Homework Assignment (Problem Set) 1:

Note, Problem Set 1 directly focuses on Modules 1 and 2; Introduction to Decision Analysis and Formulation and Solving Linear Programs.

5 questions

Rubric:

All questions worth 20 points
20 Points: Answer and solution are fully correct and detailed professionally.
16-19 Points: Answer and solution are deficient in some manner but mostly correct.
11-15 Points: Answer and solution are missing a key element or two.
1-10 Points: Answer and solution are missing multiple elements are significantly deficient/incomprehensible.
0 Points: No answer provided.

Question 1:

SteelCo manufactures three types of steel at two different steel mills. During a given month, Mill 1 has 200 hours of blast furnace time available, whereas Mill 2 has 300 hours. Because of differences in the furnaces at each mill, the time and cost to produce a ton of steel differs for each mill and are shown in the following table. Each month, SteelCo must manufacture a total of at least 400 tons of Steel 1, 500 tons of Steel 2, and 300 tons of Steel 3 to meet demand; however, the total amount of Steel 2 manufactured should not exceed the combined amount of Steel 1 and Steel 3. Also, in order to maintain a roughly uniform usage of the two mills, management's policy is that the percentage of available blast furnace capacity (time) used at each mill should be the same. That is, the relative rates of usage should be equivalent, not the absolute usage. Clearly formulate a linear program (LP) to minimize the cost of manufacturing the desired steel.

Table 1

Mill	Steel 1		Steel 2		Steel 3	
	Cost ($)	Time (Min)	Cost ($)	Time (Min)	Cost ($)	Time (Min)
Mill 1	10	20	11	22	14	28
Mill 2	12	24	9	18	10	30

During a given month, Mill 1 has 200 hours of blast furnace time available, whereas Mill 2 has 300 hours

S1_M1 + S2_M1 + S3_M1 <= 200*60 min

S1_M2 + S2_M2 + S3_M2 <= 300*60 min

Each month, SteelCo must manufacture a total of at least 400 tons of Steel 1, 500 tons of Steel 2, and 300 tons of Steel 3 to meet demand

S1_M1/(20) + S1_M2/(24) >= 400 tons of s1

S2_M1/(22) + S2_M2/(18) >= 500 tons of s2

S3_M1/(28) + S3_M2/(30) >= 300 tons of s3

the total amount of Steel 2 manufactured should not exceed the combined amount of Steel 1 and Steel 3

[S2_M1/(22) + S2_M2/(18)] <= [S1_M1/(20) + S1_M2/(24)] + [S3_M1/(28) + S3_M2/(30)]

the percentage of available blast furnace capacity (time) used at each mill should be the same

[S1_M1 + S2_M1 + S3_M1] / 200 = [S1_M2 + S2_M2 + S3_M2] / 300

And S1_M1, S2_M1, S3_M1, S1_M2, S2_M2, S3_M2 >= 0

minimize the cost of manufacturing the desired steel

Min (10)*S1_M1/20 + (12)*S1_M2/24 + (11)*S2_M1/22 + (9)*S2_M2/18 + (14)*S3_M1/28 + (10)*S3_M2/30

Solution in python notebook.

```
Pulp Solutions
Time for Steel 1, Mill 1: 8000.0 minutes
Time for Steel 2, Mill 1: 2588.2353 minutes
Time for Steel 3, Mill 1: 0.0 minutes
Time for Steel 1, Mill 2: 0.0 minutes
Time for Steel 2, Mill 2: 6882.3529 minutes
Time for Steel 3, Mill 2: 9000.0 minutes
```

Question 2:

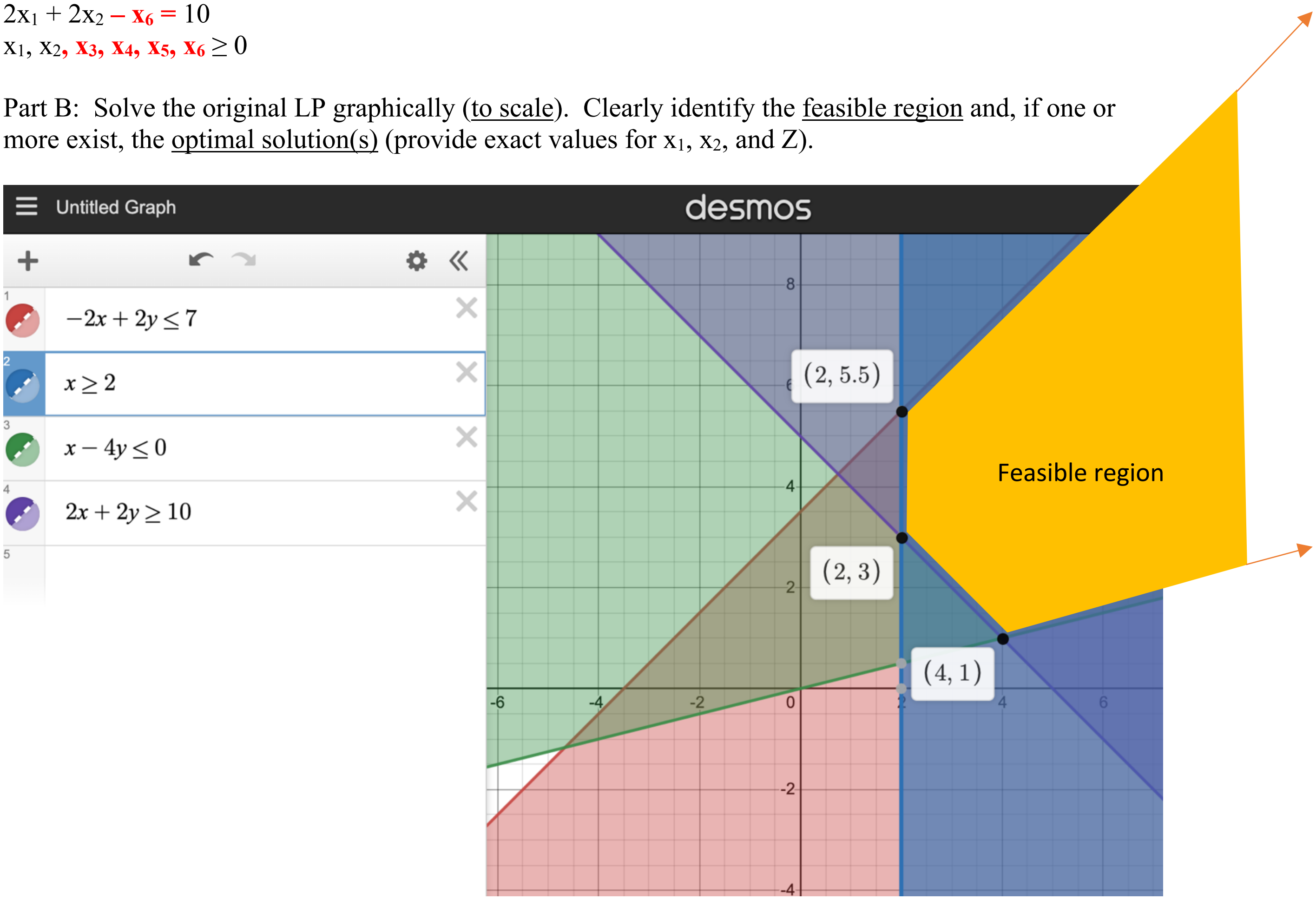
Consider the following linear program:

Max $Z = -4x_1 + 2x_2$
Subject To
$-2x_1 + 2x_2 \leq 7$
$x_1 \geq 2$
$x_1 - 4x_2 \leq 0$
$2x_1 + 2x_2 \geq 10$
$x_1, x_2 \geq 0$

Part A: Write the LP in standard equality form.

Max $Z = -4x_1 + 2x_2$
Subject To
$-2x_1 + 2x_2 + x_3 = 7$
$x_1 - x_4 = 2$
$x_1 - 4x_2 + x_5 = 0$
$2x_1 + 2x_2 - x_6 = 10$
$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

Part B: Solve the original LP graphically (to scale). Clearly identify the feasible region and, if one or more exist, the optimal solution(s) (provide exact values for x_1, x_2, and Z).



$(x_1, x_2, Z) = (2, 5.5, 3)$: Max Z = $-4(2) + 2(5.5) = -8 + 11 = 3$ (optimal solution)
$(x_1, x_2, Z) = (2, 3, -2)$: Max Z = $-4(2) + 2(3) = -8 + 6 = -2$
$(x_1, x_2, Z) = (4, 1, -14)$: Max Z = $-4(4) + 2(1) = -16 + 2 = -14$

Question 3:

At the beginning of month 1, Finco has $400 in cash. At the beginning of months 1, 2, 3, and 4, Finco receives certain revenues, after which it pays bills (see Table 2 below). Any money left over may be invested for one month at the interest rate of 0.1% per month; for two months at 0.5% per month; for three months at 1% per month; or for four months at 2% per month. So the total return for one month is 0.1%, two months is 1%, three months is 3%, and four months is 8% (no compounding). Use linear programming to determine an investment strategy that maximizes cash on hand at the beginning of month 5. Formulate an LP to maximize Finco's profit.

Table 2

Month	Revenues ($)	Bills ($)
1	400	600
2	800	500
3	300	500
4	300	250

Where x_{ij} is the money for month i to be invested for j months.

