Physica A 554 (2020) 123909

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Loss aversion, overconfidence and their effects on a virtual stock exchange

Mario A. Bertella [a,b], Jonathas N. Silva [a,*], H. Eugene Stanley [c]

[a] Department of Economics, Sao Paulo State University (UNESP), Araraquara, SP 14800-901, Brazil
[b] Swiss Federal Institute of Technology in Zurich (ETH Zurich), Zurich, 8006, Switzerland
[c] Center for Polymer Studies, Boston University, Boston, MA 02215, USA

ARTICLE INFO

Article history:
Received 14 September 2019
Received in revised form 7 December 2019
Available online 24 December 2019

Keywords:
Stock exchange
Agent-based models
Behavioral finance

ABSTRACT

This paper studies the effects of overconfidence and loss aversion in an artificial stock exchange. When we model only fundamentalists we find results that are consistent with homogeneous agent models. Adding 5% of chartists increases the stock return rate but also increases other variables, including volatility and kurtosis. We find that the inclusion of confidence in 5% of chartists raises the trading volume as empirical evidences corroborate and price volatility increases considerably. On the other hand, loss aversion in 5% of chartists substantially decreases the trading volume, although chartist traders now have a higher percentage of stocks in their portfolios, and a buy and hold strategy is adopted to mitigate losses.

1. Introduction

Financial markets exhibit behaviors that are not entirely explainable using traditional economic concepts. Despite evidence in recent decades that markets are highly efficient, a number of studies have found patterns of irrationality, inconsistency, and misjudgment when economic agents make decisions in uncertain environments (Simon [1,2]; Kahneman and Tversky, [3,4], Barberis and Thaler [5]). Behavioral finance incorporates this body of knowledge and argues that these financial phenomena can be understood using models in which agents are not always rational.

Although conventional economics uses homo economicus in its asset pricing models and assumes that trader beliefs are consistent and their actions rational, after decades of research it has become clear that the behavior of individual traders and of the stock market as a whole can no longer be understood using this framework.

Simon [1,2] suggested the term "bounded rationality" to describe a more realistic approach to human behavior. Because we cannot process an infinite amount of information, we need heuristic procedures that simplify decision-making. To overcome the difficulties of the traditional paradigm, behavioral finance examines what occurs when the assumptions underlying individual rationality are relaxed (for example, when agents do not accurately correct their expectations).

Our goal is to examine the changes in stock market price and return dynamics when two psychological variables of behavioral finance – confidence and loss aversion – are taken into account. Although there are many agent-based models in finance, very few incorporate emotional behavior. Takahashi and Terano [6] were the first to propose an agent-based model that incorporated psychological biases from behavioral finance literature. They examined two types of bias: overconfidence and loss aversion. In their paper, when the market has an equal number of fundamentalist and chartist traders, stock market prices coincide with their fundamental values. When the number of chartists is much

* Corresponding author.
E-mail address: jonathas.water@gmail.com (J.N. Silva).

https://doi.org/10.1016/j.physa.2019.123909

higher than the number of fundamentalists, prices sharply deviate from their fundamental values and fundamentalists eventually disappear from the market. There are also fundamental price deviations when investors are overconfident and chartist agents act asymmetrically when facing losses and gains. Lovric's work (Lovric [7]) is based on the Levy, Levy, & Solomon [8], henceforth LLS [8] who use a utility function compatible with empirical evidence. Lovric formalizes overconfidence, revealing that as the degree of overconfidence rises the booms and crashes become more pronounced and less frequent than when there is no overconfidence. He also proposes a stock exchange model that includes overconfidence, optimism and pessimism. Lovric finds that optimistic agents generate strong bubbles, pessimism keeps stock prices close to their fundamental values, and that overconfidence exacerbates investor sentiment. Bertella et al. [9] build an artificial market consisting of fundamentalists and chartists with a simplified utility function. Agents differ in their strategies of evaluating stock prices and exhibit different memory lengths and confidence levels. When the heterogeneity of the strategies used by agents is increased, there is an excess of volatility and kurtosis. When chartist agents become confident there is a positive correlation between average confidence and rate of return. The introduction of confidence reflects higher market volatility and the adverse effect of irrationality on market behavior. Bertella et al. [10] is similar to that by Bertella et al. [9], but it differs in the way they model confidence and how they verify the robustness of their model. As far as the present paper is concerned, the novelty is related to how we optimize the utility functions of both types of investors when they operate simultaneously on the same virtual stock exchange, the way we set confidence and loss aversion and the results obtained.

We describe the model in the second section of this paper. In the third we exhibit simulations without psychological biases. In the fourth we add confidence to the chartist (or technical) agents and observe the aggregate result. In the fifth we add loss aversion. The next section incorporates both psychological biases to the chartists. In the final section we offer our conclusions.

