APPLICATION OF LINEAR REGRESSION MODELS IN SUBSTANTIVE ANALYTICAL PROCEDURES FOR AUDITING ASSET MANAGEMENT

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ABSTRACT.

The aim of this work is to apply linear regression models to the returns of net asset values (NAV) of some assets, in order to verify if they comply with their investment policy approved if there is a benchmark to be followed, and to try to forecast the NAV for one week in order to generate an expectation. This is why this work tries to apply ISA 520 about Analytical Procedures in audit engagements, as this rule allows the use of advance statistical techniques in these procedures.

KEY WORD.

Model, asset, analytical procedure, benchmark and forecast.

JEL CODES.

C22 – Time-Series Models.
G17 – Financial Forecasting and Simulation.
M42 – Auditing.

1. REVIEW OF ISA 520 ANALYTICAL PROCEDURES.

ISA 520 Analytical procedures establishes the following:

Definition of Analytical Procedures (Ref: Para. 4)

A1. Analytical procedures include the consideration of comparisons of the entity's financial information with, for example:

- *Comparable information for prior periods.*
- *Anticipated results of the entity, such as budgets or forecasts, or expectations of the auditor, such as an estimation of depreciation.*
- *Similar industry information, such as a comparison of the entity's ratio of sales to accounts receivable with industry averages or with other entities of comparable size in the same industry.*

A2. Analytical procedures also include consideration of relationships, for example:

- *Among elements of financial information that would be expected to conform to a predictable pattern based on the entity's experience, such as gross margin percentages.*
- *Between financial information and relevant non-financial information, such as payroll costs to number of employees.*

A3. Various methods may be used to perform analytical procedures. These methods range from performing simple comparisons to performing complex analyses using advanced statistical techniques. Analytical procedures may be applied to consolidated financial statements, components and individual elements of information.

Substantive Analytical Procedures (Ref: Para. 5)

A4. The auditor's substantive procedures at the assertion level may be tests of details, substantive analytical procedures, or a combination of both. The decision about which audit procedures to perform, including whether to use substantive analytical procedures, is based on the auditor's judgment about the expected effectiveness and efficiency of the available audit procedures to reduce audit risk at the assertion level to an acceptably low level.

A5. The auditor may inquire of management as to the availability and reliability of information needed to apply substantive analytical procedures, and the results of any such analytical procedures performed by the entity. It may be effective to use analytical data prepared by management, provided the auditor is satisfied that such data is properly prepared.

The Reliability of the Data (Ref: Para. 5(b))

A12. The reliability of data is influenced by its source and nature and is dependent on the circumstances under which it is obtained. Accordingly, the following are relevant when determining whether data is reliable for purposes of designing substantive analytical procedures: (a) Source of the information available. For example, information

may be more reliable when it is obtained from independent sources outside the entity;(b) Comparability of the information available. For example, broad industry data may need to be supplemented to be comparable to that of an entity that produces and sells specialized products; (c) Nature and relevance of the information available. For example, whether budgets have been established as results to be expected rather than as goals to be achieved; and (d) Controls over the preparation of the information that are designed to ensure its completeness, accuracy and validity. For example, controls over the preparation, review and maintenance of budgets.

A13. The auditor may consider testing the operating effectiveness of controls, if any, over the entity's preparation of information used by the auditor in performing substantive analytical procedures in response to assessed risks. When such controls are effective, the auditor generally has greater confidence in the reliability of the information and, therefore, in the results of analytical procedures. The operating effectiveness of controls over non-financial information may often be tested in conjunction with other tests of controls. For example, in establishing controls over the processing of sales invoices, an entity may include controls over the recording of unit sales. In these circumstances, the auditor may test the operating effectiveness of controls over the recording of unit sales in conjunction with tests of the operating effectiveness of controls over the processing of sales invoices. Alternatively, the auditor may consider whether the information was subjected to audit testing. ISA 500 establishes requirements and provides guidance in determining the audit procedures to be performed on the information to be used for substantive analytical procedures.

A14. The matters discussed in paragraphs A12(a)–A12(d) are relevant irrespective of whether the auditor performs substantive analytical procedures on the entity's period-end financial statements, or at an interim date and plans to perform substantive analytical procedures for the remaining period. ISA 330 establishes requirements and provides guidance on substantive procedures performed at an interim date.

Evaluation Whether the Expectation Is Sufficiently Precise (Ref: Para. 5(c))

A15. Matters relevant to the auditor's evaluation of whether the expectation can be developed sufficiently precisely to identify a misstatement that, when aggregated with other misstatements, may cause the financial statements to be materially misstated, include:

- *The accuracy with which the expected results of substantive analytical procedures can be predicted. For example, the auditor may expect greater consistency in comparing gross profit margins from one period to another than in comparing discretionary expenses, such as research or advertising.*
- *The degree to which information can be disaggregated. For example, substantive analytical procedures may be more effective when applied to financial information on individual sections of an operation or to financial statements of components of a diversified entity, than when applied to the financial statements of the entity as a whole. • The availability of the information, both financial and non-financial. For example, the auditor may consider whether financial information, such as budgets or forecasts, and non-financial information, such as the number of units produced or sold, is available to design substantive analytical*

procedures. If the information is available, the auditor may also consider the reliability of the information as discussed in paragraphs A12–A13 above.

According to the description above this work will be focused on the use of advanced statistical techniques, in particular linear regression models in asset management in order to verify the compliance with investment policies that follow a benchmark during one year, and to forecast the weekly returns and the net asset value (NAV) for making an expectation. In particular, these models will be applied to an investment fund managed by Santander Asset Management, S.A., SGIIC, a pension plan managed by Ibercaja Pensión, S.A.U. and a prevision social plan managed by Bestinver Gestión, S.A. SGIIC, all of which are located in Spain. The data used in this work is attached in Annex I and consists of 52 weeks from week 42 of 2022 until week 41 of 2023.

Traditionally, in order to be assured about the compliance of the investment policy with the objective of following a particular benchmark, auditors used to compare daily returns of the NAV against the returns of the index for a complete year. If the differences were below 2%, then the auditors could conclude that the investment policy was followed, but this approach will not be developed in this work. However, in order to make the analysis simpler, we use weekly data in this work instead of daily data.

2. ANALYSIS OF THE ASSETS AND ITS BENCHMARKS, AND PROPOSED MODELS.

2.1. Santander Renta Fija Flotante, Investment Fund.

Consulting the data included in the brochure that appears in CNMV (Spanish Securities Regulator), this is a short term fixed income fund in Euros managed by Santander Asset Management, S.A. It uses the Bloomberg Barclays EURO Floating Rate Notes TR index as a benchmark. This is why we propose a linear regression model, where the dependent variable is the weekly return of the NAV of the fund and the independent variable is the weekly return of the index. The weekly returns are calculated as follows:

Weekly return of the NAV:

$$r_t = \ln\left(\frac{NAV_t}{NAV_{t-1}}\right) \cdot 100, \text{ [1]}$$

Where:

- NAV_t is the NAV in period t.
- NAV_{t-1} is the NAV in period $t-1$.

