock in those economies (i.e., high relative adjustment costs). Moreover, those high-interest economies tend to be countries in which US affiliates have lower fundamental marginal productivities, yet higher investment opportunities and market valuations as inferred from within-firm Tobin's $q$ estimates.

Finally, contextualizing findings, my structural estimates suggest that US firms have excelled in capitalizing on investment opportunities in foreign markets through affiliates, which, on average, offer higher returns than domestic investments since the 1980s. These estimates complement a large literature focused on understanding capital flows across countries, particularly from rich to developing nations, following the seminal work by Lucas (1990). Moreover, the larger affiliate returns that emerge from matching currencies are consistent with a salient, yet untargeted, feature in the multinational literature that finds subsidiaries generally outperforming domestic firms (Guadalupe, Kuzmina, and Thomas (2012)). Collectively, the microfounded predictability of currencies and structural results suggest that exchange rates are not entirely disconnected from fundamentals, a common belief since Meese and Rogoff (1983). Indeed, Hassan, Mertens, and Zhang (2016) are early proponents of connections to country-wide production, yet this paper distinctly reveals a first connection to MNE cross-border investment $q$-theory and a forecastability of currency changes that is neither present nor explained by country aggregates.

## 1.1 Relation to prior literature

My paper contributes to several literatures. First, this paper contributes to the literature on production-based asset pricing pioneered by Cochrane (1991, 1996). This body of literature

[6] Portfolio-level cross-sectional carry trade spreads are an order of magnitude larger than the average spread an investor can earn from any two given countries (Lustig and Verdelhan (2007)). The carry trade arises from the disconnection between interest rates and exchange rates, contrary to uncovered interest rate parity Fama (1984)).

has played a key role in molding our comprehension of how a firm's investment decisions are connected to its return on equity. This literature has seen large empirical support by comparing model-implied returns and firm values against their realized counterparts and using data on listed firms. Some examples include the pricing of the aggregate US stock market (Cochrane (1991) and Merz and Yashiv (2007)), the cross-section of US stock returns and firm values (Cochrane (1996), Gomes, Yaron, and Zhang (2006), Liu et al. (2009), Belo et al. (2013), Gonçalves, Xue, and Zhang (2020), Delikouras and Dittmar (2022), Choy, Lewis, and Tan (2023), Li, Ma, Wang, and Yu (2023), Belo, Gala, Salomao, and Vitorino (2022), and Alfaro, Graniero, and Schneider (2024)), and the cross-section of firm values in different countries around the world (Belo, Li, Salomao, and Vitorino (2023)). I differ by showing theoretically that the above models are a special application in a closed-economy of the multicountry MNE $q$-theory framework introduced in this paper. As such, this work expands investment-based asset pricing to the all-important economic question of the microfoundations of exchange rates. Taken to the data, a parsimonious specification of the preference-free model characterizes currencies through US affiliate fundamentals.[7]

Second, my paper complements an extensive body of work in international macroeconomics and finance that examines equilibrium exchange rates from a SDF and consumption perspective. Given that the data are generally at odds with standard SDFs (Backus and Smith (1993), Brandt, Cochrane, and Santa-Clara (2006)), several different frameworks have been proposed to address these disconnections, e.g., models with long-run risk (Colacito and Croce (2011, 2013) and Colacito, Croce, Ho, and Howard (2018)).[8] Although these frameworks offer compelling resolutions, they depend on highly correlated SDFs, a feature that is difficult to test because SDFs depend on hard-to-measure probabilities of long-run risk, habit ratios, or rare disasters (Lustig and Verdelhan (2019)).[9] I contribute by uncovering that equilibrium exchange rates are tied not only to the consumption side of the economy, and SDFs therein, but also to production, and MRTs therein, of and within representative MNEs. Abstracting from imposing specific structures on SDFs, I pursue a complementary new route that infers exchange rates from overlooked broad US multinational aggregates and through production functions of home and foreign affiliates. Deviating from the conventional modeling of firms as representative

[7] Other examples of single country models on asset prices and quantities Jermann (1998), Gomes, Kogan, and Zhang (2003), Zhang (2005), Cooper (2006), Belo (2010), Papanikolaou (2011), Belo, Lin, and Bazdresch (2014), Crouzet and Eberly (2023).

[8] Rare disasters (Farhi and Gabaix (2016)), segmented markets (Gabaix and Maggiori (2015)), habit (Stathopoulos (2016)).

[9] Recent contributions, and references therein, include Maggiori (2017), Hassan and Mano (2018) and Itskhoki and Mukhin (2021). For a review see Hassan and Zhang (2021).

standalones, this MNE $q$-theory of investment approach succeeds in pricing the cross-section of currency carry trade (Lustig and Verdelhan (2007)) through the innovative cross-section of the fundamentals of US foreign affiliates.[10]

Lastly, to the best of my knowledge, this paper is the first attempt to characterize exchange rate markets in closed form through the returns on investments of US MNE parents and foreign affiliates, and their unified link to US equity markets. In this objective, this work connects to the MNE literature, such as McGrattan and Prescott (2010), who study affiliate returns and the US current account, yet in a model where real exchange rates are fixed at one and unlinked to US equities. Other related agendas include the effects of exchange rate volatility on foreign direct investment (FDI) (e.g., Goldberg and Kolstad (1995)), early work on international diversification and risk sharing (Errunza, Hogan, and Hung (1999)), and the equity premia of MNEs (Fillat and Garetto (2015)).[11] This paper provides a novel bridge between the MNE literature and the broader international macro-finance literature on SDFs, as well as both currency and equity asset pricing.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework. Section 3 describes the econometric methodology, including data sources. Results are in section 4 and include both reduced-form and structural tests. Section 5 concludes.

## 2 The theory of multinational production-based exchange rates

Markets are complete and integrated. Integration means that domestic investors can trade all foreign assets through the exchange rate market, and vice versa. For parsimony, there are two goods, each produced and consumed locally in each of two countries. The foreign good is produced both by the foreign affiliate of the home firm and by the headquarters of the foreign MNE. A similar scenario takes place at home for the home good. To ease intuition, this two-country world economy is presented first and then extended to include a third country representing the rest of the world (ROW). This extension ensures that investment opportunities in the ROW, involving trillions of US dollar flows for US MNEs and likely important for equilibrium exchange rate dynamics, are not overlooked in the value maximization problem of MNEs that allocate optimal investments across a large set of economies at every point in time.

[10]This channel is theoretically different than in Hassan et al. (2016) who link currencies in a standalone firm setting to the aggregate capital-to-output ratios of seven industrialized nations. The link here is associated with within-firm investment flows from US MNEs to foreign affiliates (e.g., in the spirit of capital flows in Lucas (1990)), risk sharing, and connections to the equity markets through the value maximization principles of the investment-based literature. Empirically, consistent with MNE $q$-theory, currencies are forecastable by US affiliate real investment. This novel predictability is not explained by country aggregates.

[11]Fillat and Garetto (2015) find MNEs earn higher returns than non-MNEs in a model where exchange rate changes are nearly zero and disconnected from business cycles. Other work: Albuquerque (2003), Rowland and Tesar (2004), and Russ (2007).

## 2.1 Model Setup: Two-Country World Economy

### 2.1.1 The problem of the household

Complete markets imply that all variables are functions of all possible states of nature and not just of time. To simplify notation, however, I use a time subscript as shorthand without the explicit reference to states. Time is discrete, and the horizon is infinite. There are two countries, home $h$ and foreign $f$, each with a representative agent $j = \{h, f\}$ and a MNE $i = \{h, f\}$. The representative firms issue beginning-of-period equity shares $S_{i,t}$ to finance their investments over the course of period $t$. The agent $j$ decides on the equity shares to hold in firm $i, S^j_{i,t}$, such that subscripts of shares indicate the origin of equity issuance and superscripts indicate ownership. Shareholders are entitled to receive dividends $d_{i,t}$ per unit of $S^j_{i,t}$. Agent $j$ earns the competitive domestic real wage $w_{j,t}$ for providing their domestic labor $N_{j,t}$. There is no labor mobility across countries. When maximizing utility, agent $j$ chooses current consumption $C_{j,t}$ and beginning-of-next-period equities held in the two representative firms $S^j_{h,t+1}$ and $S^j_{f,t+1}$, and as purchased at the per-share prices at end-of-period $t$ values $p_{h,t}$ and $p_{f,t}$, respectively. Firm stock prices are time-varying and taken as given in the trading of shares at market prices.

Therefore, the home agent faces the following budget constraint at every period $t$

$$p_{h,t}(S^h_{h,t+1} - S^h_{h,t}) + e_t p_{f,t}(S^h_{f,t+1} - S^h_{f,t}) \leqslant w_{h,t}N_{h,t} + d_{h,t}S^h_{h,t} + e_t d_{f,t}S^h_{f,t} - C_{h,t} \tag{3}$$

where $e_t$ is the real exchange rate defined as the relative price of one unit of the foreign good available for foreign consumption, $C_{f,t}$, in terms of the home good available for domestic consumption, $C_{h,t}$. When $e_t$ increases, the foreign good appreciates and the home good depreciates. Dividends $e_t d_{f,t} S^h_{f,t}$ denote the dividend flows earned over the course of $t$ from having owned shares since the beginning of $t$ in the foreign firm, and as converted back to home consumption good units via the end-of-period exchange rate $e_t$. The term $e_t p_{f,t}(S^h_{f,t+1} - S^h_{f,t})$ denotes the expenditures (proceeds) from the purchase (sale) of equities from $t$ to $t+1$ of the foreign firm, at per-share end-of-period stock price $p_{f,t}$, and as converted back to home good units via $e_t$. Hence, the timing is such that shares $S^h_{f,t}$ are sold and $S^h_{f,t+1}$ are purchased concurrently at the end of period $t$, with a resulting change in equities held going into $t+1$ of $(S^h_{f,t+1} - S^h_{f,t})$.

Therefore, the problem of the home agent is to maximize expected lifetime utility by choosing current consumption and next-period equity shares in the two representative firms:

$$\max_{\{C_{h,t+s}, S^h_{h,t+s+1}, S^h_{f,t+s+1}\}^\infty_{s=0}} E_t[\sum_{s=0}^{\infty} \beta^s u(C_{h,t+s})] \tag{4}$$

subject to equation (3) and the initial number of shares $S_{h,0}$ and $S_{f,0}$. The first-order condition with respect to the purchase of equity shares in the foreign firm $S^h_{f,t+1}$ gives the Euler equation

$$1 = E_t[M_{h,t+1}\frac{e_{t+1}}{e_t}R^{S,f}_{t+1}] \tag{5}$$

where $R^{S,f}_{t+1} = \frac{p_{f,t+1}+d_{f,t+1}}{p_{f,t}}$ is the stock return (cum-dividend) of the foreign firm and $M_{h,t+1}$ is the SDF used at home to discount the expected payoffs of *any* asset traded in the economy.[12]

Similarly, the foreign agent in country $f$ faces an analog budget constraint at time $t$ :

$$p_{f,t}(S^f_{f,t+1} - S^f_{f,t}) + \frac{1}{e_t}p_{h,t}(S^f_{h,t+1} - S^f_{h,t}) \leqslant w_{f,t}N_{f,t} + d_{f,t}S^f_{f,t} + \frac{1}{e_t}d_{h,t}S^f_{h,t} - C_{f,t} \tag{6}$$

The first-order condition with respect to equity shares in the foreign firm $S^f_{f,t+1}$ is the Euler

$$1 = E_t[M_{f,t+1}R^{S,f}_{t+1}] \tag{7}$$

In complete markets equilibrium, (5) and (7) must hold, giving a standard risk-sharing first-order result: an *equivalence* between exchange rate growth and ratio of marginal utility growth

$$\frac{e_{t+1}}{e_t} = \frac{M_{f,t+1}}{M_{h,t+1}} \tag{8}$$

This state-by-state equivalence is long-standing in the international macro-finance literature and broadly applies to a general range of models (e.g., Backus, Foresi, and Telmer (2001)). The optimality result in (8) is enabled by the purchase and sale of equities in the two firms, which allows the agents to intertemporally substitute consumption at $t+1$ for consumption at $t$ and exploit internal risk sharing that takes place within MNEs and across borders through affiliates. Imposing economic structure on preferences in (8) gives the classic "consumption view" of exchange rates, where their fluctuations can be inferred from consumption growth aggregates and parameters.[13] Rewritten, (8) is also a statement of the future value of a foreign currency, where $e_t$ is known ex-ante at $t$, such that with standard preferences a formal test is

$$e_{t+1} = e_t\frac{M_{f,t+1}}{M_{h,t+1}} \equiv f(\Delta C_{h,t+1}, \Delta C_{f,t+1}, e_t; \gamma) \tag{9}$$

#### 2.1.2 The problem of the multinational firm

Each representative firm headquartered in country $i = \{h, f\}$ operates both domestically and abroad. The headquarter produces locally to supply consumption demand for the local good and contributes to foreign consumption demand by operating a representative affiliate in country $l \neq i$. Both the headquarter and the affiliate business units use local capital stocks and labor

[12] SDFs are unique when markets are complete, meaning that investors can invest in any contingent claim, either directly or indirectly by synthesizing claims from other securities. $j's$ SDF is $M_{j,t+1} = \frac{\beta\Lambda_{j,t+1}}{\Lambda_{j,t}}$, where $\Lambda_{j,t}$ is their Lagrange multiplier.

[13] For example, in a Breeden–Lucas–Rubenstein representative consumer model with power utility, agents have a log SDF equal to $m_{j,t+1} = \log\beta - \gamma\Delta c_{j,t+1}$ where $\gamma$ is the coefficient of relative risk aversion and $\Delta c_{j,t+1}$ denotes log consumption growth in country $j$. The econometrician can test this model by gathering data on aggregate consumption growth at home and abroad, and estimating a moment condition that links exchange rate fluctuations to data and parameters, i.e., $\frac{e_{t+1}}{e_t} \equiv f(\Delta c_{h,t+1}, \Delta c_{f,t+1}; \gamma)$.

inputs to produce. To ease presentation, the home firm and its flows and stocks are emphasized, but analog symmetric results hold for the opposite point of view. The home firm finances its total investment, $I_t^h = I_{HQ,t}^h + e_t I_{A,t}^h$, by issuing equities $p_{h,t}[(S_{h,t+1}^h - S_{h,t}^h) + (S_{h,t+1}^f - S_{h,t}^f)]$. In the former term, total investment flows, $I_t^h$, consist of investment at home done by the headquarter, $I_{HQ,t}^h$, and investment abroad by the affiliate, $I_{A,t}^h$, with flows converted back to home consumption good units via the prevailing real exchange rate, term $e_t I_{A,t}^h$. In the latter, equity shares are bought and sold by both the home and foreign investors at the prevailing stock price of the home firm, $p_{h,t}$, and noting that the shares are demanded exogenously by the investors from their first-order optimality decisions as discussed in the previous section.

The home firm's total production is $Y_t^h = Y_{HQ,t}^h + e_t Y_{A,t}^h$, with each business unit assumed to have its own Cobb–Douglass production with constant returns to scale. Output at home is $Y_{HQ,t}^h = F_{HQ,t}^h \left(K_{HQ,t}^h, N_{h,t}^{HQ,h}, Z_{HQ,t}^h\right) \equiv Z_{HQ,t}^h (K_{HQ,t}^h)^{\kappa_{HQ}^h} (N_{h,t}^{HQ,h})^{(1-\kappa_{HQ}^h)}$ and abroad $Y_{A,t}^h = F_{A,t}^h \left(K_{A,t}^h, N_{f,t}^{A,h}, Z_{A,t}^h\right) \equiv Z_{A,t}^h (K_{A,t}^h)^{\kappa_{A,t}^h} (N_{f,t}^{A,h})^{(1-\kappa_{A,t}^h)}$. Both $F_{HQ,t}^h()$ and $F_{A,t}^h()$ are business-specific functions of capital $K$ and labor $N$ inputs and shocks $Z$ to productivity. Capital stocks $K_{HQ,t}^h$ and $K_{A,t}^h$ are inputs located in the different countries where the home MNE operates. Shocks $Z_{HQ,t}^h$ and $Z_{A,t}^h$ are *vectors* of exogenous shocks to each unit that may include aggregate world and country-specific shocks, and idiosyncratic MNE- and business-level shocks. Importantly, these shocks may propagate between parents and affiliates; for instance, automation improvements abroad may spill over to the HQ and other affiliates, linking their productivities across the firm. These productivity linkages through MNE ownership channels influence the firm's equity value maximization problem, where synergies between affiliates and parents can increase firm value and make MNEs distinct from simply holding equity in an unrelated foreign firm. Labor is supplied locally by the domestic agent, such that $N_{h,t}^{HQ,h}$ is the labor supplied by the home agent (subscript) to the HQ of the home firm (superscripts) and $N_{f,t}^{A,h}$ is the foreign agent's labor (subscript) supplied to the foreign affiliate of the home firm (superscripts).[14]

Capital accumulations are standard such that investment flows increase the beginning-of-period capital stock net of depreciation, $K_{HQ,t+1}^h = I_{HQ,t}^h + (1 - \delta_{HQ,t}^h) K_{HQ,t}^h$ and $K_{A,t+1}^h = I_{A,t}^h + (1 - \delta_{A,t}^h) K_{A,t}^h$. Here, $\delta_{HQ,t}^h$ and $\delta_{A,t}^h$, represent the exogenous rates of capital depreciation for HQ and the foreign affiliate, respectively. Furthermore, each business unit incurs adjustment costs when investing in its capital stock. These costs may vary within the firm and capture

[14] Both the home and foreign agents $j$ split their full labor supply $N_{j,t}$ between all business units operating within their country, i.e., $N_{j,t} = N_{j,t}^{HQ,j} + N_{j,t}^{A,l}$ where affiliate $l \neq i$. Moreover, labor is supplied in full every period such that $N_{j,t} = 1$.

natural differences in investment costs faced in different foreign markets. These differences arise, for example, from variability in installation costs, specialization expenses associated with foreign production, and local regulation compliance costs. They may also differ from those adjustments faced by foreign MNE HQs, local standalones, or affiliates owned by third nations. Denoted by $\Phi$, these costs are increasing and convex in $I_t$, decreasing in $K_t$, and exhibit constant returns to scale in $I_t$ and $K_t$. This implies, for example, that affiliate costs are homogeneous of degree one and exhibit $\Phi^h_{A,t}(I^h_{A,t}, K^h_{A,t}) = I^h_{A,t}\partial\Phi^h_{A,t}(I^h_{A,t}, K^h_{A,t})/\partial I^h_{A,t} + K^h_{A,t}\partial\Phi^h_{A,t}(I^h_{A,t}, K^h_{A,t})/\partial K^h_{A,t}$. Precise functional forms are specified in the empirical section below. Therefore, a key assumption is:

**ASSUMPTION 1.** ***Hayashi Conditions****: Homogeneity of degree one in the production technology and adjustment cost functions of home and foreign business units within MNEs.*

The homogeneity Assumption 1 allows the derivation of a closed-form solution for the growth in exchange rates, which is a function of model parameters and US MNE production data at home and abroad. Moreover, it enables a direct and formal structural estimation of the model using observed data via GMM (e.g., akin to the single-country consumption-based tests in Hansen and Singleton (1982)), as opposed to indirectly through data simulated from the model (for example, by using simulated method of moments). It implies that the home HQ production function exhibits $F^h_{HQ,t}(K^h_{HQ,t}, N^{HQ,h}_{h,t}, Z^h_{HQ,t}) = K^h_{HQ,t}\partial F^h_{HQ,t}(K^h_{HQ,t}, N^{HQ,h}_{h,t}, Z^h_{HQ,t})/\partial K^h_{HQ,t} + N^{HQ,h}_{h,t}\partial F^h_{HQ,t}(K^h_{HQ,t}, N^{HQ,h}_{h,t}, Z^h_{HQ,t})/\partial N^{HQ,h}_{h,t}$. Moreover, HQ's MPK at every period $t$ is given by $\partial F^h_{HQ,t}/\partial K^h_{HQ,t} \equiv \kappa^h_{HQ} Y^h_{HQ,t}/K^h_{HQ,t}$, where $\kappa^h_{HQ}$ is HQ's capital share in output parameter to be estimated via GMM, while $Y^h_{HQ,t}/K^h_{HQ,t}$ is HQ's observed time-varying output-to-capital.

Dividend payout is total MNE output net of adjustment costs, investment, and labor costs:

$$\begin{aligned} D^h_t = {} & F^h_{HQ,t}\left(K^h_{HQ,t}, N^{HQ,h}_{h,t}, Z^h_{HQ,t}\right) - \Phi^h_{HQ,t}(I^h_{HQ,t}, K^h_{HQ,t}) - I^h_{HQ,t} - w_{h,t}N^{HQ,h}_{h,t} \\ & + e_t\left[F^h_{A,t}\left(K^h_{A,t}, N^{A,h}_{f,t}, Z^h_{A,t}\right) - \Phi^h_{A,t}(I^h_{A,t}, K^h_{A,t}) - I^h_{A,t} - w_{f,t}N^{A,h}_{f,t}\right] \end{aligned} \tag{10}$$

**MNE problem.** The concern of firm $i$ at every period $t$ is to maximize its *total* market value of equity by choosing optimal investments and hiring of local labor at each business unit

$$V^i_t \equiv \max_{I^i_{HQ,t+s}, I^i_{A,t+s}, K^i_{HQ,t+s+1}, K^i_{A,t+s+1}, N^{HQ,i}_{h,t+s}, N^{A,i}_{f,t+s}\}^{\infty}_{s=0}} E_t[\sum_{s=0}^{\infty} M_{i,t+s}D^i_{t+s}] \tag{11}$$

and noting that it discounts expected payouts using the appropriate local investor $i's$ SDF $M_{i,t+s}$, which is exogenous and correlated with the aggregate components in $Z^i_{HQ,t}$ and $Z^i_{A,t}$.

**MNE Euler equations for optimal investment.** The equity value maximization of the home MNE gives the following Euler equations for optimal investment at home through HQ

$$E_t[M_{h,t+1}R^{I,h}_{HQ,t+1}] = 1 \tag{12}$$

and abroad through foreign affiliates

$$E_t[M_{h,t+1}\frac{e_{t+1}}{e_t}R^{I,h}_{A,t+1}] = 1 \tag{13}$$

in which the investment returns of business units $l = \{HQ, A\}$ are time-varying and given by:

$$R^{I,h}_{l,t+1} = \frac{\frac{\partial F^h_{l,t+1}}{\partial K^h_{l,t+1}}K^h_{l,t+1} - \frac{\partial \Phi^h_{l,t+1}}{\partial K^h_{l,t+1}}K^h_{l,t+1} + q^h_{l,t+1}(1-\delta^h_{l,t+1})K^h_{l,t+1}}{q^h_{l,t}K^h_{l,t+1}} \tag{14}$$

These investment returns are the ratio of the marginal benefits of investment at $t+1$ to the marginal costs of investment at $t$. The benefits in the numerator capture [$a$] the marginal product at $t+1$ of having invested in one additional unit of capital at $t$, $\frac{\partial F^h_{l,t+1}}{\partial K^h_{l,t+1}}K^h_{l,t+1}$, [$b$] the reduction in installation costs of investing in one additional unit of capital at $t+1$, $\frac{\partial \Phi^h_{l,t+1}}{\partial K^h_{l,t+1}}K^h_{l,t+1}$, [$c$] the non-depreciated marginal continuation value of capital at time $t+1$, $q^h_{l,t+1}(1-\delta^h_{l,t+1})K^h_{l,t+1}$.

**PROPOSITION 1.** *Under Assumption 1, the first-order conditions for optimal investments imply that the appreciation into $t+1$ of foreign currencies is tied to the home MNE's returns on investment at home $R^{I,h}_{HQ,t+1}$ and abroad $R^{I,h}_{A,t+1}$ and to its total stock return $R^{S,h}_{t+1}$:*

$$\frac{e_{t+1}}{e_t} = \frac{R^{S,h}_{t+1}[q^h_{HQ,t}K^h_{HQ,t+1} + e_t q^h_{A,t}K^h_{A,t+1}] - R^{I,h}_{HQ,t+1}q^h_{HQ,t}K^h_{HQ,t+1}}{R^{I,h}_{A,t+1}e_t q^h_{A,t}K^h_{A,t+1}} \tag{15}$$

Moreover, the total ex-dividend value of multinationals, *defined* for the home MNE by $P^h_t = V^h_t - D^h_t = p_{h,t}(S^h_{h,t+1} + S^f_{h,t+1})$ (i.e., price per share times total shares outstanding), have, by first-order conditions, a closed-form $q$-theoretic *result*:

$$P^h_t \equiv q^h_{HQ,t}K^h_{HQ,t+1} + e_t q^h_{A,t}K^h_{A,t+1} \tag{16}$$

which makes the market value of MNEs evolve over time at every $t$ by the equity values of the ongoing business concerns at home $q^h_{HQ,t}K^h_{HQ,t+1}$ and abroad $e_t q^h_{A,t}K^h_{A,t+1}$. Thus, within-MNE Tobin's $q$s of HQs and foreign affiliates, which are inferable from adjustment costs parameters and investment-to-capital ratios, jointly determine the total $q$-theoretic value of MNEs.

Without loss of generality, the first-order result in (15) simplifies further by denoting the value of the business concerns at home and abroad in (16) with weights $\omega^h_{HQ,t} = \frac{q^h_{HQ,t}K^h_{HQ,t+1}}{q^h_{HQ,t}K^h_{HQ,t+1} + e_t q^h_{A,t}K^h_{A,t+1}}$ and $\omega^h_{A,t} = \frac{e_t q^h_{A,t}K^h_{A,t+1}}{q^h_{HQ,t}K^h_{HQ,t+1} + e_t q^h_{A,t}K^h_{A,t+1}}$, such that shares $\omega^h_{HQ,t} + \omega^h_{A,t} = 1$, simplifying (15) to:

$$\frac{e_{t+1}}{e_t} = \frac{R^{S,h}_{t+1} - R^{I,h}_{HQ,t+1}\omega^h_{HQ,t}}{R^{I,h}_{A,t+1}(1-\omega^h_{HQ,t})} \tag{17}$$

Equations (17) and (16) comprise the key scientific contribution of the $q$-theory of multinationals introduced in this paper, which nest 2 foundational results in the broad financial markets literature. The latter in (16) nests the seminal single-country result in Tobin (1969) of the equivalence of Tobin's $q$, under the Hayashi (1982) conditions, to both marginal $q$ and to the ratio between the market value of a (domestic) asset and its replacement cost, $q^h_{HQ,t} \equiv P^h_t / K^h_{HQ,t+1}$. The former in (17) is a generalization of the well-established result in single countries of the state-by-state *equivalence* between stock returns and (domestic) investment returns $R^{S,h}_{t+1} = R^{I,h}_{HQ,t+1}$. That is, rearranging (17) and when there is no equity stake abroad $\omega^h_{HQ,t} = 1$, reduces (17) to $R^{S,h}_{t+1} = R^{I,h}_{HQ,t+1} \equiv f(\frac{I^h_{HQ,t}}{K^h_{HQ,t}}, \frac{I^h_{HQ,t+1}}{K^h_{HQ,t+1}}, \frac{Y^h_{HQ,t+1}}{K^h_{HQ,t+1}}; production\ parameters)$. Cochrane (1991) shows that US home MRTs, $R^{I,h}_{HQ,t+1}$, constructed from technological parameters and aggregate US $I/K$ ratios dated both at time $t$ and $t+1$, largely match the time-series variation in the US stock market return, with an annual correlation of 45%. Cochrane (1996) and Liu et al. (2009) build on such result to price the cross-section of US stock returns. Notably, the international framework here allows, as a special case, for firms to be standalone entities in each country, implying $R^{S,i}_{t+1} = R^{I,i}_{HQ,t+1}$ in each country $i$ as is broadly typical in the international asset pricing literature, yet permits the richer case of MNEs that involve the relative role of within-firm affiliate MRTs and the link to exchange rates via (17).

As such, the MNE $q$-theory result in (17) establishes a first connection between the investment-based asset pricing literature and exchange rate asset pricing, portraying currency changes as functions of technological parameters and observable equity returns and fundamental characteristics of MNEs. Importantly, Proposition 1 imposes *no* assumptions on the functional forms of international preferences and infers exchange rates from a multinational producer's first-order conditions. It parallels the consumption approach which uses no production assumptions (i.e., valid for any technology) and infers currencies from consumers' first-order conditions. Therefore, just as exchange rates in (8) can be inferred using data at $t$ and $t+1$ on aggregate real consumption growth and by estimating utility parameters such as risk aversion, $e_{t+1} \equiv f(\Delta C_{h,t+1}, \Delta C_{f,t+1}, e_t; \gamma^h, \gamma^f)$, currencies in (15) are inferable from aggregates at $t$ and $t+1$ on home and affiliate investment-to-capital and output-to-capital ratios and by estimating technological parameters such as capital's share and adjustment costs, $e_{t+1} \equiv f(\frac{I^h_{HQ,t}}{K^h_{HQ,t}}, \frac{I^h_{HQ,t+1}}{K^h_{HQ,t+1}} \frac{Y^h_{HQ,t+1}}{K^h_{HQ,t+1}}, \frac{I^h_{A,t}}{K^h_{A,t}}, \frac{I^h_{A,t+1}}{K^h_{A,t+1}}, \frac{Y^h_{A,t+1}}{K^h_{A,t+1}}, R^{S,h}_{t+1}, e_t; production\ params.)$. Therefore, Proposition 1 posits a new, yet intuitive, supply-based perspective on bilateral exchange rate determination, analogous to the classical demand view in (8). The relationship between these complementary views and their risk sharing implications is discussed in the introduction. [15]

Lastly, at the risk of semantics, the SDF literature typically refers to (8) as stating that exchange rates are "pinned down" by the marginal utilities of foreign and domestic households, which may suggest a non-trivial causal link from consumption behavior to currency dynamics. A more nuanced interpretation, however, might be to ask whether observed equilibrium exchange rate changes are *consistent* with investors' consumption and optimizing behavior. Similarly, the production approach can be viewed as asking whether exchange rates are consistent with MNEs' global investment and optimizing behavior, a testable implication explored in section 4.

#### 2.1.2.1 Mechanisms intuition: $q$s, $MPK$s, and offsetting role of exchange rates

What are the production-based economic mechanisms tied to fluctuations in currencies? Fluctuations in (17) are tied to three time $t+1$ return components of the home firm: its stock return $R^{S,h}_{t+1}$ and its return on investment at home $R^{I,h}_{HQ,t+1}$ and abroad $R^{I,h}_{A,t+1}$. Ceteris paribus, foreign currencies appreciate upon low relative returns from investing at home $\downarrow R^{I,h}_{HQ,t+1}$.

The result also gives first-order intuition on the link with affiliate investment returns: all else equal, currencies play a crucial *offsetting* role between marginal costs and benefits of investing abroad. To ease intuition, examine (15) in a 2-period setting, with $t+1$ investments set to zero:

$$\frac{e_{t+1}}{e_t} = \frac{R^{S,h}_{t+1}[q^h_{HQ,t}K^h_{HQ,t+1} + e_t q^h_{A,t}K^h_{A,t+1}] - \kappa^h_{HQ}Y^h_{HQ,t+1}}{e_t\kappa^h_A Y^h_{A,t+1}} \tag{18}$$

Exchange rates at $t+1$ are a function of five terms, two of which relate to the foreign affiliate: [$i$] the value at time $t$ of the affiliate, $e_t q^h_{A,t}K^h_{A,t+1}$, i.e., price of affiliate capital times its quantity, [$ii$] the time $t+1$ marginal productivity of the affiliate, $e_t\kappa^h_A Y^h_{A,t+1}$.

First, from term [$i$], if the value at $t$ of the affiliate increases due to high foreign investment opportunities $\uparrow q^h_{A,t}$, the foreign currency is expected to appreciate into $t+1$. Note that $q^h_{A,t}$, is the marginal $q$ of investing abroad: the marginal rate of transformation between installed and uninstalled capital (Cochrane (1996)). This term is not only the marginal cost of investing abroad but also reflects the investment opportunities offered by the foreign market. Second, note that the affiliate's Tobin's $q$ in the numerator is increasing in the affiliate's $I/K$ ratio. Hence, all else equal, upon higher investment today at time $t$ by foreign affiliates (i.e., implicitly a reflection of higher relative investment opportunities abroad within the firm $\uparrow q^h_{A,t}$), the foreign

[15] Both approaches involve economic concepts of optimal *intertemporal* allocation of resources. Whereas demand links exchange rates to intertemporal marginal rates of substitution—which involve investors' willingness to substitute consumption at time $t+1$ for consumption at $t$ and are therefore rates of change concepts—production involves investment returns. These returns also represent rates of change, involving intertemporal MRTs of capital from $t$ to $t+1$ and expressed in terms of the consumption good produced in the applicable country. These analogous approaches naturally involve information at $t$ and $t+1$ to infer currency changes from $t$ to $t+1$. (17) further exploits the *intratemporal* concept of optimal allocation within MNE's and across borders.

currency is expected to appreciate into $t+1$. Why? To *offset* the higher relative marginal benefits realized at time $t+1$ of foreign affiliate capital. These marginal benefits are captured by term $[ii]$, $e_t \kappa_A^h Y_{A,t+1}^h$, appearing in the denominator, and which reflect the marginal output created at $t+1$ from having invested in one additional unit of real affiliate capital at $t$.

In short, exchange rates play a crucial offsetting role between marginal costs in the numerator and marginal benefits in the denominator of investing abroad through affiliates. The empirical section presents reduced-form evidence that supports the novel predictability of exchange rates consistent with this mechanism, showing a *positive* correlation with lagged affiliate $I/K$ ratios. The above 2-period discussion still applies in the richer multiperiod setting (for brevity, see discussion in Internet Appendix IA.C.1). The key difference is that exchange rate appreciation is also linked to next period's $I/K$ ratios at home and abroad—e.g., due to the continuation value of affiliate capital included in time $t+1$ affiliate Tobin's $q$s, which further influence affiliate marginal benefits of investment. Consistent with this mechanism, the empirical section also presents evidence of currency depreciation tied to the contemporaneous $I/K$ and $Y/K$ ratios of affiliates, with a *negative* correlation as predicted. The model replicates these correlations.

# 3 Econometric Methodology

The novel framework can be extended to include other inputs and complexities.[16] Yet a key feature of the parsimonious specification is its ability to infer exchange rates from broad US capital investment data via production functions. This section describes the structural procedure. As described below, in *all* tests of the results section 4, foreign affiliate aggregates for investment, output, and capital $(I, Y, K)$ are in foreign currency real terms (e.g., in constant 2015 British pounds for investments in the UK), while HQ data are in real US dollar figures.

## 3.1 Functional Forms and GMM moments

### 3.1.1 Bilateral Exchange Rates from a Three-Country World Economy

In principle, the two-country model can be estimated to infer bilateral exchange rates. However, to ensure completeness of investment opportunities available to US MNEs, I introduce the ROW. For brevity, only the key equations that result from the extended model are

[16] For example, introducing adjustment costs on labor implies that local labor inputs are no longer absorbed by the constant returns production functions through MPKs in (15). Adding labor costs extends this equation by making the returns on physical investment at home $R_{HQ,t+1}^{I,h}$ and abroad $R_{A,t+1}^{I,h}$ in (15) to become *fundamental* returns that include both inputs. That is, the affiliate return $R_{A,t+1}^{I,h}$ becomes a weighted average of the affiliate's return on capital and labor inputs. The intuition of marginal benefits vs. marginal costs of affiliate investment in inputs still applies in this extended model. My focus is only on physical capital (e.g., Cochrane (1991)) as a first approach to exchange rates, but the framework is adaptable to incorporate other inputs with frictions such as labor and intangible capital, which may improve fit to the data (e.g., Belo et al. (2022) in closed economies). Other obvious complications, such as taxes or firm bond returns, have *low-frequency effects* that change the mean of the implied returns on foreign affiliate investment while not greatly changing their correlation with *highly volatile* exchange rates.

emphasized, yet Appendix IA.C.2 presents the full setup. Building on section 2.1, the world economy is extended to include three representative agents $j = \{h, f, g\}$ and three MNEs $i = \{h, f, g\}$. The exchange rate between the home $h$ and foreign $j$ countries is denoted by $e_{j,t}$.

