Measuring and Mending LP Net Profitability

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This paper examines data on 22 automated market makers (AMM) pools since 2020 and documents that restricted-range AMM liquidity providers (LPs) generally incur losses after subtracting their gamma expense. We also document the activity and profitability of a handful of arbitrageurs across five ETH-USD pools. These data are used to demonstrate the feasibility of a mechanism designed to make LPs profitable on capital-efficient AMMs by whitelisting arbitrage traders and giving leverage to both the arbitrageurs and LPs.

Introduction

An AMM's 'impermanent loss' (IL) is the statistically certain consequence of the LP position's negative gamma. This negative drift is not large, a few basis points per day, but it accumulates to approximately 10% annually as a percentage of the unrestricted range (v2) LP market value. LPs for the most popular capital-efficient AMMs (restricted-range or v3) pools have continually generated negative net profits since inception, which is why AMM TVL peaked shortly after they were introduced in 2020. LP net returns on the capital-inefficient v2 AMMs have been modestly profitable since 2021.

DEX developers widely recognize LP net unprofitability, but its documentation is weak. Fee revenue is easy to calculate, but the LP's gamma expense—IL or LvR—is not. Most empirical estimates of v3 LP profitability do not present a stationary metric that enables comparisons across pools. For example, Crocswap (2022) presented cumulative USD losses by marking LP trades against a fixed horizon from 5 minutes to 7 days. Loesch et al. (2021), Drossos et al. (2025), Cartea et al. (2023), Heimbach et al. (2022), and Canidio and Fritsch (2025) document negative v3 LP returns using returns for specific LP positions across various pools. The LP profitability metrics are not normalized, generally presented via cumulative performance lines or scatterplots.

AMM LPs can protect themselves from IL by indirectly trading with themselves. Suppose a single LP created an AMM and created an arbitrage program that updated the AMM price to the Binance price on each 5 basis point (bp) movement. If he were the only trader and LP, he would be trading with himself in a closed system, a zero-sum game. As the asset price moved, his arbitrage account would make a profit equal to his LP position's IL. The mechanism outlined in this paper aims to simplify, maximize profitability, and minimize capital requirements for the arbitrageur's task. With a significant fee advantage, AMM-CEX price deviations would generate a profit opportunity only for the trader with zero fees. Providing margin accounts to LPs and arbitrageurs enables them to leverage each other, as statistically, their net token changes will offset each other on average. Implementing a periodic off-pool but in-AMM token swap to rebalance the arb and LPs would not affect LP liquidity or profitability, but it does significantly reduce the amount of tokens the arb and LPs need to move off and on chain to remain solvent. The arb's zero-fee would generate a barrier to entry that would increase their profits, and combined with reduced arb capital requirement, enables the LPs to capture most of the arb's profits yet still provide the arb with an attractive rate of return.

Empirical LP profitability studies have been suggestive, but the data are rarely normalized into variables that are stationary or comparable across pools. For example, cumulative net LP 'markout' PnLs are a function of both pool size and the profit rate. There is no attempt to generate standard errors, and the different PnLs by markout horizon suggest it is substantial. A v3 LP's rate of return varies by a factor of 100 depending on the size of its restricted range, making an LP's return a function of an arbitrary and unstated leverage assumption.

Given the absence of any academic consensus on how to measure IL or net LP profitability, the default practice is to present *gross* LP profits, which are revenues *without expenses*, divided by total value locked (TVL). Worse, these often include incentives by minting protocol tokens into the LP's revenue. Crypto LPs have become accustomed to seeing 50% APYs on most DEXes while losing money after expenses.

This paper demonstrates how the most straightforward IL metric, the change in pool tokens and the end-of-day price, is consistent with other estimates that utilize liquidity. For the v2 pools, the choice of IL metric does not matter because liquidity does not change significantly over a day. For v3, however, IL estimates are plagued by subtle variance-liquidity change correlations that bias these estimates (e.g., E(x*y) <> E(x)*E(y)). To avoid the ambiguity of an LP *returns* applied to v3 pools where LPs within the pool have a wide distribution of leverage, instead of an LP net APY, we normalize the IL with fee revenue, generating a stationary variable that enables comparisons between all types of AMM. The stationary variable cost ratio applies to any holding duration and has the same interpretation, whether the numeraire token is ETH or a stablecoin. Using data on 22 pools over the past five years, we document that v2 LPs make a net profit, while v3 LPs do not, at statistically significant levels; latency and fee tier do not significantly impact this outcome.

Prominent solutions to rectify the LP unprofitability include lower latency (e.g., Arbitrum vs. Ethereum), dynamic fees that scale with volatility (TraderJoe), end-of-trade fill prices that eliminate IL on single trades (Thorchain), single-sided liquidity (Thorchain), and dynamic LP-range strategies (Charm's Alpha Vaults). These have been implemented for years and have not generated capital-efficient, profitable LPs.

An oracle-updated price could mitigate LP IL (Im et al. 2024, Krishnamachari et al. 2021), but they are vulnerable to hacks, and it is non-trivial to continuously align the oracle's incentive with a pool's LPs. Coinbase and Chainlink are large enough companies that their franchise value would exceed any potential gain from misreporting prices, aligning their incentives as oracles with the DEX. However, these companies are susceptible to censorship because Coinbase is directly regulated, and Chainlink's executives live in the US and are thus ultimately subject to standard financial regulations under Dodd-Frank's expansive powers. Regulated administration is unacceptable for any long-run decentralized exchange (DEX).[1] Decentralization is essential to crypto because it cannot be centralized, despite government preferences.

Frequent batch auctions have been proposed by Budish, Cramton, and Shim (2015) to replace the standard centralized limit order book in traditional finance (tradFi). Auctions are attractive solutions because classic auction theory shows that auctions maximize seller revenue in various forms (English, Dutch, and double auctions). An AMM could auction off the right to trade first in a block (Jososo, 2022), aggregate all buy and sell orders in a block to transact at a single price (Ramseyer et al. 2024; Canidio and Fritsch 2025), or the right to become the exclusive owner of fee revenue over a fixed period (Adams, Moallemi, and Robinson 2024). The intuition behind an auction's efficiency is similar to that of Budish et al. (2015): arb competition in speed and gas payments to block builders incurs a deadweight cost for the exchange, as these arb expenses do not go to LPs. Auctions transfer off-contract competition into price competition that benefits LPs.

Centralized limit order book have not made significant movement towards auctions in spite of their academic popularity. Budish, Lee, and Shim (2024) hypothesize that reluctance arises because imitators would quickly eliminate the profit bump, leaving the innovator to bear the large fixed costs of creating a new exchange for a fleeting reward, as imitators would recapture market share. However, investment banks have created numerous dark pools with minimal start-up costs, each with its unique trading protocols and order types. None has experimented with frequent batch auctions.

Tradfi's revealed preference for CLOBs is more likely because exchanges realize that frequent auctions generate attack surfaces that not only hinder their efficiency but also expose the market to legal liability. Several papers have identified scenarios where frequent auctions allow informed traders to manipulate auction prices (Eibelshauser et al., 2022; Ausubel et al., 2014; Kagel and Levin, 2001). The small number of market makers participating in a repeated game makes collusion a reasonable default assumption. For example, in the 1990s, NASDAQ market makers were found guilty of colluding on stock quotes by enforcing a collusive equilibrium that discouraged any dealer from quoting stocks in spreads below 1/4 of a dollar. If NASDAQ dealers could enforce an unwritten illegal equilibrium with over 20 dominant players, a handful of abs with zero legal risk would have the will and ability to collude and avoid perfectly competitive outcomes that auction models generate. As documented in this paper, AMMs have, at most, a handful of arbs who set their prices, and its agents are worldwide, pseudonymous, and

[1] In the US, the Swap Execution Facility (SEF) regulation in the Dodd-Frank (2010) covers any American citizen or resident controlling any exchange regardless of the exchange's country of domicile.

uncensorable. A frequent auction would elevate the level of collusion that currently exists because arbs would have little fear of legal liability.

Many theoretical solutions rely on the assumption that in perfectly competitive markets, arbitrage profits would be zero. Several mechanisms eliminate arbitrageur profits, assuming perfectly competitive arbitrageurs, including zero-latency blocks, auctions, and trades executed at the end-of-trade price. As the LPs and arbitrageurs play a zero-sum game, a zero-profit arbitrageur implies that the LP offsets their IL completely through the arb's fees. In practice, the low-latency blockchains, AMMs that apply end-of-trade prices, or auctions have not generated capital-efficient, profitable LP positions. While everyone recognizes that, in practice, no market has perfect competition, this paper outlines the limiting principle that has allowed and will continue to allow arbitrageurs to maintain a tacit collusion, preventing perfect competition.

This paper illustrates how vertically integrating arbs into LPs can reduce the profit leakage to arbs. Any decentralized AMM will exist on a blockchain with latency 100 times greater than any centralized exchange. This is because decentralized blockchains require consensus mechanisms, which necessitate messaging across a sufficiently broad region to avoid country-specific censorship. For any token with liquid CEX markets, the DEX price will follow the CEX price and not vice versa because of its lower latency. This makes the arb's objective a straightforward technical problem as opposed to the nebulous alpha skill needed to set prices on CEX exchanges successfully. AMM arbs use liquid centralized exchanges like Binance as their target price because no other alpha is needed or adds value.

As there are only a handful of arbitrageurs for any one pool, they represent the left-tail of a power-law distribution in high-frequency trading competence. Maintaining a basic arbitrage bot requires a certain level of competence, including integrating price feeds to extract the true price, generating the lowest-latency connections to RPCs, understanding how blockchain builders respond, and batching off-AMM transactions to minimize CEX trading costs. Given this barrier to entry, the handful of remaining arbitrageurs then focus primarily on non-price competition to avoid the zero-profit equilibrium.

Instead of capturing arb profits via fees or auctions, one can tax the arb's profit directly and add this to the LP revenue. This aligns the arbs and LPs, who then both want to maximize the arb's profit. The arbs need a sufficient post-tax APY, and the data on arb activity shows that even if the arbs keep only 10% in this new scheme, they can still generate returns well above 20% on the required capital. Such returns are sufficient to ensure arbs will exist. The high tax rate and liquidation protocol will only appeal to those engaging in high-frequency AMM trading, discouraging traders from using the arb account to merely take positions without incurring a fee.

To see economic feasibility at a high level, consider a standard example of how to estimate annualized IL on a v2 pool, which is $\sigma^2/8$. For a token with a 4% daily volatility, the IL annualizes to a 7.3% APY. An arb with 5% of the v2 pool's market cap would have the capital to push the AMM price 10% in either direction, which would cover most days. Giving the arb 5:1 leverage implies the arb uses 100 times less capital than the v2 LP market value. The zero-fee arb's return on capital would then be 73% (7.3%*0.10*100). This is only a lower bound, as the benefit of correcting AMM noise trades that push the AMM price away from the CEX price would result in additional profit. Empirically, we see that v3 arbs generate profits well above the LP's IL in many pools, highlighting that the IL represents a lower bound of gross arb profitability. The handful of successful arbs should be able to generate attractive returns sufficient for their existence.

The net result is to turn the arb into an oracle, as it effectively updates the AMM price to the CEX price, utilizing a modest amount of capital for a modest profit. Unlike an oracle, however, this mechanism has no centralization point that can be censored. If an arb maliciously pushes the AMM price away from the true price, other arbs can immediately reap profits by reverting the price. Arb accounts are pseudonymous, replaceable, and disciplined by a decentralized liquidation mechanism if insufficiently collateralized.

This paper is split into the following sections. First, it derives three different formulas for estimating the LP's IL, which highlight the mirror nature of arb PnL and the IL. While all three IL metrics are consistent and empirically correlated, we find the 'markout IL' better than alternatives because it avoids the issues created when estimating 'liquidity' on v3 pools. We argue that the stationary metric IL/fees is superior to rates of return and dollar estimates of IL, and highlight the relevance of noise traders to the profitability of arbs and LPs. In Section 2, we empirically document LP performance using the ratio of IL/fees and find that LPs make a profit on v2 pools but lose money on v3 pools at a high level of statistical significance. Section 3 empirically documents the activity and profitability of current AMM arbs, their share of AMM volume, and their pre- and post-gas-fee APYs. Section 4 uses the data on arbs and LPs to show the feasibility of this approach. We outline the unique insolvency risk introduced by leveraging LPs and how liquidators can address this. Lastly, we highlight the simplicity of extending leverage to traders, which enables the use of perpetual contracts (perps) and stablecoins.

We will present pools using ETH and USD as my token pair, though the following generalizes to any token pair. It is easier to think about the price of ETH in terms of USD, while using the abstract notation of tokens A and B makes the numeraire in 'price' ambiguous. We will also refer to arbitrageurs as arbs, as arbitrageur has too many syllables to be mentioned so often.

Section 1: Deriving IL Estimates

1.1 AMM Axioms

A constant product AMM is based on the constant product equation $x \cdot y = k$, where x and y are token amounts in the pool. Defining k as $liquidity^2$, which is also a constant, makes subsequent results more intuitive, as the sum of LP liquidity equals the total pool liquidity. We will refer to *liquidity* as *liq* in these equations. References to ETH and USD refer to the pool quantities unless otherwise noted. Thus, for a standard ETH-USD pool, we have

$$ETH \cdot USD = liq^2 \tag{1.1}$$

Price is defined as the marginal ratio of USD needed to buy a unit of ETH; the price of ETH is the ratio of pool USD over pooled ETH.

$$price = p = \frac{USD}{ETH} \tag{1.2}$$

Given this definition of p, we can substitute and derive the following pool equations that allow us to refer to the pool via the tuple (*liq, p*) or (*USDC, ETH*).

1.1.1 Pool balances

$$\begin{aligned} USDC &= liq\sqrt{p} \\ ETH &= \frac{liq}{\sqrt{p}} \end{aligned} \tag{1.3}$$

1.1.2 LP position Value

$$LpVal = USDC + ETH \cdot p = liq\sqrt{p} + \frac{liq}{\sqrt{p}} \cdot p \tag{1.4}$$

or

$$LpVal = 2 \cdot liq \cdot \sqrt{p} \tag{1.5}$$

1.1.3 Greeks

The sensitivity of the LP's position to a change in the ETH price is its delta, which is the first derivative of the LP position value with respect to the price.

$$delta = \Delta = \frac{\partial V}{\partial p} = \tfrac{1}{2} \cdot 2 \cdot liq \cdot p^{-\frac{1}{2}} = \frac{liq}{\sqrt{p}} = ETH \tag{1.6}$$

The LP's gamma is its delta sensitivity to a change in price or the second derivative of its pool position value.

$$gamma = \Gamma = \frac{\partial^2 V}{\partial p^2} = \frac{\partial \Delta}{\partial p} = \frac{\partial ETH_{pool}}{\partial p} = \frac{\partial\left(\frac{liq}{\sqrt{p}}\right)}{\partial p} = -\frac{1}{2} \cdot liq \cdot p^{-\frac{3}{2}} = -\frac{liq}{2 \cdot p^{3/2}} \tag{1.7}$$

1.2 IL Definition

The LP's initial ETH and USD deposit amounts are called the HODL portfolio, referring to the infamous drunken tweeter who recommended users 'HODL' during a Bitcoin drawdown, a misspelling of hold, which conveniently abbreviates the phrase 'hold on for dear life.'

$$LpVal_1 = USD_1 + ETH_1 \cdot p_1 \tag{1.8}$$

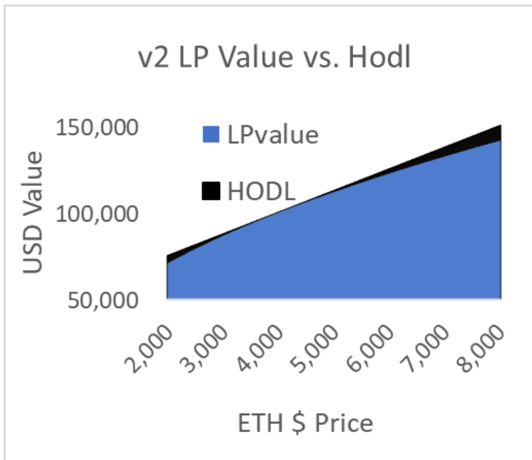
$$HODL_1 = USD_0 + ETH_0 \cdot p_1 \tag{1.9}$$

$$IL_t = LpVal_t - HODL_t \tag{1.10}$$

The initial LP position equals the HODL portfolio at inception, but the LP position value is a linear function of the square root of price (equation (1.5)), implying that without fee revenue, the HODL portfolio value will always be greater than or equal to the subsequent LP value. The difference between the LP value and HODL is called 'impermanent loss' or IL because, like an unrealized loss, the price could revert to its initial value and eliminate the loss.

