Innovation and Financial Disclosure*

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ABSTRACT: We examine how financial disclosure policy affects a firm manager's strategy to innovate within a two-period bandit problem featuring two production methods: an old method with a known probability of success, and a new method with an unknown probability. Exploring the new method in the first period provides the manager with decision-useful information for the second period, thus creating a real option that is unavailable under exploiting the old known production method. Voluntary disclosure of the firm's financial performance provides the manager with another option to potentially conceal initial failure from the market. The interaction of these two options determines the manager's incentive to explore. In equilibrium, a myopic manager who cares about the interim market price may over- or under-explore compared to the optimal exploration strategy that maximizes firm value. Our analysis shows that firms operating in an environment with voluntary disclosure early in the trial stage and mandated requirement later are most motivated to explore, while firms subject to early mandated disclosure and late voluntary disclosure are least likely to do so. We also provide empirical predictions about the link between the disclosure environment and the intensity and efficiency of corporate innovation.

KEYWORDS: Innovation, bandit problem, exploration and exploitation, voluntary disclosure, real option, disclosure option
JEL Classification: D82, O30, O31, O32, O33, M41

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1 Introduction

In modern economies, firms constantly face the strategic choice between exploitation of existing technologies and exploration of new methods. While exploitation offers familiarity and certainty of the status quo, exploration could lead to innovation, discovery, and improved efficiency if successful. Firms must make optimal decisions on how much to explore, as both under-exploration and over-exploration result in a loss of efficiency. Many factors influence firms' incentive to innovate and explore; firms facing financial markets can be especially affected by the associated disclosure requirements. In this paper, we study the effects of disclosure policy on how firms innovate. Specifically, we examine a myopic manager's strategy in choosing exploration under different disclosure requirements and describe the scenarios that foster value-maximizing innovation decisions.

We use a two-period and two-armed bandit problem as in Dye (2004) and Manso (2011) to study a firm's strategic decision of innovation.[1] The first period represents a trial on a small scale, while the second period represents a full-scale commercial launch. There are two production methods available for the firm to use: an old method and a new method. The old method has been tried and tested in the past, with a known probability of success. The probability of success of the new method, however, is unknown. The new method is ex-ante believed to be less profitable than the old method, but the exact profitability can only be learnt after trying, and may turn out to be higher than the profitability of the old method. The manager of the firm can choose to explore the new method or exploit the old method. In equilibrium, the firm decides whether to explore in the first period—if the new method is successful, the firm will find it optimal to use it in the second period; if the new method fails, the firm can still switch to the old conventional method. Exploration thus provides the firm with a real option in the second period. If the firm decides to use the old method in the

[1] The bandit problem is often used to analyze problems involving the tension between exploration and exploitation, i.e., trying out new arms to find the one with the highest payoff versus using the arm with the known payoff. For a discussion of the bandit problems in more general forms, see Bergemann and Valimaki (2006).

first period, it will not switch to the new method later as starting to explore in the second period is not optimal. To capture the firm's innovation activity, we define exploration as the manager's choice to try out the new method in the first period.

The cash flows associated with the second period are a multiple of those in the first period, capturing the potential growth from the trial stage to the full launch of the business. While these cash flows are not directly observable, the firm's accounting system enables the manager to learn the financial outcome of the operations, whether success or failure, with a certain probability at the end of each period.[2] The manager could subsequently disclose the outcome to the investors. There are two types of disclosure scenarios: mandated full disclosure or voluntary disclosure, with the firm having the option to either conceal information or disclose truthfully (Dye, 1985; Jung and Kwon, 1988). These two disclosure scenarios can be interpreted in terms of a firm's status as either public or private. While a public firm faces mandated disclosure requirements, a privately-held firm has the discretion to disclose voluntarily. In either case, after observing the disclosure (or non-disclosure), investors form expectations about the firm's future cash flows. These expectations are reflected in what we refer to as the market price. For a public firm, this price is listed on a stock exchange; for a private firm, it would be represented by the firm's valuation as determined by investors. There is a short-term price formed after the first period, as well as a long-term price after the second period. In the absence of managerial myopic preference for price maximization, the first-best exploration strategy that maximizes the long-term firm value depends on the growth of cash flows from the first to the second period and has a simple single-threshold feature. Intuitively, exploring the new method is only worthwhile if the potential second-period profit growth is sufficiently high; otherwise, the firm should simply choose to exploit the old method.

[2]The financial success or failure of a new project or product is not always observed immediately, especially at trial stage. Even a project that has attained technical success is not necessarily immediately profitable. As the cost incurred and benefits received are not always easily identified and traced to each single project in a large company with multiple business lines, a better accounting system would allow the manager to observe the future profitability of the individual projects more clearly.

As a baseline, we first examine a benchmark where there is only one disclosure and the long-term firm's price is perfect, i.e., the price at the end of the second period is formed *after* the final cash flow is realized, and thus equals the liquidation firm value. This setup allows us to focus on the disclosure tension in the first period to develop the basic intuition. In equilibrium, the firm's exploration strategy is characterized with two thresholds (and three regions), depending on the potential growth in cash flows at the full-scale commercial stage in the second period. At the two ends, the manager either always explores or never explores. In the middle region, she adopts a mixed strategy of "exploring with a probability". In this benchmark setting, the manager explores strictly less than under the first-best scenario, but is more likely to explore the new method under voluntary disclosure than under mandated disclosure. Lacking the disclosure option under the mandatory regime, the myopic manager favors the known old method with a higher ex ante probability of success to boost the price. The short-term advantage of old method thus discourages the myopic manager from exploration.

The benchmark demonstrates two forces at work simultaneously. Exploration provides a real option for the manager to try out a new method. If the new method fails in the first round, the manager could always revert to the conventional method. Voluntary disclosure provides another option for the manager to potentially inflate the investors' expectation of the firm value by not reporting an initial failure. These two options work together to encourage exploration. When exploration generates a first-period success, the voluntary disclosure rewards the myopic manager by improving first-period pricing; when exploration results in a first-period failure, the voluntary disclosure partially protects the manager by allowing to conceal the bad news. Voluntary disclosure, therefore, gives the manager more incentive to explore by allowing the firm to hide the short-term disadvantage of exploration while taking advantage of the learning effect of exploration for the long-term.

In the full model, the second-period price is formed before realization of the final cash flow and based on the second disclosure. Compared to the benchmark setting, now the timing of disclosure also plays a role. We show that firms explore the most with disclosure mandated only in the second period, and they explore the least with disclosure mandated only in the first period. Under our interpretation of the disclosure scenarios as firms' status of being public or private, our results would indicate, all else equal, a private company expecting to go public would have the highest incentive to innovate, while a public firm expecting to delist would innovate the least. This is because allowing voluntary disclosure at the early stage of innovation provides the manager with the disclosure option, thereby encouraging exploration. In contrast, disclosure mandated at the later stage ensures that the second-period price more accurately reflects the final operating results, enabling the manager to fully benefit from the long-term success of exploration. Further, the information benefit of voluntary disclosure at this late stage is absent because the real production option (of switching to another method) is already sunk at that point.

The situation becomes more nuanced when the disclosure policy remains constant over time, for instance, when a public firm stays public or a private firm remains private. These two scenarios combine elements that provide conflicting incentives for the manager to explore, since disclosure mandated in the first period hinders exploration but disclosure mandated in the second period encourages exploration. We show that a constant voluntary disclosure scenario induces more innovation than a constant mandated disclosure scenario if the manager is sufficiently myopic. The opposite is true when the manager is sufficiently forward-looking *and* the firm's internal information quality in the second period is not sufficiently high.

We also examine the efficiency of innovation and show that the firm could both over- or under-explore in equilibrium. If the firm's internal information quality is significantly higher at the second stage as compared to the first stage, i.e., if the firm's manager can better discern the profitability of the project, then the manager has incentives to show success at the second period and may be overzealous in exploring the new method at the beginning of the first period.[3] In this case, when the size of the commercial launch of the project is limited, over-innovation becomes a problem, and the disclosure scenario with mandated disclosure early and voluntary disclosure late would achieve the highest efficiency by countering the manager's incentive to over-explore. Conversely, if the firm's information system is more precise at the first stage, only under-exploration can happen in equilibrium, as the manager is more concerned about showing the short-term success instead of exploring the new method. In this case, the disclosure scenario that mandates disclosure late would induce a level of exploration closest to the optimal level.

Our model relies on two critical assumptions to capture the tension in exploration. First, the manager in our model is somewhat myopic, and cares about both the short-term and long-term prices of the firm. Managerial myopia exists for many reasons, such as manager's short employment horizon, or shareholders' need to sell a portion of the firm to the next generation of shareholders. In our model, managerial myopia underlies the conflict between long-term benefit and short-term cost of exploration. Second, the investors receive the firm's report on operational outcome but do not observe the method being used. This friction could stem from the firm's difficulty in communicating the method in a credible manner, or simply the insurmountable cost of disclosing the firm's proprietary information. In reality, full disclosure of a new technology or production method itself is often both complex and costly, making it practically infeasible for the firm.

Our results lend themselves to empirical tests. The growing empirical literature on innovation and disclosure does not provide conclusive evidence on the link between disclosure and innovation. Our predictions could shed some light by offering potential cross-sectional analysis. By interpreting firms' exogenous disclosure requirements in terms of their public or private status, we make several empirical predictions regarding corporate innovation and

[3] A firm's cost accounting information for innovation projects could indeed improve from the initial trial to the commercial launch later. During the trial phase of a project, costs are generally project-specific, focusing on design, testing, and development. In contrast, during subsequent large-scale production, costs become more operational, encompassing raw materials, labor, and logistics. Costs can be more directly attributed to each unit of product, allowing for more precise cost-per-unit calculations for profitability analysis.

efficiency. Specifically, we anticipate that private firms planning to go public will consistently innovate more, whereas public firms intending to delist will generally innovate less, compared to firms that either remain public or stay private. Moreover, among firms that maintain their status, private firms tend to innovate more than public firms when their managers exhibit some degree of myopia—such as when compensation contracts emphasize short-term or price-based incentives. Furthermore, we predict that private firms aiming to go public will typically achieve higher innovation efficiency, whereas public firms planning to delist will likely exhibit the lowest efficiency.

The vast literature on exploration and innovation can be dated back to Schumpeter (1942), who argued that exploration is the key to economic development and corporate profitability.[4] While there are some studies that examine optimal innovation through contracting or delegation (e.g., Manso, 2011; Dutta and Fan, 2012; Laux and Ray, 2020; Baldenius and Yang, 2023, etc.)[5], there is very little research on the effect of disclosure policy on innovation, with the exception of Laux and Stocken (2018). Laux and Stocken (2018) examine the stringency and enforcement of accounting regulation on an entrepreneur's incentive to engage in research and development. In their model, the entrepreneur decides how much effort to exert into the R&D activities before issuing a financial report, a regulatory agency then checks the report and imposes a penalty on the entrepreneur if the report is found to be incompliant with the reporting standards. The optimal reporting standard is determined by the trade-off between the investment efficiency it induces and the cost of regulatory compliance. Our paper differs from that of Laux and Stocken (2018) in both the research question and the modeling approach. We focus on the dynamic property of disclosure policy, while Laux and

[4]See Hall and Lerner (2010) for a detailed review on corporate innovation and financing.

[5]Baldenius and Yang (2023) also follow Manso (2011) in adopting a two-period and two-armed bandit problem. In their paper, an agent is delegated with the implementation of a project and could communicate the interim state of the project to the principal through cheap talk. They show that experimentation and communication are complements and a firm delegating the project to the manager actually experiments more due to the benefits provided by the strategic communication. Their setting is significantly different from ours, as they focus on internal agency conflicts, while we focus on the effects of financial disclosure to investors in the financial market.

Stocken (2018) focus on the effect of regulatory enforcement. Further, we model exploration using a bandit problem and disclosure as a voluntary disclosure as per Dye (1985) and Jung and Kwon (1988), while Laux and Stocken (2018) follow Dye (2002).

Our study is also related to prior research on project selection and disclosure. Ben-Porath et al. (2018) examine a manager's choice between two projects, with one project being riskier and ex-ante inferior. If the manager could commit to full disclosure, she would choose the safer project. However, if the manager can disclose the interim signal voluntarily, she always chooses the riskier project because voluntary disclosure allows her to benefit from a good signal but temporarily hide the bad signal. In Guttman and Meng (2020), while the option value of voluntary disclosure is still present, observing the interim signal helps the manager make the investment decision more efficiently. The manager therefore must trade off these two effects when choosing the projects. Our model is different from Ben-Porath et al. (2018) and Guttman and Meng (2020) in both results and the mechanism. We study the effect of disclosure on a strategy choice of exploration vs exploitation rather than a project selection. Due to the real option embedded in the bandit problem, the method can be reversed after the first period. Therefore there is no method that is ex-ante preferred uniformly.[6] Further, the manager in Ben-Porath et al. (2018) and Guttman and Meng (2020) would be better off if she could commit to full disclosure at the interim stage. In our model, in contrast, full early disclosure will lead to even greater loss of efficiency. Furthermore, we examine the effect of disclosure in a dynamic setting with potential changes in the disclosure regime and show that the interim disclosure option can be value-improving or value-destroying.[7]

Lastly, our research contributes to the literature on real effects. Generally, real effects models involve a myopic manager who could undertake a costly investment to inflate the

[6] In fact, project selection can be understood as a subgame of strategy choice. In the second period of our model, after learning the outcome from the first period, the manager has two methods to choose from but with one being strictly more efficient. The game would then proceed just like Ben-Porath et al. (2018) and Guttman and Meng (2020) if we allow another round of voluntary disclosure and price formation.

[7] Our model also relates to the models of Arya and Glover (2003) and Caskey and Hughes (2012), who consider abandonment options in optimal contracting settings. Different from these papers, we explicitly model a bandit problem, consider its dynamics, and study the voluntary disclosure option of the manager.

interim signal and manipulate market expectation.[8] Many of these studies require the accounting disclosure to be mandatory and subject to bias. One exception is Wen (2012), in which the manager has the option to voluntarily disclose the investment information, or to withhold unfavorable information by pooling with the manager who did not undertake the investment. Wen (2012) finds that voluntary disclosure can bring both positive and negative effects on the firm's investment efficiency. Another important paper related to ours is Gigler, Kanodia, Sapra, and Venugopalan (2014). They find that increased frequency of disclosure may exacerbate managerial short-termism. They consider two scenarios in which the interim disclosure is either mandated or not but do not allow for voluntary disclosure. In contrast, we consider different disclosure regimes, where disclosure can be mandated in different times in a dynamic setting. Thus, we focus on the interaction of the voluntary disclosure option and the real option and establish under which reporting environment the manager chooses the most efficient action. Furthermore, our research questions are different. While Gigler et al. (2014) examine whether disclosure should be frequent or infrequent, we focus on the timing of disclosure, i.e., we compare reporting scenarios in which disclosure can be mandated early or late.

2 Model

2.1 Bandit Problem

Following Manso (2011), we construct a two-armed and two-period bandit problem to capture a manager's decision about exploration and exploitation. One arm of the bandit is an old conventional method and the other a new method, indexed as $i = \{o, n\}$. The old method has a known probability of success, while the new method's probability of success is unknown until the method is tried out. There are two periods of operations, indexed as $t = \{1, 2\}$.

[8] For more detailed discussion, see Kanodia and Sapra (2016) for a review on how different accounting measures interact with firms' investment decisions in the presence of managerial myopia.

At the beginning of each period, the manager decides whether to explore the new method or exploit the old one by choosing $a_t \in \{o, n\}$. Since there are two periods in the game, without considering the interim signals, there are four combinations of actions that could occur in equilibrium: $\{a_1 = n, a_2 = n\}$, $\{a_1 = n, a_2 = o\}$, $\{a_1 = o, a_2 = o\}$, and $\{a_1 = o, a_2 = n\}$. The first (last) two combinations involve exploring the new (old) method in the first period.

The output of period t, denoted as v_t, can be either a success (S) or failure (F), with $F < S$. We adopt the notation of Manso (2011) to capture the uncertainty of the output given a particular action. The probability of success under the old method, $Pr(v_t = S|a_t = o)$, $t \in \{0, 1\}$, denoted as p_o, is a known time-invariant positive constant. The first period output realization v_1 therefore does not change the manager's assessment of the success probability under the old method. The probability of success under the new method, $Pr(v_t = S|a_t = n)$, is a random variable denoted as $\tilde{p}_n$ with $0 < \tilde{p}_n < 1$. The unconditional expectation of the probability of success under the new method, denoted as p_n, is

$$p_n \equiv E[\tilde{p}_n] = E[Pr(v_t = S|a_t = n)], \tag{1}$$

for $t \in \{1, 2\}$, where $0 < p_n < 1$.

