Kay Quiballo | MSDS 460 | 04.25.2022

Homework Assignment (Problem Set) 3:

Note, Problem Set 3 directly focuses on Modules 5 and 6: Integer Programs, Nonlinear and Multiobjective Programming.

5 questions

Rubric:

All questions worth 30 points
30 Points: Answer and solution are fully correct and detailed professionally.
26-29 Points: Answer and solution are deficient in some manner but mostly correct.
21-25 Points: Answer and solution are missing a key element or two.
1-20 Points: Answer and solution are missing multiple elements are significantly deficient/incomprehensible.
0 Points: No answer provided.

1. An engineer at Fertilizer Company has synthesized a sensational new fertilizer made of just two interchangeable basic raw materials. The company wants to take advantage of this opportunity and produce as much as possible of the new fertilizer. The company currently has $40,000 to buy raw materials at a unit price of $8000 and $5000 per unit, respectively. When amounts x_1 and x_2 of the basic raw materials are combined, a quantity q of fertilizer results given by: $q = 4x_1 + 2x_2 - 0.5x_1^2 - 0.25x_2^2$

Part A: Formulate as a constrained nonlinear program. Clearly indicate the variables, objective function, and constraints.

x_1 = units of basic raw material #1
x_2 = units of basic raw material #2

Max $q = 4*x_1 + 2*x_2 - 0.5*x_1^2 - 0.25*x_2^2$ (produce as much as possible of the new fertilizer)
Subject to
$8000*x_1 + 5000*x_2 <= 40{,}000$ (cannot exceed purchasing power)
$x_1, x_2 >= 0$ (non-negative)

Part B: Solve the Program (provide exact values for all variables and the optimal objective function).

x1 = 3.15789473684211
x2 = 2.94736842105263
q = 11.3684210526316

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2. The area of a triangle with sides of length a, b, and c is $\sqrt{s(s-a)(s-b)(s-c)}$, where s is half the perimeter of the triangle. We have 60 feet of fence and want to fence a triangular-shaped area.

Part A: Formulate the problem as a constrained nonlinear program that will enable us to maximize the area of the fenced area, with constraints. Clearly indicate the variables, objective function, and constraints.

Hint: *The length of a side of a triangle must be less than or equal to the sum of the lengths of the other two sides.*

a = length of side #1 of a triangle
b = length of side #2 of a triangle
c = length of side #3 of a triangle

Max [(a+b+c)/2] * sqrt([[(a+b+c)/2]-a) * ([[(a+b+c)/2]-b) * ([[(a+b+c)/2]-c) (area)
Subject to
a + b + c <= 60 (60 feet of fence)
a <= b + c (length <= sum of other 2)
b <= a + c (length <= sum of other 2)
c <= a + b (length <= sum of other 2)
a, b, c >= 0

Part B: Solve the Program (provide exact values for all variables and the optimal objective function).

a = 20.00003836 ~ 20 ft
b = 19.99987143 ~ 20 ft
c = 20.0000905 ~ 20 ft
area = 173.2050824 ~ 173.21 sqft

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3. The Tiny Toy Company makes three types of new toys: the tiny tank, the tiny truck, and the tiny turtle. Plastic used in one unit of each is 1.5, 2.0 and 1.0 pounds, respectively. Rubber for one unit of each toy is 0.5, 0.5, and 1.0 pounds, respectively. Also, each tank uses 0.3 pounds of metal and the truck uses 0.6 pounds of metal during production. The average weekly availability for plastic is 16,000 pounds, 9,000 pounds of metal, and 5,000 pounds of rubber. It takes two hours of labor to make one tank, two hours for one truck, and one hour for a turtle. The company allows no more than 40 hours a week for production (priority #1). Finally, the cost of manufacturing one tank is \$7, 1 truck is \$5 and 1 turtle is \$4; a target budget of \$164,000 is initially used as a guideline for the company to follow.

a) Minimize over-utilization of the weekly available supply of materials used in making the toys and place twice as much emphasis on the plastic (priority #2)

b) Minimize the under and over-utilization of the budget. Maximize available labor hour usage (priority #3).

Formulate the above decision problem as a single linear goal program. Clearly identify your achievement vector (i.e., hierarchy of priority levels for the goals). Do not solve.

