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Shape classification using complex network and Multi-scale Fractal Dimension

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A B S T R A C T

Shape provides one of the most relevant information about an object. This makes shape one of the most important visual attributes used to characterize objects. This paper introduces a novel approach for shape characterization, which combines modeling shape into a complex network and the analysis of its complexity in a dynamic evolution context. Descriptors computed through this approach show to be efficient in shape characterization, incorporating many characteristics, such as scale and rotation invariant. Experiments using two different shape databases (an artificial shapes database and a leaf shape database) are presented in order to evaluate the method, and its results are compared to traditional shape analysis methods found in literature.

1. Introduction

Shape is a feature of great importance in human communication. It is one of the most important visual attributes of an object and the first used to perform object identification and classification (Loncaric, 1998; Torres et al., 2003).

Shape analysis is a classical problem in the pattern recognition area. Over the years, several methods for shape description and recognition have been suggested. Basically, these methods can be classified into three approaches: region-based, boundary-based and skeleton-based approaches. The region-based methods consider all image object to compute a set of descriptors which characterize that shape (Zhenjiang, 2000; Khotanzad and Hong, 1990; Hu, 1962). These methods can be applied to generic shapes, but they fail to distinguish among objects that are too similar. The boundary-based methods are specified and more efficient for shapes described by their contours (Mokhtarian and Bober, 2003). Methods such as Fourier descriptors (Mehtre et al., 1997; Wallace and Wintz, 1980; Osowski and Nghia, 2002), Curvature Scale Space (CSS) (Mokhtarian and Bober, 2003, 1993), wavelet descriptors (Chuang and Kuo, 1996), Multi-scale Fractal Dimension (Torres et al., 2003; Plotze et al., 2005; Bruno et al., 2008), Inner Distance (Ling and Jacobs, 2007) and Learning Graph Transduction (Yang et al., 2008) are included in this category. These techniques consider the shape as a set of ordered coordinates, and so, the lack of points, missing parts or even occlusion of a shape region could affect the results. Skeleton-based methods, such as Bai et al. (2008), Bai and Latecki (2008), consider only the media axis of the shape during its analysis. For this reason, these methods usually present better results than contour or others shape methods in the presence of occlusion and articulation of parts. Besides, the information about local thickness of the shape is lost during the skeleton computing. Such information is vital to distinguish solid shapes and linear shapes.

Multi-scale Fractal Dimension is a complexity analysis method which allows shape identification by analysing its irregularity pattern. It consists in estimating a curve that represents the changes in shape complexity as we change the visualization scale. Shape complexity is straight related to the irregularity pattern presented by the shape and, respectively, the amount of the space the shape occupies (Chaudhuri and Sarkar, 1995; Lange et al., 1996; Plotze et al., 2005; Emerson et al., 1999; Gonzalez and Woods, 2002; Tricot, 1995).

Presented paper proposes a novel approach to the shape boundary analysis problem using the Complex Network Theory (Albert and Barabási, 2002; Boccaletti et al., 2006; Costa et al., 2007; Dorogovtsev and Mendes, 2003) and Multi-scale Fractal Dimension (Torres et al., 2003; Plotze et al., 2005). Points in the shape contour are considered as a set of vertices and modeled as a network. Then, topological features, derived from the dynamics of the network growth, are correlated to physical aspects of the shape. Multi-scale Fractal Dimension curve is computed from degree measurements of the network. Result is a curve representing network complexity that may be used to identify and distinguish shape objects.

This paper is organized as follows: Section 2 presents an overview of the Complex Network Theory and some network measurements found in the literature. This section also details how to

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model a shape as a complex network. Section 3 shows how Fractal Dimension can be estimated from a complex network while Section 4 describes how the Multi-Scale Fractal Dimension is computed and how to compose a shape signature based in its descriptors. In Section 5, the performance of the proposed descriptors is evaluated using linear discriminant analysis (LDA) (Everitt and Dunn, 2001) in an experiment based on image classification. For this, two image databases were considered: (i) generic artificial shapes and (ii) plant leaves shapes. Proposed descriptors are also compared with traditional shape analysis methods found in Literature. Conclusions are finally presented in Section 6.

