[1] (20 pts) For Random Variables X and Y, they have joint distribution P(X,Y) over values [1:4] given below,

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | X=1 | X=2 | X=3 | X=4 |
| Y=1 | 1/8 | 1/16 | 1/32 | 1/32 |
| Y=2 | 1/16 | 1/8 | 1/32 | 1/32 |
| Y=3 | 1/16 | 1/16 | 1/16 | 1/16 |
| Y=4 | 1/4 | 0 | 0 | 0 |

1. What is the marginal distribution of X, P(X) and Y, P(Y) ? [5 pts]
2. What is the entropy, H(X) and H(Y) ? [5 pts]
3. What is the conditional entropy H(X|Y), and H(Y|X) ? [5 pts]
4. What is the mutual info I(X, Y) ? [5 pts]

[2] (20 pts) Given a set of symbols A={a1, a2, a3, a4, a5 }, and their probability, P= [0.4 0.2 0.2 0.1 0.1],

1. What is the entropy of this source ? compute their Huffman code, show the steps and the prefix tree. What is the difference between the average coding length and entropy ?
2. If the measured source symbol distribution is P’=[0.30, 0.25 0.20, 0.15 0.1], design a new Huffman code for this distribution, and what is the relative entropy between P and P’ ? what is the average coding length difference of this new code ?

[3] (30 pts), For the given image lena.png, compute the distribution of the residual, from the prediction scheme: , compute the predicted image residual error, . (Hint, use Matlab imfilter() ), and answer the following questions,

1. Plot the distribution of the residual error image *e,* find the double sided Geometry Distribution that best fits this error distribution for *W*=[1/3, 1/3, 1/3 , 0] [15 pts]

b) Design a Golomb codes with proper segment size *m*, that best fits the statistics of this residual image. [15 pts]