Probability

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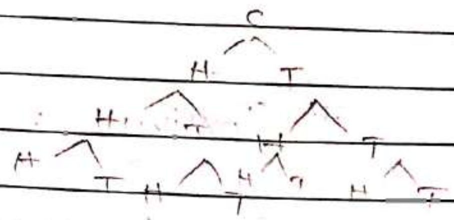
1) What is the probability of getting a head while tossing a coin ~~once~~
   i) once
   ii) twice
   iii) thrice



$\longrightarrow$

Experiment :- Tossing a coin

i) Sample Space = {H, T} H = Head, T = Tail

No. of all possible outcomes $n(S) = 2$

event of getting a head — $A = \{H\}$

$$n(A) = \{1\}$$

probability of getting the favourable event

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$$ //

ii) Tossing a coin twice
Sample Space = {HH, HT, TH, TT}

$n(S) = 4$ while tossing a coin twice

event of getting a head — $A = \{HT, TH, HH\}$

$$n(A) = \{3\}$$

probability of getting the favourable event,

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

iii) Tossing a coin thrice

Sample Space = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

$n(S) = 8$

event of getting a head while tossing a coin thrice. — A = {HHH, HHT, HTH, HTT, THH, THT, TTH}

$n(A) = 7$

Probability of getting the favourable event,

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$$

Types of events:

1) Exhaustive events :-
An event consisting of all various possibility is called Exhaustive event

2) Mutually exclusive events
Two (or) more events are said to be a mutually exclusive if the happening of one event "prevent" the Simultaneous happening of other.

3) Independent events :- Two (or) more events are said to be independent if the happening (or) none happening does not effect (or) prevent the happening (or) non-happening of an event.

4)

let $p \rightarrow$ probability of an favourable event(E) = probability of Success =

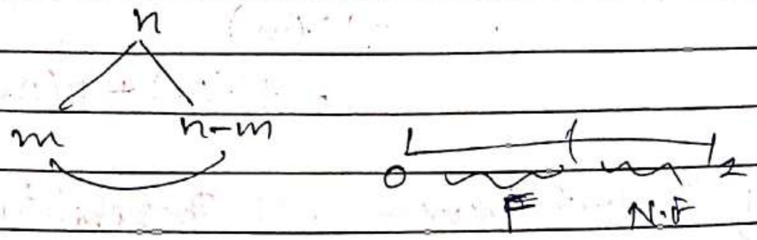
$$P(E) = p = \frac{\text{no. of favourable events}}{\text{Sample space}}$$

$$= \frac{n(E)}{n(S)}$$

$$= \frac{m}{n}$$

5) Probability of a non-favourable event (NF) = Probability of failure = $q = 1 - p$

Note :-



$$\therefore \quad P(F) + P(NF) = \frac{m}{n} + \frac{n-m}{n}$$

$$= \frac{n}{n} = 1$$

$$p + q = 1$$

$$q = 1 - p$$

* Axioms of probability

i) $P(E) >= 0$

ii) $0 \leq P(E) \leq 1$

iii) Probability of an sure event = 1,
Probability of an impossible event = 0.

iv) Addition theorem of probability

If A & B are two events then
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Case I

If A & B are mutually exclusive.
~~P(A)~~ o P(A or B)

$$A \cap B = \phi$$

$$\Rightarrow P(A \cap B) = 0$$

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v) Generalization

If $A_1, A_2, \ldots A_n$ are mutually exclusive
$P(A_1 \text{ or } A_2 \text{ or } \ldots A_n) = P(A_1 \cup A_2 \cup A_2 \ldots \cup A_n)$
$= P(A_1) + P(A_2) + \ldots + P(A_n)$

vi) Product theorem of Probability

If A & B are independent events then
$P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B)$

4) Find the probability of getting
i) a number greater than 2
ii) odd number
iii) even number
when a die is thrown?

→

The set of all possible outcomes
$S = \{1, 2, 3, 4, 5, 6\}$
$n(S) = 6$

i) Let,
$E_1 \to$ event of getting number greater than 2

$E_1 = \{3, 4, 5, 6\}$
$n(E_1) = 4$

The probability of getting a number greater than 2.

$$p(E_1) = \frac{n(E)}{n(S)} = \frac{\cancel{4}^{2}}{\cancel{6}_{3}} = \frac{2}{3}$$

ii) let, $E_2 \to$ event of getting an odd number.

$E_2 = \{1, 3, 5\}$
$n(E_2) = 3$

$$\therefore p(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

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iii) Let E_3 be the event of getting an even number.

$$E_3 = \{2, 4, 6\}$$
$$n(E_3) = 3$$

$$\therefore P(E_3) = \frac{n(E_3)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(5) Find the probability of getting
i) a total more than 10
ii) — " — < 10
iii) — " — = 10
iv) getting same number in both
v) getting sum as > 7
vi) — " — > 12
when two dice thrown simoultaneosly?

→

The set of all possible outcomes when two dice are thrown together

$$S = \left\{ \begin{matrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{matrix} \right\}$$

$$n(S) = 36$$

i) Let,

$E_1 \rightarrow$ event of getting total more than 10

$\therefore E_1 = \{(5,6), (6,5), (6,6)\}$

$m = n(E_1) = 3$

~~P(E_1)~~

Probability of getting favourable event

$$p(E_1) = \frac{m}{n} = \frac{n(E_1)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

ii) Let,

$E_2 \rightarrow$ event of getting total less than 10

$$\therefore E_2 = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1) (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$$

$n(E_2) = 30$

Probability of getting favourable events

$$p(E_2) = \frac{m}{n} = \frac{n(E_2)}{n(S)} = \frac{30}{36} = \frac{5}{6}$$

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iii) let, $E_3 \rightarrow$ event of getting total equal to 10.

$$E_3 = \{(5,5), (6,4), (4,6)\}$$

$\therefore n(E_3) = 3$ $\therefore n(S) = 36$

The probability of getting favourable event

$$P(E_3) = \frac{m}{n} = \frac{\cancel{3}^{1}}{\cancel{36}_{12}} = \frac{1}{12}$$

iv) let, $E_4 \rightarrow$ event of getting the same number in both

$$E_4 = \{(1,1); (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

$$n(E_4) = 6$$

The probability of getting favourable events

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

v) let, $E_5 \rightarrow$ event of getting sum as >7

$E_5 = \{(2,6), (3,5), (3,6), (4,4), (4,5), (4,6), (5,3), (5,4), (5,5), (5,6), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$$n(E_5) = 15$$

The probability of getting favourable events

$$p(E_5) = \frac{n(E_5)}{n(S)} = \frac{15}{36}$$

vi) Let, E_6 be the event of getting sum as > 2

$$E_6 = \{\phi\}$$

$$n(E_6) = 0$$

7)366

52
7)366

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6) Find the probability of getting 53 Sundays in a leap year.

→

A leap year contains 366 days in which 52 weeks already contains 52
$\left(\frac{366}{7} = 52 \text{ weeks} + 2 \text{ days}\right)$ Sundays.

The remaining 2 days will be a combination of

i) Monday & Tuesday
ii) Tuesday & wednesday
iii) wednesday & Thursday
iv) Thursday & Friday
v) Friday & Saturday
vi) Saturday & Sunday
vii) Sunday & monday.

This 7 combinations make a sample space $S \Rightarrow n(S) = 7$

∴ ~~No. of favourable~~

Let,

$E \rightarrow$ event of getting a Sunday

$E = \{(\text{Saturday \& Sunday}), (\text{Sunday \& monday})\}$

$n(E) = 2$

∴ Probability of getting a favourable event

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{7}$$

(7) A ~~Bag~~ Box contains 3 white, 5 black & 6 red balls. what is the probability of getting a ball is either red or white?

→

Total No. of balls in a box. = 14 balls

$\therefore n(S) = 14$

Probability of getting a red balls = $\frac{6}{14} = \frac{3}{7}$

Probability of getting a white balls = $\frac{3}{14}$

$\therefore$ Since events are mutually exclusive. by addition theorem of probability their intersection is 0.

$\therefore$ Probability of getting a red (or) white ball is $= \frac{6}{14} + \frac{3}{14}$.

$\Rightarrow P(A \text{ or } B) = P(A \cup B) = \frac{6}{14} + \frac{3}{14}$

$= \frac{9}{14}$

(8) An urn contains 2 white & 2black balls. and a 2nd Urn contains 2white & 4 black balls. if one ball is drawn at random from each Urn, what is the probability that they are of the Same Colour.

→

	White	Black	total	Probability
1st urn	2	2	4	
2nd urn	2	4	6	

	P(white)	P(black
1st urn	2/4	2/4
2nd Urn	2/6	4/6

Probability of getting both the balls are white

$$\left(\frac{2}{4} \times \frac{2}{6}\right) + \left(\frac{2}{4} \times \frac{4}{6}\right)$$

gives independent events

P(w) in Urn I × P(w) in urn II

$$\frac{2}{4} \times \frac{2}{6} =$$

~~Probability of getting either both are black (or) both are white.~~

* Probability of getting both the ball are white $= \frac{2}{4} \cdot \frac{2}{6}$

$$P(A \text{ and } B) = P(A_1) \cdot P(B_1)$$
$$= \frac{2}{4} \cdot \frac{2}{6} = \left(\frac{2}{4}\right)\left(\frac{2}{6}\right)$$

* Probability of getting both the balls are black $= \frac{2}{6} \cdot \frac{4}{6}$

$$P(A \text{ and } B) = P(A_2) \cdot P(B_2)$$
$$= \left(\frac{2}{6}\right) \cdot \left(\frac{4}{6}\right)$$

Probability of getting either both are white (or) Black

$P(A_1 \text{ and } B_1)$ (or) $P(A_2 \text{ and } B_2)$

$$P(A \text{ or } B) = P(A_1 \text{ and } B_1) + P(A_2 \text{ and } B_2)$$

$$= \frac{2}{4} \times \frac{2}{6} + \frac{2}{4} \times \frac{4}{6}$$

$$= \frac{1}{6} + \frac{2}{6}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

(9) An urn1 contains 2 white & 4 black balls & a 2nd urn contains 5 white & 7 black balls. A ball is "transfered" from 1 to 2, then a ball is drawn from 2. Find the probability that it is white

→

	W·B	B·B	Total.
urn 1^{st}	2	4	6
urn 2^{nd}	5	7	12

Case I,

1 → urn II → Black

The transfered ball is black = 13

probability of transfering 1 black ball.

in Urn I = P(B) = $\frac{4}{6}$

P(B)

~~After~~ transfering 1 black ball

	W·B	B·B	total
urn II	5	7+1 = 8	12+1 = 13

∴ probability of getting a white ball after transferring = $\frac{5}{13}$ = P(W)

Since the transferring & drawing are independent events,

probability of transferring a black ball and getting a white ball =

$P(B \cap W) = P(B) \cdot P(W) = \frac{4}{6} \times \frac{5}{13}$ //

Case II

1 → Urn II → white ball.
The transfered ball is white
probability of transfering 1 white ball in Urn II $= \frac{2}{6} = p(w_1)$

After transfering one white ball.