Different from the old method, the first-period outcome obtained under the new method is informative about the success probability of the second-period outcome if new method is continued.[9] Under the new method, the realization of the first-period output as a success contains two separate pieces of news: that the first period trial is a success, and that the next-period output has a higher probability of success if the new method is continued. Following Manso (2011), we directly specify the updated success probability under the new method conditional on the first-period outcome, denoted as $p_{n,v}$ where $v \in \{S, F\}$. We thus have

$$0 < p_{n,F} < p_n < p_{n,S} < 1, \tag{2}$$

[9]While the random output v_1 and v_2 are independent under the new method, the random outcome from the first period v_1 is correlated with the unknown probability $\tilde{p}_n$.

where $p_{n,F} \equiv E[\tilde{p}_n | v_t = F, a_t = n]$ and $p_{n,S} \equiv E[\tilde{p}_n | v_t = S, a_t = n]$. Similar to Manso (2011), we assume that the unconditional probability of success under the new method is lower than under the old one and that, conditional on success in the first period, the updated probability is higher. Formally,

$$p_{n,F} < p_n < p_o < p_{n,S}. \tag{3}$$

We also assume that the manager does not learn from the output under the old method about the success probability under the new method, that is $E[\tilde{p}_n | v_t = F, a_t = o] = E[\tilde{p}_n | v_t = S, a_t = o] = p_n$. This completes the basic model setup of the two-period two-arm bandit problem.[10]

The total cash flow associated with the project is $v = v_1 + \gamma v_2$ and is realized at the end of the second period.[11] We use the parameter γ to capture the difference in the cash flow implication between the two periods. In projects involving experimentation and innovation, the production scales between early trial and subsequent full-scale launch could be significantly different. For example, a pharmaceutical company could conduct a small regional trial before making the product available globally. The cash flow generated in the second period would therefore be amplified from that of the first period, even though the product itself is the same.[12] An orphan drug would typically have a small γ due to its limited market demand.

[10]This description closely follows the intuitive style of Manso (2011), which avoids fully specifying the true outcomes of random variable $\tilde{p}_n$. An alternate, perhaps more complete, specification would involve fully specifying the random variable $\tilde{p}_n$. Suppose $\tilde{p}_n \in \{p_H, p_L\}$ where $0 < p_L < p_H < 1$ and the unconditional probability $Pr(\tilde{p}_n = p_H) = \alpha$. Under this complete specification, we recover the unconditional success probability p_n as $p_n \equiv \alpha p_H + (1-\alpha)p_L$ and the learning follows the Bayes' rule:

$$p_{n,S} = \frac{\alpha p_H}{\alpha p_H + (1-\alpha)p_L} p_H + \frac{(1-\alpha)p_L}{\alpha p_H + (1-\alpha)p_L} p_L$$

In this case, any assumptions on $\{p_o, p_n, p_{n,S}, p_{n,F}\}$ can be equivalently regenerated by corresponding assumptions on $\{p_o, p_H, p_L, \alpha\}$.

[11]For example, if the firm fails in the first period but achieves success in the second period, the total cash flow would be $F + \gamma S$.

[12]Alternatively, we can specify the outcome in the first period as $v_1 = \{S, F\}$ and the outcome in the second period as $v_2 = \{\gamma S, \gamma F\}$, with γ capturing the amplification effect from the first period to the second period.

2.2 Information Structure and Disclosure Environment

The manager is not always informed about the result of the operations. In every period, with probability $g_t \in (0, 1)$, she gets to observe the financial outcome of the exploration/exploitation action and receives a perfect signal of the result $s_t = v_t$; with probability $1 - g_t$, she does not obtain any information and receives $s_t = \varnothing$. The manager's signal $s_t \in \{\varnothing, S, F\}$ is a noisy signal of v_t, while the probability g_t can be understood as the quality of the firm's internal information system. For example, a firm's cost accounting system largely determines how project costing and overhead allocations are performed. A better costing system could help the manager see how costs are attributed to a project and how profitable the project is, while a costing system of poor quality would lump all costs together and make it difficult to differentiate the results of each individual project.

Further, we allow $g_1 \neq g_2$ to capture the difference of the firm's information system quality at the trial and the full-scale stages of product launch, that is, g depends on the project stage. Since cost accounting information is produced on demand and often varies with the decision context, g_1 and g_2 could differ because cost accounting changes as the project moves from the initial stage to commercial launch. For example, during the initial phase of a project, costs are generally exploratory and experimental, focusing on design, testing, and development. These costs might include salaries for research staff, prototype materials, and patent application fees. In contrast, during large-scale production, costs become more operational, encompassing raw materials, labor for manufacturing, quality control, and logistics. The cost allocation may also change. In the initial trial stage, costs are more project-specific and may be difficult to allocate directly to individual products or services. However, in the commercial production stage, costs can be more directly attributed to each unit of product, allowing for more precise cost-per-unit calculations. [13]

[13] In order to avoid additional complexity, we assume, however, that the probability g_t is independent of the method a_t. If we did otherwise, we would introduce additional aspect of learning as the very fact of disclosure would be informative about the used method. In the robustness check (available upon request), we show that the main result of the paper still obtains when the probabilities are method-dependent but sufficiently

The firm faces an exogenous disclosure environment and may be required to disclose information to the public. While the production method is unobservable by outsiders and the manager is unable to disclose it due to high proprietary costs, the firm's financial information disclosure requirements can still be determined by its status as either public or private. A publicly-listed firm may be mandated to disclose, whereas a privately-held firm can choose whether or not to disclose information at its own discretion. In fact, a startup firm that might choose to disclose voluntarily could be required to do so once it goes public. Conversely, a public firm that is mandated to disclose would no longer have this requirement once it transitions to being a private entity.

If the financial disclosure is mandated, the firm's manager must always report $R_t = s_t$. That is, the manager cannot withhold any information from the firm's investors, and the investors know that the manager must truthfully report all information she has, including when she has no information. If disclosure is not mandated, the manager has the option to truthfully disclose $R_t = s_t$ or to withhold information by reporting $R_t = \varnothing$. Following Dye (1985) and Jung and Kwon (1988), we assume that disclosure is always truthful if the firm chooses to disclose. If there is no disclosure, it could be that the manager does not have any information or that she chooses to withhold unfavorable information.

One may question why information can be verified in a mandatory environment but not in a discretionary one. In a mandated environment, nondisclosure signals a lack of information. In contrast, in a discretionary environment, nondisclosure might indicate either no information or unfavorable information. We argue that this assumption is reflective of different disclosure regulations and their legal implications. Mandating disclosure typically entails predetermined reporting guidelines, compelling a firm to gather specific data for compliance.[14] In contrast, the discretionary regime does not have these guidelines, exempting firms from any specific data collection. This absence means a firm's information endowment

similar.

[14]For instance, mandated inventory disclosure would specify the LIFO or FIFO method, and R&D disclosure would dictate expensing or capitalization.

might remain unverifiable, even in a legal setting, due to the lack of data.

In the next stage, a price, P_t, is formed by investors after they observe R_t at the end of each period. This price does not necessarily have to represent a formal stock price of a listed firm; alternatively, it can be understood as the investors' expectation of the firm's value, especially in the case of a privately-held firm. The manager in our model is myopic and cares about both short-term price P_1 and long-term price P_2. The degree of myopia is denoted by δ. The manager is risk-neutral and wishes to maximize the expected value of U at the points of decision for production and disclosure, where

$$U = P_1 + \delta P_2. \tag{4}$$

The variable $0 \leq \delta < +\infty$ captures the manager's degree of myopia. When $\delta = 0$, the manager is infinitely myopic and only cares about the short-term price P_1; when $\delta \to \infty$, the manager has zero myopia and infinitely cares about the long-term price P_2.[15]

2.3 Time Line

Figure 1 summarizes the time line of events. In the beginning of the first period, the manager decides whether to explore the new method or not by taking action $a_1 = n$ or $a_1 = o$. Then the manager observes the result of the operations $s_1 = v_1$ with probability g_1 and obtains no information $\varnothing$ with probability $1 - g_1$. The manager then may disclose the information to the investors in a report R_1. At the end of the first period, the investors receive the disclosed information and form the first price P_1. In the second period, the manager decides which method to use, observes the result and decides whether to disclose the information. At the end of the second period, the investors form the second price P_2.

[15] Alternatively, we could specify the manager's objective function as $U = (1 - \delta')P_1 + \delta' P_2$, with $\delta' \in [0, 1)$, which can be expressed as $\frac{U}{(1-\delta')} = P_1 + \frac{\delta'}{(1-\delta')} P_2$. This alternative specification generates exactly the same results, just with a different scale. When $\delta' \to 1$, the effect is equivalent to $\delta \to +\infty$, implying that the manager puts all incentive weight to P_2; when $\delta' = 0$, the effect is equivalent to $\delta = 0$, with the manager becoming infinitely myopic.

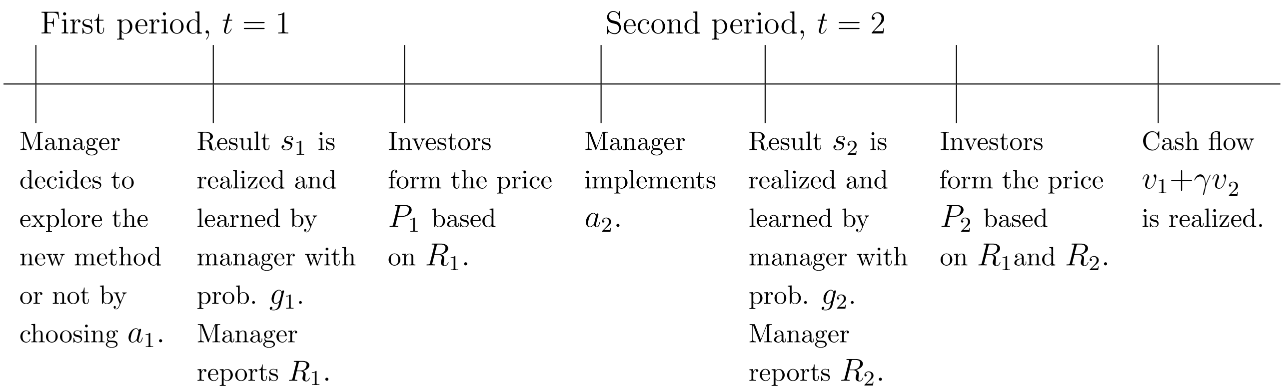

Figure 1: Time line of events.

3 First-Best Strategy

We first examine the optimal strategy that maximizes the firm value, which reduces the model to the basic bandit problem without managerial myopia and the friction created by disclosure. The manager chooses whether to use the old method, $a_1 = o$, or the new method, $a_1 = n$, to maximize the expected firm value $E[V] = E[v_1 + \gamma v_2]$ (and thus does not care about any interim market price).

If the manager chooses to explore the new method in the first period, she will continue with the new method if the outcome is S and switch to the old method if the outcome is F or $\varnothing$. We call this choice the *exploration strategy*. The exploration strategy involves creating a real option as the manager can choose whether to proceed with the new method or revert to the old method based on the first-period result.

If the manager chooses to exploit the old method in the first period, she will always continue with the old method. We label this choice the *exploitation strategy*. Indeed, trying the new method in the second period for the first time (i.e., $\{a_1 = o, a_2 = n\}$) generates strictly lower expected payoffs. We denote the manager's probability of exploration, i.e., choosing the new method $a_1 = n$ in the first period, as q. Accordingly, $1-q$ is the probability that the manager picks the exploitation strategy.

We denote the first-best optimal strategy of exploration as the probability $q^{FB} \in [0, 1]$ of choosing $a_1 = n$ that maximizes the expected firm value. To find q^{FB}, we compare the manager's expected payoff when $a_1 = o$ and when $a_1 = n$. Recall that $a_1 = o$ impliles $a_2 = o$ for certain. Therefore, the payoff of exploitation is

$$E[V|a_1 = o] = E[v_1 + \gamma v_2|a_1 = o, a_2 = o] = (1 + \gamma) (p_o S + (1 - p_o)F) \ . \tag{5}$$

If the manager chooses $a_1 = n$, she will also choose $a_2 = n$ provided that the trial in the first period is successful ($v_1 = S$) and that the outcome is observed by the manager (with probability g_1). Otherwise, the manager switches back to the old method in the second period, i.e. $a_2 = o$. The payoff of choosing exploration is

$$\begin{aligned} E[V|a_1 = n] =& p_n S + (1 - p_n)F + \\ & \gamma \left[p_n g_1 (p_{n,S} S + (1 - p_{n,S})F) + (1 - p_n g_1) (p_o S + (1 - p_o)F)\right] \ . \end{aligned} \tag{6}$$

To maximize the expected firm value, the manager should try the new method if and only if the expected benefit of exploration is higher, i.e. $q^{FB} = 1$ if $\Delta_V \geq 0$, where

$$\Delta_V \equiv E[V|a_1 = n] - E[V|a_1 = o] = \left[\underbrace{\gamma p_n g_1 (p_{n,S} - p_o)}_{\text{real-option benefit of exploration}} - \underbrace{(p_o - p_n)}_{\text{cost of exploration}} \right] (S - F), \tag{7}$$

The expression above shows that exploring entails a cost of a lower probability of success in the first period together with a benefit of a higher future probability of success, given that the trial is successful and the result is observed (the real-option value of the exploration strategy). Lemma 1 provides the optimal exploration strategy.

Lemma 1 *The manager's optimal strategy for exploration that maximizes the firm value is*

q^{FB}, *with* $q^{FB} = 0$ *when* $\gamma < \gamma^{FB}$ *and* $q^{FB} = 1$ *when* $\gamma \geq \gamma^{FB}$, *where*

$$\gamma^{FB} = \frac{p_o - p_n}{p_n g_1 (p_{n,S} - p_o)}.$$

The intuition of Lemma 1 is straightforward. The manager must trade off the potential cost and benefit of creating the real option of exploration at $t = 1$. The critical factor of decision is thus γ, the scale parameter indicating the size of the second-period cash flow potential. When γ is sufficiently large, the manager will always prefer to experiment with the new method. Conversely, when γ is small, the benefit of trying out new method is insignificant. The firm is thus better off taking the safer choice of the known old method.

4 Single-Disclosure Benchmark

To better understand the effects of the disclosure in the first period, we start with a benchmark where the second-period price is perfect, i.e., the cash flow $v_1 + \gamma v_2$ is realized and all uncertainties are resolved *before* the second price P_2 is formed. This renders the manager's second disclosure moot, as investors perfectly observe the second-period results. The manager's objective function (4) thus becomes $U = P_1 + \delta V$. Specifically, we compare the firm's equilibrium behavior in both mandated and discretionary disclosure environments. This comparison can alternatively be interpreted as examining the behavior of public and private firms due to their different disclosure requirements.

4.1 Mandated Full Disclosure

Under mandated disclosure, the manager of a public firm must report the observed outcomes fully and truthfully. We denote the market belief about the probability that the manager explores the new method as q^M. The conditional probability that the firm explores the new method when the manager reports success after the first period, i.e. when $R_1 = S$, is

$$\Pr\left(a_1 = n|S, q^M\right) = \frac{1}{1 + \frac{(1-q^M)p_o}{q^M p_n}}, \tag{8}$$

and the conditional probability of the firm using the old method when $R_1 = S$, is $1 - \Pr\left(a_1 = n|S, q^M\right)$. When the manager reports $R_1 = S$, the investors update their belief about the expected future cash flow conditional on the report. This results in the market price

$$\begin{aligned} P_1\left(S, q^M\right) = S &+ \left(\Pr\left(a_1 = n|S, q^M\right) p_{n,S} + \left(1 - \Pr\left(a_1 = n|S, q^M\right)\right) p_o\right) \gamma S \\ &+ \left(1 - \left(\Pr\left(a_1 = n|S, q^M\right) p_{n,S} + \left(1 - \Pr\left(a_1 = n|S, q^M\right)\right) p_o\right)\right) \gamma F. \end{aligned} \tag{9}$$

Since the manager cannot credibly communicate the choice a_1, the market must estimate the possible second-period outcome for both new and old methods. The first term S in the equation (9) above is the reported outcome of the first period; the second term is the expected cash flow of successful outcome generated in the second period, weighted by the conditional probabilities of success from the new and old methods; and the third term is the expected cash flow of failure weighted by the conditional probabilities from the two methods.

The market price P_1 when the manager reports $R_1 = F$ is

$$P_1\left(F, q^M\right) = F + p_o \gamma S + (1 - p_o)\gamma F, \tag{10}$$

as a failure in the first period implies the new method would not be used in the second period. The expected cash flow in the second period is thus only associated with the old method. The market price when the manager reports no information, i.e. $R_1 = \varnothing$, is

$$P_1\left(\varnothing, q^M\right) = q^M\left(p_n S + (1 - p_n)F\right) + \left(1 - q^M\right)\left(p_o S + (1 - p_o)F\right) + p_o \gamma S + (1 - p_o)\gamma F. \tag{11}$$

Under mandated disclosure, no information being reported implies the manager has not obtained any discernible signal about the outcome. The stock price is thus the weighted average of payoffs from using the old and new methods in the first period, and the expected payoff from using the old method in the second period.

