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Fourier series

1. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin x)$

where $x \in (-\pi, \pi)$ or $(0, 2\pi)$

$a_0 = \int_{-\pi}^{\pi} f(x)\,dx$	or	$\int_0^{2\pi} f(x)\,dx$.
$a_n = \int_{-\pi}^{\pi} f(x)\cos nx\,dx$	or	$\int_0^{2\pi} f(x)\cos nx\,dx$
$b_n = \int_{-\pi}^{\pi} f(x)\sin nx\,dx$	or	$\int_0^{2\pi} f(x)\sin nx\,dx$

① Find the fourier series of $f(x) = x$ in $(-\pi$ to $\pi)$

$$a_0 = \int_{-\pi}^{\pi} x\,dx$$

$$= \left[\frac{x^2}{2}\right]_{-\pi}^{\pi} = \frac{\pi^2}{2} - \frac{(-\pi^2)}{2} = 0$$

$$a_n = \int_{-\pi}^{\pi} \underset{u}{x}\ \underset{v'}{\cos nx}\,dx$$

$$= \left[x\frac{\sin nx}{n} - \int 1 \cdot \frac{\sin nx}{n}\,dx\right]_{-\pi}^{\pi}$$

$$= \left[\frac{x\sin nx}{n} - \frac{1}{n}\left(-\frac{\cos nx}{n}\right)\right]_{-\pi}^{\pi}$$

$\sin n\pi = 0$
$\cos n\pi = (-1)^n$
$\cos(-\theta) = \cos\theta$
$\sin(-\theta) = -\sin\theta$

$$= \left[\frac{\pi \overset{0}{\sin n\pi}}{n} + \frac{\cos n\pi}{n^2}\right] - \left[\frac{-\pi \overset{0}{\sin(-n\pi)}}{n} + \frac{\cos(-n\pi)}{n^2}\right]$$

$$= \frac{\cos n\pi}{n^2} - \frac{\cos n\pi}{n^2} = 0$$

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$\int uv = u\int v - \int u' \int v$

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$$b_n = \int_{-\pi}^{\pi} x \sin nx \, dx$$

$$= \left[x\left(\frac{-\cos nx}{n}\right) + \int 1 \cdot \frac{\cos nx}{n} \, dx \right]_{-\pi}^{\pi}$$

$$= \left| \frac{-x\cos nx}{n} + \frac{\sin nx}{n^2} \right|_{-\pi}^{\pi}$$

$$= \left[\frac{-\pi\cos n\pi}{n} + \cancel{\frac{\sin n\pi}{n}} \right] - \left[\frac{-\pi\cos(-n\pi)}{n} + \cancel{\frac{\sin(-n\pi)}{n^2}} \right]$$

$$= \frac{-\pi\cos n\pi}{n} - \frac{\pi\cos n\pi}{n}$$

$$= \frac{-2\pi\cos n\pi}{n}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{-2\pi}{n} \cos n\pi \sin nx$$

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Q.24 $f(x) = kx(\pi - x) \quad 0 < x < 2\pi$

$$a_0 = \int_0^{2\pi} kx(\pi - x)\,dx.$$

$$k\int_0^{2\pi} (\pi x - x^2)\,dx.$$

$$= k\left[\pi\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{2\pi}$$

$$= k\left[\pi/2\,(2\pi)^2 - \frac{(2\pi)^3}{3}\right]$$

$$= k\left[\frac{\pi}{\cancel{2}}\cancel{4}^{2}\pi^2 - \frac{8\pi^3}{3}\right]$$

$$= k\left[2\pi^3 - \frac{8\pi^3}{3}\right]$$

$$= k\,\frac{6\pi^3 - 8\pi^3}{3}$$

$$= k\,\frac{-2\pi^3}{3}$$

$$\boxed{= \frac{-2k\pi^3}{3}}$$

$$a_n = \int_0^{2\pi} kx(\pi - x)\cos nx\,dx.$$

$$= k\int_0^{2\pi} \underbrace{(\pi x - x^2)}_{u}\,\underbrace{\cos nx}_{v}\,dx$$

$$= k\left[(\pi x - x^2)\frac{\sin nx}{n} - \int (\pi - 2x)\frac{\sin nx}{n}\,dx\right]_0^{2\pi}$$

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$$= k\left[(\pi x - x^2)\frac{\sin nx}{n} - \frac{1}{n}\left\{(\pi - 2x)\left(-\frac{\cos nx}{n}\right) - \int(-2)\left(-\frac{\cos nx}{n}\right) dx\right\}\right]_0^{2\pi}$$

$$k\left[(\pi x - x^2)\frac{\sin nx}{n} + (\pi - 2x)\frac{\cos nx}{n^2} + \frac{2}{n^2}\left[\frac{\sin nx}{n}\right]\right]_0^{2\pi}$$

$$= k\left[(\pi . 2\pi - 2\pi^2)\cancel{\frac{\sin 2n\pi}{n}}^{0} + (\pi - 2\pi)\frac{\cos 2n\pi}{n^2} + \frac{2}{n^3}\cancel{\sin 2n\pi}^{0}\right]$$

$\cos 2n\pi = 1$, $\sin 2n\pi = 0$

$$- \left[\cancel{\frac{0 \sin 0}{n}}^{0} + (\pi - 0)\frac{\cos 0}{n^2} + \cancel{\frac{2}{n^3}\sin 0}^{0}\right]$$

$$= k\left[\frac{-\pi}{n^2}\right] - \left[\frac{\pi}{n^2}\right]$$

$$= k\,\frac{-2\pi}{n^2} = \frac{-2k\pi}{n^2}$$

$$b_n = \int_0^{2\pi} kx(\pi - x)\sin nx\, dx$$

$$= k\int_0^{2\pi} \underbrace{(\pi x - x^2)}_{u}\ \underbrace{\sin nx}_{v}\, dx.$$

$$= k\left[(\pi x - x^2)\frac{\cos nx}{n} - \int(\pi - 2x)\frac{\cos nx}{n}\, dx\right]_0^{2\pi}$$

$$= k\left[(\pi x - x^2)\frac{\cos nx}{n} - \frac{1}{n}\int \underbrace{(\pi - 2x)}_{u}\ \underbrace{\frac{\cos nx}{n}}_{v}\, dx\right]_0^{2\pi}$$

$$= k\left[(\pi x - x^2)\frac{\cos nx}{n} - \frac{1}{n}\left\{(\pi - 2x)\frac{\sin nx}{n} - (-2)\frac{\sin nx}{n}\, dx\right\}\right]_0^{2\pi}$$

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Now Formula $\left[u\int v - u'\iint v + u''\iiint v - \cdots \right]_a^b$

$$K\left[(\pi x - x^2)\left(\frac{-\cos nx}{n}\right) - (\pi - 2x)\left(\frac{-\sin nx}{n^2}\right) + (-2)\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$K = \left[-(\pi - x^2)\frac{\cos nx}{n} + \frac{1}{n}(\pi - 2x)\frac{\sin nx}{n} - \frac{2\cos nx}{n^3}\right]_0^{2\pi}$$

$$= K\left[-(2\pi^2 - 4\pi^2)\frac{\cos 2n\pi}{n} + \frac{1}{n^2}(\pi - 4\pi)\sin 2n\pi - \frac{2\cos 2n\pi}{n^3}\right]$$

$$= K\left[-(2\pi^2 - 4\pi^2)\frac{\cos 2n\pi}{n} + \frac{1}{n^2}(\pi - 4\pi)\cancel{\sin 2n\pi}^{0} - \frac{2\cos 2n\pi}{n^3}\right] -$$

$$\left[-\frac{0\cos 0}{n} + \frac{1}{n^2}(\pi - 0)\sin 0 - \frac{2\cos 0}{n^3}\right]$$

$$= K\left[2\pi^2\frac{1}{n} + 0 - \frac{2}{n^3}\right] - K\left[0 + 0 - 2/n^3\right]$$

$$= \frac{2K\pi^2}{n} - \cancel{\frac{2K}{n^3}} + \cancel{\frac{2K}{n^3}}$$

$$= \frac{2K\pi^2}{n}$$

Q3) $f(x) = x^2 \qquad 0 < x < 2\pi$

$$a_0 = \int_0^{2\pi} x^2\,dx$$

$$\left|\frac{x^3}{3}\right|_0^{2\pi} = \left[\frac{(2\pi)^3}{3} - \frac{0^3}{3}\right]$$

$$= \left[\frac{8\pi^3}{3} - 0\right]$$

$$= \frac{8\pi^3}{3}$$

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$$a_n = \int_0^{2\pi} \underset{u}{x^2} \underset{v}{\cos nx}\, dx.$$

$$= \left[x^2 \frac{\cos nx}{n} - \int 2x \right.$$

$$a_n = \int_0^{2\pi} \underset{u}{x^2} \underset{v}{\cos nx}\, dx$$

$$= \left[\underset{u}{x^2} \frac{\sin nx}{n} - \int \underset{u'}{2x} \underset{v}{\frac{\sin nx}{n}}\, dx \right]_0^{2\pi}$$

$$= \left[x^2 \frac{\sin nx}{n} - \left(2x \frac{\cos nx}{n} \right. \right.$$

$$= \left[(2\pi)^2 \frac{\sin n(2\pi)}{n} - \left[2x \frac{-\cos nx}{n} - 2\left(\frac{-\cos nx}{n}\right) \right] \right]_0^{2\pi}$$

$$= \left[(2\pi)^2 \frac{\sin n\pi}{n} - \left(2(2\pi) - \frac{\cos 2n\pi}{n} + 2\frac{\cos 2n\pi}{n} \right) \right]$$

$$= - (0)^2 \frac{\sin(0)}{n} - \left[2(0) - \frac{\cos n(0)}{n} + 2\left(\frac{\cos 0}{n}\right) \right]$$

$$= 4\pi^2 - 4\pi + 2$$

$$= 4\pi^2 - 4\pi + 3$$

$$\int \underset{u}{x^2} \underset{v}{\cos nx}\, dx$$

$$\int u\,v - u' \int v + u'' \int\int v$$

$$\left[\left(x^2\right)\left(\frac{\sin nx}{n}\right) - (2x)\left(-\frac{\cos nx}{n^2}\right) + (2)\left(-\frac{\sin nx}{n^3}\right) \right]_0^{2\pi}$$

$$= \left[(2\pi)^2 \frac{\sin 2n\pi}{n} + 2(2\pi) \frac{\cos 2n\pi}{n^2} - \frac{2 \sin 2n\pi}{n^3} \right] -$$

$$\left[(0)^2 \frac{\sin(0)}{n} + 2(0) \frac{\cos 0}{n^2} - \frac{\sin(0)}{n^3} \right]$$

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$$\cancel{(2\pi)^2}\ 0 + \frac{4\pi}{n^2} - 0 - 0$$

$$= \frac{4\pi}{n^2}$$

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$$b_n = \int_0^{2\pi} \underset{u}{x^2} \underset{v}{\sin nx}\, dx$$

$$= \cancel{\left[(x^2)\frac{\cos nx}{n} + 2x\left(\frac{-\sin nx}{n}\right) + 2\left(\frac{-\sin nx}{n}\right)\right]}$$

$$\left[(x^2)\left(\frac{-\cos nx}{n}\right) - (2x)\left(\frac{-\sin nx}{n^2}\right) + (2)\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \left[(2\pi)^2\left(-\frac{\overset{1}{\cos 2n\pi}}{n}\right) + 2(2\pi)\frac{\overset{0}{\sin 2n\pi}}{n^2} + 2\frac{\overset{1}{\cos 2n\pi}}{n^3}\right] -$$

$$\left[0^2\left(-\frac{\overset{1}{\cos 0}}{n}\right) + 2(0)\frac{\overset{0}{\sin(0)}}{n^2} + 2\frac{\overset{1}{\cos 0}}{n^3}\right]$$

$$= \left[\frac{4\pi^2}{n} - 0 + \frac{2}{n^3}\right] - \left[\frac{1}{n} + 0 + \frac{2}{n^3}\right]$$

$$= \frac{4\pi^2}{n} + \cancel{\frac{2}{n^3}} - \frac{1}{n} - \cancel{\frac{2}{n^3}}$$

$$\frac{4\pi^2}{n}$$

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Find the fourier series of

$$f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum a_n \cos nx + b_n \sin nx$$

$$a_0 \int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \int_{-\pi}^{\pi} f(x) \cos nx\,dx$$

$$b_n = \int_{-\pi}^{\pi} f(x) \sin nx\,dx.$$

$$a_0 = \int_{-\pi}^{0} (-\pi)\,dx + \int_{0}^{\pi} x\,dx$$

$$= -\pi \left| x \right|_{-\pi}^{0} + \left| x^2/2 \right|_{0}^{\pi}$$

$$= -\pi (0 - (-\pi)) + (\pi^2/2 - 0)$$

$$= -\pi^2 + \frac{\pi^2}{2}$$

$$= -\frac{\pi^2}{2}$$

$$\cancel{a_0 = \int_{-\pi}^{0} (-\pi)} \quad a_n = \int_{-\pi}^{0} (-\pi) \cos nx\,dx + \int_{0}^{\pi} \underset{u}{x} \underset{v}{\cos nx}\,.dx$$

$$= -\pi \left| \frac{\sin x}{n} \right|_{-\pi}^{0} + \left| x \left(\frac{\sin nx}{n} \right) - 1 \left(-\frac{\cos nx}{n^2} \right) \right|_{0}^{\pi}$$

$$-\pi \left(\cancel{\frac{\sin 0}{n}} - \cancel{\frac{\sin(-n\pi)}{n}} \right)^{0} + \left[\pi \cancel{\frac{\sin n\pi}{n}}^{0} + \frac{\cos n\pi}{n^2} \right]$$

$$- \left[\cancel{\frac{0 \sin 0}{n}}^{0} + \frac{\cos 0}{n^2} \right]$$

$$= \frac{\cos n\pi}{n^2} - \frac{1}{n^2}$$

$$= \frac{\cos n\pi - 1}{n^2} - \frac{1}{n^2}$$

$$= \frac{\cos n\pi - 1}{n^2} = \begin{cases} -2/n^2 & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}$$

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$$b_n = \int_{-\pi}^{0} (-\pi)\sin nx\,dx + \int_{0}^{\pi} x\sin nx\,dx$$

$$= \left[(-\pi)\left(\frac{-\cos nx}{n}\right)\right]_{-\pi}^{0} + \left[x\left(\frac{-\cos nx}{n}\right) - (1)\left(\frac{-\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$= \pi\left(\frac{\cos 0}{n} - \frac{\cos(-n\pi)}{n}\right) + \left\{-\frac{\pi\cos n\pi}{n} + \cancel{\frac{\sin n\pi}{n^2}}\right\} - \left\{\cancel{-\frac{0\cos 0}{n}} + \cancel{\frac{\sin 0}{n^2}}\right\}$$

$$= \pi\left(\frac{\cos 0}{n} - \frac{\cos(-n\pi)}{n}\right) + \left\{-\frac{\pi\cos n\pi}{n}\right\}$$

$$= \pi\left(\frac{1-\cos(n\pi)}{n}\right) - \frac{\pi}{n}\cos n\pi$$

$$= \frac{\pi}{n}\left(-\frac{2\pi}{n}\cos n\pi\right)$$

$$= \frac{\pi}{n}\left(1 - 2\cos n\pi\right)$$

$$= \begin{cases} \frac{-3\pi}{n} & \text{if } n \text{ is odd} \\ \frac{-\pi}{n} & \text{if } n \text{ is even} \end{cases}$$

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Q2 $f(x) = x(2l-x) \quad x \in (0, 2l)$

formula $a_0 = \int_0^{2l} f(x)\, dx$

$a_n = \int_0^{2l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$

$b_n = \int_0^{2l} f(x) \sin\left(\frac{n\pi x}{l}\right) dx$

$a_0 = \int_0^{2l} x(2l-x)\, dx$

$a_0 = \int_0^{2l} 2lx - x^2\, dx$

$a_0 = \int_0^{2l} 2lx - \int_0^{2l} x^2 dx$

$= 2l \int_0^{2l} x dx - \left|\frac{x^3}{3}\right|_0^{2l}$

$= 2l \left|\frac{x^2}{2}\right|_0^{2l} - \frac{8l^3}{3}$

$= \cancel{2}l\left(\frac{4l^2}{\cancel{2}} - \frac{8l^3}{3}\right)$

$= 4l^3 - \frac{8l^3}{3}$

$= \frac{12l^3 - 8l^3}{3} = \frac{4l^3}{3}$

$a_n = \int_0^{2l} x(2l-x) \cos\left(\frac{n\pi x}{l}\right) dx.$

$= (2lx - x^2) \cos\left(\frac{n\pi x}{l}\right) dx$

$= \left[(2lx - x^2)\frac{l}{n\pi} \sin\left(\frac{n\pi x}{l}\right) - (2l - 2x)\left(\frac{l}{n\pi}\right)^2 \left(-\frac{\cos n\pi x}{l}\right)\right.$

$\left. + (-2) \left(\frac{l}{n\pi}\right)^3 \left(-\sin \frac{n\pi x}{l}\right)\right]_0^{2l}$

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$$\left[2l(2l) - (2l)^2 \frac{1}{n\pi} \sin\left(\frac{n\pi(2l)}{l}\right) - \left(2l - 2(2l)\right)\left(\frac{1}{n\pi}\right)^2 \left(-\frac{\cos n\pi(2l)}{l}\right) + (-2)\left(\frac{1}{n\pi}\right)^3 \left(-\sin\frac{n\pi(2l)}{l}\right) \right]$$

$$4l^2 + 2l - 4l\left(\frac{1}{n\pi}\right)^2 \left(-\frac{\cos n\pi(2l)}{l}\right)$$

$$(2l-2l)\frac{l^2}{n^2\pi^2}\cos\left(\frac{n\pi x}{l}\right)\Big]_0^{2l}$$

$$\{2l - 2(2l)\}\frac{l^2}{n^2\pi^2}\cos\left(\frac{n\pi 2l}{l}\right) - (2l - 0)\frac{l^2}{n^2\pi^2}\cos(0)$$

$$= (-2l)\frac{l^2}{n^2\pi^2} - 2l\frac{l^2}{n^2\pi^2} = -\frac{4l^3}{n^2\pi^2}$$

$$b_n = \int_0^{2l} x(2l - x)\sin\left(\frac{n\pi x}{l}\right) dx$$

$$b_n = \int_0^{2l} (2lx - x^2)\sin\left(\frac{n\pi x}{l}\right) dx$$

$$= (2lx - x^2)(1/n\pi)\left(-\cos\left(\frac{n\pi x}{l}\right)\right) - (2l - 2x)\left(\frac{l}{\pi}\right)^2\left(-\sin\left(\frac{n\pi x}{l}\right)\right) + (-2)\left(\frac{l}{n\pi}\right)^3 \left(\cos\left(\frac{n\pi x}{l}\right)\right)$$

$$= (2lx - x^2)(1/n\pi)\left(-\cos\left(\frac{n\pi 2l}{l}\right)\right) - 2\left(\frac{l^3}{n^3\pi^3}\right)\cos\left(\frac{n\pi x}{l}\right)\Big]_0^{2l}$$

u =

$2lx - x^2$ (1)

$2l - 2x$ (2)

-2 (3)

0

v:

$\sin\left(\frac{n\pi x}{l}\right)$

$\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)}$ (1)

$\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}$ (2)

$\frac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}$ (3)

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$$\left(2l(2l) - (2l)^2\right)\frac{l}{n\pi}\left(-\cos\frac{n\pi(2l)}{l}\right) - 2\left(\frac{l^3}{n^3\pi^3}\right)\left(\cos\frac{n\pi(2l)}{l}\right)$$

$$-\left(2l(0) - (0)^2\right)\left(\frac{l}{n\pi}\right)\left(-\cos\left(\frac{n\pi(0)}{l}\right)\right) - 2\left(\frac{l^3}{n^3\pi^3}\right)\left(\cos\frac{n\pi(0)}{l}\right)$$

$$4l^2 - 4l^2\left(\frac{l}{n\pi}\right) - 2\left(\frac{l^3}{n^3\pi^3}\right) + 2\left(\frac{l^3}{n^3\pi^3}\right)$$

$$= 0$$

$f(1) = x^2 - 2 \qquad -2 < x < 2$

$$a_0 = \int_{-2}^{2} \frac{x^3}{3} - 2x \Big]_{-2}^{2}$$

$$= \left[\frac{2^3}{3} - 2(2) - \frac{(-2)^3}{3} - 2(-2)\right]$$

$$= \left(\frac{8}{3} - 4 - \frac{8}{3} + 4\right]$$

$$= \frac{4}{3} - \frac{12}{3}$$

$$= \frac{8}{3}$$

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$$a_n = \int_{-2}^{2} (x^2-2)\cos\left(\frac{n\pi x}{2}\right)dx$$

$$a_n = \left[2x\left(\frac{2}{n\pi}\right)^2 \cos\left(\frac{n\pi x}{2}\right)\right]_{-2}^{2}$$

$$= \left[2(2)\frac{4}{n^2\pi^2}\cos(n\pi)\right] - \left[2(-2)\frac{4}{n^2\pi^2}\cos(-n\pi)\right]$$

$$= \frac{16}{n^2\pi^2}\cos n\pi + \frac{16}{n^2\pi^2}\cos n\pi$$

$\cos(-\theta) = \cos\theta$

$$= \frac{32}{n^2\pi^2}\cos n\pi$$

$$b_n = \int_{-2}^{2} (x^2-2)\sin\left(\frac{n\pi x}{2}\right)dx$$

$$b_n = \quad x^2-2 \rightarrow \frac{-\cos\left(\frac{n\pi x}{2}\right)}{n\pi/2}$$

$$2x = \frac{-\sin\left(\frac{n\pi x}{2}\right)}{(n\pi/2)/2}$$

$$2 = \frac{\cos\left(\frac{n\pi/2\, x}{2}\right)}{(n\pi/2)/3}$$

$$\int_{-2}^{2} (x^2-2)\left(-\cos\left(\frac{n\pi x}{2}\right)\right)\left(\frac{2}{n\pi}\right) - 2x\left(\cancel{-\sin\left(\frac{n\pi x}{2}\right)}\right)^{0}\left(\frac{2}{n\pi}\right)^2$$

$$+ 2\left(\frac{2}{n\pi}\right)^3\left(\cos\left(\frac{n\pi x}{2}\right)\right)$$

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$$= (x^2-2)\left(-\cos\left(\frac{n\pi x}{2}\right)\right)\left(\frac{2}{n\pi}\right) - 2\left(\frac{2}{n\pi}\right)^3 \cos\left(\frac{n\pi x}{2}\right)$$

$$= \left[(2^2-2)\left(-\cos\left(\frac{2\pi(2)}{2}\right)\right) - 2\left(\frac{2}{n\pi}\right)^3 \cos\left(\frac{n\pi(2)}{2}\right)\right.$$

$$\left. - \left[(-2)^2-2)\left(-\cos\left(\frac{n\pi(-2)}{2}\right)\right)\left(\frac{2}{n\pi}\right) - 2\left(\frac{2}{n\pi}\right)^3 \cos\left(\frac{n\pi(-2)}{2}\right)\right]\right.$$

$$(4-2) - 2 + \left(\frac{8}{n^3\pi^3}\right) - (4-2)\left(\frac{2}{n\pi}\right) - \left(\frac{8}{n^3\pi^3}\right)$$

$$2 - 2\left(\frac{8}{n^3\pi^3}\right) - \left(4 - \frac{4}{n\pi}\right)$$

$$= 0\left(\frac{8}{n^3\pi^3}\right) - 0$$

$$= 0$$

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DIFFERENTIAL EQUATION

General form of second order lineal D E

$$\frac{d^2y}{dx^2} + P\left(\frac{dy}{dx}\right) + Q(y) = R(x)$$

where P Q R are constants

RULES FOR FINDING COMPLEMENTORY FUNCTION

1) Replace D by M where $D = \frac{dy}{dx}$ then we get Q E in M i,e $m^2 + Pm + Q = 0$

2) Solve this eqn and get two roots $\alpha, \beta = m$

=> if α, β are real then $y(x) = C_1 e^{\alpha x} + C_2 e^{\beta x}$

=> if α, β are complex then it will $m = a \pm ib$

$$y(x) = e^{ax}\{C_1 \cos bx + C_2 \sin bx\}$$

if $\alpha = \beta$ then $y(x) = (C_1 + C_2 x) e^{\alpha x}$

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eg. $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Auxiliary eqn is

$(m^2+m)y = 0$

$m^2+m=0$

$m(m+1)=0$

$m = 0, -1$ → otherwise

$\therefore y(x) = C_1 e^{0x} + C_2 e^{-1\times x}$

$= C_1 + C_2 e^{-x}$

$ax^2+bx+c=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a=1 \quad b=1 \quad c=0$