Urn II	white	black	total
	$5+1=6$	7	$12+1=13$

Probability of getting white ball $p(w) = \frac{6}{13}$
Since tranfering & drawing are independent events.
Probability of transfering & getting a white ball is $p(w_1 \cap w) = p(w_1) \cdot p(w)$

$$= \frac{2}{6} \times \frac{6}{13}$$

∴ The probability of getting white ball $(w_1 \cap w)$ (or) $(B \cap w)$

$$P(A \cup B) = P(A) + P(B) = \frac{4}{6} \times \frac{5}{13} + \frac{2}{\cancel{6}} \times \frac{\cancel{6}}{13}$$

$$= \frac{2}{3} \times \frac{5}{13} + \frac{2}{13}$$

$$= \frac{10}{39} + \frac{2}{13}$$

$$= \frac{10+6}{39} = \frac{16}{39}$$

Since case I & Case II are ~~Mutul~~ Mutually exclusive.
Probability of getting white ball is $\frac{16}{39}$

(11) There are 10 students of which 3 are graduate. If a committee of 5 to be formed. what is the probability that.
i) only 2 graduates
ii) atleast 2 graduates

$\longrightarrow$

Total No. of Students = 10
No. of graduates = 3
No of Non-graduates = 7

Since a committee of 5 is to be formed, the no. ways of selecting 5 from 10 students $= {}^{10}C_5$

$\therefore$ Sample Space $= {}^{10}C_5 = n$

i) Bo only 2 graduates

To find the probability that only 2 graduates out of 3

The No. of ways of selecting 2 graduates out of 3 $= {}^{3}C_2$

To form a group / committee of 5 with 2 graduates we need 3 non-graduates.

^{3}C $\qquad$ $\cancel{\frac{4255}{210\times4}}$ $\cancel{840}$ $\qquad$ $^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$

Total no. of ways of selecting 3 Non-graduates out of 7 $= {}^{7}C_{3}$

Since these two are independent events the total no. of ways of selecting 2 graduates & 3 non-graduates

$$\frac{{}^{3}C_{2} \times {}^{7}C_{3}}{{}^{10}C_{5}} = m$$

∴ The probability that there are only 2 graduates out of 5

$$\frac{m}{n} = \frac{{}^{3}C_{2} \times {}^{7}C_{3}}{{}^{10}C_{5}}$$

$$= \cancel{\frac{3\times7\times6\times5\times4}{10\times9\times8\times7\times6}}$$

$$= \cancel{\frac{1}{12}}$$

${}^{3}C_{2} = \frac{3\times2\times1}{2\times1} = 3$

${}^{7}C_{3} = \frac{7\times6\times5\times\cancel{4\times3\times2\times1}}{4!\;3\times2\times1} = 35$

${}^{10}C_{5} = \frac{10\times9\times8\times7\times6}{1\times2\times3\times4\times5} = 252$

35×3

$\Rightarrow {}^{3}C_{2} = \frac{3\times2\times1}{2\times1} = 3$

$\Rightarrow {}^{7}C_{3} = \frac{7!}{(7-3)!\,3!} = \frac{n!}{(n-r)!\,r!}$

$= \frac{7!}{4!\,3!} = \frac{7\times6\times5\times4!}{4!\,3!} = \frac{210}{3\times2\times1} = \frac{210}{6} = 35$

$\Rightarrow {}^{10}C_{5} = \frac{10!}{5!\,5!} = \frac{10\times9\times8\times7\times6\times5!}{5!\,5!} = \frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1}$

$= 252$

$\therefore \frac{3\times35}{252} = \frac{105}{252} = \frac{5}{12}$ //

$\frac{15 \times 12}{30}$

$\frac{36 \times 7}{252}$

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5

G 2, NG 3 ; G 3, NG 2

${}^3C_2 \quad {}^7C_3 \quad {}^3C_3 \quad {}^7C_2$

ii) atleast 2 graduates

$p(\geq 2 \text{ graduates}) = P(2G \,\&\, 3\,NG) + P(3G \,\&\, 2NG)$

The probability of getting 2 graduates and 3 Non-graduates is $\frac{5}{12}$ (case I)

The total no. of getting 3 graduates out of $3 = {}^3C_3$

The total no. of getting 2 Non-graduates out of $7 = {}^7C_2$

The probability of getting 3 graduates and 2 Non-graduates.

$$= \frac{{}^3C_3 \times {}^7C_2}{{}^{10}C_5}$$

$${}^7C_2 = \frac{7 \times \overset{3}{\cancel{6}} \times \cancel{5 \times 4 \times 3 \times 2 \times 1}}{\cancel{5!}\,\underset{1}{\cancel{2!}}} = 21$$

$${}^{10}C_5 = \frac{10!}{5!\,5!} = 252$$

$$\therefore \frac{21}{252} = \frac{1}{12}$$

$$= \frac{5}{12} + \frac{1}{12} = \frac{6}{12}$$

(13) If two numbers are selected from the set of numbers {0, 1, 2, 3 - - - , 9} find the chance that their sum is equal to 10?

→

No. Sample Space $n(s) = {}^{10}C_2$

given,
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9} in this two numbers should be selected from 0 to 9

Total No. of ways of selecting two numbers is ${}^{10}C_2$

The different possibilities of getting sum of 2 numbers is 10 are
{(1, 9), (2, 8), (3, 7), (4, 6)}
$= 4 = m$

∴ The number of diff possibility = 4

The required probability of getting sum equal to $10 = \frac{m}{n} = \frac{4}{{}^{10}C_2}$

$$= \frac{4}{\frac{10!}{8!\,2!}} = \frac{4}{\frac{\cancel{10}^{5} \times 9 \times \cancel{8!}}{\cancel{8!}\,\cancel{2!}}} = \frac{4}{45}$$

(14) From 6 positive & 8 negative numbers, 4 numbers are choosen at random without replacement & multiplied. What is the probability that the product is positive?

$8+6=14$ $\quad 14\times$

$\Longrightarrow$

$$n(S) = {}^{14}C_4 = \frac{14\times13\times12\times11\times\cancel{10!}}{\cancel{10!}\,4!} = \frac{14\times13\times12\times11}{4\times3\times2\times1}$$
$$= \frac{14!}{10!\,4!} \qquad = 1001$$

$14\times13\times12$

P- +ve

N= -ve

$$P\left[(4P \text{ and } 0N) \text{ or } (0P \text{ and } 4N) \text{ or } (2P \text{ and } 2N)\right]$$

$$= \left(\frac{{}^6C_4 \times {}^8C_0}{{}^{14}C_4}\right) + \left(\frac{{}^6C_0 \times {}^8C_4}{{}^{14}C_4}\right) + \left(\frac{{}^6C_2 \times {}^8C_2}{{}^{14}C_4}\right)$$

i) $4P$ and $0N = {}^6C_4 \times {}^8C_0$
$$= \frac{6!}{2!\,4!} = \frac{\cancel{6}\times5\times\cancel{4!}}{\cancel{2!}\,\cancel{4!}} = 15$$

ii) $0P$ and $4N = {}^6C_0 \times {}^8C_4$
$$= \frac{8!}{4!\,4!} = \frac{8\times7\times6\times5\times\cancel{4!}}{4!\,\cancel{4!}} = \frac{\cancel{8}\times7\times\cancel{6}\times5}{\cancel{4}\times\cancel{3}\times\cancel{2}\times1}$$
$$= 7\times2\times5$$
$$= 70$$

iii) $2P$ and $2N = {}^6C_2 \times {}^8C_2$
$$= \frac{6!}{4!\,2!} \times \frac{8!}{6!\,2!} = \frac{\cancel{6}\times5\times\cancel{4!}}{\cancel{4!}\,\cancel{2!}} \times \frac{\cancel{8}\times7\times\cancel{6!}}{\cancel{6!}\,\cancel{2!}}$$
$$= 15 \times 28$$
$$= 420$$

$$\frac{m}{n} = \frac{15+70+420}{1001} = \frac{505}{1001}$$

15) A bag contains 10 white and 3 red balls while another bag contains 3 white & 5 red balls, two balls are drawn at random from 1^{st} bag and put it in a 2nd bag. Then a ball is drawn at random from 2nd bag. what is the probability ~~of getting~~ that it is white ball?

→

	White (w)	(R) Red.	Total
Bag 1^{st}	10	3	13
Bag 2^{nd}	3	5	8

Two balls are drawn at random from 1^{st} Bag (B_1) and put it in 2^{nd} Bag (B_2)

The No. of ~~possible~~ selecting 2 balls from 13 balls in $B_1 = {}^{13}C_2$ ways.

Bag 2

Case (i)	Case (ii)	case (iii)
W & W	R & R	W & R.
3+2 W, 5R = 5 w	3W, 5+2 = 7R	3+1, 5+1 = 4W = 6R
$P(w) = \frac{5}{10}$	$P(w) = \frac{3}{10}$	$P(w) = \frac{4}{10}$

$$\left(\frac{{}^{10}C_2}{{}^{13}C_2} \times \frac{5}{10}\right) + \left(\frac{{}^{3}C_2}{{}^{13}C_2} \times \frac{3}{10}\right) + \left(\frac{{}^{10}C_1 \times {}^{3}C_1}{{}^{13}C_2} \times \frac{4}{10}\right)$$

i) Probability of drawing 2 white ball from bag 1

$$= \frac{{}^{10}C_2 \times {}^{3}C_0}{{}^{13}C_2} \times \frac{5}{10}$$

$$= \frac{\frac{10!}{8!\,2!}}{\frac{13!}{11!\,2!}} \times \frac{5}{10}$$

$$= \frac{\frac{\cancel{10}^{5} \times 9 \times \cancel{8!}}{\cancel{8!} \times \cancel{2} \times 1}}{\frac{13 \times \cancel{12}^{6} \times \cancel{11!}}{\cancel{11!} \times \cancel{2} \times 1}} \times \frac{5}{\cancel{10}}$$

$$= \frac{5 \times 9}{13 \times 6} \times \frac{5}{10} = \frac{\cancel{45}^{9}}{78} \times \frac{5}{\cancel{10}_{2}}$$

$$= \frac{225}{780} = \frac{45}{156}$$

$$= \frac{\cancel{30}}{\cancel{156}}$$

ii) probability of drawing 2 balls (1 Red & 1 white)

$$= \frac{{}^{10}C_1 \; {}^{3}C_1}{{}^{13}C_2} \times \frac{4}{10}$$

$$= \frac{\cancel{30}^{3}}{78} \times \frac{4}{\cancel{10}_{1}}$$

$$= \frac{12}{78}$$

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iii) Probability of drawing 2 Red balls

$$= \frac{{}^{10}C_0 \; {}^{3}C_2}{{}^{13}C_2} \times \frac{3}{10}$$

$$= \frac{\frac{3!}{1!\,2!}}{78} \times \frac{3}{10}$$

$$= \frac{\frac{3 \times 2!}{2!}}{78} \times \frac{3}{10}$$

$$= \frac{3}{\cancel{78}_{26}} \times \frac{\cancel{3}^{1}}{10}$$

$$= \frac{3}{260}$$

Probability of event that white ball is drawn from 2^{nd} bag

$$= \frac{45}{156} + \frac{12}{78} + \frac{3}{260}$$

$$= \frac{45(5) + 12(10) + 3(3)}{780}$$

$$= \frac{225 + 120 + 9}{780}$$

$$= \frac{354}{780}$$

16) 3 groups of children contained respectively

i) 3 girls and 1 boy, ~~2 girls~~

ii) 2 girls and 2 boys

iii) 1 girl and 3 boys.

one child is selected at random in each group. find the probability of selecting ~~one~~ 1 girl and 2 boys.