2. Complex network

Nowadays, complex networks have become a topic of great interest in many fields of science (Albert and Barabási, 2002; Newman, 2003; Dorogovtsev and Mendes, 2003). This interest is due to the capacity of complex networks represent many real-world systems, natural structures and computer vision topics. However, this is still an unexplored field with few references in the literature (Costa, 2004).

Studies conducted by Flory (1941), Rapoport (1951), Rapoport (1953), Rapoport (1957), Erdös and Rényi (1959), Erdös and Rényi (1960), Erdös et al. (1961) settle the basis of this research area, which can be understood as an intersection between graph theory and statistical mechanisms. This grants a truly multidisciplinary nature to this area (Costa et al., 2007). Recent motivation in complex network research is due to the investigations about Small-World Networks performed by Watts and Strogatz (1998) and Barabási and Albert characterizing Scale-Free models (Barabási and Albert, 1999).

The main reason for complex network popularity lays in its flexibility and generality to represent any given structure, natural or discrete (such as lists, trees, networks and images (Costa, 2004)), including those undergoing dynamic changes of topology (Costa et al., 2007).

Many papers use complex networks to represent real structures. These studies include investigations of the problem representation as a complex network, analysis of topological characteristics and feature extraction (Barry, 2005): texts can be modeled by using Complex Networks, so that different texts can be distinguished using the correlation between network parameters and text quality (Antiqueira et al., 2005). Textures, when modeled in such way, present their complex texture patterns represented by network connectivity. So, a feature vector based in traditional connectivity measurements allows texture characterization and classification (Thomas Chalumeau et al., 2006).

This work uses the Complex Network Theory in a similar approach to the cited works above. The focus is the shape boundary analysis and its identification, a feature of great importance in human communication.

2.1. Shape contour as a complex network

Literature presents various methods to analyze images and objects using the shape boundary. Most of them consider shape as a sequence of connected points where the order of these points expresses some meaning. Thus, the information of a shape contour can be described as a list S, $S = [s_1, s_2, \ldots, s_N]$, of N size where $s_i = (x_i, y_i)$ are discrete numerical values representing the coordinates of point i of the contour.

To apply complex networks theory to the problem, a graph representation of the shape contour is necessary (Fig. 1). A graph $G = (V, E)$ is built, where each point of the contour $s \in S$ corresponds to a vertex $v \in V$ in the graph G. A set of non-directed edges $E : V \times V$ binding each pair of vertices is also built. As in the work of Belongie et al. (2002), the set E is achieved by calculating the Euclidean distance between each pair of points of the contour:

$$d(s_i, s_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}. \tag{1}$$

Hence, the network is represented by the $N \times N$ weight matrix W

$$W(i,j) = w_{ij} = d(s_i, s_j) \tag{2}$$

normalized into the interval $[0, 1]$,

$$W = \frac{W}{\max\limits_{w_{ij} \in W}}. \tag{3}$$

At this stage, each network vertex presents the same number of connections, i.e., a regular behavior. Nevertheless, this network does not present any relevant property for the proposed application, not even being a complex network. Following sections describe how to transform this network in a complex network as also the property of our interest.

2.2. Network degree

Degree (or *connectivity*) of a vertex v_i, $deg(v_i)$, is defined as being the number of edges in the network that are bound to v_i, i.e., the number of incident edges in v_i. Let $\partial v_i = \{v_j \in V | (v_i, v_j) \in E\}$ the set of neighbors of v_i, the degree of a vertex v_i is defined as

$$deg(v_i) = |\{e \in E | v_i \in e\}| = \left|\{v_j \in V | \{v_i, v_j\} \in E\}\right| = |\partial v_i|, \tag{4}$$

where $|A|$ denotes the cardinality (number of elements) of a set A (Wuchty and Stadler, 2003).

From degree distribution analysis, several measurements can be computed from a network. Three measurements considered for the proposed application are the *minimum degree*

$$Min(G) = \min_i deg(v_i), \tag{5}$$

the *Average Degree*

$$Av(G) = \sum_{v_i \in V} \frac{deg(v_i)}{|N|}, \tag{6}$$

and the *maximum degree*

$$Max(G) = \max_i deg(v_i). \tag{7}$$

2.3. Dynamic evolution

Different networks may present a large range in theirs characteristics. This makes the modeling of the dynamics of a complex network a difficult task. Moreover, a network characterization is not complete without considering the interaction between structural and dynamical aspects (Boccaletti et al., 2006).

