Kay Quiballo | MSDS 460 | 04.19.2022

Homework Assignment (Problem Set) 2:

Note, Problem Set 2 directly focuses on Modules 3 and 4: Linear Programming and the Economic Interpretation of the Dual and Sensitivity Analysis, and Network Models.

4 Questions

Rubric:
All questions worth 37.5 points
37.5 Points: Answer and solution are fully correct and detailed professionally.
25-37 Points: Answer and solution are deficient in some manner but mostly correct.
15-24 Points: Answer and solution are missing a key element or two.
1-14 Points: Answer and solution are missing multiple elements are significantly deficient/incomprehensible.
0 Points: No answer provided.

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Question 1:

1A. Write the general dual problem associated with the given LP.
(Do not transform or rewrite the primal problem before writing the general dual)

Maximize $-4x_1 + 2x_2$
Subject To
$4x_1 + x_2 + x_3 = 20$
$2x_1 - x_2 \geq 6$
$x_1 - x_2 + 5x_3 \geq -5$
$-3x_1 + 2x_2 + x_3 \leq 4$
$x_1 \leq 0$, $x_2 \geq 0$, x_3 unrestricted

Short answer (without transforming/rewriting)
Minimize $20p_1 + 6p_2 - 5p_3 + 4p_4$
Subject To
$4p_1 + 2p_2 + 1p_3 - 3p_4 \geq -4$
$p_1 \quad - p_2 \quad - p_3 + 2p_4 \geq 2$
$p_1 \qquad + 5p_3 \ + p_4 \geq 0$
p_1 ~ unrestricted
$p_2, p_3 \leq 0$
$p_4 \geq 0$

1B. Given the following information for a product-mix problem with three products and three resources.

Primal Decision Variables: x_1 = number of unit 1 produced; x_2 = # of unit 2 produced; x_3 = # of unit 3 produced

Primal Formulation:

Max Z (Rev.) =		$25x_1$	$+ 30x_2$	$+ 20x_3$		
Subject To	$8x_1$	$+ 6x_2$	$+ x_3$		≤ 50	(Res. 1 constraint)
	$4x_1$	$+ 2x_2$	$+ 3x_3$		≤ 20	(Res. 2 constraint)
	$2x_1$	$+ x_2$	$+ 2x_3$		≤ 25	(Res. 3 constraint)
		x_1, x_2, x_3			≥ 0	(Nonnegativity)

Dual Formulation:

Min W =		$50\pi_1$	$+ 20\pi_2$	$+25\pi_3$
Subject To	$8\pi_1$	$+ 4\pi_2$	$+2\pi_3 \geq 25$	
	$6\pi_1$	$+ 2\pi_2$	$+\pi_3 \geq 30$	
	π_1	$+ 3\pi_2$	$+2\pi_3 \geq 20$	
	π_1, π_2, π_3		≥ 0	

Optimal Solution:

Optimal Z = Revenue = \$268.75	
$x_1 = 0$ (Number of unit 1)	Dual Var. Optimal Value = 22.5 (Surplus variable in 1st dual constraint)
$x_2 = 8.125$ (Number of unit 2)	Dual Var. Optimal Value = 0 (Surplus variable in 2nd dual constraint)
$x_3 = 1.25$ (Number of unit 3)	Dual Var. Optimal Value = 0 (Surplus variable in 3rd dual constraint)
Resource Constraints:	
Resource 1 = 0 leftover units	Dual Var. Optimal Value = 3.125 = π_1
Resource 2 = 0 leftover units	Dual Var. Optimal Value = 5.625 = π_2
Resource 3 = 14.375 leftover units	Dual Var. Optimal Value = 0 = π_3

1Bi. What is the fair-market price for one unit of Resource 3?
Dual Var. Optimal Value for Resource 3 = \$0

1Bii. What is the meaning of the surplus variable value of 22.5 in the 1st dual constraint with respect to the primal problem?
Our current optimal solution has 0 of unit 1. In order to begin manufacturing unit 1 in the optimal solution, the revenue value must increase by 22.5 (from 25 to 47.5).