$\longrightarrow$

	G_1	G_2	G_3	Independent events
Case i	G	B	B	$\frac{3}{4} \times \frac{\cancel{2}^{1}}{\cancel{4}_{2}} \times \frac{3}{4} = \frac{9}{32}$
Case ii	B	G	B	$\frac{1}{4} \times \frac{\cancel{2}^{1}}{\cancel{4}_{2}} \times \frac{3}{4} = \frac{3}{32}$
Case iii	B	B	G	$\frac{1}{4} \times \frac{\cancel{2}^{1}}{\cancel{4}_{2}} \times \frac{1}{4} = \frac{1}{32}$

$$= \frac{9}{32} + \frac{3}{32} + \frac{1}{32}$$

$$= \frac{9+3+1}{32}$$

$$= \frac{13}{32}$$

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17) 2 Dice are thrown find the probability of
i) getting an odd number on the 1 and a multiple of 3 on the other.
ii) one of the dice showed 3 and sum on the 2 dice is 9
iii) Sum on the 2 dice is 9.
iv) Sum should be 13

→

Sample space $n(S) = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$n(S) = 36$

i) The set of all possibilities of getting an odd number on 1 and a multiple of 3 on the other.

$n(E) = \{(1,3), (1,6), (3,3), (3,6), (5,3), (5,6)$
$(3,1), \cancel{(3,3)}, (3,5), (6,1), (6,3), (6,5)\}$

$n(E) = 11$

$\therefore P = \frac{n(E)}{n(S)} = \frac{11}{36}$

//

ii) The set of all posibilities of getting one of the dice showed 3 and sum on the 2 dice is 9.

$E_1 = \{(3,6), (6,3)\}$

$n(E_1) = 2$

$\therefore$ Probability of $P(E_1) = \frac{2}{36}$

iii) The set of all possibilities of getting sum on the 2 dice is 9

$E_2 = \{(3,6), (4,5), (5,4), (6,3)\}$

$n(E_2) = 4$

Probability $P(E_2) = \frac{4}{36} = \frac{1}{9}$

iv) The set of all possibilities of getting sum should be 13

$E_3 = \{\}$

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Conditional probability

Let, A & B → 2 events
probability of the happening of B when the event A has already happened is called the conditional probability.
Denoted by $P(B/A)$.

$P(B/A)$ = Probability of B given A

$$= \frac{\text{Probability of occurrence of both A \& B}}{\text{probability of occurrence of A}}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \quad \text{llly} \quad P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Rule

i) $$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(B/A) \cdot P(A)$$

llly

$$P(A \cap B) = P(A/B) \cdot P(B)$$

If A and B are independent events then,

$$\therefore P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap B) = P(A/B) \cdot P(B)$$

Baye's theorem on Conditional Probability

Let, $A_1, A_2, A_3 \ldots A_n \longrightarrow$ a set of exhaustive & mutually exclusive events of the sample space S with $p(A_i) \neq 0$ for each i.

If A is any other event associated with A_i, ~~$A \subset$~~

$$A \subset \bigcup A_i \quad , \quad P(A) \neq 0$$

$$P(A_i/A) = \frac{P(A_i)\, P(A/A_i)}{\sum_{i=1}^{n} P(A_i)\, P(A/A_i)}$$

Problems

① 3 machines A, B, C Produces respectively 60%, 30%, 10% of the total no. of items in a factory. The percentages of defective outputs of these machines are respectively are 2%, 3%, 4%. An item is selected at random and is formed to be defected. Find the probability that item is produced by C?

$$P(A) = \frac{60}{100} \quad , \quad P(B) = \frac{30}{100} \quad , \quad P(C) = \frac{10}{100}$$

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Company	A	B	C
Produces	60%	30%	10%
Defective item	2%	3%	4%

Probability of selecting an item

i) from machine A
$$p(A) = \frac{60}{100}$$

ii) from machine B = $P(B) = \frac{30}{100}$

iii) from machine C = $P(C) = \frac{10}{100}$

Probability of selecting an defective item

i) from machine A = $P(D/A) = \frac{2}{100}$

ii) —"— B = $P(D/B) = \frac{3}{100}$

iii) —"— C = $P(D/C) = \frac{4}{100}$

By Baye's theorem,

Probability of selecting defective item from factory C

$$P(C/D) = \frac{P(C) \cdot P(D/C)}{P(A)P(D/A) + P(B)P(D/B) + P(C)P(D/C)}$$

$$= \frac{\frac{10}{100} \times \frac{4}{100}}{\left(\frac{60}{100} \times \frac{2}{100}\right) + \left(\frac{30}{100} \times \frac{2}{100}\right) + \left(\frac{10}{100} \times \frac{4}{100}\right)}$$

$$= \frac{\frac{40}{10000}}{\frac{12+60+40}{10000}}$$

$$= \frac{40}{112}$$

(2) In a Bolt factory there are four machine A, B, C, D manufacturing respectively 20%, 15%, 25%, 40% of the total production out of these 5%, 4%, 3%, 2% are defective. If a Bolt drawn at random was found defective. What is the probability that it was manufactured by A (or) D?

→

Company	A	B	C	D
production, item	20%	15%	25%	40%
defective item	5%	4%	3%	2%

	A	B	C	D	
The probability of item	$2/100$	$15/100$	$25/100$	$40/100$	$\Rightarrow P(A)$
The probability of defective item	$\frac{5}{100}$	$\frac{4}{100}$	$\frac{3}{100}$	$\frac{2}{100}$	$\Rightarrow P(E/A)$

let,

E → the event of selecting a defective Bolt

Probability that it was manufactured by A(or) D

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$$P(A \cup D/E) = P(A \cup D) \cap P(E/A \cup D)$$

$$P(A \cup D/E) = P(A/E) + P(D/E) \quad \text{—— (1)}$$

$$P(A/E) = \frac{P(A)\,P(E/A)}{P(A)\cdot P(E/A) + P(B)\,P(E/B) + P(C)\cdot P(E/B) + P(D)\,P(E/D)}$$

$$= \frac{\frac{20}{100} \times \frac{5}{100}}{\frac{20}{100} \times \frac{5}{100} + \frac{15}{100} \times \frac{4}{100} + \frac{25}{100} \times \frac{3}{100} + \frac{40}{100} \times \frac{2}{100}}$$

$$= \frac{\frac{100}{10000}}{\frac{100+60+75+80}{10000}} = \frac{100}{315}$$

$$P(D/E) = \frac{P(D)\,P(E/D)}{P(A)\,P(E/A) + P(B)\,P(E/B) + P(C)\,P(E/C) + P(D)\,P(E/D)}$$

$$= \frac{\frac{40}{100} \times \frac{2}{100}}{\frac{100+60+75+80}{10000}}$$

$$= \frac{80}{315} \quad //$$

(1) $\Rightarrow$ $\therefore$ $P(A \cup B / E) = P(A/E) + P(B/E)$

$$= \frac{100}{315} + \frac{80}{315}$$

$$= \frac{180}{315} = 0.57$$

(3) The chance that a doctor will diagonise a disease correctly is 60% the chance that a patient will die after a correct diagonise is 40% and the chance by death by wrong diagonise is 70% if a patient die's: what is the chance that his disease was correctly diagonise?

$\longrightarrow$

Let,

A $\rightarrow$ the ~~same~~ event of correct diagonise is

$P(A) = 60\% = \frac{60}{100}$, $P(B) = 60 - 100 = 40$

B $\rightarrow$ the event of correct diagonise is

$P(E/A)\ 1 - \frac{60}{100} = \frac{100-60}{100} = \frac{40}{100}$

C $\rightarrow$ the event of death by wrong diagonise

$P(E/B) = \frac{70}{100}$

$\therefore$ The probability of the chance that his disease was correctly diagonised.

$$P\left(\frac{A}{E}\right) = \frac{P(A)\cdot P\left(\frac{E}{A}\right)}{P(A)\,P\left(\frac{E}{A}\right) + P(B)\,P\left(\frac{E}{B}\right)}$$

$$= \frac{\frac{6}{100} \times \frac{40}{100}}{\frac{60}{100} \times \frac{40}{100} + \frac{40}{100} \times \frac{70}{100}}$$

$$= \frac{\frac{240}{10{,}000}}{\frac{2400}{10{,}000} + \frac{2{,}800}{10{,}000}}$$

$$= \frac{\frac{240}{\cancel{10{,}000}}}{\frac{5200}{\cancel{10{,}000}}}$$

$$= \frac{240}{5200}$$

$$= \frac{24}{520}$$

$$= \frac{6}{130}$$

$$= \frac{3}{65}$$

(4) A bag contains three coins, one of which is two headed and the other two are normal and fair. A coin is chosen at random from the bag and tossed four times in succession. If ~~toward~~ headup each time, what is the probability that this is the two headed coin.

$\longrightarrow$

Let,

$c_1 \rightarrow$ two headed coin.

$c_2, c_3 \rightarrow$ nolmal coins.

$E \rightarrow$ the event of getting 4 heads in succession

$P(C_1/E) = ?$

$\therefore P(c_1) = P(C_2) = P(C_3) = 1/3$

$P(E/c_1) = 1$ since c_1 is a two headed coin.

$P(E/c_2) = P(E/c_3) = 1/2 \cdot 1/2 \cdot 1/2 \cdot 1/2$

$= 1/6$

$P(E) = P(C_1)\, P(E/c_1) + P(C_2)\, P(E/c_2) + P(C_3)\, P(E/c_3)$

$= \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot \frac{1}{16} + \frac{1}{3} \cdot \frac{1}{16}$

$= \frac{3}{8}$ //

$P(C_1/E) = \frac{P(C_1) \cdot P(E/c_1)}{P(E)} = \frac{1/3 \cdot 1}{3/8}$

$= \frac{8}{9}$ //

$\therefore$ The required probability is $\frac{8}{9}$ //

(v) There

(5) Three major points A, B, C are contending for power in the elections of a state and the chance of their winning the election is in the ratio 1:3:5. The parties A, B, C respectively have probabilities of banning the online lottery 2/3, 1/3, 3/5. What is the probability that there will be a ban on the online lottery in the state? What is the probability that the ban is from the party C?