2. Model

Although our model is inspired by the work of LLS [8], unlike their model – which incorporates rational, informed investors and believers in an efficient market – we incorporate fundamentalist investors (who base their expectations on fundamental values) and chartist investors (who base their expectations on past prices). Although fundamentalists and rational informed investors are similar, chartist and efficient market believers are not. Chartists believe that markets are not efficient and that by extrapolating trends money can be made. We will first describe fundamentalist behavior and then chartist behavior.

2.1. Fundamentalists

Fundamentalists estimate the stock price (the fundamental value) P^e_{t+1} according to the expected flow of future dividends (the Gordon model),

$$P^e_{t+1} = \frac{D_t(1+Z)(1+g)}{k-g}. \tag{1}$$

Here D_t is the dividend in time t, Z a random variable of the dividend that has a uniform distribution, g the dividend growth rate, and k the discount factor.

During each period, fundamentalists decide what percentage of wealth to invest in stocks and what percentage in risk-free bonds. To determine this percentage, they maximize the expected utility of wealth $EU(W^i_{t+1})$ using the expression

$$EU(W^i_{t+1}) = EU\left\{W^i_t\left[(1-x)(1+r_f) + xR_{t+1}\right]\right\}, \tag{2}$$

where W^i_t is agent wealth at time t, x the percentage of wealth invested in stocks, r_f the interest rate of the risk-free asset, and R_{t+1} a random variable of the return rate plus one influenced by Z, a random variable of the dividend. The return rate is calculated

$$R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t} = \frac{\frac{D_t(1+Z)(1+g)+D_t(1+g)}{k-g} + D_t(1+Z)}{P_t}. \tag{3}$$

The utility function compatible with the empirical evidence, as shown in the works of Gordon, Paradis, & Rorke [11], Friend & Blume [12], Kroll, Levy, & Rapoport [13] and Levy [14], indicates a function that is CRRA — Constant Relative Risk Aversion, which is a particular case of decreasing absolute risk aversion, DARA. The utility function used in this work (a power function), $U(W)$, has this property and is expressed by

$$U(W) = \frac{W^{1-\alpha}}{1-\alpha}, \tag{4}$$

where α is the risk aversion degree.

Thus the expected utility function is

$$EU(W_{t+1}^i) = \frac{(W_t^i)^{1-\alpha}}{1-\alpha} \times \int_{Z_1}^{Z_2} \left[(1-x)(1+r_f) + x \left(\frac{\frac{D_t(1+Z)(1+g)+D_t(1+g)}{k-g} + D_t(1+Z)}{P_t} \right) \right]^{1-\alpha} f(Z)dZ. \quad (5)$$

This is integrated over all possible values of the random variable Z between Z_1 and Z_2. If we consider that variable Z has a uniform distribution on the interval $[Z_1, Z_2]$, then the above expression becomes, according to Levy, Levy, & Solomon [8],

$$EU(W_{t+1}^i) = \frac{(W_t^i)^{1-\alpha}}{(1-\alpha)(2-\alpha)} \frac{1}{Z_2 - Z_1} \left(\frac{k-g}{1+k} \right) \frac{P_t}{xD_t} (\varphi_2 - \varphi_1), \quad (6)$$

where $\varphi_j = \left[(1-x)(1+r_f) + \frac{x}{P_t} \left(\frac{k-g}{1+k} \right) D_t(1+Z_j) \right]^{2-\alpha}$.

2.2. Chartists

Unlike fundamentalists, chartists form their expectations based on the past. Using three levels of memory ($m = 5$, $m = 10$, and $m = 20$ business days), the price and dividend expectations of chartists will be, according to Takahashi & Terano,

$$P_{t+1}^e = P_{t-1}(1 + a_{t-m})^2, \quad (7)$$

$$D_{t+1}^e = D_{t-1}(1 + a_{t-m}), \quad (8)$$

where $a_{t-m} = \frac{1}{m} \sum_{m=1}^{m} \left(\frac{P_{t-1}}{P_{t-m-1}} - 1 \right)$. Their expected utility function is

$$EU(W_{t+1}^i) = \frac{(W_t^i)^{1-\alpha}}{1-\alpha} \sum_{j=1}^{m} P_r^i(R_{t+1} = R_{t-j}) \left[(1-x)(1 + r_f + xR_{t-j}) \right]^{1-\alpha}, \quad (9)$$

where $P_r^i(R_{t+1} = R_{t-j}) = \frac{1}{m}$ (a uniform distribution) is the probability that agent i of the return in $t+1$ is equal to the return in $t-1$ and $R_{t-j} = \frac{P_{t-1}+D_{t-1}}{P_{t-j}}$.