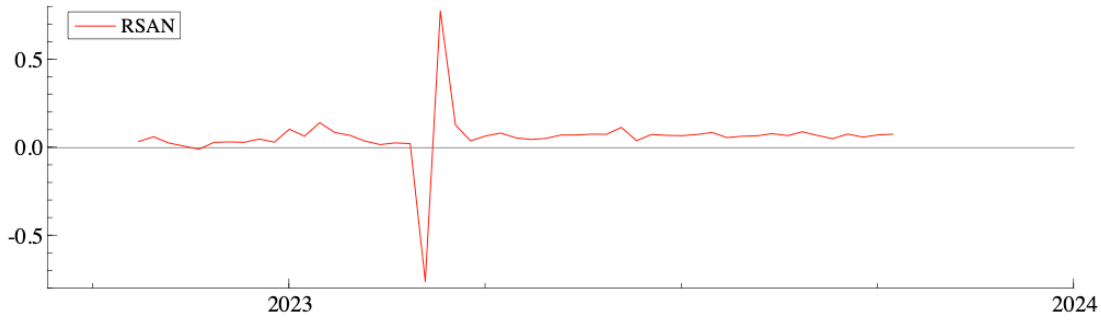
Weekly return of the index:

$$r_I = \ln\left(\frac{I_t}{I_{t-1}}\right) \cdot 100, \text{ [2]}$$

Where:

- I_t is the value of the index in period t.
- I_{t-1} is the value of the index in period $t-1$.

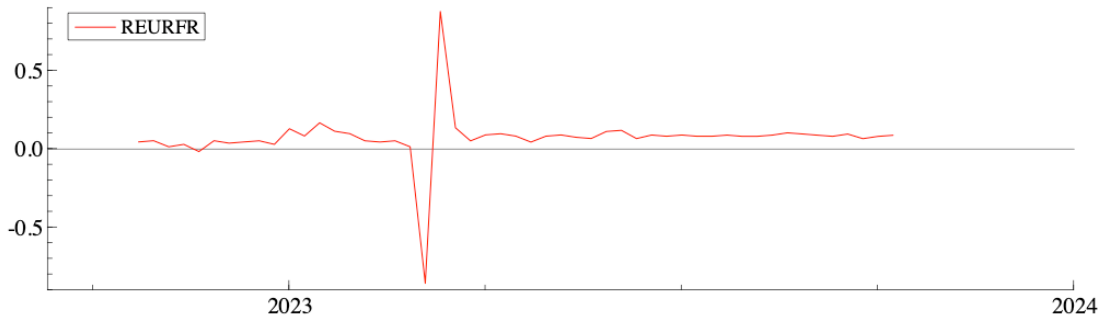
In the following graph, the weekly return evolution of the fund can be seen:



Graph 1. Weekly returns of Santander Renta Fija Flotante, Investment Fund. *Font: Santander Asset Management.*

At first glance, it can be seen that the data time series is stationary in mean and variance. However, there are fluctuations in the weeks for numbers 10 and 11 of 2023.

Additionally, in the following graph it can be seen the weekly return of the index:



Graph 2. Weekly returns of Bloomberg Barclays EURO Floating Rate Notes TR index.
Font: Bloomberg.

As mentioned above, this time series also is stationary in mean and variances and also shows fluctuations in the weeks for number 10 and 11 of 2023.

This is why it is proposed the following linear regression model:

$$r_t = \beta_0 + \beta_1 \cdot r_I + u_t, [3]$$

Where:

- r_t is the return of the fund.
- r_I is the return of the index.
- u_t is the random disturbance.

In order to perform the estimation of the model we used the software OxMetrics 8 and in particular it's module PcGive. The models will be estimated by ordinary least squares method (OLS). In order to evaluate the forecast capacity of the models we save the last value of the data in order to make a contrast with the forecast done with the software.