In the chart below, the HODL position shows the value of the LP's initial portfolio across a range of ETH prices. The curvature of the LP's value, in blue, is because the LP value is linear in the square root of the price, while the linear HODL portfolio is linear in the price. Since the HODL value line is tangent to the LP position at the initial LP deposit price, the IL is reflected in the black-banded difference. The initial price is at the tangency point, so the increasing black band reflects the incentive that prompts arbs to move the pool price to its new level.

Figure 1.1



The subtlety of the gamma expense stems from the fact that this expense is relatively small for a v2 LP position compared to the delta risk, especially over short periods, such as a week. An asset with 5% daily volatility generates an annualized 11% gamma expense for a v2 AMM, but this drift is a mere 3 bps per day. On v2 AMMs, few LPs hedge their delta in practice, so the second-order negative gamma drift is generally ignored.

We can substitute and rearrange the canonical IL HODL definition in (1.10) to generate a formula for IL given the change in pool quantities applied to the end-of-period price. The IL as a function of the HODL and LP position values can be expressed using changes in pool token quantity and the ending price. The estimate is the LP's profit or loss from that period's trades, ignoring the LP's PnL from its initial token quantities.

$$IL_{0,1} = \left(USD_1 + ETH_1 \cdot p_1\right) - \left(USD_0 + ETH_0 \cdot p_1\right) \tag{1.11}$$

$$IL_{0,1} = USD_1 - USD_0 + p_1 \cdot \left(ETH_1 - ETH_0\right) \tag{1.12}$$

$$IL = \Delta USD + p_1 \cdot \Delta ETH \tag{1.13}$$

1.3 Markout PnL

Uniswap v3 swap events present token changes that include fees, so that they represent LP profitability as opposed to IL. However, it is easy to convert LP PnL into IL by subtracting the fees. The LP's profitability is its fee revenue plus its IL (I define the IL as strictly negative here).

$$LpPnl = fees + IL \tag{1.14}$$

$$IL = LpPnl - fees \tag{1.15}$$

A common way to capture IL is via markout PnLs, as in equation (1.14). This method compares AMM pool token changes, including fees. It calculates pool profitability by marking each trade with a price taken over a constant horizon, such as 4 hours, 1 day, or a week. This calculates the LP's PnL on each trade.

$$MarkoutPnl = \Delta ETH \cdot \left(p_{t+x} - fillprice_t \right) \tag{1.16}$$

Uniswap pool swap event logs contain the gross number of tokens going in and out of the pool. The token change amount going into the pool includes the fee that is set aside before the AMM swap. This implies that the swap price calculated from the ratio of token amounts in the swap event log is effectively the swap fill price plus the fee.

$$fillPrice = -\frac{\Delta USDC_t^{w/\,fee}}{\Delta ETH_t^{w/\,fee}} \tag{1.17}$$

Substituting equation (1.17) into equation (1.16), and algebra, then leads to

$$markoutPnL = \sum_{t=1}^{T} \Delta ETH_t^{w/\,fee} \cdot fillPrice_{t+X} + \Delta USDC_t^{w/\,fee} \tag{1.18}$$

Crypto prices are efficient for liquid coins; there is no predictable autocorrelation on the hourly or daily horizon. This means the expected price in one hour, day, or month is equal to its current price for all markout horizons. Pulling AMM transactional data and joining it with down-sampled CEX price data is laborious. However, since all markout horizons have the same expected value, our choice of horizon is arbitrary; therefore, we can pick one that is most convenient, such as the ending price over a day. The expected markout PnL is the same using end-of-day prices and any arbitrary horizon.

$$\mathbb{E}\sum_{t=1}^{T} \Delta ETH_t \left(Price_{t+X} - txPrice_t \right) = \mathbb{E}\sum_{t=1}^{T} \Delta ETH_t \left(eodPrice_t - txPrice_t \right) \tag{1.19}$$

Summing the net token changes from swap event logs over a day and then using the end-of-day price is what we refer to as the pool's *markout PnL*.

$$MarkoutPnl = \Delta USD^{w/\,fee} + p_{eod} \cdot \Delta ETH^{w/\,fee} - feeRate \cdot \sum_t abs\left(\Delta USD^{w/\,fee} \right) \tag{1.20}$$

This is identical to Eq. (1.13), except that the token change amounts here include fees, so we need to remove them explicitly. While fees can be paid in ETH or USD, if the ETH fees are converted into USD at the end of the day, the noise generated by the fluctuation in the ETH price over the day is insignificant compared to a one-sided daily fee revenue estimate. This generates the markout IL measurement.

$$markoutIL \approx markoutPnL - feeRate \cdot \sum_t abs\left(\Delta USD^{w/\,fee}\right) \tag{1.21}$$

The same IL estimate can be derived as the PnL for an LP who hedges their ETH position [see Appendix A.1]. This should be intuitive because benchmarking the LP position against HODL is equivalent to assuming the LP hedged (it removes the LP's initial ETH delta).

1.4 Equivalent IL formulas

Using equation (1.13) for IL, we can substitute the pool token quantities, as calculated using equation (1.3), along with prices and liquidity determined by the pool quantity formulas above. Substituting and some algebra generates IL as a function of liquidity, initial, and ending prices.

$$IL = -liq \cdot \frac{\left(\sqrt{p_1} - \sqrt{p_0}\right)^2}{\sqrt{p_0}} \tag{1.22}$$

Using similar substitutions, we can generate twelve functions for IL that have been presented, which are identical if liquidity is constant.

1. $LPvalue - HODLvalue$
2. Δs w/o fees: $\Delta usd_{0,1} + p_1 \cdot \Delta eth_{0,1}$
3. Δs w/fees: $fees - \Delta usd_{0,1} + p_1 \cdot \Delta eth_{0,1}$
4. $\Delta eth_{0,1} \cdot \left(p_1 - \sqrt{p_0 p_1}\right)$
5. $hodl_1 \cdot \left(\frac{2\sqrt{p_1/p_0}}{1 + p_1/p_0} - 1\right)$
6. $\frac{hodl_0}{2} \cdot \left(\sqrt{p_1/p_0} - p_1/p_0 - 1\right)$
7. $-p_1 \cdot eth_0 \cdot \left(usd_1 - usd_0\right)^2 / usd_0$
8. $eth_0 \cdot \left(p_1 - p_0\right) + \left(LP_1 - LP_0\right)$
9. $LPval_0 \cdot \left(\sqrt{p_1/p_0} - \frac{\left(p_1/p_0 + 1\right)}{2}\right)$
10. $liq \cdot \frac{\left(\sqrt{p_1} - \sqrt{p_0}\right)^2}{\sqrt{p_0}}$
11. $\sqrt{p_0 \cdot liq^2} \cdot \left(p_1/p_0 + 1 - 2 \cdot \sqrt{p_1/p_0}\right)$

12. $liq \cdot \sqrt{p_0} \cdot \left(1-\sqrt{p_1/p_0}\right)^2$

1.5 Arbitrage Profit and IL

An AMM contains more than LPs and arbs. Noise traders are essential for markets to exist (Grossman and Stiglitz, 1980; Milgrom and Stokey, 1982), as otherwise, a market maker would never be able to make a profit. While essential, noise traders have nothing to do with the LP's expected IL, given that noise trader net demand cancels out over time by definition. This allows us to assume that only arbitrageurs are trading with the pool when modeling IL, because the net pool token changes that alter prices are non-zero.

Given the arb's net token change is the inverse of the pool's token change, the arb PnL must be the negative of the Pool's IL. Given the IL is non-positive, this implies the arb PnL is non-negative.

$$ArbPnL = -\Delta USD - p_1 \cdot \Delta ETH = -IL \tag{1.23}$$

Alternatively, we can derive this by using the arb's profit on each trade over a period.

$$ArbPnL = \Delta ETH_{arb} \cdot \left(p_1 - fillPrice\right) \tag{1.24}$$

Algebra shows that this is equivalent to the initial IL equation (1.13) (see A.3). Thus, we have HODL minus LP value, hedged LP PnL (see A.1), markout IL, and the inverse of arb profit as equivalent measurements of IL derived under different motivations.

1.6 Black-Scholes LP Gamma Expense

Black and Scholes (1973) derived the standard option formula based on an arbitrage argument applied to continuous time. They create a portfolio with option V that hedges its first-order risk by shorting the underlying based on the option's delta.

$$RisklessPortfolio = V - \Delta \cdot p \tag{1.25}$$

By continuously rebalancing its hedge, the portfolio is riskless. Thus, the hedged portfolio's return on the LHS must equal the riskless return on required capital on the RHS.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 p^2 \frac{\partial^2 V}{\partial p^2} = r \cdot \left(V - p \cdot \frac{\partial V}{\partial p}\right) \tag{1.26}$$

The LHS is time decay plus the gamma term that reflects the second-order adjustment in a Taylor expansion of a nonlinear function. The RHS represents the cost of capital, where r is the interest rate, and the term in brackets represents the value of the option and its hedge.

Assuming interest rates are zero for clarity, we can see that the option's time decay, referred to as theta, equals the adjustment for the negative gamma.

$$theta = \frac{\partial V}{\partial t} = -\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 V}{\partial p^2} \tag{1.27}$$

Replacing the second derivative with the Greek letter gamma, Γ, we get the classic equation of theta.

$$\theta = -\frac{\Gamma}{2}\sigma^2 p^2 \tag{1.28}$$

By applying the Black-Scholes theta formula, we can calculate an LP's instantaneous loss by substituting the LP's gamma using liquidity and price.

$$\Theta_{lp} = gammaExpense = \frac{\left(-\frac{liq}{2 \cdot p^{3/2}}\right)}{2} \cdot p^2 \cdot \sigma^2 = \frac{liq \cdot \sqrt{p} \cdot \sigma^2}{4} \tag{1.29}$$

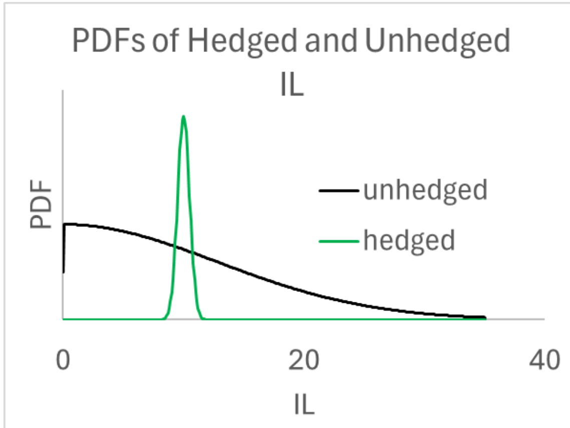
This is the LP's loss in value over time due to gamma. σ^2 refers to the period in question, so if we had an annualized volatility applied to a day, it would be $\sigma^2/365$. This is also known as the loss-versus-rebalancing (LVR) estimate of expected IL (Milionis et al. 2024). It applies to continuous time as opposed to the finite time intervals for the moment generating functions of power payoffs, and thus underestimates gamma because third-order terms are irrelevant at infinitesimal horizons. While a moment generating function (MGF) generates a dominant estimate for discrete intervals like days or years, as a practical matter, the above equation is virtually equivalent up to monthly horizons (see A.4).

1.7 Daily Hedged Returns Generate Efficient Estimates

A 24-hour pool markout can be reframed as the PnL of an LP who hedges each of their trades 24 hours in the future, locking in the profit or loss on that swap. Estimates of IL outside the Black-Scholes/MGF estimates are often presented with multiple markout horizons, and their dispersion highlights the standard errors in these estimates. Many object to IL because they believe it only applies to those LPs who hedge or take profits at the assumed frequency, and most LPs do not hedge. However, the primary reason for assuming a daily hedge is that it generates more efficient estimates of IL rather than accurately reflecting LP behavior.

A hedging strategy based on past returns has a zero expected return if we make the standard assumption that prices are weak-form efficient. Adding a hedge to an option does not change the expected return of an option position, but it does change its variance [A.2]. Similarly, the expected LP PnL is the same whether the LP hedges or not, but the hedged LP PnL has an order of magnitude less variance, making it more efficient.

Figure 1.2



While efficiency motivates the use of the highest-frequency data available, market microstructure issues related to fees imply there should be a lower-bound on the markout horizon and its presumed frequency of LP hedging. For example, initial estimates of the size effect in 1980 were a massive 20% annually, which was biased upwards due to the random use of the bid or ask price as the closing price on low-priced stocks. This was where low-priced stocks, priced at $2, would have a ¼ bid-ask spread, comparable to an 11.5% spread. In crypto, the largest fees are generally 0.3%, and given that the 5-second volatility is around 3 bps, markout horizons under a minute will be biased.

Assuming daily LP hedging—whether they hedge or not—is a convenient and efficient compromise. This uses an average markout horizon or hedging frequency of 12 hours. An estimate based on daily hedging will have a lower standard error than one based on weekly or monthly hedging.

1.8 Stationary Metrics

A v2 LP's profit can be normalized by its market value, $2{\cdot}liq\sqrt{p}$. Given the Black-Scholes LP's gamma expense is also a linear function of $liq\sqrt{p}$, the ratio of gamma expense as a percent of v2 value annualizes to the simple metric $\sigma^2/8$. A v2 pool's TVL can be derived simply from its price and total volatility. In contrast, estimating the v3 pool's active TVL is a function not merely of the current price and liquidity but of the distribution of the various LP positions in the pool, many of which are inactive. Even if we could estimate the active TVL subset within a v3 pool, the individual LP positions would have various leverage amounts: A v3 pool with 5% range generates a 20x leverage; a 10% range generates 10x leverage.

While users want simple APYs, any v3 LP APY would use an arbitrary leverage assumption. Instead of targeting an LP's APY, we can focus on the variable cost ratio, which is the ratio of variable costs to fee revenue. The price variance and liquidity of an AMM generate the LP's primary variable costs, the gamma expense generated by their negative convexity, and its IL. Measuring the ratio of this cost over revenue is an intuitive way of thinking about LP profitability.

$$Variable\ Cost\ Ratio = \frac{IL}{fee\ revenue} \tag{1.30}$$

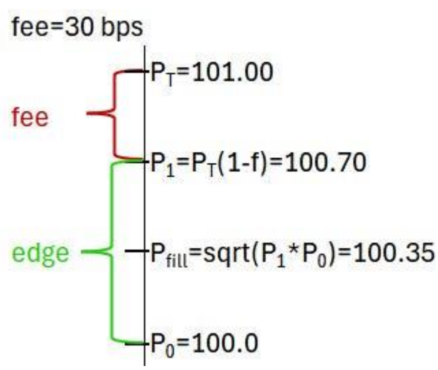
The valuable aspect of the variable cost ratio is that it is independent of time and size. They are meaningful in a clear and precise way: an expense-to-revenue ratio above 1.0 loses money for LPs; a ratio below 1.0 makes a profit.

1.9 Edge and Arb Profit

The arbs pay the fee to the LPs, so the greater the fees the arb pays, the greater the amount of the LP's IL that is offset. However, the degree to which the fee covers the LP's total IL varies based on the fee rate, and the arb's trade size relative to AMM liquidity, which determines the price movement generated by the arb (i.e., end-of-trade vs. pre-trade price).

To see this, consider Figure 1.3 below where the fee is 30 bps, and the edge is 70 bps. The arb's optimal end-of-trade pool price is 100.7, because, at that point, his incremental fill price, including fees, will be the true price, 101.0 (100.70 + 30 bp fee), so no more profit.