Although modeling the dynamics of a complex network is a difficult task, additional information about structure and dynamic of complex networks can be yielded by applying a transformation over the original network and, in the sequel, by computing its properties (Costa et al., 2007). This transformation can be performed in many ways. Applying a threshold t over the edges E, in order to select E^*, $E^* \subseteq E$, so yielding a new network $G^* = (V, E^*)$ is a simple and straight approach. In this approach, each edge of E^* has a weight equal or smaller than t and this δ_t transformation is represented as

$$E^* = \delta_t(E) = \{e \in E | w(e) \leqslant t\}. \tag{8}$$

This δ_t transformation allows to study network properties at intermediate steps of its evolution, i.e., the properties of the sub-networks yielded as the maximum weight for its edges increases. By

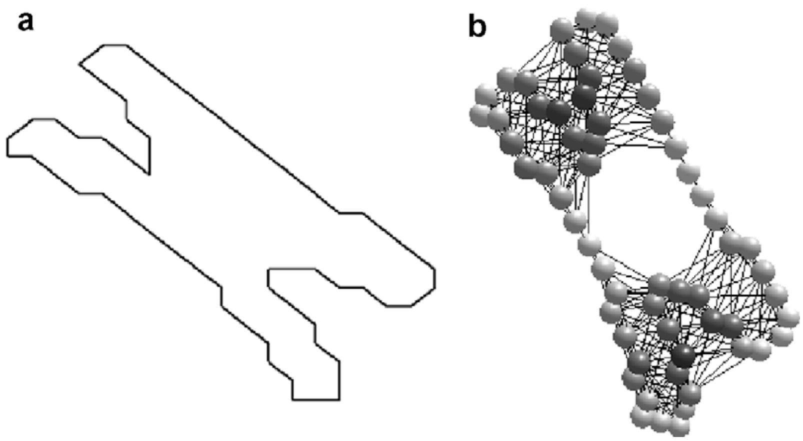

Fig. 1. Example of a shape contour modeled as a complex network.

using different threshold values it is possible to achieve a richer set of measurements that describes the network dynamics. So, network characterization is performed using various δ_{T_i} transformations, where T_{min} is the initial threshold value for T_i, and its value is incremented in a regular interval T_{inc} until a maximum threshold T_{max}.

3. Complex network fractal dimension

Literature defines the fractal dimension as a measure of complexity of fractal objects. It may also be understood as a characterization of self-similarity in these objects. The fractal dimension is represented as a non-integer number that quantifies the density of the fractal in the metric space, and it is commonly used as a tool to identify how complex a fractal is, allowing its comparison with another fractal (Mandelbrot, 2000; Tricot, 1995; Schroeder et al., 1996).

Complex networks may also be described in terms of complexity. In fact, many real networks are self-similar fractals (Song et al., 2007; Kim et al., 2007; Barabási and Albert, 1999; Feder, 1988). A common approach to estimate the fractal dimension d from complex networks is based on the power law that exists between the vertices degree k and the distance l used to compute the degree. This approach can reveal interesting network topological features, especially in terms of self-similarity or fractal characteristics, and it is defined as

$$k \approx l^d, \tag{9}$$

For our application, we consider the degree k as a value normalized into the interval $[0,1]$. This is achieved dividing the degree value by the total number of vertices in the network, N. As the degree, we consider one of the proposed measurements computed from the network: *minimum degree*, *average degree* or *maximum degree*. The distance l is the threshold value T_i used to compute such measurement. So, the fractal dimension of our complex network G is defined as

$$k(T_i) \approx (T_i)^d, \tag{10}$$

$$d = \lim_{T_i \to 0} \frac{\log k(T_i)}{\log T_i}, \tag{11}$$

$$k(T_i) \in \{Min(G_{T_i})/N, Av(G_{T_i})/N, Max(G_{T_i})/N\}, \tag{12}$$

Degree based measurements allow to study physical properties of the shape by the complexity of its point connections in the network. Saliences or others contour characteristics (such as, the proximity of another contour section or a straight section) reach a different number of connections according to the threshold used (Fig. 2). These characteristics disturb the way the degree curve grows as the threshold used is also increased. This additional information about shape peculiarity added to the curve makes the method very sensitive to small changes in the network structure and, as a consequence, the shape.