Substituting the market prices derived above into the manager's expected utility function when she chooses the old method and when she chooses the new method, we show that the benefit of exploration for the utility of the manager $E[U|a_1 = o] - E[U|a_1 = n]$ is determined by the expression

$$E[U|a_1 = n] - E[U|a_1 = o] = \delta\Delta_V - \underbrace{g_1 \left(p_o - p_n\right) \left(P_1(S, q^M) - P_1(F, q^M)\right)}_{\text{short-term price effect of exploration}}. \qquad (12)$$

The first part of this expression highlights the trade-off between exploration and exploitation for a value-maximizing manager, captured by the long-term value benefit of exploration Δ_V derived in (7) multiplied by the coefficient δ that denotes the degree to which the myopic manager cares about the firm value. In contrast to the net value benefit, the second term represents an additional cost of exploration that arises because of manager's myopic incentive to maximize short-term price. For any investors' belief about the probability of exploration q^M, the manager has an additional incentive to exploit rather than explore because exploitation increases the chance of the price moving upwards from $P_1(S, q^M) - P_1(F, q^M)$. The short term success is observed more often when the manager does not explore, captured by the term $g_1 \left(p_o - p_n\right)$. This short-term price pressure creates an incentive for the manager to under-explore. The cost-benefit trade-off gives rise to a unique equilibrium with a mixed strategy for exploration described in the following lemma.

Lemma 2 *Under mandated full disclosure with $P_2 = V$, there exist boundaries $\underline{\gamma}^M \in (0, +\infty)$ and $\overline{\gamma}^M \in (\underline{\gamma}^M, +\infty]$ such that the manager never explores, i.e., $q^M = 0$, when $\gamma \leq \underline{\gamma}^M$; the manager explores with probability $q^M \in (0, 1)$ when $\gamma \in (\underline{\gamma}^M, \overline{\gamma}^M)$; and the* *manager always explores, i.e.,* $q^M = 1$*, when* $\gamma \geq \overline{\gamma}^M$*.*

The equilibrium strategy of exploration is characterized as a combination of both pure and mixed strategies, depending on the parameter γ. Similar to Lemma 1, when γ is sufficiently small (large), the cost of exploration (exploitation) is strictly higher than the benefits, and the manager always chooses the old (new) method. When γ lies in the middle range with $\underline{\gamma}^M < \gamma < \overline{\gamma}^M$, the manager resorts to a mixed strategy and chooses exploration with a probability $q^M \in (0, 1)$. The closed form solutions to the threshold values $\underline{\gamma}^M$ and $\overline{\gamma}^M$ and the value of the mixed exploration strategy q^M are provided in the appendix.

4.2 Discretionary Disclosure

Under discretionary disclosure, the manager can voluntarily decide to disclose the first-period outcome or to conceal it. In equilibrium, the manager prefers to disclose a successful outcome S and withhold the failure F, provided $g_1 < 1$. We denote the investors' belief about the probability that the manager chooses exploration strategy in the discretionary disclosure regime as q^D. Intuitively, the functional forms of conditional probability and market price that the firm explores the new method when the manager reports success after the first period, i.e. $R_1 = S$, are the same as under mandated disclosure. However, when the manager does not disclose anything, i.e. $R_1 = \varnothing$, the market's information updating is more complex. No disclosure could imply that the manager truly has not obtained any information, or that the first-period trial has failed and the manager chooses to withdraw that information. The stock price when $R_1 = \varnothing$ is

$$P_1\left(\varnothing, q^D\right) = \Pr\left(v_1 = S|\varnothing, q^D\right) S + \left(1 - \Pr\left(v_1 = S|\varnothing, q^D\right)\right) F + p_o\gamma S + \left(1 - p_o\right)\gamma F, \quad (13)$$

where the first two terms represent the expected payoff from the first period and the last two terms represent the payoffs from the second period. Since the manager will always continue with the old method after the trial in the first period ends in failure or no information, the payoff in the second period is only associated with using the old method. Further, the probability of success conditional on no disclosure is

$$\Pr\left(v_1 = S|\varnothing, q^D\right) = \frac{\Pr\left(v_1 = S, \varnothing|q^D\right)}{\Pr\left(\varnothing|q^D\right)} = \frac{1}{1 + \frac{q^D(1-p_n)+(1-q^D)(1-p_o)}{(1-g_1)(q^D p_n+(1-q^D)p_o)}}\,.$$

We establish the unique equilibrium under discretionary disclosure in the following lemma.

Lemma 3 *Under discretionary disclosure with $P_2 = V$, there exist boundaries $\underline{\gamma}^D \in (0, +\infty)$ and $\overline{\gamma}^D \in (\underline{\gamma}^D, +\infty]$ such that the manager never explores, i.e., $q^D = 0$, when $\gamma \leq \underline{\gamma}^D$; the manager explores with probability $q^D \in (0, 1)$ when $\gamma \in (\underline{\gamma}^D, \overline{\gamma}^D)$; and the manager always explores, i.e., $q^D = 1$, when $\gamma \geq \overline{\gamma}^D$.*

Similar to the equilibrium with mandated full disclosure, q^D is characterized with a combination of pure and mixed strategies and depends on γ, the scale of second-period payoff compared to the first-period payoff.

4.3 Comparing discretionary and mandated disclosure

We proceed by comparing the effects on the manager's incentive to pursue exploration of mandatory disclosure and discretionary disclosure, or alternatively of a public firm that is required to always disclose and a private firm that is not.

Lemma 4 *The thresholds of γ in different disclosure environments are presented as follows:*

$$\gamma^{FB} < \underline{\gamma}^D < \underline{\gamma}^M\,, \qquad \overline{\gamma}^D \leq \overline{\gamma}^M\,,$$

and, consequently, there is more exploration in the discretionary disclosure environment than in the mandatory disclosure environment.

The key insight from Lemma 4 is that early mandatory disclosure requirement reduces the manager's incentive to explore. The threshold γ^{FB} for the first-best strategy is strictly lower than both $\underline{\gamma}^M$ and $\underline{\gamma}^D$, indicating that with myopia, there is always under-exploration as compared to the first-best, independently of the disclosure scenario. Further, unlike in the pure strategy in the first-best benchmark, the manager here adopts a two-threshold strategy, exploring with some probability in the middle range of γ. The two thresholds under discretionary disclosure are strictly lower than two respective thresholds under mandatory disclosure, i.e. $\underline{\gamma}^D < \underline{\gamma}^M$ and $\overline{\gamma}^D < \overline{\gamma}^M$, implying that mandated disclosure reduces the manager's incentive to explore.

We summarize the effects of disclosure requirement on exploration and firm value in Proposition 1 and provide a graphic illustration in Figure 2.

Proposition 1 *In equilibrium, the manager under-explores regardless of the disclosure scenario. Allowing discretionary disclosure induces more exploration and generates higher expected firm value than when disclosure is mandated, i.e.,* $q^{FB} \geq q^D \geq q^M$*, and* $E[V|q^{FB}] \geq E[V|q^D] \geq E[V|q^M]$.

Proposition 1 implies that the myopia-induced under-exploration can be mitigated through the interaction of disclosure option with the real option of learning embodied in the exploration strategy. While the long-term value benefits of exploration in the mandatory and discretionary disclosure environments are the same in this benchmark setting, the manager has strictly higher incentive to explore in an environment with discretionary disclosure because the disclosure option protects her from a negative short-term price reaction in case of observed failure in the first period.

Importantly, without the presence of the real option the effect of disclosure option would go away. To see this, assume that the action of the manager is irreversible in that the manager has to choose the same action in the second period as in the first period ($a_2 = a_1$). Then the learning through exploration cannot affect the manager's second period action and

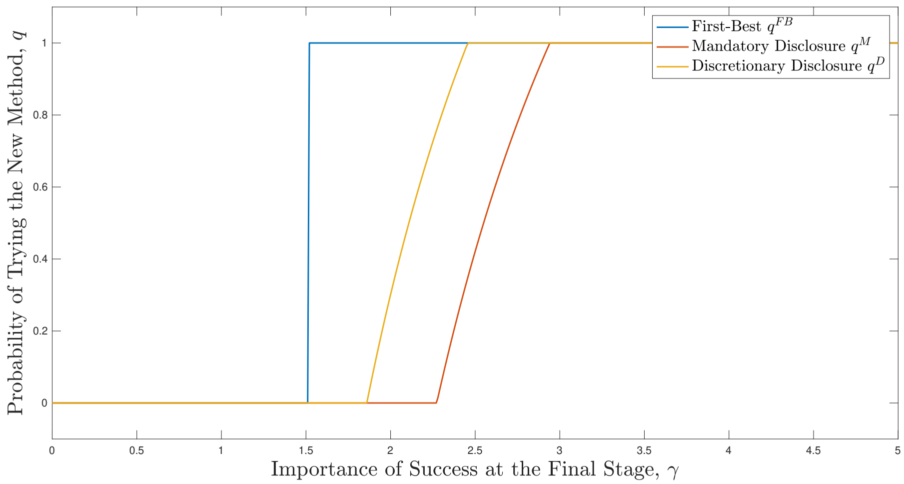

Figure 2: Comparing exploration strategy in the single-disclosure benchmarks with mandatory and discretionary disclosure. Parameters: $p_o = 0.6$, $p_n = 0.55$, $p_{n,S} = 0.9$, $g_1 = 0.2$, $\delta = 0.4$.

the value-maximizing strategy for the firm is to exploit the old method with probability 1 ($a_2 = a_1 = o$), regardless of disclosure regimes. In other words, in the absence of the real option of learning, the disclosure option becomes irrelevant.

To further study how the disclosure option interacts with the real option of exploration, we examine the effect of g_1, the internal quality of the firm's information system in the first period, on the equilibrium outcome of exploration in different disclosure environments. The result is summarized in Proposition 2.

Proposition 2 *Under both first-best and mandatory disclosure scenarios, the equilibrium probabilities of exploration q^{FB} and q^M increase in the quality of firm's information system g_1. However, under discretionary disclosure, the probability of exploration q^D increases in the quality of firm's information system g_1 when $g_1 < \underline{g}$ and decreases in g_1 when $g_1 > \overline{g}$.*

The value-maximizing probability of exploration always increases in g_1, the quality of the firm's information system. Since the manager only wants to maximize the true long-term firm value, a higher g_1 improves the efficiency of the manager's decision in the second period.

Under mandatory disclosure, the myopic manager cares about the interim market price but has to disclose all information truthfully. Her incentive to explore thus still increases in g_1.

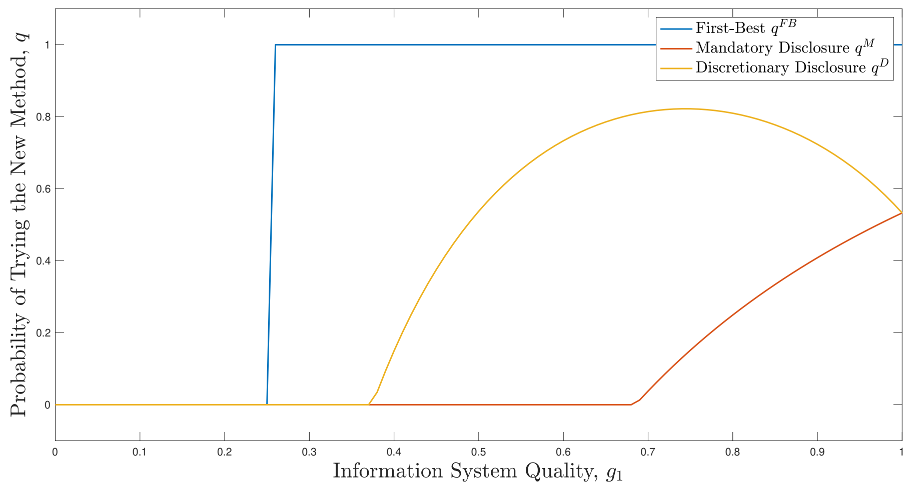

Figure 3: Comparative statics of exploration strategy. Parameters: $p_o = 0.6$, $p_n = 0.55$, $p_{n,S} = 0.9$, $\gamma = 1.2$, $\delta = 0.4$.

Under discretionary disclosure, the manager has both the option to learn about the new method and the option to disclose the interim information. Therefore, g_1 now has two effects on the manager's incentive to explore. First, a higher g_1 increases the value of the real option by improving the firm's subsequent operational efficiency—the firm is more likely to make the correct decision in the second period. Second, a higher g_1 reduces the value of the disclosure option due to a lowered probability for the manager to hide the bad result by pooling with the managers who do not observe information. While the first effect increases the manager's incentive to try out the new method, the second effect could dampen her incentive to pursue exploration. Consistent with this logic, we find that the first effect dominates and the manager explores more when the information system initially improves, but as g_1 becomes higher, the second effect dominates and the manager explores less.

We know from Proposition 1, however, that there is always more exploration in equilibrium under discretionary disclosure than under mandatory disclosure. Thus, the combined value of the two options under discretionary disclosure still dominates the value of the the single option under mandatory disclosure. Figure 3 summarizes these findings by presenting the manager's exploration strategy q in different scenarios.

5 Dynamic Disclosure

In this section, we return to the full model and examine the setting in which the second-period market price is formed before the final cash flows are realized, thus $P_2 \neq V$. Compared to the single-disclosure benchmark with $P_2 = V$, the manager's second disclosure is also strategic, making this a dynamic disclosure setting. In this case, the timing of the disclosure becomes important, as demonstrated by prior studies. For example, Guttman, Kremer, and Skrzypacz (2014) show in their setting that later disclosures result in higher market price than early disclosures. When to mandate disclosure could be as important for firm value as whether to mandate disclosure at all. Aghamolla and An (2021) study a setting where firm value and manager's information change over time. They show that the manager may be willing to disclose negative information initially for the long-term option in disclosure. In our paper, we examine how mandatory disclosure at different stages of innovation process affects the amount of innovation and efficiency.

We consider four disclosure scenarios that the firm in our model may face: 1) disclosure is only mandated in early stage, 2) disclosure is only mandated in late stage, 3) disclosure is mandated both early and late, and 4) disclosure is not mandated at all. We denote these four scenarios with d, where $d = MD, DM, MM, DD$. These reporting regimes can be understood in the context of the firm's status as either public or private. While a public firm is likely to face mandated reporting and a private firm has discretionary reporting in both project stages, changing their status would also change their reporting requirements. For example, a startup firm that is initially privately-held could go public after the first stage of the project, resulting in a shift from discretionary to mandated disclosure. Similarly, a publicly-traded firm could choose to go private, thereby switching from mandated to discretionary reporting.[16]

5.1 Effects on Exploration

In the following analyses, we derive and compare the manager's exploration strategies under each of these different disclosure scenarios. Each disclosure scenario is characterized by its own equilibrium probability of exploration q^d.

Lemma 5 *Consider disclosure scenarios $d = MD, DM, MM, DD$, in which disclosure is mandated in the first period, second period, both periods, and neither period, respectively. There exist boundaries $\underline{\gamma}^d \in (0, +\infty)$ and $\overline{\gamma}^d \in (\underline{\gamma}^d, +\infty]$ such that the manager never explores, i.e., $q^d = 0$, when $\gamma \leq \underline{\gamma}^d$; the manager explores with probability $q^d \in (0, 1)$ when $\gamma \in (\underline{\gamma}^d, \overline{\gamma}^d)$; and the manager always explores, i.e., $q^d = 1$, when $\gamma \geq \overline{\gamma}^d$.*

In the dynamic setting, exploration is still associated with benefits and costs similar to the previous cases, and the manager's optimal strategy depends on the same fundamental cost-benefit trade-off captured in γ. Lemma 5 shows that, the manager uses similar threshold strategies for exploration even when disclosure in each of the two periods can be either discretionary or mandatory. Specifically, the manager never chooses to explore when the future benefits are small, i.e., γ is below the lower boundary value. On the contrary, she always chooses to explore when γ is above the upper boundary. When γ is between the two boundaries, the manager takes a mixed strategy and explores with a positive probability.

Proposition 3 *Among the four possible disclosure scenarios $d = MD, DM, MM, DD$, disclosure mandated only in the second period always results in the highest exploration probability and disclosure mandated only in the first period always results in the lowest probability, while*

[16] For the purpose our our research questions, we assume the disclosure scenarios are given exogenously rather than chosen by firms. This is consistent with the reality, where changes in firm status are typically motivated by financing needs.

disclosure mandated in both periods and neither period result in exploration probabilities in between. That is, $q^{DM} \geq q^{MM} \geq q^{MD}$ *and* $q^{DM} \geq q^{DD} \geq q^{MD}$.