Figure 1.3



The arb's net profit after fees is just the value of his position at the new true price minus the fill price and the fees he paid.

$$netProfitPerTrade = \Delta E \cdot \left(p_T - p_{fill} - p_{fill} \cdot fee \right) \tag{1.31}$$

The relation of the initial price to the arb's ending price implicitly defines the arb's edge.

$$p_T = p_0 \cdot \left(1 + fee + edge\right) \tag{1.32}$$

On an AMM, his trade or fill price is the geometric mean of his trade's initial and ending price, which, for price changes less than 1%, is effectively the *edge/2*.

$$p_{fill} = p_0 \cdot (1 + edge/2) \tag{1.33}$$

Substituting for p_{fill} and p_T generates the arb's net profit as a function of the edge.

$$netProfitPerTrade = \Delta E \cdot \left(p_0 \cdot (1 + fee + edge) - p_0 (1 + edge/2) - p_0 \cdot (1 + edge/2) \cdot fee \right) \tag{1.34}$$

We can ignore the *fee*edge/2* term, as it will be of second-order insignificance. The other fee terms cancel out, so the arb's profit per trade is a function of edge only, not the fee.

$$netProfitPerTrade = \Delta E \cdot p_0 \cdot \left(\frac{edge}{2} \right) \tag{1.35}$$

Given constant product AMM math, ΔE equals one-half the edge times the amount of ETH in the pool. Note that given *liquidity/sqrt(p)* is the amount of ETH in the pool, this implies a price change of *x%* implies a pool's ETH change of *x/2%*. As the *edge* is in percent, this generates an unambiguous metric for ETH traded.

$$\Delta E = \left({liq}/{\sqrt{p}} \right) \cdot \left({edge}/{2} \right) \tag{1.36}$$

Combining equations (1.35) and (1.36) shows that the arb profit is linear in the square of the trade impact.

$$netProfitPerTrade = liq \cdot \sqrt{p} \cdot \left({edge}/{2} \right)^2 \tag{1.37}$$

An arb trade is triggered when the price is at least '*fee + edge*' away from the current price. If there were no fee, the number of times his edge would trigger a trade is a well-known result in stochastic calculus, the expected first passage time of a geometric Brownian motion to hit a fixed percentage barrier *b* given return volatility σ.

$$E[\tau] = \frac{b^2}{\sigma^2} \tag{1.38}$$

The expected number of trades would be the inverse of the expected first passage time to hit a barrier. If we define the barrier as edge plus fees and invert it to estimate the number of trades in a period, we have

$$NumberTrades_{day} = \frac{\sigma_{day}^2}{(edge + fees)^2} \tag{1.39}$$

The fee complicates this because once an arb pushes the price to p_{true} *+fee* or p_{true} *- fee*, the new barriers are asymmetric. At that point, the AMM price will be closer to the CEX price on one side and further on the other, which biases the simple formulation above by a factor of fee/edge [see Appendix A.5]. For purposes here, it is sufficient to show the general approximation, which fits simulations reasonably well, and highlights the general relation between the arbitrageur's net profit after fees as a percent of the LP's gamma expense, as a function of edge and fees.

Multiplying the number of trades in equation (1.39) and the profit per trade in equation (1.35), we get the following result for the arb's net profit after fees for a given edge and fee.

$$arbNetProfit_{day} = \frac{\sigma_{day}^{2} \cdot liq \cdot \sqrt{p}}{4} \cdot \left(\frac{edge}{fee + edge} \right)^{2} \tag{1.40}$$

Note that the LP's Black-Scholes gamma expense is the leftmost ratio. If *edge* > 0 and *fee* = 0, the arb's net profit equals the LP's IL, which makes sense because the LPs are getting zero fee revenue from the arbs. If *edge* = 0 and *fee* >0, the arb's net profit is zero, meaning the arb paid fees equal to his gross profit. The fact that the arb's net profit approaches zero as his edge approaches zero is what drives the result that latency should lower LP losses, in that in those derivations, the edge is generated by an exogenous process where greater latency generates larger trading opportunities.

The arbitrage profit equation highlights that arbs can avoid the perfectly competitive equilibrium by maintaining a minimal edge significantly greater than zero. The arb's ability to make comparable net profits on the low-latency chains like Arbitrum, where block times are 48 times faster, implies it is not reasonable to assume active arbs will be perfectly competitive.

1.10 Edge and Perfectly Competitive Arbitrageurs

Economic theory has little to say about the nature of equilibrium profits, that is, profits above the standard CAPM returns on capital. For AMM arbs, the high-frequency nature of their strategy, combined with daily hedging or rebalancing, generates a zero CAPM beta, and so, the risk-free rate of return.

An obvious adjustment would be to incorporate the arb's costs, as capturing DEX mispricing takes a considerable investment in APIs, pricing algorithms, etc. However, at the moment of an arb's decision to trade, these are sunk costs, irrelevant to the arb's trade decision, which has a virtually zero marginal cost. In standard economic theory, there is no stable equilibrium in industries facing zero marginal costs and positive fixed costs.

The best theories about economic profits involve barriers to entry and collusion. Collusion requires a small set of players because the costs of monitoring and the benefits of cheating increase exponentially as the number of players increases. The barriers can be regulatory, such as the duopoly of Standard & Poor's and Moody's, or railroads that have become natural monopolies due to the modern political impracticability of laying new tracks.

In modern market making, the barrier involves the hardware and software required to respond quickly to price anomalies. The winner-takes-all nature of arbitrage rewards only those who are first, and a power-law distribution in proficiencies tends to produce a small set of potential winners. Those with average competency cannot see and react to these mispricings at the scale needed to make arbitrage possible.

The arb maximizes his profit by waiting as long as possible so that the CEX-DEX mispricing edge is most significant. If an asset moves 5.0% in a day and the fee is 0.30%, his ideal trade places an order at the end of the day, capturing an edge of 4.70%. If he trades every time a 0.31% deviation occurs, he will trade more frequently but generate a lower profit after fees.

Competition forces the arbs to operate on a sufficiently small edge so that only a select few can sustainably play the game. Given the small set of active arbs, at some point, they no longer compete on edge. Instead, they focus on speed and selectivity. If the arbs implemented a strategy of perfect competition, they would anticipate zero profits, so they have a strong incentive to avoid this equilibrium. This incentivizes the fortunate few competitors to find an equilibrium where the arb's edge or trade size stays at a sufficiently large number to generate a net profit after fees. [see Appendix A.6]

1.11 Arb and LP Profit with Noise Traders

While price-setting arbitrage PnL mirrors the LP's IL, noise trading generates a different source of arb profits. To see this, consider the case where the price of token A is stable at $100.0. A noise trader sells 1 unit of A and pushes the AMM price down to $99.0 for a fill price of $99.5 (in AMMs, the fill price is the same as the mean of the start and end price). The arb buys 1 unit, which pushes the AMM price back to $100.0. Another noise trader pushes the price to $101.0 for a fill price of $100.5. The arb sells and pushes the price back to $100.0.

Table 1.1

Example of Noise Trade Arbitrage

		AMM Credits and Debits			
StartPrice	action	arb	noise	LP	endPrice
$100.0	noise sell	$0.0	$99.5	$99.5	$99.0
$99.0	arb buy	$99.5	$0.0	$99.5	$100.0
$100.0	noise buy	$0.0	$100.5	$100.5	$101.0
$101.0	arb sell	$100.5	$0.0	$100.5	$100.0
	Net	$1.0	$1.0	$0.0	

The net result for the noise trader, arb, and LP is zero change in their token positions. Pre-fees, the noise trader loses $1, the arb makes $1, and the LP is unaffected. If we add fees, this transfers some wealth to the LPs from the arb and noise traders. The arb is not competing with the LP on those trades that move the DEX price away from the CEX price. Instead, it is competing with other arbs and other noise traders.

While it is helpful to define noise traders as generating zero net demand, in practice, retail traders wishing to buy will find the DEX more attractive if its price is closer to the ‘CEX price – fee’ instead of ‘CEX price + fee’. Thus, noise trades will overlap somewhat with the arb traders, especially given front ends that route swaps to the pools with the best price.

Section 2: LP Empirical Data

Data comes from 22 pools across four chains: Ethereum main net, Arbitrum, Base, and Avalanche C-chain. These are all Uniswap pools and two Trader Joe pools. The most prominent pools are the original ETH-USDC main net pools: v2 30 bp introduced in May 2020, and the v3 5 and 30 bp pools introduced in May 2021. There are ten ETH-USD pools, with three using USDT and seven using USDC. Only the Maker-DAI v2 pool was discontinued before the present.

Table 2.1

Data Description

asset	numeraire	type	fee	chain	start	ADV USD
ETH	USDC	v3	5	Ethereum	2021-05	374,491,226
ETH	USDC	v3	5	Arbitrum	2022-11	67,297,469
BTC	ETH	v3	5	Arbitrum	2024-10	62,864,235
ETH	USDC	v3	30	Ethereum	2021-05	57,485,065
ETH	USDT	v3	5	Arbitrum	2024-09	54,882,358
BTC	ETH	v3	5	Ethereum	2023-01	53,324,603
ETH	USDC	v3	5	Base	2023-09	43,858,647
ETH	USDT	v3	30	Ethereum	2023-01	20,471,639
AVAX	USDC	v3	22	Avalanche	2023-04	17,468,221
BTC	ETH	v3	30	Ethereum	2023-01	14,619,856
AVAX	USDC	v3	4	Avalanche	2024-12	11,981,475
LINK	ETH	v3	30	Ethereum	2022-12	9,562,643
UNI	ETH	v3	30	Ethereum	2024-01	7,351,524
AVAX	USDC	v3	5	Avalanche	2023-07	6,994,177
MKR	ETH	v3	30	Ethereum	2024-01	4,353,727
PEPE	ETH	v3	30	Ethereum	2024-01	4,013,669
SHIB	ETH	v3	30	Ethereum	2023-04	822,486
ANYONE	ETH	v3	100	Ethereum	2024-07	266,160
ETH	USDC	v2	30	Ethereum	2020-05	25,909,870
ETH	USDT	v2	30	Ethereum	2022-03	7,387,752
PEPE	ETH	v2	30	Ethereum	2024-01	7,008,511
MKR	DAI	v2	30	Ethereum	2024-01	5,951,547
ETH	USDC	v2	30	Base	2024-07	1,738,579

Note: Fees are in basis points. All data were collected through 4/30/25

2.1 IL Estimates

For the IL estimates, we use three metrics that use slightly different inputs.

Markout IL metric

$$\sum_{day} \Delta USD^{w/\ fee} + p_{EOD} \cdot \sum_{day} \Delta ETH^{w/\ fee} - feeRate \cdot \sum_{i \in day} abs\left(\Delta USD_i\right) \tag{1.41}$$

This formula dominates because it uses only the token amounts in swap event logs and the end-of-day price. The others utilize liquidity, which varies significantly over a day for v3 pools and is correlated with price and volatility, both contemporaneously and serially. For pools without USD, one uses the token that serves as the numeraire in the price (for most non-stablecoin pools, this is typically ETH). For liquid pools, using the last trade price as the end-of-day price is as effective as using a CEX price, as the AMM price will generally be within the fee of the CEX price, and whether this is near the +fee or -fee price point is random and so cancels out. This makes generating a daily IL easier.

Price-liquidity IL metric

$$IL_{day}^{liq-prc} = liq \cdot \sqrt{p_{eod-1}} \cdot \left(1 - \sqrt{p_{eod} / p_{eod-1}}\right)^2 \tag{1.42}$$

This metric utilizes a single liquidity estimate, along with end-of-day prices according to GMT timestamps.

Variance-liquidity IL metric

$$\frac{liq \cdot \sqrt{p} \cdot \sigma^2}{4} \tag{1.43}$$

The key here is to use a variance estimate derived from non-AMM data and applied to monthly data, which serves as an independent check on the price-liquidity estimate derived from daily AMM data. The most significant difference from the price-liquidity estimate is that it multiplies a monthly variance by a monthly *liq·sqrt(p)* estimate, as opposed to the daily variance implicit in equation (1.42). For v3 pools, there are significant contemporaneous and serial correlations between liquidity and the square root of price, which leads to bias if estimated separately. To mitigate this, we average the transaction-level product of liquidity and the square root of price and then multiply this by a monthly variance estimate using CEX data.

$$IL_{day}^{B-S} = \frac{liq \cdot \sqrt{p} \cdot \sigma^2}{4} = \frac{\sigma^2}{4} \cdot \frac{\sum_{d \in month} liq_d \cdot \sqrt{p_d}}{daysInMon} \tag{1.44}$$

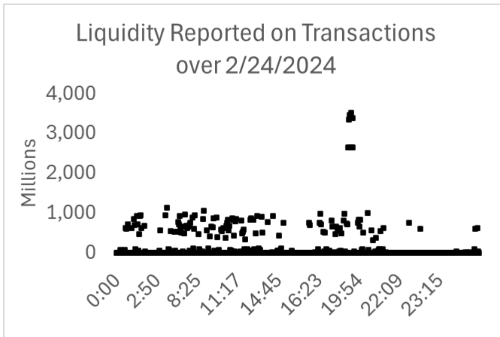
2.2 v3 Pool Data Adjustments

V2 Pools have relatively stable liquidity, typically remaining unchanged on most days. In contrast, v3 pool liquidity data changes drastically. Figure 2.1 below reports the liquidity for each transaction on February 24, 2025, for the 5bp USDC-ETH pool, where there were 10,000 transactions. There are 137 swaps where the liquidity was *100 times* greater than the swap immediately before and after. This dramatically affects the mean liquidity, and they should be excluded.

Figure 2.1

JIT Trade Intraday Pattern

These data are from the ETH-USDC 5bp v3 mainnet Uniswap pool. The median liquidity on this day was around 10 million, so the dots around 1000 all represent 100-fold temporary increases in liquidity.



These extreme observations reflect 'just-in-time'(JIT) liquidity, which adds and removes capital within a narrow band, typically 10 basis points wide. These occur sequentially, just before and after a targeted swap, but since they occur within the same block, they have identical timestamps. A standard JIT LP transaction is shown in Table 2.2.

Table 2.2

JIT trade sequence

Sequence of actual transactions on the ETH-USDC 5bp v3 mainnet Uniswap pool. Liquidity rises from 16 million to 833 million for one swap.

			Pool Changes				
action	Block	tx Index	USDC	ETH	liquidity	price	Price Range
Swap	22,047,558	68	18.39	-0.01	16,972,778	1,923.25	
LP add	22,047,559	0	13,338,661	2,373			1921.81-23.75
Swap	22,047,559	1	-221,446	115	833,627,952	1,923.22	
LP w/d	22,047,559	2	-13,121,723	-2,486			1921.81-23.75
Swap	22,047,560	25	42,931	-22.31	16,972,778	1,923.44	

The JIT LP represented 98% of the liquidity on the trade, so he effectively captured all the fees on the targeted trade, worth around $110. The LP effectively bought 115 ETH at 1923.20 USD, acting more like a trader engaging in an RFQ. Interestingly, this JIT LP was one of the arbs. Such transactions are more like arbitrage trades than LP returns.

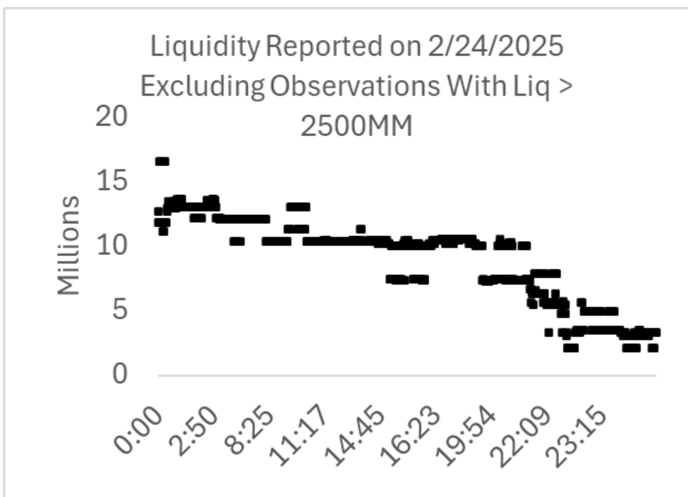
Without the JIT LP transactions, the AMM price would have moved further from the CEX price, generating an opportunity for arbs to sell back down to the CEX price. In the long run—over an hour or a day—the AMM price will be back to within the fee of the CEX price regardless of the JIT transaction. Standard passive LPs do not reap the fees from these trades, and the liquidity in these trades is irrelevant to the liquidity provided by standard, passive LPs.