→

	A	B	C	
banning online	2/3	1/3	3/5	

$P(A) = \frac{1}{9}$, $P(B) = \frac{3}{9} = \frac{1}{3}$, $P(C) = \frac{5}{9}$

Let,
E → an event of banning the online lottery

$\therefore P(E/A) = 2/3$, $P(E/B) = 1/3$, $P(E/C) = 3/5$

$P(E) = P(A)\,P(E/A) + P(B)\,P(E/B) + P(C)\,P(E/C)$

$= \frac{1}{9} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{1}{3} + \frac{5}{9} \cdot \frac{3}{5}$

$= \frac{14}{27}$

$\therefore$ The required probability is $\frac{14}{27}$

(6) An office has 4 secretaries handling respectively 20%, 60%, 15% and 5% of the files of all government reports. The probability that they misfile such reports are respectively 0.05, 0.1, 0.1 and 0.05. Find the probability that the misfiled report can be blamed on the first secretary.

→

Let,

A_1, A_2, A_3, A_4 → 4 secretaries of the office, respectively handaling 20%, 60%, 15%, 5% of the files.

Hence we have,

$$P(A_1) = \frac{20}{100} = 0.2$$

$$P(A_2) = \frac{60}{100} = 0.6$$

$$P(A_3) = \frac{15}{100} = 0.15$$

$$P(A_4) = \frac{5}{100} = 0.05$$

Let,

E → the event of misfiling a report by the secretaries.

$$\therefore P(E/A_1) = 0.05$$
$$P(E/A_2) = 0.1$$
$$P(E/A_3) = 0.1$$
$$P(E/A_4) = 0.05$$

we need to find $P(A_1/E)$ and we have, by Baye's theorem.

$$P(A_1/E) = \frac{P(A_1)\cdot P(E/A_1)}{P(A_1)\cdot P(E/A_1) + P(A_2)\cdot P(E/A_2) + P(A_3)\cdot P(E/A_3) + P(A_4)\, P(E/A_4)}$$

$$= \frac{\frac{20}{100}\times 0.05}{\frac{20}{100}\times 0.05 + \frac{60}{100}\times 0.1 + \frac{15}{100}\times 0.1 + \frac{5}{100}\times 0.05}$$

$$= \frac{\frac{1}{100}}{\frac{1}{100} + \frac{0.6}{100} + \frac{1.5}{100} + \frac{0.25}{100}}$$

$$= \frac{\frac{1}{100}}{\frac{3.35}{100}} = \frac{1}{3.35}$$

(7) Three machines A, B, C, Produces 50%, 30% and 20% of the items in a factory. The percentage of defective outputs are 3, 4, 5. If an item is selected at random. What is the probability that it is defective? What is the probability that it is from A?

→

let, A, B, C → the probability of items produced.

$P(A) = \frac{50}{100}$, $P(B) = \frac{30}{100}$, $P(C) = \frac{20}{100}$

let, E → be the event of getting defective output of A, B, C

$P(E/A) = \frac{3}{100}$

$P(E/B) = \frac{4}{100}$

$$P(E/C) = \frac{5}{100}$$

the probability of finding the item is defective and choosen from A is

$$P(A/D) = \frac{P(A)\,P(E/A)}{P(A)\,P(E/A) + P(B)\,P(E/B) + P(C)\,P(E/C)}$$

$$= \frac{\frac{50}{100} \times \frac{3}{100}}{\frac{50}{100} \times \frac{3}{100} + \frac{30}{100} \times \frac{4}{100} + \frac{20}{100} \times \frac{5}{100}}$$

$$= \frac{\frac{150}{100}}{\frac{150}{10000} + \frac{120}{10000} + \frac{100}{10000}}$$

$$= \frac{\frac{150}{\cancel{10000}}}{\frac{150 + 120 + 100}{\cancel{10000}}}$$

$$= \frac{150}{370}$$

(8) Random Variables

In a random experi If a real variable is associated with every outcome then it is called a random variable (or) a Strocastic variable.

A random variable is a function that assigns a real number to every sample point in the sample space of a random experiment.

Notation.

Random variables are denoted by X, Y, Z.

The set of all real numbers of a random variable X is called the range of X.

Example — 1

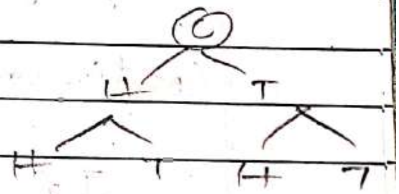
Let tossing a coin be a random experiment.

outcomes = {H, T}

Let, the real number 1 is associated to H & 0 for tail (T)

Let, X → random variable defined in such a way that X (H) = 1 & X (T) = 0

The range of X = {0, 1}



Example — 2

Tossing 2 coins be the random experiment.

⟹ sample space = {HH, HT, TH, TT}

Let, X → random variable which denotes the number of heads

outcomes	HH	HT	TH	TT
Random variables	2	1	1	0

Range of X (2, 1, 1, 0) = (2, 1, 0)

let, Y → no. of tail

outcomes	HH	HT	TH	TT
Random variables of 'X	0	1	1	2

Range of $Y = \{0, 1, 2\}$

Discreate Variables

If a random variable takes finite (or) countably infinite number of values then it is called Discreate Variable.

Countably ⇒ Sequence of real numbers

Range of Discreate variable :- finite.

Continuous random variable

If a random variable takes non-countable infinite number of values then it is called Continuous random variable (or) Non-Discreate Variable.

The Range of continuous random variable x is an interval of x

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Discrete random variable	Continuous random variable
· Tossing a coin · Throwing a die	· Weight of articles · height of pillers.

Discrete probability distribution

If for each value x_i of a discrete random variable X, be assigned a real number $p(x_i)$ such that,
i) $p(x_i) \geq 0$
ii) $\sum_i p(x_i) = 1$
Then the function $p(x)$ is called a probability function.
If the probability that X takes the value x_i is $p_i \Rightarrow P(X = x_i) = p(x_i)$
$= p_i$

The set of values $[x_i, p(x_i)]$ is called a discrete probability distribution of the discrete random variable X. The function $p(x)$ is called a probability density function or (or) mass function.
The Distribution function $f(x)$ defined by $p(X \leq x) = \sum_{i=1}^{x} p(x_i)$, x being an integer, is called the cumulative distribution function.

Problems.

(1) A coin is tossed twice a random variable x represent the number of heads find the discrete probability distribution for x, also find mean and variance.

$\longrightarrow$

let,

$x \rightarrow$ random variable which denote number of heads.

A coin tossed twice is random variable

Sample Space $= \{HH, HT, TH, TT\}$

$\therefore$ X takes the value

$X = \{\underset{x_1}{0}, \underset{x_2}{1}, \underset{x_3}{2}\}$

O/P	HH	HT	TH	TT
X	2	1	1	0

$p_i = P(X = x_i)$

$P(X=0) = P(X = \text{no. heads}) = \frac{1}{4}$

$P(X=1) = P(X = H\&T \ (\text{or}) \ X = T\&H)$

$= P(HT \cup TH)$

$= P(HT) + P(TH)$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$ //

$P(X=2) = P(X = HH \ (\text{or}) \ X = TT)$

$= P(HH) \cancel{+ P(TT)}$

$= \frac{1}{4}$

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The discrete probability distribution for X as follows.

$X = x_i = (0,1,2)$	$X = 0$	$X = 1$	$X = 2$
$P(X = x_i)$ $p(x_i) = p_i$	$P(X=0) = p(0) = p_0 = \frac{1}{4}$	$P(X=1) = p(1) = p_1 = \frac{1}{2}$	$P(X=2) = p(2) = p_2 = \frac{1}{4}$

Hence $p(x_i) > 0$

$$\sum p(x_i) = p_0 + p_1 + p_2$$
$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{4}$$
$$= 1$$

$$\boxed{\text{Mean} = \mu = \sum x_i \, p(x_i)}$$

$$= \left(0 \times \frac{1}{4}\right) + \left(1 \times \frac{1}{2}\right) + \left(2 \times \frac{1}{4}\right)$$
$$= \frac{1}{2} + \frac{1}{2}$$
$$= \frac{2}{2}$$
$$= 1$$

$$\boxed{\text{Variance} = V = \sigma^2 = \sum (x_i - \mu)^2 \, p(x_i)}$$

$$= \left((0-1)^2 \cdot \frac{1}{4}\right) + \left((1-1)^2 \cdot \frac{1}{2}\right) + \left((2-1)^2 \cdot \frac{1}{4}\right)$$

$$= \left(-1 \cdot \frac{1}{4}\right) + \left(0 \cdot \frac{1}{2}\right) + \left(1 \cdot \frac{1}{4}\right)$$

$$= -\frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2}$$

(2) Find the value of K such that the following distributions represents a finite probability distributions hence find its mean & S.D also find $p(x \leq 1)$; $p(x > 1)$; $p(-1 < x \leq 2)$

x_i	-3	-2	-1	0	1	2	3
$p(x_i)$	K	2K	3K	4K	3K	2K	K

$\longrightarrow$

To find K

Since the total probability = 1

i,e; $\Sigma p(x) = 1 \Rightarrow K + 2K + 3K + 4K + 3K + 2K + K = 1$

$\Rightarrow 16K = 1$

$$\boxed{K = \frac{1}{16}}$$

Hence, the discrete probability is

x_i	-3	-2	-1	0	1	2	3
$p(x_i)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

Mean = $\mu = \Sigma x\, p(x) = \left(-3 \times \frac{1}{16}\right) + \left(-2 \times \frac{2}{16}\right) + \left(-1 \times \frac{3}{16}\right) +$

$\left(0 \times \frac{4}{16}\right) + \left(1 \times \frac{3}{16}\right) + \left(2 \times \frac{2}{16}\right) + \left(3 \times \frac{1}{16}\right)$

$= \frac{1}{16}\left[-3 - 4 - 3 + 0 + 3 + 4 + 3\right]$

$= \frac{1}{16} \cdot 0$

$= 0$

Variance

$\sigma^2 = \Sigma (x_i - \mu)^2\, p(x_i)$

$= \left[(-3)^2 \times \frac{1}{16}\right] + \left[(-2)^2 \times \frac{2}{16}\right] + \left[(-1)^2 \times \frac{3}{16}\right] +$

$0 + \left[1^2 \times \frac{3}{16}\right] + \left[2^2 \times \frac{2}{16}\right] + \left[3^2 \times \frac{1}{16}\right]$

$= \left[9 \times \frac{1}{16}\right] + \left[4 \times \frac{2}{16}\right] + \left[1 \times \frac{3}{16}\right] + \left[1 \times \frac{3}{16}\right] +$

$\left[4 \times \frac{2}{16}\right] + \left[9 \times \frac{1}{16}\right]$

$$= \cancel{\frac{1}{16}\left[9+4+1+1+4+9\right]} = \frac{1}{16}\left[9+8+3+0+3+8+9\right]$$

$$= \cancel{\frac{1}{16} \cdot 28} = \frac{40}{16}$$

$$= \cancel{\frac{28}{16}} = \frac{5}{2}$$

$$p(x \le 1) = p(x=-3) + p(x=-2) + p(x=-1) + p(x=0) + p(x=1)$$

$$= \frac{1}{16} + \frac{2}{16} + \frac{3}{16} + \frac{4}{16} + \frac{3}{16}$$

$$= \frac{13}{16}$$

$$p(x>1) = p(x=2) + p(x=3)$$

$$= \frac{2}{16} + \frac{1}{16}$$

$$= \frac{3}{16}$$

$$p(-1 < x \le 2) = p(x=0) + p(x=1) + p(x=2)$$

$$= \frac{4}{16} + \frac{3}{16} + \frac{2}{16}$$

$$= \frac{9}{16}$$

③ Show that the following distribution represents a discrete probability distribution. find the mean and variance.