3. Simulations

3.1. Artificial stock market with fundamentalists and chartists

Market Dynamics

Market dynamics begins with a set of initial conditions consisting of a starting price of stock P_0, an initial dividend D_0, the wealth W_0^i and the number of N_0^i shares held by each investor at time $t = 0$. In the first period of time ($t = 1$), interest is paid on the security and dividend D_1 is realized and paid. Investors then submit their demand orders, N_0^i, and the market clearing price P_1 is determined. After the clearing price is set, the new W_1^i wealth and the number of N_1^i shares held by each investor are calculated. This completes one period of time. This process is repeated several times as market dynamics develops.

Pricing mechanism

The pricing mechanism of the model is based on the temporary market equilibrium. Fundamentalists and chartists determine the ideal proportion of stocks to maximize the expected utility of their wealth in the next period. However, expected utility is a function of price, which is still unknown in the current period. Therefore, investors need to determine optimal wealth ratios to invest in risky asset $x(P_h)$ and its demand for $N_t^i(P_h)$ shares from a hypothetical price, P_h. The equilibrium price P_t is calculated from the adjustment of x and P variables that maximize the utility function of investors. This numerical process takes as initial value P_h and the respective $x(P_h)$ from which the maximization will adjust to the optimal value P_t and x_t subject to the constraint

$$\sum \frac{x_f W_f}{P_f} + \sum \frac{x_c W_c}{P_c} = N, \quad (10)$$

where N is the total supply of stocks, x the percentage of wealth invested in stocks, W agent wealth, P stock price, and subscripts f and c refer to fundamentalists and chartists, respectively.

The hypothetical price is calculated as follows: For each utility function, we replace the price with an expected price (the fundamental price for fundamentalists and the expected price for chartists with memories of 5, 10, and 20). We do the same with the constraint equation to find x_f and x_c. We then substitute both values in the constraint equation to find the hypothetical price, P_h. Then we use x_f, x_c and P_h as initial estimates to optimize both utility functions at time t. Note

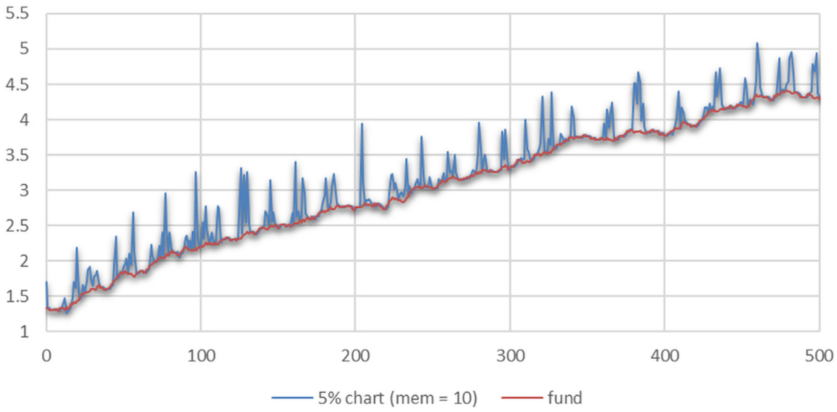

Fig. 1. Time step evolution of equilibrium price (EQP) in log with 5% of chartists (m = 10) (Blue Line). Fundamental price (Red Line). *Source:* Own creation.

that the total investor wealth we use in this maximization process, W_h^i, is the total wealth before the trade (W_{bt}^i) plus the gains or losses due to changes in stock price ($P_h - P_{t-1}$) (LLS [8]), i.e.,

$$W_h^i = W_{bt}^i + N_{t-1}^i(P_h - P_{t-1}). \tag{11}$$

The next simulations show the stock price and the return rate in terms of fundamentalist and chartist configurations in this artificial stock market. We present them step by step in order to show that the inclusion of chartists with equal and different memories leads to the "puzzles" or "anomalies" that we usually find in real stock exchanges, such as excess volatility, heavy trading volume, excess kurtosis, and so on.

3.1.1. Fundamentalists (95%) and chartists (5%) with equal memory ($m = 10$)

Fig. 1 shows the log price of shares over time when all chartists ("technical traders") have the same memory $m = 10$, but when 95% are fundamentalists and 5% are chartists.

It shows that the presence of only 5% chartists causes cyclical price dynamics with periodic booms and crashes. The economic intuition is that when the market is bullish, i.e., the stock price is increasing, the fundamentalists sell additional stocks as market price is above its fundamental value (the stock is expensive), and at the same time, chartists notice that, as the stock market price increases, they make money because they buy stocks at one price and sell them at a higher one. Therefore, they buy stocks aggressively pushing share price upwards. On the other hand, when the market is bearish, the fundamentalists buy stocks because they are cheap and the chartists sell them, as in their memories they have decreasing stock prices. The periodogram of this series (Fig. 7(a)) shows that the price cycle takes, on average, approximately 18 units of time, i.e., it takes 18 units of time for the price to move away from its fundamental value and eventually come back to this level.