The results are the following:

```
EQ( 1) Modelling RSAN by OLS
       The estimation sample is: 2022(43) - 2023(40)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant          -0.00504554   0.001728    -2.92  0.0053   0.1508
REURFR               0.883717   0.009032     97.8  0.0000   0.9950

sigma                0.011295  RSS             0.00612371816
R^2                  0.995011  F(1,48) =      9573 [0.000]**
Adj.R^2              0.994907  log-likelihood        154.243
no. of observations         50  no. of parameters           2
mean(RSAN)          0.0595572  se(RSAN)              0.158268

AR 1-4 test:      F(4,44)    =  0.50105 [0.7351]
ARCH 1-4 test:    F(4,42)    =   1.0194 [0.4084]
Normality test:   Chi^2(2)   =   1.8419 [0.3981]
Hetero test:      F(2,47)    =  0.66364 [0.5197]
Hetero-X test:    F(2,47)    =  0.66364 [0.5197]
RESET23 test:     F(2,46)    =   1.2534 [0.2951]
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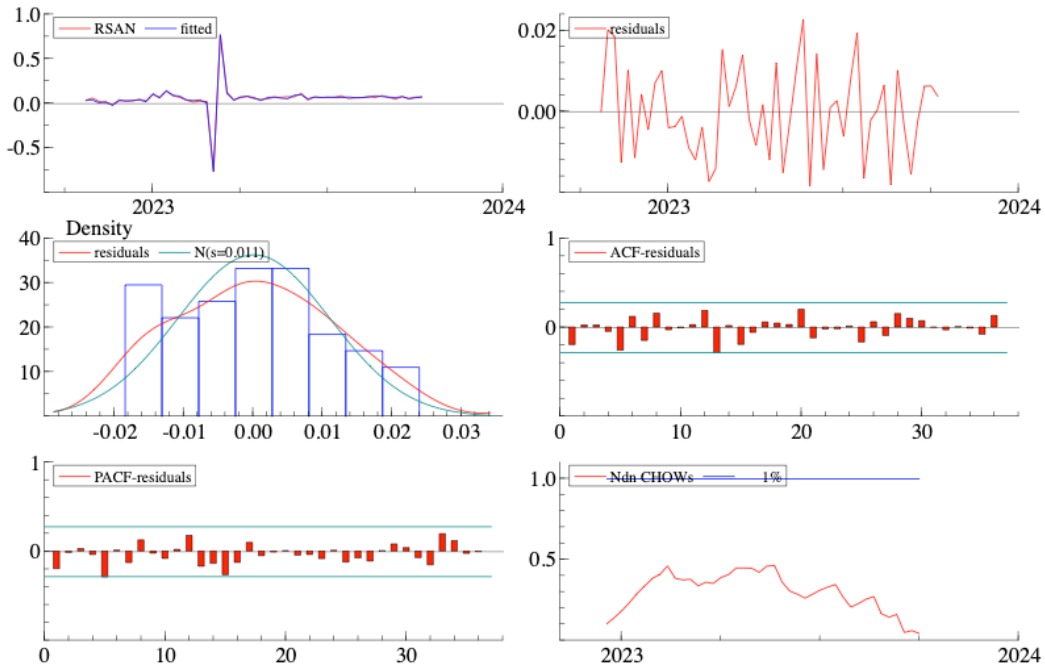
Figure 1. Results for Santander Renta Fija Flotante, Investment Fund.



Graph 3. Residuals for Santander Renta Fija Flotante, Investment Fund.

The estimated equation is the following:

$$\hat{r}_{SAN} = -0.01 + 0.88 \cdot r_{REURFR} \qquad R^2 = 0.99$$
$$(97.8) \qquad F = 9{,}573$$

The results above show that the estimator is significant (|t-value| > 2) and precise (|t-value| > 8). The determination coefficient R^2 is 0.99, greater than 0.85, which is elevated in order to perform adequate forecasts. There are not problems of autocorrelation, heteroscedasticity, normality and functional form.

On the other hand, the graph of the residuals show that the original series and the fitted are very similar, the residuals do not show dispersion. The histogram of frequencies show a normal distribution, and the ACF and PACF show white noise. The break-even point graph for Chow test does not show possible structural changes.

According to these results, we can conclude that this asset complies with its investment policy and its benchmark.

The next step will be to perform the forecast:

`Dynamic (ex ante) forecasts for RSAN (SE based on error variance only)`

Horizon	Forecast	SE	Actual	Error	t-value	-2SE	+2SE
2023(41)	0.0735595	0.0113	0.077208	-0.0036485	0.323	0.050969	0.09615

Figure 2. Forecast of the return of Santander Renta Fija Flotante, Investment Fund.

It can be seen that the error is less than 5%, so we can conclude that the forecast is adequate. On the other hand, if we present the forecast of the NAV[1] the results are the following:

Horizon	Forecast	Actual	Error	Error%
2023(41)	100.854044	100.85745	-0.00340643	-0.00338%

Figure 3. Forecast of the NAV of Santander Renta Fija Flotante, Investment Fund.

It can also be concluded that the forecast of the NAV is adequate as the error is less than 5%.

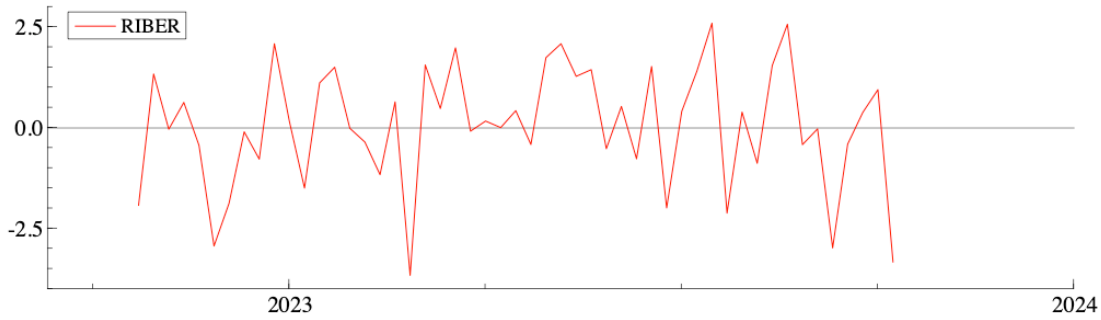
[1] In order to calcule the forecast NAV, it can be used the function goal seek from Excel to clear the following equation:

$$ln\left(\frac{\hat{r}^{*}_{2023-41}}{100.77961}\right)\cdot 100 = 0.0735595 \Rightarrow \boxed{\hat{r}^{*}_{2023-41} = 100.854044}$$

2.2. Ibercaja Pensiones Bolsa USA, Pension Plan.

According to the information included in the document of fundamental data for the participant that appears in the website of Ibercaja, this is a pension plan that invests at least 75% of it's portfolio in equity securities from the Standard & Poor's 500 index and it is included in the pension fund Plan Ibercaja de Pensiones Bolsa. This is why we use the S&P 500 as a benchmark. The pension fund is managed by Ibercaja Pensión, S.A.U.

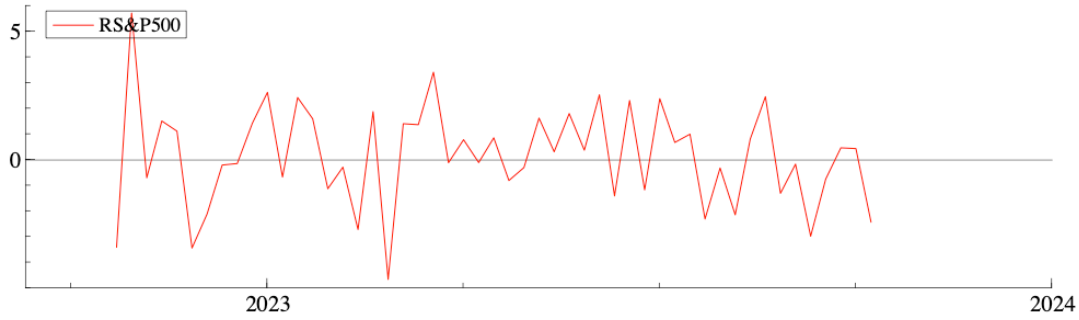
In the following graph, it can be seen the weekly return evolution of the pension plan:



Graph 4. Weekly returns of Ibercaja Pensiones Bolsa USA, Pension Plan. *Font: Ibercaja Pensión, S.A.U.*

The graph shows that the time series is stationary in mean and variance. At first glance, no fluctuations are identified.

Additionally, in the following graph the weekly return evolution of the index can be seen:



Graph 5. Weekly returns of S&P 500 index. *Font: Yahoo Finance.*

The graph shows that the time series is stationary in mean and variance too. Initially, no fluctuations are identified.

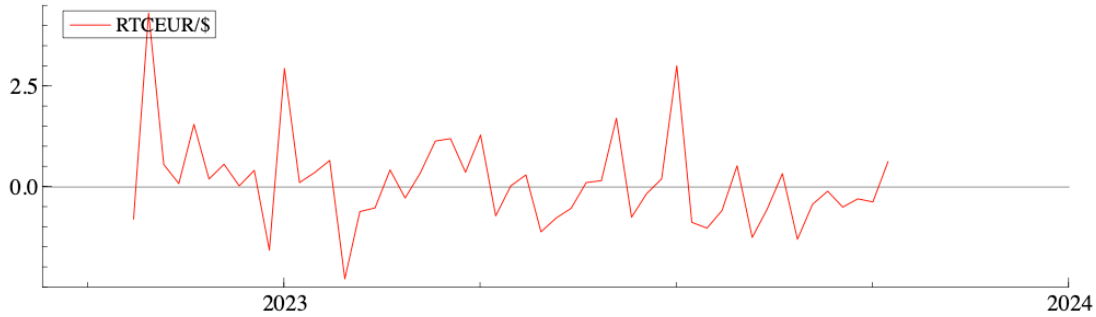
As this pension plan invests in an index that is in US$ (dollars) and it's NAV is in Euros, it is proposed to include the weekly variation of the €/US$ exchange rate as an independent variable too, as it is formulated in the following way:

$$ER\dot{€}/\$_t = \ln\left(\frac{ER€/\$_t}{ER€/\$_{t-1}}\right) \cdot 100, \text{ [4]}$$

Where:

- $ER€/\$_t$ is the €/$ exchange rate in period t.
- $ER€/\$_{t-1}$ is the €/$ exchange rate in period $t - 1$.

In the following graph, the weekly variation of the exchange rate can be seen:



Graph 6. Weekly variation of the €/US$ exchange rate. *Font: Bank of Spain.*

The graph shows that the time series is stationary in mean and variance too. At first, no fluctuations are identified.

This is why it is proposed the following linear regression model:

$$r_t = \beta_0 + \beta_1 \cdot r_I + \beta_2 \cdot ER\dot{€}/\$_t + u_t, \text{ [5]}$$

Where:

- r_t is the return of the plan.
- r_I is the return of the index.
- $ER\dot{€}/\$_t$ is the weekly variation of the €/US$ exchange rate.
- u_t is the random disturbance.

The results of the model are the following:

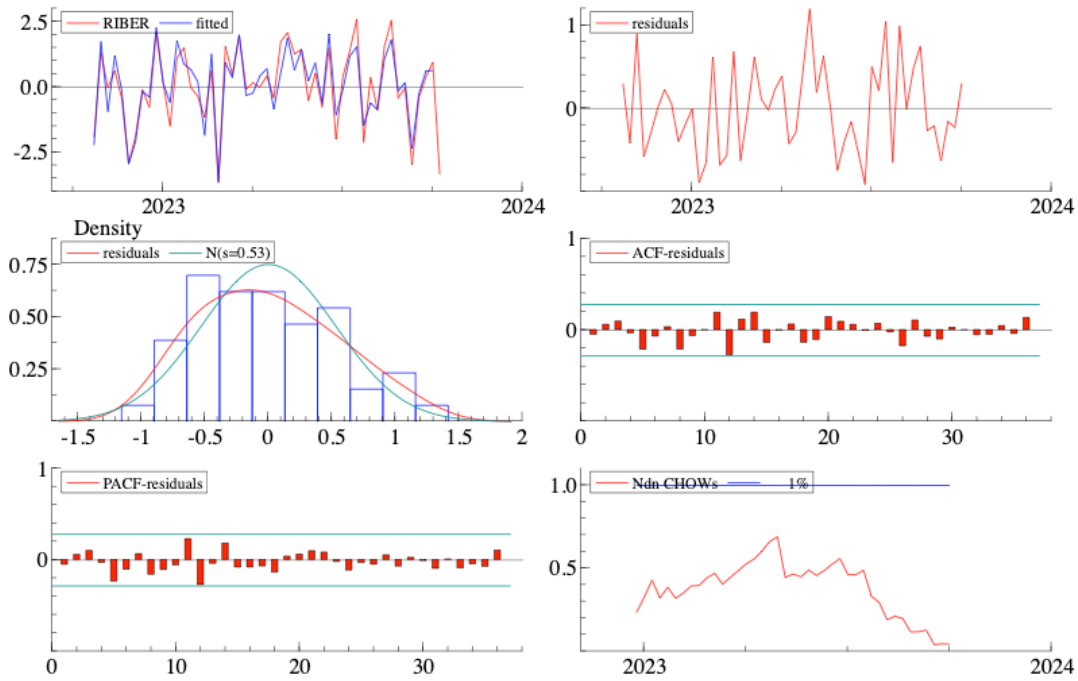
```
EQ( 2) Modelling RIBER by OLS
       The estimation sample is: 2022(43) - 2023(40)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            0.0213969   0.07789    0.275  0.7847   0.0016
RS&P500              0.821480   0.04786     17.2  0.0000   0.8624
EREUR/$             -0.688506   0.08060    -8.54  0.0000   0.6082

