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Midterm Exam 2

1) f(x) is one to one AND onto

Proof:

One to one: $g(x_1) = g(x_2)$

$$\frac{5x_1 - 4}{3x_1 + 7} = \frac{5x_2 - 4}{3x_2 + 7}$$

$$(5x_1 - 4)(3x_2 + 7) = (5x_2 - 4)(3x_1 + 7)$$

$$15x_1x_2 - 12x_2 + 35x_1 - 28 = 15x_1x_2 - 12x_1 + 35x_2 - 28$$

$$\Rightarrow 35x_1 - 12x_2 = 35x_2 - 12x_1$$

$$\Rightarrow 47x_1 = 47x_2$$

$$\Rightarrow x_1 = x_2 \quad \checkmark$$

onto: $m \in R$

$$m = \frac{5x-4}{3x+7} \Rightarrow m(3x+7) = 5x-4$$

$$\Rightarrow 3xm + 7m = 5x - 4 \Rightarrow 5x - 3xm = 7m + 4$$

$$\Rightarrow x(5-3m) = 7m+4$$

$$\Rightarrow x = \frac{7m+4}{5-3m} \Rightarrow \boxed{\frac{7x+4}{5-3m}}$$

$$f^{-1}(x) = \frac{7x+4}{5-3m}$$

f(x) is one to one $\checkmark$
f(x) is onto $\checkmark$

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2) 365 days = 1 year
365 different birthdays

$$\left\lceil \frac{365 \cdot 2 + 1}{365} \right\rceil = \left\lceil \frac{731}{365} \right\rceil = \lceil 2.0027397 \rceil = 3$$

We need 731 people in a group to have atleast 3 people with the same birthday

3)
$$f(x) = \frac{8x^6 - 13x^3(\log x)^2 + 6x^4}{(x^2+2)(3x-1)} \Rightarrow \text{Let } \lim_{x\to\infty} \frac{f(x)}{x^n} = z$$

z is some number

$$\Rightarrow \lim_{x\to\infty} \frac{8x^6 - 13x^3(\log x)^2 + 6x^4}{x^n(x^2+2)(3x-1)} = \lim_{x\to\infty} \left|\frac{f(x)}{x^n}\right|$$

$$\Rightarrow \lim_{x\to\infty} \frac{8x^6 - 13x^3(\log x)^2 + 6x^4}{x^n(3x^3 - x^2 + 6x - 2)} = \lim_{x\to\infty} \left|\frac{f(x)}{x^n}\right|$$

$$\Rightarrow \lim_{x\to\infty} \frac{8x^6 - 13x^3(\log x)^2 + 6x^4}{3x^{n+3} - x^{n+2} + 6x^{n+1} - 2x^n} = \lim_{x\to\infty} \left|\frac{f(x)}{x^n}\right|$$

$$\Rightarrow \lim_{x\to\infty} \frac{x^6}{x^{n+3}} \left\{ \frac{8 - 13\frac{(\log x)^2}{x^3} + \frac{6}{x^2}}{3 - \frac{1}{x} + \frac{6}{x^2} - \frac{2}{x^3}} \right\} = \lim_{x\to\infty} \left|\frac{f(x)}{x^n}\right|$$

If we plug in... $x \to \infty$

$$\frac{6}{x^2} = 0, \quad -\frac{1}{x} = 0, \quad \frac{6}{x^2} = 0, \quad -\frac{2}{x^3} = 0$$

so we use L'Hopitals Rule

$$\lim_{x\to\infty} \frac{(\log x)^2}{x^3} = \lim_{x\to\infty} \frac{2\log x}{3x^3} = \lim_{x\to\infty} \frac{\frac{2}{x}}{9x^2} = 0$$

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3) continued...

$$\left\{ \frac{8 - 13\frac{(\log x)^2}{x^3} + \frac{6}{x^2}}{3 - \frac{1}{x} + \frac{6}{x^2} - \frac{2}{x^3}} \right\} \Rightarrow \frac{8}{3}$$

so... $\lim_{x\to\infty} \left| \frac{f(x)}{x^n} \right| = 2$

$$\Rightarrow \lim_{x\to\infty} \frac{x^6}{x^{n+3}} \left\{ \frac{8 - 13\frac{(\log x)^2}{x^3} + \frac{6}{x^2}}{3 - \frac{1}{x} + \frac{6}{x^2} - \frac{2}{x^3}} \right\} = 2$$

$$= \frac{8}{3} = 2$$

therefore $\lim_{x\to\infty} \frac{x^6}{x^{n+3}}$ is finite when $n+3 \geq 6$.

$n \geq 3$ then $\lim_{x\to\infty} \left| \frac{f(x)}{x^n} \right| = 2$

$\forall n \geq 3$:

$O(x^n)$ estimate order of $f(x)$

the smallest integer n for a $O(x^n)$ is $3 = n$.

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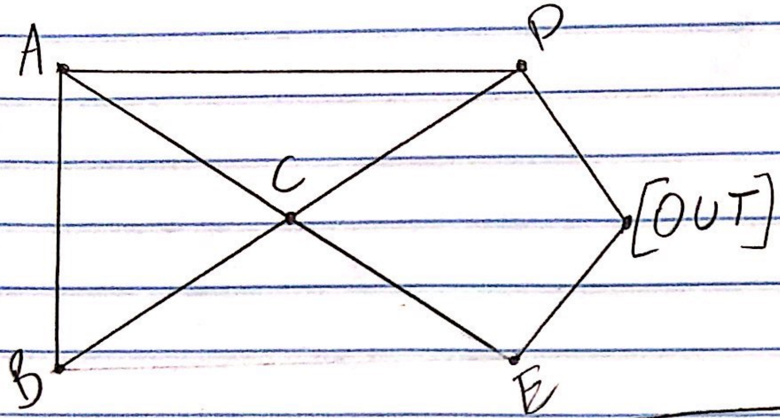
4)



Degrees

A = 4, B = 3, C = 4, D = 3, E = 2

2 odd Degrees

Euler Path : DACA BA[OUT] DCEB

There are 2 odd degrees. Therefore NO Euler circuit.

5) Dijkstra's Algorith:

Path = AECJ for shortest path (33)

Set = EDBCFGJ

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6) $f:(0,1) \to (-15,-8)$

$f(x) = 7x - 15$

f is one to one and onto:

cardinality of $(-15,-8)$ is the same as $(0,1)$

if $(0,1)$ is not countable then proven.

Let $g:(0,1) \to \mathbb{R}$

$g(x) = \cot\left(\frac{\pi x}{2}\right) \Rightarrow g$ is one to one and onto

g is bijective and $\mathbb{R}$ is not countable

$\Rightarrow (0,1)$ is a set that is not countable

hence $(-15,-8) \subset \mathbb{R}$ is not countable.

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