In Table 5 (where it summarizes all the data) we see that when only 5% of the traders in the model are chartists with a single memory ($m = 10$), there is a substantial increase in the average rate of return (from 4% to 8%), but also an increase in the standard deviation (from 0.05 to 0.56) in comparison with the case of having only fundamentalists in the stock market. The average rate of return and its standard deviation are higher because the return values sharply change within a few time periods. When 5% of the traders are chartists, the return distribution is more leptokurtic than when there are only fundamentalists. The tails are longer and heavier and the distribution is asymmetrical.

Although most models of homogeneous and rational agents predict no trading volume, a zero return autocorrelation, and a price volatility equal to or less than the volatility of the underlying stock value defined by Shiller [15] to be the present value of all future dividends, the empirical evidence differs greatly: (a) The trading volume can be extremely high (Karpoff [16]; Admati and Pfleiderer [17]); (b) Stock prices are excessively volatile in relation to dividends (Shiller [15]).

Because most standard models of the rational-representative agent cannot explain these empirical behaviors, they are designated as "anomalies". They are caused by aspects of investor behavior not present in agent models, such as experimentally documented deviations from investor behavior, investor rationale, and heterogeneity.

To verify excess volatility between equilibrium and fundamental prices, we follow Shiller [15] and LLS [8]. Fig. 1 shows that both series have a trend that we must detrend using the regression

$$\ln P_t = gt + h + \varepsilon_t, \tag{12}$$

to determine the price growth rate (in which g and h are constants, ε_t is the disturbance term), and where we calculate the detrended price,

$$P_t = \frac{P_t}{e^{\hat{g}t}}. \tag{13}$$

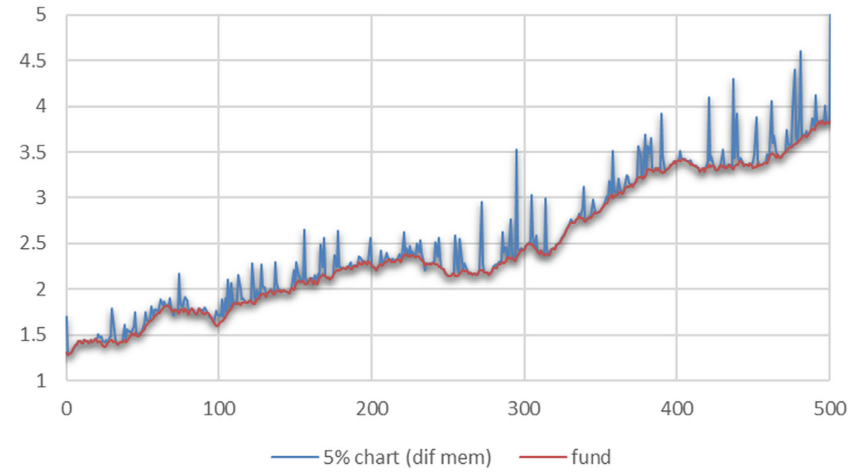

Fig. 2. Log of equilibrium price with 5% of chartists according to the time step (different memories) (Blue Line). Fundamental price (Red Line). *Source:* Own creation.

For 50 simulations of 5000 periods with 5% chartist investors (with the same memory $m = 10$), we find an average standard deviation of the equilibrium price, $\sigma(P_t)$, of 96.94, and an average standard deviation of the fundamental price, $\sigma(P_t^f)$, of 79.11 — an excess volatility of approximately 23%. Besides that, 5% inclusion of chartists ($m = 10$) generate a trading volume of about 1708 shares out of 10,000 per period, a significant 17% of the total (see Table 6). On the other hand, when there are only fundamentalists, our simulation shows that there is no trading volume, and no excess volatility, results that are consistent with homogeneous models.

As one of the stylized facts of asset returns is related to absence of return autocorrelation (the others are heavy tails, gain/loss asymmetry, volatility clustering, among others; for a discussion on this issue, see Cont [18]), we run 50 simulations of 5000 periods for different lags $j = 1 \ldots 40$. Following Campbell, Lo, and Mackinlay [19] and LLS [8], we calculate the autocorrelated returns for each,

$$R_t = \alpha_j + \beta_j R_{t-j} + \varepsilon, \tag{14}$$

where R_t is the return of t, j is the lag, α and β are the parameters, and ε the disturbance term. For each simulation we run this regression for different lags $j = 1 \ldots 40$. For every lag we calculate the average autocorrelation and average t-value considering 50 simulations, in addition to the number of significant positive and negative t-values. Unlike what happens when there are only fundamentalists in which the returns are uncorrelated according to the random-walk hypothesis (see Table A.1 in Appendix), when we have 5% of chartists with equal memory, although values are close to zero and therefore not economically significant, in lag 1 most simulations (44) indicate significant negative autocorrelation, and in lag 2 most (37) show significant positive autocorrelation. In the other lags, the majority of samples do not show significant autocorrelation (see Table A.2 in Appendix).