sigma                0.547096  RSS                14.0677741
R^2                  0.862986  F(2,47) =       148 [0.000]**
Adj.R^2              0.857156  log-likelihood       -39.2435
no. of observations        50  no. of parameters           3
mean(RIBER)          0.114854  se(RIBER)             1.44755

AR 1-4 test:      F(4,43)    =  0.19773 [0.9382]
ARCH 1-4 test:    F(4,42)    =  0.18609 [0.9444]
Normality test:   Chi^2(2)   =   2.6530 [0.2654]
Hetero test:      F(4,45)    =  0.53575 [0.7101]
Hetero-X test:    F(5,44)    =  0.43538 [0.8214]
RESET23 test:     F(2,45)    = 0.047078 [0.9541]
```

Figure 2. Results for Ibercaja Pensiones Bolsa USA, Pension Plan.



Graph 7. Residuals for Ibercaja Pensiones Bolsa USA, Pension Plan.

The estimated equation is the following:

$$\hat{r}_{IBER} = \underset{}{0.02} + \underset{(17.2)}{0.82} \cdot r_{S\&P500} - \underset{(-8.54)}{0.69} \cdot \dot{ER€/\$}_t \qquad R^2 = 0.86 \qquad F = 148$$

The results above show that the estimators are significant (|t-value| > 2) and precise (|t-value| > 8), and significant jointly (F-value > 4). The determination coefficient R^2 is 0.86, greater than 0.85, which is elevated in order to perform adequate forecasts. There are not problems of autocorrelation, heteroscedasticity, normality and functional form neither.

On the other hand, the graph of the residuals show that the original series and the fitted are very similar, the residuals do not show dispersion. The histogram of frequencies show a normal distribution, and the ACF and PACF show white noise. The break-even point graph for Chow test does not show possible structural changes.

According to these results, we can conclude that this asset complies with its investment policy and its benchmark too.

The forecast is shown in the following way:

```
Dynamic (ex ante) forecasts for RIBER (SE based on error variance only)
```

Horizon	Forecast	SE	Actual	Error	t-value	-2SE	+2SE
2023(41)	-2.40634	0.54710	-3.33192	0.92558	-1.692	-3.5005	-1.3121

Figure 4. Forecast of the return of Ibercaja Pensiones Bolsa USA, Pension Plan.

The error is less than 5%, so we can conclude that the forecast is adequate too. On the other hand, if we present the forecast of the NAV[2] the results are the following:

Horizon	Forecast	Actual	Error	Error%
2023(41)	44.6408026	44.229488	0.4113146	0.92996%

Figure 5. Forecast of the NAV of Ibercaja Pensiones Bolsa USA, Pension Plan.

It can also be concluded that the forecast of the NAV is adequate as the error is less than 5%.

[2] As previously, we clear the following equation using the goal seek function from Excel:

$$ln\left(\frac{\hat{r}^*_{2023-41}}{45.728003}\right)\cdot 100 = -2.40634 \Rightarrow \boxed{\hat{r}^*_{2023-41} = 44.6408026}$$

2.3. Bestinver Futuro, Individual Social Prevision Plan.

Looking at the information included in the document of fundamental data for the associate that appears on the website of Bestinver, this is an individual social prevision plan that invests at least 75% of it's portfolio in equity securities worldwide and the rest in fixed income securities, and it is included in the Voluntary Social Prevision Entity[3] Bestinver Individual. It uses 50% of the BBG Barclays Euro Aggr. Bond 1-10 Year Total Return index and 50% of the MSCI World Net TR EUR index as a benchmark. This entity is managed by Bestinver Gestión, S.A. SGIIC. This is why it is proposed to calculate a replicated index that consists on the following:

$$RI_t = 50\% \cdot BBG\ Barclays\ Euro_t + 50\% \cdot MSCI\ World\ Net_t, \text{[6]}$$

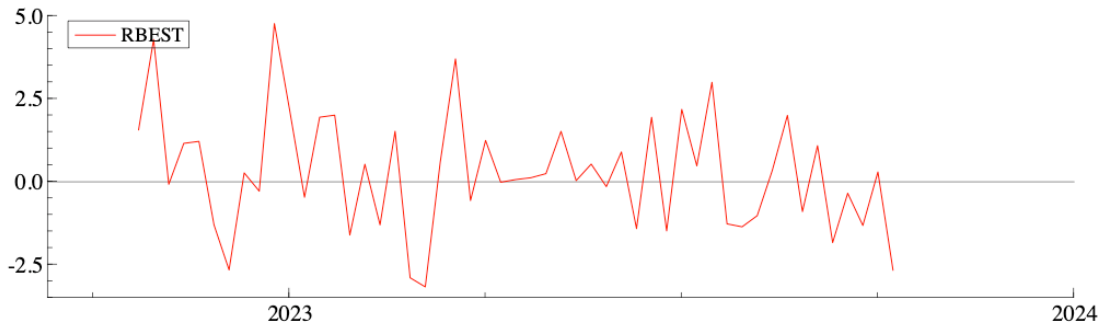
Consequently, the weekly return of this replicated index can be calculated as follows:

$$r_{RI} = ln\left(\frac{RI_t}{RI_{t-1}}\right) \cdot 100, \text{[7]}$$

Where:

- RI_t is the replicated index in period t.
- RI_{t-1} is the replicated index in period $t - 1$.

In the following graph, the weekly return evolution of the individual social prevision plan can be seen:

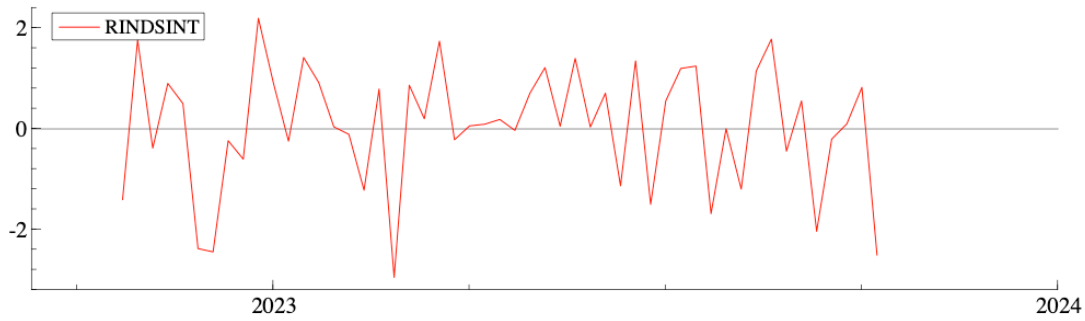


Graph 8. Weekly returns of Bestinver Futuro, Individual Social Prevision Plan. *Font: Bestinver Gestión, S.A. SGIIC.*

The graph shows that the time series is stationary in mean and variance too. At the start, no fluctuations are identified.

[3] A Voluntary Social Prevision Entity is an asset similar to a pension fund which is managed in the Basque Country and is regulated according to Basque Law. They act as a legal entity, which is different to pension funds, which they are not.

In the following graph, the weekly return evolution of the replicated index can be seen:



Graph 8. Weekly returns of the replicated index. *Font: Own.*

The graph shows that the time series is stationary in mean and variance too. At the outset, no fluctuations are identified.

This is why the following linear regression model is proposed:

$$r_t = \beta_0 + \beta_1 \cdot r_{RI} + u_t, [8]$$

Where:

- r_t is the return of the individual social prevision plan.
- r_{RI} is the return of the replicated index.
- u_t is the random disturbance.

The results of the model are the following:

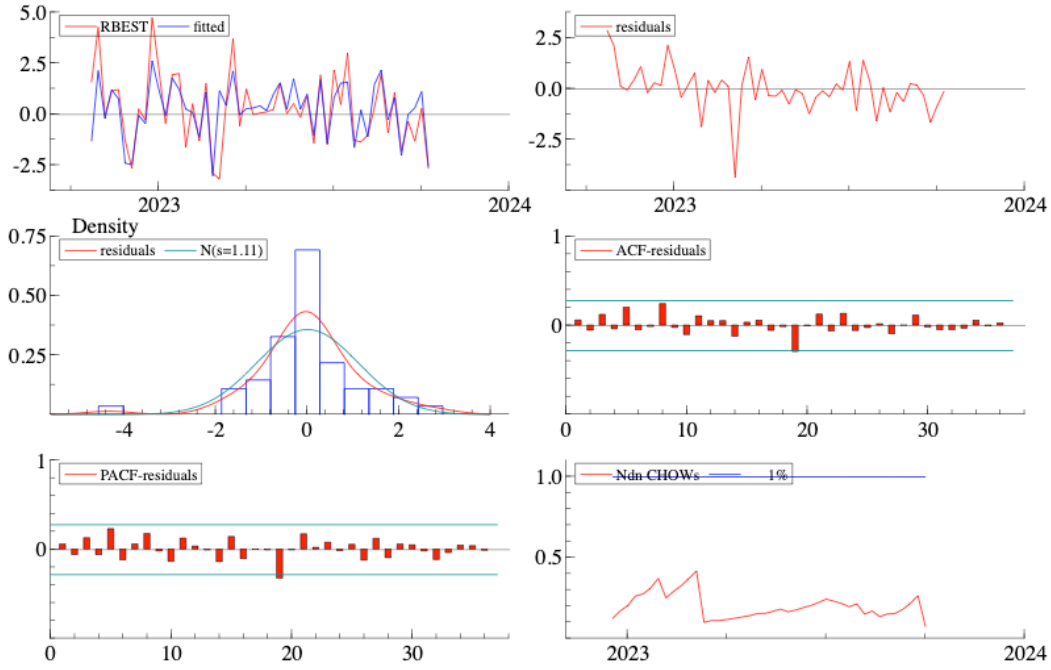
```
EQ( 3) Modelling RBEST by OLS
       The estimation sample is: 2022(43) - 2023(40)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant             0.215810     0.1626     1.33  0.1908   0.0354
RRI                   1.09912     0.1385     7.93  0.0000   0.5673

sigma                 1.14559  RSS                62.9940776
R^2                  0.567318  F(1,48) =     62.94 [0.000]**
Adj.R^2              0.558304  log-likelihood      -76.7224
no. of observations         50  no. of parameters          2
mean(RBEST)          0.328158  se(RBEST)             1.72372