8+9]

x	10	20	30	40
$p(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

→ We observe that $p(x) > 0$ for all x
and $\sum p(x) = 1$

Mean $= \mu = \sum p(x) \cdot x = \frac{10 + 60 + 90 + 40}{8}$

$= \frac{200}{8}$

$= 25$

Variance

$V = \sum (x - \mu)^2 \cdot p(x)$

$= \left[(10-25)^2 \cdot \frac{1}{8}\right] + \left[(20-25)^2 \cdot \frac{3}{8}\right] + \left[(30-25)^2 \cdot \frac{3}{8}\right] + \left[(40-25)^2 \cdot \frac{1}{8}\right]$

$\cancel{= 25 \cdot \frac{1}{8} + 225 \cdot \frac{3}{8} +}$

$= \left[(15)^2 \cdot \frac{1}{8}\right] + \left((5)^2 \cdot \frac{3}{8}\right) + \left((5)^2 \cdot \frac{3}{8}\right) + \left((15)^2 \cdot \frac{1}{8}\right)$

$= \frac{225}{8} + \frac{75}{8} + \frac{75}{8} + \frac{225}{8}$

$= \frac{600}{8}$

$= 75$

4) The p.d.f of a variable X is given by the following table:

x	0	1	2	3	4	5	6
$P(x)$	K	3K	5K	7K	9K	11K	13K

for what value of K, this represents a valid probability distribution? Also find $P(x \geq 5)$ and $P(3 < x \leq 6)$

$\longrightarrow$

To find K,

Since the total probability is 1

i,e, $\sum P(x) = 1$

$\Rightarrow K + 3K + 5K + 7K + 9K + 11K + 13K = 1$

$$49K = 1$$

$$\boxed{K = \frac{1}{49}}$$

$$P(x \geq 5) = P(5) + P(6)$$
$$= 11K + 13K$$
$$= 24K$$
$$= 24\left(\frac{1}{49}\right)$$
$$= \frac{24}{49}$$

$$P(3 < x \leq 6) = P(4) + P(5) + P(6)$$
$$= 9K + 11K + 13K$$
$$= 33K$$
$$= 33\left(\frac{1}{49}\right)$$
$$= \frac{33}{49}$$

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11
13

5) The probability distribution of finite random variable X is given by the following table

x_i	−2	−1	0	1	2	3
$P(x_i)$	0.1	K	0.2	2K	0.3	K

find value of K, mean and variance.

→ We have, $P(x_i) \geq 0$ and $\sum P(x_i) = 1$ for a probability distribution.

$\sum P(x_i) = 1$ requires $4K + 0.6 = 1$
$\Rightarrow K = 0.1$

Mean

$$\mu = \sum x_i P(x_i) = -0.2 - 0.1 + 0.2 + 0.6 + 0.3$$
$$= 0.8$$

Variance

$$\sigma^2 = \sum x_i^2 \cdot P(x_i) - \mu^2$$
$$= (0.4 + 0.1 + 0.2 + 1.2 + 0.9) - (0.8)^2$$
$$= 2.16$$

6) A random variable X has the following probability function for various values of x.

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

i) Find k ii) evaluate $P(x<6)$, $P(x \geq 6)$ and $P(3<x\leq 6)$

Also find the probability distribution & the distribution function of X.

→

$$P(x) \geq 0 \text{ \& } \Sigma P(x) = 1$$

The first condition is satisfied for $k \geq 0$ and we have to find k such that $\Sigma P(x) = 1$

i.e; $0+k+2k+2k+3k+k^2+2k^2+(7k^2+k)=1$

$10k^2+9k-1=0$

(or) $(10k-1)(k+1)=0$

(or) $k=\frac{1}{10}$ & $k=-1$

If $k=-1$ the first condition fails & hence $k \neq -1$

$\therefore k=\frac{1}{10}$

Hence,

x	0	1	2	3	4	5	6	7
$P(x)$	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{1}{50}$	$\frac{17}{100}$

Now,

$P(x<6) = P(0)+P(1)+P(2)+P(3)+P(4)+P(5)$

$= 0+\frac{1}{10}+\frac{1}{5}+\frac{1}{5}+\frac{3}{10}+\frac{1}{100}$

$= \frac{81}{100} = 0.81$

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$P(x \geq 6) = P(6) + P(7)$

$= \frac{1}{50} + \frac{17}{100}$

$= \frac{19}{100}$

$= 0.19$

$P(3 < x \leq 6) = P(4) + P(5) + P(6)$

$= \frac{3}{10} + \frac{1}{100} + \frac{1}{50}$

$= \frac{33}{100}$

$= 0.33$

The probability distribution is.

x	0	1	2	3	4	5	6	7
$P(x)$	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

The distribution function of X is $f(x) = P(X \leq x) = \sum_{i=1}^{x} P(x_i)$ is called commulative distribution of the same as follows.

x	0	1	2	3	4	5
$f(x)$	0	0+0.1 = 0.1	0.1+0.2 = 0.3	0.3+0.2 = 0.5	0.5+0.3 = 0.8	0.8+0.01 = 0.81

6	7
0.81+0.02 = 0.83	0.83+0.17 = 1

7) A random variable X takes the values $-3, -2, -1, 0, 1, 2, 3$ such that $P(X=0) = P(X<0)$ & $P(X=-3) = P(X=-2) = P(X=-1) = P(X=1) = P(X=2) = P(X=3)$.

Find the probability distribution.

$\longrightarrow$

Let, the probability distribution be, $[x_i, p(x_i)]$ (or) $[X, P(X)]$

$X = x$	-3	-2	-1	0	1	2	3
$P(x) = p(x)$	$p_1 = p(x=-3) = p(-3)$	$p_2 = p(-2)$ p_1	$p_3 = p(-1)$ p_1	$p_4 = p(0)$ $3p_1$	$p_5 = p(1)$ p_1	$p_6 = p(2)$ p_1	$p_7 = p(3)$ p_1

~~$P(X \geq 0) = 4$~~

So that the total probability $= 1$

i.e., $p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 = 1$ — (*)

$P(X=0) = P(X<0)$

$= P(X=-3) + P(X=-2) + P(X=-1)$

$\boxed{p_4 = p_1 + p_2 + p_3}$ — (1)

Also given,

$P(X=-3) = P(X=-2) = P(X=-1) = P(X=\phi) = P(X=2) = P(X=3)$

$\Rightarrow p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7$ — (2)

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(2) in (1) $\Rightarrow p_4 = p_1 + p_2 + p_1 = 3p_1$

$$\Rightarrow \boxed{p_4 = 3p_1} \quad \text{—— (3)}$$

(4) $\Rightarrow$ (2) & (3) in (*)

$$p_1 + p_1 + p_1 + 3p_1 + p_1 + p_1 + p_1 = 1$$

$$\Rightarrow 9p_1 = 1$$

$$\boxed{p_1 = \frac{1}{9}}$$

$X = x$	-3	-2	-1	0	1	2	3
$P(x)$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{3}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
Cummulative Probability $F(x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	$\frac{9}{9} = 1$

8) From a sealed box containing a dozen apple, it was found that 3 apples are perished. obtaine the probability distribution of the perished apple when 'x' apples are drawn at random. also find mean and variance of the distribution. (12)

$\rightarrow$

Let $X \rightarrow$ random variable which denotes no. of perished apples

2 apples are drawn at random.

$\therefore X = 0, 1, 2$

Sample Space = S = ~~$^{12}C_2$~~ drawing 2 apples out of 12 ~~(ants)~~

$n(S) = {}^{12}C_2$

$$P(X=0) = \frac{^{9}C_2}{^{12}C_2}$$

$$= \frac{\cancel{9}^{3} \times \cancel{8}^{2}}{\cancel{12}_{\cancel{4}_1} \times 11} = \frac{6}{11}$$

$$P(X=1) = \frac{^{3}C_1 \times {}^{9}C_1}{^{12}C_2} = \frac{\cancel{3}^{1} \times 9}{\cancel{12}_{4} \times 11} = \frac{9}{22}$$

$$P(\cancel{X=2}) = \frac{^{3}C_2}{^{12}C_2} = \frac{^{3}C_2}{^{12}C_2} = \frac{\frac{3!}{2!}}{66} = \frac{\frac{3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}}}{66} = \frac{3}{66} = \frac{1}{22}$$

The probability distribution is given by

X	0	1	2
P(X)	$\frac{6}{11}$	$\frac{9}{22}$	$\frac{1}{22}$
Cumulative Probability	$\frac{6}{11}$	$\frac{21}{22}$	$\frac{22}{22} = 1$

$\therefore \frac{6}{11} + \frac{9}{22} = \frac{22}{22} = 1$

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9) If the random variable x takes the value 1, 2, 3, 4, $2P(x=1) = 3P(x=2) = P(x=3) = 5P(x=4)$. Find probability distribution?

→

let the probability distribution be $[x, P(x)]$

x	1	2	3	4
$p(x_i)$	p_1	p_2	p_3	p_4

So that

$$p_1 + p_2 + p_3 + p_4 = 1$$

Also given,

$$2p_1 = 3p_2 = p_3 = 5p_4$$

$$2p_1 = 3p_2 \Rightarrow \boxed{p_2 = \frac{2}{3}p_1}$$

lly $\boxed{2p_1 = p_3}$

$$2p_1 = 5p_4 \Rightarrow \boxed{p_4 = \frac{2}{5}p_1}$$

$$2p_1 + \frac{2}{3}p_1 + 2p_1 + \frac{2}{5}p_1 = 1$$

$$\frac{15p_1 + 10p_1 + 30p_1 + 6p_1}{15} = 1$$

$$\frac{61p_1}{15} = 1$$

$$61p_1 = 15 \quad \boxed{p_1 = \frac{15}{61}}$$

$$p_2 = \frac{2}{3} p_1 = \frac{2}{3} \times \frac{15}{61} = \frac{10}{61}$$

$$p_3 = 2 p_1 = 2 \times \frac{15}{61} = \frac{30}{61}$$

$$p_4 = \frac{2}{5} p_1 = \frac{2}{5} \times \frac{15}{61} = \frac{6}{61}$$

X	1	2	3	4
$P(X)$	$15/61$	$10/61$	$30/61$	$6/61$
commulative probability $F(x)$	$15/61$	$25/61$	$55/61$	$61/61 = 1$

Repeated Trails

A random expirement only 2 outcomes categorized as success & failure, is called a Bernoulli trail.