3.1.2. Fundamentalists and chartists with different memories (m = 5, 10 and 20)

Fig. 2 shows the dynamics of the log prices when traders are 95% fundamentalists and 5% chartists and have memories $m = 5$, 10, and 20.

In contrast to the situation where all the chartists have the same memory, Table 5 shows that all the data increase substantially due to the heterogeneity of chartist memory. As there are three memories, they change their price and dividend expectations differently. Therefore, when the market is booming (dropping), the investor of shorter memory is pushing the price upwards (downwards) faster than the other two types of traders, completely changing the stock price dynamics in relation to the single memory chartist above, as shown in Fig. 2. The periodogram of this price time series (Fig. 7(a)) indicates that the price cycle takes, on average, 24 units of time, i.e., with different memories the cycle price is greater than when it is a single memory.

As Table 6 shows, the volume of traded shares when memories are heterogeneous is higher than when they are the same. With 5% of chartists with heterogeneous memories, 1963 shares are traded on average per period, approximately 20% of the total. On the other hand, the price volatility is much higher than in the previous case. As regards the return autocorrelation, most simulations for each lag do not show significant autocorrelation (Table A.3-see Appendix).

We next examine the behavior of the share price and share rate of return when we assign confidence to only 5% of the chartist traders.

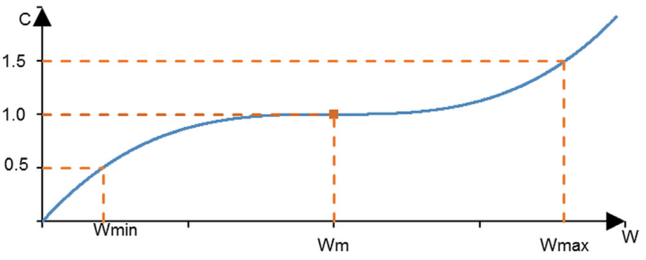

Fig. 3. Confidence function of wealth.
Source: Own creation.

4. Overconfidence

Overconfidence has been widely studied by psychologists because it is ubiquitous in human society. The confidence intervals that agents assign to their quantity estimates (for example, the S&P 500 index) are narrow. Alpert and Raiffa [20] found that 98% of the agent confidence intervals contained the true index only 60% of the time. Agents tend to be hugely overconfident. Events they judge to be 100% certain actually occur only $\approx$ 80% of the time, and events they believe impossible occur $\approx$ 20% of the time (Fischhoff, Slovic, and Lichtenstein, [21]). Overconfidence can also emerge when there is a self-attribution bias, a tendency to attribute success to talent. Thus when things go wrong it is attributed to poor luck, not ineptitude. Investors thus often become overly confident after experiencing successful stock market trading periods (Gervais and Odean, [22]).

Using a logistic function, Lovric [7] assumes that confidence changes based on the success of return predictions. On the other hand, Takahashi and Terano [6] model confidence by assuming that overconfident agents underestimate the risk of the stock. Differently, we assume that agent confidence (C) increases (decreases) as their wealth (W) increases (decreases). It formally corresponds to the cubic function

$$C(W) = c_1 W^3 + c_2 W^2 + c_3 W + c_4, \tag{15}$$

where

$$\begin{aligned} c_1 &= \frac{C_{max} - C_n}{3W_m^2 + a^3 - 3W_m a^2 - W_m^3},\ c_2 = \frac{3W_m(C_{max} - C_n)}{3W_m^2 + a^3 - 3W_m a^2 - W_m^3}, \\ c_3 &= \frac{3W_m^2(C_{max} - C_n)}{3W_m^2 + a^3 - 3W_m a^2 - W_m^3},\ c_4 = \frac{3C_n W_m^2 a + C_n a^3 - 3C_n a^2 W_m - W_m^3 C_{max}}{3W_m^2 + a^3 - 3W_m a^2 - W_m^3}, \end{aligned} \tag{16}$$

and

- C_{max} = maximum confidence value;
- C_n = neutral confidence value;
- W_m = weighted average value of wealth based on memory, where the most recent values of W have greater weight by the expression $W_m = \frac{\sum_{i=1}^{mem}\left(\frac{W_{t-i}}{i^{\alpha}}\right)}{\sum_{i=1}^{mem}\left(\frac{1}{i^{\alpha}}\right)}$ where $\alpha = 2$;
- $a = W_{max}$, if $(W_{max} - W_m) > (W_m - W_{min})$; otherwise $a = 2W_m - W_{min}$;
- W_{max} = maximum wealth of m memories;
- W_{min} = minimum wealth of m memories.