AR 1-4 test:      F(4,44)   =  0.32020 [0.8629]
ARCH 1-4 test:    F(4,42)   =  0.25508 [0.9049]
Normality test:   Chi^2(2)  =   20.799 [0.0000]**
Hetero test:      F(2,47)   =  0.89649 [0.4149]
Hetero-X test:    F(2,47)   =  0.89649 [0.4149]
RESET23 test:     F(2,46)   =   5.4020 [0.0078]**
```

Figure 6. Results for Bestinver Futuro, Individual Social Prevision Plan.



Graph 9. Residuals for Bestinver Futuro, Individual Social Prevision Plan.

The estimated equation is the following:

$$\hat{r}_{BEST} = 0.22 + 1.10 \cdot r_{RI} \qquad R^2 = 0.57$$
$$(7.93) \qquad F = 62.94$$

These results show that the estimator is significant (|t-value| > 2), but not precise (|t-value| < 8). The determination coefficient R^2 is 0.57, less than 0.85, which is very low in order to perform adequate forecasts. There are not problems of autocorrelation and heteroscedasticity but there are problems with normality and functional form.

The graph of the residuals show that the original series and the fitted are not similar and the residuals show outliers. The histogram of frequencies do not show a normal distribution as asymmetry exists on the left, but the ACF and PACF show white noise. The break-even point graph for Chow test does not show possible structural changes.

In order to solve this special problem, it is necessary to include dummy variables, specifically for the non-normality and functional form.

The results of the model are the following:

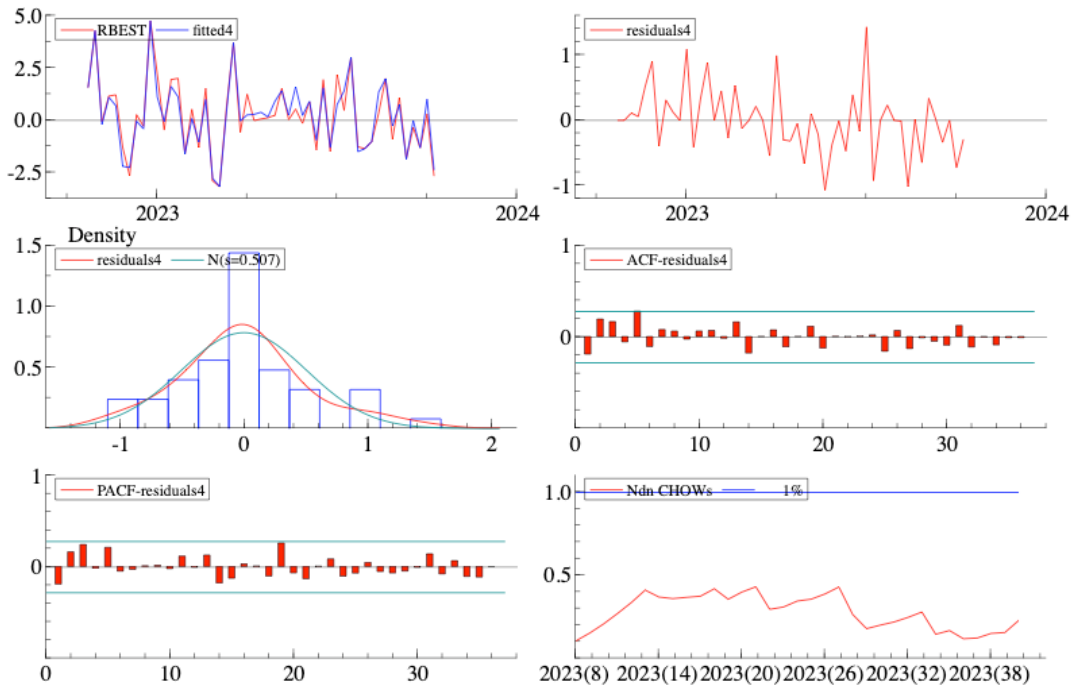
```
EQ( 4) Modelling RBEST by OLS
       The estimation sample is: 2022(43) - 2023(40)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant             0.194631    0.09027     2.16  0.0373   0.1065
RRI                   1.00705    0.07894     12.8  0.0000   0.8067
D(2022-44)            2.78422     0.5946     4.68  0.0000   0.3598
D(2022-45)            2.32061     0.6017     3.86  0.0004   0.2761
D(2023-1)             2.36505     0.6107     3.87  0.0004   0.2777
D(2023-6)            -1.84146     0.5848    -3.15  0.0031   0.2027
D(2023-11)           -4.23926     0.5891    -7.20  0.0000   0.5704
D(2023-13)            1.75805     0.6014     2.92  0.0057   0.1798
D(2023-30)            1.54972     0.5934     2.61  0.0127   0.1488
D(2023-32)           -1.55542     0.5847    -2.66  0.0113   0.1536
D(2023-40)           -1.61200     0.5848    -2.76  0.0088   0.1630

sigma                0.577736  RSS                 13.0173761
R^2                  0.910589  F(10,39) =    39.72 [0.000]**
Adj.R^2              0.887663  log-likelihood       -37.3035
no. of observations        50  no. of parameters          11
mean(RBEST)          0.328158  se(RBEST)             1.72372

