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Effective classification of planar shapes based on curve segment properties

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Abstract

In this paper, a pattern classifier for recognition of planar contours is developed based on the curve bend function (CBF). The CBF makes use of both the curve bend angle which measures the bend degree of a curve segment of a contour and the type coefficient which indicates whether the curve segment is convex or concave, i.e., the related corner on the contour is an inner or outer angle. The information of a contour obtained by its CBF is sufficient to represent its main features. The classifier is designed by using the properties of the CBF directly, and its training process is simpler than other kinds of classifiers, such as neural networks. Our experimental results demonstrate that the classifier is robust for planar shape classification. © 1997 Published by Elsevier Science B.V.

Keywords: Corner point; Curve bend function; Planar shape classification

1. Introduction

Shape or contour representation and recognition is one of the central problems in computer vision. The main information of a planar shape is concentrated at corner (or *critical*) points which have high curvature. Many methods based on searching for critical points have been developed in the past for contour representation (methods, such as Fourier descriptors, which do not locate corner points along a contour, are not considered here). The methods can be classified into three classes: the first class includes methods which search for corner points using some significant measure other than curvature, such as the methods in (Rosenberg, 1972, 1974; Fischler and Bolles, 1986; Ogawa, 1989; Arcelli and Ramella, 1993). These methods can locate the position of a critical point, but they cannot evaluate the magnitude of the angle.

The second class includes methods in which the curvature is evaluated after transforming the contour to the Gaussian scale space (Mokhtarian and Mackworth, 1992; Rattarangsi and Chin, 1992; Ueda and Suzuki, 1993). One advantage of these methods is that they are suitable for extracting features from a noisy curve. However, a long computing time is needed to construct the Gaussian scale space. These methods also cannot estimate the magnitude of an angle.

The third class comprises methods in which a curvature is employed directly to search for critical points in the original picture space (Rosenfeld and Johnston, 1973; Rosenfeld and Weszka, 1975; Davis, 1977; Teh and Chin, 1989; Zhu and Chirlian, 1995). These methods locate critical points on a contour. Some of them can also evaluate the magnitude of an angle (Rosen-

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feld and Johnston, 1973; Rosenfeld and Weszka, 1975; Teh and Chin, 1989). However, they are not able to identify directly whether an angle on a contour is an inner or outer one. This is because they cannot distinguish angles α and $(360° - \alpha)$ if they both exist on a contour. There is one parameter that needs to be assigned initially in Rosenfeld and Johnston's, and Rosenfeld and Weszka's methods. There is no such requirement for Teh and Chin's method, but, it does not always work correctly (Fu and Yan, to appear). It is well known that information which includes only critical points and sizes of angles of a contour is not enough to determine the contour in general. In terms of human perception, a complete description of a contour should comprise the following four aspects:

1. critical point where the contour has a local maximum significant measure (curvature or an alternative quantity which is proportional to the curvature, such as the chord height),
2. magnitude of angles,
3. property of angles (i.e., inner or outer),
4. property of edges of angles (convex or concave).

Based on the combination of a quantity called *the curve bend angle* and a quantity known as *the type coefficient of a curve segment*, the curve bending function (CBF) method has been presented (Fu and Yan, to appear), which solves the first three problems and partly solves the last problem. The information obtained by the CBF includes the key features of a contour. In this paper, a simpler classifier of contours is designed based on the information of contours obtained by the CBF. A brief review of the CBF is given in Section 2. In Section 3, the new classifier is presented. The experimental results and conclusions are given in Sections 4 and 5, respectively.

2. Planar shape description

2.1. The representation of planar shape by CBF

A contour (planar shape) is the boundary of an object represented by the binary image

$$I(x, y) = \begin{cases} 1 & \text{if } (x, y) \in \text{object}, \\ 0 & \text{otherwise}. \end{cases} \tag{1}$$

The points along a contour can be traced and labeled anticlockwise from an arbitrary starting point, and represented by an array: $\Omega = \{S_k;\ k = 0, 1, \dots, N-1\}$, where N is the total number of points (the size of the contour). Hereafter, Ω denotes a contour and S_0 and S_{N-1} represent the starting and ending points, respectively. Since a contour of an object is a closed curve, we can consider that $S_{N+k} = S_k$, $S_{-k} = S_{N-k}$, $k = 0, \dots, N-1$. Thus, the contour Ω is treated as a periodic function with variable k and period N.

To facilitate the discussion, we begin with several definitions. A portion of a contour, composed of $2J + 1$ points $S_{i-J}, \dots, S_i, \dots, S_{i+J}$ is denoted by $\Omega_i(J)$, where $J = \alpha N$ $(0 < \alpha < 1)$ is a positive integer called the *supported length* or *step length* and α is known as the *supported rate*. Let ρ_{ik} denote the distance from S_k to the line segment $S_{i-J}S_{i+J}$, where $i - J \leqslant k \leqslant i + J$, and let C_i denote the centroid of $S_{i-J}S_{i+J}$ with coordinate (c_{ix}, c_{iy}).

Definition 1. The angle $\beta_i^J = \angle S_{i-J}S_iC_i$ is known as a *curve bend angle* (CBA) at point S_i of Ω.

Definition 2. If there is an integer $l > 0$ such that $i - J \leqslant k - l \leqslant k + l \leqslant i + J$ and $\rho_{ik} > \rho_{ij}$ for $k - l \leqslant j \leqslant k + l$ and $j \neq k$, then S_k is known as a *local critical point* of Ω.

Definition 3. If there is no more than one local critical point on $\Omega_i(J)$, then $\Omega_i(J)$ is known as a *simple curve segment* (SCS). For a given α if each $\Omega_i(J)$, $i = 0, \dots, N-1$, is an SCS, the contour Ω is referred to as a *compound simple curve* (CSC) with the supported length J.

Definition 4. The *type coefficient* of $\Omega_i(J)$ is defined as

$$r_i = 2I(c_{ix}, c_{iy}) - 1. \tag{2}$$

By combining the curve bend angle and type coefficient, a function known as the *Curve Bend Function* (CBF) can be defined and its properties are sufficient to provide the main features of a contour. Suppose Ω is a CSC with the supported length J, then the CBF related to the contour Ω is defined as

$$\chi(S_i) = r_i \cos(\beta_i^J), \quad S_i \in \Omega, \tag{3}$$

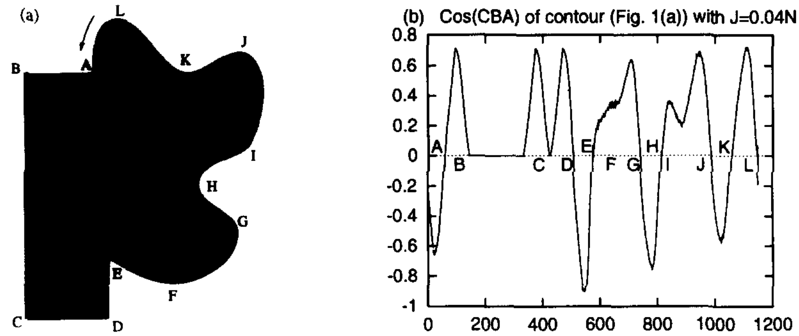

Fig. 1. (a) The contour used in our experiment. (b) The graph of the CBF of the contour.

where r_i is given by Eq. (2),

$$\cos(\beta_i^J) = \rho(S_i, C_i)/\rho(S_{i-J}, S_i),$$

and $\rho(x, y)$ represents the distance between points x and y. The critical points on a contour correspond to the peaks of the CBF's graph of the contour. The procedures for locating critical points of a contour based on its CBF are summarized below.

1. *Select supported rate α and get step length $J = \alpha N$* (in general, $\alpha = 0.01$–0.05).
2. *Extend the domain of the contour:*

$$S_{N+i} = S_i, \quad i = 0, 1, \ldots, N-1 \tag{4}$$

and

$$S_{-i} = S_{N-i}, \quad i = 0, 1, \ldots, N-1. \tag{5}$$

3. *Determine $\chi(S_i)$ by computing r_i and $\cos(\beta_i^J)$.*
4. *Determine critical points based on $\chi(S_i)$:* point $S_i \in \Omega$ is considered a critical point if there is a $0 < L < J$ such that

$$|\chi(S_i)| \geqslant |\chi(S_{i\pm l})|, \quad l = 1, \ldots, L/2, \tag{6}$$

and

$$|\chi(S_i)| > |\chi(S_{i\pm l})| > \theta, \quad l = L/2+1, \ldots, L, \tag{7}$$

where θ is a threshold used to remove angles close to 180° since these angles are very sensitive to noise. The parameter L is selected to be larger than 20. Due to the error of calculation, it is possible that there are two or more equal maximum values of $|\chi(S_i)|$ around a critical point, so the relational operator "$\geqslant$" is used in Eq. (6).

5. *Suppress redundant points.* Due to condition (6), it is possible that there are some redundant points found as critical points obtained in Step 4. The reason is that there may be more than one critical point in the curve segment $\Omega_i(L) = \{S_{i-L}, \ldots, S_i, \ldots, S_{i+L}\}$ and $\Omega_i(L) \subset \Omega_i(J)$, while $\Omega_i(J)$ is an SCS in which there is no more than one critical point, as is $\Omega_i(L)$. So all critical points in $\Omega_i(L)$ except the one in the middle must be suppressed. After the above procedure a set of critical points of Ω can be obtained and we denote them as $S_{i_1}, \ldots, S_{i_K}$.
6. *Compute the magnitude of an angle.* If S_{i_k} is a vertex, then the size of the angle is $2\beta_{i_k}^J$, where

$$\beta_{i_k}^J = \cos^{-1}(|\chi(S_{i_k})|). \tag{8}$$

The angle is an inner one if $\chi(S_{i_k}) > 0$, a straight one if $\chi(S_{i_k}) = 0$ and an outer one if $\chi(S_{i_k}) < 0$. Note that an angle on a contour is twice that of the related CBA.

7. *Identify the property of $\Omega_{i_k}(J)$.* $\Omega_{i_k}(J)$ is convex if $r_{i_k} = 1$, otherwise it is concave.

2.2. Example

Experiments on different types of contours were also carried out to verify the effectiveness of the CBF method. One example is shown below. The contour shown in Fig. 1(a) is used to perform the experiment with the supported rate $\alpha = 0.04$, and its CBF is illustrated in Fig. 1(b). The critical points of the contour obtained by the CBF are shown in Fig. 2 denoted by small squares. The angles (or corners) of the contour, and their magnitudes are given in Table 1. The notation "X" in the table means that the angle does not

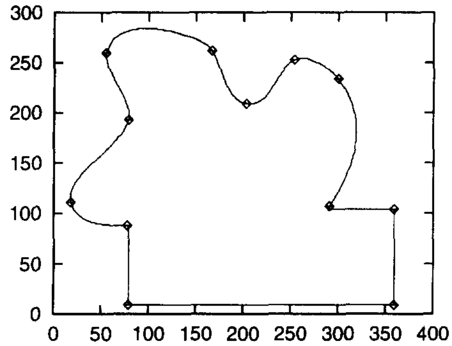

Fig. 2. Critical points of the contour (Fig. 1(a)) found by the CBF with $J = 0.04N$.

Table 1
The angles of the contour in Fig. 1(a)

Actual sizes	CBF-method		
	$J = 0.02N$	$J = 0.04N$	$J = 0.05N$
A corner	−94.98°	−98.92°	−104.71°
$\angle B = 90°$	90°	90°	90°
$\angle C = 90°$	90°	90°	90°
$\angle D = 90°$	90°	90°	90°
E corner	−43.05°	−50.74°	−54.82°
F corner	153.53°	X	X
G corner	118.74°	100.04°	96.08°
H corner	−114.51°	−82.77°	−80.53°
I corner	141.98°	137.25°	136.44°
J corner	123.14°	93.95°	85.63°
K corner	−128.14°	−111.13°	−109.45°
L corner	114.30°	87.09°	79.92°

exist in the case. We can see that the critical points exactly coincide with the actual vertexes of the angles (or corners) of the contour. The magnitudes of angles on the contour are also evaluated accurately. Thus, a contour Ω is traced effectively by a curve-polygon (CPG) whose vertexes are the critical points found by the CBF method.

3. Contour classifier based on the CBF

As discussed in Section 2, a CPG can be used to approximate a contour with high accuracy if the supported rate α is suitably selected, i.e., the supported length $J = \alpha N$ is used such that each $\Omega_i(J) = \{S_{i-J}, \dots, S_{i+J}\}$, $i = 0, \dots, N-1$, is an SCS. Thus, the problem of contour recognition is reduced to classification of the CPGs. A CPG can be represented by a K-dimensional vector called CPG-vector defined as

$$V(\alpha, \theta, K) = (V_1 \ V_2 \ \dots \ V_K)^{\mathrm{T}}, \qquad (9)$$

where α is the supported rate, θ is the threshold of angle, K is the total number of vertices of the CPG, and T indicates the transpose. The component V_k is a 2-dimensional vector given by

$$V_k = (v_k(1) \ v_k(2))^{\mathrm{T}}, \quad k = 1, \dots, K, \qquad (10)$$

where

1. $v_k(1) = |\chi(S_{i_k}|$, here S_{i_k} is the kth vertex of the CPG,
2. $v_k(2) = r_{i_k}$, the type coefficient of Ω_{i_k}, here $\Omega_{i_k}(J) = \{S_{i_k-J}, \dots, S_{i_k}, \dots, S_{i_k+J}\}$.

Note that the components of V_k are linear and invariant under scaling and rotation. Let $V^m(\alpha, \theta, K_m)$ and $V^s(\alpha, \theta, K_s)$ denote the model and sample contour's CPG-vectors, where $V^m(\alpha, \theta, K_m) = (V_1^m \ \dots \ V_{K_m}^m)$ and $V^s(\alpha, \theta, K_s) = (V_1^s \ \dots \ V_{K_s}^s)$, and K_m and K_s represent the vertex number of the model and sample contours, respectively. If $K_m < K_s$, we define $v_k^m(1) = a_1$, $v_k^m(2) = a_2$, $k = K_m + 1, \dots, K$; otherwise $v_k^s(1) = a_1, v_k^s(2) = a_2, k = K_s+1, \dots, K$, where $K = \max\{K_m, K_s\}$. a_1 and a_2 can be any real values, but in order to reduce the overlap of components of vertices between the model and sample contours, a_1 and a_2 should be chosen such that $a_1 \neq v_k^s(1)$ and $a_2 \neq v_k^s(2)$ as $k > K_m$ if $K_m < K_s$, and $a_1 \neq v_k^m(1)$ and $a_2 \neq v_k^m(2)$ as $k > K_s$ if $K_s < K_m$, because if the model and sample contours are in the same class we should have $K_m = K_s$, otherwise $K_m \neq K_s$ in general. In our experiments, we chose $a_1 = 10$ and $a_2 = 2$.

After the above procedure, the CPGs of both the model and the sample are approximated by K-dimensional vectors denoted by $V^m(K)$ and $V^s(K)$, respectively.

3.1. Matching rate

To define a similarity measure between model and sample shapes based on their CPG-vector, $V^m(K)$ and $V^s(K)$, we define

$$v_{k+K}^s(i) = v_k^s(i), \quad k = 0, \dots, K-1, \ i = 1, 2, \qquad (11)$$

where $V^s(K) = (V_1^s \ \dots \ V_K^s)$, $V_k^s = (v_k^s(1) \ v_k^s(2))^{\mathrm{T}}$, $k = 0, \dots, K-1$. Let $V^s(K, k)$ denote a CPG-vector $(V_{1+k}^s, \dots, V_{K+k}^s)$ which is a permutation of the components of $V^s(K)$ without changing the relative location between the components.

A quantity called *the matching rate* between model CPG-vector and sample CPG-vector denoted by R is defined as

$$R = \max_{k \in \{0,\ldots,K-1\}} \{V^m(K) \odot V^s(K,k)\}, \qquad (12)$$

where $V^m(K) \odot V^s(K,k)$ is the dot-product between $V^m(K)$ and $V^s(K,k)$, which is defined as

$$V^m(K) \odot V^s(K,k) = \frac{1}{K}\sum_{l=1}^{K} f_l(k),$$
$$k = 0,\ldots,K-1, \qquad (13)$$

and

$$f_l(k) = \begin{cases} 1 & \text{if } ((|v_l^m(1) - v_{l+k}^s(1)| < d) \\ & \quad \cap (v_l^m(2) = v_{l+k}^s(2))), \\ 0 & \text{otherwise}, \end{cases} \qquad (14)$$

where d is the threshold.

Clearly, $0 \leqslant R \leqslant 1$, and the larger the value of R, the more similar are the model and sample shapes. If the model and sample belong to the same class, then R is close or equal to 1, otherwise R has a small value. A critical value $\bar{R}$ of R can be determined by experiment, so a sample is classified into a class as model if $R > \bar{R}$.

3.2. Training procedure

The training of our system is simple because only three parameters, the supported rate α, the thresholds θ in Eq. (7) and d in Eq. (14), need to be selected. They are chosen based on the information of individual models.

All operations of the contour classifier are explained in Fig. 3.

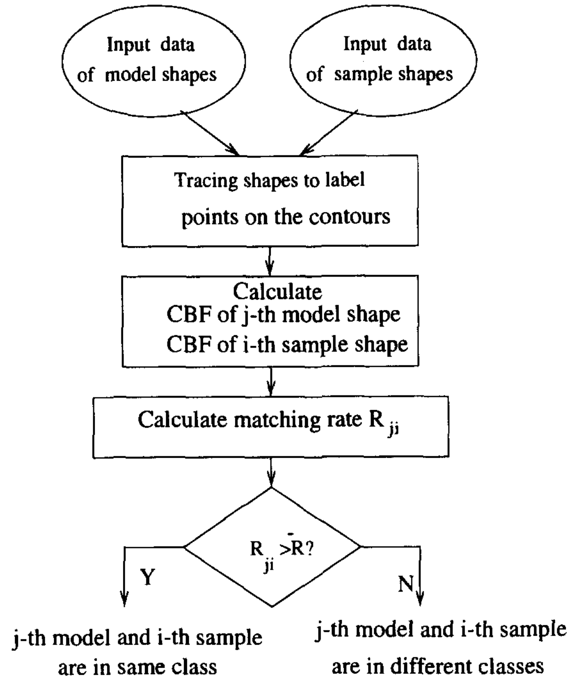

Fig. 3. Diagram of the contour classifier.

Table 2
The parameters used in our experiments

Parameter	md. 1	md. 2	md. 3	md. 4	md. 5
α	0.03	0.01	0.04	0.02	0.045
θ	0.3	0.2	0.3	0.25	0.3
d	0.2	0.08	0.08	0.12	0.15

4. Experimental results

The shapes in Fig. 4 are used to verify the effectiveness and reliability of the proposed scheme for classifying shapes. The shapes belong to five different classes, and each class has eight shapes which may be different in scale, orientation or both. In our system three parameters, the supported rate α, and the thresholds θ in Eq. (7) and d in Eq. (14), need to be determined in a training process. Using the information of the models, the training process is simple. The parameters used in our experiments are given in Table 2. Note that there are many other suitable values of the parameters for solving the same classification problem.

The experimental results given in Table 3 indicate that all shapes are classified correctly, if the critical value $\bar{R}$ is selected to be 0.7.

5. Conclusion

The experimental results demonstrate that our system works well in solving the shape recognition problem. Although the training process is carried out on a small number of samples, once trained, the classifier will classify new shapes correctly. This is because the representation of a contour obtained by the CBF includes the main features of the contour, i.e., the corners

Table 3
The matching rates

Sms.	md. A	md. B	md. C	md. D	md. E	Sms.	md. A	md. B	md. C	md. D	md. E
1	1.00	0.10	0.18	0.36	0.35	21	0.40	0.10	1.00	0.27	0.41
2	0.80	0.10	0.20	0.36	0.29	22	0.40	0.10	1.00	0.36	0.35
3	1.00	0.10	0.20	0.36	0.24	23	0.50	0.10	1.00	0.36	0.35
4	1.00	0.10	0.20	0.45	0.29	24	0.50	0.10	1.00	0.45	0.29
5	1.00	0.10	0.20	0.45	0.35	25	0.36	0.18	0.36	1.00	0.29
6	1.00	0.10	0.20	0.45	0.29	26	0.30	0.18	0.40	0.91	0.24
7	1.00	0.10	0.20	0.27	0.29	27	0.40	0.18	0.30	1.00	0.24
8	1.00	0.20	0.18	0.45	0.29	28	0.36	0.10	0.27	1.00	0.29
9	0.40	1.00	0.20	0.27	0.29	29	0.36	0.09	0.27	1.00	0.35
10	0.40	1.00	0.20	0.27	0.29	30	0.36	0.18	0.27	0.91	0.35
11	0.40	1.00	0.20	0.27	0.29	31	0.20	0.18	0.40	1.00	0.24
12	0.40	1.00	0.20	0.27	0.29	32	0.30	0.18	0.40	0.73	0.24
13	0.40	0.90	0.20	0.27	0.29	33	0.31	0.13	0.24	0.28	1.00
14	0.40	1.00	0.20	0.27	0.29	34	0.29	0.12	0.24	0.19	0.83
15	0.40	1.00	0.20	0.27	0.29	35	0.33	0.07	0.22	0.22	0.83
16	0.40	1.00	0.20	0.27	0.29	36	0.31	0.13	0.24	0.28	1.00
17	0.40	0.09	1.00	0.36	0.35	37	0.31	0.12	0.24	0.18	1.00
18	0.50	0.10	1.00	0.45	0.29	38	0.31	0.13	0.24	0.28	1.00
19	0.40	0.08	1.00	0.45	0.41	39	0.33	0.12	0.24	0.22	1.00
20	0.40	0.10	1.00	0.36	0.41	40	0.33	0.12	0.27	0.24	1.00

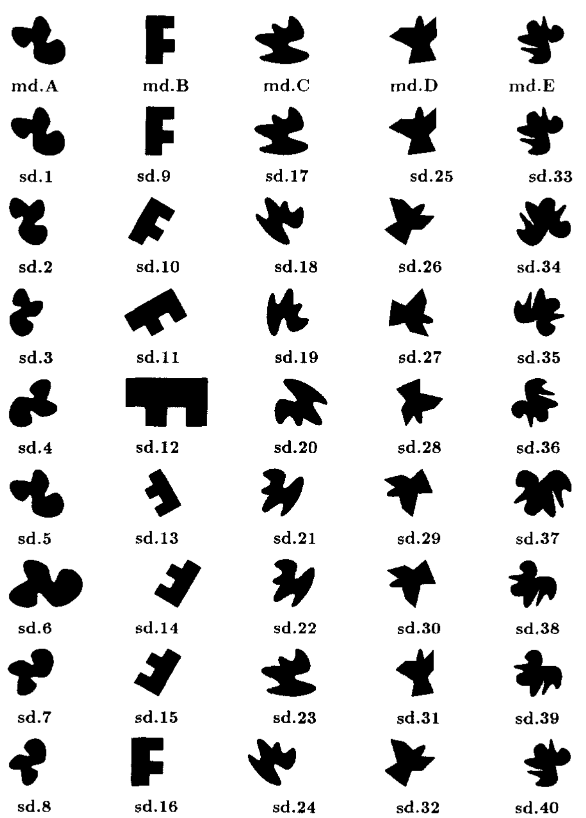

Fig. 4. The contours used in our experiments.

and properties (inner or outer) of the related angles of a contour, which are determined accurately. Besides, the magnitude of an angle can be estimated with small error, and it is invariant under scaling when the supported rate α is given. Since a contour Ω is treated as a periodic function in our system, the measure of an angle is invariant under rotation as well. The matching rate given in the last section depends heavily on the number of vertices K. Shapes in the same class should have an identical K, and the vertices of a contour can be found accurately if θ is suitably selected. In fact α, θ and d can be easily chosen based on the information of a related model shape. Compared with other classifiers, such as the neural network based method (Fu and Yan, 1996), the training process of this method is simpler and less computational time is needed.

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