Note:
Max objective → minimize n
Min objective → minimize p

a = tanks
b = trucks
c = turtle

Convert to Goals

plastic: $1.5a + 2.0b + 1.0c \sim 16{,}000$	(priority #2 – minimize over-utilization x2 emphasis)
rubber: $0.5a + 0.5b + 1.0c \sim 5{,}000$	(priority #2 – minimize over-utilization)
metal: $0.3a + 0.6b + 0.0c \sim 9{,}000$	(priority #2 – minimize over-utilization)
labor: $2.0a + 2.0b + 1.0c <= 40$	(priority #1 – maximize labor | priority #3 maximize labor?)
budget: $7.0a + 5.0b + 4.0c \sim 164{,}000$	(priority #3 – minimize under/over-utilization)

Goal Programming Form

plastic: $1.5a + 2.0b + 1.0c + n_1 - p_1 = 16{,}000$
rubber: $0.5a + 0.5b + 1.0c + n_2 - p_2 = 5{,}000$
metal: $0.3a + 0.6b + 0.0c + n_3 - p_3 = 9{,}000$
labor: $2.0a + 2.0b + 1.0c + n_4 - p_4 = 40$
budget: $7.0a + 5.0b + 4.0c + n_5 - p_5 = 164{,}000$
$n_i, p_i >= 0$ for all i

Achievement Vector (based on priorities)

$$lexmin \begin{pmatrix} n4 \\ 2p1 + p2 + p3 \\ n5 + p5 - p4 \end{pmatrix}$$

4. (source) XYZ Company is planning an advertising campaign for its new product. The media considered are television and radio. Rated exposures per thousand dollars of advertising expenditure are 10,000 for TV and 7,500 for radio. Management has agreed that the campaign cannot be judged successful if total exposures are under 750,000. The campaign would be viewed as superbly successful if 1 million exposures occurred. In addition, the company has realized that the two most important audiences for its product are persons 18 to 21 years of age and persons 25 to 30 years of age. The following table estimates the number of individuals in the two age groups expected to be exposed to advertisements per $ 1,000 of expenditures:

Exposures per $1000

Age	Television	Radio
18-21	2,500	3,000
25-30	3,000	1,500
(Total)	10,000	7,500

Management has rank ordered five goals it wishes to achieve, arranged from highest to lowest priorities.

a) Achieve total exposures of at least 750,000 persons.
b) Avoid expenditures of more than $100,000.
c) Avoid expenditures of more than $70,000 for television advertisements.
d) Achieve at least 1 million total exposures.
e) Reach at least 250,000 persons in each of the two age groups, 18-21 and 25-30 years. In addition, management realizes and wishes to account for the fact that the purchasing power of the 25-30 age group is twice that of the 18-21 age group.

Formulate the above decision problem as a single linear goal program. Clearly identify your achievement vector (i.e., hierarchy of priority levels for the goals). Do not solve.

t = television expenditures (in thousands)
r = radio expenditures (in thousands)

Convert to Goals

a) $10{,}000t + 7{,}500r >= 750{,}000$	(exposures at least 750k)
b) $t + r <= 100$	(expenditures less than 100k)
c) $t <= 70$	(tv expenditures less than 70k)
d) $10{,}000t + 7{,}500r >= 1{,}000{,}000$	(exposures at least 1mil)
e1) $2{,}500t + 3{,}000r >= 250{,}000$	(18-21 exposures at least 250k)
e2) $3{,}000t + 1{,}500r >= 250{,}000$	(25-30 exposures at least 250k)
e3) [purchasing power]	

Goal Programming Form

a) $10{,}000t + 7{,}500r + n_1 - p_1 = 750{,}000$
b) $t + r + n_2 - p_2 = 100$
c) $t + n_3 - p_3 = 70$
d) $10{,}000t + 7{,}500r + n_4 - p_4 = 1{,}000{,}000$
e1) $2{,}500t + 3{,}000r + n_5 - p_5 = 250{,}000$
e2) $3{,}000t + 1{,}500r + n_6 - p_6 = 250{,}000$
$n_i, p_i >= 0$ for all i