$x = \frac{-1 \pm \sqrt{1^2-4\times1\times0}}{2\times1}$

$= \frac{-1 \pm 1}{2}$

$= 0, -1$

Q2) solve 1. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$ 3. $(D^2-6D+9)y=0$

2. $\frac{d^2y}{dx^2} + y = 0$ 4. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

5) $(D^2+4D+3)y=0$

1) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$(m^2-2m)y=0$ $(m^2-2m)=0$

$m(m-m)y=0$ $m(m-m)=0$

$m=0, \quad m=1$

$y(x) = m^2y - 2my + 1 = 0$

$y(m^2-2m+1)y=0$

$m^2-2m+1=0$

$a=1 \quad b=2 \quad c=1$

$y = (C_1 + C_2 x)e^x$

$m^2+1=0$

$m = 0 \pm \sqrt{0^2-1}$

$\frac{-2 \pm \sqrt{2^2-4\times1\times1}}{2\times1}$

$= \frac{-2 \pm \sqrt{0}}{2}$

$= 1, 1$

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2) $\frac{d^2y}{dx^2} + y = 0$

$(m^2 y + y) = 0$

$y(m^2+1) = 0$

$m^2 + 1 = 0$

$m = \frac{0 \pm \sqrt{0^2 - 4 \times 1 \times 1}}{2}$

$= \frac{\pm\sqrt{-4}}{2}$

$= \frac{\pm\sqrt{-1 \cdot 4}}{2}$

$= \frac{\pm \cancel{2}\sqrt{-1}}{\cancel{2}}$

$= \pm i$

$m = i, -i$

$y(x) = e^{0x}(c_1 \cos 1x + c_2 \sin 1x)$

3) $(D^2 - 6D + 9)y = 0$

$(m^2 y - 6my + 9)y$

$y(m^2 - 6m + 9) = 0$

$a = 1 \quad b = -6 \quad c = 9$

$\frac{+6 \pm \sqrt{-6^2 - 4 \times 1 \times 9}}{2 \times 1}$

$= \frac{6 \pm \sqrt{36 - 36}}{2}$

$\frac{6 \pm \sqrt{0}}{2}$

$(3, 3) = m$

$y(x) = (c_1 + c_2 x)e^{3x}$

Classic

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4) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5y = 0$

$a = 1 \quad b = 4 \quad c = 5$

$$\frac{-4 \pm \sqrt{4^2 - 4(5)(1)}}{2 \times 1}$$

$$= \frac{-4 \pm \sqrt{16 - 4(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm \sqrt{-1 \cdot 4}}{2}$$

$$= \frac{-\cancel{4}^{2} \pm 2\sqrt{-1}}{\cancel{2}_{1}}$$

$$= \frac{-4 \pm 2i}{2}$$

$$\frac{-4+2}{2} = \frac{-\cancel{2}}{\cancel{2}} = -1$$

$$\boxed{m = \frac{-\cancel{4}^{2} + 2i}{\cancel{2}} , \frac{-\cancel{4}^{2} - 2i}{\cancel{2}_{1}}}$$

$$\frac{-4-2}{2} = -\frac{6}{2}$$

$m = -2 \pm i$

$y(x) = e^{-2x}(c_1 \cos ix + c_2 \sin ix)$

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5) $(D^2+4D+3)y=0$

$(m^2+4m+3)=0$

$a=1 \quad b=4 \quad c=3$

$$\frac{-4 \pm \sqrt{4^2-4\times 3\times 1}}{2\times 1}$$

$$\frac{-4 \pm \sqrt{16-12}}{2}$$

$$\frac{-4 \pm \sqrt{4}}{2}$$

$$\frac{-4 \pm 2}{2}$$

$\Rightarrow \frac{-4-2}{2} = \frac{-\cancel{6}^{3}}{\cancel{2}}$ $\quad \Rightarrow \frac{-4+2}{2} = \frac{-\cancel{2}^{1}}{\cancel{2}}$

$\boxed{m: \quad -3, -1}$

$y = c_1 e^{-3x} + c_2 e^{-x}$.

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Method of variation of parameter

$$y'' + py' + qy = R(x)$$

Step 1 = solve

$y'' + py' + qy = 0$

we get two soln's

$y_1(x)$ and $y_2(x)$

Step 2 = Find $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$, known as wronskian $(W \neq 0)$

Step 3 = Find $c_1(x) = \int \frac{y_2 R(x)}{W} dx$

and $c_2(x) = \int \frac{y_1 R(x)}{W} dx$

$\Rightarrow y_p = c_1 y_1 + c_2 y_2$

Step 4 = soln of given DE is

$y(x) = C.F + y_p$

Q1) $(D^2 - 2D + 4)y = e^{2x} \sin x$.

Step 1 : $(D^2 - 2D + 4)y = 0$

Replace D by m

$m^2 - 2m + 4 = 0$

$(m-2)^2 = 0$

$m = 2, 2$

$C.F = (c_1 + c_2 x)e^{2x}$

$= c_1 e^{2x} + c_2 x e^{2x} \rightarrow (1)$

$y_1(x) = e^{2x}$

$y_2(x) = xe^{2x}$

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$y_2 = xe^{2x}$

$\cancel{xe^{2x}}$ $xe^{2x} + e^{2x}$

Step2

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$\begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & 2xe^{2x}+e^{2x} \end{vmatrix}$$

$$e^{2x}(2xe^{2x}+e^{2x}) - xe^{2x}(2e^{2x})$$
$$= \cancel{2xe^{4x}} + e^{4x} - \cancel{2xe^{4x}}$$
$$= e^{4x}$$

Step3 $\quad c_1 = \int \frac{y_2 R(x)}{W} dx$

$$= \int \frac{xe^{2x} e^{2x} \sin x}{e^{4x}}$$

$$= \int \underset{u}{x} \underset{v}{\sin x} \, dx$$

formula $= u\int v - u'\iint v + u''\iiint v$

$$= x\int \sin x dx - 1 \cdot \iint \sin x$$
$$= -x\cos x + \cos x$$

$$c_2 = \int \frac{y_1 R(x)}{W} dx = \int \frac{e^{2x} e^{2x}}{e^{4x}} \sin x$$

$$= -\cos x$$

Step4

$$y(x) = c_1' e^{2x} + c_2' xe^{2x} + c_1 y_1 + c_2 y_2$$

$$= c_1' e^{2x} + c_2' xe^{2x} + (-x\cos x + \cos x)e^{2x} + (-\cos x)xe^{2x}$$

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$y'' + 4y = \tan 2x$

$m^2 + 4 = 0$

$m^2 = -4$

$m = \pm\sqrt{-4}$

$= \pm 2\sqrt{-1}$

$= \pm 2i$

$CF = (c_1' + c_2' x)$

$= e^{0x}(c_1' \cos 2x + c_2' \sin 2x)$

$= c_1' \cos 2x + c_2' \sin 2x$

$y_1 = \cos 2x, \quad y_2 = \sin 2x$

Step 2 : $\begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$

$= 2\cos^2 2x - (-2\sin^2 2x)$

$= 2(\cos^2 2x + \sin^2 2x)$ $\quad \sin^2\theta + \cos^2\theta = 1$

$= 2$

Step 3

$c_1 = \int \frac{y_2 R(x)}{W}$

$= \int \frac{\sin 2x \tan 2x}{2} dx$

$c_2 = \int \frac{y_1 R(x)}{W} = \int \frac{\cos 2x \tan 2x}{2} dx$

$= \frac{1}{2} \int \sin 2x \, dx$

$= \frac{1}{2} \frac{-\cos 2x}{2} = \frac{-\cos 2x}{4}$

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Step4:- $y(x) = c_1' \cos 2x + c_2' \sin 2x + \cos 2x \int \frac{\sin^2 2x}{2\cos 2x} dx$

$+ \sin 2x \left(\frac{\cos 2x}{t}\right)$

$Q_3 = y'' - y = \frac{2}{1+e^x}$

$m^2 - 1 = 0$

$m^2 = 1$

$m = \sqrt{1}$

$= m = \pm 1$

$CF = c_1' e^x + c_2' e^{-x}$

$y_1 = e^x, \; y_2 = e^{-x}$

Step2 = $W = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}$

$\begin{cases} e^x \cdot e^{-x} \\ = e^{x-x} \\ = e^0 = 1 \end{cases}$

$-1 - 1 = -2$

Step3 = $c_1 \int \frac{y_2 R(x)}{W}$

$\int \frac{e^{-x} \, 2}{1+e^x(-2)}$

$= -\int \frac{e^{-x}}{1+e^x} dx$

$c_2 = \int \frac{e^x}{-2} \frac{2}{1+e^x} dx$

$- \int \frac{e^x}{1+e^x} dx$

$=$ Let $1+e^x = t$

$e^x dx = dt$

$-\int \frac{dt}{t}$

$= -\log t$

$= -\log(1+e^x)$

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$1(e^{3x}) + x\,3e^{3x}$

step 4 = $y(x) = c_1 e^{x} + c_2 e^{-x}$

$-e^{x}\int \frac{e^{-x}}{1+e^{x}}\,dx - e^{-x}\log(1+e^{x})$

Que $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$

step 1 :- $m^2 - 6m + 9 = 0$ $\quad \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= (3, 3)$.

$CF = (c_1 e^{3x} + c_2' e^{3}x)$

$(c_1 + c_2 x)e^{3x}$

$y_1 = e^{3x}$ $y_2 = xe^{+3x}$.

step 2 = $w = \begin{vmatrix} e^{3x} & xe^{+3x} \\ 3e^{3x} & e^{3x} + x3e^{3x} \end{vmatrix}$

$= e^{3x}(e^{3x} + x3e^{3x}) - xe^{3x}(3e^{3x})$

$\cancel{e^{2x} + x3e^{2x} - x3e^{2x}}$

$= e^{6x}$.

step 3 =

$c_1 \int \frac{y_2 R(x)}{w}$ | $c_2 \int \frac{y_1 R(x)}{w}$

$= \int \frac{xe^{3x} \frac{e^{3x}}{x^2}}{e^{6x}}\,dx$ | $c_2 = \frac{e^{3x}\left(\frac{e^{3x}}{x^2}\right)dx}{e^{6x}}$

$= \log x$ | $\int x^{-2}\,dx$

$= \frac{x^{-2+1}}{-2+1}$

$= -1/x$

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Step4 :- $y(x) = c_1' e^{3x} + c_2' x e^{3x}$
$+ e^{3x} \log x - \cancel{x} e^{3x} \frac{1}{\cancel{x}}$

85) $y'' - 2y' + y = e^x \log x$

$m^2 - 2m = 0$

$(1, 1)$

$y = (c_1 + c_2 x) e^x$

$\frac{x \cdot e^x}{1 e^x - x e^x}$

$y_1 = e^x \quad y_2 = x e^x$

Step2 :- $\begin{vmatrix} e^x & x e^x \\ e^x & e^x - x e^x \end{vmatrix}$

$e^x(e^x - x e^x) + e^x(x e^x)$

$e^{2x} - \cancel{x e^{2x}} + \cancel{x e^{2x}}$

e^{2x}

Step3 $c_1 = \int \frac{x e^x \, e^x \log x}{e^{2x}} dx$

$= \int x \log x \, dx$

$\cancel{x \int \log x - 1 \int \log x}$

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$$C_2 = \int \frac{e^x \cdot e^x \log x}{e^{2x}}$$

$$= x \log - x$$

Step 4 :-

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Undetermined co-efficients

$y'' + py' + qy = R(x)$

If $R(x) = e^{3x}$

$y_p = ae^{3x}$

If $R(x) = 2\sin x$

$y_p = a\sin x + b\cos x.$

If $R(x) = x^2$

$y_p = ax^2 + bx + c$

m	0	a & b, b=1
R(x)	x^2	sinx, cosx
y_p	$x(ax^2+bx+c)$	$(a\sin x + b\cos x)$

Q1) $(D^2+D)y = x^2+2x+4$

$m^2+m=0$

$m(m+1)=0$

$m=0, -1$

$c.f = c_1e^{0x} + c_2e^{-x} \rightarrow (1)$

$R(x) = x^2+2x+4$ is a polynomial

Let $y_p = x(ax^2+bx+c)$

$= ax^3+bx^2+cx$

$y_p' = 3ax^2+2bx+c$

$y_p'' = 6ax+2b$

wkt

$y''+y' = x^2+2x+4$

$6ax+2b+(3ax^2+2bx+c) = x^2+2x+4$

compare both sides

$3a=1$ $\quad a = 1/3$

$6a+2b=2$ $\quad 6(\frac{1}{3})+2b=2 \quad b=0$

$2b+c=4$ $\quad c=4$

$y_p = \frac{1}{3}x^3+4x \rightarrow (2)$

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Q2> $y'' + 4y' + 4y = 3\sin x + 4\cos x$, $y(0) = 1$, $y'(0) = 0$

$m^2 + 4m + 4 = 0$

$(m+2)^2 = 0$

$m = -2$, $m = -2$

$CF = (c_1 + c_1 x)e^{-2x} \rightarrow (1)$

Let $y_p = A\sin x + B\cos x$.

$y_p' = A\cos x - B\sin x$

$y_p'' = -A\sin x - B\cos x \rightarrow (2)$

$(-A\sin x - B\cos x) +$

$4(A\cos x - B\sin x) +$

$4(A\sin x + B\cos x) = 3\sin x + 4\cos x$

$= \sin x(3A - 4B) + \cos x(4A - 3B) = 3\sin x + 4\cos x.$

comparing both sides

$3A - 4B = 3$

$4A - 3B = 4$

$A = 1$

$B = 0$

$y_p = \sin x \rightarrow (3)$

$y(x) = (c_1 + c_2 x)e^{-2x} + \sin x$

$y(0) = c_1 = 1$

$y'(x) = e^{-2x}c_2 + e^{-2x}(c_1 + c_2 x)(-2) + \cos x$

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3) $(D^2+1)y = \sin x$

$m^2+1=0$	$a=0$
$m=\pm i$	$b=1$

$C.F = C_1\cos x + C_2\sin x \rightarrow (1)$

Let $y_p = x(A\sin x + B\cos x)$

$\cancel{Ax\sin x + Bx\cos x}$

$\cancel{y_p' = Ax\cos x - Bx\sin x}$

$y_p' = (A\sin x + B\cos x)1 + x(A\cos x - B\sin x)$

$y''_p = (A\cos x - B\sin x) + (A\cos x - B\sin x)1 + x(-A\sin x - B\cos x)$

$= 2A\cos x - 2B\sin x - Ax\sin x - Bx\cos x$

$= 2A\cos x - 2B\sin x = 1\cdot\sin x + 0\cdot\cos x$

$2A = 0$

$-2B = 1$

$A = 0, \quad B = -1/2$

$y_p = x\left(-\frac{1}{2}\cos x\right) \rightarrow (2)$

$y(x) = C_1\cos x + C_2\sin x - \frac{x}{2}\cos x$

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Q. System of linear eqn's

$$\left.\begin{aligned} x+y&=3 \\ 2x+3y&=5 \end{aligned}\right\}$$

$y = 3 - x$

$2x + 3(3-x) = 5$

$x =$

Gauss - Elimination method

$Ax = b$

$[A \,\vdots\, b]$

$10x_1 - x_2 + 2x_3 = 4$

$x_1 + 10x_2 - x_3 = 3$

$2x_1 + 3x_2 + 20x_3 = 7$

Diagonally dominant

$$\left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 1 & 10 & -1 & 3 \\ 2 & 3 & 20 & 7 \end{array}\right]$$

$R \to 1/10$ [∵ shift + a b/c ↓ Fraction method]

Pivot

$$\left[\begin{array}{cccc} \boxed{1} & -1/10 & 2/10 & 4/10 \\ 1 & 10 & -1 & 3 \\ 2 & 3 & 20 & 7 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{cccc} 1 & -1/10 & 2/10 & 4/10 \\ 0 & 101/10 & -6/5 & 13/5 \\ 0 & 16/5 & 98/5 & 31/5 \end{array}\right]$$

$\frac{10R_2}{101}$ [∵ shift + a b/c for fraction]

$$\left[\begin{array}{cccc} 1 & -1/10 & 2/10 & 4/10 \\ 0 & 1 & -12/101 & 26/101 \\ 0 & 16/5 & 98/5 & 31/5 \end{array}\right]$$

$R_3 \to R_3 - \frac{16}{-5} R_2$

$$\left[\begin{array}{cccc} 1 & -1/10 & 2/10 & 4/10 \\ 0 & 1 & -12/101 & 26/101 \\ 0 & 0 & 2018/101 & 543/101 \end{array}\right]$$

$R_3 \times \frac{101}{2018}$

$$\left[\begin{array}{cccc} 1 & -1/10 & 2/10 & 4/10 \\ 0 & 1 & -12/101 & 26/101 \\ 0 & 0 & 1 & 543/2018 \end{array}\right]$$

mode →

$$x_3 = \frac{543}{2018} = 0.269$$

$$x_2 - \frac{12}{101} x_3 = \frac{26}{101}$$

$$x_2 = 0.25 + (0.11)(0.269)$$
$$= 0.38$$

$$x_1 = 0.375$$

Solve equ^n using Gauss- Elimination method

$$x_1 + x_2 + x_3 = 6$$
$$3x_1 + 3x_2 + 4x_3 = 20$$
$$2x_1 + x_2 + 3x_3 = 13$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 3 & 3 & 4 & 20 \\ 2 & 1 & 3 & 13 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$
$R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 1 & 1 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -(-1) & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] \end{array}$$

$R_2 \to -R_2$

$$x_3 = 2$$
$$x_2 - x_3 = -1$$

$x_2 = 2 - 1 = 1$

$x_1 + x_2 + x_3 = 6$

$x_1 + 1 + 2 = 6$

$x_1 = 3$

Gauss Jordan Method

$x_1 + x_2 + x_3 = 1$

$4x_1 + 3x_2 - x_3 = 6$

$3x_1 + 5x_2 + 3x_3 = 4$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

$[A : I]$

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 4 & 3 & -1 & 0 & 1 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -5 & -4 & 1 & 0 \\ 0 & 2 & 0 & -3 & 0 & 1 \end{array}\right] \quad R_2 \to -R_2$$

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 5 & 4 & -1 & 0 \\ 0 & 2 & 0 & -3 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - R_2 \\ R_3 \to R_3 - 2.R_2 \end{array}$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & -4 & -3 & 1 & 0 \\ 0 & 1 & 5 & 4 & -1 & 0 \\ 0 & 0 & -10 & -11 & 2 & 1 \end{array}\right] \quad R_3 \to \frac{R_3}{-10}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -4 & 1 & -3 & 1 \\ 0 & 1 & 5 & 4 & -1 & 0 \\ 0 & 0 & 1 & 11/10 & -2/10 & -1/10 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + 4R_3 \\ R_2 \to R_2 - 5R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 14/10 & +2/10 & -4/10 \\ 0 & 1 & 0 & -15/10 & 0 & 5/10 \\ 0 & 0 & 1 & 11/10 & -2/10 & -1/10 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 14/10 & +2/10 & -4/10 \\ -15/10 & 0 & 5/10 \\ 11/10 & -2/10 & -1/10 \end{bmatrix}$$

$$b = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$$

$$x = A^{-1}b = \begin{bmatrix} \frac{14+12-16}{10} \\ \frac{-15+0+20}{10} \\ \frac{11-12-4}{10} \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ -1/2 \end{bmatrix}$$

2 M

LU decomposition method

Solve the system of eqn LU decomposition method.

Take $u_{11} = u_{22} = u_{33} = 1$

$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} = \underbrace{\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}}_{U}$$

$$= \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12}+l_{22} & l_{21}u_{13}+l_{22}u_{23} \\ l_{31} & l_{31}u_{12}+l_{32} & l_{31}u_{13}+l_{32}u_{23}+l_{33} \end{bmatrix}$$

$l_{11}=1$ $\quad l_{11}u_{12}=1$ $\quad l_{11}u_{13}=1$

$l_{21}=4$ $\quad l_{21}u_{12}+l_{22}=3$ $\quad l_{21}u_{13}+l_{22}u_{23}=-1$

$l_{31}=3$ $\quad l_{31}u_{12}+l_{32}=5$ $\quad l_{31}u_{13}+l_{32}u_{23}+l_{33}=3$

$u_{12}=1$ $\quad 4\times1+l_{22}=3$ $\quad 4\times1+(-1)u_{23}=-1$

$u_{13}=1$ $\quad l_{22}=3-4$ $\quad -u_{23}=-1-4$

$l_{22}=-1$ $\quad =-1$ $\quad u_{23}=-5$

$u_{23}=5$

$l_{32}=2$ $\quad 3\times1+l_{22}=5$ $\quad 3\times1+2\times5+l_{33}=3$

$l_{33}=-10$ $\quad l_{32}=2$ $\quad 3+10+l_{33}=3$

$$L=\begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 3 & 2 & -10 \end{bmatrix}$$

$$U=\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$Ax=b$

$LUx=b$

Let $Ux=Z$

$LZ=b$

$Z=L^{-1}b$

$x=U^{-1}Z$

Step 2: we know $LZ=b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 3 & 2 & -10 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}=\begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

$z_1=1$

$4z_2-z_2=6$

$z_2=-2$

$3z_1+2z_2-10z_3=4$

$3(1)+2(-2)-10z_3=4$

$3-4-10z_3=4$

$z_3=-1/2$

Step 3:

$$Ux = z$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -1/2 \end{bmatrix}$$

$$x_3 = -1/2$$

$$x_2 + 5x_3 = -2$$

$$x_2 + 5(-1/2) = -2$$

$$x_2 - 5/2 = -2$$

$$x_2 = -2 + 5/2$$

$$x_2 = 1/2$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1 + (1/2) + (-1/2) = 1$$

$$\therefore x_1 = 1$$

$$X = \begin{pmatrix} 1 \\ 1/2 \\ -1/2 \end{pmatrix}$$

$\Rightarrow$ $x_1 + 2x_2 + 3x_3 = 5$

$2x_1 + 8x_2 + 22x_3 = 6$

$3x_1 + 22x_2 + 82x_3 = -10$

$u_{11} = 1,\ u_{22} = 1,\ u_{33} = 1$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} = \underbrace{\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}}_{U}$$

$$= \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{13} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

$l_{11} = 1$	$l_{11}u_{12} = 2$	$l_{11}u_{13} = 3$
$l_{21} = 2$	$l_{21}u_{12} + l_{22} = 8$	$l_{21}u_{13} + l_{22}u_{23} = 22$
$l_{31} = 3$	$l_{31}u_{13} + l_{32} = 22$	$l_{31}u_{13} + l_{32}u_{23} + l_{33} = 82$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 2 & 8 & 22 & 6 \\ 3 & 22 & 82 & -10 \end{array}\right]$$

[Gauss elimination method

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 4 & 18 & -4 \\ 0 & 16 & 73 & -25 \end{array}\right]$$

~~$R_3 \to R_3 - 16R_2$~~
$R_2 \to R_2/4$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 4 & 16 & -4 \\ 0 & 16 & 73 & -25 \end{array}\right]$$

$R_3 \to \frac{R_3}{9}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 4 & -1 \\ 0 & 0 & 9 & -9 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 4 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$x_3 = -1$$
$$x_2 + 4x_3 = -1$$
$$x_2 + 4(-1) = -1$$
$$x_2 - 4 = -1$$
$$x_2 = 3$$

$$x_1 + 2x_2 + 3x_3 = 5$$
$$x_1 + 2(3) + 3(-1) = 5$$
$$x_1 + 6 - 3 = 5$$
$$x_1 = 5 - 3$$
$$x_1 = 2$$

$\therefore$ $x_1 = 2$
$x_2 = 3$
$x_3 = -1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 2 & 8 & 22 & 6 \\ 3 & 22 & 82 & -10 \end{array}\right]$$ [Gauss Jordan Method]

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}$$

$[A : I]$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 8 & 22 & 0 & 1 & 0 \\ 3 & 22 & 82 & 0 & 0 & 1 \end{array}\right]$$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 4 & 16 & -2 & 1 & 0 \\ 0 & 16 & 73 & -3 & 0 & 1 \end{array}\right]$$ $R_2 \to R_2/4$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & -1/2 & 1/4 & 0 \\ 0 & 16 & 73 & -3 & 0 & 1 \end{array}\right]$$ $R_3 \to R_3 - 16R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & -1/2 & 1/4 & 0 \\ 0 & 0 & 9 & 5 & -16/4 & 1 \end{array}\right]$$ $R_1 \to R_1 - 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -5 & 2 & -1/2 & 0 \\ 0 & 1 & 4 & -1/2 & 1/4 & 0 \\ 0 & 0 & 9 & 5 & -16/4 & 1 \end{array}\right]$$ $R_3 \to \frac{R_3}{9}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -5 & 2 & -1/2 & 0 \\ 0 & 1 & 4 & -1/2 & 1/4 & 0 \\ 0 & 0 & 1 & 5/9 & -4/9 & 1/9 \end{array}\right]$$ $R_2 \to R_2 - 4R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -5 & 2 & -1/2 & 0 \\ 0 & 1 & 0 & -49/18 & 73/36 & -4/9 \\ 0 & 0 & 1 & 5/9 & -4/9 & 1/9 \end{array}\right] \quad R_1 \to R_1 + 5R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 43/9 & -49/18 & 5/9 \\ 0 & 1 & 0 & -49/18 & 73/36 & -4/9 \\ 0 & 0 & 1 & 5/9 & -4/9 & 1/9 \end{array}\right] \quad A^{-}$$