Home firm equity value maximization implies that the exchange rate growth in (15) is now

$$\frac{e_{f,t+1}}{e_{f,t}} = \frac{\begin{bmatrix} R^{S,h}_{t+1}[q^h_{HQ,t}K^h_{HQ,t+1} + e_{f,t}q^h_{A,f,t}K^h_{A,f,t+1} + e_{g,t}q^h_{A,g,t}K^h_{A,g,t+1}] \\ -R^{I,h}_{HQ,t+1}q^h_{HQ,t}K^h_{HQ,t+1} - R^{I,h}_{A,g,t+1}e_{g,t+1}q^h_{A,g,t}K^h_{A,g,t+1} \end{bmatrix}}{R^{I,h}_{A,f,t+1}e_{f,t}q^h_{A,f,t}K^h_{A,f,t+1}} \tag{19}$$

which now incorporates the investment return of the affiliate in country $g$, $R^{I,h}_{A,g,t+1}$. The result in (19) has, as special cases, the two-country equivalence in (15) and the classical result in Cochrane (1991, 1996). To infer exchange rate growth in (19), the model uses technological parameters (details in the next section) and 10 US data inputs: the time $t+1$ US stock market return $R^{US\text{-}Mkt}_{t+1}$ and 9 aggregates associated with the $I/K$ ratios dated at $t$ and $t+1$ and $Y/K$ ratios dated at $t+1$ of each of the three US global MNE business units.

The $q$-theoretic market value of the home firm in (16) and its payout in (10) are extended:

$$P^h_t = q^h_{HQ,t}K^h_{HQ,t+1} + e_{f,t}q^h_{A,f,t}K^h_{A,f,t+1} + e_{g,t}q^h_{A,g,t}K^h_{A,g,t+1} \tag{20}$$

$$\begin{aligned} D^h_t = {} & F^h_{HQ,t}\left(K^h_{HQ,t}, N^{HQ,h}_{h,t}, Z^h_{HQ,t}\right) - \Phi^h_{HQ,t}(I^h_{HQ,t}, K^h_{HQ,t}) - I^h_{HQ,t} - w_{h,t}N^{HQ,h}_{h,t} \\ & + e_{f,t}\left[F^h_{A,f,t}\left(K^h_{A,f,t}, N^{A,f,h}_{f,t}, Z^h_{A,f,t}\right) - \Phi^h_{A,f,t}(I^h_{A,f,t}, K^h_{A,f,t}) - I^h_{A,f,t} - w_{f,t}N^{A,f,h}_{f,t}\right] \\ & + e_{g,t}\left[F^h_{A,g,t}\left(K^h_{A,g,t}, N^{A,g,h}_{g,t}, Z^h_{A,g,t}\right) - \Phi^h_{A,g,t}(I^h_{A,g,t}, K^h_{A,g,t}) - I^h_{A,g,t} - w_{g,t}N^{A,g,h}_{g,t}\right] \end{aligned} \tag{21}$$

### 3.1.2 Functional form of adjustment costs

Exchange rate growth in (19) requires the specification of the adjustment cost functions applicable to each business unit. For $HQ$'s investment at home, I consider a standard quadratic and symmetric about zero costs function, which fits well with the cross-section of both US stock returns (Liu et al. (2009)) and firm market values (Belo et al. (2022)). That is, at time $t$

$$\Phi^h_{HQ,t}(I^h_{HQ,t}, K^h_{HQ,t}) = \frac{\theta^h_{HQ}}{2}\left(\frac{I^h_{HQ,t}}{K^h_{HQ,t}}\right)^2 K^h_{HQ,t} \tag{22}$$

where $\theta^h_{HQ} > 0$ is the slope parameter that captures the size of adjustment costs. For the foreign affiliates, I assume a more general function that allows as a special case the costs in (22) but lets the data inform the researcher about potential asymmetries in costs faced by affiliates in foreign markets. For example, Abel and Eberly (1994, 1996) show that allowing for asymmetry in physical adjustment costs improves the ability of a neoclassical model to explain investment dynamics. Expanding on the asymmetries in Belo et al. (2022), for $l = \{f, ROW\}$

$$\Phi^h_{A,l,t}(I^h_{A,l,t}, K^h_{A,l,t}) = \frac{\theta^h_{A,l}}{(\nu^h_{A,l})^2}\left\{\exp\left[-\nu^h_{A,l}\left(\frac{I^h_{A,l,t}}{K^h_{A,l,t}} - \overline{\frac{I^h_A}{K^h_A}}\right)\right] + \nu^h_{A,l}\left(\frac{I^h_{A,l,t}}{K^h_{A,l,t}} - \overline{\frac{I^h_A}{K^h_A}}\right) - 1\right\} K^h_{A,l,t} \tag{23}$$

This function is smooth and homogenous of degree one, hence it satisfies the requirements of section 2.1.2 for a closed-form expression of exchange rates in (19). Parameter $\theta^h_{A,l}$ captures the size of costs as in the quadratic specification, while $\nu^h_{A,l}$ controls the degree of asymmetry. In the limit when $\nu^h_{A,l} \longrightarrow 0$, the function is symmetric about the long-run steady state foreign affiliate investment-to-capital ratio ($\overline{\frac{I^h_A}{K^h_A}}$), and asymmetrical when $\nu^h_{A,l} \neq 0$. Therefore, cross-sectional heterogeneity in potential asymmetries revolves around the steady-state investment rate of US foreign affiliates. For example, in some countries, it might be costlier to overinvest than to underinvest with respect to the steady state ($\nu < 0$), while for others, the opposite may be true ($\nu > 0$, e.g., for affiliates with high investment opportunities, having low investment rates might be relatively costlier for the firm).[17] Qualitatively, given that adjustment costs determine investment opportunities (Tobin's $q$s), deviations from steady state investment opportunities are penalized with adjustment costs at the country level of the affiliate. The size of the adjustment costs, symmetric or not, depends on the slope parameter $\theta^h_{A,l}$. Lastly, as discussed in the data section, the long-run steady state $\overline{\frac{I^h_A}{K^h_A}}$ is 18.41%, and the ratio itself for each US foreign affiliate is strictly positive.

### 3.1.3 Moment conditions

The main moment condition tests if the average real exchange rate growth observed in the data equals the average real exchange rate growth implied by the model. That is, in expectation

$$E\left[\frac{e_{f,t+1}}{e_{f,t}} - \frac{\widehat{e_{f,t+1}}}{e_{f,t}}\right] = 0 \tag{24}$$

For brevity, its lengthy equation is provided in (IA.1) of the GMM Appendix IA.A, which uses (19) and imposes functional forms. For the $ROW$ affiliate aggregates, I use the largest affiliates in a set of 25 countries with required data since the 1980s. For example, the real exchange rate of the $ROW$ is a weighted average of 25 bilateral real exchange rates, using weights based on the observed aggregate size of US affiliate capital stocks.[18] The data section provides details.

To identify the structural parameters, I employ 3 additional moment conditions derived from the model equations, each having direct counterparts in the data. The second moment compares

[17]When $\nu^h_{A,l} \longrightarrow 0$ and $\overline{\frac{I^h_A}{K^h_A}} = 0$ the function is symmetric about zero and converges to the standard quadratic in (22).

[18]This follows US Fed Board currency indexes: http://www.federalreserve.gov/pubs/bulletin/2005/winter05_index.pdf.

the model's implied valuation of the US corporate sector with its observable market values, using Tobin's $q$ as valuation measure, as is standard. Tobin's $q$ at time $t$ for the home aggregate firm is $q_t^h = P_t^h / A_{t-1}^h = [q_{HQ,t}^h K_{HQ,t+1}^h + e_{f,t} q_{A,f,t}^h K_{A,f,t+1}^h + e_{ROW,t} q_{A,ROW,t}^h K_{A,ROW,t+1}^h] / A_{t-1}^h$, where $P_t^h$ is from (20) and $A_{t-1}^h$ is the total aggregate assets of the nonfinancial corporate business sector in the US (see next section for data description). Using total assets as the denominator of $q$ is standard in empirical finance (e.g., Kaplan and Zingales (1997)) and follows the approach in Belo et al. (2013). Therefore, the second moment tests whether the average US aggregate Tobin's $q^{US}$ equals the average predicted $q^h$ by the model. That is, in expectation: $E\left[q_t^{US} - q_t^h\right] = 0$. Importantly, this valuation moment is dated at time $t$, hence using information up to year $t$ and measured with a 1-year lag with respect to the key currency moment in (24), from which bilateral exchange rate appreciation into $t+1$ is inferred.

The third moment tests whether the average capital gains return on the US corporate sector—proxied by the ex-dividend US stock return in the data, $P_{t+1}^{US}/P_t^{US}$, measurable from gross ex-dividend market returns from the Center for Research in Security Prices (CRSP)—equals the average capital gains of home firm returns predicted by the model, $P_{t+1}^h/P_t^h$. The fourth moment tests whether the average aggregate US dividend-to-price ratio—i.e., $D_{t+1}^{US}/P_t^{US}$ from CRSP—equals the average predicted dividend yield of the home firm, $D_{t+1}^h/P_t^h$, as implied by first-order conditions and derived from (20) and (21).[19] Measuring these last two moments with a lag gives materially similar results.

The four moment conditions in (IA.1), (IA.2), (IA.3), and (IA.4) in Appendix IA.A are employed to structurally estimate the parameters for each country $f$ via GMM. The set of 8 parameters is large $\Theta = (\kappa_{HQ}^h, \kappa_{A,f}^h, \kappa_{A,ROW}^h, \theta_{HQ}^h, \theta_{A,f}^h, \theta_{A,ROW}^h, \nu_{A,f}^h, \nu_{A,ROW}^h)$. To avoid an overparameterized system (e.g., Cochrane (1996)), I reduce their number to only three by setting those that are common across business units to a single estimate. That is, I estimate only one capital share in output parameter $\kappa$ instead of three. Likewise, estimate only one common parameter for the size of adjustment costs $\theta$ and one for the asymmetry in foreign affiliate costs $\nu$. Moreover, as is standard in the production literature, I do only an unconditional estimation. As explained in Belo et al. (2013), errors in moment conditions like Tobin's $q$ are measurement errors (including specification errors) which may correlate with lagged instruments, hence

[19] Costlessly adjustable labor implies that the labor inputs drop out of the optimized dividend process of the MNE and are inferable from its capital MPKs. For example, HQ's payout in (21) $F_{HQ,t}^h - \Phi_{HQ,t}^h - I_{HQ,t}^h - w_{h,t} N_{h,t}^{HQ,h}$ after optimizing and using homogeneity is $\frac{\partial F_{HQ,t}^h}{\partial K_{HQ,t}^h} - \frac{\partial \Phi_{HQ,t}^h}{\partial I_{HQ,t}^h} \frac{I_{HQ,t}^h}{K_{HQ,t}^h} - \frac{\partial \Phi_{HQ,t}^h}{\partial K_{HQ,t}^h} - \frac{I_{HQ,t}^h}{K_{HQ,t}^h}$, hence a function of HQ data on $\frac{I}{K}$ and $\frac{Y}{K}$ and of parameters.

invalidating the scaling of moments with instruments. Therefore, the system is overidentified with 3 parameters $\Theta = (\kappa, \theta, \nu)$ and 4 moments, which conveniently produces a $\chi^2$ test (Hansen (1982)) of overidentifying restrictions that can be used to assess overall model fit to the data. As is also standard, I do a one-step GMM estimation and use the identity weighting matrix to preserve economic structure of test assets following Cochrane (1996) and Belo et al. (2013). Finally, the procedure produces implied series of country-level exchange rate changes, $\widehat{\frac{e_{f,t+1}}{e_{f,t}}}$, which are then compared against realized data using standard tests, including panel and Fama and MacBeth (1973) cross-sectional regressions conducted at country and portfolio levels.

#### 3.1.4 Tests and pricing the carry trade in foreign exchange (FX) markets

The model is validated in several dimensions: [*i*] reduced-form evidence in Section 4.1, consistent with testable predictions, including novel predictabilities of exchange rates; [*ii*] structural estimation via GMM in 4.2, with formal overidentification tests and implied affiliate productivities and investment returns that align with key, yet untargeted, features in MNE literature; [*iii*] verification in 4.3 that model-implied exchange rate changes, $\widehat{\frac{e_{f,t+1}}{e_{f,t}}}$, reproduce the observed links between realized currency changes and fundamental aggregates of US production across the world. After testing the model, Section 4.4 explores whether the framework can provide insights into understanding the large currency premiums observed in currency markets from carry trade strategies (Lustig and Verdelhan (2007)). The model is applied to carry trade spreads as follows. First, I construct realized currency excess returns in gross real terms:

$$R^{FX}_{f,t+1} = \frac{e_{f,t+1}}{e_{f,t}} - \frac{(i^{US}_t + 1)}{(i^f_t + 1)} \tag{25}$$

which uses real gross exchange rate growth rates $\frac{e_{f,t+1}}{e_{f,t}}$ and real gross risk-free rates (i.e., $i^{US}_t + 1 = R^{rf}_{US,t}$ and $i^f_t + 1 = R^{rf}_{f,t}$).[20] Two key points here: [*a*] following Kremens and Martin (2019), all returns throughout the paper, including stock returns $R^{S,US}_{t+1}$ and currency returns $R^{FX}_{f,t+1}$, are in gross form—rather than log returns.[21] [*b*] From the perspective of a US home investor, the only unknown at time $t$ in the excess return $R^{FX}_{f,t+1}$ is the next period's exchange rate $e_{f,t+1}$ (or its growth into $t+1$), making it the key economic unknown variable and which is inferrable from the model. At time $t$, the current period exchange rate $e_{f,t}$ and the risk-free rates $i^{US}_t$ and $i^f_t$ are known. Therefore, second, I construct implied currency excess returns by plugging in the next-period predicted exchange rate growth $\widehat{\frac{e_{f,t+1}}{e_{f,t}}}$ and using the observed risk-free rates, i.e., $\widehat{R^{FX}_{f,t+1}} = \widehat{\frac{e_{f,t+1}}{e_{f,t}}} - \frac{(i^{US}_t+1)}{(i^f_t+1)}$. This allows for a comparison of implied and

[20] Formally, (25) is an excess return because it is a tradeable payoff in the set of traded assets whose price is 0.

[21] With currencies, approximating true returns with logs can lead to a gap as large as the risk premium itself (Engel (2016)).

realized risk premia and tests whether the framework can price the large average spreads in currency carry trades observed across a large cross-section of countries, similar to that in Lustig and Verdelhan (2007). The exercise sorts countries into portfolios based on time $t$ interest rate differentials. Note that the above procedure for carry trades does not introduce any noise in the estimation of either $\widehat{\frac{e_{f,t+1}}{e_{f,t}}}$ or $\widehat{R^{FX}_{f,t+1}}$; it simply exploits the implied currency changes to examine whether the model reproduces the historical average exchange rate movements since the 1980s, associated with the returns for a US investor in currencies who is long in high and short in low *observable* foreign interest rates.

### 3.2 Data and sample construction

BEA production data include aggregates from the National Income and Product Accounts (NIPA) tables and from the Activities of US Multinational Enterprises database. The latter offers statistics on the worldwide activities of US MNEs, which are regarded as the world's most complete and accurate of their kind and obtained from legally mandatory surveys applicable to all US legal entities with a 10% or more ownership of a foreign business. Hence, choosing the US as the home country is not only for economic relevance, but for accuracy, completeness, and necessity given data unavailability elsewhere. Affiliate statistics cover the entirety of operations, irrespective of the percentage of ownership, and cover rich required accounting items, including capital expenditures, value added, and capital stocks. They also cover other accounts, beyond the scope, such as sales, employment, research and development (R&D), assets, and debt.

The data are at the annual frequency from 1983 to 2019, and use the aggregate records of majority-owned affiliates. The final sample has a large cross-section of 44 foreign countries with up to 35 years of observations per country (1985–2019). Investment $I$ is capital expenditures, output $Y$ is value added, and capital $K$ is net property plant and equipment. In all tests of section 4, affiliate aggregates $(I, Y, K)$ are in foreign currency real terms, deflated using country-level consumer price indexes (CPIs). The $HQ$ aggregate quantities are in constant 2015 US dollars while the foreign affiliate $f$ aggregates are in constant 2015 foreign currency values. US home $HQ$ aggregates are adjusted to exclude contributions from firms operating within US borders but owned by foreign parents, e.g., deducting from US GDP figures all non-US-owned value added. Bilateral real exchange rates $e_{f,t}$ are in real US dollars per unit of real foreign currency, with an increase in $e_{f,t}$ denoting a real appreciation of the foreign currency. For comparability in the carry trade tests, I closely follow Lustig and Verdelhan

(2007) in the sample construction and data requirements.[22] For brevity, other measurement details and the use of otherwise standard data sources are in the Data Appendix IA.B.

### 3.3 Descriptive statistics of the fundamentals of US firms at home and abroad

A starting question is: How sizable is the production activity of US firms outside of US borders? Figure 1, Panel A, illustrates their global output from 1983, as measured by real sales. For presentational purposes, these amounts are plotted in constant 2015 US dollars. Total sales are substantial, increasing from \$1.5 trillion in the early 1980s to \$6.5 trillion by 2019, equivalent to half of the revenue reported by all Fortune 500 firms in 2015.[23]

Figure 1A also displays worldwide sales accounted for by shipments back to US parents from foreign affiliates (i.e., intra-firm cross-border imports by US parents). The fraction of output returned to US parents has historically been small ($\leq 9.70\%$), with an average of 6.67% since the 1980s, decreasing to 5.33% since 2000. Similarly, output produced within US borders by foreign-owned subsidiaries and returned to their foreign parents, as shown in Figure 1, Panel B, exhibits an average return fraction of just 3.77% since the 1980s. Collectively, and perhaps contrary to popular perception, BEA aggregates indicate that MNEs, whether based in the US or overseas, primarily use their affiliates to serve foreign markets. This aligns with the model's simplified depiction of foreign affiliates as producers dedicated solely to foreign consumption.[24]

Figure 2 displays the aggregate real values of value added, investment, and capital stocks (in blue solid lines left y-axis) of US foreign affiliates. The output contributed to the world economy by affiliates is sizable, exceeding \$1.3 trillion in 2019. As a fraction of total US domestic value added, this abroad output constituted up to 7% since the 1980s (red dashed line right y-axis). Moreover, investments abroad as a percentage of US domestic investment have historically varied between 7% and as large as 19%, indicating that US MNEs engage in substantial investment opportunities outside US borders.

The bottom-right plot merges the above series, illustrating the aggregate $Y/K$ and $I/K$

[22] I closely adhere to the original carry trade approach in Lustig and Verdelhan (2007), maintaining a large cross-section of countries with individual interest rates, even if they adopt a common currency during the sample period. The robustness section includes various data checks, including euro-related filters. I also avoid additional assumptions such as covered interest rate parity (CIP), often used to infer interest rate differentials from log forward discounts. For instance, CIP does not represent an accurate description of interest rates after the Great Recession (Du, Tepper, and Verdelhan (2018)), and unlike the use of true T-Bill rates, the CIP assumption substantially reduces the number of countries that have affiliate data.

[23] See Fortune 500 ranking in 2015 with a total revenue of \$12.5 trillion: https://fortune.com/ranking/fortune500/2015/.

[24] Ramondo, Rappoport, and Ruhl (2016) find that the median manufacturing foreign affiliate ships "nothing" to—and receives "nothing" from—its parent in the US. In Atalay, Hortaçsu, and Syverson (2014), a study on intra-firm trade within the US, highlights that vertical ownership is not primarily focused on facilitating physical goods movement along production chains within the firm, observing surprisingly small internal shipment shares, less than 0.1 percent on median. Although internal trade is essential for some firms and serves as a complementary channel, the data suggest that affiliate production is primarily used to serve foreign markets, enabling MNEs to overcome tariffs, trade barriers, lack of local expertise, and high transportation costs.

ratios for foreign affiliates. Output has generally exceeded capital since the 1980s by up to a factor of 1.6 (e.g., high productivity abroad per unit of affiliate capital), while investment as a fraction of foreign affiliate capital has reached as much as 25%. These $I/K$ and $Y/K$ ratios contrast markedly with the typical US domestic ratios in closed-economy models, as depicted in Figure 3. For example, the affiliate $Y/K$ ratio declined from a high of 1.6 to about 1 by the 2000s, diverging from US domestic ratios which continued to decline past 2000. These patterns highlight distinct investment opportunities and productivity levels for US affiliates compared to domestic operations, which the multinational $q$-theory model exploits. For symmetry, Appendix Figure IA.1 presents analog plots for US-based affiliates owned by foreign parents.

Finally, Table 1 presents descriptive statistics on the final panel data, spanning 1985 to 2019, used in both the reduced-form and structural estimations of the next section. For brevity, in all tables and figures that follow, I omit the home firm reference and adopt shorthand notation. For example, $I/K_{US,t+1}$ denotes the US domestic $HQ$'s $I/K$ ratio in year $t+1$, which in the model is variable $I^h_{HQ,t+1}/K^h_{HQ,t+1}$. Similarly, $Y/K_{A,f,t+1}$ and $Y/K_{A,ROW,t+1}$ serve as shorthand for US affiliate $Y/K$ ratios, which in the model are $Y^h_{A,f,t+1}/K^h_{A,f,t+1}$ and $Y^h_{A,ROW,t+1}/K^h_{A,ROW,t+1}$.

Appendix IA.C.3 briefly discusses the by-country panel, including presenting histograms and plots over time, by country, for all key variables in both ratios ($I/K$ and $Y/K$) and levels ($I, Y, K$). For presentational purposes, those amounts are plotted in constant 2015 US dollars, yet, as noted, are always measured in real foreign currency values in the tests that follow and as required by the model. Reassuringly, Figure IA.4 shows that the flows and stocks of US affiliates are substantial, totaling billions in real dollars in *each* foreign economy.[25] As discussed, to reduce influence of outliers, model-implied exchange rate growth rates are winsorized between -25% and +25%, as these bounds span the vast majority of long-run currency movements. This winsorization is applied after the GMM estimation, thereby preserving the integrity of the GMM routine, parameter estimates, their standard errors, and model fit $\chi^2$ tests.

# 4 Results

Section 4.1 presents reduced-form evidence in support of the model, 4.2 reports the structural estimation results, and 4.3 and 4.4 present formal tests for the performance of the model.

## 4.1 Reduced-form evidence of exchange rates and MNE affiliate production

Without any of the complexities of the structural GMM estimation results that follow, if there is validity to the model, one would expect to find empirical evidence in reduced-form of

[25] For example, the UK shows value added and capital of nearly 200 billion dollars in recent years, while smaller countries like Denmark and Costa Rica, each with populations around 5 million, still report substantial amounts, up to 10 billion per year.

the link between exchange rates and US global production. For example, as discussed in section 2.1.2.1, the model predicts that higher within-firm investment opportunities abroad at time $t$—captured by higher Tobin's $q$ differentials at $t$, and implicitly in the data, by higher $I/K$ differentials between foreign affiliates and the US headquarters—should lead to the foreign currency appreciating into $t+1$ to offset the higher relative marginal benefits of production abroad at time $t+1$. Thus, reduced-form regressions should show that foreign currencies, on average, appreciate from end-of-year $t$ to $t+1$ when there are higher $I/K$ differentials measured at the end-of-year $t$ between US foreign and domestic aggregates. This testable implication is examined in Table 2. The results, from panel regressions spanning 1985 to 2019, conservatively include both year and foreign country fixed effects. The former controls for all common shocks affecting currencies at every point in time and allows the specification to focus on the cross-sectional predictability of exchange rates. The country fixed effects capture any time-invariant, country-specific unobservables. Standard errors, presented in parentheses, are clustered by time and country dimensions.[26]

The results show that exchange rate growth rates are strongly positively correlated with 12-month lagged foreign affiliate investment-to-capital ratio differentials ($I/K_{A,f,t} - I/K_{US,t}$). The positive sign suggests a strong predictability of foreign currency appreciation relative to the US dollar. Column 1 runs a univariate regression, while columns 2 through 13 control for an extensive set of macroeconomic predictors. The point estimate in column 1 of 0.076 ($p$-value 0.006) indicates that, on average, foreign currencies appreciate by 7.6% following a large one-unit increase in the annual $I/K$ differential between foreign and domestic investment of US firms.[27] Equivalently, in more modest terms, a standard deviation increase in the annual $I/K$ differential (of size 0.09, see Table 1) is associated with a 0.68% annual appreciation of foreign currencies. This predictability is a novel empirical finding in the international macro-finance literature, yet is consistent with the multinational $q$-theory framework.

The predictability is robust to a large set of controls, measured as differentials at the end-of-year $t$ between the foreign country $f$ and the US economy. These controls include consumption, GDP, aggregate investment flows, inflation, interest rates, exchange rate changes, and currency excess returns (detailed in Appendix IA.C.4). Despite controls, the positive sign in column 1 remains unchanged, with the point estimate retaining a similar magnitude. Notably, neither real

[26]Results are robust to alternative clustering, such as wild and jackknife bootstraps (MacKinnon, Nielsen, and Webb (2023)).
[27]Jackknife bootstrapped standard errors with double clustering give similar significance at the 1% level ($p$-value 0.0076).

interest rate differentials nor inflation rate differentials subsume the 12-month-ahead correlation observed in column 1. The predictability is also not explained by trade activity.[28] Crucially, when investment is measured at the economy-wide level (columns 4, 5, and 6), the differential in aggregate investment between foreign countries and the US economy does not predict exchange rate fluctuations, in stark contrast to the differential in US foreign affiliate investment. These fundamental contrasting findings serve as empirical validation for the modeling of firms as MNEs rather than typical representative standalone entities. Appendix Figure IA.6 visualizes this key forecastability by plotting each country's affiliate investment rate $I/K_{A,f,t}$ against the exchange rate growth rate in the following year $t+1$.

Columns 14 and 15 in Table 2 test whether the foreign affiliate $I/K$ predictability of exchange rate changes extends to forecasting currency excess returns. The results show that while exchange rate changes are predictable, currency excess returns are not. This means that the investment policies of within-US firms align with predicted fluctuations in equilibrium exchange rates, yet do not lead to arbitrage opportunities seeking profits 1-year-ahead by sorting countries from signals obtained from US affiliate investments. This is not a prediction of MNE $q$-theory. Note that the literature on exchange rate forecasting has found it substantially more difficult to forecast pure currency appreciation than currency excess returns (Kremens and Martin (2019)); hence, the predictability from foreign affiliate $I/K$ differentials in the first set of regressions is considerably more empirically challenging. In column 15, the interest rate differential is shown to be a reliable predictor of 1-year-ahead currency excess returns, thereby confirming the cross-sectional carry trade spread as identified by Lustig and Verdelhan (2007).

The framework provides further testable implications that link exchange rate movements to contemporaneous measures of $Y/K$ and $I/K$ ratios measured at $t+1$. These aggregates are key drivers of fundamental marginal productivities, adjustment costs, and Tobin's $q$s at $t+1$, all of which relate to the marginal benefits of investment. Table 3 examines these correlations, reporting regressions of exchange rate changes (columns 1–6) and currency excess returns (7–8) on all key production variables. Recall that there are 10 key US data inputs in (19), including the US stock market return $R_{t+1}^{US\_Mkt}$, as well as the $I/K$ ratios at $t$ and $t+1$, and the $Y/K$ ratios at $t+1$, which enter as differentials between affiliates and US domestic activity.

[28]For space reasons, in untabulated results I have controlled for several measures of trade effects and find that trade does not explain the predictability in column 1 of Table 2. For example, the results are robust to controlling for differentials in the ratios of imports to GDP, exports to GDP, and net exports to GDP. In the latter, the coefficient on net exports is small at 0.022 and statistically insignificant ($p$-value of 0.82), all while affiliate $I/K$ remains highly significant.

Column 1 reports panel results of exchange rate growth rates regressed on all key foreign affiliate $f$ differentials and the US stock market return. The specification includes country fixed effects but omits year fixed effects to avoid perfect collinearity with the market. Given that US market performance correlates with the business cycle, the specification controls for US business cycle aggregates, including growth rates in consumption and GDP, and the rate of inflation. Consistent with model predictions, all key variables correlate with foreign currency appreciation with the expected signs. Affiliate investment at time $t$ is positively associated with foreign currency *appreciation* into $t+1$, attributed by the model to higher relative foreign investment opportunities at $t$. In contrast, affiliate output and investment at $t+1$ are associated with a *depreciation* of foreign currencies (negative sign in both cases). The model attributes these patterns to exchange rates fluctuating to offset the higher relative marginal benefits realized at $t+1$ from affiliate investment. All affiliate variables are significant at conventional levels. Section 4.3 shows that the structural model reproduces the sign and magnitude of these coefficients. The coefficient on the stock market return also aligns with the model's prediction, showing a positive association with foreign currencies, though it is statistically insignificant.[29]

Crucially, the investment correlations above translate into value-creating effects for the consolidated balance sheets of US-established MNEs. Intuitively, the positive forecastability of foreign currencies into $t+1$ benefits US parents because their real investments made abroad in physical capital today become more valuable one year later in the consolidated balance sheets of the US parents (denominated in US dollar terms). Moreover, the strong negative contemporaneous correlations between affiliate investment and foreign currencies are also intuitive, as higher investments abroad occur when the foreign currency is becoming relatively cheaper compared to the US dollar, i.e., a negative relationship. As such, Table 3 provides evidence that US firms tend to time their real investment flows abroad so that, on the one hand, they increase the parents' balance sheets into $t+1$ due to favorable currency movements, while on the other, these investments occur during periods when foreign currencies weaken.

Columns 2 and 3 introduce the differentials related to US affiliates in the *ROW*. The 3-country result in (19) predicts that [*i*] US investments in the ROW at time $t$ should positively correlate with an appreciation in country $f$'s foreign currency into $t+1$, and [*ii*] these currencies should depreciate (negative correlation) with investments and output rates in the ROW at $t+1$. Both columns confirm these directional relationships, validating the tradeoff channel between the

[29] Pavlova and Rigobon (2007) find a co-movement between the stock and exchange rate markets.

marginal costs and benefits of investing abroad across affiliates that offer different investment opportunities in an interconnected global economy. All production variables in column 3 have the predicted sign, with affiliate $f$'s investments and output particularly significant. This is also the case in columns 4 and 5 which add year fixed effects, with time $t$ foreign affiliate investment differentials remaining resilient in their forecastability of 1-year-ahead foreign currency appreciation, and robust at the 1% when controlling for lagged interest rate differentials in column 5. Moreover, the panel results are robust to conducting standard Fama and MacBeth (1973) (FM) cross-sectional regressions in column 6, with coefficients of similar magnitude and signs consistent with theory. These directional relationships are maintained in columns 7 (panel) and 8 (FM) which regress currency excess returns on the key variables and control for interest rate differentials. The nonlinear structural estimation that ensues exploits the linear correlations documented in this section.

## 4.2 Model structural estimation results and implied production fundamentals

This section presents the results of the structural methodology outlined in section 3.1. It discusses parameter estimates, model fit, and reasonableness of inferred production fundamentals.

Panel A of Table 4 reports the GMM parameter estimates, model fit $\chi^2$ tests, and key implied fundamentals of US foreign affiliates. The capital share in output parameter, $\kappa$, ranges for most countries of US affiliates from 0.30 to 0.35, with low standard errors (reported in brackets). The median $\kappa$ is 0.339 (average 0.334), highlighted in Panel B, which summarizes median statistics across all 44 countries. These estimates align remarkably close with typical business cycle textbook values, and serve as validity check for the novel estimation approach of the multiaffiliate and multicountry $q$-theory model.[30] In addition, the estimates are largely comparable to those in Liu et al. (2009), which range from 0.2 to 0.5 for the cross-section of US stock returns. The $\chi^2$ model fit test, with a median $p$-value of 0.515 (ranging from 0.152 to 0.989), fails to reject the model of exchange rate determination across all countries.[31]

The slope parameter $\theta$, capturing adjustment cost size, has a median estimate of 15.11 and is also generally estimated with precision. The parameter $\nu$, reflecting adjustment cost asymmetries, exhibits wide heterogeneity across foreign markets, with some countries showing positive $\nu$ (implying higher costs for underinvestment with respect to affiliate steady state investment) and others negative (indicating higher costs for overinvestment).

[30] Prominently, in Kydland and Prescott (1982) $\kappa = 0.36$ and in Rotemberg and Woodford (1992) $\kappa = 0.30$

[31] Production-based models tend to perform well in $\chi^2$ tests; e.g., Liu et al. (2009) and Belo et al. (2013) fail to reject when pricing the cross-section of stock returns and Tobin's $q$ valuation moments, respectively, with similar $p$-values. However, validation along other metrics is important, including reasonableness of parameters and implied fundamentals such as MPKs and Tobin's $q$.

To interpret these parameters economically and evaluate their reasonableness, Panel A outlines the fundamentals they imply for each foreign affiliate country. For example, the marginal product of affiliate capital in country $f$, column $MPK_{A,f}$, exhibits substantial heterogeneity, indicating that productivity of US capital abroad varies widely across foreign markets. These inferred MPKs, presented as time-series averages, are calculated at each period $t$ as $MPK_{A,f,t} = \kappa \cdot Y/K_{A,f,t}$, using the reported point estimates and observed affiliate $Y/K$ ratios. Panel B reports that the median MPK for foreign affiliates $f$ across all 44 foreign markets is 0.420, surpassing the 0.345 for US firm domestic operations at home $HQ$.

Contextualizing findings, Table 4 provides an important economic rationale for the observed increased investments in US affiliates: the marginal productivity of US capital abroad has, on average, been greater than at home since the 1980s. MPK estimates seem reasonable in size and complement an extensive literature interested in understanding capital flows across countries, especially from rich to developing nations, following the seminal work by Lucas (1990).[32]

Variation in adjustment costs faced by affiliates across the world is crucial for the performance of the model in the next section. Therefore, Panel A also reports the average size of foreign affiliate adjustment costs as a fraction of affiliates' own output, $\frac{\Phi_{A,f}}{Y_{A,f}}$, reported as a percentage ($\times 100$) per annum. As shown, there is rich heterogeneity in adjustment costs across US foreign affiliates. The fraction of output lost to adjustment costs by the median foreign affiliate is 5.69% and ranges from roughly 1% to 22% across affiliates. These estimates are largely consistent with estimates appearing in the literature. For instance, Belo et al. (2013) report similar sizes of adjustment costs as a fraction of output with a mean of 5.94% and ranging roughly from 1% to 20% when matching Tobin's $q$ moments at the industry level of US listed firms.

Affiliate estimates of Tobin's $q$ and their investment returns $R^I$ also align with reasonable sizes reported in the literature (for brevity, see discussion in Appendix IA.C.5), and importantly, their estimates vary widely across foreign markets. For example, Panel B indicates that the median foreign affiliate return on annual investments is 16.25% (average 16.28%), and ranges from -11.15% to 43.26% (Panel A). Notably, Panel B also highlights that, on average, aggregate foreign affiliates earn larger investment returns abroad ($R^I_{A,f} = 16.25\%$) than investments at home ($R^I_{HQ} = 9.32\%$), indicating US firms excel at capitalizing on foreign investment

[32] My MPK estimates focus on the directional flow of real capital investment from the US to its own foreign affiliates and from matching currencies. Hence, they differ from typical estimates that use, for example, all capital flows entering an economy. Moreover, unlike foreign country aggregates, BEA data come directly from US firms' own records (e.g., audited statements used for compliance), and do not rely on foreign governmental statistics, which may be subject to noise and other accuracy concerns. For work on MPK estimates see, for example, Caselli and Feyrer (2007) and Alfaro, Kalemli-Ozcan, and Volosovych (2008).

opportunities. Contextualizing these findings that emerge from matching currency data, the size of the estimated returns at home and abroad of MNEs align with a salient, yet untargeted, feature in the multinational literature that generally observes foreign subsidiaries outperforming domestic firms (Guadalupe et al. (2012)). This literature also highlights higher productivities of foreign affiliates, consistent with the larger MPK estimates of US affiliates discussed above.

The above estimates of affiliate productivity, adjustment costs, Tobin's $q$, and investment returns are important for the tests that follow, which examine the performance of the model in pricing the cross-section of exchange rates and observed patterns in carry trade risk premia.