Although confidence increases (decreases) when wealth in a given t period is higher (lower) than average, Fig. 3 shows that confidence does not increase (decrease) at the same rate that wealth increases (decreases), and that there is an interval in the confidence that remains relatively constant.

Fig. 4 shows the log prices when 5% of the chartist agents have confidence and different memories.

When confident chartists with different memories make up 5% of the market and the other 95% are fundamentalists, the statistics of the return rate decrease in relation to the case without confidence (Table 5). This may be because there are fewer high-return values, which decreases the number of positive tail events (the kurtosis is lower). As the confidence intensifies the activities of buying (selling) on the part of chartists when the stock exchange is booming (bearish), we can expect that the trading volume and volatility increases, which is confirmed in Table 6. The trading volume is slightly higher than when there is no confidence, and the equilibrium price becomes much more volatile than the fundamental price (see also Fig. 4). The cycle price takes, on average, 18 units of time, that is, the inclusion of confidence in the chartist agents makes the cycle become smaller than when the chartists have no confidence, as shown in Fig. 7(b).

Odean [23,24] found that overconfident investors negotiate excessively. Table 1 shows that this behavior is also found in our model. On the other hand, Lovric [7] found that overconfident investors trade less.

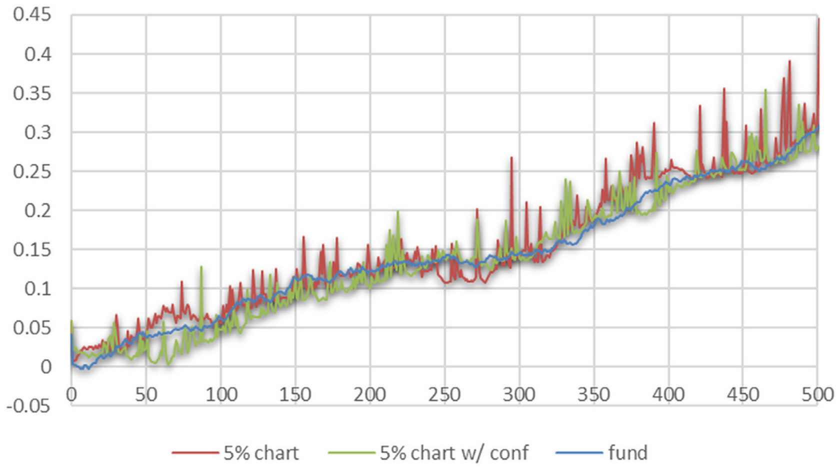

Fig. 4. Time step evolution of normalized equilibrium price in log with 5% of chartists with (Green Line) and without confidence (Red Line) (different memories). Fundamental price (Blue Line) with 100% of fundamentalists.
Source: Own creation.

Table 1
Confidence rate and trading volume by type of agent per period t.
Source: Own creation.

	Confidence rate (average)	Trading volume (per agent)
Chartist, $m = 5$	1.08	43
Chartist, $m = 10$	1.03	54
Chartist, $m = 20$	1.01	53
Fundamentalist	1	2

Table 2
Average trading volume of shares by chartist agent per period t.
Source: Own creation.

Without confidence	With confidence
46.4	50

Note that the highly confident $m = 5$ chartist executes 20% fewer trades than the $m = 10$ chartist who negotiates excessively. Table 2 shows that incorporating confidence increases the trading volume of chartist agents, already high, by approximately 8%.

With regard to the autocorrelated returns, we see that every lag does not reveal, in most simulations, significant autocorrelation (see Table A.4 in the Appendix). On the other hand, the correlation between return and confidence is approximately 60%, i.e., the increase in the rate of return accounts for 60% of the increase in the confidence index. This value is not higher since in our model confidence is a function of wealth, which we quantify as the volume of a trader's invested stocks and risk-free bonds.

5. Loss aversion

The cognitive psychologists Kahneman and Tversky [3] proposed a theory of the prospect of decision-making and reported that people tend to make decisions based on a value function rather than a utility function. This prospect theory sheds light on several aspects of real-world decision-making, for example, how people make decisions based on changes at a particular point of reference, how profit or loss affects their decision-making, and how loss affects their emotions (Kahneman and Tversky, [4], Kahneman and Tversky, [25], Kahneman and Tversky, [26]). The form of the value function ($V(x)$) is expressed as (Kahneman and Tversky, [26])

$$V(x) = \begin{cases} x^{0.88} & x \geq 0 \\ -2.25\,(-x)^{0.88} & x < 0, \end{cases} \tag{17}$$

where x is the change in wealth.