$A^{-1} \times B$

$$\begin{bmatrix} 43/9 & -49/18 & 5/9 \\ -49/18 & 73/36 & -4/9 \\ 5/9 & -4/9 & 1/9 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ -10 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}.$$

<u>cholesky method</u>

$A = LL$

$$= \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$$= \begin{bmatrix} l_{11}^2 & l_{11} l_{21} & l_{11} l_{31} \\ l_{21} l_{11} & l_{21}^2 + l_{22}^2 & l_{21} l_{31} + l_{22} l_{32} \\ l_{31} l_{11} & l_{31} l_{21} + l_{32} l_{22} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{bmatrix}$$

Eg: $x_1 + 2x_2 + 3x_3 = 5$

$2x_1 + 8x_2 + 22x_3 = 6$

$3x_1 + 22x_2 + 82x_3 = -10$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} \qquad b = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$$

$$= \begin{bmatrix} l_{11}^2 & l_{11}l_{21} & l_{11}l_{31} \\ l_{21}l_{11} & l_{11}^2 + l_{22}^2 & l_{21}l_{31} + l_{22}l_{32} \\ l_{31}l_{11} & l_{31}l_{21} + l_{32}l_{22} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{bmatrix}$$

$l_{11}^2 = 1$ $\qquad l_{11}l_{21} = 2$ $\qquad l_{11}l_{31} = 3$

$l_{21}l_{11} = 2$ $\qquad l_{21}^2 + l_{22}^2 = 8$ $\qquad l_{21}l_{31} + l_{22}l_{32} = -22$

$l_{31}l_{11} = 3$ $\qquad l_{31}l_{21} + l_{32}l_{22} = 22$ $\qquad l_{31}^2 + l_{32}^2 + l_{33}^2 = 82$

$l_{11} = 1$ $\qquad l_{21}^2 + l_{22}^2 = 8$ $\qquad l_{21}l_{31} + l_{22}l_{32} = 22$

$l_{21} = 2$ $\qquad 4 + l_{22}^2 = 8$ $\qquad 2 \times 3 + 2l_{32} = 22$

$l_{31} = 3$ $\qquad l_{22}^2 = 4$ $\qquad 2l_{32} = 22 - 6$

$l_{22} = 2$ $\qquad l_{22} = 2$ $\qquad = 16$

$l_{32} = 8$ $\qquad l_{32} = 8$

$l_{33} = 3$

$9 + 64 + l_{33}^2 = 82$

$l_{33} = 3$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 8 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 8 \\ 0 & 0 & 3 \end{bmatrix}$$

$$Ax = b$$

$$\underbrace{LL^T x}_{z} = b$$

$$Lz = b$$

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A$$

$$B = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

$\det B = adf$ [∵ Multiplication of diagonal]

Co-factor of $a = df$

" " $b = 0$

$$\text{Cofactor} = \begin{bmatrix} df & 0 & 0 \\ -bf & af & 0 \\ be-ad & -ae & ad \end{bmatrix}$$

$$\operatorname{adj} A = \begin{bmatrix} df & -bf & be-ad \\ 0 & af & -ae \\ 0 & 0 & ad \end{bmatrix}$$

Inverse of

$$L = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 5/3 & 0 \\ 1 & 4/3 & 3/5 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2/3 & 1/3 \\ 0 & 1 & 4/5 \\ 0 & 0 & 1 \end{bmatrix}$$

MODULE-4

NUMERICAL DIFFERETIATION AND INTEGRATION.

Differentiation

Forward Difference scheme

$$f'(a) = \frac{f(a+h) - f(a)}{h}$$

a - given point

h ⇒ Step length.

x	1	2	3
$f(x)$	0	0.5	0.7	0.9

If we have to begining of the number Use forward

Backward Diff scheme

$$f'(a) = \frac{f(a) - f(a-h)}{h}$$

$a-h \rightarrow a \rightarrow a+h$

Central Diff scheme

$$f'(a) = \frac{f(a+h) - f(a-h)}{2h}$$

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Note

→ If we have to use the number in the beginning of the table then use FORWARD SCHEME

→ To find the value at the end of the table we have to use BACKWARD SCHEME

→ To find the value at the Middle of the table we have to use CENTRAL SCHEME

Find $f'(x)$ for $f(x) = \frac{\sin(0.5\sqrt{x})}{x}$ at $x = 0.1$ using Forward differential scheme, backward differential scheme & central differential scheme. for $h = 0.01$

FDS, $a = 0.1$, $h = 0.01$

$$f'(a) = \frac{f(a+h) - f(a)}{h}$$

$$= \frac{f(0.1 + 0.01) - f(0.1)}{0.01}$$

$$= \frac{f(0.11) - f(0.1)}{0.01}$$

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$$= \frac{\sin(0.5\sqrt{x})}{x}$$

$$f(0.11) = \frac{\sin(0.5\sqrt{0.11})}{0.11}$$

$$= 1.5006$$

$$f(0.1) = \frac{\sin(0.5\sqrt{0.1})}{0.1}$$

$$= 1.57$$

$$= \frac{1.5006 - 1.5745}{0.01}$$

$$= -7.39.$$

BDS.

$$f'(0.1) = \frac{f(0.1) - f(0.1 - 0.01)}{0.01}$$

$$= \frac{f(\overset{x}{0.1}) - f(\overset{x}{0.09})}{0.01}$$

$$= \frac{\sin(0.5\sqrt{0.1})}{0.01}$$

$$= 15.745$$

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$$= \frac{\sin(0.5\sqrt{0.09})}{0.09}$$

$$= 16.604.$$

$$= \frac{15.745 - 16.604}{0.01}$$

$$= -85.9.$$

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2. The distance 'x' of a runner from a fixed point is measured at intervals of a sec. The data obtain is

t	0	0.5	1	1.5	2
x	0	3.65	6.80	9.90	12.15

Find the velocity obtain 0 sec, 0.5 sec, 1 sec.

Velocity, $x'(t) = \frac{dx}{dt} \frac{x(t+h)-x(t)}{h}$

at $t = 0.5, 1.5, 2.5$.

→ Forward Difference Scheme

$$x'(t) = \frac{x(t+h)-x(t)}{h}$$

Formula

$$f'(a) = \frac{f(a+h)-f(a)}{h}$$

$t=1, \quad h=0.5$

$$x'(1) = \frac{x(1+0.5)-x(1-0.5)}{2(0.5)}$$

$$= \frac{x(1.5)-x(0.5)}{2(0.5)}$$

$$= \frac{9.90-3.65}{1}$$

$$= 6.5$$

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$$x'(2) = \frac{x(2) - x(2-0.5)}{0.5}$$

Formula

$$f'(a) = \frac{f(a) - f(a-h)}{h}$$

$$= \frac{72.15 - 990}{0.5}$$

$$= 7.5$$

$$x'(0) = \frac{x(0+0.5) - x(0)}{0.5}$$

Formula

$$f'(a) = \frac{f(a+h) - f(a)}{h}$$

$$= \frac{3.65 - 0}{0.5}$$

$$= 7.3$$

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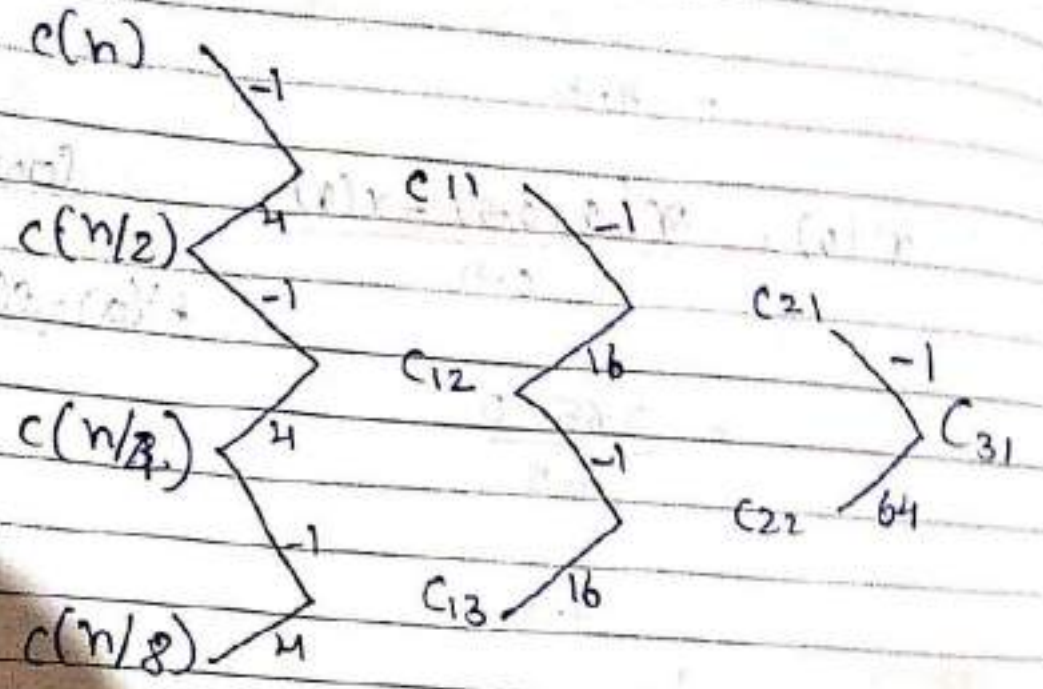
Richardson Extrapolation

Apply Central Difference scheme.

Give a, h



M=More, L=Least Acc

$\frac{4MA-LA}{3}$ $\frac{16MA-LA}{15}$ $\frac{64MA-LA}{63}$

Estimate the value of sinx using Richardson extrapolation for $a=\pi/4$, $h=\pi/3$

$a=\frac{\pi}{4}$, $h=\frac{\pi}{3}$ $f(x)=\sin x$

$$c(h)=\frac{f(a+h)-f(a-h)}{2h}$$

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$$= \frac{\sin(\pi/4 + \pi/3) - \sin(\pi/4 - \pi/3)}{2(\pi/3)} \qquad \pi = 3.14$$

cosi shift exp

$$= \frac{\sin(0.785 + 1.04) - \sin(0.785 - 1.04)}{2(1.04)}$$

$$= \frac{\sin(1.825) + \sin(0.255)}{2.08}$$

$$= \frac{0.96 + 0.252}{2.08}$$

$$= 0.584$$

$$c(h/2) = \frac{\sin(\pi/4 + \pi/6) - \sin(\pi/4 - \pi/6)}{2 \times \pi/6}$$

sine inner exp

$$= \frac{\sin(0.78 + 0.528) - \sin(0.785 - 0.523)}{2(0.523}$$

$$= \frac{\sin(1.303) - \sin(0.262)}{1.046}$$

$$= \frac{0.964 - 0.259}{1.046}$$

$$= 0.673$$

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$$c(\pi/4) = \frac{\sin(\pi/4 + \pi/9) - \sin(\pi/4 - \pi/9)}{2(\pi/9)}$$

$$= \frac{\sin(0.78 + 0.34) - \sin(0.78 - 0.34)}{2(0.34)}$$

$$= \frac{\sin(1.12) - \sin(0.44)}{0.68}$$

$$= \frac{0.90 - 0.425}{0.68}$$

$$= 0.697$$

$$c(\pi/8) = \frac{\sin(\pi/4 + \pi/12) - \sin(\pi/4 - \pi/12)}{2(\pi/9)}$$

$$= \frac{\sin(0.78 + 0.26) - \sin(0.78 - 0.26)}{2(0.26}$$

$$= \frac{\sin(1.04) - \sin(0.52)}{0.52}$$

$$= \frac{0.86 - 0.49}{0.52}$$

$$= 0.715$$

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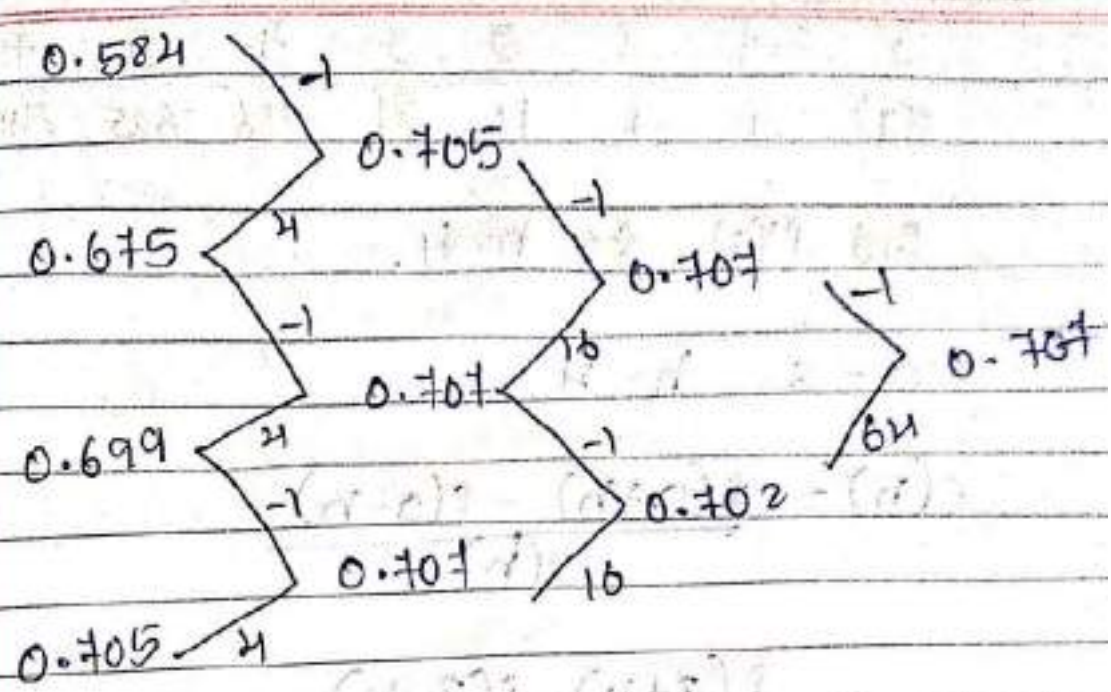

$$\frac{4(6.754) - 0.584}{3}$$

Formula $4\,\frac{MA - LA}{3}$

$$= \frac{9.1 - 0.583}{3}$$

$= 0.705$

$16\,\frac{MA - LA}{15}$

$64\,\frac{MA - LA}{63}$

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x	-1	1	2	3	4	5	7
$f(x)$	1	1	16	81	256	625	2401

find $f'(3)$ for $h=4$.

$a=3, \quad h=4.$

$$c(h) = \frac{f(a+h) - f(a-h)}{2(h)}$$

$$= \frac{f(3+4) - f(3-4)}{2(4)}$$

$$= \frac{f(7) - f(-1)}{8}$$

$$= \frac{7+1}{8}$$

$$= 1$$

$$c(h/2) = \frac{f(3+8) - f(3-8)}{2(8)}$$

$$= \frac{11+5}{16}$$

$$= \frac{16}{16}$$

$c(h) = 300$

$c(h/2) = 156$

$c(h/4) = 120$

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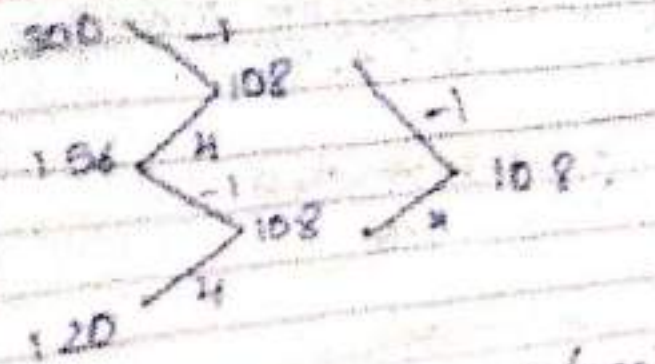

$$\frac{4(156) - 300}{3}$$

$$\frac{4(120) - 156}{3}$$

$$= 108.$$

108.

$$\frac{4(108) - 108}{3}$$

x	0.2	0.4	0.5	0.6	0.7	0.8	1.0
$f(x)$	1.4	1.9	2.1	2.4	2.6	2.9	3.5

find $f'(0.6)$ using F.E $h = 0.4$.

$$c(h) = \frac{f(a+h) - f(a-h)}{2(h)}$$

$$= \frac{f(0.6+0.4) - f(0.6-0.4)}{2(0.4)}$$

$$= \frac{1.1}{0.8}$$

$$= 2.5$$

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$$c(h/2) = \frac{f(0.6+0.8) - (0.6-0.8)}{2(0.8)}$$

$$= \frac{1.4 + 0.2}{1.6}$$

$$= 1$$

$$c(h/2) = h = 0.4$$

$$= \frac{0.4}{2}$$

$$= 0.2.$$

$$c(h/2) = \frac{f(0.6+0.2) - (0.6-0.2)}{2(0.2)}$$

$$= \frac{0.8 + 0.4}{0.4}$$

$$= 3.$$

$$c(h/4) = h = 0.4$$

$$= \frac{0.4}{4}$$

$$= 0.1$$

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$$c(x/y) = \frac{(0.6 + 0.1) - (0.6 - 0.1)}{2(0.1)}$$

$$= \underline{0.7 - 0.5}$$

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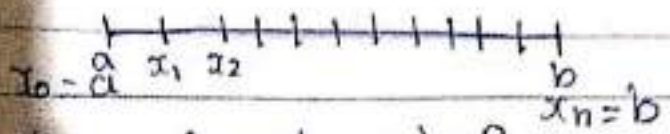
Numerical Integration

Rohan Mis

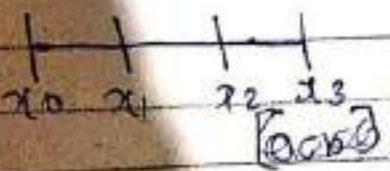
$$\int_a^b f(x)\,dx$$



h = step length = $\frac{b-a}{n}$

n denotes number of segments

$n+1$ is number of points.



let $[a,b]$ will divide into n segments & $h = \frac{b-a}{n}$ where $x_0 = a$, $x_n = b$

then the interval $[a,b]$ as $n+1$ points.

1. Composite Trapezoidal root

$$I = \int_a^b f(x)\,dx$$

$$= \frac{h}{2}\left[f(x_0) + 2\left\{f(x_1) + f(x_2) + \dots f(x_{n-1})\right\} + f(x_n)\right]$$

This rule can be apply for any n either even or odd

2. Composite Simpson

$$I = \frac{h}{3}\left[f(x_0) + 2\{f(x_2) + f(x_4) + \dots f(x_{n-2})\} + 4\{f(x_1) + f(x_3) + \dots + f(x_{n-1})\} + f(x_n) \right]$$

This rule can be apply only if n is even

Problem

1) Integrate $\int_0^1 \frac{1}{1+x^2}\,dx$ for $n=8$ by using Trapezoidal & Simpson's $\frac{1}{3}$rd rule

$h = \frac{b-a}{n}$

$a=0,\ b=1,\ n=8,\ h = \frac{1-0}{8} = \frac{1}{8}$

$f(x) = \frac{1}{1+x^2}$

	x_0	x_1	x_2	x_3	x_4	x_5
x	0	1/8	2/8	3/8	4/8	5/8
$f(x)$	1	0.988	0.94	0.87	0.8	0.7

	x_6	x_7	x_8
x	6/8	7/8	8/8
	0.64	0.56	0.5

$\frac{1}{1+x^2}$ x^2

calci

$1 \div (1 + (8 \div 8)$ shift square)

$1 \div (1 + (7 \div 8)$ shift square

calci

x_8 x_9 should not come in the formula

$$I = \frac{h}{2}\left[f(x_0) + 2\left\{f(x_1) + f(x_2) + \dots f(x_7)\right\} + f(x_8)\right]$$

$$= \frac{1}{8}/2 \left[1 + 2 \times 5.498 + 0.5\right]$$

$$= \frac{0.125}{2}\left[1 + 10.88 + 0.5\right]$$

$$= 0.0625\left[12.496\right]$$

$$= 0.7847$$

$$I = \frac{h}{3}\left[f(x_0) + 2\left\{f(x_2) + f(x_4) + f(x_6)\right\} + 4\left\{f(x_1) + f(x_3) + f(x_5) + f(x_7)\right\} + f(x_8)\right]$$

$$= \frac{1.8}{3}\left[1 + 2(2.46) + 4(3.118) + 0.5\right]$$

$$= \frac{1.8}{3}\left[1 + 4.92 + 12.472 + 0.5\right]$$

$$= \frac{1.8}{3}\left[18.892\right]$$

$$= 0.6\,(18.892)$$

$$= 0.787$$

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$I(h) = h/2 \quad h/4 \quad h/6$

2) $\int_0^2 \frac{e^x \sin x}{1+x^2} \, dx$

$h = b - a$

$2 - 0 = 2$

x	0 (x_0)	2 (x_1)
$f(x)$	0	1.34

$I(h)$ 1.34

$I(h/2)$ 1.14

$I(h/4)$ 1.005

$I(h/8)$

I_1, I_2, I_3, I_4, I_5, I_6

$\frac{4 \text{ MMA} - I_0}{3}$ $\quad \frac{16 \text{ MA} - I_0}{15}$ $\quad \frac{64 \text{ MA} - I_0}{63}$

$I(h) = \frac{h}{2}\left[f(x_0) + f(x_1)\right]$

$= \frac{2}{2}\left[0 + 1.34\right]$

$= 1\,(1.34)$

$I(h) = \underline{1.34}$

$I(h/2) = \frac{2}{2} = 1$

x	0 (x_0)	1 (x_1)	2 (x_2)
$f(x)$	0	1.14	1.34

$\frac{h}{2}\left[f(x_0) + 2f(x_1) + f(x_2)\right]$

$0.5\left[0 + 2(1.14) + 1.34\right]$

$\underline{I = 1.81}$

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$I(h/4) = \frac{2}{4} = \frac{1}{2} = 0.5$

x	0	0.5	1	1.5	2
$f(x)$	0	0.63	1.14	1.37	1.34

$\frac{h}{2}\left[f(x_0) + 2(0.63 + 1.14 + 1.37) + 1.34\right.$

$\frac{0.5}{2}(0 + 7.62)$

$= 1.905$

$I(h/8) = \frac{2}{8} = \frac{1}{4} = 0.25$

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for 0.25 ans =
1.35 [illegible] put [illegible]
0.25

$I(n/8) = \frac{2}{8} = \frac{1}{4} = 0.25$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6
x	0	0.25	0.5	0.75	1	1.25	1.5
$f(x)$	0	0.29	0.63	0.92	1.14	1.29	1.39

	x_7	x_8
x	1.75	2
f	1.39	1.34

$$\frac{h}{2}\left[f(x_0) + 2\{f(x_2) + f(x_4) + f(6)\} + 4\{f(x_1) + f(x_3) + f(x_5) + f(x_7)\} + f(x_8)\right]$$

$$= \frac{0.25}{2}\left[f(0) + [2((0.63) + (1.14) + (1.34))] + [4((0.29) + (0.92) + (1.39)] + 1.34\right.$$

$$= 0.125\left[0 + (2[3.12]) + (4[2.6]) + 1.34\right.$$

$$= 0.125\left[0 + 6.24 + 10.4 + 1.34\right]$$

$$= 0.125[17.98]$$

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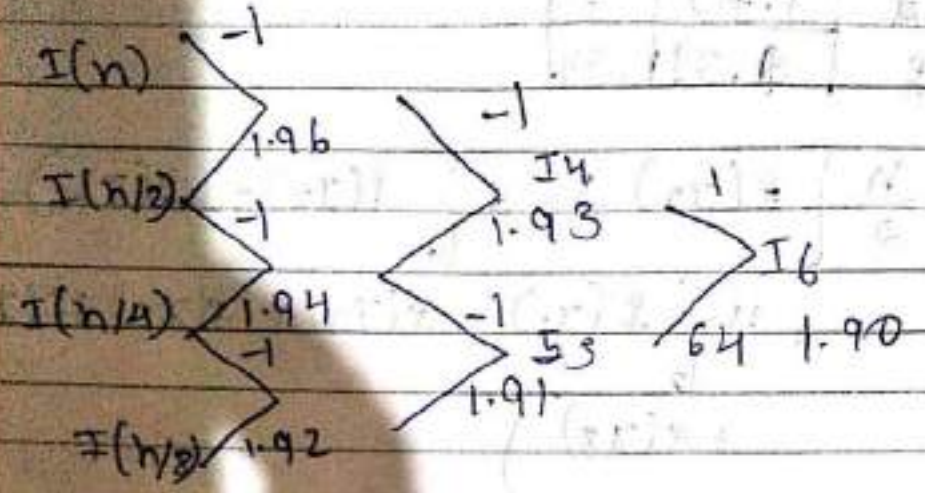
$$\frac{0.125}{2}\left[f(0) + \left[2\left(f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7)\right) + f(x_8)\right]\right.$$

$$= 0.125\,[0 + 14.06 + 1.34]$$

$$= 1.925.$$



$$\frac{4MA-2A}{3} \qquad \frac{16MA-1A}{15} \qquad \frac{64MA-2A}{63}$$

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(Module - 3)

Working procedure

1. To fit the straight line $y = a + bx$

Step 1 :- Substitute the set of n values in the equation

$\Rightarrow \Sigma y = na + b\Sigma x$

$\Rightarrow \Sigma xy = a\Sigma x + b\Sigma x^2$

These are known as normal equation

Step 2 :- Find the solution of a and b by solving system of eqn

Step 3 :- Substitute the value of a & b in $y = a + bx$

2. To fit the parabola $y = a + bx + cx^2$

Step 1 :- Substitute the set of n values in the equation

$\Rightarrow \Sigma y = na + b\Sigma x + c\Sigma x^2$

$\Rightarrow \Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$\Rightarrow \Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

Step 2 :- Solve this system of equation find the value of a, b, c.