Probability of success = p
Probability of failure = q

Bernouills theorem

The probability of x success in n trail = $^nC_x \cdot p^x \cdot q^{n-x}$

Binomial distribution

If p probability of success & q is the probability of failure then, the probability of X success out of n trails is given by $P(x) = P(X = x) = p(x) = {}^nC_x\, p^x q^{n-x}$.

This distribution is called binomial distribution (or) bernouills distribution.

Mean of the binomial distribution.

$\mu = np$

Variance $= V = \sigma^2 = npq$

Standard diviation $[S.D] = \sqrt{V} = \sqrt{\sigma^2} = \sigma = \sqrt{npq}$

Problems

① Find the binomial probability distribution which has mean 2 and variance $\frac{4}{3}$.

$\Longrightarrow$

given,

$\text{mean} = \mu = 2$

$\text{Variance} = \frac{4}{3}$

$np = 2 \quad \text{—} ①$

$npq = \frac{4}{3} \quad \text{—} ②$

$\frac{②}{①} \Rightarrow \frac{\cancel{np}q}{\cancel{np}} = \frac{4/3}{2}$

$q = \frac{4}{3 \times 2}$

$= \frac{\cancel{4}^{2}}{\cancel{6}_{3}}$

$\boxed{q = \frac{2}{3}}$

W.K.that $p+q=1$

$\Rightarrow p + \frac{2}{3} = 1 \Rightarrow 1 - \frac{2}{3} = \frac{3-2}{3}$

$\Rightarrow \boxed{p = \frac{1}{3}}$

$np = 2$

$① \Rightarrow n \cdot \frac{1}{3} = 2 \quad = \frac{n}{3} = 2$

$\Rightarrow \boxed{n = 6}$

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let, $x \rightarrow$ no. of success.

The binomial distribution for x success is given by $p(x)$

$$p(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

$$= {}^{6}C_{x}\left(\frac{1}{3}\right)^{x}\left(\frac{2}{3}\right)^{6-x}$$

The possible values for success.
$x = 0, 1, 2, 3, 4, 5, 6$.

x	0	1	2	3	4	5	6
$p(x)$	$64/729$	$192/729$	$240/729$	$160/729$	$60/729$	$12/729$	$1/729$
$F(x)$	$64/729$	$256/729$	$496/729$	$656/729$	$716/729$	$728/729$	$729/729 = 1$

$p(x=0) = {}^{6}C_{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{6-0} = {}^{6}C_{0}($

$p(x=1) = {}^{6}C_{1}\left(\frac{1}{3}\right)^{1}\left(\frac{2}{3}\right)^{6-1} = {}^{6}C_{1}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{5}$

$p(x=2) = {}^{6}C_{2}\left(\frac{1}{3}\right)^{2}\left(\frac{2}{3}\right)^{6-2} = {}^{6}C_{2}\left(\frac{1}{9}\right)\left(\frac{2}{3}\right)^{4}$

$p(x=3) = {}^{6}C_{3}\left(\frac{1}{3}\right)^{3}\left(\frac{2}{3}\right)^{6-3} = {}^{6}C_{3}\left(\frac{1}{27}\right)$

$p(x=4) = {}^{6}C_{4}\left(\frac{1}{3}\right)^{4}\left(\frac{2}{3}\right)^{6-4}$

$p(x=5) = {}^{6}C_{5}\left(\frac{1}{3}\right)^{5}\left(\frac{2}{3}\right)^{6-5}$

$p(x=6) = {}^{6}C_{6}\left(\frac{1}{3}\right)^{6}\left(\frac{2}{3}\right)^{6-6}$

H - head

(2) When a coin is tossed 4 times find the probability of getting
i) exactly one head
ii) atmost 3 heads
iii) atleast 2 heads

$\longrightarrow$

Let,

x denote no. of success, getting a H

Probability of getting a head $= p = \frac{1}{2}$

probability of getting a tail $\Rightarrow$
probability of failure $= q = \frac{1}{2}$

The possible values of x are $0, 1, 2, 3, 4$

The binomial distribution is given by

$$p(x) = {}^{n}C_x \, p^x \, q^{n-x}$$

$$= {}^{4}C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$$

i) Probability of getting exactly one head.

$$P(X=1) = P(1) = {}^{4}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{4-1}$$

$$= {}^{4}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3$$

$$= 4 \cdot \left(\frac{1}{2}\right)^4$$

$$= \frac{4}{16}$$

$$= \frac{1}{4}$$

atmost - less than or equal

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$2 \times 2 \times 2 \times 2$

ii) Probability of getting atmost 3 heads

$$P(x \leq 3) = P(0) + P(1) + P(2) + P(3)$$

$$= \left[{}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4-0}\right] + \left[{}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{4-1}\right] +$$

$$\left[{}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2}\right] + \left[{}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{4-3}\right]$$

$$= \left[{}^4C_0 \left(\frac{1}{2}\right)^4\right] + \left[{}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3\right] + \left[{}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2\right]$$

$$+ \left[{}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1\right]$$

$$= 1\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^4 + \frac{4 \times 3}{1 \times 2}\left(\frac{1}{2}\right)^4 + \frac{\cancel{4 \times 3 \times 2}}{\cancel{1 \times 2 \times 3}}\left(\frac{1}{2}\right)^4$$

$$= \left(\frac{1}{2}\right)^4 \left[1 + 4 + 6 + 4\right]$$

$$= \left(\frac{1}{2}\right)^4 [15]$$

$= \frac{1}{16}$

$P(2) + P(3) + P(4)$

$$\frac{6}{16} + \frac{4}{16} \quad \frac{1}{16} = \frac{11}{16}$$

(3) The probability that a pen manufactured by a factory is defective is 1/10 if 12 such pens are manufactured what is the probability that

i) exactly 2 are defective.
ii) atleast 2 are defective
iii) None of them are defective.

$\longrightarrow$

given,

$n = 12$ pens

probability of a defective ~~item~~ $= p = \frac{1}{10}$

$= 0.1$

probability of Non-defective pens $= q = 1 - p$

$q = 1 - \frac{1}{10} = \frac{9}{10}$ (or) $= 1 - 0.1 = 0.9$

The binomial probability distribution

$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$; $x = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$

$= {}^{12}C_{x}\left(\frac{1}{10}\right)^{x}\left(\frac{9}{10}\right)^{12-x}$

i) probability of getting exactly 2 as defective. $= P(X = 2)$

~~$P(x)$~~ $= {}^{12}C_{2}\left(\frac{1}{10}\right)^{2}\left(\frac{9}{10}\right)^{12-2}$

$\Rightarrow {}^{12}C_{2} = \frac{\overset{6}{\cancel{12}} \times 11}{1 \times \cancel{2}}\left(\frac{1}{10^{2}}\right)\left(\frac{9^{10}}{10^{10}}\right)$

$= 66\left(\frac{9^{10}}{10^{12}}\right)$

$= 66 \times 9^{10} \times \frac{1}{10^{12}}$ //

$\therefore P(X=0)+P(X=1)+P(X=2) \dots + P(X=12) = 1$

$\therefore P(X=2) + \dots P(X=12) = 1 - [P(X=0) - P(X=1)]$

ii) Probability of getting atleast 2 as defective. $= P(x \geq 2)$

$\cancel{= 1 - [P(X=0) + P(X=1)]}$

$\cancel{= 1 - [P(X=0) - P(X=1)]}$

$\cancel{= 1 - [{}^{12}C_0 (0.1)^0 (0.9)^{12} + {}^{12}C_1 (0.1)^1 (0.9)^{11}]}$

$\cancel{[0.282429536448] + 12(0.1)}$

0.3138105960

$= {}^{12}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{12-0} + {}^{12}C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{12-1}$

$= 1(1) \frac{9^{12}}{10^{12}} + \frac{12}{10^1} \frac{(9)^{11}}{10^{11}}$

$= \frac{9^{11} \, 9^1}{10^{12}} + \frac{12 \cdot 9^{11}}{10^{12}}$

$= \frac{9^{11}}{10^{12}} (9 + 12)$

$= \frac{21 \times 9^{11}}{10^{12}}$

$P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8) + P(9) + P(10) + P(11) + P(12) = 1 - [P(0) + P(1)]$

$= 1 - \frac{21 \times 9^{11}}{10^{12}}$

$= \frac{10^{12} - 21 \times 9^{11}}{10^{12}}$

$= 0.786$

(4) In a consinment of electric lamp 5% are defective. if a random sample of 8 lamps are inspected. What is the probability that one (or) more lamps are defective.

$\longrightarrow$

let, $X \rightarrow$ random variable which denotes defective lamp.

$n = 8,$

$\therefore$ the values of $x = 0, 1, 2, 3, 4, 5, 6, 7, 8$

probability of defective lamps $= p = 5\%$

$= \frac{5}{100}$

$= 0.05$

probability of non-defective lamp $= q = 1 - 0.05$

$= 0.95$

$P(x) = P(X = x) = {}^{n}c_{x} \cdot p^{x} \, q^{n-x}$

$= {}^{8}c_{x} \, (0.05)^{x} \, (0.95)^{8-x}$

$P(x \geq 1) = 1 - P(x = 0)$

$= 1 - {}^{8}c_{0} \, (0.05)^{0} \, (0.95)^{8-0}$

$= 1 - 1 \cdot 1 \cdot (0.95)^{8}$

$= 1 - 0.6634$

$= 0.3365$

(5) The probability that a person aged 60 years will live upto 70 is 0.65. What is the probability that out of 10 persons aged 60 atleast 7 of them will live upto 70.

$\longrightarrow$

let $x \rightarrow$ the no. of persons aged 60 years living upto 70 years.
for this variate we have by data
$p = 0.65$ hence $q = 0.35$

Consider $p(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$. Hence, $n=10$

We have to find $P(x \geq 7)$. That is given by
$= P(7) + P(8) + P(9) + P(10)$ $\quad x = (7, 8, 9, 10)$

$= {}^{10}C_{7}(0.65)^{7}(0.35)^{3} + {}^{10}C_{8}(0.65)^{8}(0.35)^{2} +$

${}^{10}C_{9}(0.65)^{9}(0.35) + (0.65)^{10}$

But, ${}^{10}C_{7} = {}^{10}C_{3} = \dfrac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} = 120$

${}^{10}C_{8} = {}^{10}C_{2} = \dfrac{10 \cdot 9}{1 \cdot 2} = 45$

${}^{10}C_{9} = {}^{10}C_{1} = 10$

Hence, $P(x \geq 7) = 0.5138$

6) The number of telephone lines busy at an instant of time is a binomial variate with probability 0.1 that a line is busy. If 10 lines are chosen at random, what is the probability that

i) no lines is busy
ii) all lines are busy
iii) atleast one line is busy
iv) at most 2 lines are busy.