Achievement Vector (based on priorities)

lexmin $(-p_1, n_2, n_3, -p_4, -p_5 - 2p_6)$

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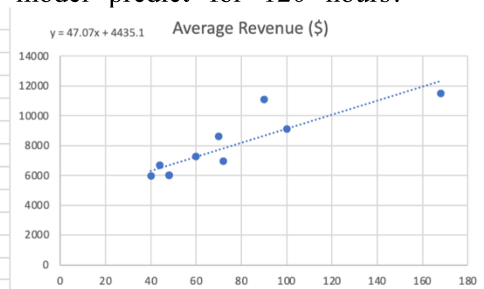
5. A large food chain owns a number of pharmacies that operate in a variety of settings. Some are situated in small towns and are open for only 8 hours a day, 5 days per week. Others are located in shopping malls and are open for longer hours. The analysts on the corporate staff would like to develop a model to show how a store's revenues depend on the number of hours that it is open. They have collected the following information from a sample of stores.

Hours of Operation	Average Revenue ($)
40	5958
44	6662
48	6004
48	6011
60	7250
70	8632
72	6964
90	11097
100	9107
168	11498

a) Use a linear function (e.g., y = ax + b; where a and b are parameters to optimize) to represent the relationship between revenue and operating hours and find the values of the parameters using the nonlinear solver that provide the **best fit** to the given data. What revenue does your model predict for 120 hours?

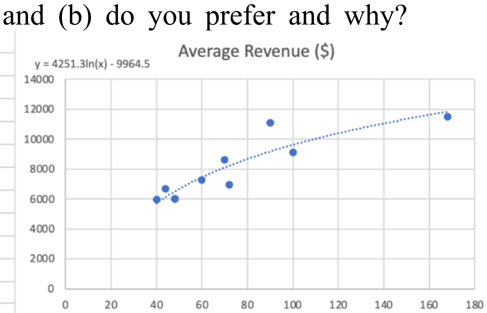
Hours of Operation	Average Revenue ($)	predicted	sq diff		Y=ax+b
40	5958	6317.903173	129530.294	A	47.0704943
44	6662	6506.18515	24278.2674	B	4435.0834
48	6004	6694.467127	476744.854	f(120)	10083.5427
48	6011	6694.467127	467127.3142		
60	7250	7259.313059	86.73306237		
70	8632	7730.018001	813571.5257		
72	6964	7824.15899	739873.4881		
90	11097	8671.427887	5883400.075		
100	9107	9142.13283	1234.315731		
168	11498	12342.92644	713900.6902		
			9249747.558		



y = ax+b (used nonlinear solver to minimize sum of squared residuals by changing A and B)
a = 47.07
b = 4435.08
revenue(120) = 10083.54

b) Suggest a two-parameter nonlinear model (e.g., $y = ax^b$; where a and b are parameters to optimize) for the same relationship and find the parameters using the Nonlinear Solver that provide the **best fit**. What revenue does your model predict for 120 hours? Which if the models in (a) and (b) do you prefer and why?

Hours of Operation	Average Revenue ($)	predicted	sq diff		Y=a*ln(x)+b
40	5958	5718.181359	57512.98042	A	4251.33399
44	6662	6123.376766	290114.9878	B	-9964.4773
48	6004	6493.291191	239405.8695	f(120)	10388.7491
48	6011	6493.291191	232604.7928		
60	7250	7441.948955	36844.40145		
70	8632	8097.29498	285909.4583		
72	6964	8217.058787	1570156.323		
90	11097	9165.716551	3729855.759		
100	9107	9613.639293	256683.373		
168	11498	11819.20498	103172.6396		
			6802260.585		



y = a*ln(x) + b (used nonlinear solver to minimize sum of squared residuals by changing A and B)
a = 4251.33
b = -9964.48
revenue(120) = 10388.75
I prefer the second model because it has a smaller sum of squared residuals.

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The point of this problem is NOT to simply build a regression model. Use nonlinear programming to determine the parameters of a regression model using an objective function and constraints. Do not use a regression tool in Excel, R or python to find this solution. Your solutions for (a) and (b) should contain a detailed spreadsheet model (where the decision variables, parameters, objective function and constraints are identified and explained), as well as answers to the questions posed. You may use Microsoft Excel, Python, or R to solve.