The threshold strategy makes it convenient for us to compare the probabilities of exploration under different disclosure scenarios. Proposition 3 summarizes how different disclosure environments affect the manager's incentive to explore by ranking them by the equilibrium amounts of exploration they induce. We first focus on the ranking of scenarios DM and MD, which can be understood as the cases of a private firm going public and a public firm becoming private, respectively. Out of the four different scenarios, DM always induces the most innovation while MD always induces the least, for all parameter values.

The most important message from Proposition 3 is that disclosure being mandated late fosters exploration while disclosure being mandated early hinders exploration. The scenario DM, where disclosure is mandated only in the second period, increases the manager's incentive to try the new method due to two reasons. First, the earlier discretionary disclosure before P_1 provides the manager with the option of hiding potential bad results from the first period (consistent with the intuition developed earlier), while the later mandated disclosure before P_2 puts more weight on the long-term benefits of exploration in the manager's payoff function, and hence encourages her to try out the new method at the exploration stage. Second, the real option of trying out the new method is no longer useful in the second period, therefore the price relief through a discretionary disclosure at $t = 2$ is not going to improve the manager's incentive to explore.

In contrast, the scenario MD where disclosure is mandated only in the second period has the opposite effect. An early mandated disclosure requirement before P_1 robs the manager of her price relief, and a late discretionary disclosure before P_2 reduces the long-term incentives to explore. Therefore, MD regime decreases the manager's incentive to explore the most severely.

The strategy of exploration implies a trade-off between possible decreased short-term performance and improved long-term performance. While discretionary disclosure being allowed in the short term provides the necessary price relief that the manager needs, disclosure mandated at the end enforces the incentive to show success in the long-term.[17]

It is clear that DM and MD have exactly the opposite effects on the manager's incentive to explore. As a result, it is easy to see that the scenarios MM and DD, which correspond to public and private firms not changing their status, have combined elements of DM and MD and give the manager conflicting incentives. In the DD scenario, early discretionary disclosure option improves a manager's incentive to explore, but late discretion in disclosure decreases the incentive. On the other hand, in the scenario MM, disclosure being mandated early hinders exploration while disclosure mandated late encourages exploration. The combined effects of these conflicting incentives lead to more subtle and complex results. As a result, MM and DD always rank in between DM and MD. The next proposition provides sufficient conditions to compare the scenarios MM and DD.[18]

Proposition 4 *If the manager is sufficiently myopic (i.e., if δ is small), then DD results in more exploration than MM.*

If the manager is sufficiently forward-looking but the internal quality of accounting information at the second stage of the project is low (i.e., if δ is large but g_2 is small), then MM results in more exploration than DD.

The results of the proposition are intuitive. When δ is small, the manager is myopic and does not internalize much the long-term price effect of exploration. Therefore, the cost of lower chance of showing long-term success because of discretionary disclosure in the second period is not so relevant for preventing exploration. In contrast, the early-disclosure option

[17] A thought experiment is to assume the second-period price being completely uninformative (i.e., pure noise), the polar opposite scenario from the single-disclosure benchmark model where the second market price is perfect. In this case, the manager will never have incentive to explore, as the long-term benefits of exploration do not materialize.

[18] We also conducted analysis for the regime in which the manager is mandated to report *both* outcomes at the end of the second period. This scenario also ranks in between DM and MD.

is still important, and therefore, the scenario DD of private firms generates more exploration than the scenario MM of public firms.

In contrast, when δ is large, the cost of lower chance of showing long-term success because of discretionary disclosure at the second stage is relevant for preventing exploration. And the chance of showing long-term success is also low (because of low g_2). Therefore, the cost of late-disclosure option outweighs the benefit of having early-disclosure option. It follows that the scenario MM of public firms results in more exploration than the scenario DD of private firms.

5.2 Effects on Firm Value

In this section, we focus on how disclosure affects the efficiency of innovation. While disclosure being mandated early maximizes exploration, it does not necessarily improve firm value. In the single-disclosure benchmark case with $P_2 = V$, only under-exploration occurs. In the case of the dynamic disclosure, however, both under- and over-exploration can happen in equilibrium, depending on the quality of the firm's information system in the second period, g_2 relative to that of the first period, g_1. The effects of the firm's information system on the exploration are presented in the next proposition.

Proposition 5 *If the quality of firm's information system in the second period does not sufficiently improve from the first period, i.e.* $g_2 \leq g_1 \frac{1+\delta}{\delta} \frac{1-p_o}{1-g_1 p_o}$*, there could be only under-exploration in equilibrium, and disclosure being mandated late weakly dominates disclosure being mandated early in efficiency for all* $\gamma > 0$*. That is,* $E[V]^{DM}$ *is the highest across four disclosure scenarios and* $E[V]^{MD}$ *is the lowest.*

Conversely, if the quality of firm's information system in the second period improves sufficiently, i.e. $g_2 > g_1 \frac{1+\delta}{\delta} \frac{1-p_o}{1-g_1 p_o}$*, there could be either under- or over-exploration in equilibrium. Specifically, disclosure being mandated late weakly dominates disclosure being mandated early in efficiency for all* $\gamma \geq \gamma^{FB}$*, and disclosure being mandated early weakly dominates disclo-*

sure being mandated late for all $\gamma < \gamma^{FB}$. That is, 1) for $\gamma \geq \gamma^{FB}$, $E[V]^{DM}$ is the highest across four disclosure scenarios and $E[V]^{MD}$ is the lowest; and 2) for $\gamma < \gamma^{FB}$, $E[V]^{DM}$ is the lowest across four disclosure scenarios and $E[V]^{MD}$ is the highest.

Proposition 5 shows that, if the firm's second-period information system quality is low enough (relative to the first period), under-exploration prevails in equilibrium, regardless of the value of γ. Intuitively, with a low g_2, there is a low chance that the second-period output is observed. Therefore, long-term price P_2 does not provide strong exploration incentives. At the same time, a high g_1 pressures the manager to exploit to show a high short-term price P_1. As a result, over-exploration does not arise for any value of γ when g_2 is not sufficiently higher than g_1, and the only concern is how to mitigate under-exploration.

In this scenario, disclosure being mandated late helps alleviate the under-exploration problem. Investors understand that no report truly means that the manager has not obtained any information, thus there is no price penalty (in P_2) for lack of observed result. In this case, the scenario DM, that corresponds to a private firm going public, outperforms all others and results in the highest firm value, while the scenario MD, that corresponds to a delisting public firm, results in the lowest value.

In contrast, when g_2 sufficiently improves from g_1, over-exploration may become a problem as the manager is too tempted to show success in the final stage of the game. When the scale parameter γ is sufficiently high, exploration is ex-ante efficient since the long-term benefit of exploration is significant. A manager in the scenario DM therefore has the highest incentive to pursue exploration and reaches a level closest to the first best, while a manager in the scenario MD is the least likely to innovate and generates the lowest firm value. On the other hand, when γ is low, exploitation is ex-ante more efficient since the long-term benefit of exploration is smaller, and over-exploration is value destroying. Thus, the manager's excessive enthusiasm to over-explore is mitigated by the reporting scenario MD of the delisting firm, which results in more efficient level of exploration. The results of Proposition

5 are summarized in Table 1.

	$\gamma < \gamma^{FB}$	$\gamma \geq \gamma^{FB}$
$g_2 \leq g_1 \frac{1+\delta}{\delta} \frac{1-p_o}{1-g_1 p_o}$	firm value $E[V]^{DM}$ is the highest, and $E[V]^{MD}$ is the lowest	
$g_2 > g_1 \frac{1+\delta}{\delta} \frac{1-p_o}{1-g_1 p_o}$	firm value $E[V]^{DM}$ is the lowest, and $E[V]^{MD}$ is the highest	firm value $E[V]^{DM}$ is the highest, and $E[V]^{MD}$ is the lowest

Table 1: Efficiency implications of four disclosure scenarios ($d = MD$, DM, MM, DD), in which disclosure is mandated in the first period, second period, both periods, and neither period, respectively. The parameter γ captures the importance of cash flow in the second period relative to first period, and g_i, where $i = 1, 2$, denotes the internal information quality in period i.

In summary, Proposition 5 shows that, in a dynamic setting, a firm facing disclosure environment DM or MD can achieve the highest innovation efficiency, depending on the source of inefficiency. Inefficiency could arise from both under- and over-exploration, and different disclosure environments could either exacerbate or mitigate the problem. Meanwhile, firms facing DD and MM, which can be understood as private and public firms that do not change their status, never deliver the most efficient level of innovation.

We now turn to comparing the levels of innovation efficiency of firms facing disclosure scenarios DD and MM. We know from Proposition 3 that these two disclosure scenarios will never induce more exploration than MD or less than DM since they give the manager conflicting incentives to explore in the two periods. The next proposition compares MM or DD and provides the sufficient conditions under which each scenario is more efficient.

Proposition 6 *If the quality of firm's information system in the second period g_2 does not sufficiently improve from that in the first period g_1, there exists a range of the manager's myopia coefficients δ for which MM results in more exploration than DD. Under these conditions, MM weakly dominates DD in efficiency if and only if $\gamma > \gamma^{FB}$.*

If the quality of firm's information system in the second period g_2 sufficiently improves from that in the first period g_1, there exists a range of manager's myopia coefficients δ

for which DD results in more exploration than MM. Under these conditions, DD weakly dominates MM if and only if $\gamma > \gamma^{FB}$.

Proposition 6 shows that the ranking between MM and DD is more complex and depends on the firm's quality of internal information system g_1 and g_2 and the manager's degree of myopia δ. The key condition is the relative change in the firm's information quality from the first period to the second period. A low g_1, the quality of the firm's internal information system at the early stages of the innovative project, increases the space for the manager to hide failure under early discretionary disclosure. Therefore, a low g_1 makes the early disclosure option valuable for stimulating exploration. On the other hand, a low g_2, the quality of the firm's internal information system at the late stages of the innovative projects, decreases the incentives of the manager to explore due to a lower probability to show long-term success. Therefore, a low g_2 makes the late mandatory disclosure valuable to stimulate exploration.

On balance, if the information quality does not sufficiently improve from the trial stage to the commercial stage of the innovative project, i.e., g_1 is relatively high and g_2 is relatively low, the relative incentive of disclosure option in the first period cannot outweigh the cost of disincentive from a low information environment in the second period. As a result, firms facing disclosure scenario MM would result in more exploration than firms facing DD, provided that the manager's myopia δ is within the specified range. Under these conditions, firms facing MM can achieve more efficient exploration than firms facing DD, when exploration is ex ante beneficial for firm value, i.e.,$\gamma > \gamma^{FB}$.

On the other hand, if g_2 is sufficiently improved from g_1, firms facing scenario DD would conduct more exploration than firms facing scenario MM. Intuition for this result is analogous to the result discussed above. Since the benefits of disclosure option in the first period outweigh the cost of low information environment in the second period, DD provides more incentives for the manager to explore. If δ and γ meet the conditions required, firms facing DD are more efficient in innovation than firms facing MM. The firm value implications of Proposition 6 are summarized in Table 2.

	$\gamma < \gamma^{FB}$	$\gamma \geq \gamma^{FB}$
g_2 doesn't sufficiently improve from g_1 and δ is moderate	$E[V]^{DD} > E[V]^{MM}$	$E[V]^{MM} > E[V]^{DD}$
g_2 improves sufficiently from g_1 and δ is moderate	$E[V]^{MM} > E[V]^{DD}$	$E[V]^{DD} > E[V]^{MM}$

Table 2: Efficiency implications of disclosure scenarios MM and DD, in which disclosure is mandated in both periods, or neither period, respectively. The parameter γ captures the importance of cash flow in the second period relative to first period, δ is the manager's degree of myopia, and g_i, where $i = 1, 2$, denotes the internal information quality in period i.

6 Empirical Implications

Accounting research on innovation has grown significantly in recent years. However, the existing literature on innovation and disclosure has yielded inconclusive and, at times, conflicting results. For instance, innovation, when proxied by patent-based measures, has been linked to a decrease in firms' disclosure frequency (Fu et al., 2020) and in media and analyst coverage (Dai et al., 2021; He and Tian, 2013). Conversely, Brown and Martinsson (2019) and Zhong (2018) suggest that patent-based measures of innovation and R&D expenditures can lead to an increase in corporate transparency. Huang et al. (2021) found that both the quantity and quality of patents are positively correlated with voluntary management forecasts and internal information quality.[19] Our analysis could help clarify these seemingly inconsistent empirical findings. Specifically, Proposition 3 shows that, all else being equal, we can compare the extent of corporate innovation through firms' status as either public or private. We thus derive the following empirical predictions:

[19]For more detailed discussions, please refer to the comprehensive literature surveys on the topic by Glaeser and Lang (2023) and Huang et al. (2021).

Prediction 1a: *Ceteris paribus, private firms that become public always innovate more than both private firms that remain private and public firms that remain public.*

Prediction 1b: *Ceteris paribus, public firms that delist always innovate less than both private firms that remain private and public firms that remain public.*

Our predictions state that firms that have a change of status behave differently from firms that stay the same. While a firm typically decides to go public or delist for financing reasons, the associated change in reporting requirements could bring forth different incentives for managers to innovate. Specifically, private startup firms that expect to go IPO are more active in innovation than their peers that plan to stay private; while public firms that expect to delist are less active than their peers that plan to stay public. Empirical research has established many measures of innovation (Glaeser and Lang, 2023), such as R&D intensity, number of patent applications, number of patents granted, etc.. We suggest that the firms' change of status, from private to public or vice versa, can serve as a proxy for their disclosure requirements.

Further, Proposition 4 yields the following predictions for firms that maintain their public or private status.

Prediction 2a: *Ceteris paribus, private firms innovate more than public firms when the managers are sufficiently myopic.*

Prediction 2b: *Ceteris paribus, public firms innovate more than private firms when the managers are moderately myopic and the firms' internal accounting quality in the commercial launch stage of innovation is not high.*

Prediction 2a states that private firms are more active in innovation compared to those public firms facing mandated disclosure, provided that their managers are sufficiently myopic. Managerial myopia can be empirically proxied by measures such as managers' age, tenure, or short-term incentives induced by compensation structure.[20] On the other hand, if the

[20]Our model does not explore the optimal design of compensation contracts in public or private firms. Therefore, if the empirical setting might involve strong endogeneity related to the firms' status as public or private, it would be more appropriate to proxy managerial myopia with measures such as age and tenure, rather than

managers are sufficiently long-term oriented, Prediction 2b states that public firms could innovate more than private firms if the quality of their internal accounting information is low in the commercial launch stage.[21]

Furthermore, innovation does not always result in higher efficiency. In fact, over-innovation could reduce firm value. While prior empirical research typically focuses on the amount of corporate innovation, our study takes it one step further and sheds light on whether innovation results in higher or lower efficiency. The results of Proposition 5, as summarized in Table 1, lead to the following empirical prediction.

Prediction 3: *Private firms that become public always achieve the highest innovation efficiency while public firms that delist have the lowest innovation efficiency, unless (i) firms' internal accounting information quality significantly improves from the trial stage to commercial launch, but (ii) the business scale of the commercial stage is not significantly large.*

Prediction 3 is directly derived from the results summarized in Table 1. Generally, private firms that later become public are the most active in innovation, which, in most cases, also leads to higher efficiency. However, as long as the business scale of the commercial stage is not significantly large (condition (ii)), over-innovation may become an issue. Moreover, when the incentive to innovate becomes excessively high – that is when the information quality is very high in the commercial stage of innovation (condition (i)) – there can be a loss of efficiency due to over-innovation. When these two conditions are both true, public firms that go private would achieve the highest innovation efficiency since the disclosure environment they face curb the managers' incentive to over-innovate. Firms that do not change their status always rank in the middle of firms that change their status in terms of innovation efficiency.

Empirically, efficiency of innovation can be measured through variables such as the number of patent citations, market value of patents, and return on investment. The potential for

the structure of compensation contracts.

[21] The latter part of the prediction is probably rare to observe in reality, since typically the profitability of a new product becomes more discernible once it transitions from trial to full-scale production phase.

business expansion or scaling can be identified in certain industry sectors, such as technology firms with readily expandable online businesses. Conversely, sectors like oil and gas present limited opportunities when introducing new technology. Certain products may also have a lower potential for scaling than others, such as orphan drugs in the pharmaceutical industry.

7 Conclusion

Innovation is one of the most crucial issues in business. Successful companies often have a track record of making balanced decisions about when to explore new technologies and products and when to exploit existing experience and knowledge. The bandit problem we build upon allows us to examine a firm's strategy choice, rather than merely a project selection. After trying the new method in the first period, the firm has the option to switch back to the old method. This reversibility gives the manager the flexibility to try out the new method and continue with it if it proves successful. We demonstrate that voluntary disclosure offers an additional option for the manager, shielding her from disclosing the failure of the initial trial. Specifically, firms operating in an environment that allows for voluntary disclosure during the trial stage and mandates disclosure at the commercial launch are more inclined to pursue innovation. Conversely, firms subject to early mandated disclosure and subsequent voluntary disclosure are less likely to engage in innovative activities.