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Question 2:

Carco manufactures cars and trucks. Each car contributes $300 to profit and each truck, $400; these profits do not consider machine rental. The resources required to manufacture a car and a truck are shown below. Each day Carco can rent up to 98 Type 1 machines at a cost of $50 per machine. The company now has 73 Type 2 machines and 260 tons of steel available. Marketing considerations dictate that at least 88 cars and at least 26 trucks be produced.

Table:

Vehicle Type	Days on Machine 1	Days on Machine 2	Tons of Steel
Car	0.8	0.6	2
Truck	1	0.7	3

Part A: Formulate the problem as a Linear Program.

x_1 = number of cars produced daily (>=0)
x_2 = number of trucks produced daily (>=0)
m_1 = Type 1 machines rented daily (>=0)

Max $300x_1 + 400x_2 – 50m_1$ (car $300 profit, truck $400 profit, machine $50 cost)
Subject to
$0.8x_1 + x_2 \leq m_1$ (m_1 type 1 machines available)
$m_1 \leq 98$ (can rent up to 98 machines per day)
$0.6x_1 + 0.7x_2 \leq 73$ (73 type 2 machines available)
$2x_1 + 3x_2 \leq 260$ (260 tons of steel available)
$x_1 \geq 88$ (make at least 88 cars)
$x_2 \geq 26$ (make at least 26 trucks)

Part B: Solve the LP (provide exact values for all variables and the optimal objective function).
Hint: The optimal objective function value is $32540
[Note, I am providing this hint because having the optimal solution is necessary to do Part C.]

Cars: 88.0
Trucks: 27.6
Machine 1: 98.0
Profit: $ 32540.0 #code in python and sensitivity analysis in excel

Kay Quiballo | MSDS 460 | 04.19.2022

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B2	optimal soln: cars	88	0	300	20	1E+30
C2	optimal soln: trucks	27.6	0	400	1E+30	25
D2	optimal soln: machine 1	98	0	-50	1E+30	350

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E10	cars min	88	-20	88	2	3
E11	trucks min	27.6	0	26	1.6	1E+30
E6	m1 machines	0	400	0	0.4	1.6
E7	m1 available	98	350	98	0.4	1.6
E8	m2 machines	72.12	0	73	1E+30	0.88
E9	steel available	258.8	0	260	1E+30	1.2

Part C: Answer the following questions from your output. *(Note, do not simply rerun the model – use the Linear Programming output and Sensitivity Analysis to explain your answers.)*

i) If cars contributed $310 to profit, what would be the new optimal solution to the problem?

Car price increase $10 from 300 to 310. Since the allowable increase is 20 (>10), the optimal solution does not change. (same solution as part B)

ii) What is the most that Carco should be willing to pay to rent an additional Type 1 machine for 1 day?

The shadow price of renting m1 is $350.

iii) What is the most that Carco should be willing to pay for an extra ton of steel?

The shadow price of steel is $0. Carco would not be willing to pay for extra steel at any price.

iv) If Carco were required to produce at least 86 cars, what would Carco's profit become?

Because the shadow price of cars is -20, Carco's profit will become 32540 + (86-88)(-20) = $32,580

v) Carco is considering producing jeeps. A jeep contributes $600 to profit and requires 1.2 days on machine 1, 2 days on machine 2, and 4 tons of steel. Should Carco produce any jeeps?

When rerunning the model with jeeps, the optimal solution changes to include jeeps, and lowers the quantity of trucks produced with a profit of $32,631. Carco only needs to to make 26 trucks, so making less of them (currently 27.6) for a higher profit margins in the jeep region is feasible.

Kay Quiballo | MSDS 460 | 04.19.2022
Question 3:

A catering company must have the following number of clean napkins available at the beginning of each of the next four days: day 1: 15, day 2: 12, day 3: 18, and day 4: 6. After being used, a napkin can be cleaned by one of two methods: fast service or slow service. Fast service costs \$0.10 per napkin, and a napkin cleaned via fast service is available for use the day after it is last used. Slow service costs \$0.06 per napkin, and a napkin cleaned via slow service is available two days after they were last used. New napkins can be purchased for a cost of \$0.20 per napkin.