Constraints

Month 1: $X_{11} + X_{12} + X_{13} + X_{14}$

$= 400 + 400 - 600$

$= 200$

Month 2: $X_{21} + X_{22} + X_{23}$

$= 800 - 500 + 1.001X_{11}$

$= 300 + 1.001X_{11}$

Month 3: $X_{31} + X_{32}$

$= 300 - 500 + 1.01X_{12} + 1.001X_{21}$

$= -200 + 1.01X_{12} + 1.001X_{21}$

Month 4: X_{41}

$= 300 - 250 + 1.03X_{13} + 1.01X_{22} + 1.001X_{31}$

$= -50 + 1.03X_{13} + 1.01X_{22} + 1.001X_{31}$

<u>Month 5</u>:

Max Z = Revenues – Bills + $1.08X_{14} + 1.03X_{23} + 1.01X_{32} + 1.001X_{41}$

Question 4:

Turkeyco produces two types of turkey cutlets for sale to fast-food restaurants. Each type of cutlet consists of white meat and dark meat. Cutlet 1 sells for $4/lb and must consist of at least 70% white meat. Cutlet 2 sells for $3/lb and must consist of at least 60% white meat. At most, 50 lb of cutlet 1 and 30 lb of cutlet 2 can be sold. The two types of turkey used to manufacture the cutlets are purchased from the GobbleGobble Turkey Farm. Each type 1 turkey costs $10 and yields 5 lb of white meat and 2 lb of dark meat. Each type 2 turkey costs $8 and yields 3 lb of white meat and 3 lb of dark meat.

Part A: Formulate an LP to maximize Turkeyco's profit.

Max Profit = sales – costs = $(4(W_1 + D_1) + 3(W_2 + D_2)) - (10T_1 + 8T_2)$

Constraints

Cutlet 1 must consist of at least 70% white meat. Cutlet 2 must consist of at least 60% white meat

$W_1 >= 0.70(W_1 + D_1)$

$W_2 >= 0.60(W_2 + D_2)$

At most, 50 lb of cutlet 1 and 30 lb of cutlet 2 can be sold

$W_1 + D_1 <= 50$

$W_2 + D_2 <= 30$

Each type 1 turkey costs $10 and yields 5 lb of white meat and 2 lb of dark meat. Each type 2 turkey costs $8 and yields 3 lb of white meat and 3 lb of dark meat.

$W_1 + W_2 <= 5T_1 + 3T_2$

$D_1 + D_2 <= 2T_1 + 3T_2$

And W_1 , W_2 , D_1 , D_2 , T_1 , $T_2 >= 0$

Part B: Solve the LP (provide exact values for all variables and the optimal objective function).

Solution in python notebook.

```
Pulp Solutions
White 1: 35.0 pounds
White 2: 18.0 pounds
Dark 1: 15.0 pounds
Dark 2: 12.0 pounds
Turkey 1: 8.6666667 turkeys (round to 9 turkeys)
Turkey 2: 3.2222222 turkeys (round to 3 turkeys)
Sales: 177.55555539999997 dollars
Sales with rounded values: 176.0 dollars
```

Question 5:

A company wants to plan production for the ensuing year to minimize the combined cost of production and inventory costs. In each quarter of the year, demand is anticipated to be 130, 160, 250, and 150 units, respectively. The plant can produce a maximum of 200 units each quarter. The product can be manufactured at a cost of $15 per unit during the first quarter, however the manufacturing cost is expected to rise by $1 per quarter. Excess production can be stored from one quarter to the next at a cost of $1.50 per unit, but the storage facility can hold a maximum of 60 units. How should the production be scheduled so as to minimize the total costs?

Part A: Formulate an LP model to minimize costs.

Quarter / Supply=S (units)	demand=D (units)	cost per unit ($)
Qa / Sa	130	15
Qb / Sb	160	16
Qc / Sc	250	17
Qd / Sd	150	18

Constraints

0 <= Storage=$\sum_{k=1}^{j} Sk - Dk$ <= 60

0 <= (Sa – 130) <= 60

0 <= (Sb – 160) + (Sa – 130) <= 60

0 <= (Sc – 250) + (Sb – 160) + (Sa – 130) <= 60

0 <= (Sd – 150) + (Sc – 250) + (Sb – 160) + (Sa – 130)<= 60

Sa, Sb, Sc, Sd <= 200

Objective Function

Quarter a costs: Sa*15 + (Sa - 130)*1.5

Quarter b costs: Sb*16 + ((Sb – 160) + (Sa – 130))*1.5

Quarter c costs: Sc*17 + ((Sc – 250) + (Sb – 160) + (Sa – 130))*1.5

Quarter d costs: Sd*18 + ((Sd – 150) + (Sc – 250) + (Sb – 160) + (Sa – 130))*1.5

Minimize Quarter a costs + Quarter b costs + Quarter c costs + Quarter d costs

Part B: Solve the LP (provide exact values for all variables and the optimal objective function).

Solution in python notebook.

```
Pulp Solutions
Supply for Quarter A: 140.0 units
Supply for Quarter B: 200.0 units
Supply for Quarter C: 200.0 units
Supply for Quarter D: 150.0 units
Total Cost: 11490.0 dollars
```