Our paper has a few caveats. First, as discussed earlier, our model assumes different verifiability of managerial information endowment under discretionary and mandatory disclosure regimes. Nondisclosure is interpreted as no information in a mandated regime, but either no information or unfavorable information in a voluntary disclosure regime. This raises the question: why can information be verified in a mandatory regime but not in a discretionary one? We argue that a mandated disclosure regime requires firms to collect information and follow procedures to prepare their accounting reports, which can later validate their genuine information endowment. A voluntary disclosure regime cannot provide the same credibility to the information firms disclose to their investors. The investors, therefore, do not form the same expectation as under a mandated regime when observing non-disclosure. Nevertheless, a model that better reconciles the difference between the two reporting regimes could represent a significant avenue for future research.

Second, the disclosure regimes in our model are exogenous, instead of being a choice made by the firm. We argue that this setup is consistent with the business reality, where firms typically decide to go public or private due to financing reasons. Moreover, our current model is not rich enough to incorporate another decision variable for the firms. However, we think it could be an interesting avenue for future research to endogenize the disclosure regime.

8 APPENDIX

This appendix contains the proofs of the lemmas and propositions.

Proof of Lemma 1:

The first-best strategy of exploration maximizes the expected long-term value $E[V] = E[v_1 + \gamma v_2]$. Let us compute the expected value if the manager exploits a traditional method in the first period 1. In this case, the second period action $a_2 = o$ is to exploit the old traditional method again independently of the result of the trial v_1, and

$$E[V|a_1 = o] = E[v_1 + \gamma v_2 | a_1 = o, a_2 = o] = p_o S + (1 - p_o)F + \gamma \left(p_o S + (1 - p_o)F\right)$$
$$= (1 + \gamma)\left(p_o S + (1 - p_o)F\right) .$$

If the manager explores the new method in the first period, i.e. $a_1 = n$, then the method will be applied for the second period providing that the trial is a success ($v_1 = S$) and it is observed by the manager (which happens with probability g_1). Otherwise, the manager switches back to the old traditional method. We have

$$E[V|a_1 = n] = E[v_1 + \gamma v_2 | a_1 = n] = E[v_1|a_1 = n] + \gamma E[v_2|a_1 = n]$$
$$= E[p_n]S + (1 - p_n)F$$
$$+ \gamma \left[p_n g_1 \left(p_{n,S} S + (1 - p_{n,S})F\right) + (1 - p_n g_1)\left(p_o S + (1 - p_o)F\right)\right] .$$

To maximize firm value, the manager should explore the new method if and only if the expected value of exploration is higher, i.e. $q^{FB} = 1$ if $E[V|a_1 = n] - E[V|a_1 = o] \geq 0$:

$$E[V|a_1 = n] - E[V|a_1 = o] = \left(-\left(p_o - p_n\right) + \gamma p_n g_1 \left(p_{n,S} - p_o\right)\right)(S - F) \tag{14}$$

i.e. $q^{FB} = 1$ if and only if

$$-\left(p_o - p_n\right) + \gamma p_n g_1 \left(p_{n,S} - p_o\right) \geq 0\,,$$

and $q^{FB} = 0$ otherwise. One can see that the cost of exploration is a lower probability of success in the first period (captured by $-\left(p_o - p_n\right)$), while the benefit is a higher future probability of success given that the trial is successful and the result is observed (captured by $\gamma p_n g_1 \left(p_{n,S} - p_o\right)$). Overall, the exploration strategy $q^{FB} = 1$ if

$$E[V|a_1 = n] - E[V|a_1 = o] \geq 0 \Leftrightarrow \gamma \geq \frac{p_o - p_n}{p_n g_1 \left(p_{n,S} - p_o\right)},$$

and it is $q^{FB} = 0$ otherwise.■

Proof of Lemma 2:

The manager chooses to explore to maximize the utility $U = P_1 + \delta V$. Denote the investors' belief of the exploration strategy of the manager as q. Then the expected utility of the manager who chooses to exploit the old traditional method in the first period is $E[U|a_1 = o] = E[P_1 + \delta V|a_1 = o] = E[P_1|a_1 = o] + \delta E[V|a_1 = o]$, and the expected utility of the manager who chooses to explore a new method in the first period is

$$E[U|a_1 = n] = E[P_1 + \delta V|a_1 = n] = E[P_1|a_1 = n] + \delta E[V|a_1 = n]\,,$$

The expected prices can be rewritten as follows:

$$E[P_1|a_1 = o] = g_1 \left(p_o P_1(S, q) + (1 - p_o) P_1(F)\right) + \left(1 - g_1\right) P_1(\varnothing, q)\,,$$
$$E[P_1|a_1 = n] = g_1 \left(p_n P_1(S, q) + (1 - p_n) P_1(F)\right) + \left(1 - g_1\right) P_1(\varnothing, q)\,.$$

Now let us compute the difference in the expected utilities of the manager conditional on exploring the new method and exploiting the old one:

$$E[U|a_1 = n] - E[U|a_1 = o] = E[P_1|a_1 = n] - E[P_1|a_1 = o] + \delta \left(E[V|a_1 = n] - E[V|a_1 = o]\right)$$
$$= -g_1 \left(p_o - p_n\right) \left(P_1(S, q) - P_1(F)\right) + \delta \left(E[V|a_1 = n] - E[V|a_1 = o]\right) .$$

One can see that the price differential creates an incentive for the manager to exploit an old method in order to show success in the first period with a higher probability (captured by the term $g_1 \left(p_o - p_n\right) \left(P_1(S, q) - P_1(F)\right)\right)$.

Substituting prices from (9) and (10) and the difference in the long-term values from (14), we have

$$E[U|a_1 = n] - E[U|a_1 = o]$$
$$= -g_1 \left(p_o - p_n\right) \left(1 + \gamma \left(p_{n,S} - p_o\right) \Pr\left(a_1 = n|S, q\right)\right) (S - F)$$
$$+ \delta \left(- \left(p_o - p_n\right) + \gamma p_n g_1 \left(p_{n,S} - p_o\right)\right) (S - F)$$
$$= \left[\delta \left(\gamma p_n g_1 \left(p_{n,S} - p_o\right) - \left(p_o - p_n\right)\right) - g_1 \left(p_o - p_n\right) \left(1 + \gamma \left(p_{n,S} - p_o\right) \Pr\left(a_1 = n|S, q\right)\right)\right] (S - F)$$
$$= \left[- \left(g_1 + \delta\right) \left(p_o - p_n\right) + \gamma g_1 \left(p_{n,S} - p_o\right) \left(\delta p_n - \left(p_o - p_n\right) \Pr\left(a_1 = n|S, q\right)\right)\right] (S - F) \ ,$$

where $\Pr\left(a_1 = n|S, q\right) = \frac{1}{1+\frac{(1-q)p_o}{qp_n}}$. The probability $\Pr\left(a_1 = n|S, q\right)$ increases in q and, consequently, $E[U|a_1 = n] - E[U|a_1 = o]$ decreases in q.

It follows that: (a) if $E[U|a_1 = n] - E[U|a_1 = o] > 0$ for $q = 1$, then the equilibrium probability of exploration q^M=1; (b) if $E[U|a_1 = n] - E[U|a_1 = o] < 0$ for $q = 0$, then the equilibrium probability of exploration q^M=0; and (c) if the conditions in (a) and (b) are not satisfied, then there exists a unique equilibrium in mixed strategies with q^M solving $E[U|a_1 = n] - E[U|a_1 = o] = 0$.

Observe that $E[U|a_1 = n] - E[U|a_1 = o]$ has the same sign as $- \left(g_1 + \delta\right) \left(p_o - p_n\right) + \gamma g_1 \left(p_{n,S} - p_o\right) \left(\delta p_n - \Pr\left(a_1 = n|S, q\right)\right)$. Now let us find conditions for (a) and (b):

(a) We have $E[U|a_1 = n] - E[U|a_1 = o] > 0$ for $q = 1$ when

$$\begin{aligned} & -(g_1+\delta)(p_o-p_n)+\gamma g_1(p_{n,S}-p_o)(\delta p_n-(p_o-p_n)\Pr(a_1=n|S,1))>0 \\ \Leftrightarrow & -(g_1+\delta)(p_o-p_n)+\gamma g_1(p_{n,S}-p_o)((1+\delta)p_n-p_o)>0\,, \end{aligned}$$

which holds when both $(1+\delta)\,p_n - p_o > 0$ and $\gamma > \frac{(\delta+g_1)(p_o-p_n)}{g_1((1+\delta)p_n-p_o)\left(p_{n,S}-p_o\right)} = \overline{\gamma}^M$.

(b) We have $E[U|a_1 = n] - E[U|a_1 = o] < 0$ for $q = 0$ when

$$\begin{aligned} & -(g_1+\delta)(p_o-p_n)+\gamma g_1(p_{n,S}-p_o)(\delta p_n-(p_o-p_n)\Pr(a_1=n|S,0))<0 \\ \Leftrightarrow & -(g_1+\delta)(p_o-p_n)+\gamma g_1(p_{n,S}-p_o)\,\delta p_n<0\,, \end{aligned}$$

which holds when $\gamma < \frac{(\delta+g_1)(p_o-p_n)}{g_1\delta p_n\left(p_{n,S}-p_o\right)} = \underline{\gamma}^M$.

(c) When the conditions in (a) and (b) are not satisfied, there exists a unique $q^M \in (0,1)$ solving

$$\begin{aligned} & -(g_1+\delta)(p_o-p_n)+\gamma g_1(p_{n,S}-p_o)\left(\delta p_n-(p_o-p_n)\Pr\left(a_1=n|S,q^M\right)\right)=0 \\ \Leftrightarrow & \Pr\left(a_1=n|S,q^M\right)=\frac{\delta p_n}{p_o-p_n}-\frac{g_1+\delta}{\gamma g_1(p_{n,S}-p_o)} \\ \Leftrightarrow & \frac{1}{1+\frac{(1-q^M)p_o}{q^M p_n}}=\frac{\delta p_n}{p_o-p_n}-\frac{g_1+\delta}{\gamma g_1(p_{n,S}-p_o)}, \end{aligned}$$

and it follows that

$$q^M = \frac{p_o}{p_o - p_n + \frac{p_n}{\frac{\delta p_n}{p_o-p_n} - \frac{g_1+\delta}{\gamma g_1\left(p_{n,S}-p_o\right)}}}. \tag{15}$$

This q^M is further simplified as

$$q^M = \frac{p_o\left(-\left(\delta+g_1\right)\left(p_o-p_n\right)+\gamma p_n g_1 \delta\left(p_{n,S}-p_o\right)\right)}{\left(p_o-p_n\right)\left(-\left(\delta+g_1\right)\left(p_o-p_n\right)+\gamma p_n g_1\left(1+\delta\right)\left(p_{n,S}-p_o\right)\right)}$$
$$= \frac{p_o\delta\left(\gamma-\underline{\gamma}^M\right)}{\left(p_o-p_n\right)\left(\gamma\left(1+\delta\right)-\delta\underline{\gamma}^M\right)}. \blacksquare$$

Proof of Lemma 3:

The manager chooses to explore to maximize the utility $U = P_1 + \delta V$. Denote the investors' belief of the exploration strategy of the manager as q. Then the expected prices can be rewritten as follows:

$$E[P_1|a_1 = o] = g_1\left(p_o P_1(S,q) + (1-p_o)P_1(\varnothing,q)\right) + \left(1-g_1\right)P_1(\varnothing,q),$$
$$E[P_1|a_1 = n] = g_1\left(p_n P_1(S,q) + (1-p_n)P_1(\varnothing,q)\right) + \left(1-g_1\right)P_1(\varnothing,q).$$

Now let us compute the difference in the expected utilities of the manager conditional on exploring the new method and exploiting the old one:

$$E[U|a_1 = n] - E[U|a_1 = o] = E[P_1|a_1 = n] - E[P_1|a_1 = o] + \delta\left(E[V|a_1 = n] - E[V|a_1 = o]\right)$$
$$= -g_1\left(p_o - p_n\right)\left(P_1(S,q) - P_1(\varnothing,q)\right) + \delta\left(E[V|a_1 = n] - E[V|a_1 = o]\right).$$

One can see that in the discretionary disclosure regime, the price differential (captured by the term $g_1\left(p_o-p_n\right)\left(P_1(S,q)-P_1(\varnothing,q)\right)$) remains but it is lower than in the mandatory disclosure regime as $(P_1(S,q) - P_1(\varnothing,q)) \leq (P_1(S,q) - P_1(F))$.

The price formed by the investors upon observing a report $R_1 = S$ is

$$P_1\left(S,q\right) = S + \left(\Pr\left(a_1 = n|S\right)p_{n,S} + \left(1 - \Pr\left(a_1 = n|S,q\right)\right)p_o\right)\gamma S \qquad (16)$$
$$+ \left(1 - \left(\Pr\left(a_1 = n|S,q\right)p_{n,S} + \left(1 - \Pr\left(a_1 = n|S,q\right)\right)p_o\right)\right)\gamma F,$$

Substituting prices from (16) and (13) and the difference in the long-term values from (14), we have

$$
\begin{aligned}
&E[U|a_1 = n] - E[U|a_1 = o] \\
&= -g_1\left(p_o - p_n\right)\left(1 - \Pr\left(v_1 = S|\varnothing, q\right) + \gamma\left(p_{n,S} - p_o\right)\Pr\left(a_1 = n|S, q\right)\right)(S - F) \\
&\quad + \delta\left(-\left(p_o - p_n\right) + \gamma p_n g_1\left(p_{n,S} - p_o\right)\right)(S - F) \\
&= \Big[-\left(g_1\left(1 - \Pr\left(v_1 = S|\varnothing, q\right)\right) + \delta\right)\left(p_o - p_n\right) \\
&\qquad + \gamma g_1\left(p_{n,S} - p_o\right)\left(\delta p_n - \left(p_o - p_n\right)\Pr\left(a_1 = n|S, q\right)\right)\Big](S - F) \\
&= \Big[\delta\left(\gamma p_n g_1\left(p_{n,S} - p_o\right) - \left(p_o - p_n\right)\right) \\
&\qquad - g_1\left(p_o - p_n\right)\left(\left(1 - \Pr\left(v_1 = S|\varnothing, q\right)\right) + \gamma\left(p_{n,S} - p_o\right)\Pr\left(a_1 = n|S, q\right)\right)\Big](S - F),
\end{aligned}
$$

where $\Pr\left(a_1 = n|S, q\right) = \frac{1}{1+\frac{(1-q)p_o}{qp_n}}$ and $\Pr\left(v_1 = S|\varnothing, q\right) = \frac{1}{1+\frac{q(1-p_n)+(1-q)(1-p_o)}{(1-g_1)(qp_n+(1-q)p_o)}}$. The probability $\Pr\left(a_1 = n|S, q\right)$ increases in q and the probability $\Pr\left(v_1 = S|\varnothing, q\right)$ decreases in q, consequently, $E[U|a_1 = n] - E[U|a_1 = o]$ decreases in q.

It follows that: (a) if $E[U|a_1 = n] - E[U|a_1 = o] > 0$ for $q = 1$, then the equilibrium probability of exploration q^D=1; (b) if $E[U|a_1 = n] - E[U|a_1 = o] < 0$ for $q = 0$, then the equilibrium probability of exploration q^D=0; and (c) if the conditions in (a) and (b) are not satisfied, then there exists a unique equilibrium in mixed strategies with q^D solving $E[U|a_1 = n] - E[U|a_1 = o] = 0$.

Now let us find conditions for (a) and (b) in a similar way as we did in the proof of Lemma 2.

(a) We have $E[U|a_1 = n] - E[U|a_1 = o] > 0$ for $q = 1$ when

$$
\begin{aligned}
&-\left(g_1\left(1-\frac{1}{1+\frac{1-p_n}{(1-g_1)p_n}}\right)+\delta\right)(p_o-p_n)+\gamma g_1\left(p_{n,S}-p_o\right)\left(\delta p_n-(p_o-p_n)\right)>0\\
\Leftrightarrow &-\left(g_1\frac{1-p_n}{1-g_1p_n}+\delta\right)(p_o-p_n)+\gamma g_1\left(p_{n,S}-p_o\right)\left((1+\delta)\,p_n-p_o\right)>0\,,
\end{aligned}
$$

which holds when both $(1+\delta)\,p_n - p_o > 0$ and $\gamma > \frac{(\delta+g_1(1-p_n(1+\delta)))(p_o-p_n)}{g_1((1+\delta)p_n-p_o)\left(p_{n,S}-p_o\right)(1-g_1p_n)} = \overline{\gamma}^D$.

(b) We have $E[U|a_1 = n] - E[U|a_1 = o] < 0$ for $q = 0$ when

$$
\begin{aligned}
&-\left(g_1\left(1-\frac{1}{1+\frac{1-p_o}{(1-g_1)p_o}}\right)+\delta\right)(p_o-p_n)+\gamma g_1\left(p_{n,S}-p_o\right)\delta p_n<0\\
\Leftrightarrow &-\left(g_1\frac{1-p_o}{1-g_1p_o}+\delta\right)(p_o-p_n)+\gamma g_1\left(p_{n,S}-p_o\right)\delta p_n<0\,,
\end{aligned}
$$

which holds when $\gamma < \frac{(\delta+g_1(1-p_o(1+\delta)))(p_o-p_n)}{\delta g_1 p_n\left(p_{n,S}-p_o\right)(1-g_1p_o)} = \underline{\gamma}^D$.

(c) When the conditions in (a) and (b) are not satisfied, there exists a unique $q^D \in (0,1)$ solving

$$
\begin{aligned}
&-\left(g_1\left(1-\Pr\left(v_1=S|\varnothing,q^D\right)\right)+\delta\right)(p_o-p_n)\\
&+\gamma g_1\left(p_{n,S}-p_o\right)\left(\delta p_n-(p_o-p_n)\Pr\left(a_1=n|S,q^D\right)\right)=0.
\end{aligned}
$$

Simplifying this expression, $q = q^D$ solves

$$
\frac{Aq^2+Bq+C}{(p_o(1-q)+p_nq)(1-g_1(p_o(1-q)+p_nq))}\Leftrightarrow Aq^2+Bq+C=0, \tag{17}
$$

where

$$A = -(1+\delta)g_1(p_o - p_n)^2\left(p_n g_1 \gamma(p_{n,S} - p_o) - (p_o - p_n)\right),$$

$$B = (p_o - p_n)^2(\delta + g_1 - 2(1+\delta)g_1 p_o)$$

$$- \gamma p_n g_1(p_{n,S} - p_o)(p_o - p_n)\left(1 - g_1 p_o + \delta(1 - 2g_1 p_o)\right),$$

$$C = \gamma\delta p_o p_n g_1(p_{n,S} - p_o)(1 - g_1 p_o) - p_o(p_o - p_n)\left(\delta + g_1\left(1 - p_o\left(1+\delta\right)\right)\right).$$

One of the condition of the case (c) is $\gamma > \underline{\gamma}^D$, and it follows that $A < 0$ and $C > 0$. Thus, q^D is the upper root of the quadratic equation $Aq^2 + Bq + C = 0$.■

Proof of Lemma 4:

We have

$$\frac{\underline{\gamma}^M}{\underline{\gamma}^D} = \frac{(\delta + g_1)(1 - g_1 p_o)}{\delta + g_1\left(1 - p_o\left(1+\delta\right)\right)} = \frac{\delta + g_1\left(1 - p_o\left(g_1 + \delta\right)\right)}{\delta + g_1\left(1 - p_o\left(1+\delta\right)\right)} > 1,$$
$$\frac{\underline{\gamma}^D}{\underline{\gamma}^{FB}} = \frac{\delta + g_1\left(1 - p_o\left(1+\delta\right)\right)}{\delta\left(1 - g_1 p_o\right)} = \frac{\delta\left(1 - g_1 p_o\right) + g_1(1 - p_o))}{\delta\left(1 - g_1 p_o\right)} > 1.$$

If $(1+\delta)\,p_n - p_o \leq 0$, then $\overline{\gamma}^M = \overline{\gamma}^D = +\infty$. Otherwise,

$$\frac{\overline{\gamma}^M}{\overline{\gamma}^D} = \frac{(\delta + g_1)(1 - g_1 p_n)}{\delta + g_1\left(1 - p_n\left(1+\delta\right)\right)} = \frac{\delta\left(1 - g_1 p_n\right) + g_1(1 - g_1 p_n)}{\delta\left(1 - g_1 p_n\right) + g_1(1 - p_n)} > 1.\blacksquare$$

Proof of Proposition 1:

When proving Lemma 2 and Lemma 3, we showed that the incentives to explore in the mandatory disclosure case

$$E[U^M|a_1 = n] - E[U^M|a_1 = o]$$

$$= -g_1\left(p_o - p_n\right)\left(P_1(S,q) - P_1(F)\right) + \delta\left(E[V|a_1 = n, q] - E[V|a_1 = o, q]\right)$$

decrease in q, and the incentives to explore in the discretionary disclosure case

$$
\begin{aligned}
&E[U^D|a_1 = n] - E[U^D|a_1 = o] \\
&= -g_1 (p_o - p_n) (P_1(S, q) - P_1(\varnothing, q)) + \delta (E[V|a_1 = n, q] - E[V|a_1 = o, q])
\end{aligned}
$$

also decrease in q. Moreover, the *difference* between the incentives to explore in the discretionary and mandatory regimes is

$$
\begin{aligned}
&\big(E[U^D|a_1 = n] - E[U^D|a_1 = o]\big) - \big(E[U^M|a_1 = n] - E[U^M|a_1 = o]\big) \\
&= g_1 (p_o - p_n) (P_1(\varnothing, q) - P_1(F)) \ ,
\end{aligned}
$$

which is positive. Thus, for any investors' belief q about exploration there are more incentives to explore in the discretionary disclosure regime. It follows that $q^{FB} \geq q^D \geq q^M$ and, consequently, $E[V|q^{FB}] \geq E[V|q^D] \geq E[V|q^M]$.■

Proof of Proposition 2:

First, recall that $q^{FB} = 1$ if $\gamma > \gamma^{FB}$ and $q^{FB} = 0$ otherwise. Since γ^{FB} decreases in g_1 (see Lemma 1), it follows that q^{FB} weakly increases in g_1. Second, it is straightforward to show that q^M defined in (15) increases in g_1.

To find conditions under which q^D increases or decreases in g_1, recall that q^D is the upper root of the quadratic equation $Aq^2 + Bq + C = 0$, as established in (17) in Lemma 3. It follows that $q^D = -n + \sqrt{n^2 + m}$, where $n = B/2A$ and $m = -C/A > 0$. We have

$$
\frac{dq^D}{dg_1} = -\frac{dn}{dg_1} + \frac{2n\frac{dn}{dg_1} + \frac{dm}{dg_1}}{2\sqrt{n^2 + m}} = \frac{\frac{dm}{dg_1} - 2q^D\frac{dn}{dg_1}}{2\sqrt{n^2 + m}}. \tag{18}
$$

We calculate

$$\frac{dm}{dg_1} = \frac{\delta p_o}{g_1^2(1+\delta)(p_o-p_n)^2}\left(-1+\frac{p_n g_1^2\gamma(p_{n,S}-p_o)(p_o-p_n)}{\delta\left(p_n g_1\gamma(p_{n,S}-p_o)-(p_o-p_n)\right)^2}(1-p_o)\right),$$

$$\frac{dm}{dg_1}-2\frac{dn}{dg_1} =$$

$$= \frac{\delta p_n}{g_1^2(1+\delta)(p_o-p_n)^2}\left(-1+\frac{p_n g_1^2\gamma(p_{n,S}-p_o)(p_o-p_n)}{\delta\left(p_n g_1\gamma(p_{n,S}-p_o)-(p_o-p_n)\right)^2}(1-p_o+\gamma(p_{n,S}-p_o))\right).$$

One can see that if $\frac{dm}{dg_1} > 0$, then $\frac{dm}{dg_1} - 2\frac{dn}{dg_1} > 0$. Consequently, $\frac{dn}{dg_1} < 0$. Observe from (18) that $\frac{dq^D}{dg_1}$ has the same sign as $\frac{dm}{dg_1} - 2q^D\frac{dn}{dg_1}$.

If $\frac{dm}{dg_1} > 0$, then the sign is positive for all q^D. This holds if

$$\begin{aligned}
&-1+\frac{p_n g_1^2\gamma(p_{n,S}-p_o)(p_o-p_n)}{\delta\left(p_n g_1\gamma(p_{n,S}-p_o)-(p_o-p_n)\right)^2}(1-p_o) > 0\\
\Leftrightarrow &-1+\frac{p_n \gamma(p_{n,S}-p_o)(p_o-p_n)}{\delta\left(p_n \gamma(p_{n,S}-p_o)-(p_o-p_n)/g_1\right)^2}(1-p_o) > 0\\
\Leftrightarrow & g_1 < \underline{g},
\end{aligned}$$

where $\underline{g}$ solves

$$\frac{p_n \gamma(p_{n,S}-p_o)(p_o-p_n)}{\delta\left(p_n \gamma(p_{n,S}-p_o)-(p_o-p_n)/\underline{g}\right)^2}(1-p_o) = 1.$$

Conversely, if $\frac{dm}{dg_1} - 2\frac{dn}{dg_1} < 0$, then the sign of $\frac{dm}{dg_1} - 2q^D\frac{dn}{dg_1}$ is negative for all q^D. This holds if

$$\begin{aligned}
&-1+\frac{p_n g_1^2\gamma(p_{n,S}-p_o)(p_o-p_n)}{\delta\left(p_n g_1\gamma(p_{n,S}-p_o)-(p_o-p_n)\right)^2}(1-p_o+\gamma(p_{n,S}-p_o)) < 0\\
\Leftrightarrow &-1+\frac{p_n \gamma(p_{n,S}-p_o)(p_o-p_n)}{\delta\left(p_n \gamma(p_{n,S}-p_o)-(p_o-p_n)/g_1\right)^2}(1-p_o+\gamma(p_{n,S}-p_o)) < 0\\
\Leftrightarrow & g_1 > \overline{g},
\end{aligned}$$

where $\overline{g}$ solves

$$\frac{p_n\gamma(p_{n,S}-p_o)(p_o-p_n)}{\delta\left(p_n\gamma(p_{n,S}-p_o)-(p_o-p_n)/\overline{g}\right)^2}(1-p_o+\gamma(p_{n,S}-p_o))=1.\blacksquare$$

Proof of Lemma 5:

Regime DM.

The manager chooses to use the old method or the new one to maximize the utility $U = P_1 + \delta P_2$. Denote the investors' belief of the exploration strategy of the manager as q. The difference in the expected utilities of the manager conditional on exploring the new method and exploiting the old is:

$$\begin{aligned}
&E[U|a_1=n]-E[U|a_1=o] \\
&=E[P_1|a_1=n]-E[P_1|a_1=o]+\delta\left(E[P_2|a_1=n]-E[P_2|a_1=o]\right).
\end{aligned}$$

The short-term prices P_1 and their conditional expectations $E[P_1|a_1=o]$ and $E[P_1|a_1=n]$ are the same functions of q as in Lemma 3 with one period discretionary disclosure.

Let us now compute the long-term prices as functions of the disclosure in the first and the second period. Observe that due to early discretionary disclosure the manager always strictly prefers to conceal the failure in the first period, so that the report $R_1 \in \{S, \varnothing\}$. The market prices formed by the investors in period 2 upon observing reports R_1 and R_2, where $R_1 \in \{S, \varnothing\}$ and $R_2 \in \{S, F, \varnothing\}$ are:

$$\begin{aligned}
P_2(S,S) &= S+\gamma S\,, \\
P_2(S,F) &= S+\gamma F\,, \\
P_2(S,\varnothing,q) &= P_1(S,q) \\
&= S+\underbrace{\left(\Pr\left(a_1=n|S,q\right)p_{n,S}+\left(1-\Pr\left(a_1=n|S,q\right)\right)p_o\right)}_{x}\gamma S+(1-x)\,\gamma F\,,
\end{aligned}$$

$$
\begin{aligned}
P_2(\varnothing, S, q) &= S \Pr(v_1 = S|\varnothing, S, q) + F(1 - \Pr(v_1 = S|\varnothing, S, q)) + \gamma S\,, \\
P_2(\varnothing, F, q) &= S \Pr(v_1 = S|\varnothing, F, q) + F(1 - \Pr(v_1 = S|\varnothing, F, q)) + \gamma F\,, \\
P_2(\varnothing, \varnothing, q) &= P_1(\varnothing, q) \\
&= \Pr(v_1 = S|\varnothing, q)\, S + (1 - \Pr(v_1 = S|\varnothing, q))\, F + p_o \gamma S + (1 - p_o)\, \gamma F\,,
\end{aligned}
$$

where $\Pr(v_1 = S|\varnothing, S, q) = \Pr(v_1 = S|\varnothing, F, q) = \Pr(v_1 = S|\varnothing, q)$ because, conditional on no disclosure in the first period, the market infers that the manager either did not receive information or received a failure. In either case, the manager uses the old method in the second period and, consequently, the report of the second period does not provide additional information whether the manager tried the new method in the first period. The expressions for $\Pr(v_1 = S|\varnothing, q)$ and $\Pr(a_1 = n|S, q)$ were defined in the proof of Lemma 3 with one period discretionary disclosure.

Taking expectations, we have

$$
\begin{aligned}
&E[P_1|a_1 = n] - E[P_1|a_1 = o] \\
&= -g_1(p_o - p_n)(1 - \Pr(v_1 = S|\varnothing, q) + \gamma(p_{n,S} - p_o)\Pr(a_1 = n|S, q))(S - F)\,,
\end{aligned}
$$

and

$$
\begin{aligned}
E[P_2|a_1 = n] &= g_1 p_n \left[p_{n,S} g_2 P_2(S,S,q) + (1 - p_{n,S}) g_2 P_2(S,F,q) + (1 - g_2) P_2(S,\varnothing,q)\right] \\
&\quad + (1 - g_1 p_n)\left[p_o g_2 P_2(\varnothing,S,q) + (1 - p_o) g_2 P_2(\varnothing,F,q) + (1 - g_2) P_2(\varnothing,\varnothing,q)\right]\,, \\
E[P_2|a_1 = o] &= g_1 p_o \left[p_o g_2 P_2(S,S,q) + (1 - p_o) g_2 P_2(S,F,q) + (1 - g_2) P_2(S,\varnothing,q)\right] \\
&\quad + (1 - g_1 p_o)\left[p_o g_2 P_2(\varnothing,S,q) + (1 - p_o) g_2 P_2(\varnothing,F,q) + (1 - g_2) P_2(\varnothing,\varnothing,q)\right]\,,
\end{aligned}
$$

so that the difference can be simplified as

$$
\begin{aligned}
&E[P_2|a_1=n]-E[P_2|a_1=o]=E[P_1|a_1=n]-E[P_1|a_1=o]\\
&+\gamma g_1 g_2\left(p_{n,S}-p_o\right)\left(p_o \Pr\left(a_1=n|S,q\right)+p_n\left(1-\Pr\left(a_1=n|S,q\right)\right)\right)(S-F)\\
&=g_1\Bigg(\gamma g_2 p_n\left(p_{n,S}-p_o\right)-\left(p_o-p_n\right)\left(1-\Pr\left(v_1=S|\varnothing,q\right)\right)\\
&-\gamma(1-g_2)(p_o-p_n)\left(p_{n,S}-p_o\right)\Pr\left(a_1=n|S,q\right)\Bigg)(S-F).
\end{aligned}
$$

Recall from Lemma 3 that probability $\Pr\left(a_1=n|S,q\right)$ increases in q and the probability $\Pr\left(v_1=S|\varnothing,q\right)$ decreases in q, consequently, both $E[P_1|a_1=n]-E[P_1|a_1=o]$ and $E[P_2|a_1=n]-E[P_2|a_1=o]$ decrease in q. Thus $E[U|a_1=n]-E[U|a_1=o]$ also decreases in q and there is a unique equilibrium with $q^{DM}\in[0,1]$.

In particular, it follows that: (a) if $E[U|a_1=n]-E[U|a_1=o]>0$ for $q=1$, then the equilibrium probability of exploration q^{DM}=1; (b) if $E[U|a_1=n]-E[U|a_1=o]<0$ for $q=0$, then the equilibrium probability of exploration q^{DM}=0; and (c) if the conditions in (a) and (b) are not satisfied, then there exists a unique equilibrium in mixed strategies with q^{DM} solving $E[U|a_1=n]-E[U|a_1=o]=0$.

(a) Substituting $q=1$ into $E[U|a_1=n]-E[U|a_1=o]$, we observe that it has the same sign as $-\left(1+\delta\right)\left(1-p_n\right)\left(p_o-p_n\right)+\gamma\left(\delta g_2 p_o-\left(1+\delta\right)\left(p_o-p_n\right)\right)\left(1-g_1 p_n\right)\left(p_{n,S}-p_o\right)$, and consequently, $E[U|a_1=n]-E[U|a_1=o]$ is positive at $q=1$ when $\left(\delta g_2 p_o-\left(1+\delta\right)\left(p_o-p_n\right)\right)>0$ and $\gamma>\overline{\gamma}^{DM}\equiv\frac{(1+\delta)(1-p_n)(p_o-p_n)}{(\delta g_2 p_o-(1+\delta)(p_o-p_n))(1-g_1 p_n)\left(p_{n,S}-p_o\right)}$. It follows then that $q^{DM}=1$ for $\gamma>\overline{\gamma}^{DM}$ providing that $\left(\delta g_2 p_o-\left(1+\delta\right)\left(p_o-p_n\right)\right)>0$. If this condition is not satisfied, the case (a) is not realized for any γ, i.e. $\overline{\gamma}^{DM}=+\infty$.