Part A: Formulate the problem as a minimum cost transportation problem.

Minimize $20(x1+x2+x3+x4) + 10(y1+y2+y3) + 6(z1+z2)$

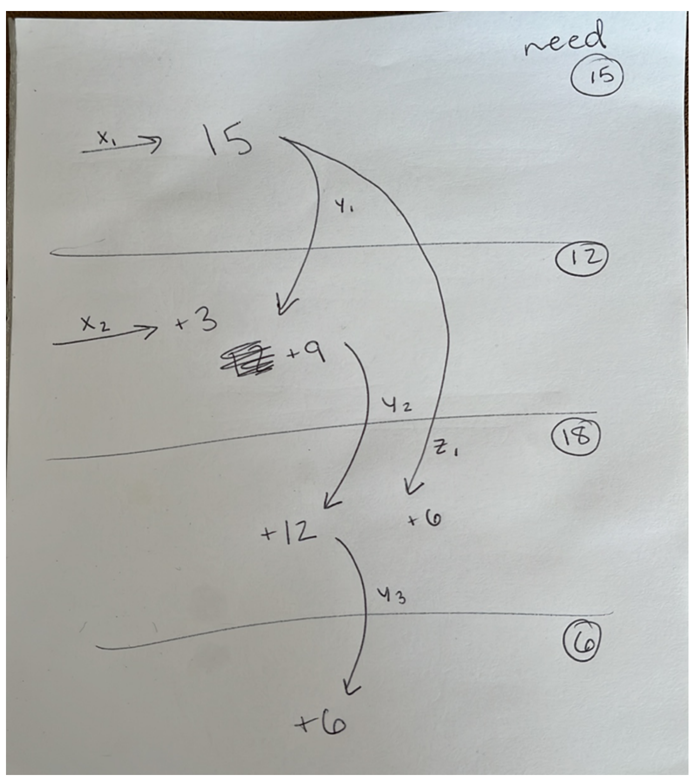
$x1-x4$ = napkins purchased on day 1-4 ($>=0$)
$y1-y3$ = napkins quick serviced on day 1-3 ($>=0$)
$z1-z2$ = napkins slow serviced on day 1-2 ($>=0$)

minimum required napkins per day
$x1 >= 15$
$x2 + y1 >= 12$
$x3 + y2 + z1 >= 18$
$x4 + y3 + z2 >= 6$

maximum reuse of napkins
$y1 + z1 <= x1$
$y2 + z2 <= x2 + y1$
$y3 <= x3 + y2 + z1$

Part B: Solve the problem (provide exact values for all variables and the optimal objective function).

```
Pulp Solutions
x1: 15.0
x2: 3.0
x3: 0.0
x4: 0.0
y1: 9.0
y2: 12.0
y3: 6.0
z1: 6.0
z2: 0.0
Cost: $ 6.66 #code in python
```



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Question 4:

A university has three professors who each teach four courses per year. Each year, four sections of marketing, finance, and production must be offered. At least one section of each class must be offered during each semester (fall and spring). Each professor's time preferences and preference for teaching various courses are given below.

	Professor 1	Professor 2	Professor 3
Fall Preference	3	5	4
Spring Preference	4	3	4
Marketing	6	4	5
Finance	5	6	4
Production	4	5	6

The total satisfaction a professor earns teaching a class is the sum of the semester satisfaction and the course satisfaction. Thus, professor 1 derives a satisfaction of 3 + 6 = 9 from teaching marketing during the fall semester.

