

### Homework 8

- 1) (a) Number of strings containing exactly 7 '1's:

$$\binom{16}{7} = \frac{16!}{(16-7)7!} = \frac{16!}{9!7!} = \boxed{11,440}$$

- (b) total 16-bit strings - 16-bits that contain 1's

$$\Rightarrow 2^{16} - \binom{16}{0} \Rightarrow 2^{16} - 1 = \boxed{65535}$$

- (c) Number of 16-bit strings containing at least 13 1's is the same as those with 13, 14, 15, and 16.

$$\binom{16}{13} + \binom{16}{14} + \binom{16}{15} + \binom{16}{16} = \boxed{697}$$

- 2) 11 points:  $\{A, B, C, D, E, F, G, H, I, J, K\}$

- (a) Number of straight lines between 2 points:

$$11C_2 = \frac{11 \times 10}{2 \times 1} = 55$$

- (b) Straight lines not going through C = 10 straight lines

$$n=1 \quad \frac{(11-1)(10-1)}{2 \times 1} = \frac{10 \times 9}{2} = \boxed{45}$$

- (c) 3 points = a triangle

$$\frac{11 \times 10 \times 9}{3 \times 2 \times 1} = \boxed{165}$$

$$3) \text{ (a) } 50c_2 = \frac{50 \times 49}{2 \times 1} = 1,225$$

Even set of integers sum to be even = 1,225

Odd set of integers sum to be even = 1,225

$$1,225 + 1,225 = \boxed{2,450}$$

(b) To have the sum of two integers be odd, one must be even and one must be odd.

one integer being odd out of 50 =  $50c_1 = 50$

one integer being even out of 50 =  $50c_1 = 50$

$$50 \times 50 = \boxed{2,500}$$

$$4) x_1 + x_2 + x_3 + x_4 = 26, \quad x_i \geq 0, \quad i = 1, 2, 3, 4$$

$$x_1 + x_2 + \dots + x_{k+1} = n$$

$$\binom{n + (k+1) - 1}{(k+1) - 1} = \binom{n+k}{k}$$

$$\Rightarrow \binom{26 + 4 - 1}{4 - 1} = \binom{29}{3} = \boxed{3,654}$$



5) range : 1- 999  
sum  $n = 8$

$$\frac{(n+1)(n+2)}{2} = \frac{(8+1)(8+2)}{2} = \frac{90}{2} = \boxed{45}$$

Check:  $\{008, 800, 080\} : 8+0=8$

$\{017, 710, 701, 071, 170, 107\} : 1+7+0=8$

$\{206, 602, 026, 062, 620, 260\} : 2+6+0=8$

3 digit integers with '0' included come in sets of 6.

The sets are as follows:  $(0, 1, 7), (0, 2, 6), (0, 3, 5), \dots$

$(0, 4, 4), (0, 0, 8), (1, 1, 6), (2, 2, 4), (3, 3, 2)$  are sets that contain 2 different integers only. They come in sets of 3.

We also have sets of 3 different integers that have a sum of 8, like:  $(1, 3, 4), (2, 5, 1)$ .

List:	008	080	170	305	424	611	} = 45 integers.
	017	107	206	314	431	620	
	026	116	215	323	440	701	
	035	125	224	332	503	710	
	044	134	233	341	512	800	
	053	143	242	350	521		
	062	152	251	404	530		
	071	161	260	413	602		