AR 1-4 test:       F(4,35)    =   1.8918 [0.1338]
ARCH 1-4 test:     F(4,42)    =   1.7372 [0.1598]
Normality test:    Chi^2(2)   =   2.8004 [0.2466]
Hetero test:       F(2,38)    =   1.8561 [0.1702]
Hetero-X test:     F(2,38)    =   1.8561 [0.1702]
RESET23 test:      F(2,37)    =  0.35101 [0.7063]
```

Figure 7. Results for Bestinver Futuro, Individual Social Prevision Plan corrected.



Graph 10. Residuals for Bestinver Futuro, Individual Social Prevision Plan corrected.

The new estimated equation is the following:

$$\hat{r}_{BEST} = \underset{}{0.22} + \underset{(12.8)}{1.01} \cdot r_{RI} + \underset{(4.68)}{2.78} \cdot D_{2022-44} + \underset{(3.86)}{2.32} \cdot D_{2022-45} + \underset{(3.87)}{2.37} \cdot D_{2023-1} -$$

$$\underset{(-3.15)}{1.84} \cdot D_{2023-6} - \underset{(-7.2)}{4.24} \cdot D_{2023-11} + \underset{(2.92)}{1.75} \cdot D_{2023-13} + \underset{(2.61)}{1.55} \cdot D_{2023-30} -$$

$$\underset{(-2.66)}{1.56} \cdot D_{2023-32} - \underset{(-2.76)}{41.61} \cdot D_{2023-40} \qquad R^2 = 0.91 \qquad F = 39.72$$

These new results above show that the estimators are significant (|t-value| > 2) and precise only in the estimator of the replicated index (|t-value| > 8), and significant jointly (F-value > 4). The determination coefficient R^2 is 0.91, greater than 0.85, which is elevated in order to perform adequate forecasts. The problems with normality and functional form have also been solved.

On the other hand, the new graph of the residuals show that the original series and the fitted are very similar, the residuals do not show dispersion, nor outliers. The histogram of frequencies show a normal distribution, and the ACF and PACF show white noise. The break-even point graph for Chow test does not show possible structural changes.

According to these results, we could conclude that for this asset complies with its investment policy and its benchmark, despite the high number of dummy variables estimated.

The forecast is shown in the following way:

Dynamic (ex ante) forecasts for RBEST (SE based on error variance only)

Horizon	Forecast	SE	Actual	Error	t-value	-2SE	+2SE
2023(41)	-2.33104	0.57770	-2.67042	0.33938	-0.587	-3.4865	-1.1756

Figure 8. Forecast of the return of Bestinver Futuro, Individual Social Prevision Plan.

The error is less than 5%, so we can conclude that the forecast is adequate. On the other hand, if we present the forecast of the NAV[4] the results are the following:

Horizon	Forecast	Actual	Error	Error%
2023(41)	16.6699847	16.613507	0.05647767	0.33995%

Figure 9. Forecast of the NAV of Bestinver Futuro, Individual Social Prevision Plan.

[4] As previously, we clear the following equation using the goal seek function from Excel:

$$ln\left(\frac{\hat{r}^*_{2023-41}}{17.063134}\right) \cdot 100 = -2.33104 \Rightarrow \boxed{\hat{r}^*_{2023-41} = 16.6699847}$$

Also, it can be concluded that the forecast of the NAV is adequate as the error is less than 5%.

3. CONCLUSIONS.

The results achieved in the section above leads to the conclusion that the investment policies and the benchmarks are being followed by the assets studied, with some reservations in the case of the individual social prevision plan.

However, the reader must be warned as the forecasts estimated must be used only with the purpose of generating an expectation, and the models must be filled with new available data. There must also be caution as the securities markets suffer from volatility, so there may be errors in further forecasts. As it is said, past returns do not imply future returns.

Additionally, other substantive procedures have to be performed, as well as an understanding and testing of the internal control system in order to have a robust conclusion in the audit of net asset value.

ANNEX I. DATA USED.

Week	*NAV Santander*	*Euro Floating rate Index*	r_{SAN}	r_{REUFR}
2022-42	97.823	130.010		
2022-43	97.858	130.070	0.036	0.046
2022-44	97.919	130.140	0.063	0.054
2022-45	97.946	130.160	0.027	0.015
2022-46	97.955	130.200	0.010	0.031
2022-47	97.947	130.180	-0.008	-0.015
2022-48	97.977	130.250	0.031	0.054
2022-49	98.010	130.300	0.033	0.038
2022-50	98.040	130.360	0.031	0.046
2022-51	98.089	130.430	0.049	0.054
2022-52	98.121	130.470	0.032	0.031
2023-1	98.225	130.640	0.106	0.130
2023-2	98.289	130.750	0.066	0.084
2023-3	98.429	130.970	0.142	0.168
2023-4	98.515	131.120	0.087	0.114
2023-5	98.585	131.250	0.071	0.099
2023-6	98.623	131.320	0.038	0.053
2023-7	98.640	131.380	0.018	0.046
2023-8	98.668	131.450	0.028	0.053
2023-9	98.692	131.470	0.024	0.015
2023-10	97.944	130.350	-0.760	-0.856
2023-11	98.709	131.500	0.777	0.878
2023-12	98.837	131.680	0.130	0.137
2023-13	98.876	131.750	0.040	0.053
2023-14	98.943	131.870	0.067	0.091
2023-15	99.026	132.000	0.084	0.099
2023-16	99.082	132.110	0.057	0.083
2023-17	99.129	132.170	0.047	0.045
2023-18	99.182	132.280	0.053	0.083
2023-19	99.253	132.400	0.072	0.091
2023-20	99.325	132.500	0.072	0.076
2023-21	99.402	132.590	0.078	0.068
2023-22	99.478	132.740	0.076	0.113
2023-23	99.594	132.900	0.116	0.120
2023-24	99.634	132.990	0.040	0.068
2023-25	99.709	133.110	0.076	0.090
2023-26	99.780	133.220	0.071	0.083
2023-27	99.848	133.340	0.068	0.090
2023-28	99.924	133.450	0.075	0.082
2023-29	100.011	133.560	0.087	0.082
2023-30	100.069	133.680	0.058	0.090
2023-31	100.134	133.790	0.066	0.082
2023-32	100.202	133.900	0.068	0.082
2023-33	100.283	134.020	0.081	0.090
2023-34	100.353	134.160	0.069	0.104
2023-35	100.444	134.290	0.091	0.097
2023-36	100.514	134.410	0.070	0.089
2023-37	100.566	134.520	0.052	0.082
2023-38	100.645	134.650	0.078	0.097