Step 3 :- Substitute the value of a & b, c in $y = a + bx + cx^2$

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1) Fit a straight line $y = a + bx$ to the following data.

x	y	xy	x^2
0	1	0	0
1	3	3	1
3	2	6	9
6	5	30	36
8	4	32	64
$\Sigma x = 18$	$\Sigma y = 15$	$\Sigma xy = 71$	$\Sigma x^2 = 110$

$c_1 \quad a_1 \quad b_1$

$15 = 5a + 18b$ $\qquad \Sigma y = na + b\Sigma x$

$71 = 18a + 110b$ $\qquad \Sigma xy = a\Sigma x + b\Sigma x^2$

$a_2 \quad b_2 \quad c_2$

Below ans use in cal Bod mode & eqn

$a = 1.626$

$b = 0.376$

$$\boxed{y = 1.626 + 0.376x}$$

2) Find the parabola $y = a + bx + cx^2$ for the following data.

x	y	x^2	x^3	x^4	xy	x^2y
-3	4.63	9	-27	81	-13.89	41.67
-2	2.11	4	-8	16	-4.22	8.44
-1	0.67	1	-1	1	-0.67	0.67
0	0.09	0	0	0	0	0
1	0.63	1	1	1	0.63	0.63
2	2.15	4	8	16	4.3	8.6
3	4.58	9	27	81	13.74	41.22
$\Sigma x = 0$	$\Sigma y = 14.86$	$\Sigma x^2 = 28$	$\Sigma x^3 = 0$	$\Sigma x^4 = 196$	$\Sigma xy = -0.11$	$\Sigma x^2y = 101.23$

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$d_1 \quad a_1 \quad b_1 \quad c_1$

$14.86 = 7a + 0b + 28c$

$-0.11 = 0a + 28b + 0c$

$101.23 = 28a + 0b + 196c$.

use the cash Rd rots & coun unknown

$a = 0.132$

$b = -0.0039$

$c = 0.4975$.

3 The following table gives the results of measurement of tacing resistors. V is velocity & r is a resistor. If r is related to V by the relation $R = a + bV + cV^2$ find a, b & c

x	y					
V	R	V^2	V^3	V^4	V.R	V^2R
20	5.5	400	8000	160000	110	2200
40	9.1	800	64000	256000	364	7280
60	14.9	3600	216000	1296000	894	536
80	22.8	6400	512000	4096000	1824	53640
100	33.3	10,000	1000000	10000000	3330	333000
120	46.0	14400	1728000	20736000	5520	
$\Sigma x = 420$	$\Sigma y = 131.6$	$\Sigma x^2 = 3564$	Σx^3	Σx^4	Σxy	Σx^2y
		35600	3528000	36400 0000	12042	

$R = 3.48 - 0.002V + 0.0029V^2$

4) Fit a relation of the $V = a + b\theta$ for the given data

x	y	xy	x^2
θ	V	θV	θ^2
0	1.80	0	0
5	1.45	7.25	25
10	1.18	11.8	100
15	1.00	15	225
$\Sigma\theta = 30$	$\Sigma V = 5.43$	$\Sigma\theta V = 34.05$	$\Sigma\theta^2 = 350$

$n = 4$

$\Sigma y = na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

$c_1 \quad a_1 \quad b_1$

$5.43 = 4a + 30b$

$c_2 \quad a_2 \quad b_2$

$34.05 = 30 + 350$

[illegible]

$y = -0.0534$

5) Find straight line to following data & find the production in 1986

x	y	xy	x^2
1941	8	15528	3767481
1951	10	19510	3806401
1961	12	23532	3845521
1971	10	19710	3884841
1981	16	31696	3922361
$\Sigma x =$ 9805	Σy 56	Σxy 109976	Σx^2 19228605

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$\Sigma y = na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

$c_1 \quad a_1 \quad b_1$

$56 = 5a + 9805b$

$109916 = 9805a + 19828605b$

$x = -302.56$

$y = 0.16$

$y = -302.56 + 0.16$

where $x = 1986$

$y = -302.56 + 0.16(1986)$

$y = 15.2$

6) V (Km/hr) and R (kg / tonne) are related by a relation the type $R = ab^{v^2}$ find by the method of least square α & b with data

x	y	x^2	xy
V	R	V^2	VR
10	8		
20	10		
30	15		
40	21		
50	30		
Σx	Σy	Σx^2	Σxy

FOR

Formula

$\Sigma RV = a\Sigma V + b\Sigma V^3 \rightarrow (2)$

$\Sigma R = na + b\Sigma V^2 \rightarrow (1)$

x	y	x^2	x^3	yx
V	R	V^2	V^3	RV
10	8	100	1000	80
20	10	400	8000	200
30	15	900	27000	450
40	21	1600	64000	840
50	30	2500	125000	1500
Σx	Σy	Σx^2	Σx^3	Σyx
150	84	5500	225000	3070

$\cancel{3070} = a_1 \quad b_1$

$84 = 5a_2 + 5500b$

$3070 = 150 + 225000$

$x = 6.716$

$y = 0.009$

7) Fit a curve $y = ax^b$ for the following

in calsi ln(1, 2, 3)

x	y	$X = \log x$	$Y = \log y$	X^2	XY
1	0.5	0	-0.69	0	0
2	2	0.693	0.693	0.480	0.48
3	4.5	1.0986	1.5050	1.205	1.64
4	8	1.386	2.07	1.920	2.86
5	12.5	1.6098	2.52	2.588	4.05
Σx	Σy	ΣX	ΣY	ΣX^2	ΣX
15	27.5	4.786	6.093	6.085	9.0

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$y = ax^b$

Taking $\log_e$ on both sides

$\log y = \log a + b \log x$

$Y = A + bX$

$Y = \log y \quad A = \log a \quad B = b \quad X = \log x$

$\boxed{a = e^A \quad b = B}$

$\Sigma Y = An + B\Sigma X$

$\Sigma XY = A\Sigma X + B\Sigma X^2$

$\underset{c_1}{6.093} = \underset{a_1}{5A} + \underset{b_1}{4.78B}$

$\underset{c_2}{9.037} = \underset{a_2}{4.78} + \underset{b_2}{6.185B}$

$a = -0.689 - 0.695$

$b = 1.988$

$a = e^A = e^{-0.695} = 0.499$

$b = 2.02$

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Rhombus Assignment

1) $\int_0^{1/2} \frac{x}{\sin x} dx$

$h = \frac{1}{2} - 0 = \frac{1}{2}$ $f(x) = \frac{x}{\sin x}$

in case i $(1/2) \div (\sin 1/2)$

x	0	1/2
$f(x)$	1	1.04

$\lim_{x \to 0} \frac{x}{\sin x} = 1$

for 0 0.001 $\div (\sin 0)$

$I_1 = \frac{1/2}{2} [1 + 1.04] = 0.5105$

$= 0.25 (2.04)$

$= 0.5106$

$h = \frac{1}{4}$

x	0	1/4	2/4
$f(x)$	1	1.01	1.04

$I_2 = \frac{1/4}{2} [1 + 2(1.01) + 1.04]$

$= \frac{0.25}{2} [1 + 4(1.01) + 1.04]$

$= 0.16 [1 + 2(1.01) + 1.04]$

$= 0.16 (1 + 2.02 + 1.04)$

$= 0.16 (4.06)$

$= 0.5075$

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$h = \frac{1}{8}$

x	0	1/8	2/8	3/8	4/8
$f(x)$	1	1.002	1.01	1.023	1.04

$$= \frac{1/8}{2}\left[1 + \left[2(1.0020 + 1.01 + 1.023) + 1.04\right]\right]$$

$$= \frac{1}{16}\left[1 + 2(3.035) + 1.04\right]$$

$$= 0.0625(1 + 6.07 + 1.04)$$

$$= 0.0625(8.11)$$

$$= 0.5068$$

x	0	1/16	2/16	3/16	4/16	5/16	6/16	7/16	8/16
$f(x)$	1	1.006	1.0027	1.005	1.01	1.016	1.023	1.032	1.04

$$= \frac{1/16}{2}\left[1 + 2\left[(1.006 + 1.0027 + 1.005 + 1.01 + 1.016 + 1.023 + 1.032)\right] + 1.04\right]$$

$$= 0.5067$$

$I_1 = 0.5105$ $\quad -1 \quad$ I_1 0.5106 $\quad -1 \quad$ I_1 0.5083 $\quad$ 0.5063

$I_2 = 0.50775$ $\quad -1 \quad$ I_2 0.5063 $\quad 16 \quad -1 \quad$ 0.5067

$I_3 = 0.5067$ $\quad -1 \quad$ I_3 0.5067 $\quad 16$

$I_4 = 0.5067$

$I_6 = 0.5067$

$$\frac{4MA - LA}{3} \quad \frac{16MA - LA}{15} \quad \frac{64MA - LA}{63}$$

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Fit the exponential curve $y = ae^{bx}$

x	y	$X = x$	$Y = \ln y$	X^2	XY
2	25	2	3.218	4	6.436
4	38	4	3.637	16	14.548
6	56	6	4.025	36	24.15
8	84	8	4.430	64	35.44
$\Sigma x = 20$	Σy	ΣX	ΣY	ΣX^2	ΣXY
	203	20	15.31	120	80.58

$n = 4$

$$y = aeb^x$$

Apply ln on both sides

$$\ln y = \ln a + bx$$

$$Y = A + Bx$$

$Y = \ln y \quad A = \ln a \quad B = b \quad X = x$

$$a = e^A$$

$$\Sigma Y = nA + B\Sigma X$$

$$\Sigma XY = A\Sigma X + B\Sigma X^2$$

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$$15.51 = 4a + 60b$$
$$80.58 = 20a + 120b$$

$A = 2.8$

$B = 0.2015$.

$a = e^{A} = 16.44$

$b = B = 0.2015$

$e^{A} = e^{2.8}$

$$\therefore \boxed{y = 16.44\, e^{0.2015x}}$$

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MODULE – 5.

(In Ranak's book).

– Method of variation of Parameter :-

$$y'' + Py' + Qy = R(x).$$

Step 1 :- solve :

$$y'' + Py' + Qy = 0.$$

we get two solutions.

$y_1(x)$ & $y_2(x)$.

Step 2 :- Find $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$, known as Wronskian ($W \neq 0$).

Step 3 :- Find $C_1(x) = \int \frac{y_2 R(x)}{W} dx$.

& $C_2(x) = \int \frac{y_1 R(x)}{W} dx$.

$\Rightarrow y_P = C_1 y_1 + C_2 y_2$.

Step 4 :- Solution of given D.E is

$$y(x) = \underset{\substack{\downarrow \\ \text{complementary function}}}{C.F} + y_P.$$

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1. $(D^2 - 2D + 4)y = e^{2x}\sin x$

$m^2 - 2m - 2m + 4$.
$m(m-2) - 2(m-2)$
$m = 2$.

Step 1:- $(D^2 - 2D + 4)y = 0$.

Replace D by m.

$m^2 - 2m + 4 = 0$.

$(m-2)^2 = 0$.

$m = 2, 2$.

(Formula for same roots)

$C.F = (C_1' + C_2' x)\, e^{2x}$

$= C_1' e^{2x} + C_2' x e^{2x}$ —— (1).

$\therefore y_1(x) = e^{2x}$.

$y_2(x) = x e^{2x}$.

x.

Step 2 :- $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$= \begin{vmatrix} e^{2x} & x e^{2x} \\ 2e^{2x} & 2x e^{2x} + e^{2x} \end{vmatrix}$

$= e^{2x}(2x e^{2x} + e^{2x}) - x e^{2x}(2e^{2x})$

$= \cancel{2x e^{4x}} + e^{4x} - \cancel{2x e^{4x}}$

$= e^{4x}$.

Step 3 :- $C_1 = \int \frac{y_2 R(x)}{W}\, dx$.

$= \int \frac{x \cancel{e^{2x}} \cancel{e^{2x}} \sin x}{e^{4x}}\, dx$

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$$= \int \underset{u}{x}\, \underset{v}{e^{-x} \sin x}\, dx.$$

$$= u\int v - u' \iint v + u'' \iiint v.$$

$$= x\int \sin x\, dx - 1 \iint \sin x.$$

$$= -x\cos x + \cos x.$$

$$C_2 = \int \frac{y_1 R(x)}{W} dx = \int \frac{\cancel{e^{2x}}\, \cancel{e^{2x}} \sin x}{\cancel{e^{4x}}}.$$

$$= -\cos x.$$

Step 4 :- $y(x) = C_1' e^{2x} + C_2' x e^{2x} + C_1 y_1 + C_2 y_2.$

$$= C_1' e^{2x} + C_2' x e^{2x} + (-x\cos x + \cos x) + (-\cos x) x e^{2x}$$

2- $y'' + 4y = \tan 2x$.

$y'' = m^2 = D^2$

$y' = m' = D.$

No $y' = 1 = -$

Step 1 :- $y'' + 4y = 0.$

Replace y by m.

m

$m^2 + 4 = 0.$

$m^2 = -4.$

$m = \pm\sqrt{-4}.$

$= \pm 2\sqrt{-1}.$

$= \pm 2i$

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$\cancel{e^{ax} + c_1 \sin}$

$a \pm ib$.

$a = 0$.

$b = 2$.

$$c.f = e^{0x}(c_1' \cos 2x + c_2' \sin 2x)$$

$$= c_1' \cos 2x + c_2' \sin 2x.$$

$y_1 = \cos 2x$, $y_2 = \sin 2x$.

Step 2 :- $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$$= 2\cos^2 2x - (-2\sin^2 2x)$$

$$= 2(\cos^2 2x + \sin^2 2x) = 2.$$

$\sin^2 2x + \cos^2 2x = 1$.

Step 3 :- $c_1 = \int \frac{y_2 R(x)}{W}$.

$$= \int \frac{\sin 2x \tan 2x}{2} dx.$$

$\frac{\sin^2 x}{\cos 2x}$

$$c_2 = \int \frac{y_1 R(x)}{W} = \int \frac{\cos 2x \tan 2x}{2} dx$$

$$= \frac{1}{2} \int \sin 2x \, dx.$$

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$$= \frac{1}{2} \frac{-\cos 2x}{2} = \frac{-\cos 2x}{4}.$$

Step 4 ÷ $y(x) = C_1' \cos 2x + C_2' \sin 2x$.

$$+ \cos 2x \int \frac{\sin^2 2x}{2\cos 2x} dx.$$

$$+ \sin 2x \left(\frac{-\cos 2x}{4} \right)$$

3. $y'' - y = \frac{2}{1+e^x}$.

Step 1 ÷

$$m^2 - 1 = 0.$$

$$\cancel{m(m-1) = 0}.$$

$$\cancel{m = 1}.$$

$$m^2 = 1.$$

$$m = \pm 1.$$

$$C.F = C_1' e^x + C_2' e^{-x}.$$

$$y_1 = e^x, \quad y_2 = e^{-x}.$$

Step 2 ÷ $W = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}$

$$= -1 - 1 = -2.$$

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Step 3 :- $C_1 = \int \frac{y_2 R(x)}{W}$

$$= \int \frac{e^{-x} \cancel{2}}{1+e^{x}(-\cancel{2})}$$

$$= -\int \frac{e^{-x}}{1+e^{x}} dx$$

$$C_2 = \int \frac{e^{x} \cancel{2}}{\cancel{-2}(1+e^{x})} dx$$

$$= -\int \frac{e^{x}}{1+e^{x}} dx$$

Let $1+e^{x} = t$

$e^{x} dx = dt$

$$C_2 = -\int \frac{dt}{t} = -\log t = -\log(1+e^{x})$$

Step 4 :- $y(x) = C_1' e^{x} + C_2' e^{-x} +$

$$\left[-e^{x} \int \frac{e^{-x}}{1+e^{x}} dx\right] - e^{-x} \log(1+e^{x})$$

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4. $y'' - 6y' + 9y = \dfrac{e^{3x}}{x^2}$.

Step 1 : $m^2 - 6m + 9 = 0$.

$m^2 - 3m - 3m + 9 = 0$.

$m(m-3) - 3(m-3) = 0$.

$m = 3, 3$.

$C.F = (C_1' + C_2' x)\, e^{3x}$

$= C_1' e^{3x} + C_2' x e^{3x}$.

$\therefore\ y_1 = e^{3x}$.

$y_2 = x e^{3x}$.

Step 2 : $W = \begin{vmatrix} e^{3x} & x e^{3x} \\ 3e^{3x} & 3x e^{3x} + e^{3x} \end{vmatrix}$

$= e^{6x}$.

Step 3 : $C_1 = \displaystyle\int \frac{y_2\, R(x)}{W}\, dx$.

$= \cancel{\displaystyle\int \frac{x e^{3x} \cdot e^{3x}}{e^{6x}\, x^2}\, dx} = \cancel{\displaystyle\int}$

$= \cancel{\displaystyle\int \frac{x e^{6x}}{e^{6x}\, x^2}\, dx} = \cancel{\displaystyle\int \frac{1}{x}\, dx}$.

$= \cancel{\displaystyle\int \frac{e^{-3x}}{x}} = \cancel{\log x\, dx}$.

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$$C_2 = \int \frac{e^{3x}\, e^{3x}}{e^{9x}\, x^2}$$

$$= \int \frac{1}{x^2}$$

$$\int \frac{x\, e^{3x}\, e^{3x}}{e^{6x}\, x^2}\, dx$$

$$= \int \frac{1}{x}\, dx$$

$$= \log x$$

Step 4 : $y(x) =$

$$C_2 = \int \frac{e^{3x}\, e^{3x}}{e^{6x}\, x^2}\, dx$$

$$= \int \frac{1}{x^2}\, dx = \int x^{-2}$$

$$= \frac{x^{-1}}{-1}$$

$$= \frac{-1}{x}$$

1

x^2

$\frac{x^{2+1}}{2}$

$\frac{x^{-2+1}}{-2+1}$

Step 4 : $y(x) = C_1' e^{3x} + C_2' x e^{3x} + e^{3x} \log x + x e^{3x} \left(-\frac{1}{x}\right)$

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5. $y'' - 2y' + y = e^x \log x$.

Step 1 : $m^2 - 2m + 1 = 0$.

$m^2 - 1m - 1m + 1 = 0$.

$m(m-1) - 1(m-1) = 0$.

$m = 1, 1$.

$C.F = (c_1' + c_2' x) e^x$.

$= c_1' e^x + c_2' x e^x$.

$\therefore \; y_1 = e^x$.

$y_2 = x e^x$.

$xe^x + e^x$.

Step 2 : $W = \begin{vmatrix} e^x & xe^x \\ e^x & xe^x + e^x \end{vmatrix}$

$= e^x(xe^x + e^x) - e^x(xe^x)$

$= \cancel{xe^{2x}} + e^{2x} - \cancel{xe^{2x}}$

$= e^{2x}$.

Step 3 : $c_1 = \int \frac{xe^x \; e^x \log x}{e^{2x}} dx$.

$= \int \frac{x \cancel{e^{2x}} \log x}{\cancel{e^{2x}}} dx$.

$= \int x \log x \, dx$

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$= x\int \log x - \int\int \log x$. $\qquad x\log x - x$.

$$C_2 = \int \frac{e^x\, e^x \log x}{e^{2x}} \cdot dx .$$

$$\boxed{D = \frac{d}{dx}}$$

$$Dy = \frac{dy}{dx} = y'$$

$$D^2 y = \frac{d^2}{dx^2} y = y''$$

$= x\log - x$.

$y(x) =$

- Undetermined coefficients :-

$y'' + Py' + Qy = R(x)$.

If $R(x) = e^{3x}$.

$y_p = ae^{3x}$.

If $R(x) = 2\sin x$.

$y_p = a\sin x + b\cos x$.

If $R(x) = x^2$.

$y_p = ax^2 + bx + c$.

Q1. $(D^2 + D)y = x^2 + 2x + 4$.

$\rightarrow m^2 + m = 0$.

$m(m+1) = 0$.

$m = 0, -1$.

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$$C.F. = C_1 e^{0x} + C_2 e^{-x}$$

$$= C_1 + C_2 e^{-x}$$

$$R(x) = x^2 + 2x + 4.$$

Let $y_P = x(ax^2 + bx + c)$.

$$= ax^3 + bx^2 + cx.$$

$$y_P' = 3ax^2 + 2bx + c.$$

If $m=0$ is the the root eq if $R(x)$ is a polynomial, x^{ly} the whole eqn. with x.

$$y_P'' = 6ax + 2b.$$

$$\begin{array}{cc} 6 & 1 \\ 2 & 3 \end{array}$$

$$b = \frac{2-2}{}$$

<u>We know</u> :-

$$y'' + y' = x^2 + 2x + 4.$$

$$6ax + 2b + (3ax^2 + 2bx + c) = x^2 + 2x + 4.$$

$$3ax^2 + x(6a + 2b) + 2b + c = 1 \cdot x^2 + 2x + 4.$$

Comparing both sides.

$$3a = 1 \quad \Rightarrow a = \frac{1}{3}.$$

$$6a + 2b = 2 \quad \Rightarrow b = 0.$$

$$2b + c = 4 \quad \Rightarrow c = 4.$$

$$y_P = \frac{1}{3}x^3 + 4x \quad \text{—— (2)}.$$

$$\therefore y(x) = C_1 + C_2 e^{-x} + \frac{1}{3}x^3 + 4x$$

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2. $y'' + 4y' + 4y = 3\sin x + 4\cos x$.

$y(0) = 1$, $y'(0) = 0$.

$m^2 + 2m + 2m + 4$.

$m(m+2)$

$\rightarrow$. $m^2 + 4m + 4 = 0$.

$(m+2)^2 = 0$.

$m = -2, -2$.

$CF = (C_1 + C_2 x)e^{-2x}$,

Let $y_p = A\sin x + B\cos x$.

$y_p' = A\cos x - B\sin x$.

$y_p'' = -A\sin x - B\cos x$.

$y'' + 4y' + 4y = 3\sin x + 4\cos x$.

$(-A\sin x - B\cos x) + 4(A\cos x - B\sin x) + 4(A\sin x + B\cos x) = 3\sin x + 4\cos x$.

$\sin x(3A - 4B) + \cos x(4A - 3B) = 3\sin x + 4\cos x$.

Comparing both sides.

$3A - 4B = 3$.

$4A - 3B = 4$.

$A = 1 \quad B = 0$.

$y_p = \sin x$ —(2).

$y(x) = (C_1 + C_2 x)e^{-2x} + \sin x$.

$y(0) = C_1 = 1$.

$y'(x) = e^{-2x}C_2 + e^{-2x}(C_1 + C_2 x)(-2) + \cos x$

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3. $(D^2+1)y = \sin x$.

$\rightarrow m^2+1=0$.

$m = \pm i$ | $a=0$. $b=1$.

C.F $= c_1 \cos x + c_2 \sin x$.

$x(A\cos x$

Let $y_p = x(A\sin x + B\cos x)$

$y_p = xA\sin x + xB\cos x$.

$y'_p = A(x\cos x + \sin x) + B(-x\sin x + \cos x)$.

$= Ax\cos x + A\sin x - Bx\sin x + B\cos x$.

$y''_p = A(-x\sin x + \cos x) + A(\cos x) - B(x\cos x + \sin x)$
$- B\sin x$.

$= -Ax\sin x + A\cos x + A\cos x - Bx\cos x -$
$B\sin x - B\sin x$.