### 4.3 Comparing realized and model-implied exchange rates

To examine the relationship between the observed and model-implied appreciation in foreign currencies, I exploit the following implication of (19): if ex post average fluctuations in foreign currencies are equal to their model-implied counterparts, then regressing those two series on variables associated with observed currency movements should produce coefficients of equal magnitude. Equivalently, the coefficients from regressing the difference between realized and implied foreign currency appreciation rates on any relevant variable should be zero.

These results are presented in Table 5. The reduced-form results of Table 3 suggest that the investment rate differential of US foreign affiliates is a good candidate testing variable, as it both forecasts (positively) and is contemporaneously correlated (negatively) with average appreciation rates of foreign currencies. Therefore, Panel A of Table 5 presents panel regressions with country and year fixed effects. The coefficients of realized exchange rate growth on lagged and current foreign affiliate investment rate differentials are significant at conventional levels, and their directional relationships are of the same sign and roughly the same magnitude in the implied exchange rate growth regressions. To test whether the coefficients are, in fact, equal, I regress the difference between the two exchange rate growth series (Realized−Implied) on the affiliate investment variables. The null of equal coefficients fails to reject with a $p$-value of 0.739. These inferences are similar in Panel B, which present coefficients from typical Fama and MacBeth (1973) regressions, which are of similar size and direction as those in the panel regressions. As stated, the $q$-theory of MNE investment links these currency patterns to the tradeoff between the intertemporal marginal costs and benefits of US investments abroad.

Importantly, Panel C adds the current output-to-capital differentials of US affiliates, which have negative signs in both the realized and implied series. In all, consistent with the intuition and testable predictions from Section 2.1.2.1, the model reproduces empirically both the forecastable (appreciation) component and the contemporaneous (depreciation) components in time $t+1$ exchange rate fluctuations associated with within-firm US affiliate marginal costs of investment at $t$ and marginal benefits at $t+1$. After deducting the implied exchange rate growth series from the realized, observed currency fluctuations have no remaining correlations with US MNE differentials in foreign affiliate investment and output activity.

### 4.4 US global production fundamentals and the carry trade in currency markets

Having tested the model in the sections above—including reduced-form validation, formal overidentification tests of model fit, and verifying its ability to reproduce the observed novel links between exchange rate changes and fundamental aggregates of US production across the world—this section explores whether the multinational $q$-theory framework can provide insights into the large premiums observed in currency markets from carry trade strategies. For example, given the sizable spread in currency returns between high- and low-interest-rate countries, this section investigates whether there is a fundamental link between the economies generating these returns and the adjustment costs and marginal productivities of US foreign affiliates operating in those countries. I show that these microfounded US fundamentals across the globe are crucial new mechanisms through which the carry trade is priced via production dynamics.

As a first test, similar to the experiments of Table 5 but now applied to the implied series of currency excess returns $\widehat{R^{FX}_{f,t+1}}$ described in Section 3.1.4, Panel A of Table 6 tests whether the model captures the large average currency risk premia associated with the carry trade by comparing the coefficients from regressions of realized and model-implied currency excess returns on lagged interest rate differentials. As shown, both the panel and Fama–MacBeth regressions indicate that foreign interest rates forecast a significant 1-year-ahead realized currency premium. The size of the slopes in these carry trade tests are comparable when using the implied return series. Economically, noting that the slope from Fama–Macbeth cross-sectional regressions is the return for an investor in a zero-investment long-short portfolio (Fama (1976)), the coefficients imply that a 10 percentage point increase in interest rate differentials yields an average return of 3.19% (realized) and 3.36% (implied) per annum. The null that these spreads are equal fails to reject with a $p$-value 0.862, indicating that after deducting implied excess returns from realized, there is no longer a forecastable component in average currency premia associated with the carry trade since the 1980s—i.e., the model reproduces the historical average exchange rate movements associated with the returns for a US investor in currencies long in high and short in low foreign interest rates.

As alternative tests, and as noted in Section 3.1.4, with risk-free rates known ex-ante at time $t$, investors can sort countries into portfolios based on interest rate differentials, creating economically large average spreads in returns at $t+1$ between high- and low-foreign-interest-rate portfolios. This approach, as noted by Lustig and Verdelhan (2007), yields a 1-year-ahead carry trade spread that exceeds by an order of magnitude those spreads achievable between any two countries. Therefore, as is typical in this literature, Panel B of Table 6 reports tests at the portfolio level of countries and examines the large carry trade spreads through their link to the fundamentals of US affiliates in those portfolios. The benefits of this portfolio aggregation approach are twofold: it not only challenges the model to replicate those larger spreads but also provides more precise estimates of both the actual risk-reward tradeoff faced by investors in $R^{FX}_{f,t+1}$ returns and the fundamentals of US affiliates associated with those countries.[33]

Sorting countries into 4 carry trade portfolios as baseline test assets, Panel B in Table 6 reports the average currency excess returns of the carry trade and the average pricing errors of the model. Pricing errors are computed as the difference between realized and model-implied currency excess returns. Consistent with the positive cross-sectional premia in Lustig and Verdelhan (2007), there is a sizable average carry trade high-minus-low (H-L) spread in excess returns of 4.48% per annum ($t = 3.83$). To ease the inspection of the pricing performance of the model, Figure 4 Panel A reports the pricing errors of the individual portfolios by plotting the average predicted against the average realized excess returns on each test asset. If the model is a good fit, the scatter points should line up as close as possible to the 45-degree line. As shown, implied carry trade returns match well their realized counterpart, and as in the data, generate monotonically increasing returns across the baseline 4 test assets, with small pricing errors.

For instance, as reported, the size of the average pricing error (realized−implied) of portfolio 1 is small at -0.20% per annum, computed from the difference between its realized (-0.10%) and model-implied (0.10%) excess return. The largest pricing error across all 4 portfolios is -0.82% per year (portfolio 4) and the smallest is 0.13% (portfolio 2), which collectively across all 4 portfolios produces a mean absolute pricing error (MAE) of only 0.36% per annum. Importantly, the model succeeds in generating a sizable implied H-L spread across the 4 test assets of 5.10% per year, which is comparable to the realized H-L spread of 4.48%, implying a small average H-L pricing error, as reported, of only -0.62% per year ($t = -0.50$). Therefore, collectively, the

[33] Black, Jensen, and Scholes (1972) provide early advocacy for using portfolios to reduce noise in estimates of average returns in equity markets. Similarly, in currencies, sorting countries into portfolios allows abstraction from currency-specific diversifiable components. To account for currency patterns, it is appropriate to increase the precision of fundamentals by using portfolios.

model succeeds in pricing each individual carry trade portfolio and its influential H-L spread. This is formally confirmed by a $\chi^2$ test for the null that the pricing errors across all portfolios are jointly zero ($p$-value 0.99 at the bottom of Panel B), failing to reject the $q$-theory model.

The model's efficacy in pricing carry trades is robust, not merely a result of selecting 4 portfolios as baseline test assets—a deeper analysis with varying levels of portfolio aggregation is provided in the mechanism section below. Figure 4 Panel B plots the H-L excess return payoff for a US investor in the carry trade (long in high and short in low foreign interest rates) since the 1980s, obtained from sorting countries into 8 portfolios. This specific aggregation yields the largest average H-L spread in the sample period at 7.54% per annum. The inferred H-L spread from production data closely matches with an average excess return of 7.27% per annum. The time-series correlation between these series is notably high at 40.7%, with the standard deviations of the spreads at 11.8% for realized and 11.9% for implied, per annum. This quantitative success in pricing currencies is reminiscent of the seminal findings in Cochrane (1991) Figure 2, where a single-country production model replicates observed patterns in the aggregate US stock market, achieving a correlation as high as 44.9% for annual returns. However, this paper's multinational $q$-theory framework highlights the far-reaching capability of production-based asset pricing to the cross-section of currencies as a new asset class.

Panel B further reports the average fundamental characteristics of each baseline carry trade portfolio.[34] Carry trade portfolios are increasing in time $t$ foreign interest rate differentials, exhibiting a high-minus-low (H-L) spread of 9.01% per annum (expressed in percentage points $\times 100$). Notably, in terms of production-based fundamentals, these portfolios also exhibit an increasing pattern in the differential between foreign affiliate $f$ and $HQ$ investment-to-capital ratios, with the investment rate differential at time $t$ ($I/K_{A,f,t} - I/K_{US,t}$) monotonically increasing across test assets, resulting in a H-L spread of 3.33% per annum ($t = 2.47$). This pattern indicates a link between interest-rate-sorted portfolios and increasing investment opportunities for US MNEs operating in those foreign markets, as evidenced by the monotonic increase in average Tobin's $q$ ($q_{A,f,t}$) across test assets, from 1.39 to 2.03. This produces a H-L spread in Tobin's $q$ for US foreign subsidiaries of 0.64 ($t = 1.93$) per annum, meaning that the market value of US affiliates relative to their replacement cost of capital in the high carry trade portfolio exceeds that in the low portfolio by a sizable margin of 0.64 per year, as reported.

Hence, countries with high (low) interest rates at time $t$, generating high (low) currency

[34] Fundamental statistics per portfolio are from the time-series average of the cross-sectional median foreign country.

excess returns at $t+1$ for US investors, tend to be those where US foreign affiliates have high (low) fundamental investment opportunities at time $t$. Moreover, these investment opportunities for US MNEs abroad continue being higher (lower) in the subsequent period at $t+1$, as evidenced by the increasing pattern in US foreign affiliate Tobin's $q$ at $t+1$, which exhibit a H-L average spread of 0.76 ($t = 2.26$) per annum. Importantly, in sharp contrast, affiliate marginal productivities ($MPK_{A,f,t+1}$) tend to decrease nearly monotonically across carry trade portfolios, declining from an average MPK of 0.51 in low-interest-rate countries to 0.40 in high. These patterns create a H-L spread in MPKs of -0.11 ($t = -4.02$) per annum, which correlates with the decreasing pattern observed in affiliate output-to-capital ratio differentials ($Y/K_{A,f,t+1} - Y/K_{US,t+1}$) across carry trade portfolios, with a H-L spread in $Y/K$ differentials of -31.38 ($t = -2.81$) per annum, expressed in percentage terms.

Collectively, carry trade portfolios tend to increase in marginal investment opportunities for US foreign affiliates (as implied by their market valuations inferred from Tobin's $q$ estimates), yet simultaneously exhibit a decreasing pattern in affiliate marginal productivities. Importantly, another crucial feature of affiliates is the heterogeneous adjustment costs they face across fundamentally distinct productive markets. The size of these costs, as a fraction of affiliates' own output ($\Phi/Y_{A,f,t+1}$), increases monotonically from an average of 1.45% (expressed as a percentage $\times 100$) for affiliates in low-interest-rate economies to 3.93% in high. This leads to a H-L spread in affiliate costs of 2.48% ($t = 1.74$) per annum, highlighting the economic importance of adjustment costs in explaining cross-sectional patterns. Altogether, the results underscore an important connection between the returns to the carry trade in currency markets and the production patterns of US foreign affiliates' fundamentals, including their investment opportunities, productivity, and adjustment costs across interest-rate-sorted economies.

Lastly, Panel B reports adjustment cost sizes for each business $HQ, f, ROW$ as a fraction of US global aggregate output ($Y^{US\_total}$, which sums US output at home and abroad). The fractions of total US output lost to affiliate adjustment costs are small, averaging 0.09% and 0.08% annually for affiliates $f$ and $ROW$, respectively. In contrast, the fraction lost to domestic $HQ$ costs is 7.73%. These adjustment cost sizes, estimated from matching currency data, seem reasonable and imply that, combined, the fraction of total US output lost to global adjustment costs on US worldwide capital investment ($\Phi^{US\_total}/Y^{US\_total}$) averages 7.90% per year.

Results indicate that the pricing of currency premia relies on the cross-sectional patterns in the fundamentals of US affiliates, including MPKs, Tobin's $q$s, and adjustment costs. The next section conducts comparative statics that explore the quantitative importance of the different $I/K$ and $Y/K$ ratios that implicitly influence those fundamentals and their link to currencies.

#### 4.4.1 Inspecting the mechanism: comparative statics

Table 7 reports the comparative statics that quantify the importance of each key input for the model's performance. Recall that the production framework links exchange rate growth to 10 key production-based aggregates. The $I/K$ inputs are crucial determinants of Tobin's $q$ and adjustment cost, while the $Y/K$ ratios are important for $MPK$s. Therefore, to quantify the mechanisms behind the model's performance, I conduct experiments that shut down one by one the role of each key input and assess the size of the resultant pricing errors relative to the baseline model. The idea is that if, say, the time $t$ foreign affiliate $I/K_{A,f,t}$ ratio is important for the time $t+1$ cross-sectional variation (spreads) in exchange rate growth rates, then by shutting down any cross-affiliate variation in $I/K_{A,f,t}$ ratios (and implicitly their effect on cross-affiliate variation in time $t$ Tobin's $q$s), the model should not perform as well as the baseline model. If the resultant pricing errors from this experiment are large relative to those of the baseline, I can infer that the ratio is quantitatively important in capturing cross-sectional spreads in carry trade currency risk premia. I design analogous experiments for all other 9 inputs (for brevity, detailed steps are in Appendix IA.C.6). For example, for the comparative statics of $HQ$, its $I/K$ and $Y/K$ ratios are set fixed at their US time-series medians, thus removing any variation over time in the rate of US investment and output at home.

In all 10 experiments, I use the parameter estimates from the baseline estimation to reconstruct the implied exchange rate growth rates and recalculate the model's pricing errors. To evaluate whether the pricing errors are larger relative to the baseline or not, the first row labeled "baseline" under model pricing errors reports the baseline pricing errors (as a percentage $\times 100$ per annum), and includes the pricing error of the H-L spread portfolio (denoted H-L$^{4p}$), and the MAE across all 4 baseline test assets (MAE$^{4p}$). I also report those H-L and MAE pricing error statistics for the case of 5 to 8 carry trade portfolios, which serve to assess model fit in those scenarios. Notably, the case of 8 portfolios produces the largest average H-L spreads observed in the data, and equal to 7.54% per annum (column H-L$^{8p}$). This latter spread represents the average excess return that the model fits well in Figure 4 Panel B, as discussed.

The results from the different experiments suggest that, in general, all 10 production inputs are important for the model's ability to explain the cross-section of exchange rates. This can be seen in the benchmark 4 portfolios, where the mean absolute pricing error (column MAE$^{4p}$)

of each experiment is larger than the baseline (which has the smallest MAE of 0.36% per year). Nonetheless, certain inputs have a more pronounced quantitative impact than others. For example, the pricing error of the H-L spread portfolio (column H-L$^{4p}$) suggests that current investment $I/K_{A,f,t}$ and future investment $I/K_{A,f,t+1}$ of affiliate $f$ are two of the largest drivers of the performance of the model, as evidenced by the large H-L average errors of 5.42% and -8.75% per annum, respectively. These errors are an order of magnitude larger than the baseline H-L pricing error of only -0.62%. Similarly, the current investment $I/K_{A,ROW,t}$ and future investment $I/K_{A,ROW,t+1}$ for the $ROW$ affiliates emerge as the second-most important fundamentals, with H-L pricing errors of -1.43% and 2.82%, respectively. This underscores that the model's quantitative success largely depends on exploiting the implied investment opportunities (Tobin's $q$) and adjustment costs not only of US affiliates in country $f$, but also in the ROW of an interconnected global economy. Thus indicating that the three-country world setting provides a better characterization of exchange rates than a two-country economy.

Lastly, the US stock market return is quantitatively important, as evidenced by the relatively large pricing errors in the H-L spread and MAE statistics of -1.05% and 5.23%, respectively, for the benchmark 4 portfolios. Consequently, the comparative statics in this section indicate that to capture average patterns in currency markets, the model relies not only on the rich cross-affiliate variations in $I/K$ and $Y/K$ ratios but also on the time variation in the US stock market. This provides internal validity to the model, as the classical result in Cochrane (1991) is a special case that, in the MNE $q$-theory of investment framework, establishes a formal first link between equity and currency markets in this literature. The quantitative importance of all 10 production-based inputs is further confirmed by experiment "All", which shuts down their joint role, yielding the largest mispricing MAE statistics across all comparative statics.

#### 4.4.2 Robustness

The Appendix section IA.C.7 discusses several additional results and robustness checks for both the model and reduced-form evidence. For example, the section explores the quantitative robustness of the baseline model in pricing the carry trade when different data filters and samples are used in the tests. To facilitate visualization of these checks, Figure IA.7 plots the pricing errors of each different check alongside those of the baseline model. The results remain qualitatively unchanged and show little quantitative variation across several meaningful robustness checks. These checks include, for example, incorporating only developed economies in the sample, excluding all euro-year countries that adopted the euro as a common currency after its introduction in 1999, and dropping the few countries where the average fraction of affiliate output shipped back to US parents exceeds 10%. Appendix Figure IA.8 plots the model's performance in pricing the carry trade at different levels of portfolio aggregation. The scatter points align well around the 45-degree line, and in each plot, the model produces economically sizable implied H-L spreads comparable to those observed in the data since the 1980s. Additional robustness tests are discussed in that section.

# 5 Concluding Remarks

This paper introduces a simple concept: when firms are multinational and maximize total firm value, exchange rates become linked to the intertemporal marginal rates of transformation for both home and foreign affiliate capital. In equilibrium, exchange rates provide the crucial offsetting link through which differences in expected returns from investing at home and abroad for multinationals are reconciled within the representative firm and align with its total stock return. This $q$-theory of MNEs is analogous to a standard international consumption-based asset pricing model, but uses multinational producers and production functions rather than consumers and stochastic discount factors. By owning equity shares in the representative MNEs, the representative consumers can diversify domestic risk through the internal risk sharing and investment optimization that takes place within MNEs. The preference-free approach infers exchange rates from observable US MNE aggregates on real investment, output, capital stocks, and US equity returns, and through the structural estimation of technological parameters.

Validating the model in reduced form, this paper documents novel correlations between the within-firm investments of US foreign affiliates and exchange rates, which translate into value-creating effects for the consolidated balance sheets of US-established MNEs. Specifically, I find evidence that US firms tend to time their real investment flows abroad such that their time $t$ investments, on the one hand, increase the parents' balance sheets into $t+1$ due to favorable foreign currency appreciation, while on the other hand, these investments occur at $t+1$ during periods when foreign currencies weaken.

Structurally estimating the model, a simple implementation of the production-based asset pricing framework performs well in pricing the cross-section of exchange rates. Tested in a large sample of 44 countries with data since the 1980s, spreads in currency carry trade are explained by the cross-sectional differences in the fundamental characteristics of US foreign affiliates. The estimates reveal a fundamental link between interest rates being relatively high in foreign countries and US foreign affiliates finding it relatively costly to expand their real productive capital stock in those economies (i.e., high relative adjustment costs). Moreover, those high-interest economies tend to be countries in which US foreign affiliates have lower fundamental marginal productivities, yet higher investment opportunities and market valuations as inferred from novel within-firm Tobin's $q$ estimates.

Moreover, the structural estimates inferred from matching currency data suggest that US firms have excelled in capitalizing on foreign investment opportunities through affiliates, which, on average, have offered higher returns than domestic investments since the 1980s. These findings align with the significant increase in foreign investments over the past four decades.

Despite the simplicity of the baseline model's specification with only real capital investment, its quantitative success suggests that modeling representative firms as multinationals, rather than standalone entities as is conventional in the broad literature on international macroeconomics and finance, is a promising avenue for new research.

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Table 1

**Descriptive statistics of country-level variables in the production-based fundamentals of exchange rates, 1985–2019**

*A. Mean, standard deviation, and percentiles*

| | Mean | S. Dev | p1 | p25 | p50 | p75 | p99 |
|---|---|---|---|---|---|---|---|
| $I/K_{A,f,t}$ | 0.20 | 0.09 | 0.06 | 0.14 | 0.19 | 0.24 | 0.52 |
| $I/K_{US,t}$ | 0.11 | 0.01 | 0.09 | 0.10 | 0.11 | 0.11 | 0.14 |
| $I/K_{A,ROW,t}$ | 0.18 | 0.04 | 0.13 | 0.16 | 0.18 | 0.21 | 0.29 |
| $Y/K_{A,f,t}$ | 1.52 | 0.80 | 0.44 | 0.97 | 1.37 | 1.81 | 4.71 |
| $Y/K_{US,t}$ | 1.03 | 0.20 | 0.80 | 0.84 | 1.00 | 1.26 | 1.35 |
| $Y/K_{A,ROW,t}$ | 1.10 | 0.12 | 0.87 | 1.02 | 1.10 | 1.17 | 1.40 |
| $R_t^{US\_Market}$ | 8.06 | 12.50 | -20.84 | 2.51 | 11.59 | 17.63 | 25.03 |
| $(e_t/e_{t-1}) - 1$ | 0.51 | 10.34 | -23.99 | -5.80 | -0.13 | 7.18 | 26.34 |
| $(e_t/e_{t-1}) - (i_{t-1}^{US} + 1)/(i_{t-1}^{f} + 1)$ | 2.00 | 11.72 | -25.26 | -5.08 | 1.54 | 9.11 | 34.34 |
| $I/K_{A,f,t} - I/K_{US,t}$ | 0.09 | 0.09 | -0.05 | 0.04 | 0.08 | 0.13 | 0.41 |
| $Y/K_{A,f,t} - Y/K_{US,t}$ | 0.49 | 0.80 | -0.64 | -0.02 | 0.34 | 0.81 | 3.65 |
| $I/K_{A,ROW,t} - I/K_{US,t}$ | 0.08 | 0.03 | 0.02 | 0.05 | 0.07 | 0.10 | 0.19 |
| $Y/K_{A,ROW,t} - Y/K_{US,t}$ | 0.07 | 0.20 | -0.21 | -0.11 | 0.16 | 0.21 | 0.45 |
| $i_{t-1}^{f} - i_{t-1}^{US}$ | 3.66 | 9.74 | -4.15 | -0.14 | 1.50 | 4.83 | 46.60 |

*B. Cross-sectional correlations*

| | $\frac{I}{K}_{A,f,t+1}$ | $\frac{Y}{K}_{A,f,t+1}$ | $i_t^f$ | $\frac{e_{t+1}}{e_t} - 1$ | $\frac{e_{t+1}}{e_t} - \frac{i_t^{US}+1}{i_t^f+1}$ | $\frac{I}{K}_{A,ROW,t}$ | $\frac{I}{K}_{A,ROW,t+1}$ | $\frac{Y}{K}_{A,ROW,t+1}$ |
|---|---|---|---|---|---|---|---|---|
| $I/K_{A,f,t}$ | 0.59 | 0.09 | 0.20 | 0.09 | 0.04 | -0.19 | -0.18 | 0.13 |
| $I/K_{A,f,t+1}$ | | 0.23 | 0.17 | -0.01 | -0.04 | -0.19 | -0.19 | 0.13 |
| $Y/K_{A,f,t+1}$ | | | 0.00 | -0.07 | -0.09 | -0.25 | -0.25 | 0.13 |
| $i_t^f$ | | | | 0.11 | 0.23 | -0.16 | -0.16 | 0.05 |
| $(e_{t+1}/e_t) - 1$ | | | | | 0.90 | -0.01 | -0.01 | 0.00 |
| $(e_{t+1}/e_t) - (i_t^{US} + 1)/(i_t^f + 1)$ | | | | | | -0.02 | -0.02 | -0.01 |
| $I/K_{A,ROW,t}$ | | | | | | | 0.98 | -0.44 |
| $I/K_{A,ROW,t+1}$ | | | | | | | | -0.41 |

**Notes:** This table reports descriptive statistics of country-level variables, including foreign affiliate data of US firms that are aggregated to the foreign country $f$ level. Data are annual from 1985 to 2019 and in *real* terms for *all* variables, and in the case of foreign affiliate aggregates these are denominated in real foreign currency values. That is, $HQ$ aggregate real quantities are in constant 2015 US dollars while the foreign affiliate $f$ real aggregates are in constant 2015 foreign currency values. Panel A reports the mean, standard deviation (S. Dev), and percentiles 1, 25, 50, 75, and 99. Panel B reports the time series average of pairwise cross-sectional correlations between all key variables. The variables include the investment-to-capital $I/K$ and output-to-capital $Y/K$ aggregate ratios for the triplet of US-owned business units in $\{US,\ f,\ ROW\}$, where "US" stands for the home headquarters within US borders, $f$ for foreign affiliates in country *f*, and ROW for foreign affiliates in the ROW. Therefore, for example, $I/K_{A,f,t}$ stands for foreign affiliates operating in country *f*, and where subscripts $A$ and $t$ denote affiliates and time $t$. Other variables are the aggregate US stock market return $R^{US\_Market}$, bilateral exchange rate growth $(e_{t+1}/e_t) - 1$, currency excess returns $(e_{t+1}/e_t) - (i_t^{US} + 1)/(i_t^f + 1)$, the interest rate differential between country $f$ and the US $i^f - i^{US}$, and the differentials in investment-to-capital and output-to-capital ratios between affiliates in $f$ *or* $ROW$ and the US. For all business units at home and abroad, investment $I$ is capital expenditures, output $Y$ is value added, and capital $K$ is net property, plant, and equipment. US firm aggregates at home $HQ$ are adjusted such that they exclude the value added, capital stock, and investment of all firms operating within the US but owned by foreign parents. That is, for example, I deduct from US GDP all aggregate value added created within the US territory by affiliates operating within the US but owned by foreign non-US investors. See section 3.2 and the Data Appendix IA.B for data sources and details.

Table 2

**Investment of foreign affiliates of US parents & forecasts of foreign currency appreciation & currency excess returns, 1985–2019**

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Panel regressions:* | $\frac{e_{t+1}}{e_t}-1$ (*columns* 1–13) | | | | | | | | | | | | | $\frac{e_{t+1}}{e_t}-\frac{i_t^{US}+1}{i_t^f+1}$ | |
| $I/K_{A,f,t}-I/K_{US,t}$ | 0.076*** | 0.089** | 0.074** | 0.077*** | 0.081*** | 0.089** | 0.090** | 0.084** | 0.091** | 0.078*** | 0.075** | 0.074** | 0.074*** | 0.025 | 0.019 |
| | (0.026) | (0.034) | (0.032) | (0.026) | (0.028) | (0.034) | (0.034) | (0.034) | (0.036) | (0.028) | (0.032) | (0.035) | (0.025) | (0.056) | (0.054) |
| $\Delta C_t^f-\Delta C_t^{US}$ | | -0.065 | | | | | | | | | | | | | |
| | | (0.213) | | | | | | | | | | | | | |
| $\Delta GDP_t^f-\Delta GDP_t^{US}$ | | | 0.281 | | | | | | | | | | | | |
| | | | (0.208) | | | | | | | | | | | | |
| $I/K_t^{f,fixed}-I/K_t^{US,fixed}$ | | | | 0.028 | | | | | | | | | | | |
| | | | | (0.062) | | | | | | | | | | | |
| $I/K_t^{f,total}-I/K_t^{US,total}$ | | | | | 0.031 | | | | | | | | | | |
| | | | | | (0.042) | | | | | | | | | | |
| $I/GDP_t^{f,nonfinA}-I/GDP_t^{US,nonfinA}$ | | | | | | -0.752 | | | | | | | | | |
| | | | | | | (0.527) | | | | | | | | | |
| $netFDI/GDP_t^f-netFDI/GDP_t^{US}$ | | | | | | | 0.116 | | | | | | | | |
| | | | | | | | (0.084) | | | | | | | | |
| $inFDI/GDP_t^f-inFDI/GDP_t^{US}$ | | | | | | | | 0.044 | | | | | | | |
| | | | | | | | | (0.045) | | | | | | | |
| $outFDI/GDP_t^f-outFDI/GDP_t^{US}$ | | | | | | | | | -0.005 | | | | | | |
| | | | | | | | | | (0.059) | | | | | | |
| $\pi_t^f-\pi_t^{US}$ | | | | | | | | | | 0.056* | | | | | |
| | | | | | | | | | | (0.030) | | | | | |
| $(e_t/e_{t-1})-1$ | | | | | | | | | | | -0.007 | | | | |
| | | | | | | | | | | | (0.075) | | | | |
| $(e_t/e_{t-1})-(i_{t-1}^{US}+1)/(i_{t-1}^f+1)$ | | | | | | | | | | | | -0.021 | | | |
| | | | | | | | | | | | | (0.060) | | | |
| $i_t^f-i_t^{US}$ | | | | | | | | | | | | | 0.055† | | 0.177** |
| | | | | | | | | | | | | | (0.035) | | (0.074) |
| Observations | 1,254 | 1,133 | 1,163 | 1,157 | 1,070 | 970 | 1,172 | 1,178 | 1,172 | 1,254 | 1,254 | 1,236 | 1,254 | 1,254 | 1,254 |
| *Adj* $R^2$ | 0.37 | 0.39 | 0.39 | 0.40 | 0.40 | 0.47 | 0.38 | 0.38 | 0.38 | 0.38 | 0.37 | 0.38 | 0.38 | 0.34 | 0.35 |
| Year fixed effects | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Country fixed effects | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |

**Notes:** This table reports in columns 1 to 13 panel regressions of bilateral real exchange rate growth $(e_{t+1}/e_t)-1$ measured at the end-of-year $t+1$ and as regressed on the 1-year lagged differential measured at the end-of-year $t$ in real investment-to-capital ratios between US foreign affiliates in country $f$ and US domestic investment at home within US borders $I/K_{A,f,t}-I/K_{US,t}$. Foreign affiliate aggregates are in foreign currency real terms (e.g., in constant 2015 British pounds for investments in the UK), while HQ data are in real US 2015 dollar figures. The results are from panel regressions from 1985 to 2019, variables are in real terms, and include both year and foreign country fixed effects. Standard errors are reported in parentheses and are double clustered in the time and country dimensions. The real exchange rate is in real US dollars per unit of real foreign currency, with an increase in $e_{f,t}$ denoting a real appreciation of the foreign currency. Column 1 runs a univariate regression, while columns 2 through 13 control for several macroeconomic predictors measured at the end-of-year $t$ between the foreign country $f$ and the US economy. The controls include real consumption growth (column 2); real GDP growth (3); real fixed capital formation growth (4); real total capital formation growth (5); real net investment in nonfinancial assets to GDP ratios (6); net FDI (inflows minus outflows) to GDP ratios, where FDI captures all investment flows to the country to acquire a lasting management interest—i.e., 10 percent or more of voting stock— and include equity capital, reinvestment of earnings, other long-term capital, and short-term capital as shown in the balance of payments (7); FDI inflows to GDP ratios (8); FDI outflows to GDP ratios (9); annual rates of inflation (10); lags for the real exchange rate growth (11) and real currency excess returns (12); real interest rates (13) to account for the predictability associated with the UIP condition. Columns 14 and 15 run panel regressions where the dependent variable is real currency excess returns. ***, **, *, and † denote statistical significance at the 1%, 5%, 10%, and 15% levels, respectively. See section 3.2 and the Data Appendix IA.B for data sources and details.

Table 3

**Production-based fundamentals of exchange rates, US production activity at home and through foreign affiliates in countries $f$ and in the rest of the world, 1985–2019**

| | (1) *Panel* | (2) *Panel* | (3) *Panel* | (4) *Panel* | (5) *Panel* | (6) *Fama-MacBeth* | (7) *Panel* | (8) *Fama-MacBeth* |
|---|---|---|---|---|---|---|---|---|
| | $\frac{e_{t+1}}{e_t}-1$ *(columns* 1-6*)* | | | | | | $\frac{e_{t+1}}{e_t}-\frac{i_t^{US}+1}{i_t^f+1}$ | |
| $I/K_{A,f,t}-I/K_{US,t}$ | 0.122** | | 0.105** | 0.130*** | 0.129*** | 0.165*** | 0.094** | 0.113*** |
| | (0.059) | | (0.044) | (0.033) | (0.033) | (0.030) | (0.038) | (0.040) |
| $I/K_{A,f,t+1}-I/K_{US,t+1}$ | -0.147** | | -0.134** | -0.098* | -0.097† | -0.148*** | -0.142* | -0.170*** |
| | (0.066) | | (0.053) | (0.057) | (0.058) | (0.045) | (0.080) | (0.055) |
| $Y/K_{A,f,t+1}-Y/K_{US,t+1}$ | -0.033** | | -0.022** | -0.020** | -0.020* | -0.005 | -0.015 | -0.006 |
| | (0.012) | | (0.010) | (0.010) | (0.010) | (0.004) | (0.015) | (0.005) |
| $R_{t+1}^{US\_Market}$ | 0.096 | 0.106 | 0.118 | | | | | |
| | (0.129) | (0.139) | (0.137) | | | | | |
| $I/K_{A,ROW,t}-I/K_{US,t}$ | | 0.235 | 0.180 | | | | | |
| | | (0.646) | (0.639) | | | | | |
| $I/K_{A,ROW,t+1}-I/K_{US,t+1}$ | | -0.736 | -0.657 | | | | | |
| | | (0.582) | (0.590) | | | | | |
| $Y/K_{A,ROW,t+1}-Y/K_{US,t+1}$ | | -0.160* | -0.136† | | | | | |
| | | (0.093) | (0.090) | | | | | |
| $i_t^f-i_t^{US}$ | | | | | 0.016 | 0.080 | 0.139* | 0.306** |
| | | | | | (0.034) | (0.055) | (0.079) | (0.127) |
| Observations | 1,250 | 1,254 | 1,250 | 1,250 | 1,250 | 1,250 | 1,250 | 1,250 |
| *Adj* $R^2$ | 0.04 | 0.06 | 0.08 | 0.40 | 0.40 | 0.11 | 0.38 | 0.21 |
| Aggregate US yearly controls | Yes | Yes | Yes | | | n/a | | n/a |
| Year fixed effects | | | | Yes | Yes | n/a | Yes | n/a |
| Country fixed effects | Yes | Yes | Yes | Yes | Yes | n/a | Yes | n/a |

**Notes:** This table reports regression results of bilateral real exchange rate growth (columns 1–6) and real currency excess returns (columns 7–8) on the key production-based real aggregates introduced in the model section 2. Data is annual from 1985 to 2019 and in real terms. Foreign affiliate aggregates are in foreign currency real terms (e.g., in constant 2015 British pounds for investments in the UK), while HQ data are in real US 2015 dollar figures. The columns labeled "Panel" are panel regressions that include either year fixed effects, foreign country fixed effects, or both. "Aggregate yearly US controls" include growth rates in consumption and GDP and the rate of inflation. Standard errors in the panel regressions are reported in parentheses and are double clustered in the time and country dimensions. Columns labeled "Fama—MacBeth" run Fama and MacBeth (1973) cross-sectional regressions, with Newey and West (1987) standard errors that account for heteroskedasticity and autocorrelation in the error structure up to a lag of 4 years. Variables include the differential in investment-to-capital $I/K$ and output-to-capital $Y/K$ ratios between US foreign affiliates in country $f$ or in the $ROW$ and US domestic activity at home within US borders. $R^{US\_Market}$ is the aggregate US stock market return, while foreign interest rate differentials are $i^f - i^{US}$. For all business units at home and abroad, investment $I$ is capital expenditures, output $Y$ is value added, and capital $K$ is net property plant and equipment. US firm aggregates at home $HQ$ are adjusted such that they exclude the value added, capital stock, and investment of all firms operating within the US but owned by foreign parents. That is, for example, I deduct from US GDP any and all aggregate value added created within the US territory by affiliates operating within the US but owned by foreign non-US investors. ***, **, *, and † denote statistical significance at the 1%, 5%, 10%, and 15% levels, respectively. See section 3.2 and the Data Appendix IA.B for data sources and details.