Graphically the value function is concave in the gain domain and convex in the loss domain, which indicates risk-favorable behavior in the field of losses and risk-averse in the field of gains. In addition, the value function is steeper

Table 3
Percentage of shares in a chartists portfolio per period t. Note: The case without loss aversion corresponds to the above analysis with 5% of chartists and different memories (Section 3.1.2).
Source: Own creation.

W/o loss aversion	W/ loss aversion
40%	53%

Table 4
Average volume of traded shares of a chartist agent per period t. Note: The case without loss aversion corresponds to the above analysis with 5% of chartists and different memories (Section 3.1.2).
Source: Own creation.

W/o loss aversion	W/ loss aversion
46.4	25.2

Table 5
Statistics of the return rate.
Source: Own creation.

	Only Fundam.	Fundam. and chartists (m = 10)	Fundam. and chartists (different memories)	Fundam. and chartists (different memories) + confidence	Fundam. and chartists (different memories) + loss aversion	Fundam. and chartists (different memories) + confidence and loss aversion
Mean	0.04	0.08	0.22	0.19	0.15	0.23
Std. dev.	0.05	0.56	3.1	0.89	0.92	2.13
Kurtosis	2.8	653	2610	42	134	887

Table 6
Trading volume and excess price volatility.
Source: Own creation.

	Only Fundam.	Fundam. and chartists (m = 10)	Fundam. and chartists (different memories)	Fundam. and chartists (different memories) + confidence	Fundam. and chartists (different memories) + loss aversion	Fundam. and chartists (different memories) + confidence and loss aversion
Trading vol.	0	17%	20%	21%	12%	13%
Exc. Volat.	0	23%	531%	3200%	1207%	843%

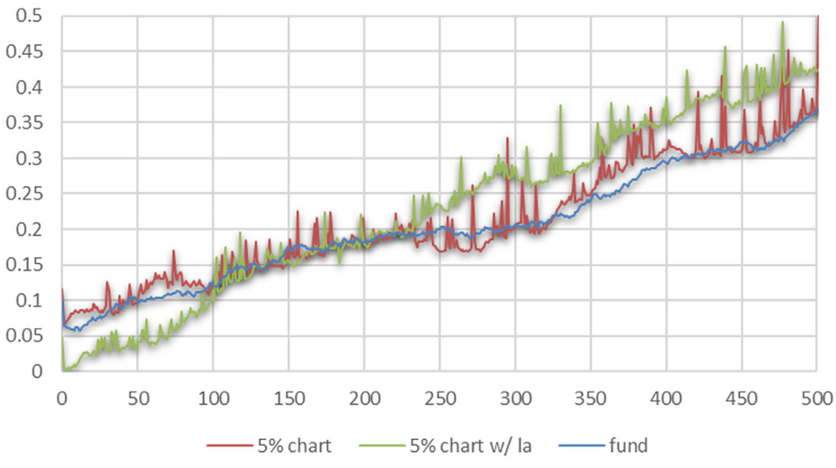

Fig. 5. Time step evolution of normalized equilibrium price in log with 5% of chartists with (Green Line) and without Loss Aversion (Red Line) (different memories). Fundamental price (Blue Line) with 100% of fundamentalists.
Source: Own creation.

in the area of losses than in the area of gains, which indicates that the value of a loss is perceived to be approximately double that of the same value of a gain.

The formalization of loss aversion in our model is

$$\text{if}\, P_t^* < P_t^e \Rightarrow P_{t+1}^{e'} = 2.25 P_{t+1}^e. \tag{18}$$

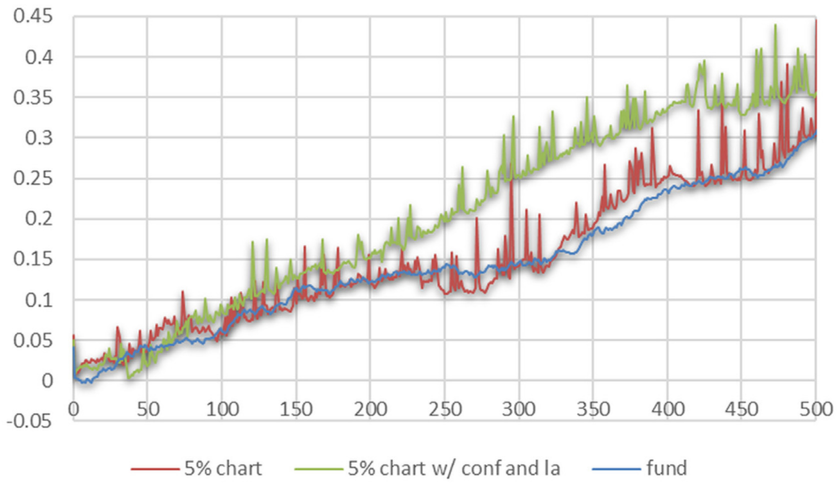

Fig. 6. Time step evolution of normalized equilibrium price with 5% of chartists with (Green Line) confidence and loss aversion and without (Red Line) confidence and loss aversion (different memories). Fundamental price (Blue Line) with 100% of fundamentalists.
Source: Own creation.