$= -Ax\sin x + 2A\cos x - 2B\sin x - Bx\cos x$.

we know :

$y'' + y = \sin x$.

$\cancel{-Ax\sin x} + 2A\cos x - 2B\sin x - \cancel{Bx\cos x} = \sin x$
$+ \cancel{xA\sin x} + \cancel{xB\cos x}$.

$2A\cos x - 2B\sin x = 1 \cdot \sin x + 0 \cdot \cos x$.

$2A = 0$.

$-2B = 1$.

$A = 0$ $B = \frac{-1}{2}$.

$y_p = x\left(\frac{-1}{2}\cos x\right)$ — (2)

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m	0	$b \neq 1$.	$b = 1$.
$R(x)$	x^n.	$\sin x, \cos x$.	$\sin x, \cos x$.
y_p.	$x(ax^2+bx+c)$	$a\sin x + b\cos x$.	$x(a\sin x + b\cos x)$

$$y(x) = C_1 \cos x + C_2 \sin x - \frac{x}{2}\cos x.$$

$-2\sqrt{3}$.
$= 2x - 3i$
$= -6i$
$2x$

4. $y'' + 2y' + 4y = 2x^2 + 3e^{-x}$

$\rightarrow$. $m^2 + 2m' + 4 = 0$.

$m^2 + 2m + 2m + 4 = 0$.

$m(m+2) + 2(m+2) = 0$.

$m = -2, -2$.

$c.f = (c_1 + c_2 x)e^{-2x}$.

$a = 1 \quad b = 2 \quad c = 4$.

$\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$.

$\frac{2 + \sqrt{4 - 4(4)}}{2} = \frac{2 - 6i}{2}$.

$2+$

Let $y_p =$

$a = 1 \quad b = 2 \quad c = 4$.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$= \frac{-2 + \sqrt{4 - 4(4)}}{2} \text{ or } \frac{-2 - \sqrt{4 - 4(4)}}{2}.$$

$$\frac{-2 + 2\sqrt{3}\,i}{2} \text{ or } \frac{-2 - 2\sqrt{3}\,i}{2}.$$

$$\cancel{2}\left(\frac{-1 + \sqrt{3}\,i}{\cancel{2}}\right)$$

$-1 + \sqrt{3}\,i$ or $-1 - \sqrt{3}\,i$.

$m = -1 \pm \sqrt{3}i$

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$$CF = e^{-x}\left(C_1 \cos\sqrt{3}\,x + C_2 \sin\sqrt{3}\,x\right)$$

$$y_p = Ax^2 + Bx + C + De^{-x}.$$

$$y_p' = 2Ax + B + 0 + \left(-De^{-x}\right).$$

$$y_p'' = 2A + 0 + 0 + De^{-x}.$$

$$= 2A + De^{-x}.$$

we know

$$y'' + 2y' + 4y = 2x^2 + 3e^{-x}.$$

$$2A + De^{-x} + 2\left(2Ax + B - De^{-x}\right) + 4\left(Ax^2 + Bx + C + De^{-x}\right)$$

$$= 2x^2 + 3e^{-x}.$$

$$2A + \cancel{De^{-x}} + 4Ax + 2B - \cancel{2De^{-x}} + 4Ax^2 + 4Bx + 4C + 4De^{-x} = 2x^2 + 3e^{-x}$$

$\cancel{2A(1+2x)}$ $\cancel{2B(1+2x)}$

$\cancel{A=0}$. $4A = 2$.

$\cancel{B=0}$. $A = \frac{2}{4} = \frac{1}{2}$

$4D = 3$.

$D = 3/4$.

$\cancel{4C=0}$.

$C = 0$.

$4Ax + 4Bx = 0$.

$B = -1/2$

$2A + 2B + 4C = 0$.

$C = 0$

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$D - 2D + 4D = 3.$

$3D = 3.$

$D = 1$

5. $(D^2 - 2D)y = e^x \sin x.$

$m^2 - 2m = 0.$

$m(m-2) = 0.$

$m = 0.$

$m = 2.$

$CF = C_1 e^{0x} + C_2 e^{2x}.$

$= C_1 + C_2 e^{2x}.$

$y_p = e^{x}(B\sin x + C\cos x)$

$= e^x(B\sin x + C\cos x)$

$= e^x B\sin x + e^x C\cos x.$

$y_p' = B(e^x\cos x + e^x\sin x) + C(-e^x\sin x + e^x\cos x)$

$= Be^x\cos x + Be^x\sin x + Ce^x\cos x - Ce^x\sin x.$

$y_p'' = B(e^x\cos x - e^x\sin x) + B(e^x\sin x + e^x\cos x)$
$+ C(e^x\cos x - e^x\sin x) - C(e^x\sin x + e^x\cos x)$

We know,

$y'' - 2y' = e^x\sin x.$

$\cancel{Be^x\cos x} - \cancel{Be^x\sin x} + \cancel{Be^x\sin x} + \cancel{Be^x\cos x} + \cancel{Ce^x\cos x}$
$- \cancel{Ce^x\sin x} - \cancel{Ce^x\sin x} - \cancel{Ce^x\cos x}$
$- \cancel{2Be^x\cos x} - 2Be^x\sin x - 2Ce^x\cos x + \cancel{2Ce^x\sin x}$
$= e^x\sin x.$

$-2Be^x\sin x - 2Ce^x\cos x = e^x\sin x + 0.$

$-2A = 1 \qquad -2B = 0$

$A = \frac{-1}{2} \quad B = 0.$

$y_p = e^x\left(-\frac{1}{2}\sin x\right).$

6. $(D^2 - 2D + 3)y = x^3 + \cos x.$

$m^2 - 2m + 3 = 0.$

$\cancel{m^2} + 3m \cancel{- 1m} + 3 \cancel{= 0}.$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$

$a = 1$
$b = -2.$
$c = 3.$

$\frac{2 \pm \sqrt{4 - 12}}{2}.$

$1 + \sqrt{2}i$ or $1 - \sqrt{2}i.$

$m = 1 \pm \sqrt{2}i.$

$C.F = e^x(C_1 \cos\sqrt{2}x + C_2 \sin\sqrt{2}x)$

$y_p = Ax^3 + Bx^2 + Cx + D + E\sin x + F\cos x.$

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Simpson $\frac{1}{3}$rd

$$I = \frac{1}{8/3}\left[1 + 2\{0.94 + 0.8 + 0.64\} + 4\{0.98 + 0.87 + 0.71 + 0.56\} + 0.5\right]$$

$$I = \frac{1}{24}\left[1 + 4.76 + 12.48 + 0.5\right]$$

$$= \frac{1}{24}\left[18.74\right]$$

$$I = 0.78$$

Evaluate the following integral using Trapezoidal Rule and Simpson's $\frac{1}{3}$rd R.

a) $\int_0^{\pi/2} (6 + 3\cos x)\,dx \qquad n = 6$

b) $\int_0^3 (1 - e^{-2x})\,dx \qquad n = 5$

c) $\int_1^2 \left(x + \frac{2}{x}\right)^2 dx \qquad h = 0.25$

d) $\int_{-3}^{5} (4x - 3)^2\,dx \qquad h = 1$

e) $\int_0^3 x^2 e^x\,dx \qquad n = 8$

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Module - 4

Numerical Differentiation and Integration

Differentiation

1. Forward Difference Scheme

$$f'(a) = \frac{f(a+h) - f(a)}{h}$$

$a \to$ given point
$h \to$ step length

2. Backward Difference Scheme

$$f'(a) = \frac{f(a) - f(a-h)}{h}$$

3. Central Difference Scheme

$$f'(a) = \frac{f(a+h) - f(a-h)}{2h}$$

Q.

x	$\boxed{1}$	2	$\boxed{3}$	4	$\boxed{5}$
$f(x)$	0	0.1	0.5	0.7	0.9

length of step $= 2 - 1 = 1$

$a = 1 \quad h = 1$

$f'(1) = 0$

$$f'(1) = \frac{f(1+1) - f(1)}{1} = \frac{f(2) - 0}{1} = 0.1$$

If we want to find a value at the beginning of the table we must use forward difference scheme.

$f(5) = 0.9$

$a = 5 \quad h = 1$

$$f'(5) = \frac{f(5) - f(5-1)}{1} = \frac{0.9 - 0.7}{1} = 0.2$$

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If we want to find a value at the end of the table we must use backward diff. scheme

If we want to find a value at the middle of the table we must use central diff. scheme

Q. Find $f'(x)$ for $f(x) = \frac{\sin(0.5\sqrt{x})}{x}$ at $x = 0.1$ using FDS, BDS, FDS for $h = 0.01$

(Use Radian Mode on Cal C)

Sol: F.D.S

$f(x) = \frac{\sin(0.5\sqrt{x})}{x}$ $\quad x = 0.1 \quad h = 0.01$

$f'(x) = \frac{f(x+h) - f(x)}{h}$

$\therefore f(x+h) = f(0.11) = \frac{\sin(0.5\sqrt{0.11})}{0.11}$

$= \frac{\sin(0.1658)}{0.11}$

$= \frac{0.165}{0.11}$

$f(x+h) = 1.5003$

$f(x) = f(0.1) = \frac{\sin(0.5\sqrt{0.1})}{0.1}$

$= \frac{\sin(0.1581)}{0.1}$

$= \frac{0.1581}{0.1}$

$= 1.5814$

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$$I(h) = \frac{0.5}{2}\left[1 + 2(0.4) + 0.5\right]$$

$$= 0.25\left[1 + 1.6 + 0.5\right]$$

$$= 0.25\left[3.1\right]$$

$$= 0.775$$

$I(h/4)$ $\quad h = \frac{h}{4} = \frac{1}{4} = 0.25$

x_0	0	0.25	0.5	0.75	1
$f(x)$	1	0.9411	0.8	0.64	0.5

$$f(0.25) = \frac{1}{1+(0.25)^2} = 0.9411$$

$$I\left(\frac{h}{4}\right) = \frac{0.25}{2}\left[1 + 2\{0.9411 + 0.8 + 0.64\} + 0.5\right]$$

$$= 0.125\left[6.2622\right]$$

$$= 0.782775$$

$I(h/8)$

$$h = \frac{h}{8} = \frac{1}{8} = 0.125$$

x_0	0	0.125	0.25	0.375	0.5
$f(x)$	1	0.9846	0.9411	0.8767	0.8

0.625	0.75	0.875	1
0.7191	0.64	0.5712	0.5

$$I\left(\frac{h}{8}\right) = \frac{0.125}{2}\left[1 + 2\{0.9846 + 0.9411 + 0.8767 + 0.8 + 0.7191 + 0.6400 + 0.5662\} + 0.5\right]$$

$$= 0.0625\left[1 + 11.0552 + 0.5\right]$$

$$= 0.7884$$

$$I\left(\frac{h}{8}\right) = 0.7884 \approx 0.75$$

$0.75 \quad I(h)$

$0.775 \quad I(h/2)$

$0.7827 \quad I(h/4)$

$0.7884 \quad I(h/8)$

$I_1 = 0.7333$

$I_2 = 0.7652$

$I_3 = 0.7853$

$I_1 = 0.7333$

$I_2 = 0.7652$

$I_3 = 0.7853$

$$\frac{4MB - LB}{3} \qquad \frac{16MB - LB}{15} \qquad \frac{64MB - LB}{63}$$

1.8140 $\quad I_1 = \frac{4(0.775)}{3} - 0.25$

1.1904 $\quad = 0.7333$

1.3904 $\quad I_2 = \frac{4(0.7827)}{3} - 0.775$

1.7654 $\quad = 0.7652$

$$I_3 = \frac{4(0.7884)}{3} - 0.7827$$

$$= 0.7853$$

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Romberg Integration

1. Evaluate $\int_0^{\pi/2} \frac{x}{\sin x} dx$ using Romberg integration

2. Use Romberg integration of order h^6 to evaluate the following integration.

(e) $\int_0^1 \frac{1}{1+x^2} dx$

$h = b - a$ $\quad x_0 = a = 0$

$h = 1 - 0 = 1$ $\quad x_1 = b = 1$

x	0	1
$f(x)$	1	0.5

$f(0) = \frac{1}{1+0^2} = 1$

$f(1) = \frac{1}{1+1^2} = \frac{1}{2} = \frac{1}{2}$

Use Trapezoidal rule

$I(h) = \frac{1}{2}[1 + 0.5]$

$= \frac{1}{2}[1.5]$

$I(h) = 0.75$

$h = \frac{h}{2} = \frac{1}{2} = 0.5$

$I(h/2)$

x	0	0.5	1
$f(x)$	1	0.8	0.5

$f(0.5) = \frac{1}{1+(0.5)^2} = 0.8$

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Richardson Exp Extrapolation

- Apply CDS
- Given a, h

$C(h)$, $C(h/2)$, $C(h/4)$, $C(h/8)$

C_{11}, C_{12}, C_{13} (4, 1, 4)

C_{21}, C_{22} (11, 16) (−1, 1)

C_{31} (64) (−1)

$\frac{4MA - LA}{3}$ $\frac{16MA - LA}{15}$ $\frac{64MA - LA}{63}$

Estimate the value of $\sin x$ using R.E for $a = \frac{\pi}{4}$, $h = \frac{\pi}{3}$ $f(x) = \sin x$.

Apply CDS $h = \frac{\pi}{3}$ $a = \frac{\pi}{4}$

$$C(h) = \frac{f(a+h) - f(a-h)}{2h} = \frac{f(\frac{\pi}{4} + \frac{\pi}{3}) - f(\frac{\pi}{4} - \frac{\pi}{3})}{2 \times \frac{\pi}{3}}$$

$$= \frac{f(7\pi/12) - f(-\pi/12)}{2 \cdot \pi/3}$$

$$= \frac{0.9659 + 0.2588}{2.0943}$$

$\frac{3\pi + \pi}{12}$

$f(-\frac{\pi}{12})$

$C(h) = 0.584$

$$C(h/2) = \frac{f(\frac{\pi}{4} + \frac{\pi}{12}) - f(\frac{\pi}{4} - \frac{\pi}{12})}{2 \cdot \pi/6}$$

$$= \frac{0.9659 + 0.2588}{2.0943}$$

$C(h/2) = 0.6752$

$$C(h/4) = \frac{f(\frac{\pi}{4} + \frac{\pi}{24}) - f(\frac{\pi}{4} - \frac{\pi}{24})}{2 \times \pi/12}$$

$$C(h/4) = \frac{f(\frac{7\pi}{24}) + 0.966 - 0.5 = 0.6}{0.5235}$$

$$C(h/4) = \frac{0.7137 - 0.6087}{0.2618}$$

$C(h/8) = 0.705$

0.5641, 0.675, 0.617, 0.705 → 0.705, 0.707, 0.707 → 0.707

Q.

x	−1	1	2	3	4	5	7
$f(x)$	1	1	16	81	256	625	2401

Find $f'(3)$ for $h = 4$.

Sol: $$C(h) = \frac{f(a+h) - f(a-h)}{2h} = \frac{f(3+4) - f(3-4)}{2 \times 4}$$

$$= \frac{f(7) - f(-1)}{8} = \frac{2401 - 1}{8}$$

$C(h) = \frac{2400}{8} = 300$

$$C(h/2) = \frac{f(3+2) - f(3-2)}{2 \times 2} = \frac{f(5) - f(1)}{4}$$

$$= \frac{625 - 1}{4} = \frac{624}{4} = 156$$

$$C(h/4) = \frac{f(3+1) - f(3-1)}{2 \times 1} = \frac{f(4) - f(2)}{2}$$

$$= \frac{256 - 16}{2} = \frac{240}{2} = 120$$

300, 156 → 102, 108 → 108

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Numerical Integration

$\int_a^b f(x)\,dx =$

$h = \frac{b-a}{n}$ = step length

n denotes number of segments

Let $[a,b]$ will divide into n segments
Then $h = \frac{b-a}{n}$ where $x_0 = a$, $x_n = b$
Then the interval $[a,b]$ has n+1 points

i) Composite Trapezoidal Rule

$$I = \int_a^b f(x)\,dx$$

$$= \frac{h}{2}\left[f(x_0) + 2\{f(x_1) + f(x_2) + \dots + f(x_{n-1})\} + f(x_n)\right]$$

This rule can be applied for any n either even or odd

ii) Composite Simpson 1/3rd Rule

$$I = \frac{h}{3}\left[f(x_0) + 2\{f(x_2) + f(x_4) + \dots + f(x_{n-2})\} + 4\{f(x_1) + f(x_3) + \dots + f(x_{n-1})\} + f(x_n)\right]$$

This rule can be applied only if n is even

Q. Integrate $\int_0^1 \frac{1}{1+x^2}\,dx$ for $n = 2$ by using Trapezoidal and Simpson 1/3rd rule

Soln: $a = 0$, $b = 1$, $n = 2$

$h = \frac{b-a}{n} = \frac{1-0}{2} = \frac{1}{2}$

$f(x) = \frac{1}{1+x^2}$

x	x_0 = 0	x_1 = 0.5	x_2 = 1
$f(x)$	1	0.8	0.5

$f(0) = \frac{1}{1+0^2} = 1$ $\quad f(1) = \frac{1}{1+1^2} = 0.5$

$$I = \frac{h}{2} = \frac{1}{2}\left[f(x_0) + 2\{f(x_1) + f(x_2) + \dots + f(x_{n-1})\} + f(x_n)\right]$$

$$= \frac{1}{4}\left[1 + 2\{0.8\} + 0.5\right]$$

$$= \frac{1}{4}\left[1 + 1.6 + 0.5\right]$$

$$I = \frac{1}{4}\left[2.6\right] = \frac{1}{4}\left[2.6\right] = 0.75$$

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Module - 3
Curve Fitting

Working Procedure

i) To fit the straight line $y = a + bx$

Step 1: Substitute the set of 'n' values in the equation :

1) $\Sigma y = na + b\Sigma x$
2) $\Sigma xy = a\Sigma x + b\Sigma x^2$

→ normal equations

Step 2: Find the solution of a and b by solving system of equations.

Step 3: Substitute the value of a and b in $y = a + bx$

ii) To fit the parabola $y = a + bx + cx^2$

Step 1: Substitute the set of 'n' values in the equation :

1) $\Sigma y = na + b\Sigma x + c\Sigma x^2$
2) $\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$
3) $\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

Step 2: Find the solution of a, b and c by solving system of equation.

Step 3: Substitute the value of a, b and c in $y = a + bx + cx^2$

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1. fit a straight line $y = a + bx$ to the following data

x	y	xy	x^2
0	1	0	0
1	3	3	1
3	2	6	9
6	5	30	36
8	4	32	64
$\Sigma x = 18$	$\Sigma y = 15$	$\Sigma xy = 71$	$\Sigma x^2 = 110$

$n = 5$

$$\underset{c_1}{15} = \underset{a_1}{5a} + \underset{b_1}{18b}$$
$$\underset{c_2}{71} = \underset{a_2}{18a} + \underset{b_2}{110b}$$

$a = 1.64$
$b = 0.376$

$x = -1.64$
$y = -0.37$

$\cancel{15 = -8.2 - 6.66}$
$\cancel{15 =}$

$$\boxed{y = 1.64 + 0.376x}$$

2. Find the parabola of the form $y = a + bx + cx^2$ for the following data

x	y	x^2	x^3	x^4	xy	x^2y
-3	4.63	9	-27	81	-13.89	41.67
-2	2.11	4	-8	16	-4.22	8.44
-1	0.67	1	-1	1	-0.67	0.67
0	0.09	0	0	0	0	0
1	0.63	1	1	1	0.63	0.63
2	2.13	4	8	16	4.3	8.6
3	4.58	9	27	81	13.74	41.22
$\Sigma x = 0$	$\Sigma y = 14.26$	28	0	196	-0.11	101.23

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$\Sigma x = 0$ $\Sigma y = 14.86$ $\Sigma x^2 = 28$ Σx^3
$\Sigma x^4 = 196$ $\Sigma xy = -0.11$ $\Sigma x^2 y = 101.23$

$n = 7$

$\Sigma y = na + b\Sigma x + c\Sigma x^2$

$14.86 = 7a + 0 + 28c$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$
$-0.11 = 0 + 28b + 0$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$
$101.23 = 28a + 0 + 196c$

$a = 0.132$
$b = -3.92$
$c = 0.497$

The following table gives the result of measurements of trained resistance, v is the velocity and 'r' is the resistance. If r is related to v by the relation $r = a + bv + cv^2$ find a, b, c.

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$$y = a e^{bx}$$

$$y = 16.8 \, e^{0.2015x}$$

Find the fourier series of f(x)
$(-\pi, \pi)$

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$$= \left[\frac{x \sin x}{n} - \frac{1}{\pi n}\left(\frac{-\cos nx}{n} \right) \right]_{-\pi}^{\pi}$$

$$= \left[\frac{\pi \sin n\pi}{n} + \frac{\cos n\pi}{n^2} \right] -$$

$$\left[\frac{-\pi \sin(-n\pi)}{n} + \frac{\cos(-n\pi)}{n^2} \right]$$

$$= \frac{\cos n\pi}{n^2} - \frac{\cos n\pi}{n^2} = 0$$

$$b_n = \int_{-\pi}^{\pi} \underset{u}{x} \underset{v}{\sin nx} \, dx$$

$$= \left[-x \cos nx + \int 1 \frac{\cos nx}{n} dx \right]_{-\pi}^{\pi}$$

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Fit the exponential curve $y = ae^{bx}$ for the following data :-

x	y	$X = x$	$Y = \ln y$	X^2	xY
2	25	2	3.218	4	6.436
4	38	4	3.637	16	14.548
6	56	6	4.025	36	24.15
8	84	8	4.430	64	35.44
		20	15.31	120	80.574

$$y = ae^{bx}$$

Apply ln on both sides

$$\ln y = \ln a + bx$$
$$Y = A + BX$$

$Y = \ln y \quad A = \ln a \quad B = b \quad X = x$

$$a = e^A$$

$$\Sigma Y = nA + B\Sigma X$$
$$\Sigma XY = A\Sigma X + B\Sigma X^2$$

$\Sigma X = 20 \quad \Sigma Y = 15.31 \quad \Sigma X^2 = 120$

$\Sigma XY = 80.574 \quad n = 4$

$$15.31 = 4A + 20B$$
$$80.574 = 20A + 120B$$

$$A = 2.8215$$
$$B = 0.2012$$

$B = b = 0.2012$

$a = e^A$

$a = e^{2.8215}$

$= 16.80$

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$\int uv = u\int v - \int u'\int v$

$\cos 2n\pi = 1$

$\sin 2n\pi = 0$

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$\sin n\pi = 0$
$\cos n\pi = (-1)^n$
$\cos(-\theta) = \cos\theta$
$\sin(-\theta) = -\sin\theta$

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

where $x \in (-\pi, \pi)$ or $(0, 2\pi)$

$$a_0 = \int_{-\pi}^{\pi} f(x)\,dx \quad \text{or} \quad \int_{0}^{2\pi} f(x)\,dx$$

$$a_n = \int_{-\pi}^{\pi} f(x)\cos nx\,dx \quad \text{or} \quad \int_{0}^{2\pi} f(x)\cos nx\,dx$$

$$b_n = \int_{-\pi}^{\pi} f(x)\sin nx\,dx \quad \text{or} \quad \int_{0}^{2\pi} f(x)\sin nx\,dx$$

Find the fourier series of $f(x) = x$ in $(-\pi, \pi)$

Sol:

$$a_0 = \int_{-\pi}^{\pi} x\,dx$$

$$= \left[\frac{x^2}{2}\right]_{-\pi}^{\pi} = \frac{\pi^2}{2} - \left(-\frac{\pi}{2}\right)^2 = 0$$

$$a_n = \int_{-\pi}^{\pi} x \cdot \cos nx\,dx$$

$$= \left[x\,\frac{\sin nx}{n} - \int 1 \cdot \frac{\sin nx}{n}\right]_{-\pi}^{\pi}$$

$$= \left[\cancel{\frac{\pi \sin n\pi}{n}}^{\,0} + \cos n\pi\right.$$

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$\therefore f(x) = kx(\pi - x) \qquad 0 < x < 2\pi$

$$a_0 = \int_0^{2\pi} kx(\pi - x)\,dx$$

$$= k\int_0^{2\pi} (\pi x - x^2)\,dx$$

$$= k\left[\frac{\pi x^2}{2} - \frac{x^3}{3}\right]_0^{2\pi}$$

$$= k\left[\frac{\pi\, 4\pi^2}{2} - \frac{8\pi^3}{3}\right]$$

$$= k\left[2\pi^3 - \frac{8\pi^3}{3}\right]$$

$$= k\left[\frac{6\pi^3 - 8\pi^3}{3}\right]$$

$$= -\frac{2\pi k \pi^3}{3}$$

$$a_n = \int_0^{2\pi} kx(\pi - x)\cos nx\,dx$$

$$= k\int_0^{2\pi} \underbrace{(\pi x - x^2)}_{u}\,\underbrace{\cos nx}_{v}\,dx$$

$$= k\left[(\pi x - x^2)\frac{\sin nx}{n} - \int (\pi - 2x)\frac{\sin nx}{n}\,dx\right]_0^{2\pi}$$

$$= k\left[(\pi x - x^2)\frac{\sin nx}{n} - \frac{1}{n}\left\{(\pi - 2x)\left(-\frac{\cos nx}{n}\right) - \int (-2)\left(-\frac{\cos nx}{n}\right)dx\right\}\right]_0^{2\pi}$$