Table 4

**GMM parameter estimates & the production-based fundamentals of exchange rates**

A: Parameters, std errs, $\chi^2$, & implied fundamentals for affiliates *f*: MPK, adjustm. costs, $q$, investm. returns

| Country | $\kappa$ | $[\kappa]$ | $\theta$ | $[\theta]$ | $\nu$ | $[\nu]$ | $\chi^2$ | $p_{\chi^2}$ | $MPK_{A,f}$ | $\Phi/Y_{A,f}$ | $q_{A,f}$ | $R^I_{A,f}$ | *Obs.* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Australia | 0.34 | 0.01 | 15.10 | 0.37 | -8.41 | 1.75 | 0.82 | 0.36 | 0.29 | 2.48 | 0.98 | 13.95 | 35 |
| Austria | 0.34 | 0.01 | 15.27 | 0.41 | -9.01 | 21.35 | 0.82 | 0.37 | 0.54 | 3.84 | 1.79 | 31.19 | 35 |
| Belgium | 0.33 | 0.09 | 13.54 | 1.67 | -68.53 | 12.46 | 0.00 | 0.99 | 0.49 | 7.78 | 3.13 | 17.81 | 35 |
| Brazil | 0.33 | 0.08 | 14.60 | 0.96 | -62.41 | 9.69 | 0.14 | 0.71 | 0.30 | 5.23 | 1.83 | 15.84 | 20 |
| Canada | 0.34 | 0.01 | 14.74 | 0.36 | -7.22 | 1.19 | 0.85 | 0.36 | 0.27 | 1.80 | 0.75 | 16.08 | 35 |
| Chile | 0.38 | 0.01 | 15.43 | 0.40 | -1.24 | 1.25 | 0.00 | 0.98 | 0.26 | 8.86 | 0.64 | -11.15 | 19 |
| China | 0.39 | 0.01 | 14.92 | 0.48 | -34.14 | 30.01 | 0.07 | 0.80 | 0.45 | 4.00 | 2.38 | 9.48 | 15 |
| Colombia | 0.32 | 0.07 | 15.16 | 1.53 | -76.64 | 38.32 | 0.17 | 0.68 | 0.33 | 5.26 | 1.69 | 18.16 | 20 |
| Costa Rica | 0.53 | 0.13 | 18.49 | 1.19 | 97.73 | 12.16 | 1.45 | 0.23 | 0.61 | 7.78 | 0.82 | -4.52 | 23 |
| Czech Republic | 0.37 | 0.01 | 15.41 | 0.45 | 11.77 | 2.38 | 0.00 | 0.99 | 0.46 | 3.28 | 0.35 | 3.40 | 17 |
| Denmark | 0.34 | 0.01 | 15.21 | 0.39 | -10.90 | 11.59 | 1.21 | 0.27 | 0.51 | 4.75 | 2.21 | 30.60 | 34 |
| Egypt | 0.38 | 0.02 | 15.15 | 0.50 | -29.52 | 2.03 | 0.00 | 0.95 | 0.27 | 16.20 | 3.69 | 17.97 | 20 |
| Finland | 0.37 | 0.01 | 15.89 | 0.53 | 1.79 | 6.02 | 0.04 | 0.84 | 0.66 | 5.99 | 1.28 | -1.84 | 18 |
| France | 0.24 | 0.08 | 5.83 | 12.13 | -59.62 | 16.12 | 0.00 | 0.97 | 0.42 | 6.90 | 2.86 | 15.81 | 35 |
| Germany | 0.31 | 0.02 | 14.88 | 1.68 | -15.48 | 98.84 | 1.06 | 0.30 | 0.51 | 1.32 | 1.39 | 35.70 | 35 |
| Greece | 0.34 | 0.01 | 15.29 | 0.39 | -6.10 | 15.37 | 0.82 | 0.37 | 0.75 | 1.06 | 1.49 | 43.26 | 35 |
| Hong Kong | 0.36 | 0.01 | 15.60 | 0.37 | 2.48 | 0.94 | 0.19 | 0.67 | 0.56 | 2.27 | 0.56 | 0.41 | 28 |
| Hungary | 0.33 | 0.02 | 14.30 | 0.99 | -90.35 | 9.83 | 0.22 | 0.64 | 0.39 | 5.38 | 1.91 | 22.37 | 19 |
| India | 0.18 | 0.01 | 0.05 | 0.23 | -39.20 | 10.14 | 0.19 | 0.67 | 0.23 | 6.41 | 1.81 | 11.22 | 26 |
| Indonesia | 0.37 | 0.00 | 14.60 | 0.10 | -5.37 | 0.00 | 0.24 | 0.62 | 0.26 | 4.11 | 0.49 | 11.30 | 14 |
| Ireland | 0.34 | 0.02 | 15.35 | 0.69 | 41.73 | 113.06 | 0.80 | 0.37 | 0.63 | 2.33 | 0.95 | 20.29 | 35 |
| Israel | 0.34 | 0.01 | 15.26 | 0.37 | -8.16 | 0.66 | 0.82 | 0.37 | 0.45 | 18.92 | 3.49 | 14.47 | 35 |
| Italy | 0.28 | 0.06 | 13.61 | 1.89 | -58.55 | 11.12 | 0.51 | 0.47 | 0.58 | 9.55 | 3.95 | 16.43 | 35 |
| Japan | 0.34 | 0.01 | 15.18 | 0.38 | -6.22 | 7.35 | 0.82 | 0.37 | 0.60 | 2.51 | 1.47 | 20.74 | 35 |
| Malaysia | 0.39 | 0.01 | 15.10 | 0.46 | -6.34 | 7.77 | 0.07 | 0.80 | 0.31 | 7.83 | 1.26 | 4.42 | 15 |
| Mexico | 0.35 | 0.01 | 15.22 | 0.43 | -3.58 | 35.24 | 0.87 | 0.35 | 0.36 | 3.16 | 1.42 | 22.45 | 33 |
| Netherlands | 0.28 | 0.06 | 12.52 | 2.80 | -51.51 | 7.90 | 0.35 | 0.56 | 0.41 | 8.64 | 3.12 | 14.63 | 35 |
| New Zealand | 0.34 | 0.01 | 15.29 | 0.38 | -7.15 | 2.11 | 0.74 | 0.39 | 0.39 | 8.43 | 1.37 | -1.23 | 34 |
| Nigeria | 0.32 | 0.02 | 15.11 | 0.79 | -3.35 | 1.62 | 0.03 | 0.88 | 0.44 | 8.55 | 2.08 | 36.69 | 15 |
| Norway | 0.34 | 0.01 | 15.18 | 0.37 | -8.13 | 0.84 | 0.83 | 0.36 | 0.27 | 4.74 | 1.18 | 6.70 | 35 |
| Philippines | 0.28 | 0.07 | 14.27 | 1.04 | -19.04 | 2.43 | 0.74 | 0.39 | 0.34 | 17.60 | 3.78 | 21.07 | 35 |
| Poland | 0.35 | 0.02 | 14.57 | 0.64 | -101.58 | 6.90 | 0.12 | 0.73 | 0.39 | 5.30 | 1.96 | 22.78 | 19 |
| Portugal | 0.36 | 0.07 | 14.02 | 0.86 | -59.06 | 4.84 | 0.32 | 0.57 | 0.72 | 22.18 | 4.54 | 29.22 | 34 |
| South Africa | 0.30 | 0.02 | 15.32 | 0.97 | -28.02 | 253.07 | 1.08 | 0.30 | 0.56 | 1.22 | 1.61 | 28.58 | 27 |
| Russia | 0.37 | 0.10 | 14.17 | 0.84 | -62.15 | 6.68 | 0.00 | 0.97 | 0.38 | 8.71 | 2.57 | 15.96 | 17 |
| Singapore | 0.33 | 0.07 | 14.29 | 1.10 | -45.36 | 6.32 | 0.02 | 0.90 | 0.42 | 18.12 | 4.06 | 18.36 | 32 |
| South Korea | 0.27 | 0.06 | 14.75 | 1.40 | -23.68 | 4.74 | 0.98 | 0.32 | 0.32 | 9.90 | 2.79 | 8.16 | 32 |
| Spain | 0.34 | 0.01 | 15.32 | 0.37 | 25.35 | 1.88 | 0.81 | 0.37 | 0.42 | 1.08 | 0.86 | -2.35 | 35 |
| Sweden | 0.34 | 0.01 | 15.23 | 0.37 | -7.70 | 0.69 | 0.82 | 0.37 | 0.60 | 7.89 | 2.54 | 7.26 | 35 |
| Switzerland | 0.32 | 0.08 | 13.52 | 1.27 | -52.05 | 4.71 | 0.01 | 0.93 | 0.69 | 18.58 | 4.58 | 23.54 | 35 |
| Taiwan | 0.40 | 0.31 | 23.11 | 5.54 | 126.00 | 2.63 | 0.59 | 0.44 | 0.52 | 8.86 | 0.79 | 4.06 | 35 |
| Thailand | 0.36 | 0.02 | 15.29 | 0.55 | -9.58 | 24.34 | 0.29 | 0.59 | 0.38 | 1.93 | 1.04 | 28.09 | 25 |
| Turkey | 0.20 | 0.08 | 12.37 | 3.76 | -24.22 | 2.81 | 2.05 | 0.15 | 0.51 | 22.62 | 3.88 | 23.99 | 30 |
| United Kingdom | 0.31 | 0.02 | 14.50 | 2.84 | -11.48 | 62.58 | 1.05 | 0.31 | 0.36 | 1.27 | 0.98 | 34.88 | 35 |

B: Cross-country medians of parameters & implied fundamentals for US multinationals' $HQ$, $f$, $ROW$

| | | | | | *Headquarters in the US* | | | | *Affiliates in country f* | | | | *Affiliates in the ROW* | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\kappa$ | $\theta$ | $\nu$ | $p_{\chi^2}$ | *Obs.* | $MPK$ | $\Phi/Y$ | $q$ | $R^I$ | $MPK$ | $\Phi/Y$ | $q$ | $R^I$ | $MPK$ | $\Phi/Y$ | $q$ | $R^I$ |
| 0.339 | 15.11 | -10.24 | 0.52 | 34 | 0.345 | 8.68 | 2.60 | 9.32 | 0.420 | 5.69 | 1.74 | 16.25 | 0.377 | 0.61 | 0.99 | 28.20 |

**Notes:** This table reports the GMM structural estimation results of the three-country production-based model. See econometric methodology details in section 3. Panel A reports the parameter estimates by country of foreign affiliates of US parents. Standard errors are reported in brackets. $\kappa$ is the capital share in output, $\theta$ captures the size of adjustment costs, and $\nu$ the asymmetry in adjustment costs of foreign affiliates. See functional forms and discussion in section 3.1.2. Column $\chi^2$ reports the overidentification test of model fit to the data, with p-values $p_{\chi^2}$. Implied fundamentals by country of foreign affiliates include marginal product of capital (MPK), adjustment costs as a fraction of affiliates' own output $\Phi/Y$ (reported as a percentage ($\times 100$) per annum), marginal $q$s, and returns on investment $R^I$ (reported as a percentage per annum). Column Obs. reports the number of observations in the time series of each aggregate foreign affiliate series with data from 1985 to 2019. Panel B reports median statistics across all 44 countries, and median estimates for the triplet of US-owned business units in $\{US, f, ROW\}$, where "US" stands for the home headquarters (HQ) within US borders, $f$ for foreign affiliates in country *f*, and ROW for foreign affiliates in the rest of the world. See section 3.2 and the Data Appendix IA.B for data sources and further details.

Table 5

**Comparing realized against production-*implied* forecasts of country-level foreign currency appreciation, 1987-2019**

A: Foreign currency appreciation: Panel regressions

| $(e_{t+1}/e_t) - 1$ | Realized | | Implied | | Realized−Implied | |
|---|---|---|---|---|---|---|
| | $\beta$ | $p$ value | $\beta$ | $p$ value | $\beta$ | $p$ value |
| $I/K_{A,f,t} - I/K_{US,t}$ | 0.113*** | 0.000 | 0.080 | 0.499 | 0.033 | 0.739 |
| $I/K_{A,f,t+1} - I/K_{US,t+1}$ | -0.097** | 0.044 | -0.195** | 0.039 | 0.099 | 0.324 |
| Observations | 1,139 | | 1,139 | | 1,139 | |
| *Adj* $R^2$ | 0.41 | | 0.61 | | 0.60 | |
| Year fixed effects | Yes | | Yes | | Yes | |
| Country fixed effects | Yes | | Yes | | Yes | |

B: Foreign currency appreciation: Fama-MacBeth regressions

| $(e_{t+1}/e_t) - 1$ | Realized | | Implied | | Realized−Implied | |
|---|---|---|---|---|---|---|
| | $\beta$ | $p$ value | $\beta$ | $p$ value | $\beta$ | $p$ value |
| $I/K_{A,f,t} - I/K_{US,t}$ | 0.147*** | 0.104 | 0.131† | 0.103 | 0.016 | 0.863 |
| $I/K_{A,f,t+1} - I/K_{US,t+1}$ | -0.102* | 0.069 | -0.130 | 0.247 | 0.028 | 0.808 |
| Observations | 1,139 | | 1,139 | | 1,139 | |
| Average $R^2$ | 0.09 | | 0.10 | | 0.10 | |
| Newey-West SEs (4yrs) | Yes | | Yes | | Yes | |

C: Foreign currency appreciation: Fama-MacBeth regressions

| $(e_{t+1}/e_t) - 1$ | Realized | | Implied | | Realized−Implied | |
|---|---|---|---|---|---|---|
| | $\beta$ | $p$ value | $\beta$ | $p$ value | $\beta$ | $p$ value |
| $I/K_{A,f,t} - I/K_{US,t}$ | 0.146*** | 0.001 | 0.146* | 0.099 | 0.000 | 0.999 |
| $I/K_{A,f,t+1} - I/K_{US,t+1}$ | -0.096* | 0.099 | -0.154 | 0.193 | 0.059 | 0.629 |
| $Y/K_{A,f,t+1} - Y/K_{US,t+1}$ | -0.004 | 0.422 | -0.002 | 0.866 | -0.002 | 0.854 |
| Observations | 1,139 | | 1,139 | | 1,139 | |
| Average $R^2$ | 0.14 | | 0.12 | | 0.14 | |
| Newey-West SEs (4yrs) | Yes | | Yes | | Yes | |

**Notes:** This table compares the realized versus production-implied forecasts of foreign currency real exchange rate growth. Observations are at the country-by-year level, from 1987 to 2019. The column "Realized−Implied" tests whether the coefficients on the realized and model-implied specifications are equal by regressing the difference between the realized and implied series on the stated regressors. These independent variables are either lagged by 1 year or measured contemporaneously to the dependent exchange rate growth rate which is measured from end-of-year $t$ to $t+1$. The regressors include the differential between the investment-to-capital $I/K$ or output-to-capital $Y/K$ ratios of US-owned foreign affiliates in country $f$ and $US$ domestic ratios. The "Panel regressions" include both country and year fixed effects, with standard errors that are double clustered in the time and country dimensions. The "Fama-MacBeth regressions" present results from Fama and MacBeth (1973) cross-sectional regressions, with Newey and West (1987) standard errors that account for heteroskedasticity and autocorrelation in the error structure up to a lag of 4 years. Foreign affiliate aggregates are in foreign currency real terms (e.g., in constant 2015 British pounds for investments in the UK), while HQ data in real US 2015 dollars. ***, **, *, and † denote statistical significance at the 1%, 5%, 10%, and 15% levels, respectively. See section 3.2 and the Data Appendix IA.B for data sources and details.

Table 6

**Pricing the carry trade: comparing realized against production-*implied* currency premia, 1987-2019**

**Panel A:** Regressions of currency excess returns on interest rate differentials

A1. Panel regressions

| $(e_{t+1}/e_t)-(i_t^{US}+1)/(i_t^f+1)$ | Realized | | Implied | | Realized−Implied | |
|---|---|---|---|---|---|---|
| | $\beta$ | $p$ value | $\beta$ | $p$ value | $\beta$ | $p$ value |
| $i_t^f - i_t^{US}$ | 0.332** | 0.012 | 0.216*** | 0.000 | 0.116 | 0.311 |
| Observations | 1,139 | | 1,139 | | 1,139 | |
| *Adj* $R^2$ | 0.42 | | 0.60 | | 0.60 | |
| Year fixed effects | Yes | | Yes | | Yes | |
| Country fixed effects | Yes | | Yes | | Yes | |

A2. Fama-MacBeth regressions

| $(e_{t+1}/e_t)-(i_t^{US}+1)/(i_t^f+1)$ | Realized | | Implied | | Realized−Implied | |
|---|---|---|---|---|---|---|
| | $\beta$ | $p$ value | $\beta$ | $p$ value | $\beta$ | $p$ value |
| $i_t^f - i_t^{US}$ | 0.319** | 0.017 | 0.336** | 0.015 | -0.016 | 0.862 |
| Observations | 1,139 | | 1,139 | | 1,139 | |
| Average $R^2$ | 0.21 | | 0.09 | | 0.05 | |
| Newey-West SEs (4yrs) | Yes | | Yes | | Yes | |

**Panel B:** Portfolio sorts, countries sorted by interest rate differentials

| B1. Returns on carry trade portfolios | Low | 2 | 3 | High | H-L | [$t$] |
|---|---|---|---|---|---|---|
| Realized currency excess returns | -0.10 | 1.65 | 2.93 | 4.38 | 4.48 | 3.83 |
| [$t$] | -0.07 | 0.99 | 2.01 | 3.43 | | |
| Model pricing errors (realized−implied) | -0.20 | 0.13 | 0.29 | -0.82 | -0.62 | -0.50 |
| [$t$] | -0.05 | 0.03 | 0.07 | -0.22 | | |

B2. Fundamental characteristics of carry trade portfolios, descriptive statistics:

| | Low | 2 | 3 | High | H-L | [$t$] |
|---|---|---|---|---|---|---|
| Observable aggregate differentials (in percentage ×100 terms) | | | | | | |
| $i_t^f - i_t^{US}$ | -0.59 | 0.75 | 2.96 | 8.42 | 9.01 | 11.03 |
| $I/K_{A,f,t} - I/K_{US,t}$ | 8.07 | 8.13 | 8.56 | 11.40 | 3.33 | 2.47 |
| $I/K_{A,f,t+1} - I/K_{US,t+1}$ | 8.18 | 8.03 | 7.80 | 11.36 | 3.18 | 2.68 |
| $Y/K_{A,f,t+1} - Y/K_{US,t+1}$ | 51.87 | 46.82 | 13.11 | 20.49 | -31.38 | -2.81 |
| Implied production fundamentals of US foreign affiliates | | | | | | |
| $q_{A,f,t}$ | 1.39 | 1.41 | 1.42 | 2.03 | 0.64 | 1.93 |
| $q_{A,f,t+1}$ | 1.36 | 1.39 | 1.57 | 2.12 | 0.76 | 2.26 |
| $MPK_{A,f,t+1}$ | 0.51 | 0.49 | 0.39 | 0.40 | -0.11 | -4.02 |
| $\Phi/Y_{A,f,t+1}$ | 1.45 | 1.60 | 1.92 | 3.93 | 2.48 | 1.74 |
| $R^I_{A,f,t+1}$ | 31.05 | 28.16 | 19.05 | 25.4 | -5.65 | -0.98 |

B3. Average model fit and implied size of adjustment costs

| $MAE$ | $p_{\chi^2}$ | $\Phi_{HQ}/Y^{US\ total}$ | $\Phi_{A,f}/Y^{US\ total}$ | $\Phi_{A,ROW}/Y^{US\ total}$ | $\Phi^{US\ total}/Y^{US\ total}$ |
|---|---|---|---|---|---|
| 0.36 | 0.99 | 7.73 | 0.09 | 0.08 | 7.90 |

**Notes:** Panel A compares realized versus production-implied forecasts of real currency excess returns to the carry trade trading strategy. Observations are at the country-by-year level from 1987 to 2019. The tests are similar to the panel and Fama-MacBeth regressions described in Table 5. Panel B presents portfolio-level tests, where carry trade portfolios are formed following Lustig and Verdelhan (2007), by sorting countries every year $t$ into 4 portfolios based on end-of-year $t$ interest rate differentials, with annual rebalancing. Average currency excess returns and model pricing errors are reported as a percentage (×100) per annum. t-statistics are Newey and West (1987) robust to heteroskedasticity and autocorrelation in error structure up to a lag of 4 years. Fundamental characteristics include: interest rate differentials (in percentage), differentials in investment-to-capital $I/K$ and output-to-capital $Y/K$ ratios between US foreign affiliates and domestic activity at home. Implied affiliate fundamentals include: average marginal $q$, marginal product of capital (MPK), adjustment costs as a fraction of affiliates' own output $\Phi/Y$ (as a percentage) and returns on investment $R^I$ (as a percentage). Mean absolute pricing errors (MAE) across all portfolios, $p$-value of a $\chi^2$ test for the null hypothesis that all pricing errors are jointly zero, and average size of adjustments costs of each business unit $\{HQ, f, ROW\}$ as a fraction of total US output $Y^{US\ total}$ are at the bottom. See sections 3 and Data Appendix IA.B for details.

Table 7

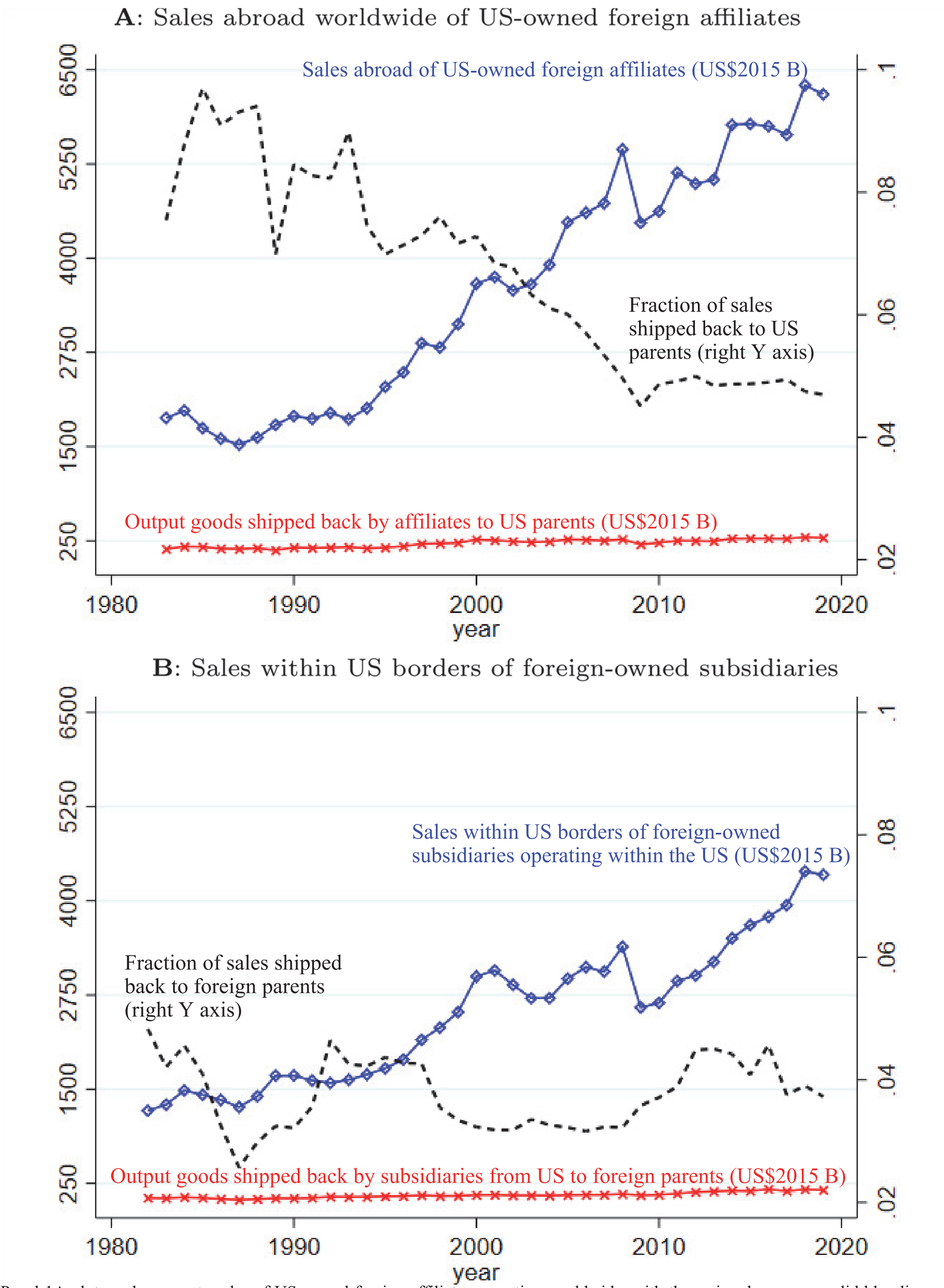
**Comparative statics: pricing the carry trade and quantifying the importance of the production-based fundamentals, 1987–2019**

*Carry trade portfolios*

| | Low | 2 | 3 | High | H-L$^{4p}$ | | H-L$^{5p}$ | | H-L$^{6p}$ | | H-L$^{7p}$ | | H-L$^{8p}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Realized currency excess returns | -0.10 | 1.65 | 2.93 | 4.38 | 4.48 | | 5.65 | | 6.68 | | 7.18 | | 7.54 | |
| Model pricing errors (realized−implied) | Low | 2 | 3 | High | H-L$^{4p}$ | $MAE^{4p}$ | H-L$^{5p}$ | $MAE^{5p}$ | H-L$^{6p}$ | $MAE^{6p}$ | H-L$^{7p}$ | $MAE^{7p}$ | H-L$^{8p}$ | $MAE^{8p}$ |
| Baseline | -0.20 | 0.13 | 0.29 | -0.82 | -0.62 | 0.36 | 0.51 | 0.85 | 1.36 | 1.09 | 0.40 | 1.47 | 0.27 | 1.50 |
| $\overline{I/K_{A,f,t}}$ | -9.82 | -7.18 | -6.28 | -4.40 | 5.42 | 6.92 | 5.23 | 6.93 | 6.32 | 6.91 | 5.68 | 6.92 | 5.79 | 6.87 |
| $\overline{I/K_{A,f,t+1}}$ | 12.53 | 7.98 | 7.38 | 3.78 | -8.75 | 7.92 | -7.32 | 7.94 | -6.67 | 7.99 | -6.53 | 7.92 | -7.59 | 7.89 |
| $\overline{Y/K_{A,f,t+1}}$ | -0.13 | 0.53 | 0.27 | -0.67 | -0.54 | 0.40 | 0.64 | 0.82 | 1.46 | 0.98 | 0.91 | 1.19 | 0.83 | 1.20 |
| $\overline{I/K_{US,t}}$ | 1.25 | 1.09 | 1.49 | 0.15 | -1.10 | 0.99 | -0.94 | 0.99 | -0.30 | 1.23 | -1.01 | 1.27 | -1.07 | 1.33 |
| $\overline{I/K_{US,t+1}}$ | 0.74 | 0.40 | 1.41 | 0.73 | -0.01 | 0.82 | 0.31 | 0.93 | 0.84 | 1.48 | 0.09 | 1.41 | -0.16 | 1.37 |
| $\overline{Y/K_{US,t+1}}$ | -0.61 | -0.43 | 0.32 | -0.31 | 0.30 | 0.42 | 0.70 | 0.89 | 1.01 | 1.24 | -0.23 | 1.59 | -0.36 | 1.64 |
| $\overline{I/K_{A,ROW,t}}$ | -16.41 | -15.04 | -14.7 | -17.84 | -1.43 | 16.00 | -1.56 | 15.98 | -0.59 | 16.01 | -0.99 | 15.94 | -0.77 | 15.96 |
| $\overline{I/K_{A,ROW,t+1}}$ | 20.2 | 19.02 | 21.18 | 23.02 | 2.82 | 20.85 | 3.71 | 20.91 | 4.28 | 20.90 | 4.19 | 20.87 | 3.94 | 20.84 |
| $\overline{Y/K_{A,ROW,t+1}}$ | -0.44 | 0.07 | 0.29 | -0.84 | -0.39 | 0.41 | 0.49 | 0.89 | 1.33 | 1.07 | 0.37 | 1.53 | 0.24 | 1.54 |
| $\overline{R_{t+1}^{US\_Market}}$ | 5.43 | 5.23 | 5.87 | 4.38 | -1.05 | 5.23 | -1.30 | 5.21 | 0.17 | 5.31 | -0.98 | 5.22 | -1.40 | 5.31 |
| $\overline{All}$ | 24.73 | 25.29 | 25.51 | 25.86 | 1.13 | 25.35 | 1.92 | 25.36 | 2.55 | 25.37 | 3.05 | 25.36 | 3.16 | 25.35 |

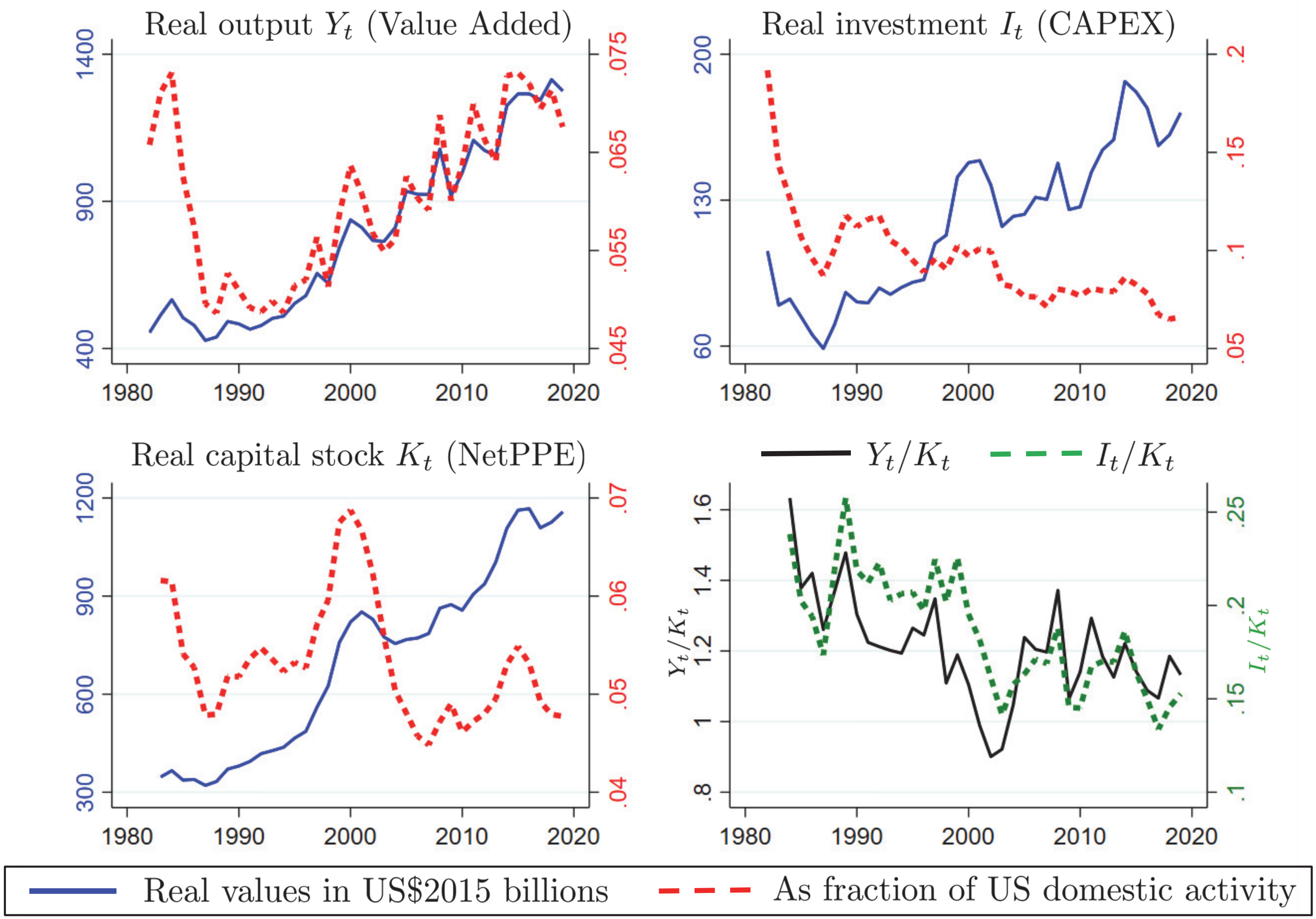
**Notes:** This table reports pricing errors of the production-based asset pricing model from eleven comparative statics. Each experiment is labelled with an overbar, starting with experiment $\overline{I/K_{A,f,t}}$ and ending with $\overline{All}$. Each experiment shuts down one by one the role of a given input to assess its quantitative importance for the performance of the model in fitting currency data. If the resultant pricing errors of the given comparative static are larger than those of the "baseline" model, then the given input is inferred to be quantitatively important in capturing cross-sectional variation in average carry trade currency premia. The experiment $\overline{I/K_{A,f,t}}$ shuts down any cross-affiliate variation in $I/K_{A,f,t}$ ratios by fixing the ratio for all affiliates and every point in time to its long-run steady investment ratio. The other experiments are designed analogously. In each experiment, I use the parameter estimates from the baseline estimation to reconstruct the implied exchange rate growth rates and recalculate the model's pricing errors. This allows direct comparison with the pricing errors of the "baseline" specification. Experiment $\overline{All}$ shuts down the collective joint role of all 10 production-based inputs. The test assets are at the portfolio level of countries and for forecasts of real currency excess returns. The average currency excess returns from the carry trade are reported in the top row (as a percentage ($\times$100) per annum) for the 4 carry trade benchmark portfolios shown earlier in Table 6, along with the average H-L spread across the 4 portfolios (denoted H-L$^{4p}$) and H-L spreads across 5 to 8 carry trade portfolios. The H-L portfolio is the excess return payoff to a US investor in the carry trade (long in high and short in low foreign interest rates). To assess whether the pricing errors of the comparative statics are larger or not relative to the baseline, the first row "baseline" under model pricing errors reports the baseline model pricing errors (reported as a percentage ($\times$100) per annum) and presented earlier in Table 6, and include the individual pricing errors for each of the 4 benchmark portfolios, the pricing error of the H-L spread portfolio (denoted H-L$^{4p}$), and the MAE across all 4 baseline test assets (denoted MAE$^{4p}$). The H-L and MAE pricing error statistics for the case of 5 to 8 carry trade portfolios are reported, accordingly, and used to assess model fit to the different choices of portfolios. In all comparisons the realized and implied forecasts of real currency excess employ data from 1987 to 2019. Foreign affiliate aggregates are in foreign currency real terms (e.g., in constant 2015 British pounds for investments in the UK), while HQ data are in real US 2015 dollar figures. See sections 3 and 4.4.1 for estimation methodology and details.