Including JIT trades overstates LP profitability, so we exclude them when analyzing v3 pools. This can be done by simply checking if the swaps immediately before and after a swap had liquidity 5 or 10 times lower than the swap itself. At a minimum, one should take the median rather than the mean of the liquidity derived from swaps within a given time window.

Even after removing the JIT swaps, v3 liquidity over a day can be highly variable compared to v2 pools. On that same day of 2/24/25, for the ETH-USDC pool, liquidity varies from 16 to 3 million over the day. Thus, while removing liquidity outliers helps, the variability of v3 liquidity will still be considerably higher than on the v2 pools, where intraday liquidity is effectively constant.

Figure 2.2

Real v3 Intraday Liquidity Variability Excluding JIT

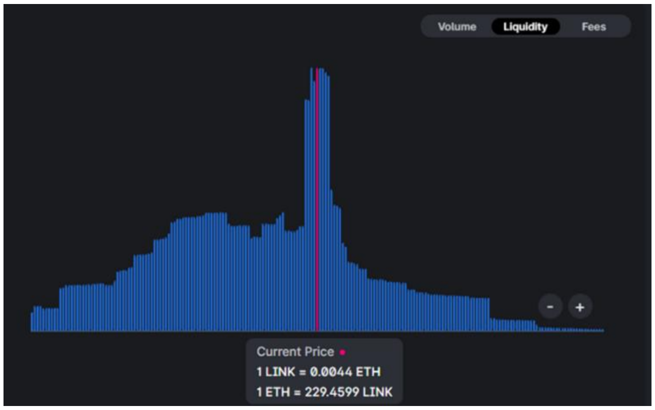


V3 liquidity is usually largest around the current price, as shown in Figure 2.3 below. When prices move quickly, the AMM price often deviates from the peak liquidity as LPs are less active than arbs. This highlights a bias in one of our gamma expense metrics.

Figure 2.3

V3 Liquidity Distribution

Each column represents the pool liquidity within an LP range. Here, liquidity is concentrated around the current price, with more liquidity at lower prices than at higher ones.

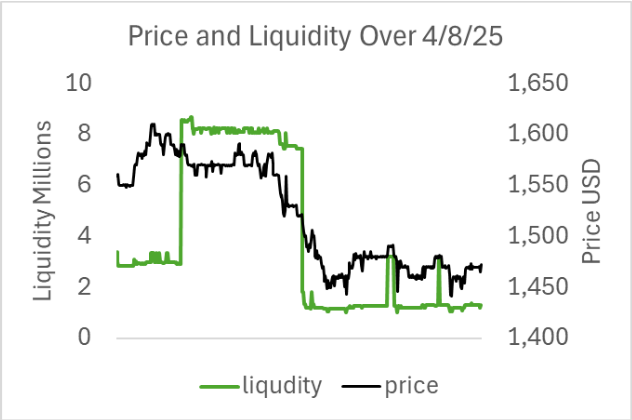


On 4/28/25, the markout IL estimate was -$303k, while the estimate using liquidity, starting and ending prices was only -$116k. The price fell 5.6% over the day, with most of the price action concentrated within a few midday hours.

Figure 2.4

Intraday liquidity and price changes

Active liquidity and price changes over a day on the ETH-USDC 5bp v3 mainchain Uniswap pool.



On this day, several LP adds occurred early in the day, which increased liquidity from 2.93 million to 8.56 million. However, these adds primarily had lower price bounds of at least $1,522, so when the price fell below that, liquidity fell to 1.8 million, where it stayed for the rest of the day.

The price-liquidity IL estimate understates IL on this day because it applied an average liquidity that was significantly lower than the pool's actual liquidity during most of that day's price change. Given that v3 liquidity tends to peak around the current price, if the price moves more than a standard deviation, the LPs will not adjust quickly, resulting in a higher IL for the markout compared to the price-liquidity estimate. A large price change early in the day, to a price with significantly less liquidity, applies lower liquidity to the day's price change, which explains why the markout method generates higher IL estimates than the price-liquidity metric (see A.7).

2.3 IL Estimate Comparison

The ETH-USDC v2 and v3 pools are the largest and oldest Uniswap pools, making them ideal for examining the robustness of the IL estimates. The IL/fee ratio estimates for the v2 pool are virtually identical, as expected, given that v2 pool liquidity does not change much intraday. For the v3 pool, we observe a 9% higher calculation using the markout vs. price-liquidity estimate, reflecting the bias discussed directly above, where liquidity falls significantly after large price movements.

The variance-liquidity estimate is 10% greater than the markout method due to correlations between *liquidity·price* and realized variance, which is significant over a week, both contemporaneously and serially. Milionis et al. (2024) argue in favor of the Black-Scholes estimate because it assumes instant hedging, thereby eliminating the market risk acquired over finite horizons. However, the hedged LP's position is symmetrically distributed around zero, so its market risk within a daily markout is independent and diversifies to insignificance over several months. To put this in context, variance data taken from daily, minute, and second data can vary by 10% applied to the same month; all estimates are noisy. The nature of the liquidity-volatility correlation and its stability are not immediately apparent and may be interesting, but that would be a distraction in this paper.

Table 2.3

IL/Fee Ratio for ETH-USDC v2 and v3 pools

	markout	price-liq	variance-liq	Observations
v3	1.27	1.17	1.34	48
t-stat m=1	5.11	3.67	6.03	
v2	0.50	0.51	0.49	59
t-stat m=1	-16.89	-17.34	-17.90	

Note: the v3 pool is the ETH-USDC v3 5bp Uniswap mainnet pool; v2 is the ETH-USDC v2 30 bp Uniswap mainnet pool. V2 pool data from 2020-25, v3 from 2021-25. T-stats for hypothesis IL/fee = 1.

Both ETH-USDC pools have sufficient observations to generate estimates with standard errors that indicate the v3 LPs incur a loss. In contrast, the v2 LPs generate a profit at highly statistically significant levels. The results below remain the same regardless of which IL estimator is used or whether an average of the estimates is used. For simplicity, we will use the markout IL estimator below.

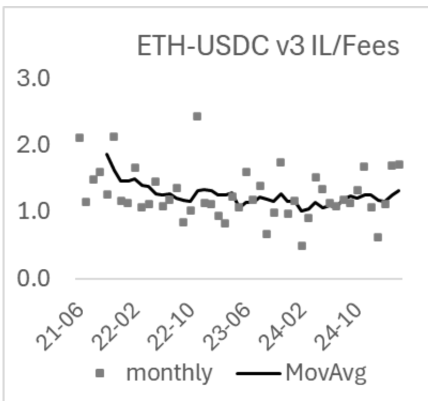
2.4 Subsample variability

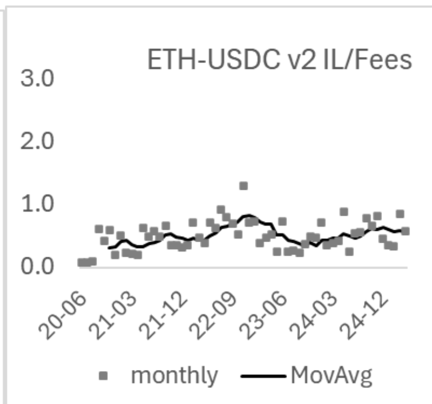
Using just the markout estimate, the IL/fee ratio variability by month is compared to its 6-month moving average. The general pattern of being above or below one is relatively stable, although month-to-month, there is substantial variation. The IL/fee ratio does not appear to be trending since these pools started in 2020 and 2021.

Figure 2.5

Monthly and 6-Month Moving Average of IL/Fees

IL and Fee data were estimated daily, which were then used to create monthly averages, and subsequently, a moving average of the monthly averages was calculated.





2.5 Overall pool performance

Applying the markout IL estimate, we see that the IL/fee ratio is significantly below 1.0 for four of the five v2 pools. The only exception is the Maker-Dai v2 pool, which incurred a significant LP loss during its brief existence and was subsequently terminated in 2024. Of the 18 v3 pools, 13 show statistically significant IL/fee ratios greater than 1, indicating the LPs lose money. Three of the five remaining pools generate a statistically significant LP profit, though these are the smallest pools in the sample. The profitable v2 pools are considerably smaller than the v3 pools ($48MM vs. $811MM ADV), implying that a plausible hope for an extreme return is more attractive than an inevitable modest return. Aggregating the monthly IL and fees for the v2 and v3 pools via an ADV weighting, the unprofitability of v3 LPs and the profitability of v2 LPs are highly significant.

Table 2.4

Lifetime Average Monthly IL/fees by Pool

IL and Fee data were first calculated by day, then monthly as the average of those days, and then the months were treated as observations for these pools

type	pair	fee	chain	count	IL/Rev	t-stat =1	ADV $MM
v2	ETH-USDC	30	Ethereum	60	0.51	-16.35	25.91
v2	ETH-USDT	30	Ethereum	38	0.51	-13.63	7.39
v2	PEPE-ETH	30	Ethereum	16	0.74	-3.79	7.01
v2	MKR-DAI	30	Ethereum	9	1.74	3.45	5.95
v2	ETH-USDC	30	Base	10	0.27	-8.76	1.73
v2			total	133	0.60	-10.81	48
v3	ETH-USDC	5	Ethereum	48	1.32	4.41	374.23
v3	ETH-USDC	5	Arbitrum	30	1.13	2.37	67.14
v3	BTC-ETH	5	Arbitrum	7	1.48	2.29	62.45
v3	ETH-USDC	30	Ethereum	48	1.16	3.85	57.49
v3	ETH-USDT	5	Arbitrum	8	1.20	2.59	54.61
v3	BTC-ETH	5	Ethereum	28	1.26	2.44	53.37
v3	ETH-USDC	5	Base	20	1.23	2.52	43.77
v3	ETH-USD	30	Ethereum	28	1.11	1.65	20.45
v3	AVAX-USDC	22	Avalanche	25	1.31	5.84	17.46
v3	BTC-ETH	30	Ethereum	28	1.23	3.28	14.57
v3	AVAX-USDC	6	Avalanche	5	1.63	7.07	12.27
v3	LINK-ETH	30	Ethereum	29	1.04	0.55	9.55
v3	UNI-ETH	30	Ethereum	16	1.12	0.68	7.36
v3	AVAX-USDC	5	Avalanche	22	1.26	2.56	6.98
v3	MKR-ETH	30	Ethereum	16	1.33	2.12	4.30
v3	PEPE-ETH	30	Ethereum	16	0.85	-2.08	4.01
v3	SHIB-ETH	30	Ethereum	25	0.63	-5.59	0.83
v3	ANYONE-ETH	100	Ethereum	10	0.70	-4.53	0.27
v3			total	409	1.16	6.91	811

2.6 v2 vs v3

A more direct comparison of v2 and v3 LP performance compares pools with identical token pairs and fees, all on the Ethereum mainnet. This uses the intersection of their monthly data, excluding data when one of the pools is not active. There are four ETH-USD pools, two using USDC and two using USDT, with 30bp fees for v2 and v3 pools. USDT and USDC are almost perfect substitutes, making comparisons appropriate. For all five comparisons, the v2 LP profitability is statistically different than the v3 LP profitability. The v2 pools are all smaller than the v3 pools.

Table 2.5

Token Pairs that trade with identical 30 bp fees on Ethereum Mainnet

pair	IL/Fees v2	v3	Months	Difference	t-stat
PEPE-ETH	0.74	0.85	16	-0.11	-2.6
ETH-USDC/C	0.55	1.16	48	-0.61	-18.2
ETH-USDC/T	0.51	1.11	28	-0.60	-11.5
ETH-USDT/C	0.51	1.16	38	-0.65	-17.4
ETH-USDT/T	0.47	1.11	28	-0.65	-12.2

2.7 5 vs. 30 bp

Using identical v3 pools with the same token pairs on the same chain, there is a weak PnL dominance for the higher-fee pools, with the ETH-USD pool showing a statistically significant difference. These poor-performing pools all had more volume than the better-performing pools, which makes sense because the lower-fee pools generate more arb trading opportunities, which dominate volume. The Avalanche pools use two TraderJoe pools, the original AVAX-USDC pool with a 20 bp fee, Uniswap's AVAX-USDC 5bp pool, and TraderJoe's recent 2 bp fee pool. TraderJoe's fees vary by asset volatility, but in practice, this adjustment adds just 2 bps to their base fee.

Table 2.6

Token pools with the same token pair on v3 but with different fees

pair	5 bp	22/30 bp	Months	Difference	t-stat
AVAX-USDC TJ-TJ	1.63	1.46	5	0.17	1.32
AVAX-USDC Uni-TJ	1.26	1.29	22	-0.02	-0.23
ETH-USDC	1.32	1.16	48	0.16	2.52
BTC-ETH	1.26	1.23	28	0.03	0.42

2.8 Latency

A prominent prediction was that lower latency would lead to better LP performance. Using v3 pools with identical token pairs and fees, we see an ambiguous pattern. On two of the four pools, the faster chain has a lower IL/fee ratio, and none of the differences are statistically significant. All LPs in these pools lose money.

Table 2.7

V3 pools with the identical token pairs and fees trading on different blockchains

pair	fast chain	Eth	Arb/Base	Months	Difference	t-stat
ETH-USDC	Arb	1.23	1.13	30	0.10	1.59
ETH-USDC/T	Arb	1.31	1.20	8	0.11	1.22
ETH-USDC	Base	1.19	1.23	20	-0.04	-0.38
BTC-ETH	Arb	1.31	1.48	7	-0.16	-1.45

Note: In the second row, the Ethereum chain pool uses USDC while Arbitrum uses USDT

2.9 v2 APY

The IL/fee ratio is a meaningful measure of LP profitability that can be applied across v2 and v3 pools. The variable nature of v3 LP leverage within a pool makes any APY arbitrary. However, given v2 LP costs are consistently lower than their fees, it is helpful to see what this implies in terms of the more intuitive APY. Initially, net APYs were attractive, though not close to the 40%+ APYs one regularly sees on the web.

Over the past four years, these profitable LP positions have generated APYs of 5-9%, which appears to be a reasonable return for a passive investment on the blockchain. Once v3 pools were introduced in 2021, it seemed that the v2 pools would eventually disappear. However, they experienced a volume resurgence in 2023, though they gave most of that back in 2025.

Table 2.8

LP Net APY on Original ETH-USDC 30 bp Uniswap v2 mainnet pool

	mktcap-weighted gross APY	mktcap-weighted net APY	TVL $MM
2020	16%	9%	162
2021	33%	19%	262
2022	15%	6%	199
2023	8%	5%	121
2024	10%	2%	345
2025	15%	7%	112

Note: data start in June 2020, end in April 2025.

Section 3: Arbitrageur Stats

A standard CLOB LP can adjust its bid-ask prices without a trade, and it invests in models and hardware to adjust prices before arbitrageurs pick up its stale prices. The AMM requires arbs to trade with the AMM to update its prices. It is unhelpful to characterize them as picking off stale AMM prices because that is the only way prices are updated. There are alternatives to CPAMMs, such as CLOBs or Oracle-updated exchanges, but these introduce centralization and potential attack surfaces for censorship.

Here, we focus on the price-setting arbs, as opposed to accounts engaged in various Maximal Extractive Value (MEV) tactics, such as sandwich attacks or multi-hop trades that arbitrage across multiple pools in a single transaction. The arbs who dominate AMM trading volume target the CEX price and respond to deviations between the AMM and CEX prices.