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$$= K\left[(\pi x - x^2)\frac{\sin nx}{n} + (\pi - 2x)\frac{\cos nx}{n^2} + \frac{2}{n^2}\left(\frac{\sin nx}{n}\right)\right]_0^{2\pi}$$

$$= K\left[(\pi 2\pi - 2\pi^2)\cancel{\frac{\sin 2n\pi}{n}}^{\to 0} + (\pi - 2\pi)\frac{\cos 2n\pi}{n^2} + \cancel{\frac{2}{n^3}\sin 2n\pi}^{\to 0}\right]$$

$$- \left[\cancel{\frac{0 \sin 0}{n}}^{\to 0} + (\pi - 0)\frac{\cos 0}{n^2} + \cancel{\frac{2}{n^3}\sin}\right]$$

$$= K\left[\frac{-\pi}{n^2}\right] - \left[\frac{\pi}{n^2}\right]$$

$$= K\,\frac{-2\pi}{n^2}$$

$$= \frac{-2K\pi}{n^2}$$

$$b_n = \int_0^{2\pi} Kx(\pi - x)\sin nx\,dx$$

$$= K\int_0^{2\pi}(\underbrace{\pi x - x^2}_{u})\underbrace{\sin nx}_{v}\,dx$$

$$= K\left[(\pi x - x^2)\left(\frac{-\cos \pi x}{\pi}\right) - \int(\pi - 2x)\left(\frac{-\cos \pi x}{n}\right)dx\right]_0^{2\pi}$$

$$\int uv = u\int v - u'\iint v + u''\iiint v$$

↳ polynomial

$$= \frac{2\pi}{n^2} + \frac{2\pi}{n^2} + 2\pi \; 2 \times 0$$

$$= \frac{4\pi}{n^2}$$

$$b_n = \int_0^{2\pi} (\pi - x)^2 \sin nx \, dx$$

u	v
$\pi^2 + x^2 - 2\pi x$	$\sin nx$
$2x - 2\pi$	$-\frac{\cos nx}{n}$
2	$-\frac{\sin nx}{n^2}$
0	$\frac{\cos nx}{n^3}$

$$= \pi^2 + (2\pi)^2 - 2\pi$$

$$= (\pi - x)^2 \left(8 - \frac{\cos nx}{n}\right) - (2x - 2\pi)\left(-\right.$$

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$$= k\left[-(\pi x - x^2)\frac{\cos \pi x}{n} + \frac{1}{n}(\pi - 2x)\frac{\sin \pi x}{n} - \frac{2\cos nx}{n^3}\right]_0^{2\pi}$$

$$= k\left[-(2\pi^2 - 4\pi^2)\frac{\cos 2n\pi}{n} + \frac{1}{n^2}(\pi - 4\pi)\sin 2n\pi - \frac{2\cos 2n\pi}{n^3}\right]$$

$$= k\left[-0\frac{\cos 0}{n} + \frac{1}{n^2}(\pi - 0)\sin 0 - \frac{2\cos 0}{n^3}\right]$$

$$= k\left[2\pi^2\frac{1}{n} + 0 - \frac{2}{n^3}\right] - k\left[0 + 0 - \frac{2}{n^3}\right]$$

$$= \frac{2k\pi^2}{n} - \frac{2k}{n^3} + \frac{2k}{n^3}$$

$$b_n = \frac{2k\pi^2}{n}$$

$$\cancel{f(x) = x^2 \quad 0}$$

4. $f(x) = x^2 \qquad 0 < x < 2\pi$

$$a_0 = \int_0^{2\pi} x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^{2\pi} = \left[\frac{(2\pi)^3}{3}\right] - \left[\frac{(0)^3}{3}\right]$$

$$= \frac{8\pi^3}{3}$$

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1st $x^2 - 2\pi x$ $\quad \frac{2x^2}{2}$ $\quad (1-x)^{(2)}$ $\quad 2(1-x)(2)$

$2x - 2\pi$ $\quad \frac{(1-x)^2}{2}$ $\quad 2(1-x)(-1)$

$$= -\pi\left(\frac{\sin 0}{n}\right)^{\to 0} -$$

$$= -\pi\left(\frac{\sin 0}{n}^{\to 0} - \frac{\sin(-n\pi)}{n}^{\to 0}\right) +$$

$$\left[\pi\frac{\sin n\pi}{n}^{\to 0} + \frac{\cos n\pi}{n^2}\right] - \left[\frac{0\sin 0}{n}^{\to 0} + \frac{\cos 0}{n^2}\right]$$

$$= \frac{\cos n\pi}{n^2} - \frac{1}{n^2}$$

$$= \frac{\cos n\pi - 1}{n^2} = \begin{cases} -2/n^2 & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}$$

$$b_n = \int_{-\pi}^{0}(-\pi)\sin nx\,dx + \int_{0}^{\pi} x\sin nx\,dx$$

$$= \left|(-\pi)\left(\frac{-\cos nx}{n}\right)\right|_{-\pi}^{0} + \left|x\left(\frac{-\cos nx}{n}\right) - (1)\left(\frac{-\sin nx}{n^2}\right)\right|_{0}^{\pi}$$

$u \quad Sv \quad -u' \quad Sv$

$$= \pi\left(\frac{\cos 0}{n} - \frac{\cos(-n\pi)}{n}\right) + \left\{\frac{-\pi\cos n\pi}{n} + \frac{\sin n\pi}{n^2}^{\to 0}\right\} - \left\{\frac{-0\cos 0}{n}^{\to 0} + \frac{\sin 0}{n^2}^{\to 0}\right\}$$

$$= \pi\left(\frac{1-\cos(n\pi)}{n}\right) - \frac{\pi}{n}\cos n\pi$$

$$= \frac{\pi}{n} - \frac{2\pi}{n}\cos n\pi - \frac{\pi}{n}(1 - 2\cos n\pi)$$

$$= \begin{cases} 3\pi/n & \text{if } n \text{ is odd} \\ -\pi/n & \text{if } n \text{ is even} \end{cases}$$

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$= \frac{4\pi}{n^2}$ $\{\because \cos 2n\pi = 1$

$(-1)^n$

$$f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum a_n \cos nx + b_n \sin nx$$

$$a_0 = \int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \int_{-\pi}^{\pi} f(x) \cos nx\,dx$$

$$b_n = \int_{-\pi}^{\pi} f(x) \sin nx\,dx$$

$$a_0 = \int_{-\pi}^{0} (-\pi)\,dx + \int_{0}^{\pi} x\,dx$$

$$= -\pi \left| x \right|_{-\pi}^{0} + \left| \frac{x^2}{2} \right|_{0}^{\pi}$$

$$= -\pi (0 - (-\pi)) + \left(\frac{\pi^2}{2} - 0\right)$$

$$= -\pi^2 + \frac{\pi^2}{2} + = -\frac{\pi^2}{2}$$

$$a_n = \int_{-\pi}^{0} (-\pi \cos$$

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$$a_n = \int_0^{2\pi} \underset{u}{x^2} \underset{v}{\cos nx}\, dx$$

$$= \left[x^2 \int \cos nx - \int 2x \cdot \frac{\sin nx}{n} \right]_0^{2\pi} dx$$

$$= \left[\frac{x^2 \sin nx}{n} - \frac{2}{n} \int 2x \left(\frac{-\cos nx}{n} \right) \right]_0^{2\pi}$$

$$= \left[\frac{x^2 \sin nx}{n} - \frac{2}{n} \left\{ \frac{2x^2}{2} \left(\frac{-\sin nx}{n} \right) \right. \right.$$

$$= \frac{x^2 \sin nx}{n} -$$

$$= \left[\frac{x^2 \sin nx}{n} - \frac{2}{n} \left[x \cdot \left(-\frac{\cos nx}{n} \right) - \int 1 \cdot -\frac{\cos nx}{n} \right] \right]$$

$$= \left[\frac{x^2 \sin nx}{n} - \frac{2}{n} \left[-\frac{x \cos nx}{n} - \frac{1}{n} \left[-\frac{\sin nx}{n} \right] \right] \right]_0^{2\pi}$$

$$= \frac{x^2 \sin nx}{n} + \frac{2}{n} \frac{x \cos nx}{n} + \frac{2}{n} \frac{\sin nx}{n^2} \Big]_0^{2\pi}$$

$$= \frac{x^2 \sin nx}{n} + \frac{2x \cos nx}{n^2} + \frac{2 \sin nx}{n^3}$$

$$= \left[\frac{2\pi \sin 2\pi n}{n} + \frac{4\pi \cos 2\pi n}{n^2} + \frac{4\pi \sin 2\pi n}{n^3} \right.$$

$$- \left[\frac{0 \sin n0}{n} + \frac{2 \times 0 \cos n0}{n^2} + \frac{4\pi \sin 0n}{n^3} \right]_0^{2\pi}$$

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$f(x) = (\pi - x)^2 \qquad 0 < x < 2\pi$

$a_0 = \int_0^{2\pi} f(x)\,dx$

$a_n = \int_0^{2\pi} f(x) \cos nx\,dx$

$b_n = \int_0^{2\pi} f(x) \sin nx\,dx$

※ formulae

$\int dx = x$

$\int x^n dx = \frac{x^{n+1}}{n+1}$

$\int \cos ax = \frac{\sin ax}{a}$

$\int \sin ax = -\frac{\cos ax}{a}$

$\int uv\,dx = u\int v - u'\iint v + u''\iiint v \ldots$

$a_0 = \int_0^{2\pi} (\pi - x)^2 dx$

$= \int_0^{2\pi} (\pi^2 + x^2 - 2\pi x)\,dx$

$= \left[\pi^2 x + \frac{x^3}{3} - 2\pi \frac{x^2}{2}\right]_0^{2\pi}$

$= \pi^2(2\pi) + \frac{(2\pi)^3}{3} - \pi(2\pi)^2 - \left[\pi^2 x + \frac{0^3}{3} - 2\pi\frac{(0)^2}{2}\right]$

$= 2\pi^3 + \frac{8\pi^3}{3} - 4\pi^3$

$= \left(2 + \frac{8}{3} - 4\right)\pi^3$

$a_0 = \frac{2\pi^3}{3}$

$u = (\pi - x)^2$

$u = \pi^2 + x^2 - 2\pi x$

$u' = 2x - 2\pi$

$u'' = 2$

$a_n = \int_0^{2\pi} \underset{u}{(\pi - x)^2} \underset{v}{\cos nx}\,dx$

$= \int_0^{2\pi}$

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$u\int v \quad - \quad u'\iint v$

$$= \Big[(\pi - x)^2 \int \cos nx\,dx - \underbrace{2(\pi - x)(-1)}_{u'} \iint \cos nx\,dx$$

$+ \; u''\iiint v$

$$+ 2(-1)(-1)\iiint \cos nx\,dx\Big]_0^{2\pi}$$

$$= \Big[(\pi - x)^2\left(\frac{\sin nx}{n}\right) + 2(\pi - x)\left(-\frac{\cos nx}{n \times n}\right)$$

$$+ 2\left(-\frac{\sin nx}{n^2 \cdot n}\right)\Big]_0^{2\pi}$$

$u \qquad v \qquad \cancel{2(\pi - x)(-1)}$

$(\pi - x)^2 \qquad \cos nx$

$u \quad \pi^2 + x^2 - 2\pi x \longrightarrow \frac{\sin nx}{n} \quad \int v$

$u' \quad 2x - 2\pi \longrightarrow -\frac{\cos nx}{n^2} \quad \iint v$

$u'' \quad 2 \longrightarrow -\frac{\sin nx}{n^3} \quad \iiint v$

$u''' \quad 0$

$$= \Big[(\pi^2 + (2\pi)^2 - 2\pi(2\pi))\frac{\sin 2n\pi}{n}^{0} + [2(2\pi) - 2\pi]$$

$$\frac{\cos 2n\pi}{n^2}^{1} - 2\frac{\sin 2n\pi}{n^3}^{0}\Big]_0^{2\pi}$$

$$= \Big[(\pi^2 + 0 - 0)\frac{\sin 0}{n}^{0} + (0 - 2\pi)\frac{\cos 0}{n^2}^{1}$$

$$- 2\frac{\sin 0}{n^3}^{0}\Big]$$

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TRICK: Cancel all the sin terms where you see, because $\sin \pi, 2\pi, 0, 4\pi$ are zero.

Q. $f(x) = x(2l - x)$ $\quad x \in (0, 2l)$ or $x \in (-l, l)$

$$a_0 = \int_0^{2l} f(x)\,dx$$

$$a_n = \int_0^{2l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

$$b_n = \int_0^{2l} f(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

$$a_0 = \int_0^{2l} x(2l - x)\,dx$$

$$= \int_0^{2l} (2lx - x^2)\,dx$$

$$= \int_0^{2l} 2lx\,dx - \int_0^{2l} x^2\,dx$$

$$= 2l \int_0^{2l} x\,dx - \left|\frac{x^3}{3}\right|_0^{2l}$$

$$\cancel{= \frac{2l\,x^2}{2} - \frac{x^3}{3}}$$

$$= 2l \left|\frac{x^2}{2}\right|_0^{2l} - \left|\frac{x^3}{3}\right|_0^{2l}$$

$$= lx^2 - \frac{x^3}{3}$$

$$= l(2l)^2 - \frac{8l^3}{3}$$

$$= 4l^3 - \frac{8l^3}{3}$$

$$= 4l^3$$

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$$a_n = \int_0^{2l} x(2l-x)\cdot\cos\left(\frac{n\pi x}{l}\right)dx$$

u	v
$2lx - x^2$	$\cos\left(\frac{n\pi x}{l}\right)$
$2l - 2x$	$\frac{\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)}$
-2	$-\frac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}$
0	$-\frac{\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}$

$\cos(ax) = \frac{\sin(ax)}{a}$

$\cos\left(\frac{n\pi x}{l}\right) = \frac{\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)}$

$\sin\left(\frac{n\pi x}{l}\right) = \frac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)}$

$$a_n = \left[(2lx - x^2)\frac{l}{n\pi}\cancel{\sin\left(\frac{n\pi x}{l}\right)}^{0} - (2l-2x)\left(\frac{l}{n\pi}\right)^2\left(-\cos\left(\frac{n\pi x}{l}\right)\right) + (-2)\left(\frac{l}{n\pi}\right)^3\cancel{\left(-\sin\frac{n\pi x}{l}\right)}^{0}\right]_0^{2l}$$

$$= \left[(2l-2x)\frac{l^2}{n^2\pi^2}\cos\left(\frac{n\pi x}{l}\right)\right]_0^{2l}$$

$$= \{2l - 2(2l)\}\frac{l^2}{n^2\pi^2}\cancel{\cos\left(\frac{n\pi 2l}{l}\right)}^{1} - (2l-0)\frac{l^2}{n^2\pi^2}\cos(0)$$

$$= (-2l)\frac{l^2}{n^2\pi^2} - 2l\frac{l^2}{n^2\pi^2} = -\frac{4l^3}{n^2\pi^2}$$

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$$b_n = \int_0^{2l} (2lx - x^2) \cdot \sin\left(\frac{n\pi x}{l}\right) dx$$

u	v
$2lx - x^2$	$\sin\left(\frac{n\pi x}{l}\right)$
$2l - 2x$	$-\frac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)}$
-2	$-\frac{\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}$
0	$\frac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}$

$$b_n = (2lx - x^2)\frac{l}{n\pi}\left(-\cos\left(\frac{n\pi x}{l}\right)\right) - (2l - 2x)\frac{l^2}{n^2\pi^2}\left(-\sin\left(\frac{n\pi x}{l}\right)\right) + (-2)\frac{l^3}{n^3\pi^3}\left(\cos\left(\frac{n\pi x}{l}\right)\right)$$

$$= \left[(2lx - x^2)\frac{l}{n\pi}\left(-\cos\left(\frac{n\pi x}{l}\right)\right) + (-2)\frac{l^3}{n^3\pi^3}\left(\cos\left(\frac{n\pi x}{l}\right)\right)\right]_0^{2l}$$

$$= (4l^2 - 4l^2)\frac{l}{n\pi}\left(-\cos\left(\frac{n\pi 2l}{l}\right)\right) + (-2)\frac{l^3}{n^3\pi^3}\left(\cos\left(\frac{n\pi 2l}{l}\right)\right) - \left[2l(0) - (0)^2\frac{l}{n\pi}\right]$$

$$= -\frac{2l^3}{n^3\pi^3}$$

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$$a_n = \int_{-2}^{2} x^2 - 2 \; \cos\left(\frac{n\pi x}{2}\right) dx$$

$x^2 - 2$	$\cos\left(\frac{n\pi x}{2}\right)$
$2x$	$\frac{\sin\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)}$
2	$\frac{-\cos\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)^2}$
0	$\frac{-\sin\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)^3}$

$$a_n = (x^2-2)\frac{2}{n\pi}\left(\sin\left(\frac{n\pi x}{2}\right)\right) - 2x\left(\frac{2}{n\pi}\right)^2\left(-\cos\left(\frac{n\pi x}{2}\right)\right) + 2\left(\frac{2}{n\pi}\right)^3\left(-\sin\left(\frac{n\pi x}{2}\right)\right)$$

$$= 4x - 2x\left(\frac{4}{n^2\pi^2}\right)\left(-\cos\left(\frac{n\pi x}{2}\right)\right)$$

$$= -2(2)\frac{4}{n^2\pi^2}\left(-\cos\left(\frac{n\pi(2)}{2}\right)\right) - (-2)(-2)\frac{4}{n^2\pi^2}\left(-\cos\left(\frac{n\pi(-2)}{2}\right)\right)$$

$$= -\frac{16}{n^2\pi^2}(-1) - \frac{16}{n^2\pi^2}(-1)$$

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$$\left(-\cos\left(\frac{n\pi(0)}{l}\right)\right) + (-2)\frac{l^3}{n^3\pi^3}\left(\cos\left(\frac{n\pi(0)}{l}\right)\right)$$

$$\Rightarrow \frac{2l^3}{n^3\pi^3} - \frac{2l^3}{n^3\pi^3}$$

$$= 0$$

$-l < x < l$

$f(x) = x^2 - 2 \qquad (-2 \le x < 2)$

$l = 2$

$$a_0 = \int_{-2}^{2} f(x)\,dx$$

$$a_n = \int_{-2}^{2} f(x)\cos\left(\frac{n\pi x}{2}\right)dx$$

$$b_n = \int_{-2}^{2} f(x)\sin\left(\frac{n\pi x}{2}\right)dx$$

$$a_0 = \int_{-2}^{2} x^2 - 2\,dx$$

$$= \int_{-2}^{2} x^2\,dx - \int_{-2}^{2} 2\,dx$$

$$= \left|\frac{x^3}{3} - 2x\right|_{-2}^{2}$$

$$= \left[\frac{2^3}{3} - 2(2)\right] - \left[\frac{(-2)^3}{3} - 2(-2)\right]$$

$$= \left[\frac{8}{3} - 4\right] - \left[-\frac{8}{3} + 4\right]$$

$$= \frac{8}{3} - 4 + \frac{8}{3} - 4$$

$$= \frac{16}{3} - 8$$

$$= -\frac{8}{3}$$

$\frac{16}{3} - 8$

$\frac{16 - 24}{3}$

2

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$$= \frac{16}{n^2\pi^2} + \frac{16}{n^2\pi^2}$$

$$= \frac{32}{n^2\pi^2}$$

$$b_n = \int_{-2}^{2} x^2 - 2 \sin\left(\frac{n\pi x}{2}\right) dx$$

$x^2 - 2$	$\sin\left(\frac{n\pi x}{2}\right)$
$2x$	$-\frac{\cos\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)}$
2	$-\frac{\sin\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)^2}$
0	$\frac{\cos\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)^3}$

$$b_n = (x^2-2)\frac{2}{n\pi}\left(-\cos\left(\frac{n\pi x}{2}\right)\right) - (2x)\left(\frac{2}{n\pi}\right)^2 \left(-\sin\left(\frac{n\pi x}{2}\right)\right) + 2\left(\frac{2}{n\pi}\right)^3\left(\cos\left(\frac{n\pi x}{2}\right)\right)$$

$$= \left[(x^2-2)\frac{2}{n\pi}\left(-\cos\left(\frac{n\pi x}{2}\right)\right) + 2\left(\frac{2}{n\pi}\right)^3 \left(\cos\left(\frac{n\pi x}{2}\right)\right)\right]_{-2}^{2}$$

$$= (4-2)\frac{2}{n\pi}\left(-\cos\left(\frac{n\pi 2}{2}\right)\right) + 2\left(\frac{8}{n^3\pi^3}\right)\left(\cos\left(\frac{n\pi 2}{2}\right)\right) - \left[(4-2)\frac{2}{n\pi}\left(-\cos\left(\frac{n\pi(-2)}{2}\right)\right)\right]$$

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Module - 5

$F = ma$

$m\frac{dv}{dt} = F$ $\qquad$ $\frac{d^2x}{dt^2} = 4$

$$\boxed{\frac{d^2x}{dt^2} = 0}$$

Homogeneous D.E

General form of second order linear D.E

$$\frac{d^2y}{d^2x} + P\frac{dy}{dx} + Qy = R(x)$$

where P, Q are constants

If $R(x) = 0$ then it is a homogeneous D.E

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$$+ 2\left(\frac{8}{n^3\pi^3}\right)\left[\cos\left(\frac{n\pi(-2)}{2}\right)\right]$$

$$= -\frac{4}{n\pi} + \frac{16}{n^3\pi^3} - \left(-\frac{4}{n\pi}\right) + \frac{16}{n^3\pi^3}$$

$$= \cancel{-\frac{4}{n\pi} + \frac{16}{n^3\pi^3} + \frac{4}{n\pi} - \frac{16}{n^3\pi^3}}$$

$$= 0$$

$$= -\frac{4}{n\pi} + \frac{16}{n^3\pi^3} + \frac{4}{n\pi} - \frac{16}{n^3\pi^3}$$

$$= 0$$

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2) $\frac{d^2y}{dx^2} + y = 0$

$m^2 y + y = 0$

$m^2 + 1 = 0$

$m = \frac{0 \pm \sqrt{0^2 - 4 \times 1 \times 1}}{2}$

$= \frac{\pm \sqrt{-1 \times 4}}{2}$

$= \frac{\pm 2\sqrt{-1}}{2}$

$m = \pm i \Rightarrow a \pm ib \Rightarrow 0 \pm i \times 1$

$y(x) = e^{0x}\{c_1 \cos x + c_2 \sin x\}$

$y(x) = c_1 +$

3) $(D^2 - 6D + 9)y = 0$

$(m^2 - 6m + 9)y = 0$

$m^2 - 6m + 9 = 0$

$m^2 - 3m - 3m + 9 = 0$

$m(m-3) - 3(m-3) = 0$

$m - 3 = 0 \qquad m - 3 = 0$

$m = 3 \qquad m = 3$

$m = 3, 3$

$9m^2$

$3 \quad 3$

4) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 y + 4my + 5y = 0$

$y(m^2 + 4m + 5) = 0$

$m^2 + 4m + 5 = 0$

$m = \frac{-4 \pm \sqrt{(-4)^2 - 4 \times 1 \times 5}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4 \pm \sqrt{-1 \times 4}}{2}$

$= \frac{-4 \pm 2\sqrt{-1}}{2} = -4 \pm \frac{2\sqrt{-1}}{2}$

$m = \frac{(-2 \pm \sqrt{-1})}{2}$

$m = -2 \pm \sqrt{-1}$

$m = (-2 + \sqrt{-1}, -2 - \sqrt{-1})$

$= (-2 + i, -2 - i)$

$-2 \pm i \Rightarrow -a \pm bi$

$y(x) =$

Rules for finding complimentary function

Step 1: Replace D by m where $D = \frac{d}{dx}$ then we get quadratic equation in 'm' that is $m^2 + Pm + Q = 0$

Step 2: Solve this equation and get two roots that is α and β $(m = \alpha, \beta)$

i) If α, β are real then
$y(x) = c_1 e^{\alpha x} + c_2 e^{\beta x}$ If $\alpha = \beta$ then $y(x) = c_1 + c_2 x e^{\alpha x}$

ii) If α, β are complex then
$m = a \pm ib$
$y(x) = e^{ax}\{c_1 \cos bx + c_2 \sin bx\}$

