DAYANANDA SAGAR UNIVERSITY

USN No:

I Semester B.C.A. Examinations – December 2016

Course Title: Mathematics – I | **Course Code:** 16CA101

Duration: 03 Hours | **Date:** 20-12-2016

Time: 10:00 AM to 01:00 PM | **Max Marks:** 60

Note:
1. **Answer 5 full questions choosing one from each section**
2. **Draw neat sketches wherever necessary**
3. **Missing Data may be suitably assumed**

SECTION – 1

1.a. By reducing to the echelon form, find the rank of the matrix (04 Marks)

$$A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

1.b. Find the Eigen values of the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ (04 Marks)

1.c. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ and find its inverse. (04 Marks)

OR

2.a. Using matrix inversion method, solve the system of equations: (05 Marks)
$3x + y - 2z = 3$,
$2x - 3y - z = -3$,
$x + 2y + z = 4$

2.b. Reduce to the normal form and hence find the rank of the matrix (07 Marks)

$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

SECTION – 2

3.a. Prove the De Morgan's laws (03 Marks)

(i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

(ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

3.b. In a group of 50 students, 15 students study Mathematics, 8 students study Physics, 6 study Chemistry and 3 study all these three subjects. Prove that 27 or more students study none of the subjects. (05 Marks)

3.c. For non-empty sets *A, B and C* prove that: (04 Marks)

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

OR

4.a. Define the following: (03 Marks)

(i) Relation on a set A

(ii) Equivalence relation on a set *A*

4.b. Let $A = \{1,2,3,4,5,6,7,8,9,10,11,12\}$. On this set, define the relation R by $(x, y) \in R$ *if and only if* $x - y$ *is a multiple of* 5. Is R an equivalence relation? (05 Marks)

4.c. Let a function $f : R \to R$ be defined by $f(x) = \begin{cases} 3x - 5 & for\ x > 0 \\ -3x + 1 & for\ x \le 0 \end{cases}$.

Determine $f(0), f(-1), f^{-1}(-1), f^{-1}(\{-5,5\})$. (04 Marks)

SECTION – 3

5.a. State the Leibnitz's rule for the n^th^ derivative of a product of two functions. Using the same, find the n^th^ derivative of $y = x^2 \log 4x$. (03 Marks)

5.b. If $z = x^2 \tan^{-1}(\frac{y}{x}) - y^2 \tan^{-1}(\frac{x}{y})$, show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$ (05 Marks)

5.c. State Euler's theorem for a homogeneous function of two variables *x* and *y*. Using the same prove that prove that if (04 Marks)

$$u = \tan^{-1}(\frac{x^3 + y^3}{x - y})\ then\ xu_x + yu_y = \sin 2u.$$

OR

6.a. If $x = a\cos^3 t,\ y = a\sin^3 t$ find the value of $\frac{dy}{dx}$. (03 Marks)

6.b. Examine for maxima and minima the function $x^5 - 5x^4 + 5x^3 - 1$. (05 Marks)

6.c. Expand $\log(1 + \cos x)$ up to the term containing x^4 using Maclaurin's series. (04 Marks)

SECTION – 4

7.a. Solve $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$. (02 Marks)

7.b. Solve $(y^2 + 2xy)dx + (2x^2 + 3xy)dy = 0$ (05 Marks)

7.c. Solve $\frac{dy}{dx} + xy = xy^3$ (05 Marks)

OR

8.a. Solve $\{y(1+\frac{1}{x}) + \cos y\}dx + \{x + \log x - x\sin y\}dy = 0$ (04 Marks)

8.b. Solve $(1+y^2)dx = (\tan^{-1} y - x)dy$ (05 Marks)

8.c. Solve $\frac{dy}{dx} = (x+y)^2$ (03 Marks)

SECTION – 5

9.a. For any elements a, b in a group G, prove that: (03 Marks)

(i) $(a^{-1})^{-1} = a$ (ii) $(ab)^{-1} = b^{-1}a^{-1}$

9.b. Let G be the set of real numbers not equal to -1 and * be defined by a*b=a+b+ab. Prove that $(G, *)$ is an abelian group. (04 Marks)

9.c. Define subgroup of a group G. Prove that the intersection of two subgroups of a group is a subgroup of the group. (05 Marks)

OR

10.a. Let R be a commutative ring with unity. Prove that R is an integral domain if and only if for all $a, b, c \in R$ *where* $a \neq 0$, $ab = ac \Rightarrow b = c$. (04 Marks)

10.b. Define the following: (03 Marks)

(i) A ring R (ii) A commutative ring R

10.c. Let H be a sub group and let K be a normal sub group of a group G. Prove that, HK is a sub group of G, where $HK = \{hk/h \in H, k \in K\}$. (05 Marks)