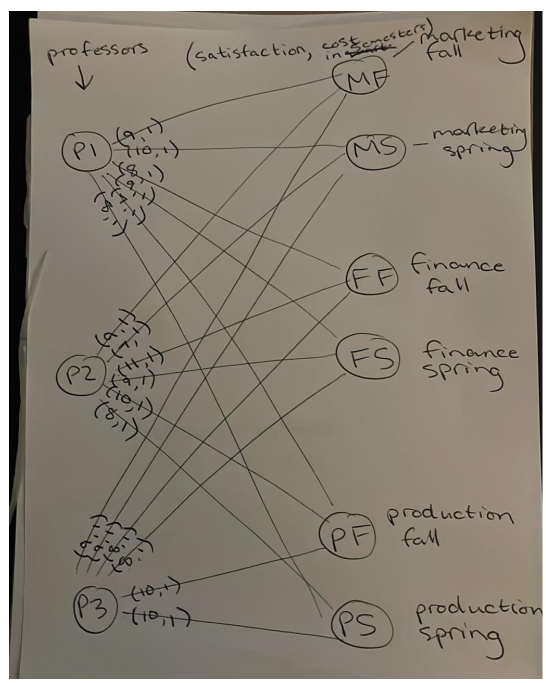
Part A: Formulate the problem as a minimum cost network flow problem that can be used to assign professors to courses so as to maximize the total satisfaction of the three professors. Draw the network and identify the nodes and arcs.

variable = [professor_semester_course] | e.g p1_f_m = professor1_fall_marketing (1 if teaching, 0 if not)

Maximize [satisfaction]
p1_f_m*9 + p1_f_f*8 + p1_f_p*7 + p1_s_m*10 + p1_s_f*9 + p1_s_p*8
+ p2_f_m*9 + p2_f_f*11 + p2_f_p*10 + p2_s_m*7 + p2_s_f*9 + p2_s_p*8
+ p3_f_m*9 + p3_f_f*8 + p3_f_p*10 + p3_s_m*9 + p3_s_f*8 + p3_s_p*10

teaches 4 classes
p1_f_m + p1_f_f + p1_f_p + p1_s_m + p1_s_f + p1_s_p = 4
p2_f_m + p2_f_f + p2_f_p + p2_s_m + p2_s_f + p2_s_p = 4
p3_f_m + p3_f_f + p3_f_p + p3_s_m + p3_s_f + p3_s_p = 4

at least one section
p1_f_m + p2_f_m + p3_f_m >= 1
p1_f_f + p2_f_f + p3_f_f >= 1
p1_f_p + p2_f_p + p3_f_p >= 1
p1_s_m + p2_s_m + p3_s_m >= 1
p1_s_f + p2_s_f + p3_s_f >= 1
p1_s_p + p2_s_p + p3_s_p >= 1



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Part B: Solve the problem (provide exact values for all variables and the optimal objective function).

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Pulp Solutions

Professor 1, Fall, Marketing: 1.0
Professor 1, Fall, Finance: 0.0
Professor 1, Fall, Production: 0.0
Professor 1, Spring, Marketing: 1.0
Professor 1, Spring, Finance: 1.0
Professor 1, Spring, Production: 1.0
Professor 2, Fall, Marketing: 1.0
Professor 2, Fall, Finance: 1.0
Professor 2, Fall, Production: 1.0
Professor 2, Spring, Marketing: 0.0
Professor 2, Spring, Finance: 1.0
Professor 2, Spring, Production: 0.0
Professor 3, Fall, Marketing: 1.0
Professor 3, Fall, Finance: 0.0
Professor 3, Fall, Production: 1.0
Professor 3, Spring, Marketing: 1.0
Professor 3, Spring, Finance: 0.0
Professor 3, Spring, Production: 1.0

Satisfaction, Professor 1: 36.0
Satisfaction, Professor 2: 39.0
Satisfaction, Professor 3: 38.0
Total Satisfaction: 113.0

Prof. 1 teaches Marketing in Fall, and Marketing & Finance & Production in Spring.
Prof. 2 teaches Marketing & Finance & Production in Fall, and Finance in Spring.
Prof. 3 teaches Marketing & Production in Fall, and Marketing & Production in Spring.

# solutions in python
```

	P1	P2	P3
Fall	M	M,F,P	M,P
Spring	M,F,P	F	M,P

	P1	P2	P3
Marketing	F,S	F	F,S
Finance	S	F,S	
Production	S	F	F,S

	Marketing	Finance	Production
Fall	P1,P2,P3	P2	P2,P3
Spring	P1,P3	P1,P2	P1,P3