Eg:- $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Auxiliary equation is:

$(m^2 + m)y = 0$
$m^2 + m = 0$
$\cancel{m^2 + m = 0}$
$m(m+1) = 0$
$m = 0, -1$

$\therefore y(x) = c_1 e^{0x} + c_2 e^{-1x}$
$= c_1 + c_2 e^{-x}$

$ax^2 + bx + c = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$a = 1, b = 1, c = 0$
$m = \frac{-1 \pm \sqrt{1^2 - 4(1)(0)}}{2(1)}$
$m = \frac{-1 \pm 1}{2}$
$m = 0, -1$

Solve:

1) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

2) $\frac{d^2y}{dx^2} - 6 + y = 0$

3) $(D^2 - 6D + 9)y = 0$

4) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

1) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$
$(m^2 - 2m + 1)y = 0$
$m^2y - 2my + y = 0$
$y(m^2 - 2m + 1) = 0$
$m^2 - 2m + 1 = 0$
$m^2 - m - m + 1 = 0$
$m(m-1) - 1(m-1) = 0$
$(m-1) = 0$
$m = 1$

$m - 1 = 0$
$m = 1$

$y(x) = (c_1 + c_2 x)e^{x}$

$c = 1$

$\frac{-2 \pm \sqrt{4 - 4}}{2}$

$\frac{-2 \pm 0}{2}$

$-1 - 1$

$-1 - 1$

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5) $(D^2 + 4D + 3)y = 0$

$(m^2 + 4m + 3)y = 0$

$m^2 + 4m + 3 = 0$

$m^2 + 1m + 3m + 3 = 0$

$m(m+1) + 3(m+1) = 0$

$m + 1 = 0 \qquad m + 3 = 0$

$m = -1 \qquad m = -3$

$m = (-1, -3)$

Method of Variation of Parameter

$y'' + \cancel{py} \; Py' + Qy = R(x)$

Step 1: Solve

$y'' + Py' + Qy = 0$

we get two solutions

$y_1(x)$ and $y_2(x)$

Step 2: Find $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$, known as Wronskian. $(W \neq 0)$

Step 3: Find $C_1(x) = \int \frac{y_2 R(x)}{W} dx$

and $C_2(x) = \int \frac{y_1 R(x)}{W} dx$

$\Rightarrow y_p = C_1' y_1 + C_2' y_2$

Step 4: Solution of given D.E is

$y(x) = C.F + y_p$

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papergrid

Date: $e^0 = 1$

$e^{a+b} = e^a e^b$

$\frac{d(e^{ax})}{dx} = ae^{ax}$

$e^{a-b} = e^a / e^b$

$\int \frac{1}{x} = \log x$

$(D^2 - 2D + 4)y = e^{2x}\sin x$

Step 1: $(D^2 - 2D + 4)y = 0$

Replace D by m

$m^2 - 2m + 4 = 0$

$(m-4)^2 = 0$

$m = 2, 2$

$C.F = (c_1 + c_2 x)e^{2x}$

$= c_1 e^{2x} + c_2 x e^{2x}$ —— ①

$y_1(x) = e^{2x}$

$y_2(x) = \underset{u}{x}\,\underset{v}{e^{2x}}$

$y_1' = 2e^{2x}$

Step 2: $y_2' = x2e^{2x} + e^{2x}$

Step 2: $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_1' \end{vmatrix}$

$= \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix}$

$= e^{2x}(2xe^{2x} + e^{2x}) - xe^{2x}(2e^{2x})$

$= 2xe^{4x} + e^{4x} - 2xe^{4x}$

$= e^{4x}$

Step 3: $c_1 = \int \frac{y_2 R(x)}{W} dx$

$= \int \frac{xe^{2x} e^{2x} \sin x}{e^{4x}} dx$

$= \int \underset{u}{x} \cdot \underset{v}{\sin x}\, dx$

$= u\int v - u'\iint v + u''\iiint v$

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MODULE-4

NUMERICAL DIFFERETIATION AND INTEGRATION.

Differentiation

Forward Difference scheme

$$f'(a) = \frac{f(a+h) - f(a)}{h}$$

a - given point

h ⇒ Step length.

x	1	2	3
$f(x)$	0	0.5	0.7	0.9

If we have to begining of the number Use forward

Backward Diff scheme

$$f'(a) = \frac{f(a) - f(a-h)}{h}$$

$a-h \rightarrow a \rightarrow a+h$

Central Diff scheme

$$f'(a) = \frac{f(a+h) - f(a-h)}{2h}$$

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Note

→ If we have to use the number in the beginning of the table then use FORWARD SCHEME

→ To find the value at the end of the table we have to use BACKWARD SCHEME

→ To find the value at the Middle of the table we have to use CENTRAL SCHEME

Find $f'(x)$ for $f(x) = \frac{\sin(0.5\sqrt{x})}{x}$ at $x = 0.1$ using Forward differential scheme, backward differential scheme & central differential scheme. for $h = 0.01$

FDS, $a = 0.1$, $h = 0.01$

$$f'(a) = \frac{f(a+h) - f(a)}{h}$$

$$= \frac{f(0.1 + 0.01) - f(0.1)}{0.01}$$

$$= \frac{f(0.11) - f(0.1)}{0.01}$$

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$$= \frac{\sin(0.5\sqrt{x})}{x}$$

$$f(0.11) = \frac{\sin(0.5\sqrt{0.11})}{0.11}$$

$$= 1.5006$$

$$f(0.1) = \frac{\sin(0.5\sqrt{0.1})}{0.1}$$

$$= 1.57$$

$$= \frac{1.5006 - 1.5745}{0.01}$$

$$= -7.39.$$

BDS.

$$f'(0.1) = \frac{f(0.1) - f(0.1 - 0.01)}{0.01}$$

$$= \frac{f(\overset{x}{0.1}) - f(\overset{x}{0.09})}{0.01}$$

$$= \frac{\sin(0.5\sqrt{0.1})}{0.01}$$

$$= 15.745$$

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$$= \frac{\sin(0.5\sqrt{0.09})}{0.09}$$

$$= 16.604.$$

$$= \frac{15.745 - 16.604}{0.01}$$

$$= -85.9.$$

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2. The distance 'x' of a runner from a fixed point is measured at intervals of a sec. The data obtain is

t	0	0.5	1	1.5	2
x	0	3.65	6.80	9.90	12.15

Find the velocity obtain 0 sec, 0.5 sec, 1 sec.

Velocity, $x'(t) = \frac{dx}{dt} \frac{x(t+h)-x(t)}{h}$

at $t = 0.5, 1.5, 2.5$.

→ Forward Difference Scheme

$$x'(t) = \frac{x(t+h)-x(t)}{h}$$

Formula

$$f'(a) = \frac{f(a+h)-f(a)}{h}$$

$t=1, \quad h=0.5$

$$x'(1) = \frac{x(1+0.5)-x(1-0.5)}{2(0.5)}$$

$$= \frac{x(1.5)-x(0.5)}{2(0.5)}$$

$$= \frac{9.90-3.65}{1}$$

$$= 6.5$$

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$$x'(2) = \frac{x(2) - x(2-0.5)}{0.5}$$

Formula

$$f'(a) = \frac{f(a) - f(a-h)}{h}$$

$$= \frac{72.15 - 990}{0.5}$$

$$= 7.5$$

$$x'(0) = \frac{x(0+0.5) - x(0)}{0.5}$$

Formula

$$f'(a) = \frac{f(a+h) - f(a)}{h}$$

$$= \frac{3.65 - 0}{0.5}$$

$$= 7.3$$

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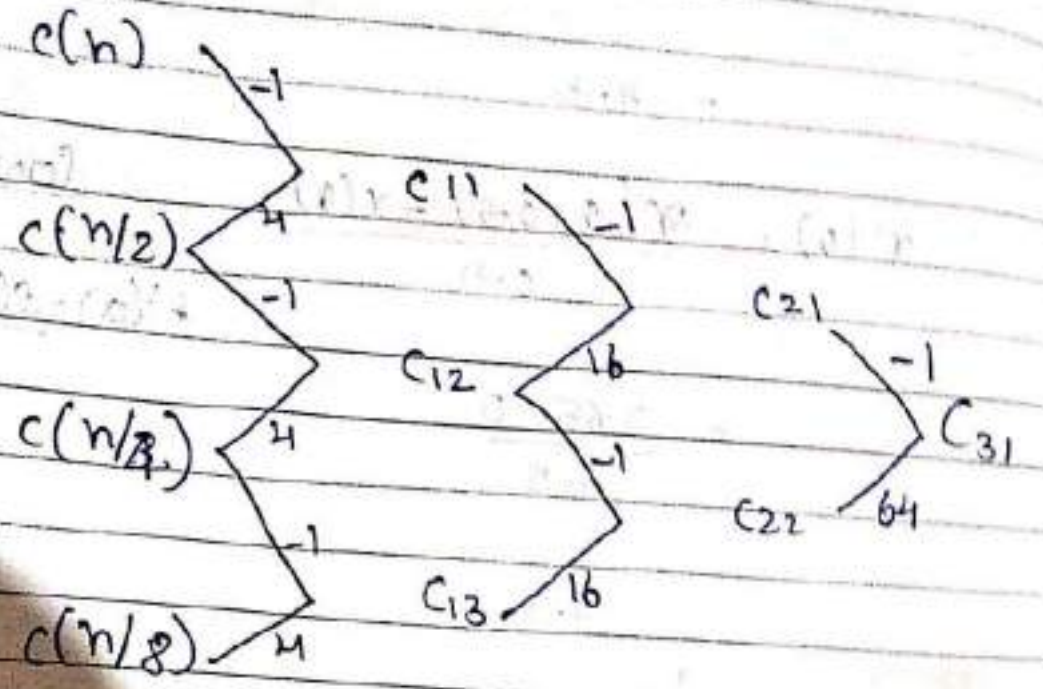
Richardson Extrapolation

Apply Central Difference scheme.

Give a, h



M=More, L=Least Acc

$\frac{4MA-LA}{3}$ $\frac{16MA-LA}{15}$ $\frac{64MA-LA}{63}$

Estimate the value of sinx using Richardson extrapolation for $a=\pi/4$, $h=\pi/3$

$a=\frac{\pi}{4}$, $h=\frac{\pi}{3}$ $f(x)=\sin x$

$$c(h)=\frac{f(a+h)-f(a-h)}{2h}$$

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$$= \frac{\sin(\pi/4 + \pi/3) - \sin(\pi/4 - \pi/3)}{2(\pi/3)} \qquad \pi = 3.14$$

cosi shift exp

$$= \frac{\sin(0.785 + 1.04) - \sin(0.785 - 1.04)}{2(1.04)}$$

$$= \frac{\sin(1.825) + \sin(0.255)}{2.08}$$

$$= \frac{0.96 + 0.252}{2.08}$$

$$= 0.584$$

$$c(h/2) = \frac{\sin(\pi/4 + \pi/6) - \sin(\pi/4 - \pi/6)}{2 \times \pi/6}$$

sine inner exp

$$= \frac{\sin(0.78 + 0.528) - \sin(0.785 - 0.523)}{2(0.523}$$

$$= \frac{\sin(1.303) - \sin(0.262)}{1.046}$$

$$= \frac{0.964 - 0.259}{1.046}$$

$$= 0.673$$

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$$c(\pi/4) = \frac{\sin(\pi/4 + \pi/9) - \sin(\pi/4 - \pi/9)}{2(\pi/9)}$$

$$= \frac{\sin(0.78 + 0.34) - \sin(0.78 - 0.34)}{2(0.34)}$$

$$= \frac{\sin(1.12) - \sin(0.44)}{0.68}$$

$$= \frac{0.90 - 0.425}{0.68}$$

$$= 0.697$$

$$c(\pi/8) = \frac{\sin(\pi/4 + \pi/12) - \sin(\pi/4 - \pi/12)}{2(\pi/9)}$$

$$= \frac{\sin(0.78 + 0.26) - \sin(0.78 - 0.26)}{2(0.26}$$

$$= \frac{\sin(1.04) - \sin(0.52)}{0.52}$$

$$= \frac{0.86 - 0.49}{0.52}$$

$$= 0.715$$

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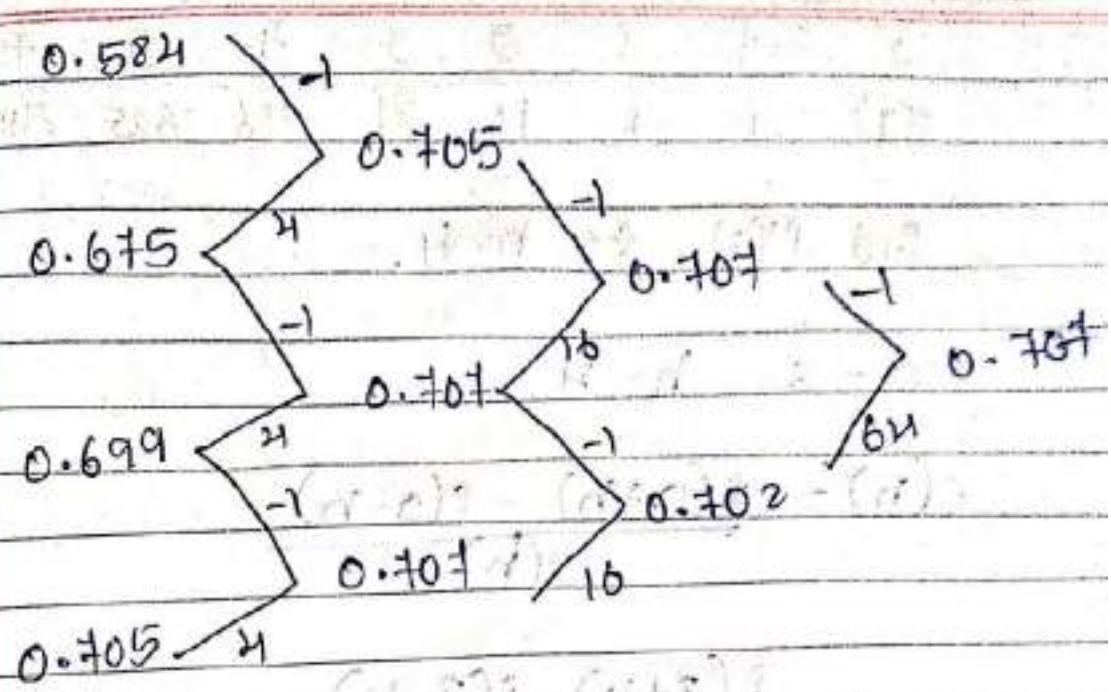

$$\frac{4(6.754) - 0.584}{3}$$

Formula $4\,\frac{MA - LA}{3}$

$$= \frac{9.1 - 0.583}{3}$$

$= 0.705$

$16\,\frac{MA - LA}{15}$

$64\,\frac{MA - LA}{63}$

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x	-1	1	2	3	4	5	7
$f(x)$	1	1	16	81	256	625	2401

find $f'(3)$ for $h=4$.

$a=3, \quad h=4.$

$$c(h) = \frac{f(a+h) - f(a-h)}{2(h)}$$

$$= \frac{f(3+4) - f(3-4)}{2(4)}$$

$$= \frac{f(7) - f(-1)}{8}$$

$$= \frac{7+1}{8}$$

$$= 1$$

$$c(h/2) = \frac{f(3+8) - f(3-8)}{2(8)}$$

$$= \frac{11+5}{16}$$

$$= \frac{16}{16}$$

$c(h) = 300$

$c(h/2) = 156$

$c(h/4) = 120$

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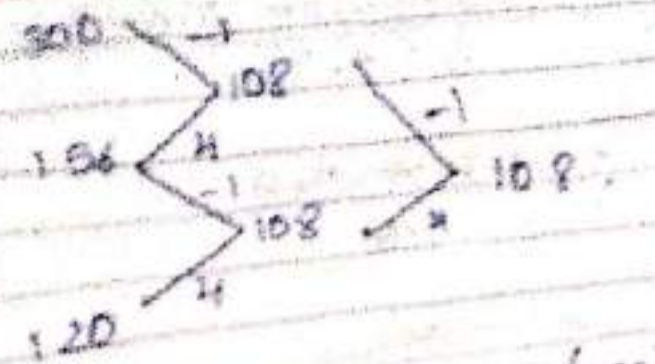

$$\frac{4(156) - 300}{3}$$

$$\frac{4(120) - 156}{3}$$

$$= 108.$$

108.

$$\frac{4(108) - 108}{3}$$

x	0.2	0.4	0.5	0.6	0.7	0.8	1.0
$f(x)$	1.4	1.9	2.1	2.4	2.6	2.9	3.5

find $f'(0.6)$ using F.E $h = 0.4$.

$$c(h) = \frac{f(a+h) - f(a-h)}{2(h)}$$

$$= \frac{f(0.6+0.4) - f(0.6-0.4)}{2(0.4)}$$

$$= \frac{1.1}{0.8}$$

$$= 2.5$$

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$$c(h/2) = \frac{f(0.6+0.8) - (0.6-0.8)}{2(0.8)}$$

$$= \frac{1.4 + 0.2}{1.6}$$

$$= 1$$

$$c(h/2) = h = 0.4$$

$$= \frac{0.4}{2}$$

$$= 0.2.$$

$$c(h/2) = \frac{f(0.6+0.2) - (0.6-0.2)}{2(0.2)}$$

$$= \frac{0.8 + 0.4}{0.4}$$

$$= 3.$$

$$c(h/4) = h = 0.4$$

$$= \frac{0.4}{4}$$

$$= 0.1$$

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$$c(x/y) = \frac{(0.6 + 0.1) - (0.6 - 0.1)}{2(0.1)}$$

$$= \underline{0.7 - 0.5}$$

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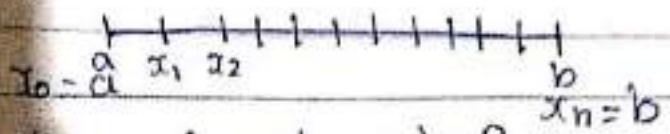
Numerical Integration

Rohan Mis

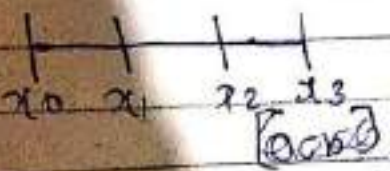
$$\int_a^b f(x)\,dx$$



h = step length = $\frac{b-a}{n}$

n denotes number of segments

$n+1$ is number of points.



let $[a,b]$ will divide into n segments & $h = \frac{b-a}{n}$ where $x_0 = a$, $x_n = b$

then the interval $[a,b]$ as $n+1$ points.

1. Composite Trapezoidal root

$$I = \int_a^b f(x)\,dx$$

$$= \frac{h}{2}\left[f(x_0) + 2\left\{f(x_1) + f(x_2) + \dots f(x_{n-1})\right\} + f(x_n)\right]$$

This rule can be apply for any n either even or odd

2. Composite Simpson

$$I = \frac{h}{3}\left[f(x_0) + 2\{f(x_2) + f(x_4) + \dots f(x_{n-2})\} + 4\{f(x_1) + f(x_3) + \dots + f(x_{n-1})\} + f(x_n) \right]$$

This rule can be apply only if n is even

Problem

1) Integrate $\int_0^1 \frac{1}{1+x^2}\,dx$ for $n=8$ by using Trapezoidal & Simpson's $\frac{1}{3}$rd rule

$h = \frac{b-a}{n}$

$a=0,\ b=1,\ n=8,\ h = \frac{1-0}{8} = \frac{1}{8}$

$f(x) = \frac{1}{1+x^2}$

	x_0	x_1	x_2	x_3	x_4	x_5
x	0	1/8	2/8	3/8	4/8	5/8
$f(x)$	1	0.988	0.94	0.87	0.8	0.7

	x_6	x_7	x_8
x	6/8	7/8	8/8
	0.64	0.56	0.5

$\frac{1}{1+x^2}$ x^2

calci

$1 \div (1 + (8 \div 8)$ shift square)

$1 \div (1 + (7 \div 8)$ shift square

calci

x_8 x_9 should not come in the formula

$$I = \frac{h}{2}\left[f(x_0) + 2\left\{f(x_1) + f(x_2) + \dots f(x_7)\right\} + f(x_8)\right]$$

$$= \frac{1}{8}/2 \left[1 + 2 \times 5.498 + 0.5\right]$$

$$= \frac{0.125}{2}\left[1 + 10.88 + 0.5\right]$$

$$= 0.0625\left[12.496\right]$$

$$= 0.7847$$

$$I = \frac{h}{3}\left[f(x_0) + 2\left\{f(x_2) + f(x_4) + f(x_6)\right\} + 4\left\{f(x_1) + f(x_3) + f(x_5) + f(x_7)\right\} + f(x_8)\right]$$

$$= \frac{1.8}{3}\left[1 + 2(2.46) + 4(3.118) + 0.5\right]$$

$$= \frac{1.8}{3}\left[1 + 4.92 + 12.472 + 0.5\right]$$

$$= \frac{1.8}{3}\left[18.892\right]$$

$$= 0.6\,(18.892)$$

$$= 0.787$$

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$I(h) = h/2 \quad h/4 \quad h/6$

2) $\int_0^2 \frac{e^x \sin x}{1+x^2} \, dx$

$h = b - a$

$2 - 0 = 2$

x	0 (x_0)	2 (x_1)
$f(x)$	0	1.34

$I(h)$ 1.34

$I(h/2)$ 1.14

$I(h/4)$ 1.005

$I(h/8)$

I_1, I_2, I_3, I_4, I_5, I_6

$\frac{4 \text{ MMA} - I_0}{3}$ $\quad \frac{16 \text{ MA} - I_0}{15}$ $\quad \frac{64 \text{ MA} - I_0}{63}$

$I(h) = \frac{h}{2}\left[f(x_0) + f(x_1)\right]$

$= \frac{2}{2}\left[0 + 1.34\right]$

$= 1\,(1.34)$

$I(h) = \underline{1.34}$

$I(h/2) = \frac{2}{2} = 1$

x	0 (x_0)	1 (x_1)	2 (x_2)
$f(x)$	0	1.14	1.34

$\frac{h}{2}\left[f(x_0) + 2f(x_1) + f(x_2)\right]$

$0.5\left[0 + 2(1.14) + 1.34\right]$

$\underline{I = 1.81}$

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$I(h/4) = \frac{2}{4} = \frac{1}{2} = 0.5$

x	0	0.5	1	1.5	2
$f(x)$	0	0.63	1.14	1.37	1.34

$\frac{h}{2}\left[f(x_0) + 2(0.63 + 1.14 + 1.37) + 1.34\right.$

$\frac{0.5}{2}(0 + 7.62)$

$= 1.905$

$I(h/8) = \frac{2}{8} = \frac{1}{4} = 0.25$

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for 0.25 ans =
1.35 [illegible] put [illegible]
0.25

$I(n/8) = \frac{2}{8} = \frac{1}{4} = 0.25$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6
x	0	0.25	0.5	0.75	1	1.25	1.5
$f(x)$	0	0.29	0.63	0.92	1.14	1.29	1.39

	x_7	x_8
x	1.75	2
f	1.39	1.34

$$\frac{h}{2}\left[f(x_0) + 2\{f(x_2) + f(x_4) + f(6)\} + 4\{f(x_1) + f(x_3) + f(x_5) + f(x_7)\} + f(x_8)\right]$$

$$= \frac{0.25}{2}\left[f(0) + [2((0.63) + (1.14) + (1.34))] + [4((0.29) + (0.92) + (1.39)] + 1.34\right.$$

$$= 0.125\left[0 + (2[3.12]) + (4[2.6]) + 1.34\right.$$

$$= 0.125\left[0 + 6.24 + 10.4 + 1.34\right]$$

$$= 0.125[17.98]$$

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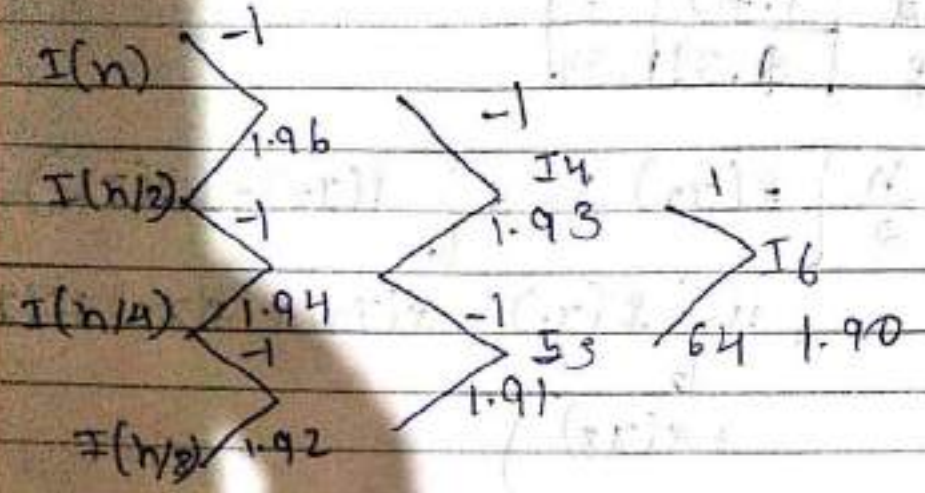
$$\frac{0.125}{2}\left[f(0) + \left[2\left(f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7)\right) + f(x_8)\right]\right.$$

$$= 0.125\,[0 + 14.06 + 1.34]$$

$$= 1.925.$$



$$\frac{4MA-2A}{3} \qquad \frac{16MA-1A}{15} \qquad \frac{64MA-2A}{63}$$

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(Module - 3)

Working procedure

1. To fit the straight line $y = a + bx$

Step 1 :- Substitute the set of n values in the equation

$\Rightarrow \Sigma y = na + b\Sigma x$

$\Rightarrow \Sigma xy = a\Sigma x + b\Sigma x^2$

These are known as normal equation

Step 2 :- Find the solution of a and b by solving system of eqn

Step 3 :- Substitute the value of a & b in $y = a + bx$

2. To fit the parabola $y = a + bx + cx^2$

Step 1 :- Substitute the set of n values in the equation

$\Rightarrow \Sigma y = na + b\Sigma x + c\Sigma x^2$

$\Rightarrow \Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$\Rightarrow \Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

Step 2 :- Solve this system of equation find the value of a, b, c.