(b) Substituting $q=0$ into $E[U|a_1=n]-E[U|a_1=o]$, we observe that it has the same sign as $-\left(1+\delta\right)\left(1-p_o\right)\left(p_o-p_n\right)+\gamma\delta g_2 p_n\left(1-g_1 p_o\right)\left(p_{n,S}-p_o\right)$, and consequently, $E[U|a_1=n]-E[U|a_1=o]$ is negative at $q=0$ when $\gamma<\underline{\gamma}^{DM}\equiv\frac{(1+\delta)(1-p_o)(p_o-p_n)}{\delta g_2 p_n(1-g_1 p_o)\left(p_{n,S}-p_o\right)}$. It follows then that $q^{DM}=0$ for $\gamma<\underline{\gamma}^{DM}$.

(c) Due to monotonicity of the expression $E[U|a_1 = n] - E[U|a_1 = o]$ in q when the conditions in (a) and (b) are not satisfied, there exists a unique $q^{DM} \in (0,1)$ solving $E[U|a_1 = n] - E[U|a_1 = o] = 0$.■

Regime MD.

The proof is analogous to the proof above. We denote the investors' belief of the exploration strategy of the manager as q. The difference in the expected utilities of the manager conditional on exploring the new method and exploiting the old is:

$$
\begin{aligned}
&E[U|a_1 = n] - E[U|a_1 = o] \\
&= E[P_1|a_1 = n] - E[P_1|a_1 = o] + \delta \left(E[P_2|a_1 = n] - E[P_2|a_1 = o] \right) .
\end{aligned}
$$

The short-term prices P_1 and their conditional expectations $E[P_1|a_1 = o]$ and $E[P_1|a_1 = n]$ are the same functions of q as in Lemma 2 with one period mandatory disclosure.

Let us now compute the long-term prices as functions of the disclosure in the first and the second period. Observe that the manager always strictly prefers to conceal the failure in the second period, so that the report $R_2 \in \{S, \varnothing\}$. The market prices formed by the investors in period 2 upon observing reports R_1 and R_2, where $R_1 \in \{S, F, \varnothing\}$ and $R_2 \in \{S, \varnothing\}$ are:

$$
\begin{aligned}
P_2(S,S) &= S + \gamma S\,, \\
P_2(F,S) &= F + \gamma S\,, \\
P_2(\varnothing, S, q) &= q\left(p_n S + (1-p_n)F\right) + (1-q)\left(p_o S + (1-p_o)F\right) + \gamma S\,, \\
P_2(S, \varnothing, q) &= S + \gamma \left(\Pr\left(v_2 = S|S, \varnothing, q\right) S + \left(1 - \Pr\left(v_2 = S|S, \varnothing, q\right)\right) F \right)\,, \\
P_2(F, \varnothing, q) &= F + \gamma \left(\Pr\left(v_2 = S|F, \varnothing, q\right) S + \left(1 - \Pr\left(v_2 = S|F, \varnothing, q\right)\right) F \right)\,, \\
P_2(\varnothing, \varnothing, q) &= q\left(p_n S + (1-p_n)F\right) + (1-q)\left(p_o S + (1-p_o)F\right) \\
&\quad + \gamma \left(\Pr\left(v_2 = S|\varnothing, \varnothing, q\right) S + \left(1 - \Pr\left(v_2 = S|\varnothing, \varnothing, q\right)\right) F \right)\,,
\end{aligned}
$$

where

$$
\begin{aligned}
\Pr\left(v_2 = S|S,\varnothing,q\right) &= \frac{\Pr\left(v_2 = S; S,\varnothing,q\right)}{\Pr\left(S,\varnothing,q\right)} \\
&= \frac{qp_n g_1 p_{n,S}(1-g_2) + (1-q)p_o g_1 p_o(1-g_2)}{qp_n g_1\left((1-g_2)+g_2(1-p_{n,S})\right) + (1-q)p_o g_1\left((1-g_2)+g_2(1-p_o)\right)} \\
&= \frac{qp_n p_{n,S}(1-g_2) + (1-q)(p_o)^2(1-g_2)}{qp_n\left(1-g_2 p_{n,S}\right) + (1-q)p_o\left(1-g_2 p_o\right)}
\end{aligned}
$$

is the probability that the value $v_2 = S$ given that there was a disclosure of S in the first period and no disclosure in the second period, and

$$
\Pr\left(v_2 = S|F,\varnothing,q\right) = \Pr\left(v_2 = S|\varnothing,\varnothing,q\right) = \frac{p_o(1-g_2)}{1-g_2+g_2(1-p_o)} = \frac{p_o(1-g_2)}{1-g_2 p_o},
$$

is the probability that the value $v_2 = S$ given that there was a disclosure of F in the first period or no disclosure in the first period. Observe that in either case the manager will not repeat a new method in the second period as the method either resulted in failure or the manager did not observe the result at all. Consequently, whether the manager tried the new method in the first period or not is irrelevant for the probability of receiving S in the new period.

Taking expectations, we have

$$
E[P_1|a_1 = n] - E[P_1|a_1 = o] = -g_1\left(p_o - p_n\right)\left(1 + \gamma\left(p_{n,S} - p_o\right)\Pr\left(a_1 = n|S,q\right)\right)\left(S - F\right),
$$

and

$$
\begin{aligned}
E[P_2|a_1 = n] = g_1 p_n & \left[p_{n,S} g_2 P_2(S,S,q) + (1-p_{n,S} g_2)P_2(S,\varnothing,q)\right] \\
& + g_1\left(1-p_n\right)\left[p_o g_2 P_2(F,S,q) + (1-p_o g_2)P_2(F,\varnothing,q)\right] \\
& + (1-g_1)\left[p_o g_2 P_2(\varnothing,S,q) + (1-p_o g_2)P_2(\varnothing,\varnothing,q)\right],
\end{aligned}
$$

$$E[P_2|a_1 = o] = g_1 p_o \left[p_o g_2 P_2(S,S,q) + (1-p_o g_2) P_2(S,\varnothing,q)\right]$$
$$+ g_1 (1-p_o) \left[p_o g_2 P_2(F,S,q) + (1-p_o g_2) P_2(F,\varnothing,q)\right]$$
$$+ (1-g_1) \left[p_o g_2 P_2(\varnothing,S,q) + (1-p_o g_2) P_2(\varnothing,\varnothing,q)\right] ,$$

and the difference can be simplified as

$$E[P_2|a_1 = n] - E[P_2|a_1 = o] = g_1(S-F) \left(\gamma g_2 p_n \left(p_{n,S} - p_o\right) - \left(p_o - p_n\right)\right)$$
$$+ g_1(S-F)\gamma \left(1 - g_2 p_o\right)\left(p_o - p_n\right) \Pr\left(v_2 = S|F,\varnothing,q\right)$$
$$- g_1(S-F)\gamma \left(p_o \left(1 - p_o g_2\right) - p_n \left(1 - p_{n,S} g_2\right)\right) \Pr\left(v_2 = S|S,\varnothing,q\right).$$

Recall from Lemma 3 that probability $\Pr(a_1 = n|S,q)$ increases in q and, consequently, $E[P_1|a_1 = n] - E[P_1|a_1 = o]$ decreases in q. Observe also that $E[P_2|a_1 = n] - E[P_2|a_1 = o]$ only depends on q through $\Pr(v_2 = S|S,\varnothing,q)$, which increases in q. Since $E[P_2|a_1 = n] - E[P_2|a_1 = o]$ decreases in $\Pr(v_2 = S|S,\varnothing,q)$, it follows that $E[P_2|a_1 = n] - E[P_2|a_1 = o]$ also decreases in q. The above argument establishes that $E[U|a_1 = n] - E[U|a_1 = o]$ also decreases in q, and there is a unique equilibrium with $q^{MD} \in [0,1]$.

In particular, it follows that: (a) if $E[U|a_1 = n] - E[U|a_1 = o] > 0$ for $q = 1$, then the equilibrium probability of exploration q^{MD}=1; (b) if $E[U|a_1 = n] - E[U|a_1 = o] < 0$ for $q = 0$, then the equilibrium probability of exploration q^{MD}=0; and (c) if the conditions in (a) and (b) are not satisfied, then there exists a unique equilibrium in mixed strategies with q^{MD} solving $E[U|a_1 = n] - E[U|a_1 = o] = 0$.

(a) Substituting $q = 1$ into $E[U|a_1 = n] - E[U|a_1 = o]$, we observe that it has the same sign as

$$- (1+\delta)\left(1 - g_2 p_{n,S}\right)\left(p_o - p_n\right)$$
$$+ \gamma \left(\delta \left(p_n \left(1 - g_2 p_{n,S}\right) - p_o(1-g_2)\right) - \left(1 - g_2 p_{n,S}\right)\left(p_o - p_n\right)\right)\left(p_{n,S} - p_o\right),$$

and consequently, $E[U|a_1 = n] - E[U|a_1 = o]$ is positive at $q = 1$ when

$$\left(\delta\left(p_n\left(1 - g_2 p_{n,S}\right) - p_o(1 - g_2)\right) - \left(1 - g_2 p_{n,S}\right)\left(p_o - p_n\right)\right) > 0$$

and $\gamma > \overline{\gamma}^{MD} \equiv \frac{(1+\delta)\left(1-g_2 p_{n,S}\right)(p_o-p_n)}{\left(\delta\left(p_n\left(1-g_2 p_{n,S}\right)-p_o(1-g_2)\right)-\left(1-g_2 p_{n,S}\right)(p_o-p_n)\right)\left(p_{n,S}-p_o\right)}$. It follows then that $q^{MD} = 1$ for $\gamma > \overline{\gamma}^{MD}$ providing that $\delta\left(1 - \frac{p_o(1-g_2)}{p_n\left(1-g_2 p_{n,S}\right)}\right) > \frac{p_o-p_n}{p_n} > 0$. If this condition is not satisfied, the case (a) is not realized for any γ, i.e. $\overline{\gamma}^{MD} = +\infty$.

(b) Substituting $q = 0$ into $E[U|a_1 = n] - E[U|a_1 = o]$, we observe that it has the same sign as $-(1+\delta)(1 - g_2 p_o)(p_o - p_n) + \gamma\delta g_2 p_n (1 - p_o)(p_{n,S} - p_o)$, and consequently, $E[U|a_1 = n] - E[U|a_1 = o]$ is negative at $q = 0$ when $\gamma < \underline{\gamma}^{MD} \equiv \frac{(1+\delta)(1-g_2 p_o)(p_o-p_n)}{\delta g_2 p_n(1-p_o)\left(p_{n,S}-p_o\right)}$. It follows then that $q^{MD} = 0$ for $\gamma < \underline{\gamma}^{MD}$.

(c) Due to monotonicity of the expression $E[U|a_1 = n] - E[U|a_1 = o]$ in q when the conditions in (a) and (b) are not satisfied, there exists a unique $q^{MD} \in (0, 1)$ solving $E[U|a_1 = n] - E[U|a_1 = o] = 0$.

The proofs for the regimes MM and DD are analogous.■

Proof of Proposition 3:

To prove this proposition, we compare the incremental incentives to try the new method, $E[U|a_1 = n] - E[U|a_1 = o]$, for any belief of the market q about the probability of the manager trying the new method. in particular, let us denote by $\Delta^{ij} = E[U|a_1 = n] - E[U|a_1 = o]$ the incremental incentive to explore under the disclosure regime ij, where $i, j \in \{D, M\}$.

We will show that $\Delta^{MD} \leq \Delta^{DD} \leq \Delta^{DM}$ and that $\Delta^{MD} \leq \Delta^{MM} \leq \Delta^{DM}$. Given that Δ^{DM} and Δ^{MD} decrease in q as established in Lemma 5 and that there are unique equilibria in the regimes MD and DM (q^{MD} and q^{DM}, respectively), it follows that all equilibria q^{DD} and q^{MM} in the disclosure regimes DD and MM compare to q^{MD} and q^{DM} as described in the Lemma.

Let us first show that $\Delta^{MD} \leq \Delta^{MM} \leq \Delta^{DM}$. We start with computing Δ^{MM}, which is the incremental incentive to try a new method $E[U|a_1 = n] - E[U|a_1 = o]$ in the regime where disclosure is mandated both in period 1 and period 2.

The short-term prices P_1 and their conditional expectations $E[P_1|a_1 = o]$ and $E[P_1|a_1 = n]$ are the same functions of q as in Lemma 2 with mandatory disclosure in the first period. Let us now compute the long-term prices formed by the investors in period 2 upon observing reports R_1 and R_2, where $R_1 \in \{S, F, \varnothing\}$ and $R_2 \in \{S, F, \varnothing\}$:

$$
\begin{aligned}
P_2(F,F) &= F + \gamma F\,, \\
P_2(F,S) &= F + \gamma S\,, \\
P_2(F,\varnothing,q) &= F + \gamma\left(p_o S + (1-p_o)F\right)\,, \\
P_2(\varnothing,F,q) &= q\left(p_n S + (1-p_n)F\right) + (1-q)\left(p_o S + (1-p_o)F\right) + \gamma F\,, \\
P_2(\varnothing,S,q) &= q\left(p_n S + (1-p_n)F\right) + (1-q)\left(p_o S + (1-p_o)F\right) + \gamma S\,, \\
P_2(\varnothing,\varnothing,q) &= q\left(p_n S + (1-p_n)F\right) + (1-q)\left(p_o S + (1-p_o)F\right) + \gamma\left(p_o S + (1-p_o)F\right)\,, \\
P_2(S,F) &= S + \gamma F\,, \\
P_2(S,S) &= S + \gamma S\,, \\
P_2(S,\varnothing,q) &= P_1(S,q) \\
&= S + \underbrace{\left(\Pr\left(a_1 = n|S,q\right)p_{n,S} + \left(1 - \Pr\left(a_1 = n|S,q\right)\right)p_o\right)}_{x}\gamma S + (1-x)\,\gamma F\,.
\end{aligned}
$$

Taking expectations, we have

$$
E[P_1|a_1 = n] - E[P_1|a_1 = o] = -g_1\left(p_o - p_n\right)\left(1 + \gamma\left(p_{n,S} - p_o\right)\Pr\left(a_1 = n|S,q\right)\right)(S - F)\,,
$$

and

$$
\begin{aligned}
E[P_2|a_1=n] &= g_1 p_n \left[p_{n,S} g_2 P_2(S,S,q) + (1-p_{n,S}) g_2 P_2(S,F,q) + (1-g_2) P_2(S,\varnothing,q)\right] \\
&\quad + g_1 (1-p_n) \left[p_o g_2 P_2(F,S,q) + (1-p_o) g_2 P_2(F,F,q) + (1-g_2) P_2(F,\varnothing,q)\right] \\
&\quad + (1-g_1) \left[p_o g_2 P_2(\varnothing,S,q) + (1-p_o) g_2 P_2(\varnothing,F,q) + (1-g_2) P_2(\varnothing,\varnothing,q)\right] , \\
E[P_2|a_1=o] &= g_1 p_o \left[p_o g_2 P_2(S,S,q) + (1-p_o) g_2 P_2(S,F,q) + (1-g_2) P_2(S,\varnothing,q)\right] \\
&\quad + g_1 (1-p_o) \left[p_o g_2 P_2(F,S,q) + (1-p_o) g_2 P_2(F,F,q) + (1-g_2) P_2(F,\varnothing,q)\right] \\
&\quad + (1-g_1) \left[p_o g_2 P_2(\varnothing,S,q) + (1-p_o) g_2 P_2(\varnothing,F,q) + (1-g_2) P_2(\varnothing,\varnothing,q)\right] .
\end{aligned}
$$

We substitute the expected prices and compute Δ^{MM}. After comparing it to Δ^{DM} derived in Lemma 5, we have $\Delta^{DM} - \Delta^{MM} = (1+\delta)\, g_1 (p_o - p_n)(S-F) \Pr(v_1 = S|\varnothing, q) \geq 0$. After comparing it to Δ^{MD} derived in Lemma 5, we have

$$
\Delta^{MM} - \Delta^{MD} = \frac{\delta g_1 g_2 (1-g_2) \gamma p_o p_n (p_{n,S} - p_o)(p_o^2 (1-q) + p_n p_{n,S} q)(S-F)}{(p_o(1-q) + p_n q)(p_o (1-q)(1-g_2 p_o) + p_n q (1 - g_2 p_{n,S}))} \geq 0.
$$

Summing up, $\Delta^{MD} \leq \Delta^{MM} \leq \Delta^{DM}$ for any $q \in [0,1]$ and, consequently, $q^{MD} \leq q^{MM} \leq q^{DM}$.

Let us now show that $\Delta^{MD} \leq \Delta^{DD} \leq \Delta^{DM}$. We start with computing Δ^{DD}, which is the incremental incentive to try a new method $E[U|a_1 = n] - E[U|a_1 = o]$ in the regime where disclosure is discretionary both in period 1 and period 2.

The short-term prices P_1 and their conditional expectations $E[P_1|a_1 = o]$ and $E[P_1|a_1 = n]$ are the same functions of q as in Lemma 3 with discretionary disclosure in the first period. Let us now compute the long-term prices formed by the investors in period 2 upon observing reports R_1 and R_2, where $R_1 \in \{S, \varnothing\}$ and $R_2 \in \{S, \varnothing\}$ (recall that F is always concealed, and S is revealed).

$$
\begin{aligned}
P_2(S,S) &= S + \gamma S\,, \\
P_2(\varnothing,S,q) &= S\Pr(v_1 = S|\varnothing,S,q) + F\left(1 - \Pr(v_1 = S|\varnothing,S,q)\right) + \gamma S\,, \\
P_2(S,\varnothing,q) &= S + \gamma\left(\Pr(v_2 = S|S,\varnothing,q)\,S + \left(1 - \Pr(v_2 = S|S,\varnothing,q)\right)F\right)\,, \\
P_2(\varnothing,\varnothing,q) &= S\Pr(v_1 = S|\varnothing,\varnothing,q) + F\left(1 - \Pr(v_1 = S|\varnothing,\varnothing,q)\right) \\
&\quad + \gamma\left(\Pr(v_2 = S|\varnothing,\varnothing,q)\,S + \left(1 - \Pr(v_2 = S|\varnothing,\varnothing,q)\right)F\right)\,.
\end{aligned}
$$

Here, the $P_2(\varnothing,S,q)$ is the same as in the DM regime and $P_2(S,\varnothing,q)$ is the same as in the MD regime Taking expectations, we have

$$
\begin{aligned}
&E[P_1|a_1 = n] - E[P_1|a_1 = o] \\
&= -g_1(p_o - p_n)\left(1 - \Pr(v_1 = S|\varnothing,q) + \gamma(p_{n,S} - p_o)\Pr(a_1 = n|S,q)\right)(S - F)\,,
\end{aligned}
$$

and

$$
\begin{aligned}
E[P_2|a_1 = n] &= g_1 p_n\left[p_{n,S}g_2 P_2(S,S,q) + (1 - p_{n,S}g_2)P_2(S,\varnothing,q)\right] \\
&\quad + (1 - g_1 p_n)\left[p_o g_2 P_2(\varnothing,S,q) + (1 - p_o g_2)P_2(\varnothing,\varnothing,q)\right]\,, \\
E[P_2|a_1 = o] &= g_1 p_o\left[p_o g_2 P_2(S,S,q) + (1 - p_o g_2)P_2(S,\varnothing,q)\right] \\
&\quad + (1 - g_1 p_o)\left[p_o g_2 P_2(\varnothing,S,q) + (1 - p_o g_2)P_2(\varnothing,\varnothing,q)\right]\,,
\end{aligned}
$$

We substitute the expected prices and compute Δ^{DD}. After comparing it to Δ^{DM} derived in Lemma 5, we have

$$
\Delta^{DM} - \Delta^{DD} = \frac{\delta g_1 g_2(1 - g_2)\gamma p_o p_n(p_{n,S} - p_o)\left(p_o^2(1 - q) + p_n p_{n,S}q\right)(S - F)}{(p_o(1 - q) + p_n q)\left(p_o(1 - q)(1 - g_2 p_o) + p_n q(1 - g_2 p_{n,S})\right)} \geq 0.
$$

Observe also that $\Delta^{DM} - \Delta^{DD} = \Delta^{MM} - \Delta^{MD}$, which we derived earlier. It follows then that $\Delta^{DD} - \Delta^{MD} = \Delta^{DM} - \Delta^{MM} \geq 0$, where inequality has been established earlier in the lemma.

Summing up, $\Delta^{MD} \leq \Delta^{DD} \leq \Delta^{DM}$ for any $q \in [0,1]$ and, consequently, $q^{MD} \leq q^{DD} \leq q^{DM}$.■

Proof of Proposition 4:

Consider first the scenario MM with mandatory disclosure. By substituting the prices derived above in Lemma 5 and Proposition 3, we obtain that

$$\begin{aligned}
&E[U|a_1 = n] - E[U|a_1 = o] \\
&= E[P_1|a_1 = n] - E[P_1|a_1 = o] + \delta \left(E[P_2|a_1 = n] - E[P_2|a_1 = o] \right) \\
&= g_1 \left[\gamma \delta g_2 p_n (p_{n,S} - p_o) - (p_o - p_n)(1+\delta) \right] (S - F) \\
&\ - g_1 \gamma (1 + \delta(1 - g_2))(p_o - p_n)(p_{n,S} - p_o) \Pr(a_1 = n|S, q^{MM})(S - F).
\end{aligned}$$

Denote the last expression as $\Phi^{MM}(q^{MM})$. Since $\Pr(a_1 = n|S, q^{MM}) = \frac{1}{1 + \frac{(1-q^{MM} p_o)}{q^{MM} p_n}}$ is increasing in q^{MM}, $\Phi^{MM}(q^{MM})$ is decreasing in q^{MM}. After the analysis similar to to the cases of MD and DM in Lemma 5, we conclude that q^{MM} is a unique solution to $\Phi^{MM} = 0$ when $\gamma \in \left[\underline{\gamma}^{MM}, \overline{\gamma}^{MM}\right]$, $q^{MM} = 1$ for $\gamma > \overline{\gamma}^{MM}$, and $q^{MM} = 0$ for $\gamma < \underline{\gamma}^{MM}$, where $\overline{\gamma}^{MM} = \overline{\gamma}^{DM} \frac{1 - g_1 p_n}{1 - p_n}$ and $\underline{\gamma}^{MM} = \underline{\gamma}^{DM} \frac{1 - g_1 p_o}{1 - p_o}$.

Now consider the scenario DD with discretionary disclosure. The difference between the utilities the manager obtains from exploring versus exploiting in period 1 is

$$
\begin{aligned}
&E[U|a_1 = n] - E[U|a_1 = o] \\
&= g_1(\gamma\delta g_2 p_n(p_{n,S} - p_o) - (1+\delta)(p_o - p_n))(S - F) \\
&- g_1\gamma(p_o - p_n)(p_{n,S} - p_o)\Pr(a_1 = n|S, q^{DD})(S - F) \\
&+ g_1\delta\gamma(p_o - p_n)(1 - g_2 p_o)\Pr(v_2 = S|\varnothing, \varnothing, q^{DD})(S - F) \\
&+ g_1(1+\delta)(p_o - p_n)\Pr(v_1 = S|\varnothing, q^{DD})(S - F) \\
&- g_1\delta\gamma(p_o(1 - g_2 p_o) - p_n(1 - g_2 p_{n,S}))\Pr(v_2 = S|S, \varnothing, q^{DD})(S - F).
\end{aligned}
$$

Denote this expression as $\Phi^{DD}(q^{DD})$, and derive

$$
\begin{aligned}
\Pr(a_1 = n|S, q^{DD}) &= \frac{1}{1 + \frac{(1-q^{DD}p_o)}{q^{DD}p_n}}, \\
\Pr(v_2 = S|\varnothing, \varnothing, q^{DD}) &= \frac{1}{1 + \frac{1-p_o}{(1-g_2)p_o}}, \\
\Pr(v_1 = S|\varnothing, q^{DD}) &= \frac{1}{1 + \frac{q^{DD}(1-p_n)+(1-q^{DD})(1-p_o)}{(1-g_1)(q^{DD}p_n+(1-q^{DD})p_o)}}, \\
\Pr(v_2 = S|S, \varnothing, q^{DD}) &= \frac{1}{1 + \frac{q^{DD}p_n(1-p_{n,S})+(1-q^{DD})p_o(1-p_o)}{(1-g_2)(q^{DD}p_n p_{n,S}+(1-q^{DD})p_o^2)}}.
\end{aligned}
$$

Since $\Pr(a_1 = n|S, q^{DD})$ and $\Pr(v_2 = S|S, \varnothing, q^{DD})$ are increasing in q^{DD} and $\Pr(v_1 = S|\varnothing, q^{DD})$ is decreasing in q^{DD}, $\Phi^{DD}(q^{DD})$ is decreasing in q^{DD}. After the analysis similar to to the cases of MD and DM in Lemma 5, we conclude that q^{DD} is a unique solution to $\Phi^{DD}(q^{DD}) = 0$ when $\gamma \in \left[\underline{\gamma}^{DD}, \overline{\gamma}^{DD}\right]$, $q^{DD} = 1$ for $\gamma > \overline{\gamma}^{DD}$, and $q^{DD} = 0$ for $\gamma < \underline{\gamma}^{DD}$, where $\overline{\gamma}^{DD} = \overline{\gamma}^{MD}\frac{1-p_n}{1-g_1p_n}$ and $\underline{\gamma}^{DD} = \underline{\gamma}^{MD}\frac{1-p_o}{1-g_1p_o}$.

Observe that, given the monotonicity of the functions $\Phi^{DD}(q^{DD})$, we have that the probability q^{DD} of exploration in the scenario DD with discretionary disclosure is higher than in the scenario MM with mandatory disclosure, $q^{DD} > q^{MM}$ if $\Phi^{DD}(q^{MM}) > 0$, that is if

$$
\begin{aligned}
& g_1(\gamma\delta g_2 p_n(p_{n,S} - p_o) - (1+\delta)(p_o - p_n))(S-F) \\
& - g_1\gamma(p_o - p_n)(p_{n,S} - p_o)\Pr(a_1 = n|S, q^{MM})(S-F) \\
& + g_1\delta\gamma(p_o - p_n)(1 - g_2 p_o)\Pr(v_2 = S|\varnothing, \varnothing, q^{MM})(S-F) \\
& + g_1(1+\delta)(p_o - p_n)\Pr(v_1 = S|\varnothing, q^{MM})(S-F) \\
& - g_1\delta\gamma(p_o(1 - g_2 p_o) - p_n(1 - g_2 p_{n,S}))\Pr(v_2 = S|S, \varnothing, q^{MM})(S-F) > 0
\end{aligned}
$$

We next substitute $\Pr(a_1 = n|S, q^{MM})$ in the expression above from the expression $\Phi(q^{MM}) = 0$ and observe that the modified expression still decreases in q^{MM}. We proceed by concluding that if the modified expression is positive even for $q^{MM} = 1$, then $q^{DD} > q^{MM}$. Alternatively, we conclude that if the expression is negative even for $q^{MM} = 0$, then $q^{DD} < q^{MM}$.

These allows us to derive the sufficient condition for $q^{DD} > q^{MM}$ to be $C_1 < 0$, where

$$
\begin{aligned}
C_1 = & (1+\delta)(1 - g_2 p_{n,S})(p_o - p_n)\left(\delta(1-p_n)(1-g_2) - p_n(1-g_1)\right) \\
& + \gamma\delta(1 - g_1 p_n)(1-g_2)(p_{n,S} - p_o)\left((p_o - p_n) + g_2 p_n p_{n,S} + \delta\left(p_o(1-g_2) - p_n(1 - g_2 p_{n,S})\right)\right),
\end{aligned} \tag{19}
$$

and the sufficient condition for $q^{DD} < q^{MM}$ to be $C_2 > 0$, where

$$
\begin{aligned}
C_2 = & (1+\delta)(1 - g_2 p_o)(p_o - p_n)\left(\delta(1-p_o)(1-g_2) - p_o(1-g_1)\right) \\
& - \gamma\delta p_n(1 - g_1 p_o)(1-g_2) g_2 (p_{n,S} - p_o)\left(-p_o + \delta(1-p_o)\right) > 0,
\end{aligned} \tag{20}
$$

The proof follows from observing that the condition (19) is satisfied for small δ and the condition (20) is satisfied for large δ and small g_2.■

Proof of Proposition 5:

Recall that if $\underline{\gamma}^{DM} \geq \gamma^{FB}$ then $q^{DM} = q^{DD} = q^{MD} = q^{MM} = 0$ for all $\gamma < \gamma^{FB}$, which is the value-maximizing exploration strategy as $q^{FB} = 0$ as well. In contrast, for $\gamma > \gamma^{FB}$ the optimal strategy of exploration is $q^{FB} = 1$, and thus the regime that provides more exploration is optimal, which is the regime DM of early discretionary disclosure and mandated disclosure later on in the second period. Summing up, DM weakly dominates all other disclosure regimes for all $\gamma > 0$ when $\underline{\gamma}^{DM} \geq \gamma^{FB} \iff \underline{\gamma}^{DM}/\gamma^{FB} \geq 1 \iff \frac{g_1}{g_2}\frac{1+\delta}{\delta}\frac{1-p_o}{1-g_1 p_o} \geq 1$.

If $\underline{\gamma}^{DM} < \gamma^{FB}$ then $q^{DM} = q^{DD} = q^{MD} = q^{MM} = 0$ for all $\gamma < \underline{\gamma}^{DM}$, which is the value-maximizing exploration strategy as $q^{FB} = 0$ as well. In contrast, for $\gamma \in \left[\underline{\gamma}^{DM}, \gamma^{FB}\right)$ the optimal strategy of exploration is $q^{FB} = 0$, while the regime DM provides $q^{DM} > 0$. Consequently, the regime that provides less exploration is optimal, which is the regime MD of early mandatory disclosure and discretionary disclosure later on in the second period. Finally, for $\gamma \geq \gamma^{FB}$ the optimal strategy of exploration is $q^{FB} = 1$, and thus the regime that provides more exploration is optimal, which is the regime DM of early discretionary disclosure and mandated disclosure later on in the second period. Summing up, when $\underline{\gamma}^{DM} < \gamma^{FB} \iff \frac{g_1}{g_2}\frac{1+\delta}{\delta}\frac{1-p_o}{1-g_1 p_o} < 1$, then the disclosure regime DM that mandates late disclosure weakly dominates all other regimes for all $\gamma \geq \gamma^{FB}$, and the disclosure regime MD that mandates early disclosure weakly dominates all other regimes for all $\gamma < \gamma^{FB}$.■

Proof of Proposition 6:

The proof follows from the proof of Proposition 4. We showed there that the sufficient condition for $q^{DD} > q^{MM}$ is $C_1 < 0$ in (19) and the sufficient condition for $q^{DD} < q^{MM}$ is $C_2 > 0$ in (20). We now analyze these two conditions.

We start with $C_2 > 0$. One can see that $C_2 > 0$ for all γ if

$$\delta(1-p_o)(1-g_2) - p_o(1-g_1) > 0 \quad \text{and} \quad -p_o + \delta(1-p_o) < 0,$$

which is equivalent to $\delta \in \left(\frac{p_o(1-g_1)}{(1-p_o)(1-g_2)}, \frac{p_o}{1-p_o}\right)$, and the interval is well-defined for $g_2 < g_1$.

We now move to analyzing the condition $C_1 < 0$. One can see that $C_1 < 0$ for all γ if

$$\delta(1-p_n)(1-g_2) - p_n(1-g_1) < 0 \text{ and}$$
$$(p_o - p_n) + g_2 p_n p_{n,S} + \delta\left(p_o(1-g_2) - p_n(1-g_2 p_{n,S})\right) < 0,$$

which is equivalent to

$$\delta < \frac{p_n(1-g_1)}{(1-p_n)(1-g_2)},$$
$$\delta > \frac{(p_o - p_n) + g_2 p_n p_{n,S}}{-p_o(1-g_2) + p_n(1-g_2 p_{n,S})}, \text{ and}$$
$$p_n(1-g_2 p_{n,S}) > p_o(1-g_2).$$

Thew last condition can be simplified as $g_2 > (p_o - p_n)/(p_o - p_n p_{n,S})$, where $(p_o - p_n)/(p_o - p_n p_{n,S}) < 1$. In order to ensure that the interval for δ is well-defined, we require that

$$\frac{p_n(1-g_1)}{(1-p_n)(1-g_2)} > \frac{(p_o - p_n) + g_2 p_n p_{n,S}}{-p_o(1-g_2) + p_n(1-g_2 p_{n,S})},$$

which is further simplified as

$$g_1 < g_2 - \frac{(1-g_2)((p_o - p_n) + g_2 p_n(p_{n,S} - p_o))}{p_n(-p_o(1-g_2) + p_n(1-g_2 p_{n,S}))}.$$

The last condition can be achieved when g_2 is sufficiently high (as stipulated by the condition $g_2 > (p_o - p_n)/(p_o - p_n p_{n,S})$. Summing up, there indeed exists an interval of δ for which the condition $C_1 < 0$ is satisfied for any γ providing that g_2 is sufficiently large and g_1 is sufficiently low.

The last thing to observe is that the conditions on δ, g_1, and g_2 are independent of γ. Therefore one concludes that, under these conditions, the disclosure regime that maximizes innovation is weakly preferred as long as $\gamma > \gamma^{FB}$. This finishes the proof.■

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