2023-39	100.706	134.740	0.060	0.067
2023-40	100.780	134.850	0.073	0.082
2023-41	100.857	134.970	0.077	0.089

Week	*NAV Ibercaja*	*S&P 500 Index*	*ER€/$*	r_{IBER}	$r_{S\&P500}$	*ER·€/$*
2022-42	43.176	3,901.060	0.995			
2022-43	42.354	3,770.550	0.987	-1.922	-3.403	-0.797
2022-44	42.925	3,992.930	1.031	1.339	5.730	4.322
2022-45	42.911	3,965.340	1.037	-0.033	-0.693	0.561
2022-46	43.183	4,026.120	1.038	0.633	1.521	0.087
2022-47	42.999	4,071.700	1.054	-0.428	1.126	1.559
2022-48	41.757	3,934.380	1.056	-2.931	-3.431	0.199
2022-49	40.984	3,852.360	1.062	-1.869	-2.107	0.567
2022-50	40.944	3,844.820	1.062	-0.096	-0.196	0.028
2022-51	40.629	3,839.500	1.067	-0.773	-0.138	0.413
2022-52	41.487	3,895.080	1.050	2.090	1.437	-1.569
2023-1	41.551	3,999.090	1.081	0.155	2.635	2.947
2023-2	40.936	3,972.610	1.083	-1.492	-0.664	0.111
2023-3	41.397	4,070.560	1.087	1.119	2.436	0.360
2023-4	42.025	4,136.480	1.094	1.506	1.606	0.660
2023-5	42.022	4,090.460	1.069	-0.007	-1.119	-2.284
2023-6	41.875	4,079.090	1.063	-0.350	-0.278	-0.610
2023-7	41.392	3,970.040	1.057	-1.159	-2.710	-0.519
2023-8	41.661	4,045.640	1.062	0.647	1.886	0.425
2023-9	40.164	3,861.590	1.059	-3.659	-4.656	-0.274
2023-10	40.797	3,916.640	1.062	1.563	1.416	0.349
2023-11	40.994	3,970.990	1.075	0.483	1.378	1.142
2023-12	41.817	4,109.310	1.088	1.988	3.424	1.203
2023-13	41.784	4,105.020	1.092	-0.080	-0.104	0.367
2023-14	41.855	4,137.640	1.106	0.171	0.791	1.293
2023-15	41.860	4,133.520	1.098	0.010	-0.100	-0.717
2023-16	42.039	4,169.480	1.098	0.428	0.866	0.027
2023-17	41.867	4,136.250	1.101	-0.410	-0.800	0.300
2023-18	42.604	4,124.080	1.089	1.745	-0.295	-1.114
2023-19	43.504	4,191.980	1.081	2.090	1.633	-0.774
2023-20	44.064	4,205.450	1.075	1.281	0.321	-0.529
2023-21	44.706	4,282.370	1.076	1.446	1.813	0.112
2023-22	44.476	4,298.860	1.078	-0.516	0.384	0.158
2023-23	44.714	4,409.590	1.097	0.533	2.543	1.711
2023-24	44.373	4,348.330	1.088	-0.766	-1.399	-0.751
2023-25	45.055	4,450.380	1.087	1.526	2.320	-0.166
2023-26	44.170	4,398.950	1.089	-1.985	-1.162	0.202
2023-27	44.353	4,505.420	1.122	0.414	2.392	3.013
2023-28	44.980	4,536.340	1.112	1.405	0.684	-0.877
2023-29	46.165	4,582.230	1.101	2.601	1.007	-1.021
2023-30	45.200	4,478.030	1.095	-2.114	-2.300	-0.583
2023-31	45.378	4,464.050	1.100	0.393	-0.313	0.528
2023-32	44.981	4,369.710	1.087	-0.878	-2.136	-1.253
2023-33	45.685	4,405.710	1.081	1.553	0.820	-0.544
2023-34	46.875	4,515.770	1.084	2.573	2.467	0.333
2023-35	46.681	4,457.490	1.070	-0.415	-1.299	-1.299
2023-36	46.670	4,450.320	1.066	-0.023	-0.161	-0.431
2023-37	45.301	4,320.060	1.065	-2.978	-2.971	-0.103
2023-38	45.120	4,288.050	1.059	-0.400	-0.744	-0.499
2023-39	45.295	4,308.500	1.056	0.387	0.476	-0.293
2023-40	45.728	4,327.780	1.052	0.951	0.446	-0.370
2023-41	44.229	4,224.160	1.059	-3.332	-2.423	0.635

Week	*NAV Bestinver*	*MSCI World Net Index*	*Euro Treasury Index*	*RI*	r_{BEST}	r_{RI}
2022-42	14.481	403.370	157.880	280.625		
2022-43	14.709	397.150	156.270	276.710	1.564	-1.405
2022-44	15.354	406.020	157.240	281.630	4.290	1.762
2022-45	15.342	402.570	158.550	280.560	-0.076	-0.381
2022-46	15.522	407.670	158.540	283.105	1.161	0.903
2022-47	15.712	409.610	159.450	284.530	1.219	0.502
2022-48	15.509	396.630	159.060	277.845	-1.301	-2.378
2022-49	15.102	385.510	156.770	271.140	-2.657	-2.443
2022-50	15.143	385.520	155.490	270.505	0.269	-0.234
2022-51	15.100	383.130	154.640	268.885	-0.282	-0.601
2022-52	15.839	392.710	157.010	274.860	4.773	2.198
2023-1	16.190	397.030	157.710	277.370	2.196	0.909
2023-2	16.115	395.510	157.880	276.695	-0.468	-0.244
2023-3	16.432	403.980	157.310	280.645	1.949	1.417
2023-4	16.765	408.450	158.040	283.245	2.010	0.922
2023-5	16.499	410.070	156.660	283.365	-1.604	0.042
2023-6	16.586	410.170	155.940	283.055	0.529	-0.109
2023-7	16.372	404.140	155.120	279.630	-1.299	-1.217
2023-8	16.623	409.780	153.920	281.850	1.524	0.791
2023-9	16.149	391.730	155.590	273.660	-2.896	-2.949
2023-10	15.645	393.270	158.820	276.045	-3.171	0.868
2023-11	15.740	394.440	158.760	276.600	0.610	0.201
2023-12	16.335	405.450	157.480	281.465	3.709	1.744
2023-13	16.243	403.230	158.490	280.860	-0.564	-0.215
2023-14	16.447	405.390	156.670	281.030	1.247	0.061
2023-15	16.445	406.310	156.270	281.290	-0.012	0.092
2023-16	16.456	406.000	157.640	281.820	0.067	0.188
2023-17	16.476	405.420	158.060	281.740	0.122	-0.028
2023-18	16.517	409.290	158.210	283.750	0.247	0.711
2023-19	16.770	417.300	157.150	287.225	1.520	1.217
2023-20	16.776	418.440	156.310	287.375	0.035	0.052
2023-21	16.866	424.570	158.280	291.425	0.535	1.399
2023-22	16.842	425.220	157.850	291.535	-0.144	0.038
2023-23	16.994	429.930	157.290	293.610	0.899	0.709
2023-24	16.755	422.950	157.670	290.310	-1.414	-1.130
2023-25	17.084	431.340	157.170	294.255	1.945	1.350
2023-26	16.833	423.740	156.020	289.880	-1.481	-1.498
2023-27	17.205	425.980	157.020	291.500	2.185	0.557
2023-28	17.288	432.490	157.550	295.020	0.482	1.200
2023-29	17.815	439.820	157.630	298.725	3.001	1.248
2023-30	17.590	430.140	157.340	293.740	-1.268	-1.683
2023-31	17.353	430.330	157.170	293.750	-1.357	0.003
2023-32	17.177	423.450	157.070	290.260	-1.022	-1.195
2023-33	17.235	429.710	157.530	293.620	0.341	1.151
2023-34	17.584	439.830	157.980	298.905	2.004	1.784
2023-35	17.427	437.710	157.460	297.585	-0.899	-0.443
2023-36	17.618	441.630	156.850	299.240	1.089	0.555
2023-37	17.298	429.930	156.500	293.215	-1.833	-2.034
2023-38	17.238	429.090	156.150	292.620	-0.346	-0.203
2023-39	17.013	429.980	155.850	292.915	-1.316	0.101
2023-40	17.063	434.030	156.650	295.340	0.296	0.824
2023-41	16.614	419.920	156.130	288.025	-2.670	-2.508

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