AMM users can be broken into three groups. First, there are the router contract swaps that process retail orders, such as the front ends offered by Uniswap, 0x, MetaMask Swap, and 1Inch. These are classic noise traders because of the latency generated by the front end, and the extra fees necessarily exclude anyone attempting CEX-DEX arbitrage. This is why trade flow coming from retail brokerage front ends, such as Robinhood, can be sold to companies like Citadel, as outsiders can be confident that these are noise trades, not toxic flow. Router addresses are easy to spot because they are at the top of the sender list by volume, prominently labeled, and verifiable contracts on blockchain scanners, allowing their users to audit and trust them. They tend to generate high gas use because they frequently wrap or unwrap tokens or engage in multi-hop transactions. The second group is the arb accounts. These are all contracts, as opposed to EOAs, and they are not verifiable on explorers. Lastly, there is the ubiquitous 'other' category, which includes MEV bots engaging in everything but CEX-DEX arbitrage, among other unknown objectives.

Identifying arbitrage addresses starts with finding those addresses that generate the most pool volume. One can then pull up these addresses, and routers, and MEV bots are usually identified as such by Etherscan and other explorers. Arbitrageur accounts arise and expire within any time frame, so it is essential to note whether a dominant address generating a large amount of volume in, say, 2024, stopped sending orders in December. This usually implies a different address that replaced its activity, which may not be apparent when viewing the aggregate 2024 volume. We initially included all chains, but isolating arbs on the lower latency chains is more difficult because it is more common for arbs to arise or exit. The main results were not substantively different, so we concentrated on the Ethereum chain subsample where we could be confident in identifying arbitrage trades.

The data refers to the 12 months from April 1, 2024, through April 30, 2025, because my base data did not include the 'sender' address, and repulling this amount of data requires a considerable amount of time. We focused on 5 ETH-USD Ethereum mainnet Uniswap pools, as they contained considerable overlap in their arbs. Four of the pools have 30 bp fees, the other 5 bps, and two are v2, while three are v3 pools. This identified 6 arb addresses.

3.1 CEX to DEX Price

Table 3.1 below displays the weighted average AMM-to-CEX price. End-of-trade prices were linked to the second down-sampled Coinbase data. The percentages were adjusted so that if the CEX price was 100.00, with a fee of 0.30%, an arb buy would push the price up, an end-of-trade price of 99.70 generates a percent of +0.30%; if the trader sold down to 100.30, that price would generate a +0.30%. Arbs are pushing the price towards the CEX price, which is the profit-maximizing tactic.

This table is important for two reasons. First, it highlights that AMM prices follow CEX prices, not vice versa. If these AMMs were involved in price discovery, they would trade past the CEX price and then watch or push the CEX price to follow. We could present more data to that effect, but the point is beyond dispute and would be redundant given this data.

Table 3.1

End-of-Trade Price Difference to CEX Price

The data are the 'prcDex/prcCex -1' for sales and 'prcDex/prcDex -1' for buys. The CEX price is taken from the second of the trade timestamp. A profit-maximizing trade, assuming the CEX price is the true price, is +fee. Data were from Feb 4, 2025 through Apr 30, 2025 for these various trader groupings, using at least 1000 observations for each. Data are weighted averages. Equal-weighted data for the arbs are within a basis point of these data.

trader type	ETH-USDC v3 5bp	ETH-USDC v3 30bp	ETH-USDT v3 30bp	ETH-USDT v2 30bp	ETH-USDC v2 30 bp
other	-0.12%	0.10%	0.08%	-0.14%	-0.06%
router	-0.17%	-0.04%	-0.07%	-0.20%	-0.16%
arb#1	0.06%	0.29%	0.29%	0.28%	0.28%
arb#2	0.05%	0.30%	0.30%	0.30%	0.31%
arb#3	0.07%	0.30%	0.30%	0.28%	0.28%
arb#4	0.05%	0.29%	0.29%	#N/A	#N/A
arb#5	0.05%	0.30%	0.29%	0.29%	0.28%
arb#6	0.06%	#N/A	#N/A	#N/A	#N/A

The other reason the arb's end-of-trade CEX/DEX price deviation is essential is that it highlights that they are not engaged in the canonical two-legged arbitrage that most people imagine when they hear about arbitrage. The lowest fees on CEXes are 2 bps for takers. A two-legged arbitrage trade, where one trade is on the blockchain and the other on a CEX, requires the arb to be a taker. The explicit fee for institutional traders is a minor part of their overall trading costs, which also include price impact and the bid-ask spread. The all-in cost of trading \$10,000 to \$100,000 in ETH—standard arb trade sizes—is at least 5 bps on liquid CEXes. If the arbs hedged immediately on a CEX, they would expect their hedge to cost them 5 bps, and they would target end-of-trade prices of +10 and +35 bps instead of +5 and +30 bps.

3.2 Arb Trading Volume

In practice, price-setting arbs engage in 'stat-arb,' which takes advantage of the law of large numbers. Given that prices follow a martingale, and these arbs treat the CEX price as the true price, hedging periodically versus every trade does not affect the arb's total profit, pre-fee. Given that hedging is costly, it is prudent to batch hedges so that one hedges only when a position has reached a specific absolute size, based on capital or risk tolerance, or at the end of each trading day (which explains why trade volume increases in the last half-hour). Furthermore, if the hedge trade timing is discretionary, one can employ various high-frequency trading tactics to minimize hedge costs. This would not require difficult alpha but rather the more feasible goal of minimizing trading costs. Such a strategy cannot be applied if one is hedging each AMM trade instantly. The incidental intraday position accumulated by an arb will vary symmetrically around zero, and so over a few months, this risk diversifies away to insignificance over a year. In Table 3.2 below, we observe that the daily net-to-gross traded amount for arbs ranged from 7% to 20%. Thus, by hedging once a day, one removes 80 to 93% of hedging costs.

Table 3.2

Activity by Arbitrageurs, Routers, and Other

group	daily $ gross traded	daily $ net traded	Average trade size $	daily % volume
		ETH-USDC 5 bp v3		
Total	266,703,614	8,339,626	40,196	
other	66,985,790	7,021,001	33,610	24%
router	30,145,039	5,789,569	9,116	12%
arbs	169,572,785	11,489,965	134,032	64%
		ETH-USDC 30 bp v3		
Total	20,918,291	3,245,649	45,038	
other	3,146,067	712,270	32,553	14%
routers	1,611,489	491,800	10,613	9%
arbs	15,394,595	2,599,498	80,827	72%
		ETH-USDT 30 bp v3		
Total	30,784,072	4,249,452	41,035	
other	4,390,219	1,017,000	30,708	13%
routers	3,099,116	875,453	9,279	11%
arbs	22,120,809	3,486,964	103,560	70%
		ETH-USDC 30 bp v2		
Total	6,472,253	442,058	2,551	
other traders	1,530,776	341,547	2,101	16%
routers	2,257,635	297,766	1,296	41%
arbs	2,652,246	500,144	18,942	42%
		ETH-USDT 30 bp v2		
Total	7,804,069	560,990	2,537	
other traders	1,658,738	321,737	2,198	17%
routers	2,986,481	350,958	1,328	43%
arbs	3,108,339	532,960	22,567	40%

The second thing to notice from Table 3.2 is that the arb trading volume ranges from 64% to 72% of pool volume on v3 pools and only 40-42% on v2 pools. Noise trading, reflected by routers and ‘other’, is essential for any asset market, and the profitable v2 pools have significantly more noise traders. Other researchers have noted arbitrage volume percentages ranging from 20% (Cao et al., 2023) to 80% (Heimbach et al., 2024).

3.3 Arb PnL

The daily PnL was calculated by summing the net tokens from the swap event logs each day and then converting the net ETH into USD using the end-of-day (GMT) price. These PnLs are from the LP’s perspective, so the negative arb PnL represent profits from the perspective of the arbitrageurs. The arb collective consistently makes a net profit. While one can speculate on why v3 LPs exist, given that they lose money, an obvious explanation is that they are either lazy or misinformed. Arbitrageurs are neither. The ‘other’ and router-based traders consistently lose money, as expected, given that they are making uninformed trades and paying a fee. These data exclude front-end fees, such as Uniswap's 25 bp fee instituted in April 2024 or MetaMask's 87.5 bp fee.

Table 3.3

Arbitrageurs, Routers, and Other Profitability

This is the profitability from the LP's perspective. Router refers to traders send from aggregator front ends like UniswapX, 1inch. Gross profit is pnl + fees. IL is the total's fees minus PnL (total PnL = fees - IL).

group	PnL	fees	% Volume	gross Profit / IL
ETH-USDC 5 bp v3				
Total	-30,532	133,352	100%	
other	58,814	35,942	27%	
router	41,636	15,073	11%	
arbTot	-130,981	82,337	62%	130%
ETH-USDC 30 bp v3				
Total	-16,401	62,755	100%	
other	2,574	10,507	17%	
routers	1,665	4,834	8%	
arbTot	-20,640	47,413	76%	86%
ETH-USDT 30 bp v3				
Total	-16,380	92,902	100%	
other	10,803	14,574	16%	
routers	5,220	9,329	10%	
arbTot	-32,403	68,999	74%	93%
ETH-USDC 30 bp v2				
Total	8,585	15,920	100%	
other	9,924	3,503	22%	
routers	6,261	6,088	38%	
arbTot	-7,699	6,405	40%	192%
ETH-USDT 30 bp v2				
Total	10,401	23,412	100%	
other	12,408	5,031	21%	
routers	7,452	8,959	38%	
arbTot	-9,532	9,488	41%	146%

The arb collective generates a gross profit significantly greater than the IL for three of these pools. This implies the arbs were making profits off noise trades that temporarily pushed the AMM price away from the CEX price. Thus, noise trader-generated profits were at least 30 to 92% of IL in these pools. For the smaller v2 pools where gross profit was less than IL, there are relatively few noise traders, which implies that noise traders generated some of the AMM price setting. Given that aggregators direct traders to the cheapest AMM, this process acts as a crude price-setting arbitrage mechanism.

3.4 Arb Gas Stats

Subsamples of swap transaction gas data by arb and pool show the arbs' gas pricing strategy varied wildly. Data from the arbs were relatively consistent across the pools, so for brevity, Table 3.4 presents only the average data by arb across all pools. In general, arbs make single-hop trades, reflected by their relatively low gas usage compared to the routers. I found an account that appeared to be an arbitrage account, using 10 times the typical arbitrage account's gas. However, further examination revealed that these transactions involved several tokens, making a straightforward interpretation of their ETH/USD profitability meaningless. Therefore, I removed this account. Gas prices paid varied from 3 to 113 gwei. This generates USD transaction costs ranging from $1 to $45.

Table 3.4

Average Gas Usage and Price by Arbitrageur

These data are from a subsample of the period

	average			
	Gas Price	Gas Used	$/tx	# obs
other	11.9	1,165,552	41.7	41,424
routers	3.2	394,788	3.8	37,481
arb#2	31.8	129,292	12.3	7,205
arb#3	113.8	134,141	45.8	5,008
arb#4	3.0	125,650	1.1	3,162
arb#5	43.0	177,070	22.8	2,456
arb#7	66.7	140,537	28.1	913

3.5 Arb APYs

Arb profitability is consistently profitable when excluding gas costs estimated via their average gas statistics. To generate an APY, we used the average absolute daily net USD traded times three, a standard worst-case scenario capital assumption. If the arb hedged once a day, the average absolute net amount traded, in USD, is the average USD value needed to generate a day's trades. Accounting for volatility, the arb needs more than that amount of capital for days when their net position change is above average. Arbs could invoke intraday position limits throughout the day, thereby using less capital but generating more transaction costs. Assuming the arb uses 3 times their average daily absolute net position is reasonable; a high-level estimate of the arb's capital helps generate an estimate of arb APYs.

The pre-gas APYs generated an average of an impressive 82% across all arbs and pools. Arbe #3 does lose money, and there is a wide disparity in arb APYs. The after-gas APYs average 68% across all pools, which is significantly lower but still significantly above any hurdle rate.

Table 3.5

Pre-gas and Post-Gas PnL and Pro-Forma APY by Arbitrageur and Pool

Average Daily Stats April 2024-April 2025

arb	$ pnl	# trades	$ tx cost	$ pnl after gas	avg net traded	APY pre-gas	APY w/ gas
			ETH-USDC 5 bp v3				
arb#1	41,264	372	13	36,454	8,653,657	58%	51%
arb#2	20,060	467	39	1,940	1,608,974	152%	15%
arb#3	66,180	230	1	66,001	5,398,965	149%	149%
arb#4	660	177	19	2,710	3,135,877	3%	-11%
arb#6	12,921	147	13	11,026	2,624,212	60%	51%
			ETH-USDC 30 bp v3				
arb#1	9,416	87	11	8,436	1,739,172	66%	59%
arb#2	717	53	8	321	422,865	21%	9%
arb#3	9,016	38	4	8,877	1,158,997	95%	93%
arb#4	1,623	21	19	1,230	544,805	36%	27%
			ETH-USDT 30 bp v3				
arb#1	13,221	86	9	12,432	2,183,076	74%	69%
arb#2	6,216	65	20	4,923	1,201,887	63%	50%
arb#3	12,463	39	1	12,442	1,338,401	113%	113%
arb#4	769	26	19	266	619,764	15%	5%
			ETH-USDC 30 bp v2				
arb#1	3,512	51	5	3,260	255,405	167%	155%
arb#2	156	34	20	523	86,576	22%	-74%
arb#3	7,511	44	1	7,458	285,410	320%	318%
			ETH-USDT 30 bp v2				
arb#1	2,970	48	9	2,528	294,706	123%	104%
arb#2	1,062	39	39	455	124,408	104%	-45%
arb#3	5,778	67	1	5,741	305,414	230%	229%

Simple APY uses raw PnL in the numerator and three times the average net traded amount in the denominator. The daily PnL is multiplied by 365 to annualize it.
Net APY* subtracts the daily gas costs, and applies a 5 bp cost to the average net USD traded, assuming the arb would have to trade out of the acquired position.

In Section 2.9, we saw that the average LP APY on the ETH-USD main net v2 pools is around 8%, which is a reasonable equilibrium rate of return. The net after-gas APYs for most arbs are well above an equilibrium rate of return (i.e., 50%), even with a capital estimate that is probably biased upward and thus understates the APY. This suggests that there are barriers to entry due to the technical competence required to filter CEX data, send orders to the RPCs, and efficiently hedge. Though the arb's task is straightforward, as with most activities, it requires skills that are beyond those of the average person.

Two additional rules would help generate less chaotic arb competition. First, a maximum gas price would focus the arb on competing on latency and having an efficient algorithm for filtering CEX prices into an actual price, and prevent leakage to validators/sequencers/etc. Second, a minimum trade size. While, in theory, a zero fee implies the arbs will capture the complete IL regardless of their edge when fees are zero, a trade minimum targeting a 5 bp price impact would simplify the arb's game space, making their objective more straightforward.

Section 4: IL-Eliminating Policy Particulars and Estimated PnL

Rectifying the v3 LP unprofitability centers on giving arbs zero-fee trading in exchange for a 90% profit tax. Applying this to the current pool activity gives us information on its feasibility. It expects there to be a handful of arbitrageurs, just like today, where arb accounts are treated no differently than other traders, as there are economies of scale in arbitrage due to the value of saving CEX transaction costs by netting trades over a day. All that is needed is for the arbs to generate sufficient returns, not that they should exceed today's returns.

4.1 Pro forma LP Profits with Proposed Rule

We can estimate the LP's new profit by applying a 90% tax rate to the empirical data on the arb's gross profit, which consists of their fees plus net profit. Also included in LP revenue are fees from the noise traders, particularly from routers and 'other.' Finally, we subtract the IL to determine the net LP profit under the proposed rule.

This estimate will be biased low because arb behavior will change in response to their lower fee, as their historical activity was optimal given the fees of 5 or 30 basis points. Arbitrageurs will more effectively dominate arbitrage opportunities, whether generated by CEX movements or noise traders who skew the AMM price away from the CEX price.