Step 3 :- Substitute the value of a & b, c in $y = a + bx + cx^2$

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1) Fit a straight line $y = a + bx$ to the following data.

x	y	xy	x^2
0	1	0	0
1	3	3	1
3	2	6	9
6	5	30	36
8	4	32	64
$\Sigma x = 18$	$\Sigma y = 15$	$\Sigma xy = 71$	$\Sigma x^2 = 110$

$c_1 \quad a_1 \quad b_1$

$15 = 5a + 18b$ $\qquad \Sigma y = na + b\Sigma x$

$71 = 18a + 110b$ $\qquad \Sigma xy = a\Sigma x + b\Sigma x^2$

$a_2 \quad b_2 \quad c_2$

Below ans use in cal Bod mode & eqn

$a = 1.626$

$b = 0.376$

$$\boxed{y = 1.626 + 0.376x}$$

2) Find the parabola $y = a + bx + cx^2$ for the following data.

x	y	x^2	x^3	x^4	xy	x^2y
-3	4.63	9	-27	81	-13.89	41.67
-2	2.11	4	-8	16	-4.22	8.44
-1	0.67	1	-1	1	-0.67	0.67
0	0.09	0	0	0	0	0
1	0.63	1	1	1	0.63	0.63
2	2.15	4	8	16	4.3	8.6
3	4.58	9	27	81	13.74	41.22
$\Sigma x = 0$	$\Sigma y = 14.86$	$\Sigma x^2 = 28$	$\Sigma x^3 = 0$	$\Sigma x^4 = 196$	$\Sigma xy = -0.11$	$\Sigma x^2y = 101.23$

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$d_1 \quad a_1 \quad b_1 \quad c_1$

$14.86 = 7a + 0b + 28c$

$-0.11 = 0a + 28b + 0c$

$101.23 = 28a + 0b + 196c$.

use the cash Rd rots & coun unknown

$a = 0.132$

$b = -0.0039$

$c = 0.4975$.

3 The following table gives the results of measurement of tacing resistors. V is velocity & r is a resistor. If r is related to V by the relation $R = a + bV + cV^2$ find a, b & c

x	y					
V	R	V^2	V^3	V^4	V.R	V^2R
20	5.5	400	8000	160000	110	2200
40	9.1	800	64000	256000	364	7280
60	14.9	3600	216000	1296000	894	536
80	22.8	6400	512000	4096000	1824	53640
100	33.3	10,000	1000000	10000000	3330	333000
120	46.0	14400	1728000	20736000	5520	
$\Sigma x = 420$	$\Sigma y = 131.6$	$\Sigma x^2 = 3564$	Σx^3	Σx^4	Σxy	Σx^2y
		35600	3528000	36400 0000	12042	

$R = 3.48 - 0.002V + 0.0029V^2$

4) Fit a relation of the $V = a + b\theta$ for the given data

x	y	xy	x^2
θ	V	θV	θ^2
0	1.80	0	0
5	1.45	7.25	25
10	1.18	11.8	100
15	1.00	15	225
$\Sigma\theta = 30$	$\Sigma V = 5.43$	$\Sigma\theta V = 34.05$	$\Sigma\theta^2 = 350$

$n = 4$

$\Sigma y = na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

$c_1 \quad a_1 \quad b_1$

$5.43 = 4a + 30b$

$c_2 \quad a_2 \quad b_2$

$34.05 = 30 + 350$

[illegible]

$y = -0.0534$

5) Find straight line to following data & find the production in 1986

x	y	xy	x^2
1941	8	15528	3767481
1951	10	19510	3806401
1961	12	23532	3845521
1971	10	19710	3884841
1981	16	31696	3922361
$\Sigma x =$ 9805	Σy 56	Σxy 109976	Σx^2 19228605

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$\Sigma y = na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

$c_1 \quad a_1 \quad b_1$

$56 = 5a + 9805b$

$109916 = 9805a + 19828605b$

$x = -302.56$

$y = 0.16$

$y = -302.56 + 0.16$

where $x = 1986$

$y = -302.56 + 0.16(1986)$

$y = 15.2$

6) V (Km/hr) and R (kg / tonne) are related by a relation the type $R = ab^{v^2}$ find by the method of least square α & b with data

x	y	x^2	xy
V	R	V^2	VR
10	8		
20	10		
30	15		
40	21		
50	30		
Σx	Σy	Σx^2	Σxy

FOR

Formula

$\Sigma RV = a\Sigma V + b\Sigma V^3 \rightarrow (2)$

$\Sigma R = na + b\Sigma V^2 \rightarrow (1)$

x	y	x^2	x^3	yx
V	R	V^2	V^3	RV
10	8	100	1000	80
20	10	400	8000	200
30	15	900	27000	450
40	21	1600	64000	840
50	30	2500	125000	1500
Σx	Σy	Σx^2	Σx^3	Σyx
150	84	5500	225000	3070

$\cancel{3070} = a_1 \quad b_1$

$84 = 5a_2 + 5500b$

$3070 = 150 + 225000$

$x = 6.716$

$y = 0.009$

7) Fit a curve $y = ax^b$ for the following

in calsi ln(1, 2, 3)

x	y	$X = \log x$	$Y = \log y$	X^2	XY
1	0.5	0	-0.69	0	0
2	2	0.693	0.693	0.480	0.48
3	4.5	1.0986	1.5050	1.205	1.64
4	8	1.386	2.07	1.920	2.86
5	12.5	1.6098	2.52	2.588	4.05
Σx	Σy	ΣX	ΣY	ΣX^2	ΣX
15	27.5	4.786	6.093	6.085	9.0

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$y = ax^b$

Taking $\log_e$ on both sides

$\log y = \log a + b \log x$

$Y = A + bX$

$Y = \log y \quad A = \log a \quad B = b \quad X = \log x$

$\boxed{a = e^A \quad b = B}$

$\Sigma Y = An + B\Sigma X$

$\Sigma XY = A\Sigma X + B\Sigma X^2$

$\underset{c_1}{6.093} = \underset{a_1}{5A} + \underset{b_1}{4.78B}$

$\underset{c_2}{9.037} = \underset{a_2}{4.78} + \underset{b_2}{6.185B}$

$a = -0.689 - 0.695$

$b = 1.988$

$a = e^A = e^{-0.695} = 0.499$

$b = 2.02$

Scanned by CamScanner

Rhombus Assignment

1) $\int_0^{1/2} \frac{x}{\sin x} dx$

$h = \frac{1}{2} - 0 = \frac{1}{2}$ $f(x) = \frac{x}{\sin x}$

in case i $(1/2) \div (\sin 1/2)$

x	0	1/2
$f(x)$	1	1.04

$\lim_{x \to 0} \frac{x}{\sin x} = 1$

for 0 0.001 $\div (\sin 0)$

$I_1 = \frac{1/2}{2} [1 + 1.04] = 0.5105$

$= 0.25 (2.04)$

$= 0.5106$

$h = \frac{1}{4}$

x	0	1/4	2/4
$f(x)$	1	1.01	1.04

$I_2 = \frac{1/4}{2} [1 + 2(1.01) + 1.04]$

$= \frac{0.25}{2} [1 + 4(1.01) + 1.04]$

$= 0.16 [1 + 2(1.01) + 1.04]$

$= 0.16 (1 + 2.02 + 1.04)$

$= 0.16 (4.06)$

$= 0.5075$

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$h = \frac{1}{8}$

x	0	1/8	2/8	3/8	4/8
$f(x)$	1	1.002	1.01	1.023	1.04

$$= \frac{1/8}{2}\left[1 + \left[2(1.0020 + 1.01 + 1.023) + 1.04\right]\right]$$

$$= \frac{1}{16}\left[1 + 2(3.035) + 1.04\right]$$

$$= 0.0625(1 + 6.07 + 1.04)$$

$$= 0.0625(8.11)$$

$$= 0.5068$$

x	0	1/16	2/16	3/16	4/16	5/16	6/16	7/16	8/16
$f(x)$	1	1.006	1.0027	1.005	1.01	1.016	1.023	1.032	1.04

$$= \frac{1/16}{2}\left[1 + 2\left[(1.006 + 1.0027 + 1.005 + 1.01 + 1.016 + 1.023 + 1.032)\right] + 1.04\right]$$

$$= 0.5067$$

$I_1 = 0.5105$ $\quad -1 \quad$ I_1 0.5106 $\quad -1 \quad$ I_1 0.5083 $\quad$ 0.5063

$I_2 = 0.50775$ $\quad -1 \quad$ I_2 0.5063 $\quad 16 \quad -1 \quad$ 0.5067

$I_3 = 0.5067$ $\quad -1 \quad$ I_3 0.5067 $\quad 16$

$I_4 = 0.5067$

$I_6 = 0.5067$

$$\frac{4MA - LA}{3} \quad \frac{16MA - LA}{15} \quad \frac{64MA - LA}{63}$$

Scanned by CamScanner

Fit the exponential curve $y = ae^{bx}$

x	y	$X = x$	$Y = \ln y$	X^2	XY
2	25	2	3.218	4	6.436
4	38	4	3.637	16	14.548
6	56	6	4.025	36	24.15
8	84	8	4.430	64	35.44
$\Sigma x = 20$	Σy	ΣX	ΣY	ΣX^2	ΣXY
	203	20	15.31	120	80.58

$n = 4$

$$y = aeb^x$$

Apply ln on both sides

$$\ln y = \ln a + bx$$

$$Y = A + Bx$$

$Y = \ln y \quad A = \ln a \quad B = b \quad X = x$

$$a = e^A$$

$$\Sigma Y = nA + B\Sigma X$$

$$\Sigma XY = A\Sigma X + B\Sigma X^2$$

Scanned by CamScanner

$$15.51 = 4a + 60b$$
$$80.58 = 20a + 120b$$

$A = 2.8$

$B = 0.2015$.

$a = e^{A} = 16.44$

$b = B = 0.2015$

$e^{A} = e^{2.8}$

$$\therefore \boxed{y = 16.44\, e^{0.2015x}}$$

Scanned by CamScanner

1. Roots of non linear equation

1. Bisection Method

a) $x^3 + x^2 - 3x - 3 = 0$

$x^3 + x^2 - 3x = 3$

$f(x) = x^3 + x^2 - 3x - 3 = 0$

$f(1) = 1 + 1 - 3 - 3 = -4$

$f(2) = 8 + 4 - 6 - 3 = +3$

The root lies in $(1, 2)$

Step 1: $x_1 = 1$ $x_2 = 2$

$$x_3 = \frac{x_1 + x_2}{2} = \frac{3}{2} = 1.5$$

$f(1.5) = (1.5)^3 + (1.5)^2 - 3 \times 1.5 - 3 = -1.875$ (-ve)

1	1.5	2
-ve	-ve	+ve

Step 2: $x_1 = 1.5$ $x_2 = 2$

$x_3 = 1.75$

$f(1.75) = 0.1715$ +ve

1.5	1.75	2
-ve	+ve	+ve

Step 3: $(1.5, 1.75)$

$x_3 = 1.625$

$f(x_3) = -0.9433$ -ve

1.625	1.75
-ve	+ve

Step 4: $x_1 = 1.625$ $x_2 = 1.75$ $x_3 = 1.6875$

$f(1.6875) = -0.4094$

1.5	1.6875	1
-ve	-ve	+ve

Conv: till 0.000

2. Regular false method

Formula $x_3 = x_2 - f(x_2)\dfrac{x_2 - x_1}{f(x_2) - f(x_1)}$

Eg: $x^3 + x^2 - 3x - 3 = 0$

	x_1	x_2
	1	2
$f(x)$	-4	3

$$x_3 = 2 - 3\left(\frac{2-1}{3-(-4)}\right)$$

$$= 2 - 3\left(\frac{1}{7}\right)$$

$$= 2 - \frac{3}{7} = 1.57$$

$$f(x_3) = (1.57)^3 + (1.57)^2 - 3 \times 1.57 - 3$$

$$= -1.375207$$

IInd step: (1.57, 2)

	x_1	x_2
x	1.57	2
$f(x)$	-1.375	3

$$x_3 = 2 - 3\left(\frac{2 - 1.57}{3 - (-1.375)}\right)$$

$$x_3 = 1.705$$

$$f(x_3) = (1.705)^3 + (1.705)^2 - 3 \times 1.705 - 3$$

$$= -0.2514$$

x	1.705	2
$f(x)$	-0.2514	3

$x_3 = 1.727$

$f(x_3) = (1.727)^3 + (1.727)^2 - 3 \times 1.727 - 3$

$f(x_3) = -0.0476$

IVth step: (1.727, 2)

x	1.727	2
$f(x)$	-0.0476	3

$x_3 = 1.7312$

$f(x_3) = -0.00804$

A root of the eqn $f(x) = x^3 - 5x + 1$ lies in the interval (0, 1). perform 4 iterations of regular false method to get the root.

$f(x) = x^3 - 5x + 1$ lies in $(0, 1)$ x_1 x_2

$f(0) = 0 - 5(0) + 1 = 1$

$f(1) = 1 - 5 + 1 = -3$

	x_1	x_2
	0	1
$f(x)$	1	-3

step 1: $x_3 = 1 - (-3)\left(\frac{1-0}{-3-1}\right)$

$= 1 + 3\left(\frac{1}{-4}\right)$

$= 1 - \frac{3}{4} = \frac{4-3}{4} = \frac{1}{4} = 0.25$

$f(x_3) = (0.25)^3 - 5(0.25) + 1$

$= -0.234375$

$(0^+, 0.\bar{2}5)$

$x_1 = 0 \qquad x_2 = 0.25$

$f(x_1) = 1 \qquad f(x_2) = -0.234$

$$x_3 = 0.25 - (-0.234375)\left(\frac{0.25 - 0}{-0.234 - 1}\right)$$

$$= 0.25 + 0.234375\left(\frac{0.25}{-1.234375}\right)$$

$$= \cancel{0.25 + 0.0585} - \left(\frac{0.05859375}{1.234375}\right)$$

$= 0.25 + 0.234375(-0.202531645)$

$= 0.25 - 0.047468354$

$= 0.202531646$

$f(x_3) = 8.307659285 - 1.012658237 + 1$

$= 8.307659285 - 2.012658237$

$= 6.295001055$

Newton - Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

eg: $f(x) = x^3 - 5x + 1 = 0$

$f(0) = 1 > 0 \qquad f(1) = 1 - 5 + 1$

$= -3 < 0$

Root Lies in $(0, 1)$

$x_1 = 0.5$ (Mid-pt)

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$f(x_1) = (0.5)^3 - 5 \times 0.5 + 1 = -1.375$

$f'(x) = 3x^2 - 5$

$f'(x_1) = 3 \times (0.5)^2 - 5 = -4.25$

$$x_2 = 0.5 - \left(\frac{-1.375}{-4.25}\right)$$

$x_2 = 0.1764$

2nd step: $x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$

$$= 0.1764 - \frac{(-1.87651096826)}{(-4.90664912)}$$

$= 0.1764 - 0.3824424617 = -0.2060424617$

$f(x_2) = (0.1764)^3 - 5 \times 0.1764 + 1$

$= \cancel{5.489031744 - 0.8821 + 1}$

$= 0.005489031744 - 1.882$

$= -1.87651096826$

$f'(x_2) = 3x^2 - 5$

$= 3(0.1764)^2 - 5$

$= 3(0.03111696) - 5$

$= 0.09335088 - 5$

$= -4.90664912$

Con:

perform 4 iterations of Newton-Raphson method to obtain the approximate value of $(17)^{1/3}$ with initial guess $x_0 = 2$

→ $(17)^{1/3}$ $\quad x_0 = 2$

$x = (17)^{1/3}$

$x^3 = 17$

$x^3 - 17 = 0$

$f(x) = x^3 - 17 \qquad f'(x) = 3x^2$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^3 - 17}{3x_n^2}$$

Step 1: $x_0 = 2$

$$x_1 = 2 - \frac{2^3 - 17}{3 \times (2)^2} = 2 - \frac{8-17}{3 \times 4} = 2 - \frac{(-9)}{12}$$

$$= 2.75 \qquad = 2 + 0.75$$

$$x_2 = x_1 - \frac{x_1^3 - 17}{3x_1^2}$$

$$= 2.75 - \frac{(2.75)^3 - 17}{3 \times (2.75)^2}$$

$$= 2.75 - \frac{20.796875 - 17}{3 \times 7.5625}$$

$$= 2.75 - \frac{3.796875}{22.6875}$$

$$= 2.75 - 0.1673553719$$

$$= 2.5826446281$$

$x_3 = 2.57$ $\qquad$ $x_4 = 2.57$

Picard's iteration method

$$x_{n+1} = \phi(x_n)$$

Eg: $x^3 - 5x + 1 = 0$

$$x^3 + 1 = 5x$$

$$\boxed{x = \frac{x^3+1}{5}}$$

$$\phi(x) = \frac{x^3+1}{5}$$

$$\boxed{x_{n+1} = \frac{x_n^3 + 1}{5}}$$

$x_0 = 0.5$

$x_1 = \frac{(0.5)^3+1}{5} = 0.225$

$x_2 = \frac{x_1^3+1}{5} = 0.20227$

$x_3 = \frac{x_2^3+1}{5} = 0.20165$

$x_4 = \frac{x_3^3+1}{5} = 0.20163$

$e^x - 3x = 0$

$e^x = 3x$

$x = \frac{e^x}{3}$

$x^3 - 17$

$x = (17)^{1/3}$

Can't do

Muller's method

x_0	x_1	x_2
$f(x_0)$	$f(x_1)$	$f(x_2)$

$h_0 = x_1 - x_0$ $\quad$ $h_1 = x_2 - x_1$

$\delta_0 = \frac{f(x_1) - f(x_0)}{h_0}$ $\quad$ $\delta_1 = \frac{f(x_2) - f(x_1)}{h_1}$

$a = \frac{\delta_1 - \delta_0}{h_1 + h_0}$ $\quad$ $b = ah_1 + \delta_1$ $\quad$ $c = f(x_2)$

$x_3 = x_2 - \frac{2c}{b \pm \sqrt{b^2 - 4ac}}$

1. perform 2 iterations of Muller's method for finding the root of $x^3 - 5x + 1 = 0$ in the interval $(0,1)$.

→ $f(x) = x^3 - 5x + 1 = 0$ $\quad$ $(0,1)$

$x_0 = 0 \quad x_1 = 0.5 \quad x_2 = 1$

$f(x_0) = 1 \quad f(x_1) = 1.375 \quad f(x_2) = -3$

$h_0 = x_1 - x_0$ $\quad$ $h_1 = x_2 - x_1$

$= 0.5 - 0$ $\quad$ $= 1 - 0.5$

$= 0.5$ $\quad$ $= 0.5$

$$\delta_0 = \frac{f(x_1) - f(x_0)}{h_0}$$

$$= \frac{-1.375 - 1}{0.5}$$

$$= -4.75$$

$$\delta_1 = \frac{f(x_2) - f(x_1)}{h_1}$$

$$= \frac{-3 - (-1.375)}{0.5}$$

$$= -3.25$$

$$a = \frac{-3.25 - (-4.75)}{0.5 + 0.5}$$

$$= 1.5$$

$$b = ah_1 + \delta_1$$

$$= 1.5(0.5) + (-3.25)$$

$$= -2.5$$

$$c = f(x_2)$$

$$= -3$$

$$x_3 = x_2 - \frac{2c}{b \pm \sqrt{b^2 - 4ac}}$$

$$= 1 - \frac{2(-3)}{-2.5 - \sqrt{(2.5)^2 - (4)(1.5)(-3)}}$$

$$= 1 - \frac{(-6)}{-2.5 - \sqrt{6.25 - 18}}$$

$$= 1 - \frac{(-6)}{-2.5 - \sqrt{-11.75}}$$

$$= 1 - \frac{(-6)}{0.4278273002}$$

$$x_3 = 0.1914$$

new x_2

IInd step: $x_0 = 0.5$ $\quad x_1 = 1$ $\quad x_2 = 0.1914$

$f(x_0) = -1.375$ $\quad f(x_1) = -3$ $\quad f(x_2) = 0.05$

$$h_0 = x_1 - x_0$$

$$= 1 - 0.1914$$

$$= 0.5$$

$$h_1 = x_2 - x_1$$

$$= -0.8086$$

$$\delta_0 = \frac{f(x_1) - f(x_0)}{h_0}$$

$$= \frac{-3 - (-1.375)}{0.5}$$

$$= -3.25$$

$$\delta_1 = \frac{f(x_2) - f(x_1)}{h_1}$$

$$= \frac{0.05 - (-3)}{-0.8086}$$

$$= -3.772$$

$$a = \frac{\delta_1 - \delta_0}{h_1 + h_0} = \frac{3.772 - (-3.25)}{-0.8086 + 0.5}$$
$$= 1.69$$

$b = -5.13$ $\qquad x_3 = 0.2012$

2. $\cos x - xe^x = 0$ perform 2 iteration using Muller's method

→ $\cos x - xe^x = 0$ $\quad (0,1)$

1st step

$x_0 = 0$ $\qquad x_1 = 0.5$ $\qquad x_2 = 1$

$f(x_0) = 1$ $\qquad f(x_1) = 0.05358$ $\qquad f(x_2) = -2.1779$

$f(x) = \cos x - xe^x$

$f(0) = \cos(0) - (0)e^{(0)}$

$= 1 - (0)(2.71)$

$= 1$

$f(x_2) = \cos(1) - (1)e^{(1)}$

$= 0.540302305 - (1)(2.7182)$

$= -2.177897695$

$f(x_1) = \cos(0.5) - (0.5)e^{(0.5)}$

$= 0.87758 - (0.5)(1.648)$

$= 0.87758 - 0.824$

$= 0.05358$

[Mode → R
shift → ln $\quad e(0) = 0$]

$h_0 = x_1 - x_0$

$= 0.5 - 0 = 0.5$

$h_1 = x_2 - x_1$

$= 1 - 0.5 = 0.5$

$$\delta_0 = \frac{f(x_1) - f(x_0)}{h_0} = \frac{0.05358 - 1}{0.5} = -1.89284$$

$$\delta_1 = \frac{f(x_2) - f(x_1)}{h_1} = \frac{-2.1779 - 0.0538}{0.5} = -4.4634$$

$$a = \frac{\delta_1 - \delta_0}{h_1 + h_0} = \frac{-4.4634 - (-1.89284)}{0.5 + 0.5} = -2.57056$$

$b = ah_1 + \delta_1 = (-2.57056)(0.5) + (-4.4634)$ $\qquad c = (-2.177$

$= -1.28528 - 4.4634 = -5.74868$

$$x_3 = x_2 - \frac{2c}{b \pm \sqrt{b^2 - 4ac}} \qquad [\because b \text{ is } -ve]$$

$$= 1 - \frac{(2)(-2.1779)}{-5.74868 \pm \sqrt{(-5.74868)^2 - (4)(-2.57056)(-2.1779)}}$$

$$= 1 - \frac{(-4.3558)}{-5.74868 - \sqrt{33.0473 - 22.3936905}}$$

$$= 1 - \frac{(-4.3558)}{-5.74868 - \sqrt{10.6536}}$$

$$= 1 - \frac{(-4.3588)}{-5.74868 - 3.2639}$$

$$= 1 - \frac{(-4.3558)}{-9.01258}$$

$$= 1 - 0.483302228$$

$$x_3 = 0.51667772 \qquad [\because \text{New } x_2]$$

II nd step: