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Final Exam CSC230

1) (10) For all sets A, B, C show that:

$$(B - C) - (A - C) = (B - A) - C$$

Let $x \in (B - C) - (A - C)$

$\Rightarrow x \in (B - C)$ & $x \notin (A - C)$

$\Rightarrow x \in B$ & $x \in C$ & $x \in C$ or $x \in A$

$\Rightarrow x \in B$ & $x \notin A$ & $x \in C$ or $x \notin C$

$\Rightarrow x \in (B - A)$ & $x \notin C$

$\Rightarrow x \in (B - A) - C$

$$(B - C) - (A - C) \subseteq (B - A) - C$$

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2) (12)

$f: R - \{-\frac{5}{2}\} \to R - \{\frac{1}{2}\}, \quad f(x) = \frac{x+4}{2x+5};$

$g: R - \{-\frac{5}{2}\} \to R - \{\frac{3}{4}\}, \quad g(x) = \frac{2-3x}{4x+10};$

$h: R - \{-\frac{5}{2}\} \to R - \{\frac{5}{4}\}, \quad h(x) = f(x) - g(x)$

Determine if $h(x)$ is one to one and onto. If it is, find the inverse of $h(x)$.

$$h(x) = f(x) - g(x) = \frac{5x+6}{4x+10}$$

one to one: $\frac{5x_1+6}{4x_1+10} = \frac{5x_2+6}{4x_2+10}$

$$\cancel{20x_1x_2} + 24x_2 + 50x_1 + \cancel{60} = \cancel{20x_1x_2} + 24x_1 + 50x_2 + \cancel{60}$$

$$26x_1 = 26x_2$$

$$\boxed{x_1 = x_2}$$

onto:

Random number from codomain: $n = \frac{3}{4}$

$$\frac{5(3/4)+6}{4(3/4)+10} = \frac{9.75}{13} = 0.75 = \frac{3}{4}$$

Not onto because $\frac{3}{4}$ is not in the range of $h(x)$. There is no inverse.

3) (12) For $x > 0, x \in R$, find the $O()$ estimate of order of the function

$$f(x) = \frac{6x^4 + 3x^6 \log x - 5x^3 + 7x - 9}{3x^3 + 2x^2 (\log x)^2 - 5x}$$

$$|f(x)| = \left| \frac{6x^4 + 3x^6 \log x - 5x^3 + 7x - 9}{3x^3 + 2x^2 (\log x)^2 - 5x} \right|$$

$$\leq \frac{6|x^4| + 3|x^6| \cdot |x| + 5|x^3| + 7|x| + 9}{3|x^3| + 2|x^2| \cdot |x^2| + 5|x|}$$

$$\leq \frac{6|x^7| + 3|x^7| + 5|x^7| + 7|x^7| + 9|x^7|}{3|x^4| + 2|x^4| + 5|x^4|}$$

$$\leq \frac{30|x^7|}{10|x^4|} = 3|x^3|$$

$f(x)$ is $O(x^3)$, $B = 3$, $n = 3$

4) (10) Use Mathematical Induction to prove that for all positive integers n

$$6 + 13 + 20 + \cdots + (7n - 1) = \frac{n}{2}(7n + 5)$$

$\underline{n=1:}$

$$L \cdot H \cdot S = 6 \overset{\checkmark}{=} R \cdot H \cdot S = \frac{1}{2} \cdot (7+5) = 6$$

$\underline{n=m:}$

$$6 + 13 + 20 + \cdots + (7m - 1) = \frac{m}{2}(7m + 5)$$

$L \cdot H \cdot S$

$\underline{n = m+1:}$

$$6 + 13 + 20 + \cdots + (7m-1) + (7m + 7 - 1)$$

$$= \frac{m}{2}(7m+5) + (7m+6)$$

$$= \frac{(7m^2 + 5m)}{2} + (7m+6)$$

$$= \frac{7m^2 + 5m}{2} + \frac{14m + 12}{2}$$

$$= \frac{7m^2 + 5m + 14m + 12}{2}$$

$$= \frac{7m^2 + 19m + 12}{2}$$

$$= \frac{7m^2 + 12m + 7m + 12}{2}$$

$$= \frac{(m+1)(7m+12)}{2}$$

$$\boxed{= \frac{m+1}{2}(7m+12)}$$

proved by Mathematical Induction ✓

5) (12) Let $T \subset \mathbb{Z}$, $T = \{-1, 2, 7, 14, 23, 34, \ldots\}$
Prove that T is countable or uncountable.

n^{th} element $= n^2 - 2$

one to one: $f: \mathbb{N} \to T \quad f(n) = n^2 - 2$

$\Rightarrow f(m) = f(n) \Rightarrow m^2 - 2 = n^2 - 2 \Rightarrow m^2 = n^2 \Rightarrow m = n$

onto: $y \in T$, $\exists\, x \in \mathbb{Z}^+$ when $f(x) = x^2 - 2 = y$, $y = x^2 - 2$

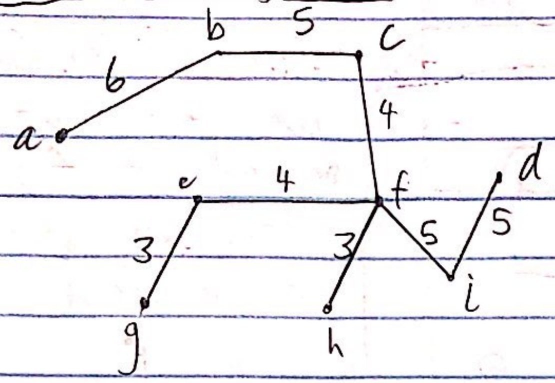
$\Rightarrow f$ is onto

$\Rightarrow f$ is bijective

$\Rightarrow \mathbb{Z}$, T are all alike

$\Rightarrow \mathbb{Z}^+$ is countable, therefore T is also countable.

6) minimal Spanning tree:



Weight = 6 + 5 + 5 + 5 + 4 + 4 + 3 + 3 = 35

7) 1) B is closest to A

$\overline{AB} = 6$, $S = \{B\}$

2) E is the 2nd closest to A

$\overline{AE} = 8$, $S = \{E, B\}$

3) Then G

$\overline{GA} = 18$, $S = \{G, E, B\}$

4) C, $\overline{AB} + \overline{BC} = 13$, $S = \{G, C, E, B\}$

5) F, $\overline{AE} + \overline{EF} = 15$, $S = \{G, F, C, E, B\}$

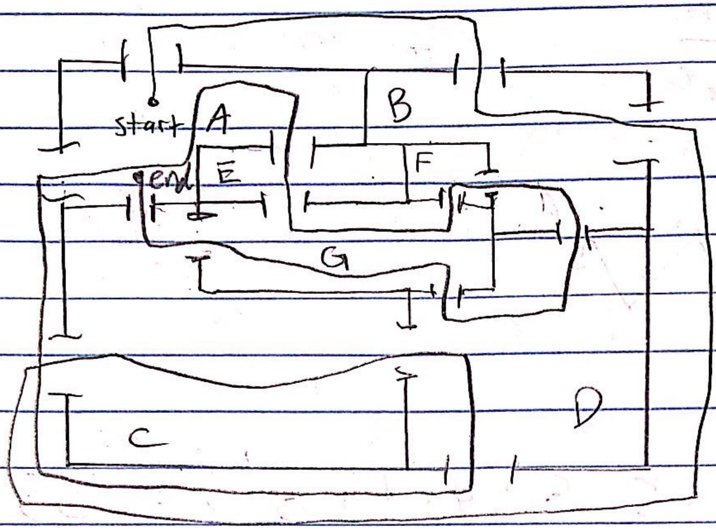
6) H, A, $GH + AG = 33$, $S = \{H, G, F, C, E, B\}$

7) L, $\overline{AE} + \overline{EF} + \overline{LF} = 34$, $S = \{L, H, G, F, C, E, B\}$

L is a part of the vertex set

so the shortest path from A - L is $\boxed{\overline{AE} + \overline{EF} + \overline{LF} = 34}$

8) (10)



For Euler Circuit
we start at A;
we end at A;
And we used
every door once.

In Euler's Path we cannot make a path from A to G. In Euler's Path we need to use every edge of the graph at least once and we are unable to do that so it is not satisfied.

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9) (12) Conference Organizers have 6 different colored mugs on a table for attendees to take.

a) how many different colored sections of 18 mugs can be displayed on a table?

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 18$$

$$x_i \geq 0, \quad 1 \leq i \leq 6$$

$$= C\binom{6+18-1}{18} = \binom{23}{18} = \frac{23!}{18!5!} = \boxed{33,649}$$

b) If the organizers always have at least 2 blue and 2 yellow mugs on the table, how many different selections of 18 mugs can be displayed?

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 14$$

$$x_i \geq 0, \quad 1 \leq i \leq 6$$

$$\binom{6+14-1}{14} = \binom{19}{14} = \frac{19!}{14!5!} = \boxed{11,628}$$