DAYANANDA SAGAR UNIVERSITY

USN No: ENG18CA0009

I Semester B.C.A. Examinations – December 2018 / January 2019

Course Title: Mathematics – I
Duration: 03 Hours
Time: 10:00 AM to 01:00 PM

Course Code: 16CA101
Date: 24-12-2018
Max Marks: 60

Note:
1. **Answer 5 full questions choosing one from each Section**
2. **Each Section carries 12 Marks**
3. **Draw neat sketches wherever necessary**
4. **Missing Data may be suitably assumed**

SECTION – 1

1.a. If $\begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 2 & x-3 \\ y-4 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 1 \\ 2 & 3 \end{bmatrix}$ then find x and y. (03 Marks)

1.b. Compute A^{-1} for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. (04 Marks)

1.c. Check the consistency of the given solution and hence find the solution $3x + y - 2z = 3$, $2x - 3y - z = -3$ and $x + 2y + z = 4$. (05 Marks)

OR

2.a. Find the eigen value and eigen vector of $\begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$. (04 Marks)

2.b. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$. (04 Marks)

2.c. Prove that "If any two rows or any two columns of a determinant are identical then the value of the determinant is zero". (04 Marks)

SECTION – 2

3.a. If $f(x) = x + 5$ and $g(x) = -3$, find the $f(g(0))$, $f(f(-5))$, $g(g(2))$. (04 Marks)

3.b. Let A = {1,2,3,4}; R = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4)} be a relation on A. Verify that R is an equivalence relation. (04 Marks)

3.c. Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} Verify that
$A \times (B \cap C) = (A \times B) \cap (A \times C)$ (04 Marks)

OR

4.a. A relation R on a collection of set of integers defined by $R = \{(x, y) : x - y \text{ is a multiple of } 3\}$. Show that R is an equivalence relation on Z. (04 Marks)

4.b. If $f(x) = \begin{cases} x^2 + 2 & when \quad x > 1 \\ 2x + 1 & when \quad x = 1 \\ 3 & when \quad x < 1 \end{cases}$ find whether f is continuous at $x = 1$. (04 Marks)

4.c. Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$. (04 Marks)

SECTION – 3

5.a. Find the nth derivative of $\log(ax + b)$. (06 Marks)

5.b. If $y = e^{m\cos^{-1}x}$ prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + m^2)y_n = 0$. (06 Marks)

OR

6.a. Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u\log u$ where, $\log u = \frac{x^3 + y^3}{3x + 4y}$. (03 Marks)

6.b. Find the derivative of $y = \frac{\cos x}{1 - \sin x}$. (03 Marks)

6.c. Expand $f(x, y) = \operatorname{Sin}x \operatorname{Sin}y$ using Taylor series at (0, 0). (06 Marks)

SECTION – 4

7.a. Solve the equation $x\frac{dy}{dx} = x^2 + 3y; x > 0$. (06 Marks)

7.b. Find the solution of the equation: $\frac{dy}{dx} + 2y\tan x = \sin x$, where $y = 0, x = \pi/3$. (06 Marks)

OR

8.a. Find the solution of the equation: $(x^2 - yx^2)\frac{dy}{dx} + (y^2 + xy^2) = 0$. (06 Marks)

8.b. Find the integrating factor of the differential equation: $(1 - y^2)\frac{dx}{dy} + yx = ay$. (04 Marks)

8.c. Find the order and degree of the equation: $\frac{d^3y}{dx^3} + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0$. (02 Marks)

SECTION – 5

9.a. Write out the multiplication table for Z_9^*. (04 Marks)

9.b. Let G_1 and G_2 be groups, and let G be the direct product $G_1 \times G_2$. Let $H = \{(x_1, x_2) \text{ in } G_1 \times G_2 | x_2 = e\}$ and $K = \{(x_1, x_2) \text{ in } G_1 \times G_2 | x_1 = e\}$. Prove that H and K are normal subgroups of G. (04 Marks)

9.c. Let H be a sub group and let k be a normal sub group of a group G. Prove that HK is a sub group of G. (04 Marks)
$HK = \{hk/h \in H, k \in K\}$.

OR

10.a. On the set G = Q of nonzero rational numbers, define a new multiplication by $a * b = ab/2$, for all a, b in G. Show that G is a group under the multiplication. (06 Marks)

10.b. Let G be a group, and suppose that a and b are any elements of G. Show that if $(ab)^2 = a^2 b^2$, then $ba = ab$. (06 Marks)

$\frac{1 (ax+b) \cdot a}{ax+b}$

$\frac{1}{x}/7 \quad \log = \frac{1}{x}$

$-(ax+b)^{-2} a \cdot a$

$y_2 =$

closed
associative
Inverse