**Figure 1: The large output worldwide of US-owned foreign affiliates 1983-2019 & the fraction of output shipped back to US parents (real US$2015 billions)**



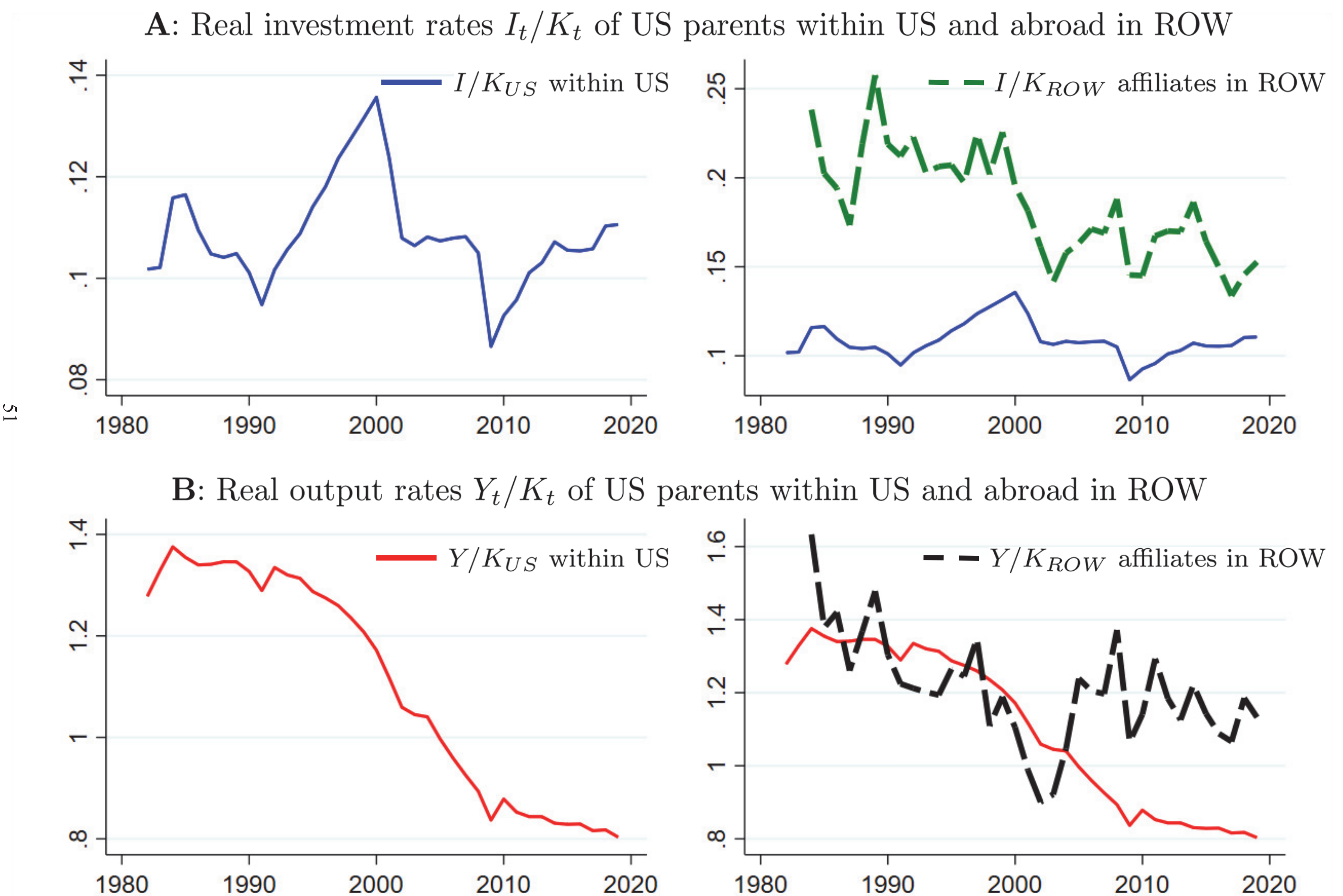

**Notes**: Panel 1A plots real aggregate sales of US-owned foreign affiliates operating worldwide, with the series shown as a solid blue line with diamond markers. For presentational purposes, foreign affiliate amounts are shown in constant 2015 US dollars, yet are always measured in real foreign currency values in all tests of the paper. The red solid line with X markers denotes total real aggregate output goods shipped back to the US parents by worldwide foreign affiliates. The black dashed line denotes the fraction of total foreign affiliate sales shipped back to US parents. Panel 1B is similar but from the perspective of US-located affiliates that are owned by foreign parents. Therefore, the black solid line in Panel 1B denotes the fraction of sales shipped back from the US to their foreign parents by US-located affiliates. Annual data are from 1983 to 2019 from the Bureau of Economic Analysis (BEA) dataset on Activities rises (MNEs). See data section for details.

## Figure 2: Output, investment, & capital stocks abroad worldwide of US-owned foreign affiliates 1983-2019



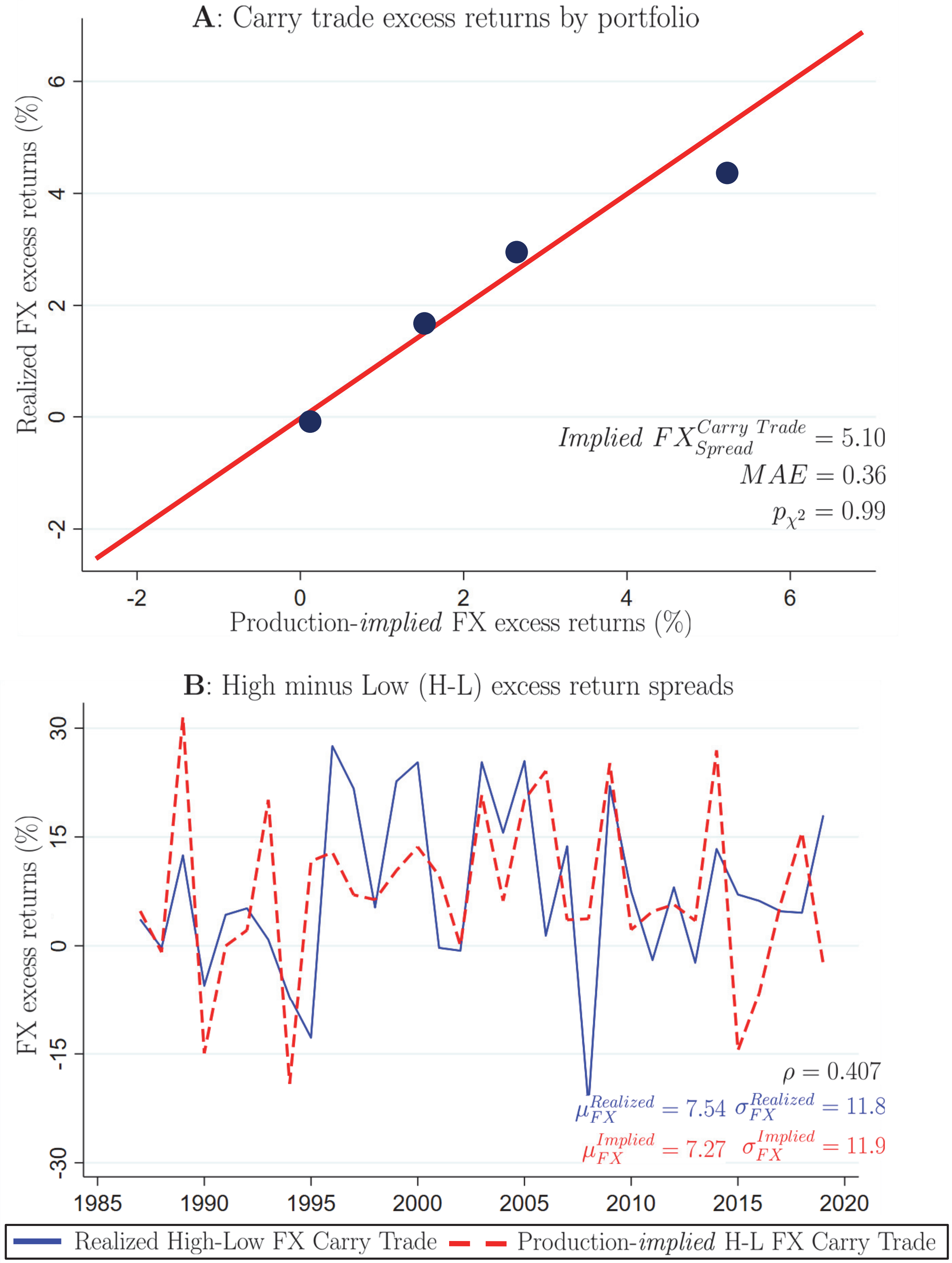

*Notes*: Aggregate real production quantities abroad *worldwide* of *US-owned foreign affiliates*. Value added, investment expenditures in physical capital, and capital stocks (net property plant and equipment) of foreign affiliates are in solid blue lines (left Y axis) from 1983 to 2019. For presentational purposes, these are shown in constant 2015 US dollars, yet are always measured in real foreign currency values in all tests of the paper. The red dashed line for value added (right Y axis) shows foreign affiliate value added as a fraction of total US domestic output created within US borders and exclusively by US-owned firms (i.e., the denominator is US GDP net of the value added created within US borders by subsidiaries owned by foreign parents). Other fractions are analogously created using exclusively US-owned domestic investment and capital at home in the denominators. The bottom right panel plots foreign affiliate output-to-capital (solid black line left Y axis) and investment-to-capital ratios (dashed green right Y axis).

**Figure 3: Investment-to-capital & output-to-capital ratios of US-owned companies: at home within US borders & abroad in the rest of the world (ROW), 1983-2019**




***Notes***: Panel A at the top presents in solid blue line the aggregate domestic investment-to-capital ratio (I/K) measured within US borders of firms exclusively owned by US parents. The series excludes any investment flows and capital stocks measured within US borders but owned by foreign parents. The green dashed line in the top right panel plots the aggregate investment-to-capital ratio of foreign affiliates located abroad and owned by US parents (this series is the same as shown and described in figure 2 bottom right panel). Panel B at the bottom presents instead value-added-to-capital-ratios (Y/K) of US parents, with the red solid line measuring output-to-capital ratios at home within US borders while in black dashed line of foreign affiliates owned by US parents. Data are annual from 1983 to 2019 and in real terms.

**Figure 4: Carry trade realized vs. production-implied currency excess returns, 1987-2019**




***Notes***: Panel A presents the average predicted vs. realized foreign exchange (FX) excess returns to the carry trade trading strategy. Returns are expressed as a percentage per annum and are reported for each of the 4 benchmark portfolios. Reported statistics include the average carry trade spread implied by the model between the high and low (H-L) portfolios, the mean absolute pricing error (MAE), and *p*-value of a *Chi*-squared test of joint significance for the null that all portfolio pricing errors are jointly zero. Panel B presents the excess return H-L spreads over time from sorting countries into 8 carry trade portfolios every year. The choice of 8 carry trade portfolios is because these yield the largest average spreads in the sample period. Reported statistics include the time series *correlation* between the realized and model-implied H-L spreads, and the *mean* and *standard deviation* of each series. Countries are sorted into portfolios every year following the methodology in Lustig and Verdelhan (2007). See econometric methodology section for details.

# Internet Appendix "Production-Based Exchange Rates: The $q$-Theory of Multinationals"

## (for Online Publication Only)

Iván Alfaro

October 2024

This Internet Appendix includes: [A] GMM Appendix for the moment conditions, [B] Data Appendix, [C] Appendix for supplementary results.

## IA.A GMM Appendix: Moment Conditions

From (19) and imposing functional forms, the bilateral exchange rate growth moment is

$$E\left[\frac{e_{f,t+1}}{e_{f,t}}-\frac{\left[\begin{array}{c} R^{S,h}_{t+1}\{([\theta^h_{HQ}\left(\frac{I^h_{HQ,t}}{K^h_{HQ,t}}\right)+1]K^h_{HQ,t+1}) \\ +e_{f,t}([\frac{\theta^h_{A,f}}{\nu^h_{A,f}}\left\{-\exp\left[-\nu^h_{A,f}\left(\frac{I^h_{A,f,t}}{K^h_{A,f,t}}-\overline{\frac{I^h_A}{K^h_A}}\right)\right]+1\right\}+1]K^h_{A,f,t+1}) \\ +e_{ROW,t}([\frac{\theta^h_{A,ROW}}{\nu^h_{A,ROW}}\left\{-\exp\left[-\nu^h_{A,ROW}\left(\frac{I^h_{A,ROW,t}}{K^h_{A,ROW,t}}-\overline{\frac{I^h_A}{K^h_A}}\right)\right]+1\right\}+1]K^h_{A,ROW,t+1})\} \\ -\{\kappa^h_{HQ}Y^h_{HQ,t+1}-\frac{\theta^h_{HQ}}{2}\left(\frac{I^h_{HQ,t+1}}{K^h_{HQ,t+1}}\right)^2K^h_{HQ,t+1}+\theta^h_{HQ}\left(\frac{I^h_{HQ,t+1}}{K^h_{HQ,t+1}}\right)I^h_{HQ,t+1} \\ +[\theta^h_{HQ}\left(\frac{I^h_{HQ,t+1}}{K^h_{HQ,t+1}}\right)+1](1-\delta^h_{HQ,t+1})K^h_{HQ,t+1}\}-e_{ROW,t+1}\{\kappa^h_{A,ROW}Y^h_{A,ROW,t+1} \\ -\frac{\theta^h_{A,ROW}}{(\nu^h_{A,ROW})^2}\left\{\begin{array}{c}\exp\left[-\nu^h_{A,ROW}\left(\frac{I^h_{A,ROW,t+1}}{K^h_{A,ROW,t+1}}-\overline{\frac{I^h_A}{K^h_A}}\right)\right] \\ +\nu^h_{A,ROW}\left(\frac{I^h_{A,ROW,t+1}}{K^h_{A,ROW,t+1}}-\overline{\frac{I^h_A}{K^h_A}}\right)-1\end{array}\right\}K^h_{A,ROW,t+1} \\ +\frac{\theta^h_{A,ROW}}{\nu^h_{A,ROW}}\left\{-\exp\left[-\nu^h_{A,ROW}\left(\frac{I^h_{A,ROW,t+1}}{K^h_{A,ROW,t+1}}-\overline{\frac{I^h_A}{K^h_A}}\right)\right]+1\right\}I^h_{A,ROW,t+1} \\ +[\frac{\theta^h_{A,ROW}}{\nu^h_{A,ROW}}\left\{-\exp\left[-\nu^h_{A,ROW}\left(\frac{I^h_{A,ROW,t+1}}{K^h_{A,ROW,t+1}}-\overline{\frac{I^h_A}{K^h_A}}\right)\right]+1\right\}+1] \\ (1-\delta^h_{A,ROW,t+1})K^h_{A,ROW,t+1}\} \end{array}\right]}{\left[\begin{array}{c} e_{f,t}\{\kappa^h_{A,f}Y^h_{A,f,t+1}-\frac{\theta^h_{A,f}}{(\nu^h_{A,f})^2}\left\{\begin{array}{c}\exp\left[-\nu^h_{A,f}\left(\frac{I^h_{A,f,t+1}}{K^h_{A,f,t+1}}-\overline{\frac{I^h_A}{K^h_A}}\right)\right] \\ +\nu^h_{A,f}\left(\frac{I^h_{A,f,t+1}}{K^h_{A,f,t+1}}-\overline{\frac{I^h_A}{K^h_A}}\right)-1\end{array}\right\}K^h_{A,f,t+1} \\ +\frac{\theta^h_{A,f}}{\nu^h_{A,f}}\left\{-\exp\left[-\nu^h_{A,f}\left(\frac{I^h_{A,f,t+1}}{K^h_{A,f,t+1}}-\overline{\frac{I^h_A}{K^h_A}}\right)\right]+1\right\}I^h_{A,f,t+1} \\ +[\frac{\theta^h_{A,f}}{\nu^h_{A,f}}\left\{-\exp\left[-\nu^h_{A,f}\left(\frac{I^h_{A,f,t+1}}{K^h_{A,f,t+1}}-\overline{\frac{I^h_A}{K^h_A}}\right)\right]+1\right\}+1](1-\delta^h_{A,f,t+1})K^h_{A,f,t+1}\} \end{array}\right]}\right]=0 \tag{IA.1}$$

The second moment condition tests if the average Tobin's $q^{US}$ of US firms observed in the aggregate data (see data section 3.2) equals the average $q^h$ predicted by the three-country production model:

$$E\left[q_t^{US} - \left(\frac{\begin{array}{c}([\theta_{HQ}^h\left(\frac{I_{HQ,t}^h}{K_{HQ,t}^h}\right)+1]K_{HQ,t+1}^h) \\ +e_{f,t}([\frac{\theta_{A,f}^h}{\nu_{A,f}^h}\left\{-\exp\left[-\nu_{A,f}^h\left(\frac{I_{A,f,t}^h}{K_{A,f,t}^h}-\overline{\frac{I_A^h}{K_A^h}}\right)\right]+1\right\}+1]K_{A,f,t+1}^h) \\ +e_{ROW,t}([\frac{\theta_{A,ROW}^h}{\nu_{A,ROW}^h}\left\{-\exp\left[-\nu_{A,ROW}^h\left(\frac{I_{A,ROW,t}^h}{K_{A,ROW,t}^h}-\overline{\frac{I_A^h}{K_A^h}}\right)\right]+1\right\}+1]K_{A,ROW,t+1}^h)\end{array}}{A_{t-1}^h}\right)\right]=0 \tag{IA.2}$$

The third moment tests whether the average ex-dividend US stock market return observed in the data (i.e., capital gains $P_{t+1}^{US}/P_t^{US}$ from CRSP) equals the average predicted ex-dividend US market return:[35]

$$E\left[\frac{P_{t+1}^{US}}{P_t^{US}} - \left(\frac{\begin{array}{c}([\theta_{HQ}^h\left(\frac{I_{HQ,t+1}^h}{K_{HQ,t+1}^h}\right)+1]K_{HQ,t+2}^h) \\ +e_{f,t+1}([\frac{\theta_{A,f}^h}{\nu_{A,f}^h}\left\{-\exp\left[-\nu_{A,f}^h\left(\frac{I_{A,f,t+1}^h}{K_{A,f,t+1}^h}-\overline{\frac{I_A^h}{K_A^h}}\right)\right]+1\right\}+1]K_{A,f,t+2}^h) \\ +e_{ROW,t+1}([\frac{\theta_{A,ROW}^h}{\nu_{A,ROW}^h}\left\{-\exp\left[-\nu_{A,ROW}^h\left(\frac{I_{A,ROW,t+1}^h}{K_{A,ROW,t+1}^h}-\overline{\frac{I_A^h}{K_A^h}}\right)\right]+1\right\}+1]K_{A,ROW,t+2}^h)\end{array}}{\begin{array}{c}([\theta_{HQ}^h\left(\frac{I_{HQ,t}^h}{K_{HQ,t}^h}\right)+1]K_{HQ,t+1}^h) \\ +e_{f,t}([\frac{\theta_{A,f}^h}{\nu_{A,f}^h}\left\{-\exp\left[-\nu_{A,f}^h\left(\frac{I_{A,f,t}^h}{K_{A,f,t}^h}-\overline{\frac{I_A^h}{K_A^h}}\right)\right]+1\right\}+1]K_{A,f,t+1}^h) \\ +e_{ROW,t}([\frac{\theta_{A,ROW}^h}{\nu_{A,ROW}^h}\left\{-\exp\left[-\nu_{A,ROW}^h\left(\frac{I_{A,ROW,t}^h}{K_{A,ROW,t}^h}-\overline{\frac{I_A^h}{K_A^h}}\right)\right]+1\right\}+1]K_{A,ROW,t+1}^h)\end{array}}\right)\right]=0 \tag{IA.3}$$

The fourth moment tests whether the average aggregate US cash-dividend-to-price ratio (i.e., $D_{t+1}^{US}/P_t^{US}$ measurable from CRSP return data) equals the average dividend yield $D_{t+1}^h/P_t^h$

[35]This third moment is different from the second one because the former is asking the model the tough hurdle of pricing the aggregate US market value jointly at two different periods of time $t$ and $t+1$ to construct a return that is consistent with data. The model can succeed at, say, pricing the average Tobin's $q$ of the US productive sector but fail (quantitatively or by means of delivering implausible estimates) in matching capital gains in CRSP records, and vice versa, hence not a redundant moment.

predicted by the model:

$$E\left[\frac{D_{t+1}^{US}}{P_t^{US}}-\left(\frac{\begin{array}{c}\kappa_{HQ}^h Y_{HQ,t+1}^h-\frac{\theta_{HQ}^h}{2}\left(\frac{I_{HQ,t+1}^h}{K_{HQ,t+1}^h}\right)^2 K_{HQ,t+1}^h-I_{HQ,t+1}^h+e_{f,t+1}\{\kappa_{A,f}^h Y_{A,f,t+1}^h\\ -\frac{\theta_{A,f}^h}{(\nu_{A,f}^h)^2}\left\{\begin{array}{c}\exp\left[-\nu_{A,f}^h\left(\frac{I_{A,f,t+1}^h}{K_{A,f,t+1}^h}-\overline{\frac{I_A^h}{K_A^h}}\right)\right]\\ +\nu_{A,f}^h\left(\frac{I_{A,f,t+1}^h}{K_{A,f,t+1}^h}-\overline{\frac{I_A^h}{K_A^h}}\right)-1\end{array}\right\}K_{A,f,t+1}^h-I_{A,f,t+1}^h\}\\ +e_{ROW,t+1}\{\kappa_{A,ROW}^h Y_{A,ROW,t+1}^h-\frac{\theta_{A,ROW}^h}{(\nu_{A,ROW}^h)^2}\\ \left\{\begin{array}{c}\exp\left[-\nu_{A,ROW}^h\left(\frac{I_{A,ROW,t+1}^h}{K_{A,ROW,t+1}^h}-\overline{\frac{I_A^h}{K_A^h}}\right)\right]\\ +\nu_{A,ROW}^h\left(\frac{I_{A,ROW,t+1}^h}{K_{A,ROW,t+1}^h}-\overline{\frac{I_A^h}{K_A^h}}\right)-1\end{array}\right\}K_{A,ROW,t+1}^h-I_{A,ROW,t+1}^h\}\end{array}}{\begin{array}{c}([\theta_{HQ}^h\left(\frac{I_{HQ,t}^h}{K_{HQ,t}^h}\right)+1]K_{HQ,t+1}^h)\\ +e_{f,t}([\frac{\theta_{A,f}^h}{\nu_{A,f}^h}\left\{-\exp\left[-\nu_{A,f}^h\left(\frac{I_{A,f,t}^h}{K_{A,f,t}^h}-\overline{\frac{I_A^h}{K_A^h}}\right)\right]+1\right\}+1]K_{A,f,t+1}^h)\\ +e_{ROW,t}([\frac{\theta_{A,ROW}^h}{\nu_{A,ROW}^h}\left\{-\exp\left[-\nu_{A,ROW}^h\left(\frac{I_{A,ROW,t}^h}{K_{A,ROW,t}^h}-\overline{\frac{I_A^h}{K_A^h}}\right)\right]+1\right\}+1]K_{A,ROW,t+1}^h)\end{array}}\right)\right]=0 \tag{IA.4}$$

## IA.B Data Appendix: Additional Data Details

Empirical results and GMM model estimations are from the perspective of a US investor. For home headquarters $HQ$ operations, aggregate US data is from the National Income and Product Accounts (NIPA) tables, available from the BEA. Data are annual. For the $HQ$ aggregates, I download nominal aggregates from the NIPA tables, adjust those aggregates for the operations of firms in the US but owned by foreign parents, and then deflate accordingly using the US CPIs to constant billions of US$2015 real values. I use CPIs to deflate US aggregates for consistency with the use of CPIs used everywhere else to deflate foreign aggregates. However, I have also used the BEA implicit price deflators for the US aggregates and have gotten similar results. For the US aggregate, the nominal annual private fixed nonresidential investment $I^{US}$ is from Table 1.1.5 Gross Domestic Product line 9. The annual private nonresidential capital stock at year-end prices, $K^{US}$ is from Table 1.1 Fixed Assets and Consumer Durable Goods line 4. Output is measured as Gross Domestic Product, $Y^{US}$ and from Table 1.5.5 Gross Domestic Product line 1. When used in robustness tests, nominal annual private nonresidential depreciation, $D_\delta^{US}$ is from Table 1.3 Fixed Assets and Consumer Durable Goods line 4.

The preceding NIPA US aggregates include the economic activity of firms within US borders but owned by foreign parents. From a US home perspective firm, in the investment model of section 2, the home aggregate economy does not include those economic activities, as they are part of the foreign representative firm. Therefore, I use BEA data on the activities of US affiliates of foreign multinational enterprises to adjust US aggregates (this data ranges back to 1977). In particular, all nominal US $HQ$ aggregate values are net of aggregate nominal values of all affiliates of foreign parents operating within the US. Thus, home output (as measured by gross product) is $Y^{HQ} = Y^{US} - Y^{World}_{A,US}$. Home investment is $I^{HQ} = I^{US} - I^{World}_{A,US}$, home nonresidential capital stock is $K^{HQ} = K^{US} - K^{World}_{A,US}$, nonresidential depreciation is $D^{HQ}_{\delta} = D^{US}_{\delta} - D^{World}_{\delta,A,US}$. Thereby, aggregate $HQ$ data exclude all economic activity of non-US-owned firms.

As for the operations abroad, aggregate foreign affiliate data of US parents are from the BEA data on the activities of US multinational enterprises. By law, all US legal entities (including firms and individuals), and referred to as "US persons", who own 10% or more of a foreign business are mandated to respond to BEA data surveys. These surveys are also mandatory for foreign legal entities ("foreign persons") that own 10% or more of a US legal entity. Affiliate statistics cover the entirety of operations, irrespective of the percentage of ownership. The aggregate foreign affiliate data at the country level of operations and I use data for majority-owned foreign affiliates (referred to as "MOFAs" by the BEA). These affiliate annual data include capital expenditures $I^{h}_{A,f}$, value added $Y^{h}_{A,f}$, net property plant and equipment $K^{h}_{A,f}$, employment, sales, research and development, and other accounts. Moreover, I restrict the data to majority-owned nonbank affiliates of US parents. The first year of foreign affiliate data for any variable is 1977, but then there is a gap for most variables and countries from 1978 through 1982. Thus, the first year of contiguous required data for the affiliate sample starts in 1983.[36] To create $ROW$ affiliate aggregates at every year $t$, I use predetermined foreign affiliate capital weights (details below) that are measured with a lag of two years in year $t-2$. Hence, foreign affiliate aggregate variables required for the GMM estimation start in 1985, and give a total of up to 35 years (1985–2019) of observations per country with affiliate data. Moreover, although an unbalanced panel of more than 75 countries is available, I restrict the sample to countries with at least 10 years of required affiliate data, which combined with other data

[36]Capital investment expenditures $I$ is continuously observable for foreign affiliates since 1977, but the capital stock data $K$ start only in 1983. Data for other inputs include, for example, R&D expenditures data for intangibles (available starting in 1989) and employee numbers (from 1983). My focus is only on physical capital (e.g., Cochrane (1991)) as a first approach to exchange rates, but the framework is adaptable to incorporate other inputs, which may improve fit to the data (e.g., Belo et al. (2022) in closed economies). I abstract from those additional complexities for parsimony and to avoid other aspects, e.g., R&D flow data start only after 1989 and creating intangible stocks adds assumptions, such as the perpetual inventory method, which are better left abstracted from at this stage of the research.

requirements such as interest rates, give a final and large cross-section of 44 foreign countries.

It is important to note how these multinational production (MP) data differ from FDI. Early literature predominantly utilized data on FDI when exploring multinationals, likely due to its ready availability for most countries. However, for measuring the actual *real* production activity of affiliates and its contribution to the value maximization problem of the whole firm, MP flows are a more appropriate empirical focus than FDI. This is because FDI represents a financial category in the Balance of Payments and is just one possible means through which affiliates finance their activities abroad. What is crucial for this paper is the *real* investment, output, and installed capital stocks of US foreign affiliates, irrespective of the financing method. For instance, if the real investment needed to expand or establish a foreign affiliate is funded from local sources (e.g., local debt), the investment would *not* be reflected in FDI flows but would appear in the MP investment flow data used in this study. The sales and capital stocks arising from those investments, both of which contribute to the total value of the MNE, are also duly accounted for in the MP data—thus, they are measured consistently with how those series are treated in the MNE $q$-theory framework.

*Measurement.* In the model, time $t$ stock variables are at the beginning of period $t$, and time $t$ flow variables are over the course of period $t$. However, in accounting records, both stock and flow variables are recorded at the end of period $t$. Therefore, as is standard and following Liu et al. (2009) and Belo et al. (2013), I adjust the timing such that, for example for the year 2010, I take time $t$ stock variables from the 2009 balance sheet, and time $t$ flow variables from the 2010 income statement. For all business units at home and abroad, investment $I$ is capital expenditures, output $Y$ is value added, and capital $K$ is net property plant and equipment. US home $HQ$ aggregates are adjusted to exclude contributions from firms operating within US borders but owned by foreign parents, e.g., deducting from US GDP figures all non-US-owned value added. Affiliate depreciation data include only total historical accumulated depreciation aggregates. Although annual depreciation flows can be inferred from these series, for simplicity, I follow Cochrane (1991, 1996) and fix the depreciation rate of capital $\delta$ when structurally estimating parameters, with $\delta$ equal to 7.2% per annum for all business units.[37] In the model, home firm Tobin's $q$ is the ratio of the total market value of the representative firm over the book value of its capital asset. In the data, US aggregate Tobin's $q_t^{US}$ is levered and

[37] Caselli and Feyrer (2007) assume 6% to infer annual capital stocks for a similarly large cross-section of countries from aggregate investment data, employing the perpetual inventory method. Merz and Yashiv (2007) estimate a 6.4% per annum (1.6% quarterly) depreciation rate from US data. Minor variations in the fixed depreciation rate yield similar results.

measured as the sum of the total market value of firms from CRSP and total credit to the private nonfinancial business sector (CRDQUSAPABIS from FRED) divided by the lagged total nonfinancial assets of the US corporate business (TABSNNCB).

The cum-dividend and ex-dividend stock returns are from the CRSP value-weighted portfolio, VWRET and VWRETX, respectively. I use monthly returns and compound them over a 12-month window to create annual returns. Moreover, as highlighted by Cochrane (1991, 1996) the stock return data are point-to-point (e.g., December to December) while the model's implied return is based on investment over the year (with a timing at about the middle of July of the year $t$). Hence, to adjust for this timing difference, I follow Cochrane (1996) and average monthly stock returns over the year such that they correspond with the timing of the model's implied return. That is, year $t$ stock return is an average of several 12-month rolling window compounded returns. In particular, I average the 12-month compounded returns ending in January $t$, February $t$, ..., December $t$, to create the annual year $t$ aggregate US stock return $R_t^{S,US}$ which is roughly timed at about the middle of the year and matches well the middle of the year $t$ annual implied return from the model. See appendix in Cochrane (1996) for details. I do this timing adjustment for both the annual cum-dividend and the ex-dividend US stock returns. The dividend-to-lagged-price ratio $\frac{D_{t+1}^{US}}{P_t^{US}}$ is then constructed by taking the difference between the above annual cum-dividend and ex-dividend returns, which follows from CRSP return definitions: $VWRET - VWRETX = (p_{t+1} + d_{t+1})/p_t - p_{t+1}/p_t = d_{t+1}/p_t$. Following Cochrane (1996), I scale this aggregate raw dividend-to-price ratio by adding 0.04, which I find improves the model's fit to the aggregate data—Cochrane (1996) also scales the raw aggregate dividend-to-price ratio by exactly 0.04.

For comparability with the original annual carry trade results in Lustig and Verdelhan (2007), I closely follow their work when measuring exchange rates, applying sample filters, and obtaining interest rate data. Accordingly, in each country, the exchange rate for year $t$ is the average daily exchange rate in December. Similarly, all CPIs are from the last month of the year, with 2015 serving as the base year, and are used to deflate aggregate quantities, exchange rates, interest rates, and annual stock market returns. Monthly CPIs and nominal exchange rate data from the International Monetary Fund's (IMF) International Financial Statistics (IFS). Bilateral real exchange rates $e_{f,t}$ are between the US and country $f$, and defined as the exchange rate in real US dollars per unit of real foreign currency, with an increase in $e_{f,t}$ denoting a real appreciation of the foreign currency. Interest rate data are from

Global Financial Data. For each year $t$, interest rates are the average interest rate in December, and use three-month government securities (e.g., US T-Bills) when available and six-month maturities for the very few cases when not. Foreign countries appearing in the sample host US affiliates that, in aggregate, report billions of US dollars in output, investment, and capital stocks (statistics below), indicating that these countries are relatively open in their capital accounts. I follow Lustig and Verdelhan (2007) in applying a filter based on the degree of a country's capital account openness, and use the Chinn–Ito index (KAOPEN) from Chinn and Ito (2006). This country-by-year index assesses the degree of a country's capital account liberalization and ranges from 0 to 1. The baseline cutoff is set at $\geq 0.10$, where country-year observations below this value may represent limitations to payments and receipts.

The real exchange rate of the ROW is calculated as a geometrically weighted average of bilateral real exchange rates in the set *ROW*. This definition follows the US Federal Reserve Board in constructing several of its indexes of foreign exchange (e.g., the real trade-weighted broad US dollar index).[38] Let the $ROW$ real exchange rate index at time $t$

$$e_{ROW,t} = e_{ROW,t-1} \times \prod_{j}^{ROW(t)} \left( \frac{e_{j,t}}{e_{j,t-1}} \right)^{w_{j,t}} \tag{IA.5}$$

where $e_{j,t}$ is the real exchange rate of country $j$ at $t$, $w_{j,t}$ is its weight at $t$, and $\sum_j w_{j,t} = 1$. The set of countries in the $ROW$ and their weights is based on the observed aggregate size of capital stocks of US foreign affiliates, which collectively account for a substantial portion of all US physical capital abroad (e.g., $\geq 90\%$). This novel capital-weighted index of foreign currencies is analogous to the trade-weighted broad US Fed index, but instead of being weighted by trade activity of commercial partners, it is weighted by US installed production capacity abroad (the correlation in their annual growth rates is 96%). The set $ROW$ consists of the 25 largest affiliate capital stocks with required data since the 1980s. Lastly, ROW aggregates for $\frac{I^h_{A,ROW,t}}{K^h_{A,ROW,t}}$ and $\frac{Y^h_{A,ROW,t}}{K^h_{A,ROW,t}}$ ratios are similarly weighted averages that use the capital-based weights.[39]

When constructing the ROW exchange rate in equation (IA.5) applicable to a given country $f$, the set of countries in the ROW always excludes country $f$'s contribution to the index (e.g., uses data only on 24 countries from ROW set of 25). For countries not included in the baseline ROW set of 25 countries, the index $e_{ROW,t}$ is taken as given from the full set of countries without adjustment. The qualitative results are unchanged and quantitatively vary little if the

[38] See: http://www.federalreserve.gov/pubs/bulletin/2005/winter05_index.pdf.

[39] Working with US firm-level data, Gonçalves et al. (2020) and Belo et al. (2022) suggest that when aggregating to the portfolio-level, it is econometrically preferable to construct portfolio aggregates of $I/K$ and $Y/K$ ratios by averaging across firms' individual ratios. I do analogously here using country-level data to form the ROW aggregate economy.

set of countries in the ROW is expanded to the top 35, use all 44, or use Fed trade weights.

National accounts data for reduced-form are from the World Bank. Consumption final expenditure annual % growth (NE.CON.TOTL.KD.ZG), GDP annual % growth (NY.GDP.MKTP.KD.ZG), gross fixed capital formation annual % growth (NE.GDI.FTOT.KD.ZG), gross (total) capital formation annual % growth (NE.GDI.TOTL.KD.ZG), net investment in nonfinancial assets as % of GDP (GC.NFN.TOTL.GD.ZS), foreign direct investment inflows as % of GDP (BX.KLT.DINV.WD.GD.ZS), foreign direct investment outflows as % of GDP (BM.KLT. DINV.WD.GD.ZS). Variable net FDI (inflows minus outflows) as % GDP is calculated by the difference between FDI inflows as % of GDP and FDI outflows as % of GDP.

For the robustness checks, from the baseline sample of 44 countries, the set of economies considered "developed" includes any country that either is a member of the Group of Ten (G10) major industrialized economies or is in the set of "developed economies" as classified by the United Nations in its World Economic Situation and Prospects (WESP) classification,[40] and include Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Poland, Portugal, Spain, Sweden, Switzerland, United Kingdom. I find similar results when using alternative classifications and fewer countries (e.g., G7 and wealthiest EU members).

The data are highly aggregated either at the country level (e.g., consumption and GDP controls) or by the BEA using all US foreign affiliates operating within foreign countries. Hence, I do not winsorize the raw data because it appears well behaved statistically (e.g., see histograms in Figure IA.5 for all key fundamental ratios). Nonetheless, winsorizing the data at conventional levels gives very similar results. As discussed and after the GMM estimation, the model-implied exchange rate growth rates are winsorized between -25% and 25% (in percentage terms per annum), as these bounds capture the vast majority of movements recorded in historical real exchange rates in the long sample (see discussion in section IA.C.3). As for other model-implied fundamentals reported in Table 4, implied Tobin's $q$s are winsorized above at $q \leq 10$, MPKs are bounded above at $MPK \leq 0.75$, adjustment costs as a fraction of each business units' own output are bounded above at $\Phi/Y \leq 0.75$ (such that in percentage terms, adjustment costs amount up to a large loss of 75% in output per year), and investment returns $R^I$ are winsorized between -75% and 75% (i.e., roughly 3 times as sizable as the smallest and largest

[40]I used the following WESP classification of countries from the last decade: https://www.un.org/en/development/desa/policy/wesp/wesp_current/2014wesp_country_classification.pdf.

annual returns in the US stock market observed since 1985; see the histogram in Figure IA.5). The winsorization step is done after the GMM estimation, hence not affecting the GMM minimization routine, its parameter estimates, their standard errors, or model fit $\chi^2$ tests.