Table 4.1 below indicates that the new net LP PnL would be significantly higher than the actual LP profit across all pools. The two smaller v3 pools have five to ten times the liquidity of the v2 pools, while the v3 noise trading volume is only about twice as large, which is why the v2 LPs have a much higher noise fees/IL ratio (~100% vs. 25%). Furthermore, the gross profits for the smaller v3 pools are approximately 83% of the IL, unlike the other pools, which significantly exceed the IL. While the new rule is better, it would not make every pool profitable for LPs, as noise trading, no AMM will generate positive LP profits. The key is that it can make some capital-efficient pools profitable, especially the largest ones.

Table 4.1

LP PNL Using Proposed Rule

All non % data are daily USD averages. Estimates are derived from the data analyzed in Section 3, using 13 months of data from April 2024 through April 2025. New LP profit equals 0.9*(arb net PnL + arb fees) + noise trader fees.

	ETH-USDC v3 5bp	ETH-USDC v3 30bp	ETH-USDT v3 30bp	ETH-USDT v2 30bp	ETH-USDC v2 30 bp
daily $ IL	170,545	80,692	115,469	13,258	10,817
Curr LP net profit	-37,193	-17,937	-22,567	2,662	12,595
proposed LP net Rev	72,456	-4,102	-304	8,869	20,165
Proposed Net APY*	11.4%	-1.5%	-0.1%	18.5%	52.5%

proposed APY* = arb capital @ 1% v2 MktCap

The new LP profitability used 20% of the v2 market capitalization for these pools, applying the mean liquidity of these pools (JIT trades excluded), divided by each pool's v2 capital, shows LP returns ranging from zero to 10.59%. For the flagship v3 pool, the LP's return is 2.3%, which is positive but not substantial. This is one of several reasons for providing LPs margin treatment because with 5x leverage, the 2.3% return can generate an attractive base return.

4.2 Pro-Forma Arb APY

The 90% tax significantly reduces arbitrage profitability before the gas fee. However, their existing returns are exceptionally high, starting at 74% annualized, which is a bug, not a feature, in these AMMs. The objective is to create a mechanism that allows them to earn enough money to motivate their ongoing price updates, and the pre-fee APY returns of 25% or more should be sufficient. With restrictions on the gas price paid by arbs, the post-fee arb net APY will better reflect the arb's true APY, which includes the fees.

Table 4.2

Arb APY Assuming Capital is 3x Average Daily Net Token Position

Non-percent data are daily averages in USD, using the same data as in Table 4.1. Proposed arb profit applies a 90% to the arb's gross profit (net profit plus arb fees).

	ETH-USDC v3 5bp	ETH-USDC v3 30bp	ETH-USDT v3 30bp	ETH-USDT v2 30bp	ETH-USDC v2 30 bp
arb % tradeVolume	64%	72%	70%	42%	40%
Current arb net profit	130,981	20,640	32,403	7,599	9,458
current APY*	82%	30%	34%	64%	99%
Proposed arb pnl	21,332	6,805	10,140	1,393	1,888
Proposed APY*	67%	50%	53%	58%	98%

current APY* = arb capital @ 5% v2 MktCap
proposed APY* = arb capital @ 1% v2 MktCap

The average arb net position is the amount that, on average, the arb must buy or sell to hedge their acquired daily position in that pool. In Table 4.2, we took the arb collective's average end-of-day net position and multiplied it by 3 to estimate their total capital, assuming they need to be prepared for a worst-case scenario. In practice, an arb could withdraw their surplus tokens and swap them into their deficit tokens using a price or token quantity rule, as opposed to simply a daily rule. The above is intended to demonstrate base feasibility without making any assumptions about how new features would improve arb performance.

4.3 Reason for v2 pools

Allocating arb profits across diverse LP ranges would be sufficiently arbitrary to incentivize costly tactics to game the arb profit allocation rule. While v2 pools are generally profitable, many users dislike their capital inefficiency. Fortunately, leveraging v2 LPs is straightforward. V3 solves a significant problem, inefficient capital in v2 AMMs, but it is not the only, or even the easiest, way of giving v2 LPs leverage. Additionally, leveraged LPs generate complementary token demand alongside leveraged arbitrage (arbs). Thus, this solution dominates the use of unlevered restricted range LPs, even if we could attribute the arbitrage profit and loss (P&L) to LP price ranges.

Restricted range pools can monitor the fees/liquidity in various price ranges, but this metric will not be related to that range's contribution to arb profits or LP IL. With v2, all liquidity is treated equally for allocating fees, allowing the arb profit to be added to the existing v2 LP revenue without issue. There will be some lumpiness created by the fact that arb profit cannot be allocated until the arb initiates a rebalancing or withdrawal, so if an LP deposits and withdraws before any arb profits are claimed, that LP will not capture any of the arb profits. However, if the LPs are v2 and the pool sweeps the arbitrage profits every few days, for long-term LPs, this level of uncertainty regarding the reclamation of arbitrage profits is tolerable.

The diversity of money-losing v3 LPs suggests there is a winner's curse among the LPs. That is, anyone acting as a v3 LP presumably believes their position generates a positive net profit. In that case, any LP would prefer to provide more liquidity in a narrower range, as this would increase their net profit. The net result of this type of thinking is to oversupply liquidity. V2's pro-rata approach, where each unit of liquidity is everywhere the same, avoids the mechanism that encourages excessive liquidity

4.4 LP Leverage

Consider the case where Alice provides liquidity for a v3 symmetric range {0.64*prc, prc/0.64} around the current price, approximately 40% above and below the current price. In this construction, Alice provides only 20% of the pool tokens needed on v2, so 80% of the initial pool tokens created are 'virtual' tokens that exist only for doing CPAMM math but do not actually exist. If the price moves outside her range, her position becomes irrelevant to swappers, as well as other LPs. This is because if the price goes down, she owns ETH, or if the price goes up and she owns USD, there are no insolvency risks. Outside her range, her funds are inactive but safe.

The 5:1 levered v2 LP has the same initial effective leverage as Alice above, 5:1. Instead of a price range, Bob gets debt instead of price bounds. Thus, initially, Bob and Alice's LPs generated the same liquidity for pricing and required the same initial deposit. These debt amounts are fixed unless paid directly, so Bob's net token position is simply the pool token amount implied by his liquidity and the price minus the debt.

Table 4.3

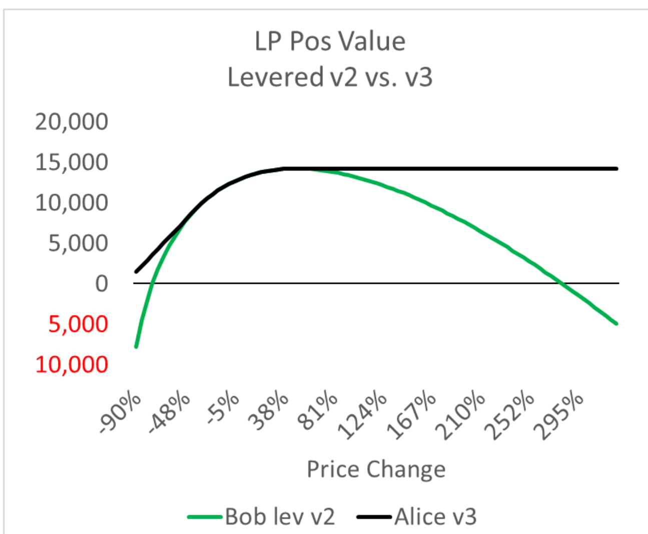
5x Levered v3 vs 5x Levered v2 LP positions

ETH Price	1,000
liquidity	1,000
ETHpool	31.62
USDpool	31,623
v2 $value	63,246

Alice's v3		Bob's leveraged v2	
pLow	640	5:1 leverage	
pHigh	1,563		
virtual ETH	25.30	ETH debt	25.30
virtual USD	25,298	USD debt	25,298
ETHnet	6.32	ETHnet	6.32
USDnet	6,325	USDnet	6,325
$value	12,649	$value	12,649

In Figure 4.1 below, we see the green and black value lines for the v3 vs. levered v2 LP positions. The 5:1 leverage ratio implicit in Alice's position, when applied directly to Bob's v2 LP position, generates the same values within Alice's range. When the price moves outside her range, Bob's position loses more value. This is because Alice's inactive position implies she has no gamma. In contrast, Bob's LP position is always active, so his gamma is negative across all prices.

Figure 4.1



It is helpful to distinguish the levered LP's primary risk to the AMM, insolvency, versus the more common risk of bankruptcy. Bankruptcy is when an enterprise's liabilities exceed its assets. In that case, ideally, the equity is wiped out, and the debtors incur a loss when they mark their debt value down to the new, lower asset value. Insolvency occurs when an entity lacks the currency required to pay its debts, which could be due to bankruptcy, but it could also be that the entity only holds ETH and needs to pay in USD. Insolvencies could be a temporary inconvenience or a sign that the entity is bankrupt. A bankrupt account can lead to a protocol run if users notice that the sum of liabilities exceeds the sum of assets. In that case, the last to withdraw bears the entire loss.

In a levered v2 AMM, the LP becomes insolvent in one of the tokens well before it becomes bankrupt. For example, with a 5:1 leverage, the price needs to fall by 75% to reach bankruptcy and 36% to reach insolvency. A symmetric relation exists on the high price end for the other token. The defaulting accounts would have sufficient value to fund the fee that incentivizes liquidation upon insolvency, but before bankruptcy.

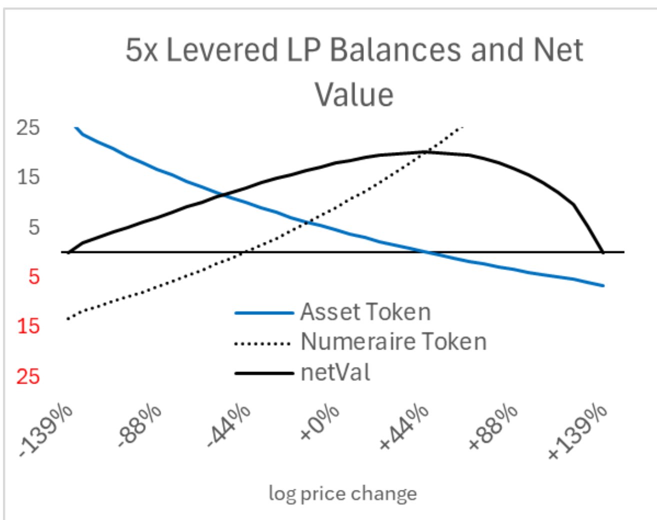
Calculating the log price returns needed to find the bankruptcy and insolvency points, from the inception of a v2 LP position, can be found via equations (4.1) and (4.2) below. Note that when *L*=5, the bankruptcy points are ±139% and the insolvency points are ±44%, as shown below in Figure 4.2.

$$BankruptcyLogReturns = \pm \ln\left(\frac{L+\sqrt{2\cdot L-1}}{L-1}\right)^2 \tag{1.45}$$

$$InsolvencyLogReturns = \pm \ln\left(1-\frac{1}{L}\right)^2 \tag{1.46}$$

Figure 4.2

The relative difference in the insolvency and bankruptcy points is shown in the symmetric log price change space. The price level of the asset or the pool's liquidity would not change the points at which these lines cross the x-axis, which is solely a function of leverage.



If our pool contained only a single LP, negative token balances would be impossible because the pool would not have the tokens to send to generate the negative balance, and the transaction would revert. This contract failure would be no different than if a v3 pool's current price were at the edge of a range, and a swap would push the price into a range with zero liquidity. With several LPs with leverage, one of them can have a negative token balance. There needs to be a mechanism that incentivizes LPs to rectify their negative balances before they affect aggregate pool solvency.

A recently insolvent LP will have a slightly negative net balance in either token, but it will still have significant total value. Below, in Table 4.4, we see the difference between a v3 and a levered v2 position with the same liquidity and initial net value. When the price falls from 1000 to 639, it is just outside Alice's range. Alice's v3 position is inactive, and her USD balance is zero, while Bob's is slightly negative, reflecting that Bob's LP position is active and his USD debt is greater than his pool USD. It is not a coincidence that the levered v2 LP position's insolvency point is at the price boundaries of the similarly levered, symmetrically ranged v3 LP position.

Table 4.4

Levered v2 Insolvent But Not Bankrupt

ETH Price	639.0
liquidity	1,000
ETHpool	39.56
USDpool	25,278
v2 $value	50,557

Alice's v3		Bob's leveraged v2	
pLow	640.0	5:1 leverage	
pHigh	1,563		
virtual ETH	n/a	ETH debt	25.30
virtual USD	n/a	USD debt	25,298
ETHnet	14.26	ETHnet	14.26
USDnet	0	USDnet	-20
$value	9,113	$value	9,093

Bob's significant ETH position provides a liquidator with the ability and incentive to liquidate an insolvent LP. The LP liquidation requirement is a negative net token balance. Liquidation is all about being first, so the incentive will be to liquidate as soon as possible, when the deficit token would still be trivial. The liquidation would involve swapping the account's ETH for 20 USD, as the DEX is built in, making it a payless transaction. Once a bot was created to monitor LPs, its marginal cost would be the gas, and so it would require a minimal fee. The defaulting LP's liquidity would be set to zero after liquidation and would have a positive balance of one of the tokens.

4.5 Netting Arb and LP Token Changes

Given an extreme daily price move of 10%, the arb would need 5% of the pool's tokens to either send 5% of the pool's ETH or USD into the pool. At the end of an extreme move, the arb would then have to withdraw half of his excess tokens and swap them into the other token on a more liquid exchange. With 5x leverage, the arb needs only 20% of 5%, or 1% of the v2 LP value, to effect a 10% price move.

The LP's net token changes mirror those of the pool's traders. If there were only arb traders, the net of the arb and LP positions would be flat across all price changes. For example, in Table 4.5 below describes a pool where liquidity is 2236, and the ETH price falls from $2000 to $1280. The LP is at the edge of default, having zero net USD, while the arb has a large short ETH position that dramatically increased his required margin. , taking the LP to the edge of default.

Table 4.5

LP and Arb Balance Sheets After Significant Price Move

Eth price	2,000				
Assets			**Liabilities&Owner's Equity**		
	quantity	value		quantity	value
liq	2,236		ETH Debt	40.00	80,000
pool ETH	50.00	100,000	USD Debt	80,000	80,000
pool USD	100,000	100,000	net ETH	10.00	20,000
			net USD	20,000	20,000
			Net Value USD		40,000

Arb Account		
	quantity	value
ETH	0.50	1,000
USD	1,000	1,000
reqMargin		200
Total Value		2,000

Eth price	1,280		With no collateral swap		
Assets			**Liabilities&Owner's Equity**		
	quantity	value		quantity	value
liq	2,236		ETH Debt	40.00	51,200
pool ETH	62.50	80,000	USD Debt	80,000	80,000
pool USD	80,000	80,000	net ETH	22.50	28,800
			net USD	0	0
			Net Value USD		28,800

Arb Account		
	quantity	value
ETH	12.00	15,360
USD	21,000	21,000
reqMargin		3,072
Total Value		5,640

Without a collateral exchange, the LP would need to withdraw its position, sell half of its ETH for USD, and redeposit. The arb would need to wrangle up 12 ETH, deposit that to rectify his deficit, and then pull out some surplus USD to get back to a balanced net position with a minimized required margin. With collateral exchange, both parties can avoid this costly and annoying activity.

The LP requires USD and has an excess of ETH, while the arb has an excess of USD and an excess of ETH. To determine the optimal token transfer amount, we calculate the excess ETH for each account by subtracting the ETH value of the USD balance from the ETH balance.

$$\text{Arb Excess ETH: } (- 12 - 21000/1280) / 2 = -14.25$$

$$\text{LP Excess ETH} = (22.5 - 0)/2 = 11.25$$

We use the smaller number in absolute value, 11.25, to avoid whipsawing an account from a deficit to a surplus via this method. Thus, in this example, we move 11.25 ETH from the LP to the arb via the LP's debt balance. Taking ETH away from the LP is equivalent to adding ETH to his debt.