$\longrightarrow$

Let,
x denote the number of telephone lines busy. For this variate we have by data,
$p = 0.1$, $q = 1 - p = 0.9$, $n = 10$

We have, $P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$$= {}^{10}C_{x} (0.1)^{x} (0.9)^{10-x}$$

i) Probability that no lines is busy =
$P(0) = (0.9)^{10} = 0.3487$

ii) Probability that all lines are busy =
$P(10) = (0.1)^{10} = 0.0000000001$

iii) Probability that atleast one line is busy
$= 1 -$ Probability of no line is busy
$= 1 - P(0)$
$= 1 - 0.3487$
$= 0.6513$

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iv) Probability that atmost 2 lines are busy

$= P(0) + P(1) + P(2)$

$= (0.9)^{10} + {}^{10}C_1 (0.1)^1 (0.9)^9 + {}^{10}C_2 (0.1)^2 (0.9)^8$

$= 0.9298$

(7) In a quiz contest of answering 'Yes' or 'No' what is the probability of guessing atleast 6 answers correctly out of 10 questions asked? Also find the probability of the same if there are 4 options for a correct answer.

→ Let, x denote the correct answer & we have in the first case,

$p = 1/2$ & $q = 1/2$

$P(x) = {}^nC_x \, p^x q^{n-x}$

$= {}^{10}C_x (1/2)^x (1/2)^{10-x}$

$= {}^{10}C_x \cdot {1/2}^{10}$

We have to find $P(x \geq 6)$

$x = 6, 7, 8, 9, 10$

$P(x \geq 6) = \frac{1}{2^{10}} \left[{}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} \right]$

$= \frac{1}{2^{10}} (210 + 120 + 45 + 10 + 1)$

$= \frac{386}{1024} = 0.377$ $\therefore P(x \geq 6) = 0.377$

In the Second case when there are 4 options.

$$p = \frac{1}{4}, \quad q = \frac{3}{4} \quad ; \quad n = 10$$

$$P(x) = {}^{10}C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{10-x}$$

$$= \frac{1}{4^{10}} \left[3^{10-x} \cdot {}^{10}C_x \right]$$

Hence, $P(x \geq 6) = P(6) + P(7) + P(8) + P(9) + P(10)$

$$= \frac{1}{4^{10}} \left[3^4 \cdot {}^{10}C_6 + 3^3 \cdot {}^{10}C_7 + 3^2 \cdot {}^{10}C_8 + 3 \cdot {}^{10}C_9 + {}^{10}C_{10} \right]$$

$$= \frac{1}{4^{10}} \left[81 \times 210 + 27 \times 120 + 9 \times 45 + 3 \times 10 + 1 \right]$$

$$= 0.019$$

Thus $P(x \geq 6) = 0.019$

(8) In sampling a large number of parts manufactured by a company, the mean number of defectives in samples of 20 is 2. Out of 1000 such samples how many would be expected to contain atleast 3 defective parts.

$\longrightarrow$

Mean $(\mu) = np = 2$, by data,
where $n = 20$

i.e; $20p = 2 \quad \therefore p = \frac{1}{10} = 0.1$

Hence $q = 1 - p = 0.9$

Let, x denote the defective part

$$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$= {}^{20}C_{x}\, (0.1)^{x} (0.9)^{20-x}$$

Probability of atleast 3 defective parts

$$= P(3) + P(4) + \cdots + P(20)$$

$$= 1 - [P(0) + P(1) + P(2)]$$

$$= 1 - \left[(0.9)^{20} + {}^{20}C_{1}(0.1)(0.9)^{19} + {}^{20}C_{2}(0.1)^{2}(0.9)^{18} \right]$$

$$= 0.323$$

Thus the number of defectives in 1000 samples is $1000 \times 0.323 = 323$

9) If the mean and standard deviation of the number of correctly answered questions in a test give to 4096 students are 2.5 and $\sqrt{1.875}$. Find an estimate of the number of candidates answering correctly

i) 8 (or) more questions

ii) 2 (or) less

iii) 5 questions.

→ We have mean $(\mu) = np$ and $S.D(\sigma) = \sqrt{npq}$ for a binomial distribution.

By data $np = 2.5$ and $\sqrt{npq} = \sqrt{1.875}$

(or)

$npq = 1.875$

Hence we have $2.5q = 1.875$

$\therefore q = 0.75$; $p = 1-q = 0.25$

Since $np = 2.5$ we have $(0.25)n = 2.5$

$n = 10$

Let, x denote the number of correctly answered questions.

$$P(x) = {}^{n}C_x p^x q^{n-x}$$

$$= {}^{10}C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{10-x}.$$

i,e; $P(x) = \frac{1}{4^{10}}\left[{}^{10}C_x (3)^{10-x}\right]$

Since, the estimate is needed for 4096 students we have,

$$4096\ P(x) = \frac{4096}{4^{10}}\left[{}^{10}C_x (3)^{10-x}\right]$$

$$= \frac{2^{12}}{2^{20}}\left[{}^{10}C_x (3)^{10-x}\right]$$

i,e; $4096\ P(x) = \frac{1}{256}\left[{}^{10}C_x (3)^{10-x}\right]$

$= f(x)$ say

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i) We have to find

$f(8)+f(9)+f(10)$

$= \frac{1}{256}\left[{}^{10}C_8 \cdot 3^2 + {}^{10}C_9 \cdot 3 + 1\right]$

$= \frac{1}{256}\left[9 \cdot {}^{10}C_2 + 3 \cdot {}^{10}C_1 + 1\right]$

$= \frac{1}{256}(436)$

$= 1.703 \approx 2$

No. of students correctly answering 8 (or) more questions is 2.

ii) We have to find $f(2)+f(1)+f(0)$

$= \frac{1}{256}\left[{}^{10}C_2 \cdot 3^8 + {}^{10}C_1 \cdot 3^9 + 3^{10}\right]$

$= \frac{3^8}{256}(45+30+9)$

$= \frac{3^8}{256}(84)$

$= 2152.8 \approx 2153$

No. of students correctly answering 2 (or) less than 2 questions is 2153.

iii) We have to find $f(5)$

$= \frac{1}{256}\left[{}^{10}C_5 \cdot 3^5\right]$

$= 239.2$

$= 239$

No. of students correctly answering 5 questions is 239.

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10) An air line knows that 5% of the people making reservations on a certain flight will not turn up. Consequently their policy is to sell 52 tickets for a flight that can only hold 50 people. What is the probability that there will be a seat for every passenger who turns up?

→

The probability (p) that a passenger will not turn up is

$p = 0.05 \quad \therefore \; q = 0.95$

Let, x denote the number of passengers who will not turn up.

$P(x) = {}^{n}c_{x}\, p^{x}\, q^{n-x} \qquad$ where $n = 52$

$\therefore \quad P(x) = {}^{52}c_{x}\,(0.05)^{x}\,(0.95)^{52-x}$

A seat is assured for every passenger who turns up if the number of passengers who fail to turn up is more than (or) equal to 2.

Hence we have to find $P(x \geq 2)$

$$P(x \geq 2) = 1 - [P(x < 2)]$$
$$= 1 - [P(x=0) + P(x=1)]$$
$$= 1 - [(0.95)^{52} + 52(0.05)(0.95)^{51}]$$
$$= 1 - 0.2595$$
$$= 0.7405$$

Probability that a seat is available for every passenger is 0.7405.

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11) Five dice were thrown 96 times and the number of times an odd number actually turned out in the experiment is given. Fit a binomial distribution to this data and calculate the expected frequencies.

No. of dice showing 1 (or) 3 (or) 5	0	1	2	3	4	5
observed frequency	1	10	24	35	18	8

→

$p =$ probability of getting 1 (or) 3 (or) 5 $= 3/6$
$= 1/2 \quad \therefore \quad q = \frac{1}{2}$

Hence,
let x denote the number of times an odd number turning out.

$P(x) = {}^{n}c_x \, p^x \cdot q^{n-x}$ where $n = 5$

hence, $P(x) = {}^{5}c_x \, (1/2)^x \, (1/2)^{5-x}$

$= \frac{1}{2^5} \, {}^{5}c_x$

This is the binomial probability distribution function.

Since 5 dice were thrown 96 times, expected frequencies are obtained from

$f(x) = 96 \cdot P(x)$ where $(x = 0,1,2,3,4,5)$

Consider, $f(x) = 96 \cdot \frac{1}{2^5} \, {}^5C_x = 3 \cdot {}^5C_x$

Hence,

$f(0) = 3 \cdot {}^5C_0 = 3$; $f(1) = 3 \cdot {}^5C_1 = 3 \times 5 = 15$

$f(2) = 3 \cdot {}^5C_2 = 3 \times 10 = 30$; $f(3) = 3 \cdot {}^5C_3 = 3 \times 10 = 30$

$f(4) = 3 \cdot {}^5C_4 = 3 \times 5 = 15$; $f(5) = 3 \cdot {}^5C_5 = 3 \times 1 = 3$

$\therefore$ The expected (theoretical) frequencies are
3, 15, 30, 30, 15, 3

12) 4 coins are tossed 100 times and the following results we obtained. Fit a binomial distribution for the data and calculate the theoretical frequencies.

No. of heads	0	1	2	3	4
frequency	5	29	36	25	5

$\rightarrow$

Let x denote the number of heads & f the corresponding frequency.
Since the data is in the form of a frequency distribution we shall first calculate mean.

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$$\text{Mean}(\mu) = \frac{\sum fx}{\sum f} = \frac{0+29+72+75+20}{100}$$

$$= \frac{196}{100}$$

$$= 1.96$$

But $\mu = np$ for the binomial distribution

Here $n = 4$

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(13) # Poisson distribution.

Poisson distribution is the limiting form of the binomial distribution when n is very large. (i,e; $n \to \infty$) and the probability of success is very small i,e; $P \to 0$ so that $np \to$ a fixed ~~fixed~~ finite constant denoted by m
i,e; $np = m$

The probability function or poisson distribution function is given by.

$$P(x) = \frac{m^x e^{-m}}{x!}$$

$$= \frac{e^{-m} m^x}{x!}, \text{ where } x = \text{poisson variant}$$

i) with $P(x \geq 0)$
ii) $\sum_{x=0}^{\infty} P(x) = 1$

Mean of the Poisson distribution

Let,
$\mu \to$ mean of the Poisson distribution.

$$\text{Mean} = \mu = E(x) = \sum_{x=0}^{\infty} x \cdot P(x)$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-m} \cdot m^x}{x!}$$

$x! = x(x-1)!$

$$= e^{-m} \cdot m \sum_{x=1}^{\infty} \frac{m^{x-1}}{(x-1)!}$$

$$= e^{-m} \cdot m \left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \cdots \right]$$

$$= e^{-m} \cdot m e^{m}$$

$$= m$$

4M

To find the S.D of the Poison distribution

let $\sigma \rightarrow$ S.D of the Poison distribution

$\longrightarrow$

let, $V(x)$ (or) $Var(x)$

where, x = Poisson Variate given by

$$\sigma^2 = V(x) = E(x^2) - [E(x)]^2$$

$$= E(x^2) - \mu^2$$

$$\boxed{\sigma^2 = E(x^2) - m^2} \text{ --- } (*)$$

To find $E(x^2)$ $\quad E(x) = \sum_{x=0}^{\infty} x \cdot P(x)$

$$E(x^2) = \sum_{x=0}^{\infty} x^2 \cdot P(x)$$

$$= \sum_{x=0}^{\infty} x^2 \cdot \frac{e^{-m} \cdot m^x}{x!}$$

$$= \sum_{x=0}^{\infty}\left([x^2-x]+x\right)\frac{e^{-m}\cdot m^x}{x!}$$

$$= \sum_{x=0}^{\infty} x(x-1)\cdot\frac{e^{-m}\cdot m^{x}}{x!} + \sum_{x=0}^{\infty} x\cdot\frac{e^{-m}\cdot m^x}{x!}$$

$$= e^{-m}\cdot m^2 \sum_{x=2}^{\infty}\frac{m^{x-2}}{(x-2)!} + m$$

$$\Rightarrow \sigma^2 = m^2+m-m^2$$

$$= m$$

$$\therefore S.D = \sqrt{\sigma^2} = \sigma = \sqrt{m}$$

$$\therefore E(X^2) = m^2+m$$

NOTE: mean and Variance are equal for Poisson Distribution.