Lastly, regarding the timing of the carry trade portfolios and following Lustig and Verdelhan (2007), currency excess return portfolios are formed every year $t$ by sorting countries into $n$ portfolios and using the real interest rate differentials $i_t^f - i_t^{US}$ (i.e., carry trade) measured at the end of year $t$ in the month of December. Portfolios are rebalanced once every year. The real currency excess return for each portfolio is measured at the end of year $t+1$ (using December $t+1$ information) by taking a simple average of the currency excess returns $R_{f,t+1}^{FX}$ of all countries within each portfolio.

## IA.C Supplementary Results

### IA.C.1 Intuition behind mechanisms in the multiperiod setting

The 2-period discussion of section 2.1.2.1 still applies in the more general multiperiod setting. The main difference is that in the richer latter setting, the exchange rate is then also linked to the next period's investment-to-capital ratios at home $\frac{I_{HQ,t+1}^h}{K_{HQ,t+1}^h}$ and abroad $\frac{I_{A,t+1}^h}{K_{A,t+1}^h}$. To see this, expanding the exchange rate equation (15) by using the full equations for the returns on investment at home and abroad in (14) reveals that the marginal benefits of capital at time $t+1$ discussed above no longer depend only on the marginal product of capital but also on other intuitive economic terms related to investment and capital at $t+1$.[41] As noted, focusing on the numerator in equation (14) for the foreign affiliate which captures the entire marginal benefit to investing abroad, we have that the marginal benefit is decomposed into [$a$] the marginal product of one unit of additional capital invested at time $t$ in the first term, $\frac{\partial F_{l,t+1}^h}{\partial K_{l,t+1}^h} K_{l,t+1}^h$, [$b$] the reduction in installation costs of investing in one additional unit of capital at $t+1$ as captured by the second term, [$c$] the non-depreciated marginal continuation value of capital at time $t+1$ in the fourth term, $q_{A,t+1}^h(1-\delta_{A,t+1}^h)K_{A,t+1}^h$. All these terms in the numerator combined allow the researcher to measure the marginal benefits of investment in a more complete way, and the implication when compared to the simpler 2-period equation (18) is that structurally inferring exchange rate fluctuations from the full-blown equation requires data on investment-to-capital ratios at time $t$ and $t+1$ and output-to-capital ratios at $t+1$.

[41] The returns on investment in (14) are, intuitively, ratios of the marginal benefits of investing in the given business unit in the numerator, and which are realized at time $t+1$, to the marginal cost of investing at $t$ in the denominator.

### IA.C.2 Three-country world economy

The world economy is extended to have three representative agents $j = \{h, f, g\}$ and three representative multinational firms $i = \{h, f, g\}$. One can think of country $g$ as the ROW, which excludes capital owned by the home firm that is not of its affiliates operating at home or in country $f$. However, to ease notation, for now it is denoted $g$. As before, each firm helps satisfy foreign consumption demand by operating fully owned affiliates in country $l \neq i$. Each firm is potentially owned by all agents through the purchase of shares.

For brevity, the home firm $i = h$ is emphasized. Let $e_{j,t}$ be the real exchange rate defined as the relative price of 1 unit of the country $j$ consumption good in terms of the home good in $h$.

The home agent's budget constraint in equation (3) is extended to

$$\begin{aligned} p_{h,t}(S^h_{h,t+1} - S^h_{h,t}) + e_{f,t}p_{f,t}(S^h_{f,t+1} - S^h_{f,t}) + e_{g,t}p_{g,t}(S^h_{g,t+1} - S^h_{g,t}) \\ \leqslant w_{h,t}N_{h,t} + d_{h,t}S^h_{h,t} + e_{f,t}d_{f,t}S^h_{f,t} + e_{g,t}d_{g,t}S^h_{f,t} - C_{h,t} \end{aligned} \tag{IA.6}$$

where the new term $e_{g,t}p_{g,t}(S^h_{g,t+1} - S^h_{g,t})$ represents agent $j = h$'s purchase of additional shares in firm $i = g$ at the prevailing stock price $p_{g,t}$, which entitles him to dividend payments $e_{g,t+1}d_{g,t+1}S^h_{f,t+1}$ in the following period (converted to the home good units using the spot exchange rate $e_{g,t+1}$).

The home agent maximizes lifetime utility by choosing current consumption and next-period equity shares in the three representative firms:

$$\max_{\{C_{h,t+s}, S^h_{h,t+s+1}, S^h_{f,t+s+1}, S^h_{g,t+s+1}\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty} \beta^s u(C_{h,t+s}) \tag{IA.7}$$

subject to (IA.6) and the initial number of shares $S_{h,0}, S_{f,0}$, and $S_{g,0}$. Agents in countries $f$ and $g$ maximize analog expected lifetime utilities.

Similar to before, deriving the full set of first-order conditions for the home agent and their associated Euler equations from the purchase of equity stakes in the home and foreign firms, we have that in equilibrium and when asset markets are complete, the following *equivalence* must hold between exchange rate growth rates and ratios of marginal rates of substitution of all three agents, i.e., a standard consumption risk-sharing implication between agents:

$$\frac{e_{f,t+1}}{e_{f,t}} = \frac{M_{f,t+1}}{M_{h,t+1}} \tag{IA.8}$$

$$\frac{e_{g,t+1}}{e_{g,t}} = \frac{M_{g,t+1}}{M_{h,t+1}} \tag{IA.9}$$

Here the information contained in exchange rates informs about all possible combinations of relative marginal utilities across countries. For example, one can back out implied ratios of other agents' SDFs by combining exchange rates $\frac{M_{f,t+1}}{M_{g,t+1}} = \frac{e_{g,t}}{e_{g,t+1}} \frac{e_{f,t+1}}{e_{f,t}}$. This indicates that exchange rates are sufficient for understanding differences in marginal utilities or pricing kernels. The reverse is true: marginal rates of substitution suffice to pin down all exchange rate fluctuations in the economy, i.e., a consumption model of exchange rate determination.

Proceeding as before, the concern of the home firm at every period $t$ is to maximize its total cum-dividend market value of equity by choosing optimally and intratemporally capital investment at home and abroad and hiring local labor for production at each business unit:

$$V_t^h = \max_{\{I^h_{HQ,t+s}, I^h_{A,f,t+s}, I^h_{A,g,t+s} K^h_{HQ,t+s+1}, K^h_{A,f,t+s+1}, K^h_{A,g,t+s+1}, N^{HQ,h}_{h,t+s}, N^{A,f,h}_{f,t+s}, N^{A,g,h}_{g,t+s}\}_{s=0}^{\infty}} E_t[\sum_{s=0}^{\infty} M_{h,t+s} D^h_{t+s}] \tag{IA.10}$$

where the total dividend payout in (10) is now:

$$\begin{aligned} D_t^h = {} & F^h_{HQ,t}\left(K^h_{HQ,t}, N^{HQ,h}_{h,t}, Z^h_{HQ,t}\right) - \Phi^h_{HQ,t}(I^h_{HQ,t}, K^h_{HQ,t}) - I^h_{HQ,t} - w_{h,t} N^{HQ,h}_{h,t} \\ & + e_{f,t}\left[F^h_{A,f,t}\left(K^h_{A,f,t}, N^{A,f,h}_{f,t}, Z^h_{A,f,t}\right) - \Phi^h_{A,f,t}(I^h_{A,f,t}, K^h_{A,f,t}) - I^h_{A,f,t} - w_{f,t} N^{A,f,h}_{f,t}\right] \\ & + e_{g,t}\left[F^h_{A,g,t}\left(K^h_{A,g,t}, N^{A,g,h}_{g,t}, Z^h_{A,g,t}\right) - \Phi^h_{A,g,t}(I^h_{A,g,t}, K^h_{A,g,t}) - I^h_{A,g,t} - w_{g,t} N^{A,g,h}_{g,t}\right] \end{aligned} \tag{IA.11}$$

and affiliate subscripts now further denote the location of the business unit. For example, $I^h_{A,g,t}$ is the investment of the subsidiary in country $g$ of the home firm $h$ (superscript is as before).

**PROPOSITION 2.** *Under Assumption 1, home firm equity value maximization gives a complete markets result for bilateral exchange rate growth between countries $f$ and home $h$:*

$$\frac{e_{f,t+1}}{e_{f,t}} = \frac{\begin{bmatrix} R^{S,h}_{t+1}[q^h_{HQ,t} K^h_{HQ,t+1} + e_{f,t} q^h_{A,f,t} K^h_{A,f,t+1} + e_{g,t} q^h_{A,g,t} K^h_{A,g,t+1}] \\ -R^{I,h}_{HQ,t+1} q^h_{HQ,t} K^h_{HQ,t+1} - R^{I,h}_{A,g,t+1} e_{g,t+1} q^h_{A,g,t} K^h_{A,g,t+1} \end{bmatrix}}{R^{I,h}_{A,f,t+1} e_{f,t} q^h_{A,f,t} K^h_{A,f,t+1}} \tag{IA.12}$$

*where these rates of foreign currency appreciation can be inferred from production data on the aggregate stock market return of the home firm, $R^{S,h}_{t+1}$, and of its marginal rates of transformation of capital at home, $R^{I,h}_{HQ,t+1}$, and abroad, $R^{I,h}_{A,f,t+1}$, and $R^{I,h}_{A,g,t+1}$.*

In this model, the ex-dividend total market value of equity of the home firm is *defined* as the sum of its ongoing business concerns in all three countries, $P^h_t \equiv V^h_t - D^h_t \equiv p_{h,t}(S^h_{h,t+1} + S^f_{h,t+1} + S^g_{h,t+1})$, which by Assumption 1, has a $q$-theoretic closed-expression result:

$$P^h_t = q^h_{HQ,t} K^h_{HQ,t+1} + e_{f,t} q^h_{A,f,t} K^h_{A,f,t+1} + e_{g,t} q^h_{A,g,t} K^h_{A,g,t+1} \tag{IA.13}$$

The *equivalence* result in (IA.12) has, as special cases, the two-country equivalence in (15) and the one-country equivalence between stock and investment returns in Cochrane (1991, 1996). This result carries through to the more general case of a larger world economy with a countable set $W$ of countries, with additional terms that enter additively in the numerator of (IA.12).

### IA.C.3 Country-by-country descriptive statistics of foreign affiliate production

Table 1 presents descriptive statistics, with Panel A reporting the mean, standard deviation, and key percentiles, and Panel B the time series average of pairwise cross-sectional correlations. Appendix Figures IA.2 and IA.3 present country-by-country $I/K$ and $Y/K$ ratios, respectively, for the trio of US-owned business units in $\{HQ,\ f,\ ROW\}$, highlighting significant variations over time and across affiliates. Figure IA.4 illustrates the aggregate *levels* of investment, output, and capital stocks of US foreign affiliates by country, and for presentational purposes in constant 2015 US dollars. All 44 countries, regardless of size, report foreign affiliate flows and stocks in the billions. Note that the exchange rate result in (17) relies on the concepts of marginal benefits and costs of investment and applies to aggregate affiliates of any size, with affiliate size playing only a weighting role. It arises from a model that connects exchange rates to the marginal productivity of the incremental (final) unit of investment by MNEs. Reassuringly, however, Figure IA.4 shows that the flows and stocks of US foreign affiliates are not small, totaling billions in real dollars across each foreign economy.

Finally, Figure IA.5 displays histograms for all key variables, highlighting a large degree of heterogeneity in foreign affiliate investment-to-capital ratios ($I/K_{A,f,t}$), which are strictly positive and range from 0.05 to 0.70. Affiliate output-to-capital ratios ($Y/K_{A,f,t}$) also show wide variation, from 0.20 to 5 across countries. Histograms are also presented for the differentials in activity of US firms abroad $f$ and at home $HQ$. Additionally, the histogram of bilateral exchange rate growth ($\frac{e_{f,t}}{e_{f,t-1}} - 1$, expressed as an annual percentage) confirms the stylized fact that exchange rates are volatile (standard deviation of 10.34% per annum), yet range quite symmetrically from -50% to +50% in the long-run sample of 44 countries from 1985 to 2019. Table 1 corroborates this symmetry, with the 1st percentile at -23.99% and the 99th at 26.34%. Consequently, I winsorize the model-implied exchange rate growth rates between -25% and +25%, as these bounds span the vast majority of historical movements. This winsorization is applied after the GMM estimation, thereby preserving the integrity of the GMM routine, parameter estimates, their standard errors, and model fit $\chi^2$ tests.[42]

### IA.C.4 Reduced-form controls, macroeconomic predictors

The controls appearing in Table 2 include the following differentials measured at the end-of-year $t$ between the foreign country $f$ and the $US$ economy: real consumption growth (column

[42]The qualitative results are unchanged and the quantitative pricing performance of the model is similar when winsorizing at -50% and 50%, hence allowing jumps (up or down) in implied exchange rates to be as large as the extreme long-run outliers.

2); real GDP growth (3); real fixed capital formation growth, which measures investment to expand a country's existing fixed capital stock (4); real total capital formation growth, which includes investment in fixed assets of the economy plus net changes in the level of inventories (5); real net investment in nonfinancial assets to GDP ratios, which includes investment in fixed assets, inventories, valuables, and nonproduced assets (6); net foreign direct investment (FDI) (inflows minus outflows) to GDP ratios, where FDI captures all investment flows to the country to acquire a lasting management interest—i.e., 10 percent or more of voting stock—in an enterprise and is the sum of equity capital, reinvestment of earnings, other long-term capital, and short-term capital as shown in the balance of payments (7); FDI inflows to GDP ratios (8); FDI outflows to GDP ratios (9); annual rates of inflation (10); lags for the real exchange rate growth (11) and real currency excess returns (12), which control for potential auto-correlations in the annual dependent variables; real interest rates (13) to account for predictability associated with uncovered interest rate parity (UIP). National accounts data appearing in the controls are downloaded from the World Bank. See Data Appendix IA.B for details.

### IA.C.5 More on the implied fundamentals of affiliates at the country level

Adjustment costs can be further examined by considering their role in determining Tobin's $q$. When firms face no adjustment costs when investing in capital (i.e., $\Phi = 0$), Tobin's $q = 1$, indicating that the stock market value of capital equals its replacement costs. In the richer case where $q > 1$, the market value of the business unit exceeds its replacement costs, suggesting high forward-looking investment or growth opportunities. Conversely, when $q < 1$, the business sells at a discount relative to its book value. Panel A of Table 4 shows foreign affiliate market values relative to replacement cost vary widely, from roughly 0.50 to 4.50, with a median Tobin's $q$ of 1.74. These estimates align with literature findings, such as Alfaro, Bloom, and Lin (2024) reporting an average Tobin's $q$ of 1.61, range 0.13 to 9, for US firms, and Belo et al. (2013) with a structural mean of 1.56, range 0.44 to 4.94, across benchmark portfolios. Given that US foreign affiliate market values are not observable, yet economically important, my structural estimates of their values through Tobin's $q$s are likely useful for follow-up work.

The fundamentals discussed above—including MPKs, adjustment costs, and Tobin's $q$—collectively characterize the returns on investment $R^I$ at home and abroad of US-established firms. These fundamentals are examined further in the mechanism section 4.4 as key drivers of the model's performance in pricing the cross-country differences in carry trade returns. Importantly, Table 4 indicates that the returns on US affiliate investment vary widely across international markets, with the median foreign affiliate earning an annual return of 16.25% (average 16.28%), and ranging from -11.15% to 43.26%. Notably, Panel B highlights that the average return on domestic investment at home $HQ$ is estimated at 9.32% per annum, which closely aligns with the mean annual return of the US stock market at 8.06% (see Table 1). In a closed economy, these returns should theoretically converge on average ($R^S = R^I_{HQ}$, as in the special case in Cochrane (1991)), and their similarity when estimated within the multinational $q$-theory framework further validates the estimation procedure's reasonableness.

### IA.C.6 Further details on the comparative statics procedure

The experiments proceed as follows. For the time $t$ investment-to-capital ratio of affiliate $f$, I set the term $\left(\frac{I^h_{A,f,t}}{K^h_{A,f,t}} - \overline{\frac{I^h_A}{K^h_A}}\right)$ in the adjustment cost equation (23), and in its related partial derivatives, to the steady state $\overline{\frac{I^h_A}{K^h_A}}$ for all affiliates $f$ at every point in time. The resulting average pricing errors from this experiment are denoted $\overline{I/K_{A,f,t}}$ in Table 7. For the comparative statics of affiliate $f$'s output-to-capital ratios, I set $Y/K_{A,f,t+1}$ equal to its long-run median value every period, thereby shutting down any cross-affiliate variation in output rates and their influence on MPKs. Regarding the $HQ$ investment-to-capital ratio $I/K_{HQ,t}$, I fix it to its time-series median, hence removing any variation over time in the rate of US domestic investment. The other comparative statics for $I/K$ and $Y/K$ ratios are designed analogously. For the US stock market return $R^{US}_{t+1}$, it is fixed to its mean value, thereby eliminating any influence in the variation over time of the US market. In each of the 10 experiments and for direct comparison with the baseline, I use the parameter estimates from the baseline estimation to reconstruct implied exchange rate changes and recalculate the model's pricing errors.

### IA.C.7 Robustness tests

This section presents additional results and robustness checks of both the model and reduced-form empirical regressions. Table IA.1 presents checks to the pricing of the carry trade when different data filters and samples are used in the tests. To facilitate visualization of these checks, Figure IA.7 plots the pricing errors of each different check alongside those of the baseline model. The results remain qualitatively unchanged and show little quantitative variation across several meaningful robustness checks, including the following. In Panel B, restricting the sample of countries to those with at least 20 years of required data for the GMM estimation, which leads to smaller average pricing errors (MAE of 0.31) due to increased precision. Panel C restricts the sample to include only developed countries,[43] while Panel D excludes all euro-year countries that adopted the euro as a common currency after its introduction in 1999. For the latter, results are very similar if euro countries are replaced instead with an aggregate euro-wide economy or with Germany as a proxy starting in 1999. Panel E restricts the sample to the period 1999 onward such that it includes post-euro-introduction years and because 1999 is roughly at the middle mark of the long sample. Panel F uses only countries that overlap with those in the original carry trade sample in Lustig and Verdelhan (2007), while Panel G is similar but further restricts the sample to the original end year 2003 of that paper.

Panel H restricts the sample to countries where the fraction of total output produced by foreign affiliates that is shipped back to their US parents is minimal. This check drops the few countries where the average fraction exceeds 10%. Consistent with the aggregates in Figure 1, the vast majority of US foreign affiliates' output is consumed abroad and not shipped back to US parents for home consumption. Countries not meeting the 10% filter are few: Mexico, Costa Rica, Nigeria, Israel, Malaysia, and Singapore. Using stricter thresholds of 7.5% and 5% gives similar results. Moreover, I find similar results to an adjustment exercise where I remeasure foreign affiliate output-to-capital ratios $Y/K_{A,f,t+1}$ but explicitly exclude the amount of affiliate output that is shipped back to US parents. The resulting pricing errors are similar, which suggests that the influence of within-firm trade does not materially influence the model's capability of matching exchange rate data.[44] Lastly, Panel I restricts the sample to countries with a large index of capital account liberalization, index $\geq 0.25$ (up from the baseline 0.10).

Complementing the results of Table 5, Appendix Table IA.2 presents Fama–MacBeth and panel regressions that compare the coefficients on the interest rate differential and the whole host of differentials in the key foreign affiliate variables. Using country-by-year observations, and the coefficients are similar across the realized and implied specifications. For example, after deducting the implied excess returns from the realized, there is no longer a forecastable component in average currency risk premia associated with interest rate differentials—hence, entirely capturing the predictable component in currency premia arising from interest rates.

Complementing Table 7, Appendix Figure IA.8 plots the model's performance in pricing the carry trade at different levels of portfolio aggregation. The scatter points line up well around the 45-degree line when sorting countries into 2 and up to 10 portfolios per year. In each

[43] Developed economies include membership in the Group of Ten (G10) or in the set of "developed economies" per the WESP classification of the UN. Alternative classifications and fewer countries (e.g., G7 and wealthiest EU members) give similar results.
[44] I thank Leonid Kogan and Andrea Vedolin for suggesting this check.

plot, the model produces economically sizable implied H-L spreads that are comparable to the payoffs seen by US investors in currency carry trade since the 1980s. Lastly, also using portfolios, Appendix Table IA.3 presents Fama–MacBeth regressions of carry trade risk premia at different portfolio levels, akin to the results in Table 5 that use country-by-year observations. For example, using 8 carry trade portfolios, the slope on the realized (implied) results suggests that a 10 percentage point increase in the foreign interest rate differential forecasts a 1-year-ahead holding-period foreign exchange return of 3.22% (3.89%) per annum. These realized and implied spreads in currency premia are economically sizable and of equal size statistically.

Table IA.1

**Robustness tests: the *production*-based pricing of the carry trade is robust to different samples and filters of countries & years**

Carry trade portfolios: currency excess returns

| Sample specification: | Implied excess returns | | | | | Realized excess returns | | | | | Model pricing errors (realized−implied) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Low | 2 | 3 | High | H-L | Low | 2 | 3 | High | H-L | Low | 2 | 3 | High | H-L | MAE |
| Baseline | 0.10 | 1.52 | 2.64 | 5.20 | 5.10 | -0.10 | 1.65 | 2.93 | 4.38 | 4.48 | -0.20 | 0.13 | 0.29 | -0.82 | -0.62 | 0.36 |
| Countries ⩾ 20 years data | 0.34 | 1.47 | 2.62 | 4.70 | 4.35 | -0.01 | 1.67 | 2.52 | 4.11 | 4.12 | -0.36 | 0.20 | -0.10 | -0.58 | -0.23 | 0.31 |
| Developed countries only | -0.12 | 1.27 | 1.41 | 3.85 | 3.97 | 0.49 | 1.23 | 2.11 | 3.42 | 2.93 | 0.61 | -0.04 | 0.70 | -0.43 | -1.04 | 0.44 |
| Excluding euro-year € countries | -0.08 | 2.45 | 3.64 | 5.20 | 5.29 | 0.32 | 2.30 | 2.40 | 5.30 | 4.98 | 0.40 | -0.15 | -1.24 | 0.10 | -0.30 | 0.47 |
| Sample year ⩾ 1999 | 0.29 | 0.72 | 2.17 | 5.64 | 5.35 | -0.72 | 0.81 | 2.15 | 4.94 | 5.66 | -1.01 | 0.09 | -0.02 | –0.70 | 0.31 | 0.46 |
| Carry trade countries Lustig & Verdelhan AER (2007) | 0.20 | 1.43 | 2.62 | 5.12 | 4.91 | -0.25 | 1.87 | 2.64 | 4.97 | 5.22 | -0.45 | 0.44 | 0.01 | -0.14 | 0.31 | 0.26 |
| Lustig & Verdelhan AER (2007) original sample ⩽ 2003 | -0.93 | 0.52 | 2.78 | 3.46 | 4.38 | -0.80 | 2.03 | 2.88 | 4.27 | 5.07 | 0.13 | 1.51 | 0.10 | 0.81 | 0.68 | 0.64 |
| Exclude ctry. w/ fraction output shipped to US parents ⩾ 0.10 | -0.13 | 1.46 | 2.66 | 5.70 | 5.83 | 0.05 | 1.47 | 2.81 | 4.49 | 4.44 | 0.18 | 0.00 | 0.15 | -1.22 | -1.40 | 0.39 |
| Large capital account openness Chinn–Ito Index ⩾ 0.25 | 0.40 | 1.06 | 2.13 | 5.07 | 4.67 | -0.15 | 1.40 | 2.90 | 3.90 | 4.05 | -0.55 | 0.34 | 0.77 | -1.17 | -0.62 | 0.71 |

**Notes:** This table reports robustness checks to the performance of the baseline model in pricing the carry trade when different data filters and samples are used in the tests. To ease the visualization of the checks reported in this table, Figure IA.7 plots the pricing errors of each check next to those of the baseline model (and which are also shown in the baseline Figure 4A and Table 6). Using the labels of the Panels presented in Figure IA.7, the robustness checks presented here include the following: Panel B restricts the sample of countries to those with at least 20 years of data required for the GMM estimation (up from the baseline 10 years of data). Panel C restricts the sample to including only developed countries. Panel D excludes all euro-year countries that adopted the euro after its introduction in 1999. Panel E is for the subsample of all observations starting in 1999. Panel F restricts the sample to include only the countries that overlap with the original carry trade sample in Lustig and Verdelhan (2007). Panel G is similar but further restricts the sample to the original end year 2003 of that paper. Panel H restricts the sample to only countries with a minimal fraction of aggregate output shipped back to US parents by foreign affiliates, i.e., countries whose fractions does not exceed 10% on average over the sample period. Panel I keeps only countries whose capital account openness is large with a threshold $\geq 0.25$ using the Chinn and Ito (2006) KAOPEN index (which ranges from 0 to 1). Implied high-minus-low carry trade spreads and mean absolute pricing error (MAE) across portfolios are reported for each check. A $\chi^2$ test fails to reject the null in each robustness check that the portfolio pricing errors are jointly zero. Baseline data are from 1987 to 2019. Results are at the portfolio level and present the average currency excess returns from the carry trade (as a percentage ($\times$100) per annum) and the average pricing errors (also as a percentage per annum) of the production-based model. Pricing errors are computed as the difference between realized and model-implied currency excess returns. The H-L portfolio is the excess return payoff to a US investor in the carry trade (long in high and short in low foreign interest rates). Carry trade portfolios are formed following Lustig and Verdelhan (2007) and by sorting countries every year $t$ into 4 portfolios based on foreign interest rate differentials $i_t^f - i_t^{US}$ measured at the end of year $t$ in the month of December. Portfolios are rebalanced once every year. The real currency excess return for each portfolio is measured at the end of year $t+1$, using December $t+1$ information, by taking a simple average of the currency excess returns of all countries within each portfolio. See section 3 and the Data Appendix IA.B for estimation and data details.

Table IA.2

**Robustness tests: Comparing realized against production-implied forecasts of *country-level* currency excess returns, 1987–2019**

| *Regression specification:* | *Fama–MacBeth (columns 1–3)* | | | *Panel regressions (columns 4–6)* | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $(e_{t+1}/e_t) - (i_t^{US}+1)/(i_t^f+1)$ | Realized | Implied | Realized−Implied | Realized | Implied | Realized−Implied |
| $i_t^f - i_t^{US}$ | 0.337** | 0.329** | 0.008 | 0.320*** | 0.242*** | 0.078 |
| | (0.123) | (0.122) | (0.095) | (0.110) | (0.046) | (0.119) |
| $I/K_{A,f,t} - I/K_{US,t}$ | 0.089* | -0.032 | 0.122* | 0.081** | 0.064 | 0.017 |
| | (0.045) | (0.051) | (0.062) | (0.039) | (0.091) | (0.093) |
| $I/K_{A,f,t+1} - I/K_{US,t+1}$ | -0.128* | -0.063 | -0.065 | -0.109 | -0.276** | 0.167* |
| | (0.066) | (0.125) | (0.120) | (0.087) | (0.113) | (0.093) |
| $Y/K_{A,f,t+1} - Y/K_{US,t+1}$ | -0.002 | -0.004 | 0.002 | -0.004 | 0.015 | -0.020 |
| | (0.005) | (0.010) | (0.011) | (0.014) | (0.012) | (0.013) |
| $I/K_{A,ROW,t} - I/K_{US,t}$ | 0.583 | -2.252 | 2.835 | -0.068 | 1.717 | -1.785† |
| | (2.170) | (5.733) | (6.090) | (0.329) | (1.272) | (1.110) |
| $I/K_{A,ROW,t+1} - I/K_{US,t+1}$ | 0.156 | -4.260 | 4.415 | 0.458† | -2.443* | 2.900** |
| | (2.946) | (5.583) | (5.918) | (0.272) | (1.339) | (1.188) |
| $Y/K_{A,ROW,t+1} - Y/K_{US,t+1}$ | -0.870 | -22.500 | 21.629 | -0.448 | 0.810 | -1.258 |
| | (10.065) | (23.785) | (33.074) | (0.314) | (0.923) | (0.897) |
| Observations | 1,139 | 1,139 | 1,139 | 1,139 | 1,139 | 1,139 |
| $R^2$ | 0.37 | 0.29 | 0.26 | 0.42 | 0.61 | 0.61 |
| Year fixed effects | n/a | n/a | n/a | Yes | Yes | Yes |
| Country fixed effects | n/a | n/a | n/a | Yes | Yes | Yes |
| Newey–West SEs (4yrs) | Yes | Yes | Yes | n/a | n/a | n/a |

**Notes:** This table extends the carry trade univariate regression analyses presented in Table 5 (Panels D and E) by running multivariate specifications. It compares the realized versus production-implied forecasts of real currency excess returns from the carry trade trading strategy, but now in addition to comparing the coefficients on the interest rate differential it also compares the slopes on the whole host of production-based differentials related to foreign affiliates of US parents. Observations are at the country-by-year level from 1987 to 2019. Foreign affiliate aggregates are in foreign currency real terms (e.g., in constant 2015 British pounds for investments in the UK), while HQ data are in real US 2015 dollar figures. The column "Realized−Implied" tests whether the coefficients on the realized and implied specifications are equal by regressing the difference between the realized and implied series. Columns 1 to 3 present results for Fama and MacBeth (1973) cross-sectional regressions, with Newey and West (1987) standard errors reported in parentheses that account for heteroskedasticity and autocorrelation in the error structure up to a lag of 4 years. Columns 4 to 6 present results for panel regressions that include both country and year fixed effects, with standard errors that are double clustered in the time and country dimensions. The independent variables include the lagged by 1-year foreign interest rate differential $i^f - i^{US}$, and for the production variables the differentials in the investment-to-capital $I/K$ and output-to-capital $Y/K$ ratios of US-owned foreign affiliates in $\{f \textit{ and } ROW\}$ and in excess of US domestic ratios. ***, **, *, and † denote statistical significance at the 1%, 5%, 10%, and 15% levels, respectively. See section 3.2 and the Data Appendix IA.B for data sources and details.

Table IA.3

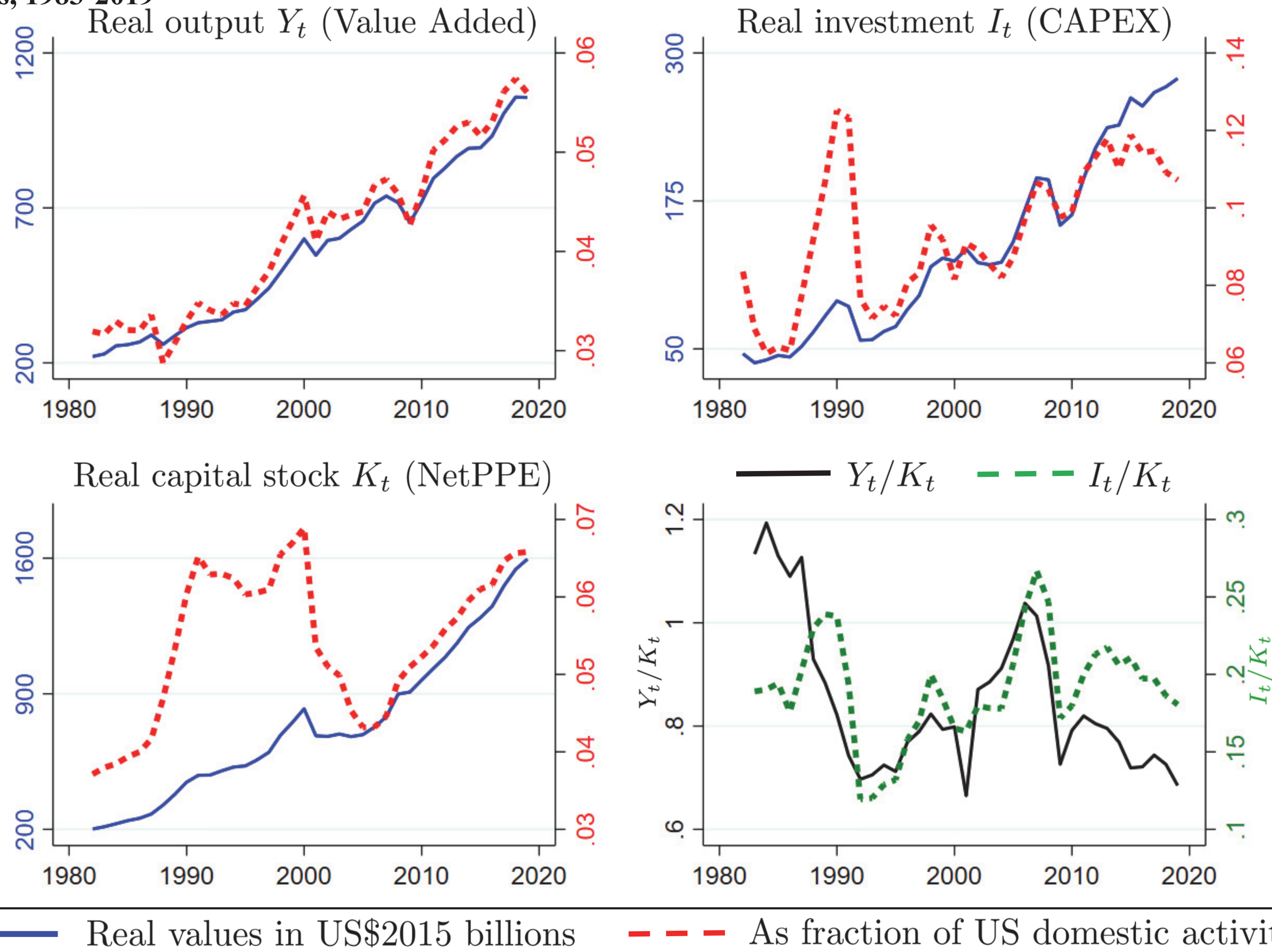
**Robustness tests: Comparing realized against production-implied forecasts of *portfolio-level* carry trade excess returns, 1987–2019**

*Cross-sectional Fama–MacBeth carry trade excess return regressions*

| $(e_{t+1}/e_t) - (i_t^{US}+1)/(i_t^f+1)$ | Realized | | Implied | | Realized−Implied |
|---|---|---|---|---|---|
| *4 portfolios:* | $\beta$ | $p$ value | $\beta$ | $p$ value | $p$ value |
| $i_t^f - i_t^{US}$ | 0.303*** | 0.002 | 0.417*** | 0.002 | 0.310 |
| | (0.092) | | (0.126) | | |
| Average *Adj* $R^2$ | 0.467 | | 0.167 | | 0.106 |
| *5 portfolios:* | $\beta$ | $p$ value | $\beta$ | $p$ value | $p$ value |
| $i_t^f - i_t^{US}$ | 0.322*** | 0.002 | 0.431*** | 0.001 | 0.270 |
| | (0.096) | | (0.116) | | |
| Average *Adj* $R^2$ | 0.450 | | 0.0822 | | 0.127 |
| *6 portfolios:* | $\beta$ | $p$ value | $\beta$ | $p$ value | $p$ value |
| $i_t^f - i_t^{US}$ | 0.337*** | 0.002 | 0.330*** | 0.004 | 0.942 |
| | (0.098) | | (0.105) | | |
| Average *Adj* $R^2$ | 0.420 | | 0.140 | | 0.133 |
| *7 portfolios:* | $\beta$ | $p$ value | $\beta$ | $p$ value | $p$ value |
| $i_t^f - i_t^{US}$ | 0.315*** | 0.003 | 0.391*** | 0.002 | 0.406 |
| | (0.098) | | (0.116) | | |
| Average *Adj* $R^2$ | 0.398 | | 0.131 | | 0.150 |
| *8 portfolios:* | $\beta$ | $p$ value | $\beta$ | $p$ value | $p$ value |
| $i_t^f - i_t^{US}$ | 0.322*** | 0.004 | 0.389*** | 0.003 | 0.495 |
| | (0.102) | | (0.122) | | |
| Average *Adj* $R^2$ | 0.391 | | 0.146 | | 0.147 |

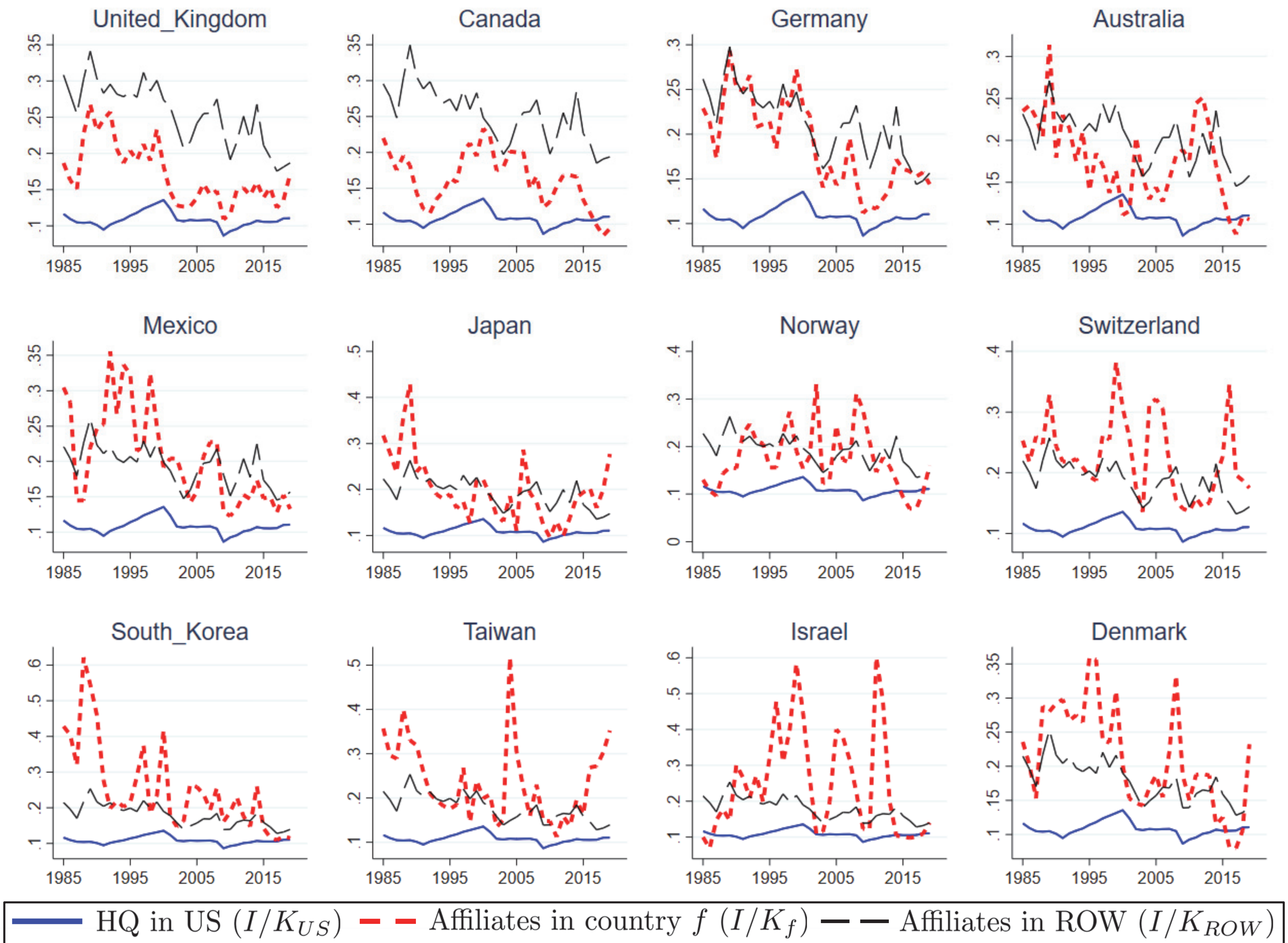
**Notes:** This table compares realized versus production-implied forecasts of real currency excess returns from the carry trade trading strategy and using different numbers of portfolios. Observations are at the portfolio level from 1987 to 2019 and from Fama and MacBeth (1973) cross-sectional regressions, with Newey and West (1987) standard errors reported in parentheses that account for heteroskedasticity and autocorrelation in the error structure up to a lag of 4 years. Foreign affiliate aggregates are in foreign currency real terms (e.g., in constant 2015 British pounds for investments in the UK), while HQ data are in real US 2015 dollar figures. Carry trade portfolios are formed following Lustig and Verdelhan (2007) and by sorting countries every year $t$ into $n$ portfolios based on foreign interest rate differentials $i_t^f - i_t^{US}$ measured at the end of year $t$ in the month of December. Portfolios are rebalanced once every year. Currency excess returns for each portfolio are measured at the end of year $t+1$ by taking a simple average of the currency excess returns of all countries within each portfolio. The column "Realized−Implied" tests whether the coefficients on the realized and implied specifications are equal by regressing the difference between the realized and implied series on the 1-year lag of the portfolio-level average differential in foreign interest rates. ***, **, *, and † denote statistical significance at the 1%, 5%, 10%, and 15% levels, respectively. See section 3.2 and the Data Appendix IA.B for data sources and details

**Figure IA.1: Output, investment, & capital stocks within US borders of US-located affiliates owned by foreign parents, 1983-2019**



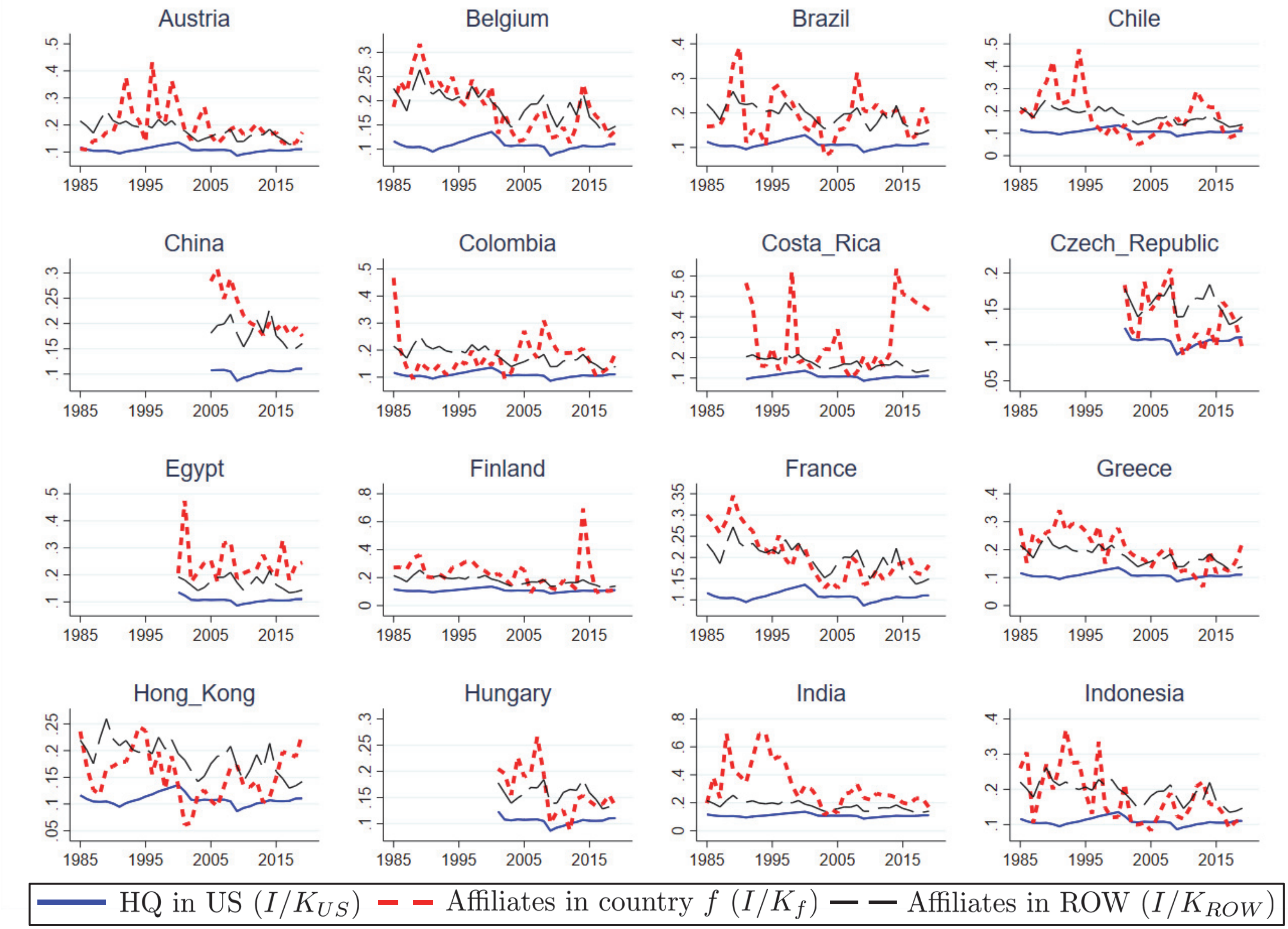

*Notes*: Aggregate real production quantities measured *within US borders* of *US-located affiliates owned by foreign parents*. Value added, investment, and capital stocks are from 1983 to 2019 in solid blue lines (left Y axis) and displayed in real constant US$2015 billions. The red dashed line for value added (right Y axis) plots the value added of US-located affiliates owned by foreign parents as a fraction of total US domestic output created within US borders and exclusively by US-owned firms (i.e., the denominator is US GDP net of the value added created within US borders by subsidiaries owned by foreign parents). Other fractions in red dashed lines for investment and capital stocks are analogously created. The bottom right panel plots output-to-capital (solid black line left Y axis) and investment-to-capital ratios (dashed green right Y axis) from the different series in this figure.

# Figure IA.2: Investment-to-capital ratios (*I/K*) of US-owned companies at home and abroad, 1985-2019



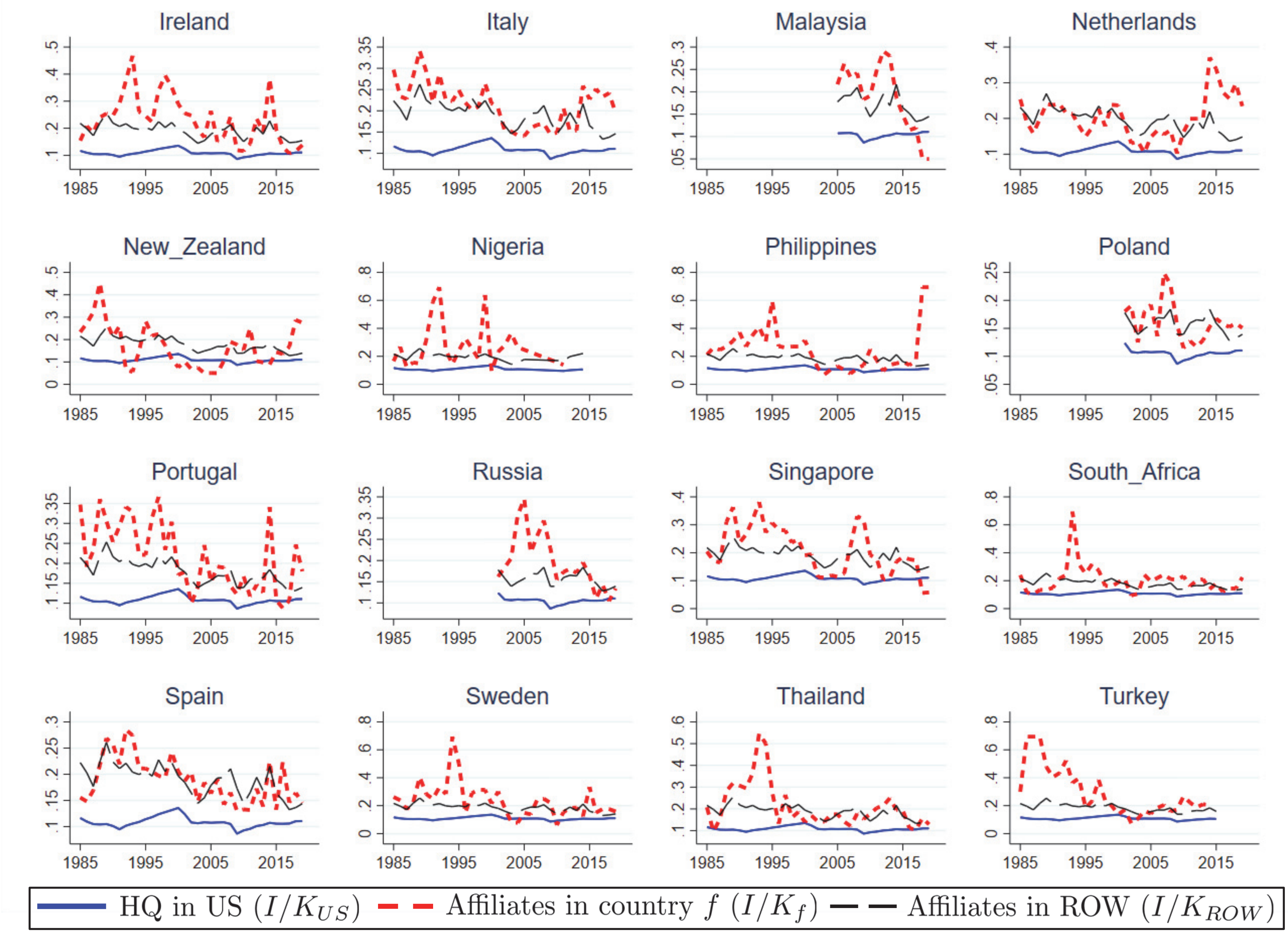

***Notes**:* Aggregate real investment-to-capital ratios (I/K) of all domestic and foreign firms owned by US parents (i.e., US "persons" including legal entities and individuals) and presented separately for i) home firms operating within US borders and denoted by headquarter (*HQ*) (in blue solid line), ii) foreign affiliates operating in foreign country *f* (red short dash), iii) foreign affiliates in the rest of the world (ROW) (black long dash). For each country *f* plot, the ROW aggregate I/K ratio is a weighted average of the I/K ratios of foreign affiliates operating elsewhere outside of *f* and home *HQ*. The ROW weights are based on the observed aggregate size of foreign affiliate capital stocks. There are 44 countries. For presentational and comparison purposes, the first set of 16 countries are presented such that the top row has countries with large foreign affiliate capital stocks, while middle and bottom rows include countries with middle- and smaller-size stocks. Remaining countries presented alphabetically. Data 1985-2019, details in data section.

# Figure IA.2: Continued



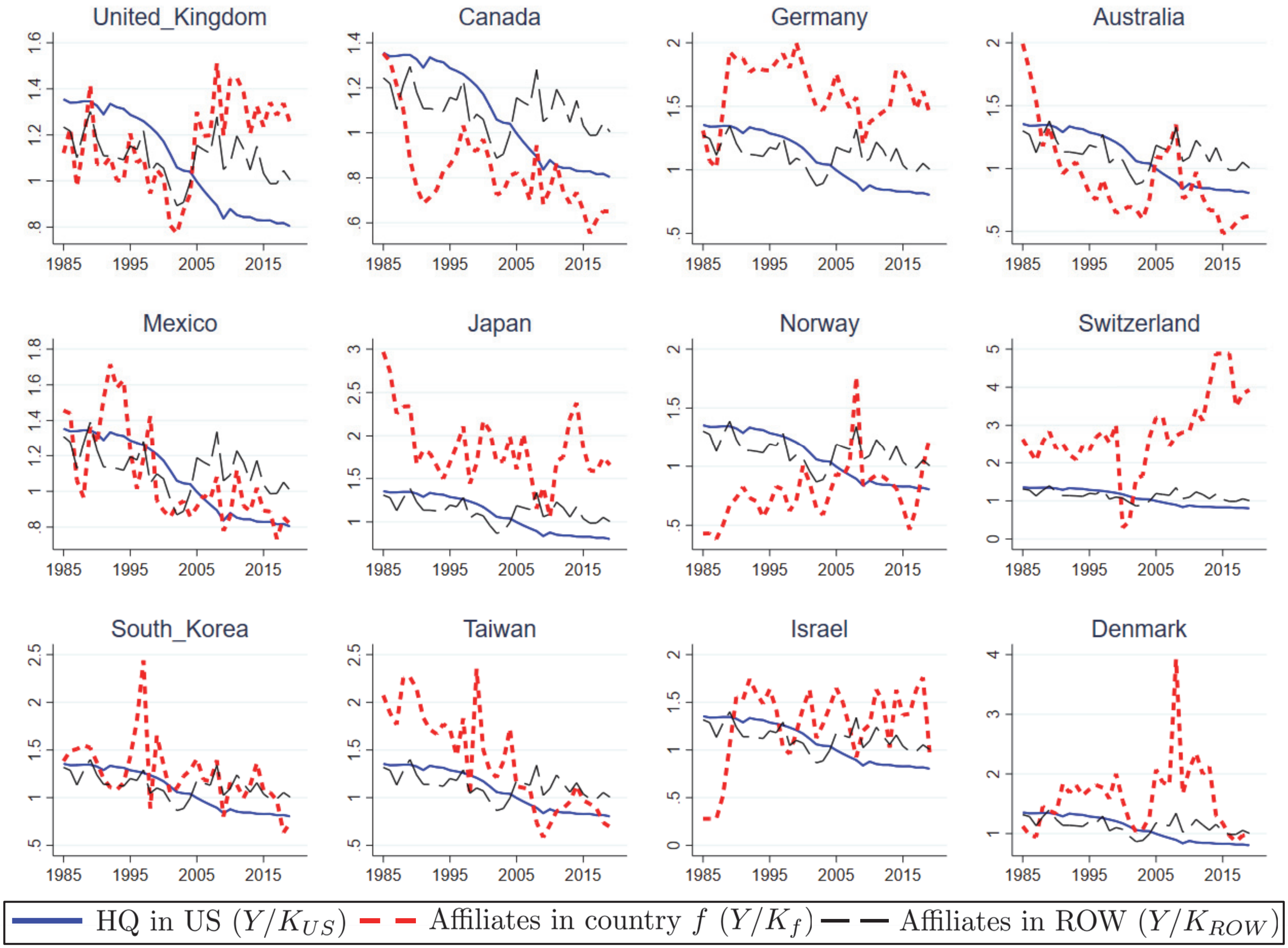

***Notes**:* Aggregate real investment-to-capital ratios (I/K) of all domestic and foreign firms owned by US parents (i.e., US "persons" including legal entities and individuals) and presented separately for i) home firms operating within US borders and denoted by headquarter (*HQ*) (in blue solid line), ii) foreign affiliates operating in foreign country *f* (red short dash), iii) foreign affiliates in the rest of the world (ROW) (black long dash). For each country *f* plot, the ROW aggregate I/K ratio is a weighted average of the I/K ratios of foreign affiliates operating elsewhere outside of *f* and home *HQ*. The ROW weights are based on the observed aggregate size of foreign affiliate capital stocks. There are 44 countries. For presentational and comparison purposes, the first set of 16 countries are presented such that the top row has countries with large foreign affiliate capital stocks, while middle and bottom rows include countries with middle- and smaller-size stocks. Remaining countries presented alphabetically. Data 1985-2019, details in data section.

# Figure IA.2: Continued



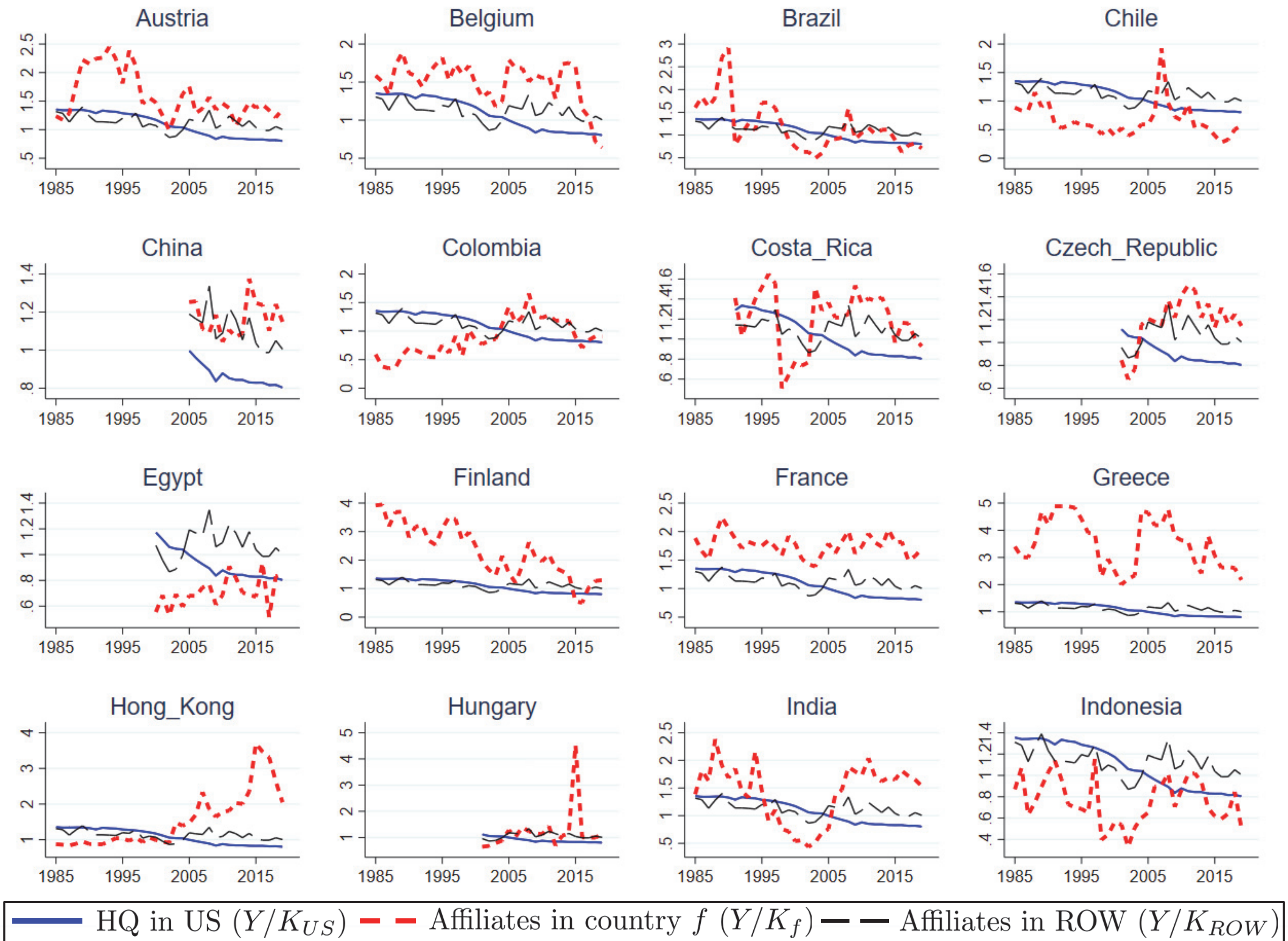

***Notes***: Aggregate real investment-to-capital ratios (I/K) of all domestic and foreign firms owned by US parents (i.e., US "persons" including legal entities and individuals) and presented separately for i) home firms operating within US borders and denoted by headquarter (*HQ*) (in blue solid line), ii) foreign affiliates operating in foreign country *f* (red short dash), iii) foreign affiliates in the rest of the world (ROW) (black long dash). For each country *f* plot, the ROW aggregate I/K ratio is a weighted average of the I/K ratios of foreign affiliates operating elsewhere outside of *f* and home *HQ*. The ROW weights are based on the observed aggregate size of foreign affiliate capital stocks. There are 44 countries. For presentational and comparison purposes, the first set of 16 countries are presented such that the top row has countries with large foreign affiliate capital stocks, while middle and bottom rows include countries with middle- and smaller-size stocks. Remaining countries presented alphabetically. Data 1985-2019, details in data section.

# Figure IA.3: Output-to-capital ratios (*Y/K*) of US-owned companies at home and abroad, 1985-2019



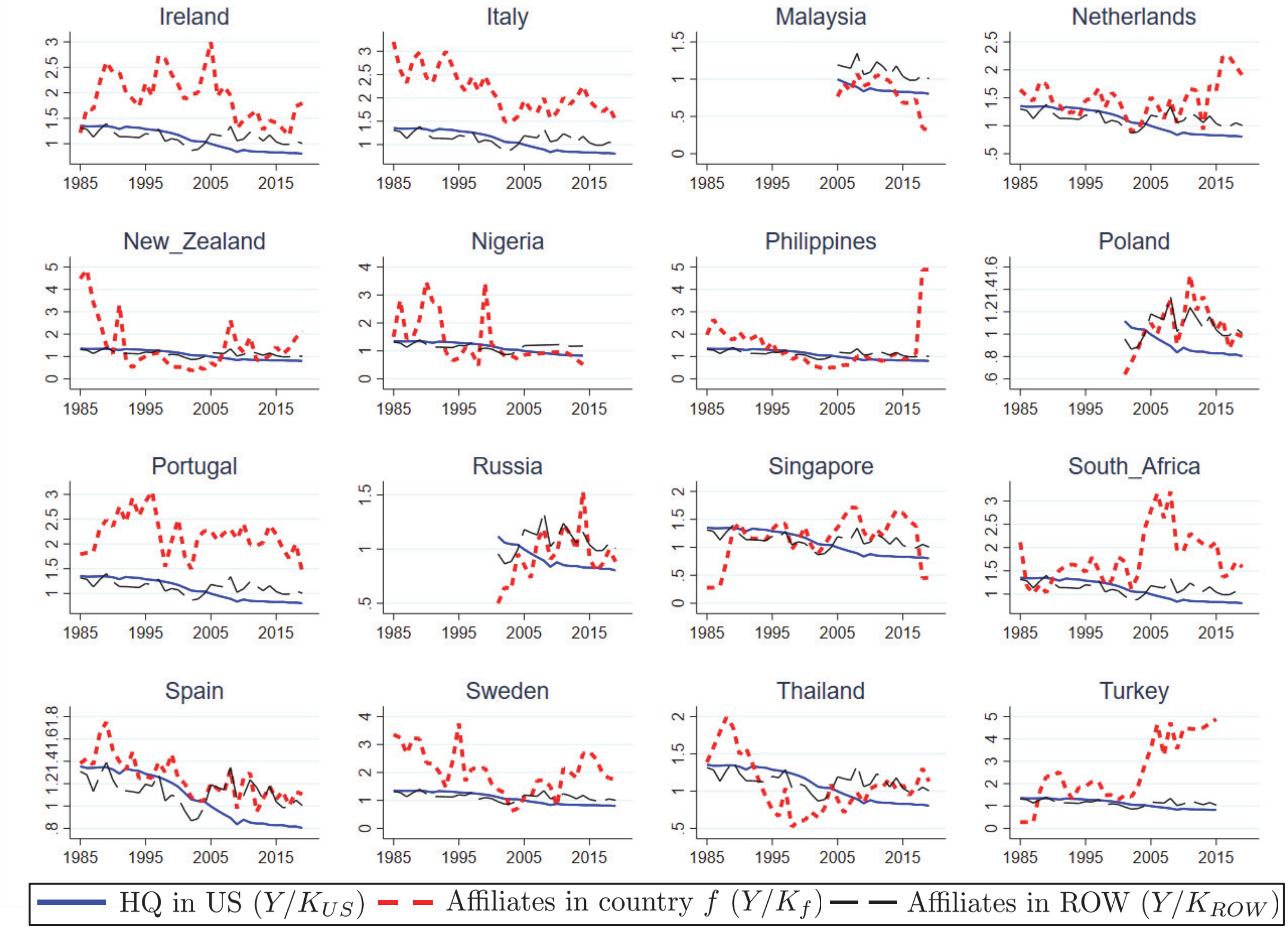

***Notes**:* Aggregate real value-added-to-capital ratios (Y/K) of all domestic and foreign firms owned by US parents (i.e., US "persons" including legal entities and individuals) and presented separately for i) home firms operating within US borders and denoted by headquarter (*HQ*) (in blue solid line), ii) foreign affiliates operating in foreign country *f* (red short dash), iii) foreign affiliates in the rest of the world (ROW) (black long dash). For each country *f* plot, the ROW aggregate Y/K ratio is a weighted average of the Y/K ratios of foreign affiliates operating elsewhere outside *f* and *HQ*. The ROW weights are based on the observed aggregate size of foreign affiliate capital stocks. There are 44 countries. For presentational and comparison purposes, the first set of 16 countries are presented such that the top row has countries with large foreign affiliate capital stocks, while middle and bottom rows include countries with middle- and smaller-size stocks. Remaining countries presented alphabetically. Data 1985-2019, details in data section.

## Figure IA.3: Continued



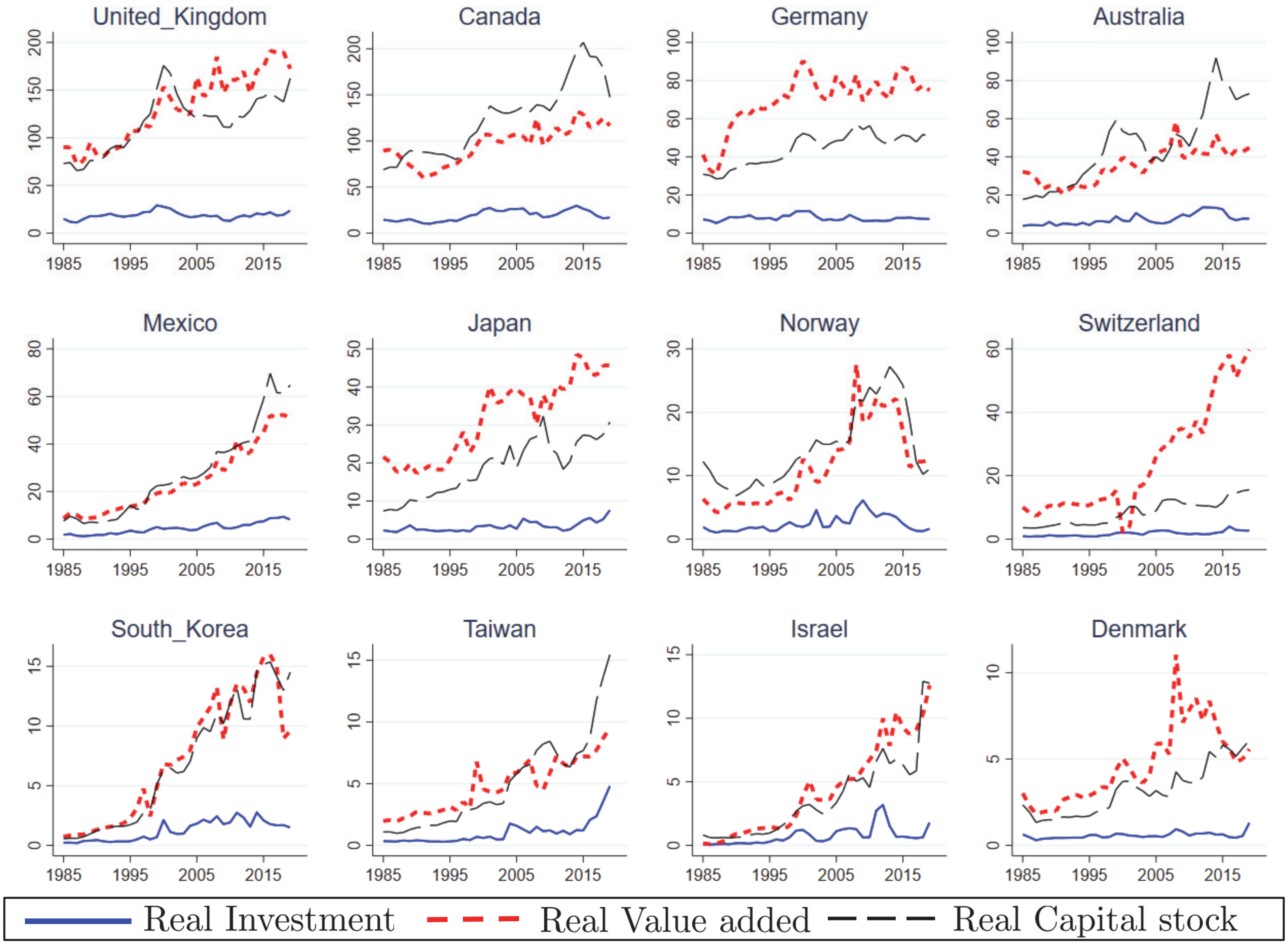

***Notes***: Aggregate real value-added-to-capital ratios (Y/K) of all domestic and foreign firms owned by US parents (i.e., US "persons" including legal entities and individuals) and presented separately for i) home firms operating within US borders and denoted by headquarter (*HQ*) (in blue solid line), ii) foreign affiliates operating in foreign country *f* (red short dash), iii) foreign affiliates in the rest of the world (ROW) (black long dash). For each country *f* plot, the ROW aggregate Y/K ratio is a weighted average of the Y/K ratios of foreign affiliates operating elsewhere outside *f* and *HQ*. The ROW weights are based on the observed aggregate size of foreign affiliate capital stocks. There are 44 countries. For presentational and comparison purposes, the first set of 16 countries are presented such that the top row has countries with large foreign affiliate capital stocks, while middle and bottom rows include countries with middle- and smaller-size stocks. Remaining countries presented alphabetically. Data 1985-2019, details in data section.

**Figure IA.3: Continued**



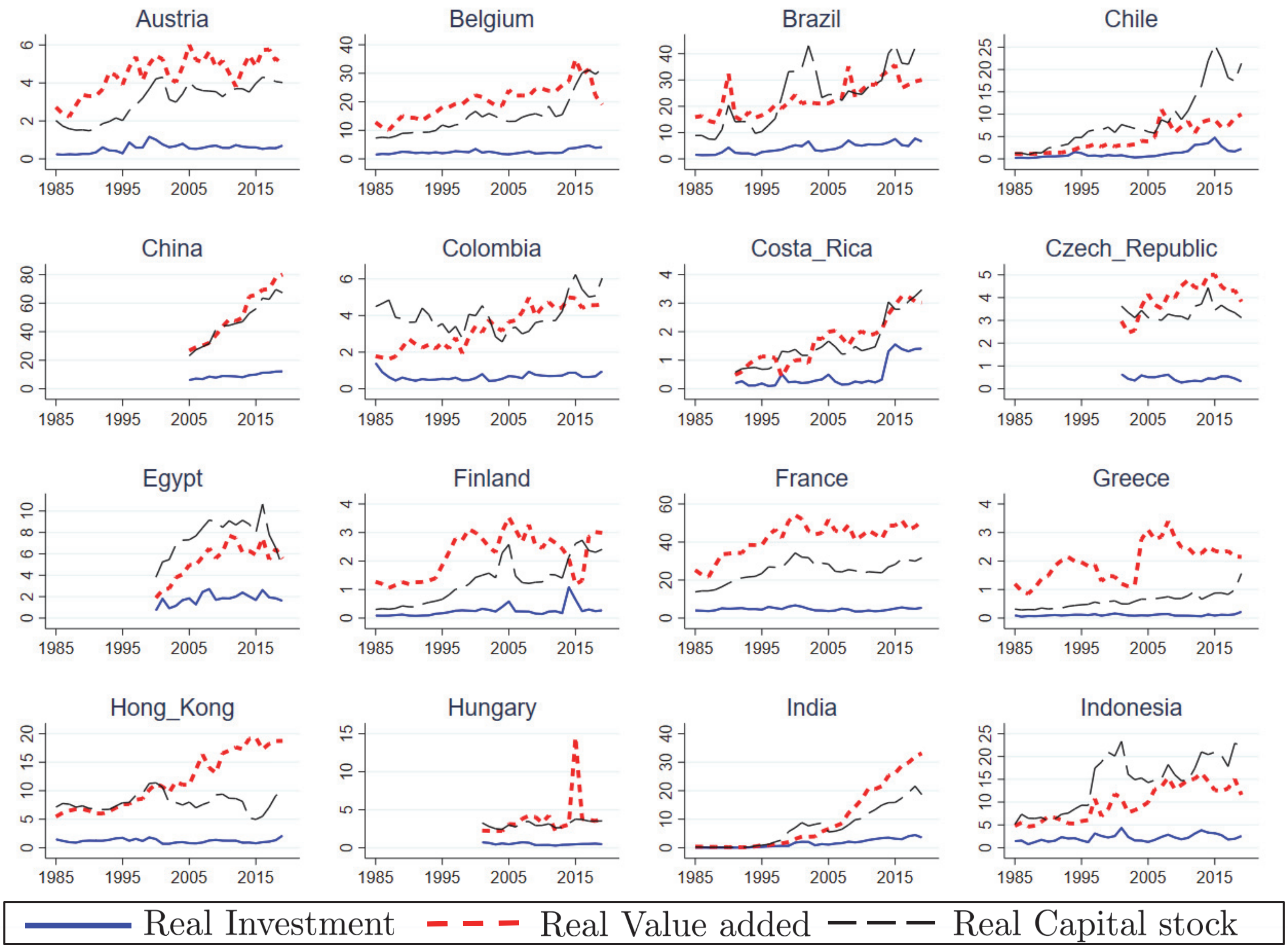

***Notes**:* Aggregate real value-added-to-capital ratios (Y/K) of all domestic and foreign firms owned by US parents (i.e., US "persons" including legal entities and individuals) and presented separately for i) home firms operating within US borders and denoted by headquarter (*HQ*) (in blue solid line), ii) foreign affiliates operating in foreign country *f* (red short dash), iii) foreign affiliates in the rest of the world (ROW) (black long dash). For each country *f* plot, the ROW aggregate Y/K ratio is a weighted average of the Y/K ratios of foreign affiliates operating elsewhere outside *f* and *HQ*. The ROW weights are based on the observed aggregate size of foreign affiliate capital stocks. There are 44 countries. For presentational and comparison purposes, the first set of 16 countries are presented such that the top row has countries with large foreign affiliate capital stocks, while middle and bottom rows include countries with middle- and smaller-size stocks. Remaining countries presented alphabetically. Data 1985-2019, details in data section.