4. Multi-Scale Fractal Dimension

Eventually, a single non-integer number d is not enough to represent all complexity of an object. The log–log curve achieved by computing the normalized distance T_i and the degree $k(T_i)$ presents more details than can be expressed by just a numeric value, as performed using line regression. Besides, non-fractal objects present finite size, and it implies that their dimension goes to zero as the visualization scales increases.

In order to solve this deficiency in the characterization of object by complexity, the Multi-Scale Fractal Dimension was developed. Different from the fractal dimension, which uses linear interpolation to estimate the angular coefficient of the log–log curve, this approach exploits the infinitesimal limit of the linear interpolation by using the derivative. By using the derivative, it is possible to find a function that binds the changes in the object complexity to the visualization scale changes (Fig. 3). In other words, this approach allows to represent an object by a curve describing the object complexity along the scale, what provides a more effective discrimination of the object (Emerson et al., 1999; Gonzalez and Woods, 2002; Plotze et al., 2005). Most of this changes in complexity are related to an irregular growing in the degree curve. This is most due to the presence of shape peculiarity and other characteristics, which saturate some vertices connections in the network for some specific thresholds. Thus, the Multi-Scale Fractal Dimension is defined as

$$d(T_i) = \frac{d \log k(T_i)}{d \log T_i}. \tag{13}$$

Computing the Multi-Scale Fractal Dimension involves to calculate the derivative of $\log k(T_i)$. This can be performed by using derivative property of the Fourier Transform, which allows to derive a given curve in its spectrum. This approach was considered instead of numeric approaches once it considers all curve information during the derivative computing (Emerson et al., 1999; Plotze et al., 2005). Otherwise, this approach tends to emphasize high frequency information and, in most of the cases, noise. So, a low pass filter, like Gaussian filter, is applied over the analyzed signal to reduce the influence of high frequency informations (Costa and Cesar, 2000; Brigham, 1988). The derivative using the Fourier transform is defined as:

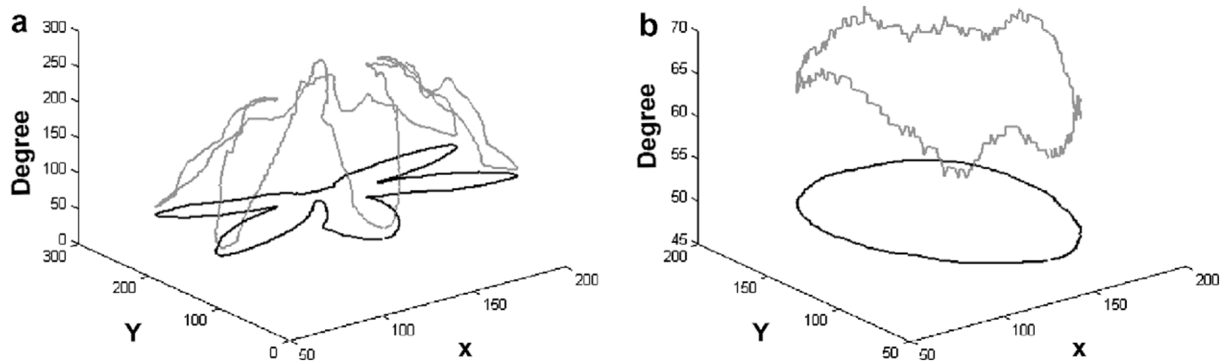

Fig. 2. Example of degree connections according to shape structure: black line represents the original shape while gray line is the vertex degree computed for a specific threshold.

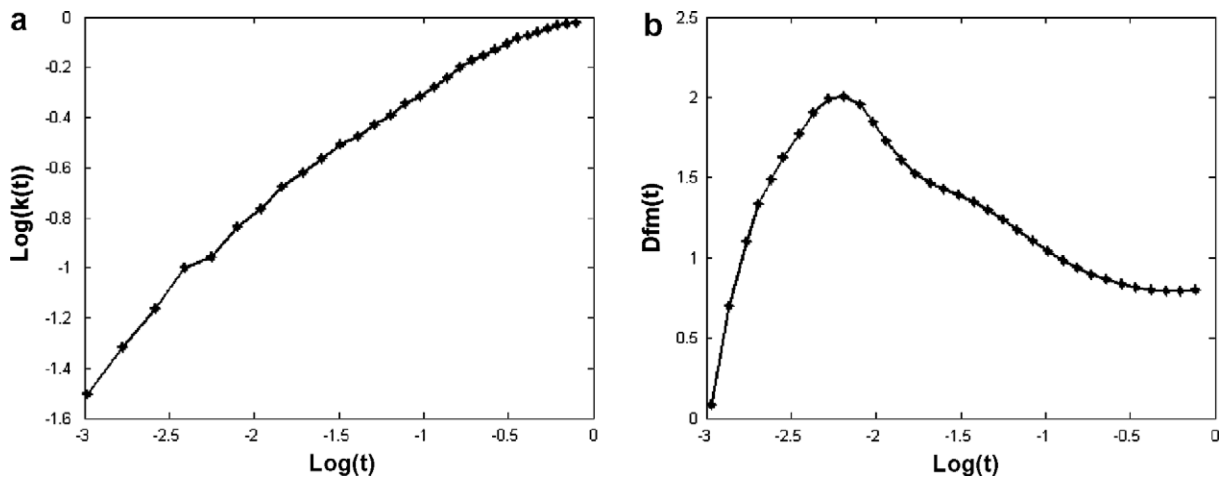

Fig. 3. (a) log–log curve and (b) Multi-scale Fractal Dimension.

$$d(T_i) = \frac{d \log k(T_i)}{d \log T_i} = F^{-1}\{F\{\log k(T_i)\}F\{g_\sigma(\log T_i)\}(j2\pi f)\} \quad (14)$$

with

$$g_\sigma(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-t^2}{2\sigma^2}\right), \quad (15)$$

where f is the frequency, j is the imaginary number and $g_\sigma(t)$ is the Gaussian function with standard deviation σ (Emerson et al., 1999; Gonzalez and Woods, 2002; Plotze et al., 2005).

In this work, the Multi-Scale Fractal Dimension descriptors are represented by a vector containing the seven coefficients of the polynomial curve, computed by regression, that fits the derivative curve. The polynomial degree was determined through a set of experiments. These experiments showed that a degree higher than seven does not improve the results.

5. Experiment and results

Experiments based on image classification were conducted to evaluate the performance of the proposed descriptors. Two image databases were considered: (i) generic artificial shapes and (ii) plant leaves shapes. The generic artificial shapes database is composed of 99 images grouped into 9 different classes with 11 samples each (Sebastian et al., 2004; Sharvit et al., 1998), where each category is composed by different variations in the shape structure. Fig. 4 shows the various classes used in the experiment. Each class present several variations of the shape, as occlusion, articulation, missing parts, etc.

The leave's database consists of 600 samples grouped into 30 classes with 20 samples each. Leaves classification is a difficult task, as the inter-class similarity is considerable while the within-class similarity is unsuitable and overlaps can occur between adjacent parts of leaves. Fig. 5 shows the various classes used in the experiment. Two other image databases were also created in order to evaluate scale and rotation invariance: one containing 4 different manifestations of each contour by scaling (a total of 2400 samples) and another containing 6 manifestations of each contour by rotation (a total of 3600 samples).

In all experiments, degree measurements are computed from the different manifestations of the complex network yielded through applying the δ transformation with an initial threshold $T_{min} = 0.025$, incremented at a regular interval $T_{inc} = 0.025$, until reaching a final threshold $T_{max} = 0.95$. These measurements are used to compute the Multi-Scale Fractal Dimension of the complex network.

The evaluation of the method is performed by statistical analysis on the descriptors using the Linear Discriminant Analysis (LDA), a supervised statistical classification method. The LDA main goal is to find a linear sub-space to project the data where the variance intra-classes is larger than inter-classes (Everitt and Dunn, 2001; Fukunaga, 1990). Basically, the LDA method attributes an observation to the class i which presents the higher conditional probability:

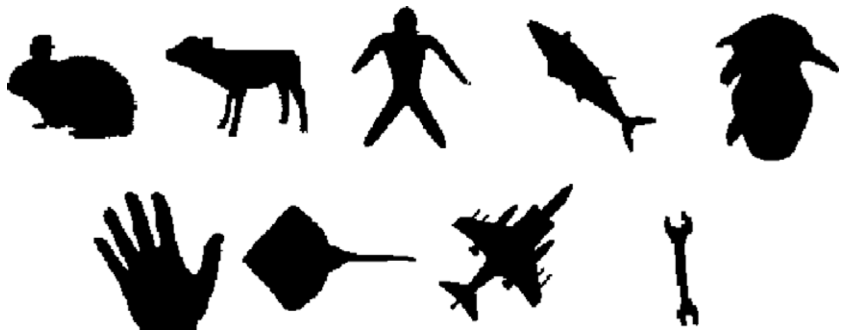

Fig. 4. Example of the artificial shapes used in the experiments.

Fig. 5. Example of the leaves images used in the experiments.

$$P(i|x) > P(j|x), \quad \forall j \neq i, \tag{16}$$

where $P(i|x)$ is the conditional probability of i given x. Assuming that this probability obeys a multivariate gaussian distribution with a single covariance matrix for all the classes, the main formula for LDA method is defined as:

$$f_i = \mu_i C^{-1} x_k^T - 0.5 \mu_i C^{-1} x_k^T + \log(p_i), \tag{17}$$

where μ_i is the mean of the descriptors of class i, C is the covariance matrix of the data set and p_i is the *a priori* probability of class i. An object is attributed to the class i which provides the higher value for f_i.

The proposed descriptor was also evaluated considering a Naive Bayes Classifier (NBC) (Theodoridis and Koutroumbas, 2003; Fukunaga, 1990). This method uses the Bayes rule to attribute an observation to the class i which presents the higher conditional probability:

$$P(i|x) = \frac{P(x|i)P(i)}{P(x)}. \tag{18}$$

In this case, we assume $P(x|i)$ as the Gaussian or normal density function:

$$P(x|i) = \frac{1}{(2\pi)^{l/2}|\Sigma_i|^{l/2}} \exp\left(-\frac{1}{2}(x-\mu_i)^T\Sigma_i^{-1}(x-\mu_i)\right), \tag{19}$$

where μ_i is the mean value of the i class, Σ_i is the $l \times l$ covariance matrix and $|\Sigma_i|$ denotes the determinant of Σ_i.

Both statistical methods were carried out over the samples using the leave-one-out cross-validation scheme. This scheme uses a single observation from the original set as the test set and the remaining observations as training set. This process is repeated until all samples have been used as test set.

Results achieved for the proposed method are compared with other shape descriptors found in the literature. The descriptors considered for this experiment are:

Fourier descriptors: a feature vector containing the 20 most significant coefficients yielded from applying the Fourier Transform over the shape contour. The method is used as described in (Gonzalez and Woods, 2002; Osowski and Nghia, 2002).

Zernike moments: a feature vector containing 20 moments (order $n = 0, \ldots, 7$), where these moments are the most significant magnitudes of a set of orthogonal complex moments of the image (Zhenjiang, 2000).

Curvature descriptors: contour is represented as a curve, where its maximum and minimum local points correspond to the direction changes in the shape contour (Wu and Wang, 1993).

Bouligand–Minkowski Multi-scale Dimension: this method employs the concept of influence area, for a given radius, to achieve a curve which describes how shape complexity changes as the radius changes. For this approach, the 50 most meaningful points of the curve were considered for shape characterization (Torres et al., 2003; Plotze et al., 2005).

Skeleton Paths: for each pair of *end Points* of the shape skeleton, the minimum path is computed. This path is sampled by M equidistant points. From each sampled point, the maximum disc size is achieved. Thus, each skeleton path is represented by a set of M disc radius, normalized by all disc radius found in the shape. Different of other methods compared, this method did not use LDA or NBC but a graph matching approach, which estimates the probability of one path to belong to a specific shape skeleton (Bai et al., 2008; Bai and Latecki, 2008). A total of $M = 50$ and $M = 15$ disc radius were used to represent each path. The optimum result achieved was considered in the comparison.

It is important to emphasize that different configurations of the methods (such as, number of descriptors) were evaluated and only the best results were considered.

5.1. Results for the artificial shapes database

At first, the proposed complexity descriptors were used in an experiment which objectives to classify a generic shapes database. This database, as previously described in Section 5, is compound of different shape categories, where each category includes samples with different manifestations of its basis shape structure. Table 1 shows the results obtained for the different degree measurements considered. Table 2 shows the results achieved for each method considered. Results expressed in these tables are referent to the amount of shapes correctly classified by the method.

Results show that *average degree* has a greater capacity to shape recognition than *minimum degree* and *maximum degree*. Otherwise, the combination of different degree based measurements increases the success rate. We note that, by combining all degree based measurements the success rates of 100.00% and 87.88% for LDA and NBC, respectively, are yielded. These results overcome the traditional shape analysis methods, such as Fourier descriptors, Zernike moments, Curvature descriptors and Bouligand–Minkowski. Exception is made only by the skeleton path, which presents a superior performance in comparison to the proposed approach when Naive Bayes Classifier is used. However, it is important to emphasize that the skeleton path does not uses LDA or NBC during its classification process. It employs a particular graph matching approach, which estimates the probability of one path to belong to a specific shape skeleton (Bai et al., 2008; Bai and Latecki, 2008), a time consuming task.

It is necessary to emphasize the presence of shapes with different variations in their structure (such as, occlusion, articulation and missing parts) in this database. These deformations are a common problem in shape image acquisition, and the results show a great efficacy of the proposed descriptors in dealing with them.

Table 1
Results for different complexity descriptors' sets and its combination for the artificial shapes database.

Combined degree features			No. of descriptors	Success rate (%)	
Minimum	Average	Maximum		LDA	NBC
X			7	71.72	75.76
	X		7	90.90	76.77
		X	7	84.85	71.72
X	X		14	94.95	82.83
X		X	14	94.95	82.83
	X	X	14	84.85	86.87
X	X	X	21	100.00	87.88

Table 2
Classification performance of various shape descriptors for the artificial shapes database.

Shape descriptor	Success rate (%)	
	LDA	NBC
Proposed method	100.00	87.88
Fourier	83.84	85.86
Zernike	91.92	84.85
Curvature	76.77	76.77
Bouligand–Minkowski	87.88	75.76
Skeleton path	100.00	

5.2. Results for the leaves database

In this second experiment, the complexity descriptors were tested using a natural images database composed by different samples of leaves. Table 3 shows the results obtained for the different degree measurements, and its combination, for the original leave's database. Table 4 shows the results achieved for each method considered, including our proposed approach using its best configuration, for different leave's databases. As in the previous experiment, results expressed in these tables are referent to the amount of shapes correctly classified by each method. As in the previous

Table 3
Results for different complexity descriptors' sets and its combination for the leave's database.

Combined degree features			No. of descriptors	Success rate (%)	
Minimum	Average	Maximum		LDA	NBC
X			7	54.50	50.50
	X		7	66.83	59.50
		X	7	67.83	60.00
X	X		14	77.00	74.50
X		X	14	77.33	74.83
	X	X	14	80.17	74.00
X	X	X	21	82.33	77.00

Table 4
Classification performance of various shape descriptors for the leave's database.

Type of experiment	Shape Descriptor	Success rate (%)	
		LDA	NBC
Original 600 images	Proposed Method	82.33	77.00
	Fourier	75.00	71.83
	Zernike	68.00	62.83
	Curvature	75.00	54.17
	Bouligand–Minkowski	73.00	63.83
	Skeleton Path	64.66	
Rotated 3600 images	Proposed Method	83.36	80.50
	Fourier	76.53	74.22
	Zernike	69.92	58.80
	Curvature	78.64	59.50
	Bouligand–Minkowski	68.19	55.30
	Skeleton Path	72.53	
Scaled 2400 images	Proposed Method	85.62	82.62
	Fourier	81.58	68.50
	Zernike	54.54	39.83
	Curvature	80.00	61.75
	Bouligand–Minkowski	74.33	59.04
	Skeleton Path	76.77	

experiment, the combination of all degree based measurements provides a more feasible shape descriptor, reaching the success rates of 82.33% and 77.00% for LDA and NBC, respectively.

The results for the original database show a higher performance of the method when compared with traditional shape analysis methods. This indicates that degree measurements might be more related to the shape aspect and its complexity than other methods considered. It is also important to emphasize that results show that the proposed method presents a great capacity of discrimination between classes, while dealing with within class variations.

The proposed descriptors also present good results in experiments considering shape transformations, such as rotation and scale. In fact, the results achieved for rotated and scaled databases demonstrate that the method presents great invariance to this kind of transformations. This invariance is due to the normalization, at interval $[0,1]$, performed over the network edges: the largest network edge (higher Euclidean distance between any two vertices) is set to weight equal to 1, while remaining edges acquire a proportional weight. This normalization acts preserving the behavior of neighbor vertices along different thresholds, and it ensures the same properties for the set of edge E^*, in spite of differences in shape rotation and scale. The differences in the success rate, in comparison to the original database, are due to the fact that now we have more samples in the transformed database, and the descriptors computed for the different versions of a shape are quite similar to the original shape if the method presents a high tolerance to these transformations. As higher is the tolerance to these transformations, higher is the success rate of the method.

Among the compared methods, we note a perceptible decrease in the performance of the skeleton Paths in comparison to the results from the previous experiment. An explanation for this behavior may lay in the fact that different plant species present leaf shape quite similar. When the skeleton is computed, these species achieve almost the same skeleton: a single path between leaf extremities (Fig. 6). Even considering the maximum disc radius at each sampled point of the skeleton, one single path does not hold enough information to yield a good leave discrimination, what may explain its result.

5.3. Computational complexity

Note that most of the computational cost of the method is spent to compute the weight matrix W, which stores the distance between each pair of network vertices. Considering a shape contour of N size, $\frac{N\times(N-1)}{2}$ operations are necessary to compute this matrix, what leads to complexity $O(N^2)$.

Another important point of the method is the computing of the Multi-scale Fractal Dimension. This is performed by using the derivative, so that, a function that binds the changes in the object complexity to the visualization scale changes is found. In order to compute the derivative, the Fourier transform is employed, what leads to complexity $O(N^2 \log N)$. Once the computational cost to achieve the weight matrix is higher than the Multi-scale curve, the computational cost of the method is considered as being $O(N^2)$.

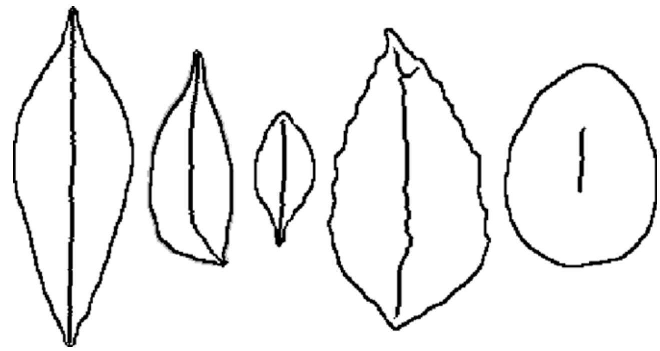

Fig. 6. Example of the leaves' skeletons for different plant species.

6. Conclusion

This paper proposes a novel pattern recognition approach based on the Complex Network Theory and complexity analysis. We have illustrated how a shape contour can be effectively represented and characterized as a complex network, in a dynamic evolution context, and how degree based measurements (such as minimum, average and maximum degrees) can be used to estimate the network complexity through Multi-Scale Fractal Dimension.

The proposed approach was experimented over different shapes databases and its results reported a powerful potential of discriminating classes, overcoming the results of traditional shape analysis methods, such as curvature, Fourier descriptors, Zernike moments, Bouligand–Minkowski and Skeleton Paths. We also demonstrated the great ability of the method in dealing with different manifestations of a shape, including rotation and scale invariance. Regarding to the Complex Network Theory, paper illustrates the potential of applying this theory to computer vision and pattern recognition problems.

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