Problems

(1) fit a poison distribution for the following data & calculate the theoretical frequency.

x	0	1	2	3	4	
frequency	122	60	15	2	1	N = 200

$\longrightarrow$

In order the construct Poisson distribution we need m.

W.K.T m is the mean of the Poisson distribution

So for the given distribution.

$$\text{Mean} = \mu = \frac{\sum f\cdot x}{\sum f}$$

$$= \frac{0 \cdot 122 + 1 \cdot 60 + 2 \cdot 15 + 3 \cdot 2 + 4 \cdot 1}{122 + 60 + 15 + 2 + 1}$$

$$= \frac{0 + 60 + 30 + 6 + 4}{200}$$

$$= \frac{100}{200}$$

$$m = \frac{1}{2} = 0.5$$

x	0	1	2	3	4
frequency $P(x)$	$\frac{e^{-0.5} \cdot (0.5)^0}{0!}$	$\frac{e^{-0.5} \cdot (0.5)^1}{1!}$	$\frac{e^{-0.5} \cdot (0.5)^2}{2!}$	$\frac{e^{-0.5} \cdot (0.5)^3}{3!}$	$\frac{e^{-0.5} \cdot (0.5)^4}{4!}$

The theolatical frequency is given by

$f(x) = 200 \cdot P(x)$ $\quad$ $f(x) = N \cdot P(x)$

$f(0) = 200 \cdot P(0)$

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(2) The probabilities of a Poisson variable taking the values 3 & 4 are equal. Calculate the probabilities of

→

Let x denotes Poisson variable and the poison probability function is given by

$$P(x) = \frac{e^{-m} m^x}{x!}$$

Also given that, $P(X=3) = P(X=4)$

$$\Rightarrow P(3) = P(4)$$

To find, $P(0)$ & $P(1)$

$x=0$ & $x=1$

To find m :- $P(3) = P(4)$

$$\Rightarrow \frac{\cancel{e^{-m}} \cdot \cancel{m^3}}{3!} = \frac{\cancel{e^{-m}} \cdot m^{\cancel{4}}}{4!}$$

$$\Rightarrow \frac{1}{3!} = \frac{m}{4!}$$

$$\Rightarrow m = \frac{4!}{3!} = \frac{4 \times \cancel{3!}}{\cancel{3!}}$$

$$\Rightarrow \boxed{m = 4}$$

Hence, $P(x) = \frac{e^{-4} \cdot 4^x}{x!}$

$\therefore P(0) = \frac{e^{-4} \cdot 4^0}{0!}$

$= \frac{e^{-4} \cdot 4^0}{0!} = e^{-4} \cdot 1$

$\therefore P(1) = \frac{e^{-4} \cdot 4^1}{1!} = 4e^{-4}$

(3) If x follows Poison law such that $P(X=2) = (2/3)\, P(X=1)$, find $P(X=0)$ and $P(X=3)$

$\rightarrow$

Let, x denote Poisson law and the Poison probability is given by

$$P(x) = \frac{e^{-m}\, m^x}{x!}$$

Also given that,

$$P(X=2) = (2/3)\, P(X=1)$$

To find, $P(0)$ & $P(3)$

$x=0$ & $x=3$

To find m:- $P(2) = P(1) = \frac{2}{3}$

$$\frac{e^{-m} \cdot m^2}{2!} = \frac{2}{3} \; \frac{e^{-m}\, m^1}{1!} \qquad \boxed{m = \frac{4}{3}}$$

$m = \frac{2 \times 2!}{3}$ $P(x=0) = \frac{e^{-m} m^x}{x!}$

$m = \frac{2 \times 2 \times 1}{3}$ $= \frac{e^{-4/3} \, 4/3^0}{0!}$

$= e^{-4/3}$

$p(x=1) = \frac{e^{-m} m^x}{x!}$ $P(x=3) = \frac{e^{-m} m^x}{x!} = \frac{e^{-4/3} (4/3)^3}{3!}$

$= \frac{e^{-4/3} (4/3)^1}{1!}$ $= \frac{e^{4/3} \frac{64}{27}}{3 \times 2 \times 1} = \frac{e^{-4/3} \, 64}{162}$

$= 4/3 \, e^{-4/3}$

(4) If x is a Poisson Variate such that ~~p(x) = 2 = 9 times that of~~ $P(x=2) = 9 \cdot P(x=4) + 90 P(x=6)$ compute the mean and Variance.

$\rightarrow$

Since x denote Poison variate then Poisson probability function is given by $p(x) = \frac{e^{-m} \cdot m^x}{x!}$

Also given that,

$P(x=2) = 9 P(x=4) + 90 P(x=6)$

$p(2) = 9 p(4) + 90 p(6)$

$\frac{e^{-m} \cdot m^2}{2!} = 9 \frac{e^{-m} \cdot m^4}{4!} + 90 \frac{e^{-m} \cdot m^6}{6!}$

$\frac{1}{2} = \frac{9 \cdot m^2}{4!} + 90 \frac{m^4}{6!}$

$\frac{1}{2} = \frac{9 m^2}{1 \times 2 \times 3 \times 4} + \frac{90 m^4}{1 \times 2 \times 3 \times 4 \times 5 \times 6}$

$$\frac{1}{2} = \frac{3m^2}{8} + \frac{m^4}{8}$$

$$\Rightarrow m^4 + 3m^2 = \frac{\cancel{8}^{4}}{\cancel{2}}$$

$$m^4 + 3m^2 = 4$$

let $m^2 = x$

$$\Rightarrow m^4 + 3m^2 - 4 = 0$$

$$\Rightarrow (x+4)(x-1) = 0$$

-4

$4 \quad -1$

$$\Rightarrow (m^2+4)(m^2-1) = 0$$

$$m^2 = -4 \quad \& \quad m^2 = 1$$

$$m = \pm 2i \quad \& \quad m = \pm 1$$

∴ m is equal to 1 because ~~as~~ m never be imaginary & negative.

∴ $\boxed{m = 1}$

Hence, Mean of the Poison distribution

Mean $= M = 1$

~~Vai~~ S.D $= \sigma = \sqrt{m} = 1$

Variance $= 1$

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(5) Prove the following recurrence relation for the Poisson distribution.

$$\cancel{P(x+1)} = \frac{m^x \cdot e^{-m}}{x!} \quad P(x+1) = \frac{m}{x+1} P(x)$$

→ $x \to$ Poisson variant.

Poisson distribution is given by $P(x) = \frac{e^{-m} m^x}{x!}$

$$\therefore P(x+1) = \frac{e^{-m} \cdot m^{x+1}}{(x+1)!}$$

$$= \frac{e^{-m} \cdot m^x \cdot m}{(x+1)(x!)}$$

$$= \frac{m}{x+1} \left(\frac{e^{-m} \cdot m^x}{x!} \right)$$

$$= \frac{m}{x+1} P(x)$$

(6) The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3 out of 1000 taxi drivers, find approximately the no. of drivers:

i) No accidents in a year.

ii) more than 3 accidents in a year.

→ let,

x denotes no. of accidents in a year

Since x is poisson variate :

Poisson probability function is given by $P(x) \frac{e^{-m} m^{x}}{x!}$

Given that mean of poisson distribution

$\mu = m = 3$

Out of 1000 we are going to find the corresponding probabilities of x. Poisson Variate

i,e', $f(x) = 1000\ P(x)$

i) Probability of no accidents in a year.

$x=0 \Rightarrow f(0) = 1000\ P(0)$ $\qquad m=3$

$= 1000 \times \dfrac{e^{-3} \cdot 3^0}{0!}$

$= 1000 \times e^{-3} \cdot 1$

$= 1000\ e^{-3}$

$= e^{-3}\ 1000$

$= 0.0497 \times 1000$

$= 49.7$

ii) Probability of more than 3 accidents in a year. i,e', $[P(X \geq 3)]$

$= 1 - P(X \leq 3)$

$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$

$= 1 - \left[\dfrac{e^{-3} \cdot 3^0}{0!} + \dfrac{e^{-3} \cdot 3^1}{1!} + \dfrac{e^{-3} \cdot 3^2}{2!} + \dfrac{e^{-3} \cdot 3^3}{3!}\right]$

$= 1 - \left[e^{-3} \cdot 1 + e^{-3} \cdot 3 + e^{-3} \cdot \dfrac{9}{2} + e^{-3} \cdot \dfrac{9}{2}\right]$

$= 1 - e^{-3}\left[1 + 3 + \dfrac{9}{2} + \dfrac{9}{2}\right]$

$= 1 - e^{-3}\left[\dfrac{2+6+9+9}{2}\right]$ — By LCM

$= 1 - e^{-3}\left[\dfrac{26}{2}\right]$

$= 1-e^{-3}\, 13$

$= 1-0.0497(13)$

$= 0.3539$

numbers of drivers with more than 3 accident in a year

$f(x>3) = 1000 \times P(x>3)$

$= 1000 \times 0.3539$

$= 353.9$

~~The~~

(7) If the probability of a bad reaction from a certain injection is 0.001. Determine the chance that out of 2000 individuals more than 2 will get a bad reaction.

→

Since probability is very small and n ~~it~~ is large ($n = 2000$), therefore we can accomodate Poisson distribution here,

given,

The probability of bad reaction $= p = 0.001$

$n = 2000$

let,

x denote the number of bad reactions

since,

$m = np = 2000 \times 0.001$

$= 2$

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$$\therefore P(x) = \frac{e^{-m} \cdot m^{x}}{x!} = \frac{e^{-2} \cdot 2^{x}}{x!}$$

The probability that the chance of more than 2 get bad reaction.

$$P(x > 2)$$

$$= 1 - P(x \leq 2)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[\frac{e^{-2} 2^{0}}{0!} + \frac{e^{-2} 2^{1}}{1!} + \frac{e^{-2} 2^{2}}{2!}\right]$$

$$= 1 - e^{-2} [1 + 2 + 4]$$

$$= 1 - 0.135(7).$$