**Figure IA.4: Foreign affiliates of US parents have real output, investment, & capital stocks in the billions of US$2015 in small and large foreign countries, 1985-2019**



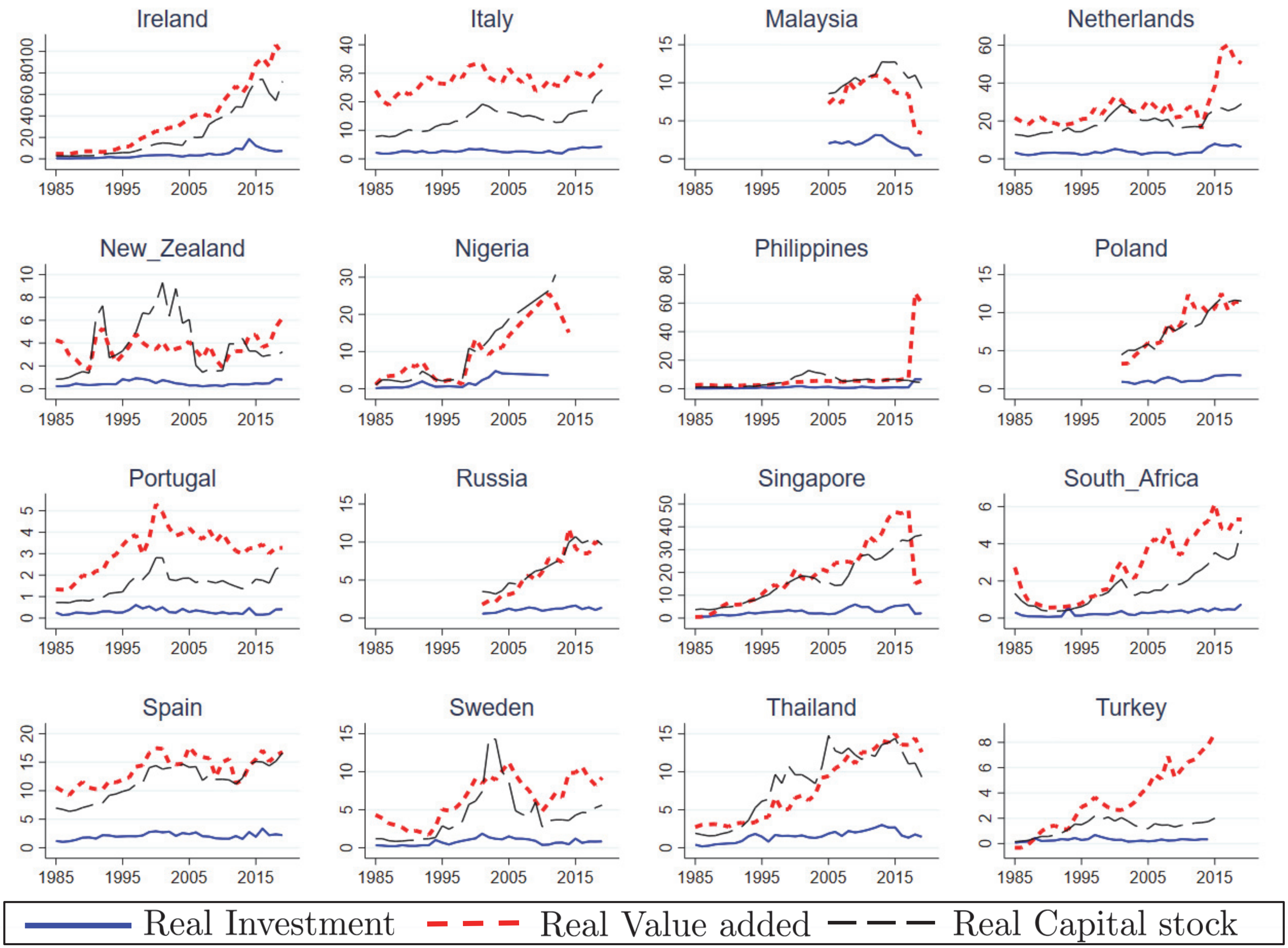

*Notes:* Aggregate real production quantities of *US-owned foreign affiliates*. Value added, investment expenditures in physical capital, and capital stocks (net property plant and equipment) of foreign affiliates are reported, for presentational purposes, in real constant US$2015 billions from 1985 to 2019, yet are always measured in real foreign currency values in all tests of the paper. Real investment flows are denoted by the solid blue line, real value added in red short dashed, and real capital stock in black long dashed. There are 44 countries. For comparison purposes, the first set of 16 countries are presented such that the top row has countries with large foreign affiliate capital stocks, while middle and bottom rows include countries with middle- and smaller-size stocks. Remaining countries presented alphabetically. See data section for details.

**Figure IA.4: Continued**



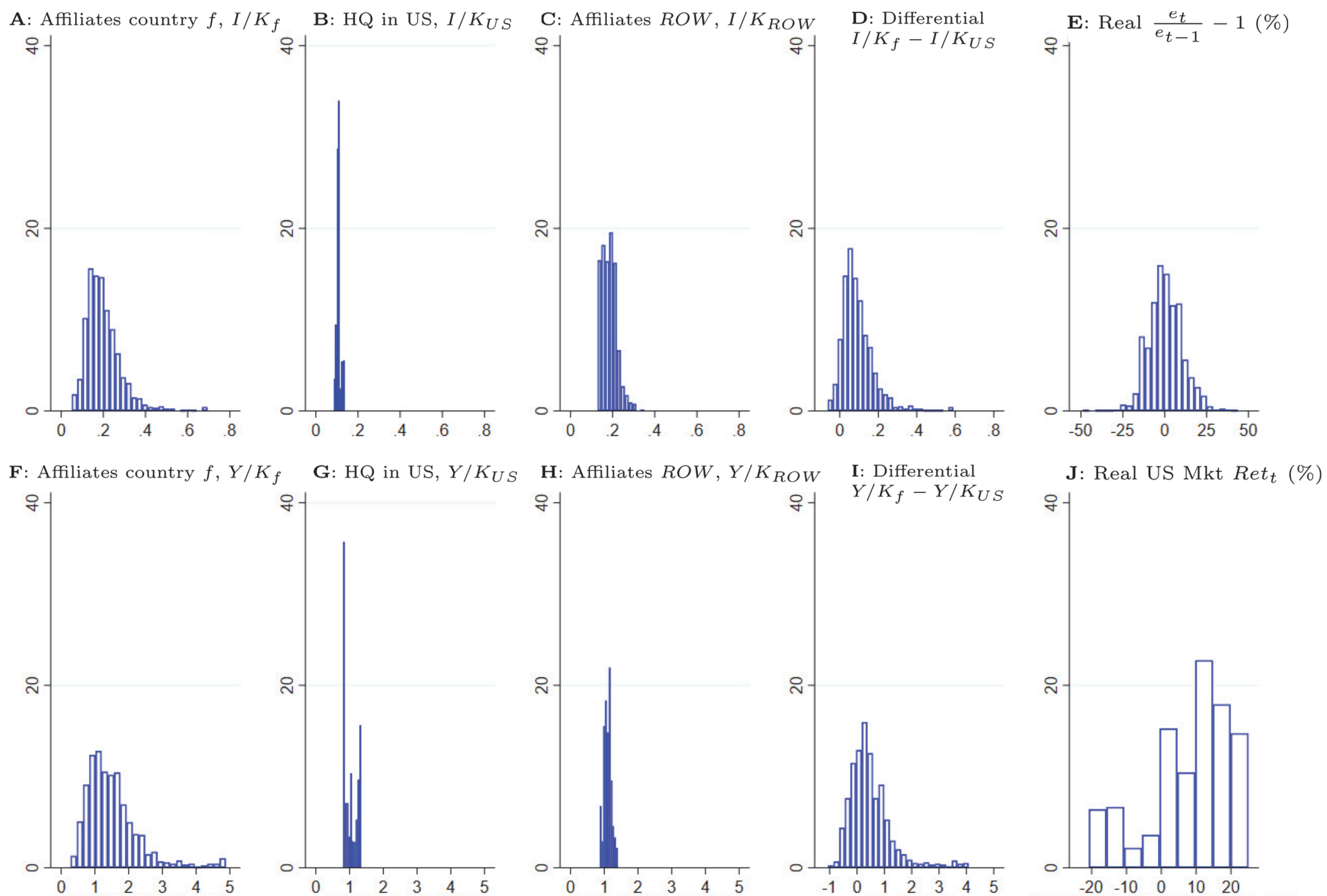

***Notes**:* Aggregate real production quantities of *US-owned foreign affiliates*. Value added, investment expenditures in physical capital, and capital stocks (net property plant and equipment) of foreign affiliates are reported, for presentational purposes, in real constant US$2015 billions from 1985 to 2019, yet are always measured in real foreign currency values in all tests of the paper. Real investment flows are denoted by the solid blue line, real value added in red short dashed, and real capital stock in black long dashed. There are 44 countries. For comparison purposes, the first set of 16 countries are presented such that the top row has countries with large foreign affiliate capital stocks, while middle and bottom rows include countries with middle- and smaller-size stocks. Remaining countries presented alphabetically. See data section for details.

## Figure IA.4: Continued



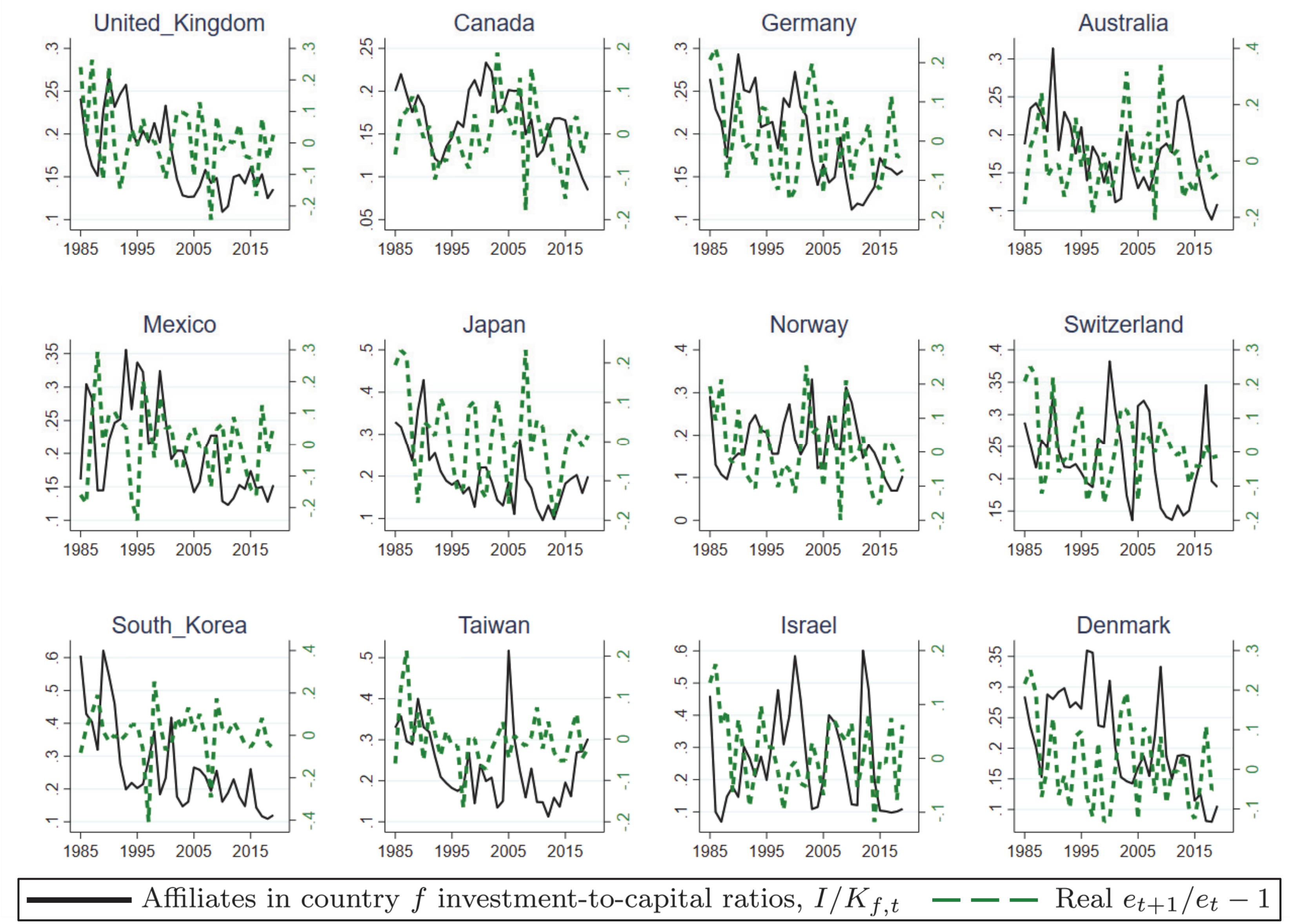

***Notes**:* Aggregate real production quantities of *US-owned foreign affiliates*. Value added, investment expenditures in physical capital, and capital stocks (net property plant and equipment) of foreign affiliates are reported, for presentational purposes, in real constant US$2015 billions from 1985 to 2019, yet are always measured in real foreign currency values in all tests of the paper. Real investment flows are denoted by the solid blue line, real value added in red short dashed, and real capital stock in black long dashed. There are 44 countries. For comparison purposes, the first set of 16 countries are presented such that the top row has countries with large foreign affiliate capital stocks, while middle and bottom rows include countries with middle- and smaller-size stocks. Remaining countries presented alphabetically. See data section for details.

**Figure IA.5 : Histograms of the production-based fundamentals of exchange rates, including US aggregates for *I/K* & *Y/K* ratios at home & abroad, the US stock market return, & foreign currency appreciation rates, 1985 to 2019**



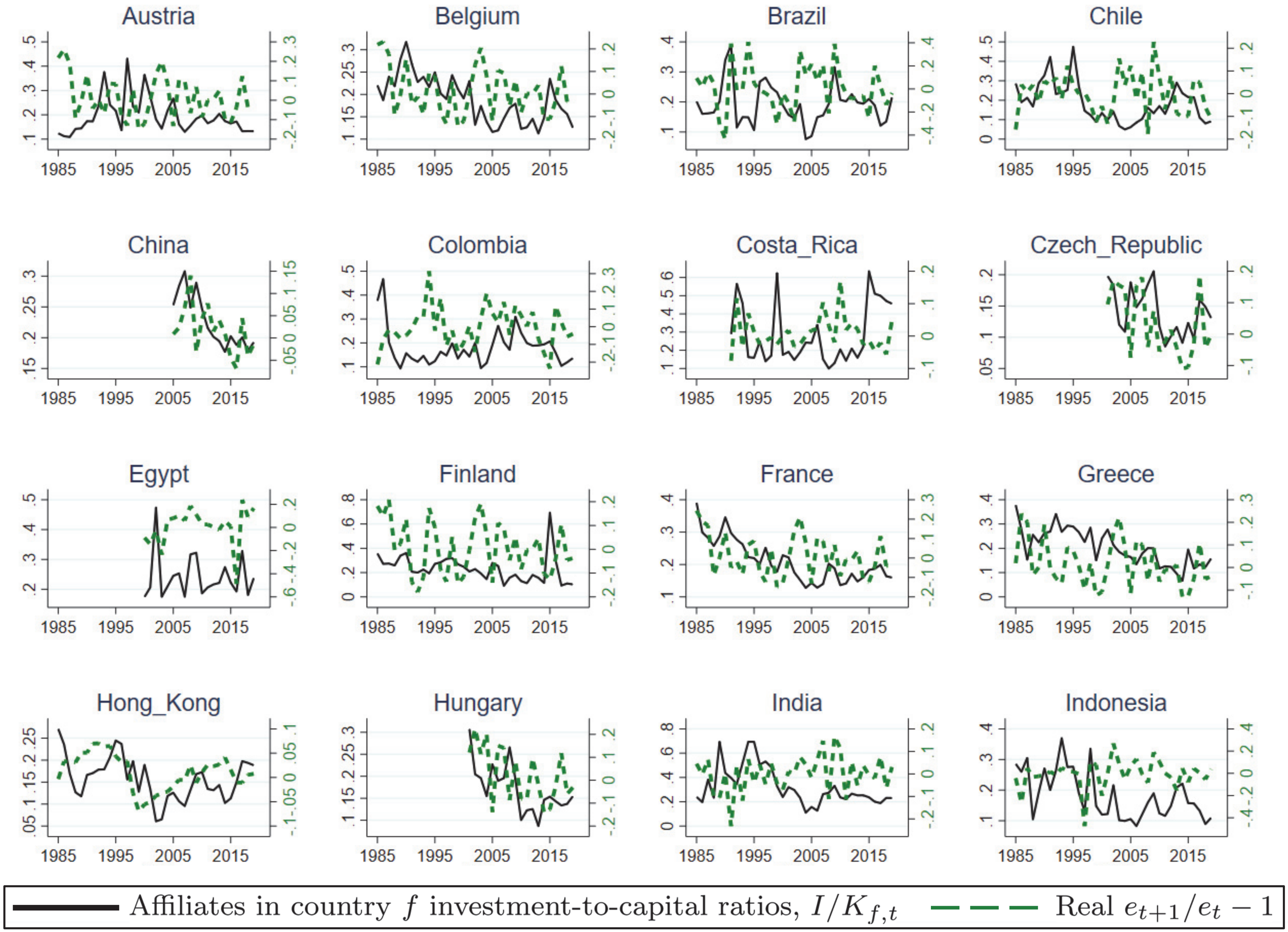

***Notes**:* Histograms. Foreign affiliate aggregates are in foreign currency real terms (e.g., in constant 2015 British pounds for investments in the UK), while HQ data are in real US dollars. Histograms for investment-to-capital *(I/K)* and value-added-to-capital ratios *(Y/K)* are for all domestic and foreign firms owned by US parents and presented separately for i) home firms operating within US borders and denoted by headquarter (*HQ*), ii) foreign affiliates operating in foreign country *f,* iii) foreign affiliates in a weighted average rest of the world economy outside of a given country *f* and home *HQ*. The ROW weights are based on the observed aggregate size of foreign affiliate capital stocks. Sample includes 44 foreign countries with aggregate affiliates of US parents and with annual data from 1985 to 2019. Histograms for annual growth rates in bilateral exchange rates between the US and foreign country *f* and the annual US stock market return from CRSP are in real terms and presented as a percentage. See data section for details.

**Figure IA.6: Foreign affiliate real investment rates & future foreign currency appreciation rates, 1985-2019**



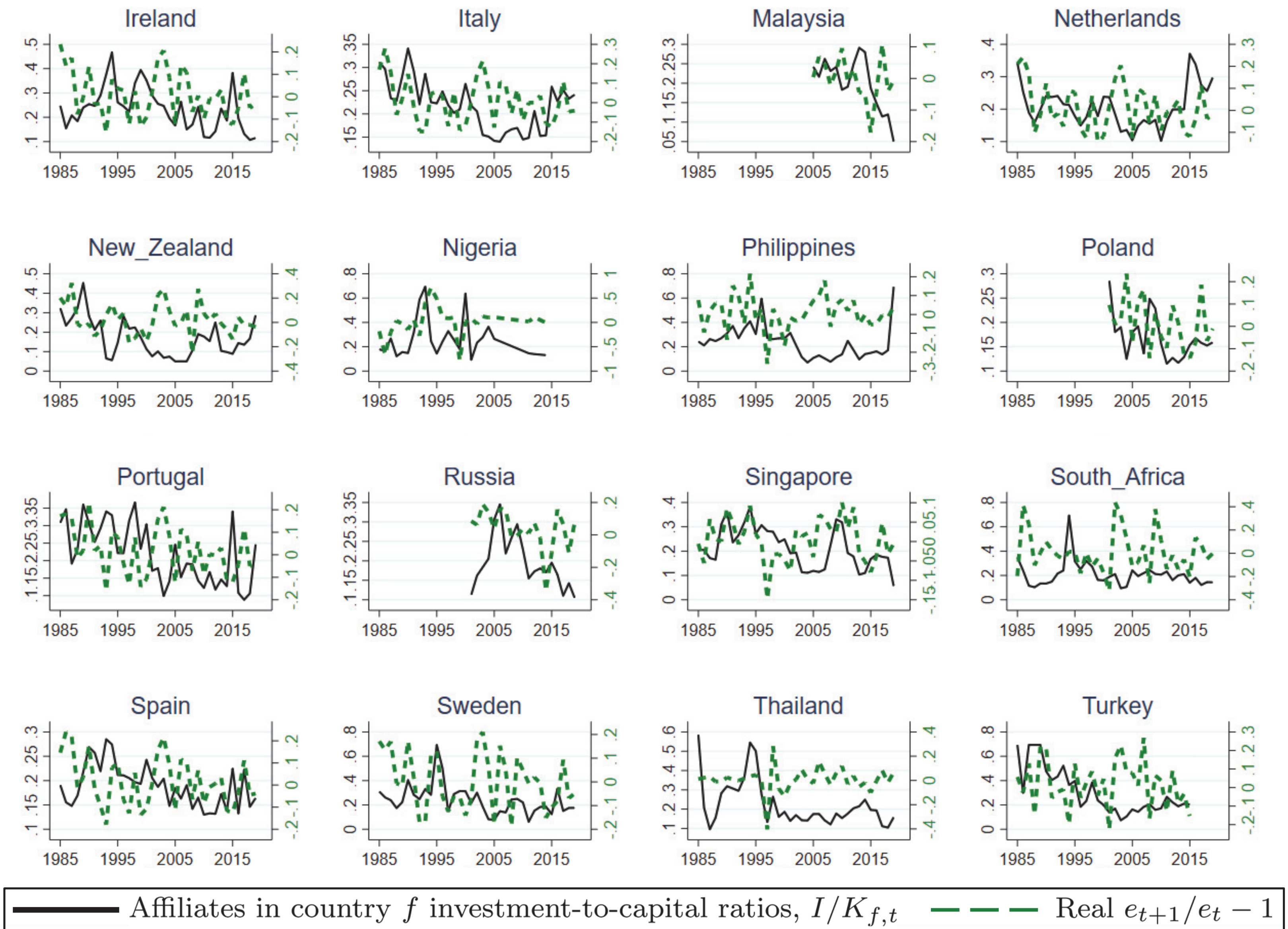

*Notes*: The figure plots (in solid black line on the left Y axis) the year *t* aggregate investment-to-capital ratio *(I/K)* of foreign affiliates owned by US parents against the year *t*+1 growth rate in bilateral real exchange rates (green dashed right Y axis). Foreign affiliate data are in foreign currency real terms (e.g., in constant 2015 British pounds for investments in the UK). Real exchange rates are in real US dollars per unit of real foreign country currency, with an increase denoting a real appreciation of the foreign currency. Sample includes 44 foreign countries with annual data from 1985 to 2019. For presentational purposes, the first set of 16 countries are presented such that the top row has countries with large foreign affiliate capital stocks, while middle and bottom rows include countries with middle- and smaller-size stocks. Remaining countries presented alphabetically. Foreign affiliate data are from the Bureau of Economic Analysis (BEA) dataset on Activities of US Multinational Enterprises (MNEs). See data section for details

## Figure IA.6: Continued



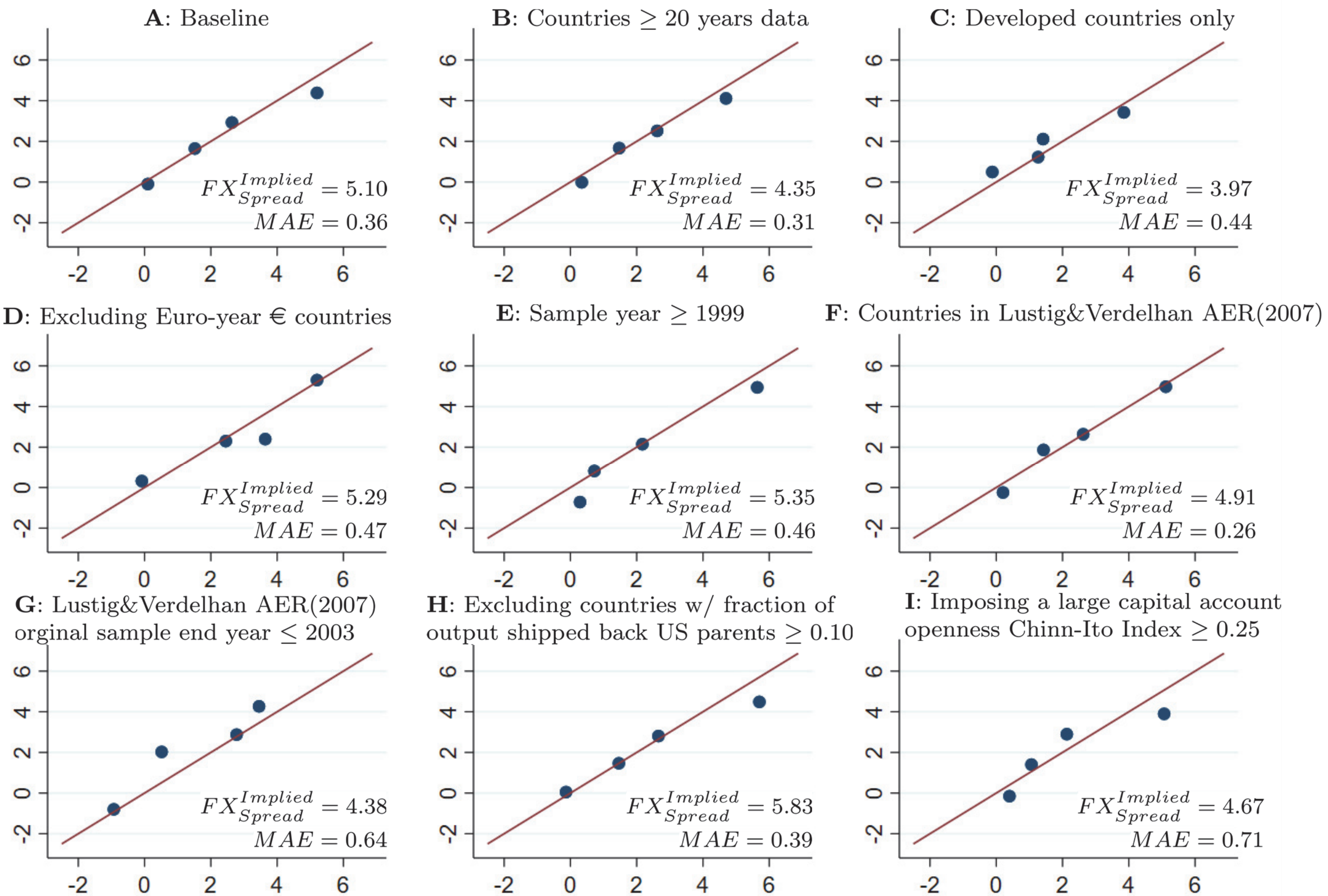

***Notes***: The figure plots (in solid black line on the left Y axis) the year $t$ aggregate investment-to-capital ratio *(I/K)* of foreign affiliates owned by US parents against the year $t$+1 growth rate in bilateral real exchange rates (green dashed right Y axis). Foreign affiliate data are in foreign currency real terms (e.g., in constant 2015 British pounds for investments in the UK). Real exchange rates are in real US dollars per unit of real foreign country currency, with an increase denoting a real appreciation of the foreign currency. Sample includes 44 foreign countries with annual data from 1985 to 2019. For presentational purposes, the first set of 16 countries are presented such that the top row has countries with large foreign affiliate capital stocks, while middle and bottom rows include countries with middle- and smaller-size stocks. Remaining countries presented alphabetically. Foreign affiliate data are from the Bureau of Economic Analysis (BEA) dataset on Activities of US Multinational Enterprises (MNEs). See data section for details

## Figure IA.6: Continued



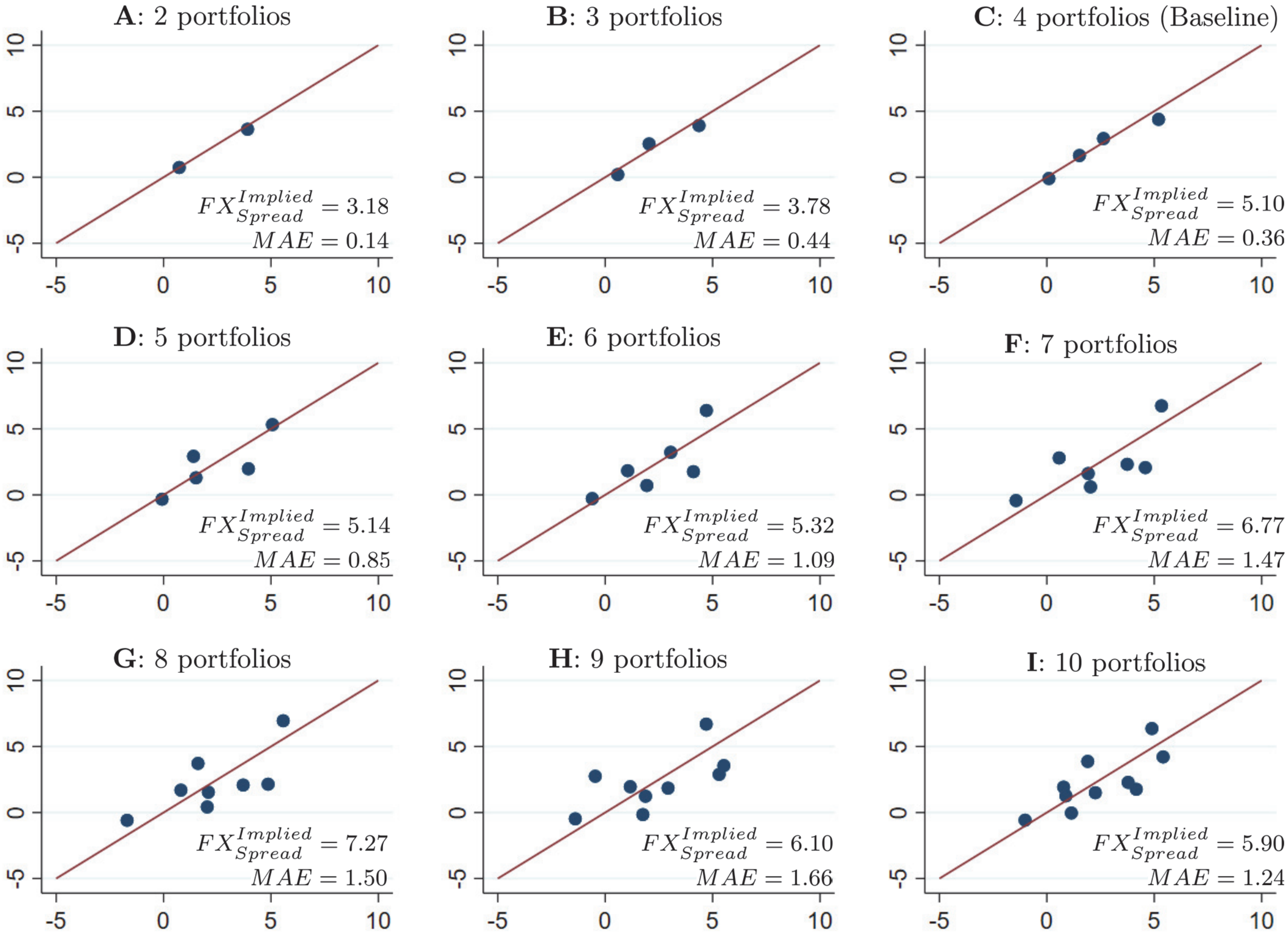

***Notes***: The figure plots (in solid black line on the left Y axis) the year $t$ aggregate investment-to-capital ratio *(I/K)* of foreign affiliates owned by US parents against the year $t$+1 growth rate in bilateral real exchange rates (green dashed right Y axis). Foreign affiliate data are in foreign currency real terms (e.g., in constant 2015 British pounds for investments in the UK). Real exchange rates are in real US dollars per unit of real foreign country currency, with an increase denoting a real appreciation of the foreign currency. Sample includes 44 foreign countries with annual data from 1985 to 2019. For presentational purposes, the first set of 16 countries are presented such that the top row has countries with large foreign affiliate capital stocks, while middle and bottom rows include countries with middle- and smaller-size stocks. Remaining countries presented alphabetically. Foreign affiliate data are from the Bureau of Economic Analysis (BEA) dataset on Activities of US Multinational Enterprises (MNEs). See data section for details

**Figure IA.7: Robustness checks to varying the sample selection & data filters when pricing the carry trade, realized vs. production-implied currency excess returns, 1987-2019**




***Notes**:* Robustness checks to the performance of the baseline model in pricing the carry trade when varying data filters and sample. Plot in panel A corresponds to the baseline specification presented in Figure 4A and Table 6. Panel B restricts the sample of countries to those with at least 20 years of data required for the GMM estimation (up from the baseline 10 years of data). Panel C restricts the sample to include only developed countries, while Panel D excludes all Euro-year countries that adopted the Euro after its introduction in 1999. Panel E is for the subsample of all observations starting in 1999. Panel F restricts the sample to include only the countries that overlap with the original carry trade sample of Lustig and Verdelhan (2007). Panel G is similar but further restricts the sample to the original end year 2003 of that paper. Panel H restricts the sample to only countries with a minimal fraction of aggregate output shipped back to US parents by foreign affiliates, i.e., countries whose fractions does not exceed 10% on average over the sample period. Panel I keeps only countries whose capital account openness is large with a threshold ≥ 0.25 using the Chinn and Ito (2006) KAOPEN index (which ranges from 0 to 1). Implied high-minus-low carry trade spreads and mean absolute pricing error (MAE) across portfolios are reported in each plot. A *Chi*-squared test fails to reject the null in each plot that all portfolio pricing errors are jointly zero.

## Figure IA.8: Robustness to different levels of portfolio aggregation when pricing the carry trade, realized vs. production-implied currency excess returns, 1987-2019




***Notes**:* Figure presents robustness checks to the performance of the model in pricing the carry trade when varying the number of portfolios. Each plot corresponds to a choice of 2 to 10 portfolios when sorting countries every year in the cross section, see Lustig and Verdelhan (2007), and based on previous year real interest rate differentials between foreign countries and the US. The baseline results of 4 portfolios is the same as the results shown in Figure 4A, see notes there for additional details. Each plot presents average predicted vs. realized foreign exchange (FX) real excess returns. Returns are expressed as a percentage per annum. Each plot presents statistics for the average implied high-minus-low carry trade spread between the extreme portfolios and the mean absolute pricing error (MAE) across portfolios. The *p*-value of a *Chi*-squared test of joint significance fails to reject the null in each plot that all portfolio pricing errors are jointly equal to zero. Cross-sectional carry trade risk premiums using portfolios follow Lustig and Verdelhan (2007). See the econometric methodology and data sections for details on construction of model-implied premiums inferred directly from observed aggregate production data at home & abroad of US firms.