For the USD transfer, we value the ETH at the post-transaction price and send that USD amount from the arb's account to reduce the LP's debt. The net result is as follows.

Table 4.6

LP and Arb Balance Sheet After Collateral Exchange

Eth price	1,280		With collateral swap		
Assets			Liabilities&Owner's Equity		
	quantity	value		quantity	value
liq	2,236		ETH Debt	51.25	65,600
pool ETH	62.50	80,000	USD Debt	65,600	65,600
pool USD	80,000	80,000	net ETH	11.25	14,400
			net USD	14,400	14,400
			Net Value USD		28,800

Arb Account		
	quantity	value
ETH	0.75	960
USD	6,600	6,600
reqMargin		192
Total Value		5,640

Note that now the LP is squared up again, with equal values of net ETH and USD. The arb still has an ETH deficit, but not as extreme as before (this is because they started with different total token amounts). The value of these two accounts is unaffected by the transfer, as is the liquidity in the pool.

In sum, the two criteria for a collateral swap are

1) Some significance of token imbalance, such as the LP's ETH excess being greater than 1% of the pool's v2 ETH amount. If this number is too high, the infrequency of transfer would generate greater 'lumpiness' in the LP's net balances over price changes. If there were no threshold, the arb could game the system, moving the price up and down repeatedly within a block, capturing 10% of the fake profits they generated on each insignificant price move.

2) The LP's net ETH excess should have the opposite sign to the arb's ETH excess. As arbitrageurs open their accounts at various times, some may have an ETH surplus, while others may have a deficit. We do not want to remove ETH from an arb that already has an ETH deficit relative to USD.

Collateral exchanges are ideal times to apply the profit-sharing formula to the arbitrage account. For this, we calculate the arb's profit, and if it is negative, we do not take anything from the arb. If the arb profit is positive, we take the profit out of whichever token the arb has an excess of. This token is then sent to the LP as if it were fee revenue. We then reset the arb's account value as the basis for future profit reclamations.

This policy would significantly reduce the capital requirement and off-contract expenses for the ARBs and LPs. If the CEX price is the true price, then the arb’s PnL is the same whether he was credited with the IL on each trade or given the full token changes. It would be like an LP hedged after every trade or once a day. This does not impact the LP or Arbs average PnL, just the volatility of their PnLs, gross and net.

4.6 Leverage implies Equity

A standard AMM has zero default risk because no one has explicit leverage. With explicit leverage, users can go bankrupt, and if the sum of these bankruptcies exceeds the contract's insurance fund, the contract would have more liabilities than assets. At that point, the first to redeem their assets receives payment in full, while the last receives nothing. This motivates a classic bank run, which leads to panic withdrawals. An insurance fund consisting of on-contract tokens that are not assets of the LP or arbs can reduce this risk to insignificance. The insurance fund should be easy to monitor, and the liquidation process should be easy to do. Such a fund would generate a real, tangible use case for the dapp’s equity tokens. In contrast, for most dapps, equity tokens generate only vague governance rights.

The insurance fund would hold the residual capital in the contract, which is the excess of total assets in the contract over the trader's and LP's assets. A portion of the liquidation and fee revenue could accrue to the insurance fund as profits add to a company's retained earnings. The equity tokens could be redeemed for this money, or a user could sell them. As the market value of a prospering contract is greater than its book value, most would choose to sell rather than redeem. At some point, the growth in these residual assets would slow down, and it would be attractive for some to redeem rather than sell these equity tokens. This makes the protocol's endgame easy, as when the contract is shut down, equity token holders will be able to redeem their tokens for the accumulated income and initial capital.

4.7 Helpful Arb Rights and Obligations

If arbs engage in Bertrand price competition and remove every arb opportunity immediately, they will waste gas that effectively transfers gross profits to validators and sequencers. As arb profits are linear in the square of the price impact of their trade, a minimum trade size as a function of liquidity would make it easier for the arbs to manage their accounts. While without a fee, arb net profits are unaffected by trade size, at some point, the excessive competition needlessly complicates the arb's objective. To prevent this, one could institute a minimum arbitrage trade size corresponding to a 5-basis point price impact. A minimum arbitrage trade size, expressed as a percentage of liquidity, would prevent arbitrageurs from wasteful competition.

In the literature on ‘hedge fund sniping,’ the focus is on shifting competition from speed-based to price-based; here, we want speed-based competition. Blockchain latency investments will not benefit from costly investments that shave milliseconds off a Binance API, as such improvements are irrelevant by orders of magnitude in the context of block times above one second. AMM arb tactics filter CEX prices effectively, and find the best times and nodes to send orders, a subject that is complex, secretive, and evolving. The winning arbitrageurs will have a comparative advantage in these tactics, which should allow them to maintain profits even without explicit barriers to entry.

A maximum gas fee on arbitrage trades would discourage sandwich attacks and prevent arbitrageurs from competing on gas, which would transfer money to blockchain validators/sequencers.

The standard fees are 5 and 30 basis points for risky pairs. Given that many front-ends charge 25 to 90 bps, and 5 bps is a good estimate of the all-in costs for trading the most liquid equities, a 5-basis-point fee is imprudent considering the current equilibrium where LPs generally lose money. Thus, a 10-basis-point fee would enable arbs to capture most of the profit opportunities from these noise trades, rather than losing them to other fee-paying traders.

4.8 Extensions

The extension to margin trading, perps, is potentially the most valuable aspect of this approach to LP capital efficiency. The margin treatment given to LPs and arbs, as well as the liquidation mechanism, can easily be extended to traders. A trader could generate a long ETH position with only 50% of its value in USDC or a short 2.0 ETH position with only 1.0 ETH deposited. One would have to add a funding rate to avoid insolvency, especially as perp traders have a long bias. Margin trading directly leads to margin positions that are converted into USD but are created by the Automated Market Maker (AMM), specifically stablecoins. See Appendix A.8 for more details.

Conclusion

The key to removing LP IL is to accept that a decentralized AMM will always be a derivative market to major CLOB exchanges for the primary tokens. The unique and novel AMM arose due to a decentralized blockchain's order of magnitude difference in latency and gas costs (centralized exchanges have no comparable gas costs). The 100-fold higher latency on blockchains transforms the arb's complex problem into an unambiguous target available to everyone: the current price on the major CLOB exchanges. The task is no mystery, so the arbs have no negotiating power to claim that they require 100%+ returns. The arb's potential gross profit is the same expense that makes LPs unprofitable. Unleashing the arbs to capture the IL more completely works because we can be confident they will have the ability and incentive to do this. This would dominate the current approach, though it is insufficient by itself to transform every pool into LP profitability, as every exchange needs a certain amount of noise trading.

Since its inception, the dominant capital-efficient AMM LPs have lost money. Most people are aware of this, but the data on this subject is sparse. Some level of decentralized AMM activity is necessary for the blockchain to thrive, making it crucial to find mechanisms that make LPs profitable. This begins with recognizing that arbs are unlikely to submit to a perfectly competitive equilibrium and explicitly designing a solution that allows arbs to generate an attractive hurdle rate. Giving the arbs zero fees, leverage, trade-size floors, and gas-price ceilings is a straightforward solution that retains decentralization, permissionless access, and pseudonymity.

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Appendix

A.1 Delta Hedged LP PnL Equals IL

The LP's position has a delta, or price risk, that can be hedged. Removing the LP's delta returns generates a much more efficient estimate of an LP's PnL. The initial LP position consists only of {ETH, USDC}, so it is long for the amount of ETH deposited. The LP's ETH position determines the amount of their delta hedge.

$$\text{Position} = \text{USD}_0 + \text{ETH}_0 * p_0$$

Given the LP's delta is the Pool's ETH, we can substitute for the pool's initial ETH amount as the LP's optimal hedge position and add this to the LP's PnL.

$$HedgedLpPnL_1 = LpValue_1 - LpValue_0 + hedge_0 \cdot (p_1 - p_0) + fees$$

$$HedgedLpPnL_1 = USD_1 + p_1 \cdot ETH_1 - \left(USD_0 + p_0 \cdot ETH_0\right) - ETH_0 \cdot \left(p_1 - p_0\right) + fees$$

$$HedgedLpPnL_1 = \Delta USD + p_1 \cdot \Delta ETH + fees$$

If we eliminate fees by using pool token change amounts that exclude fees, this is equation (1.13), which is the estimate of IL.

A.2 The Irrelevance of Hedging to Expected Gamma Return

The essence of an option is from the non-zero gamma in the payoff state, which is also called convexity or nonlinearity, a payoff that is a nonlinear function of some underlying price. This cost is independent of any transaction costs associated with hedging, including bid-ask spreads, fees, or price impact. However, hedging is desirable because it reduces the required capital as less cushion is needed for extreme drawdowns. In many cases, the risk can be virtually eliminated via the law of large numbers, but the expected benefit or cost of gamma remains unchanged.

The nice thing about option theory is that you can prove things in many different ways. This is true for most mathematically derived results, such as the fact that there are 17 ways to prove the Pythagorean theorem. The benefit of additional proofs is that, given that our minds are different, one proof might appear more intuitive than the others for quirky reasons.

The simplest way to intuit gamma is through a model that involves only algebra rather than stochastic differential equations. The binomial option pricing model (Cox, Ross, and Rubinstein, 1977) presents a riskless portfolio using a combination of a stock, bond, and option, represented by a simple tree of states.

For example, assume a call with a strike of 100 and an initial stock price of 100. The underlying stock will fluctuate by 10%, and the risk-free rate is assumed to be zero. If the stock goes up 10%, the call value is 10; if the stock goes down, the call is worthless. The portfolio can be perfectly hedged by a stock position equal to the call's delta. In the binomial model, you can calculate the hedge exactly with some simple algebra.

$$deltaHedge = \frac{callValue(up) - callValue(down)}{stockP(up) - stockP(down)}$$

Plugging in the specific parameters in our example, we have

$$deltaHedge = \frac{10 - 0}{110 - 90.91} = 0.524$$

Given a delta of 0.524, this implies the call writer hedges by going long this amount of stock. This portfolio generates the same payoff regardless of the outcome, -4.76.

	StockPriceUp	110.0	
	delta Hedge	CallOption	net
PnL	5.24	10.00	4.76

Stock Price	100

	StockPriceDown	90.91	
	delta Hedge	CallOption	net
PnL	4.76	0.00	4.76

This shows how hedging turns the option payoff into a risk-free position. Further, note that the call value is the same using either the binomial model or present value of the expected value of the payoff. As the interest rate is zero, the probability times the payoff is the discounted expected value.

$$callValue = prob \cdot payoff$$
$$callValue = 0.476 \cdot (-10) = -4.76$$

Thus, one can sell the option and accept the risk liability of \$ 10 or \$0, where \$ 10 has a 47.6% chance of occurring, or hedge the option and pay \$4.76 in the next period, regardless.

A.3 Arb Pnl and LP IL

$$ArbitragePnL = \Delta ETH^{trader} \cdot (p_1 - pFill)$$

$$\Delta ETH^{pool} = liq \cdot \left(\frac{1}{\sqrt{p_1}} - \frac{1}{\sqrt{p_0}} \right)$$
$$\Delta ETH^{trader} = -\Delta ETH^{pool} = liq \cdot \left(\frac{1}{\sqrt{p_0}} - \frac{1}{\sqrt{p_1}} \right)$$

$$pFill = abs\left(\frac{\Delta USD}{\Delta token} \right) = abs\left(\frac{liq \cdot \left(\sqrt{p_1} - \sqrt{p_0}\right)}{liq \cdot \left(\frac{1}{\sqrt{p_1}} - \frac{1}{\sqrt{p_0}} \right)} \right) = \sqrt{p_1 p_0}$$

$$ArbitragePnL = liq \cdot \left(\frac{1}{\sqrt{p_0}} - \frac{1}{\sqrt{p_1}} \right) \cdot \left(p_1 - \sqrt{p_1 p_0} \right)$$

$$ArbitragePnL = liq \cdot \frac{\left(\sqrt{p_1} - \sqrt{p_0}\right)^2}{\sqrt{p_0}} = -IL$$

A.4 Moment Generating Function for Gamma

An exact estimate of the LP's gamma expense comes from the moment generating functions. Prices are assumed to be log-normally distributed. The moment generating function for a lognormally distributed variable p is known to be

$$E\left[p^n\right] = p_0^n \cdot exp\left(n \cdot \mu + \frac{1}{2}\sigma^2 \cdot n^2\right)$$

Given the LP's value is a constant times the square root of p, if we assume a zero expected return, the mean log return is -$\sigma^2/2$. Substituting this in gives us.

$$E\left[2 \cdot liq \cdot \sqrt{p_0}\right] = 2 \cdot liq \cdot \sqrt{p_0} \cdot exp\left(-\frac{\sigma^2}{4} + \frac{\sigma^2}{8}\right) = 2 \cdot liq \cdot \sqrt{p_0} \cdot exp\left(-\frac{\sigma^2}{8}\right)$$

This implies the following LP's gamma expense.

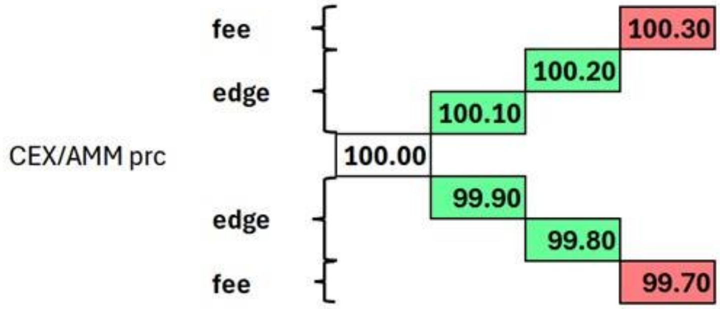
$$E\left[2 \cdot liq \cdot \sqrt{p_t}\right] - 2 \cdot liq \cdot \sqrt{p_0} = 2 \cdot liq \cdot \sqrt{p_0} \cdot \left(exp\left(-\frac{\sigma^2}{8}\right) - 1\right)$$

Simulations will confirm that the MGF approach is the correct metric for finite periods, such as months or years. The difference between these two formulas is only around 1% for monthly data and 5% for annual data. Given the standard errors, the Black-Scholes estimate works relatively well in practice.

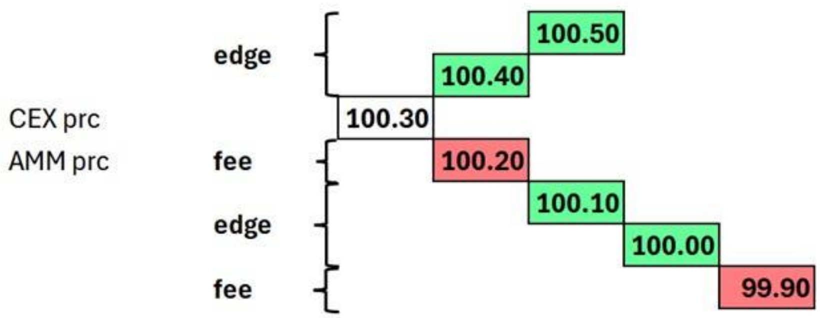
A notable property of these metrics of expected IL is that they utilize variance over a sample period, as opposed to using starting and ending prices. There are several ways to measure variance, including varying granularities and different moving average windows. While these are correlated with the twelve IL metrics listed above by substituting (p_1/p_0-1)^2 for variance, a sample variance IL is best applied to monthly data using independent volatility estimates. This allows one to generate three somewhat different metrics of historical IL, and these help highlight errors in one's data

A.5 Expected Profit with Asymmetric Barriers

For example, assume the price starts at 100.0, the edge is 0.2, and the fee is 0.1. A price move up or down of 0.3 triggers an arbitrage trade. In the example below, when the CEX price rises to the red cell at 100.3, the new post-trade AMM price would be 100.2.



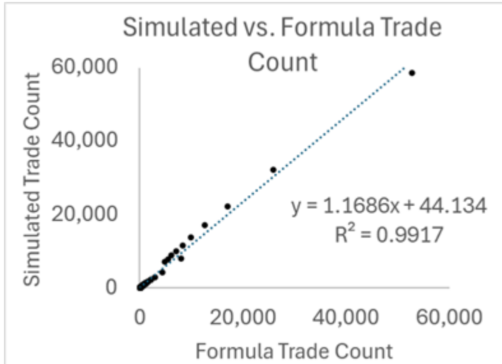
After the arbitrage trade, the boundaries triggering an arb trade are asymmetric. In the chart below, the CEX price is 100.3, but the AMM price is only 100.2 because it stopped at the CEX price minus the fee. Thus, continuing the CEX price's earlier move requires only two steps up to generate the 0.3 trigger (100.5-100.2 = 0.3); the reverting move has to first double-back over the fee buffer, then proceed through the edge, and then move by the fee amount (99.90 - 100.2= -0.3).



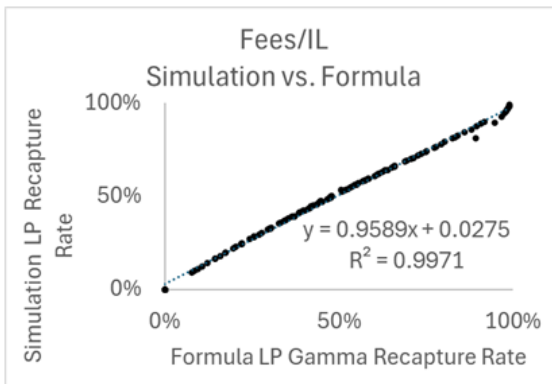
The probability of hitting a boundary decreases exponentially by distance, so moving up 0.2 *or* down 0.4 has a significantly higher probability than moving 0.3 in either direction. The simple formula for hitting a barrier does not directly apply, as it presumes symmetric barriers. The precise number can be calculated directly in a lattice, but the expression is the sum of a series of combinatoric functions multiplied by probabilities, which does not aid intuition. However, the following formula works well as a concise approximation. It takes the base case of the symmetric barriers and multiplies it by a ratio with the fee over the edge in the numerator.

$$NumberTrades_{day} = \frac{\sigma_{day}^2}{(fee + edge)^2} \cdot \frac{\left(3 + {}^{fee}\!/\!_{edge}\right)}{3}$$

We simulated an arbitrage trading strategy 10,000 times, using various fees and edges, for a fixed AMM liquidity over a day, with a total volatility of 10%. There were 260,000 periods, so each step had an average movement of 2 basis points. We then applied fee and edge combinations ranging from 0 to 100 basis points in increments of 10, resulting in a total of 121 combinations (e.g., {10, 30}, {100, 60}).



We simulated arbitrage behavior 10,000 times for 121 combinations of fees and edges over a day, with a 10% move and trades every 300 milliseconds. This is similar to the simulation mentioned above regarding the trade count. We then averaged the pool's fees divided by the day's gamma expense (aka IL) and compared it to this formula. The results show that the LP gamma recapture formula above is an excellent approximation.



For my purposes, the extra precision does not change any of my points. In practice, a lattice would calculate an even better estimate.

A.6 Arbitrageur Equilibrium and Non-Zero Profits

At some level, arbs focus on latency, not price competition with their competitors, because that is a winnable game. Many investments that help arbs, such as those that do not involve price, include RPC access, NVMe SSD storage, Field-Programmable Gate Arrays, and Kernel-bypass NICs, among others.

A power law in proficiency generates a handful of arbs that can trade at the lowest latency. They are playing a repeated game where one arb is usually making a trade when no one else wants to. The large retail trades that generate MEV interest for front-running and sandwich attacks generate much attention, but this is a different game and is irrelevant to LP IL. Arbs are targeting mispricings under 10 bps, often under 5. The parochial differences in their 'true price' aggregation model will generate random discrepancies across arbs. In practice, arbs do not compete within blocks.

The arb has to consider two extremes in an AMM with a fee. In one, he assumes no competition, and his best strategy is to wait as long as possible, so the fee, as a percentage of his gross profit, is the smallest. On the other extreme, he eliminates his competitors' profits, as well as his own, by aggressively trading on every infinitesimal profit opportunity. This is demonstrated above, but for the sake of argument, let us assume it is true.

Let us define four gradations of aggressiveness in our arb trader's strategy. In one extreme, he chooses the least competitive trading strategy that maximizes his profits, assuming he has no competitors. On the other hand, he can be considered a profit blocker, preventing both himself and other arbitrageurs from generating a net profit (i.e., after fees).

- Max-Prof: Profit from trade targeting a large trade
- Aggressive: Profit from trade targeting modest-sized trade
- More aggressive: Profit from small trades
- Zero-Prof: making an almost infinite number of super-small trades

Assuming the arbs split profits when trading the same strategy (probabilistically getting half the profit on average), as they both generate the same slippage (per the slip fee sequencer rules), the winning arb is chosen randomly. If one arb is more aggressive than the other arb, by generating a larger trade, they can then dominate arb trading and leave their competitors with nothing. However, at that point, his competitor would make a zero profit and could improve his profit by applying a more aggressive trading tactic. This generates the following normal-form game.

	Payoffs when trading simultaneously				
		Trader B			
		max-profit	aggresive	more aggres	zero-profit
Trader A	max-profit	5, 5	0, 6	0, 4	0, 0
	aggresive	6, 0	3, 3	0, 4	0, 0
	more aggres	4, 0	4, 0	2, 2	0, 0
	zero-profit	0,0	0, 0	0, 0	0, 0

Note: *Payoffs to trader A on left of comma, for trader B on right*

While there are only four possibilities above, this is intended to illustrate the basic pattern. AMMs are not constrained to minimum tick sizes, so there are an almost limitless number of potential pricing tactics (target 1 bp below the true price, 0.99 bps, etc). If traders A and B competed in a finite sequence of games, one can see how they would gravitate towards the bottom right corner equilibrium, where both make near-zero profit. That is the Nash equilibrium.

In real life, arbs often trade without competition due to random issues. For example, one arb may have a portfolio that is fully invested in a particular token, so they cannot buy more of it. More commonly, the 2-10 bp opportunities are at the edge of detection, given the randomness in their API feeds and the model they apply to extract the 'true' price from these feeds. When arbs trade without competition, their payoffs are trivial in that they do not depend on what the other player is doing. However, it is useful to present them this way to see how they add up. Below, we see a normal-form game when an arb trades without competition. The Nash equilibrium here is in the upper left, the opposite of when the arb competed.

	Payoffs when trading alone				
		Trader B			
		max-profit	aggresive	more aggres	zero-profit
Trader A	max-profit	10, 10	10, 6	10, 4	10, 0
	aggresive	6, 10	6, 6	6, 4	6, 0
	more aggres	4, 10	4, 6	4, 4	4, 0
	zero-profit	0,10	0, 6	0, 4	0, 0

Making the conservative assumption that DEX arbs compete on half of their trades (which is usually much less), their expected profit payoff per trade is the average of the payoffs in the above matrices. This shows a unique equilibrium in the upper left where they trade as if they were not competing.

		Trader B			
		max-profit	aggresive	more aggres	zero-profit
Trader A	max-profit	7.5, 7.5	5, 6	5, 4	5, 0
	aggresive	6, 5	4.5, 4.5	3, 4	3, 0
	more aggres	4, 5	4, 3	3, 3	2, 0
	zero-profit	0,5	0, 3	0, 2	0, 0

An arb could focus on the parasitic strategy of just targeting the successful arbs and immediately post a more aggressive trade that would generate a higher slip fee and get executed first. However, once the other arbs see what is happening, they could punish him in many ways. For example, if one arb acquires an excess inventory of ETH, as is natural, he expects to pay at least 5-10 basis points to sell these on another exchange. Instead, given that he expects to pay to unload the ETH anyway, he could place costly sell orders on the AMM merely to bait the price-jumping arbitrageur into making unprofitable trades. It would cost him nothing, but he would punish the penny-jumping arb who thinks these exit trades are arbitrage trades. Implementing a rule that presumes an opportunity solely based on others' trades is dangerous because these other traders can generate manipulative counter-measures.

The key to maintaining a collusion is that the viable players have some sort of edge to distance themselves from the masses. Conditional upon this excellence, they can compete among themselves on latency, risk management, and their model of distilling a true price from the many potentially relevant data points. They realized the futility of competing on edge, which would generate an equilibrium where none of them make a profit after fees.

The fastest traders with the best true price model will dominate almost everyone. The handful who can compete then play a repeated game with a handful of competitors. Given the second-order benefit from outcompeting other arbs and the zero-profit endgame, one can see why arbs do not engage in price competition as if they were bidding on assets in an auction.

A.7 Price-Liquidity IL Estimation Errors

Highlighting how these metrics differ on this day underscores the bias of using a single liquidity number for a discrete interval. Below, we see the small -117k IL estimated from the price-liquidity IL estimate. The token changes are not the actual token changes, but rather those implied by the price change and liquidity estimate.

Average Liquidity Underestimates IL for Date in Figure 2.4

			implied by prc-liq		liq*sqrt(p0)* (1-sqrt(p1/p0))^2
initial price	ending Prc	liquidity	LP ETH chg	LP USD chg	price-liq IL
1,559	1,472	3.70	2,729	4,134,831	117,028

For the markout IL estimate, we use the actual LP token changes and the ending price. The actual daily token changes are significantly greater than what was implied by the price-liq metric because most of the price change occurred when liquidity was at its peak.

Actual Token Changes and Implied IL for Date shown in Figure 2.4

	actual				Pnl - fees
ending Prc	LP ETH chg	LP USD chg	PnL	Fee Rev	Markout IL
1,472	4,733	7,197,430	230,507	101,975	332,482

A.8 Extensions

A.8.1 Leveraged Traders: perps

Consider the case where a trader deposits ETH or USD and puts on a 5:1 levered long or short position. That different starting positions generate the same long position. His default status would be easy to monitor.

Table 8.1

Leveraged Trader Position

LeverLong w/ USD

ETH price: 2,000

item	quantity	$value
ETH	0.000	0
USD	2,000	2,000
ReqMargin		0
MktValue		2,000

LeverLong w/ ETH

ETH price: 2,000

item	quantity	$value
ETH	1.000	2,000
USD	0	0
ReqMargin		400
MktValue		2,000

ETH price: 2,000

item	quantity	$value
ETH	5.000	10,000
USD	-8,000	-8,000
ReqMargin		2,000
MktValue		2,000

Short w/ USD

ETH price: 2,000

item	quantity	$value
ETH	0.000	0
USD	2,000	2,000
ReqMargin		0
MktValue		2,000

Short w/ ETH

ETH price: 2,000

item	quantity	$value
ETH	1	2,000
USD	0	0
ReqMargin		400
MktValue		2,000

ETH price: 2,000

item	quantity	$value
ETH	-5	-10,000
USD	12,000	12,000
ReqMargin		2,000
MktValue		2,000

Given that account balances are held on the AMM contract, liquidators can typically extinguish the defaulting account's negative balances on the contract. For large positions, where the deficit token is a substantial fraction of the pool, implying a significant price impact, the liquidation fee would have to make it attractive for someone to provide the capital to plug the token deficit before withdrawing the surplus token. A 20% margin requirement implies that there is sufficient value to make liquidating large accounts attractive.

A.8.2 No Oracle-based Funding Rate

By having both swap and perp functionality, the contract price would be set by arbitrage, not a funding rate linked to an oracle. This would eliminate several problems in perp protocols. Oracles needed for standard perp funding rate calculations can be hacked and censored. More importantly, the funding rate mechanism used to link perp prices with spot prices is a farce in that it is profoundly different than the funding rates applied in traditional swap markets or the basis in futures markets.

In prime broker swap accounts, the funding rate is entirely independent of the price; the assets in these accounts trade in spot markets as if they were not in swap accounts. Swap rates change slowly, like general interest rates, and are known ex ante. They apply only to overnight positions, just like financing in general. There is no reason to apply intraday funding rates because the amounts are too small to incent behavior meaningfully. The default 1 bp per 8-hour funding rate payment will not affect price-setting arbs, as these traders expect half of their trades to be buys or sells. Excluding all buys or sells because the perp price premium implies a 1 bp funding rate expense would turn the price-setting arb into one taking relatively long positions, meaning the arb would not be actively trading every wiggle in the CEX price. Such a trader would no longer be arbitraging the AMM; active arbs will ignore the funding rate. The perp funding rate mechanism only makes sense at a high level.

In futures markets, a perp and spot price implies a basis, which is like a funding rate, but this price is orders of magnitude greater than the usual 0.03% price premiums in perp markets. This larger price premium enables real arbitrage, as it covers transaction costs and can be locked in. In contrast, in perp markets, the perp premium at the time of trade means almost nothing, as the funding rate is a function of the average perp premium over a position's duration.

Several perp protocols do not even utilize the perp price premium funding mechanism, and no one seems to care. GMX and SNX apply a simple rule where the ratio of long to short positions determines the funding rate, which works fine and makes sense. Bitmex altered their perp funding rate calculation to use a default of 11% annually, given a perp-to-CEX price premium of -5 to +5 bps, and nothing changed. What previously required outrageous APYs, ranging from -54% to +54%, surely a large incentive, suddenly did not need any incentive to keep the perp price at the untethered CEX price.

Clearly, the perp price is set by a Schelling point indifferent to the perp funding rate in that the most obvious target is the spot price, and the funding rate is just there to make traders feel comfortable that it is not *merely* a Schelling point. The fact that there is a vague relation to an equilibrating mechanism seemed a necessary and sufficient condition for perp markets when users had only one coin in the protocol, as was the case with BitMex in 2016. However, when trading ETH for USDC, this mechanism is an anachronism: let 'real' arbitrage set the price.

The funding rate mechanism is a farce used by insiders to fleece retail traders when they are intoxicated with house money. If the market has just gone up, many ETH holders are sitting on significant gains, so they are not averse to paying an extra 5% for a month (i.e., only 0.018% every 8 hours). Removing this mechanism is a signal of intelligence and integrity. The absence of the 12% funding rate on regulated exchanges like Coinbase and the Chicago Mercantile Exchange highlights the non-US funding rate is a conspiratorial equilibrium on unregulated centralized exchanges. BinanceUS does not even offer perps, knowing that with regulation, the fix would be impossible, and the absence of perp funding rates on BinanceUS would highlight this.

A.8.3 StableCoin

An ETH-USDC contract with perp and Uniswap functionality naturally produces a stablecoin. As margined accounts can have negative balances, a simple way to create a stablecoin would be to deposit ETH and withdraw the USD as an AMM-specific, transferrable claim on its USD collateral (e.g., USDC or USDT), which can also be used to buy ETH on the AMM.

For example, in Table 5.3, the trader deposits ETH and withdraws his USD into his externally owned Account (EOA).

Table 8.2

Trader w/ Simple ETH Deposit

ETH price:	2,000	
item	quantity	$value
ETH	1.000	2,000
USD	0	0
ReqMargin		400
AcctMktValue		2,000
USD-EO		
TotMktValue		2,000

ETH price:	2,000	
item	quantity	$value
ETH	1.000	2,000
USD	-1,250	-1,250
ReqMargin		400
AcctMktValue		750
USD-EO	0	1,250
TotMktValue		2,000

Alternatively, the trader could create a short position and withdraw some of the USD generated. This would be similar to the Ethena strategy, where a short ETH position is created to capture the perpetual funding rate (there would be a long ETH position off-contract).

Table 8.3

Creating Stable With Short

ETH price:	2,000	
item	quantity	$value
ETH	1.000	2,000
USD	0	0
ReqMargin		400
AcctMktValue		2,000
USD-EO		
TotMktValue		2,000

ETH price:	2,000	
item	quantity	$value
ETH	-2.000	-4,000
USD	5,000	5,000
ReqMargin		800
AcctMktValue		1,000
USD-EO	1,000	1,000
TotMktValue		2,000

As long as the contract is solvent, the new stablecoin would have the value of USDC. The benefit of this approach is that it creates stablecoins more efficiently, as it does not require auctions or oracles, unlike MakerDAO, which has an attached DEX.