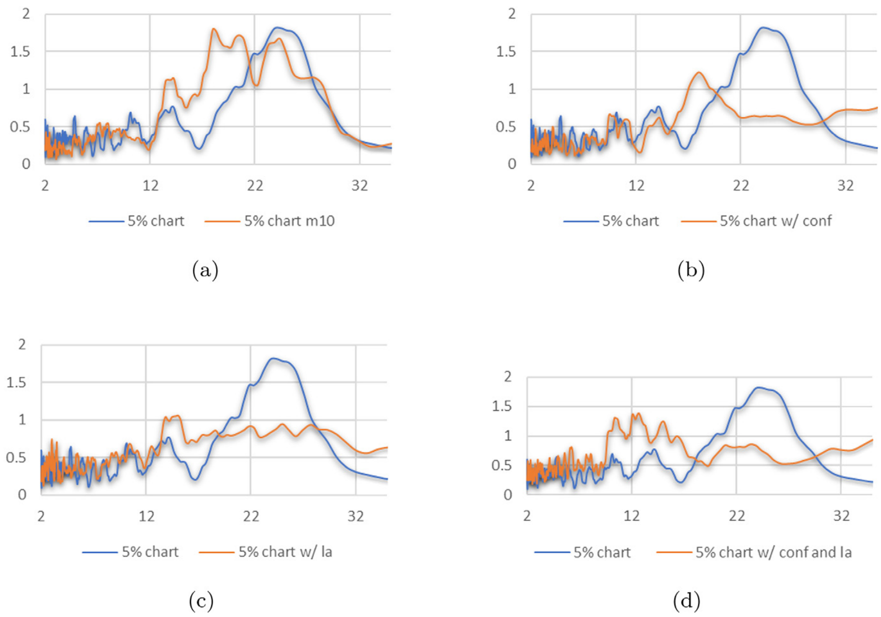

Fig. 7. Smoothed periodogram as a function of the period (in time step unity) for the log of EQP time series in different situations. The highest value peak corresponds to the period in which the series completes the dominant cycle.
Source: Own creation.

When the final equilibrium price (P_t^*) is less than the reference (or expected) price (P_t^e), we multiply the new reference (or expected) price ($P_{t+1}^{e'}$) by 2.25, i.e., when the final price of a share at a given point in time is less than the reference price, then its new reference price for the next period will be corrected by 2.25. If the final price is higher than expected, the reference price at the next moment will be the one previously predicted.

Fig. 5 shows the log equilibrium price with and without loss aversion. Note that when the market is overwhelmingly fundamentalist with only 5% percent chartists the introduction of loss aversion (at this specification) completely changes the stock price dynamics. The periodogram reveals that the cycle price takes, on average, 15 units of time (Fig. 7(c)), a little less than when the chartists are confident.

Comparing the virtual stock market with loss aversion with that which lacks it we find that loss aversion causes all values of the return rate to decrease (see Table 5). A curious reader might ask: will a chartist investor who is risk-averse in the field of earnings and risk-prone in the field of losses assemble a portfolio with a higher percentage of risk-prone stocks or risk-free assets? Remember that fundamentalist agents are risk-averse in all situations. The Table 3 answers this question.

Table 3 shows that loss aversion increases the proportion of shares in the chartist portfolio. How do we explain this apparent contradiction? One possibility is that fear of loss causes an agent to buy and sell fewer shares per period (as Table 4 confirms), keeping a larger percentage of stocks with a strategy of *buy and hold*.

Table 4 shows the changes in trading volumes of chartist agents, i.e., incorporating loss aversion decreases the chartist trading volume by approximately 45%.

Table 6 shows the average trading volume and the relationship between excess price volatility and fundamental price. When there is loss aversion the volume of stock trading is lower than when there is no loss aversion, but the volatility increases. Comparing with the case of confidence above, both values are much lower, suggesting that chartists become more cautious in trading due to loss aversion. On the other hand, when there is loss aversion the return autocorrelation falls into the same pattern seen previously, except for the fact that the first lag presents, in most simulations, significant negative autocorrelations (see Table A.5 in the Appendix).

6. Confidence and loss aversion

When we add both psychological biases in the chartists at the same time, we find that the statistics of the return rate increase considerably compared to the situation when the chartists have only one psychological bias, which it seems a counter intuitive result (see Table 5). The Fig. 6 shows the price dynamics (in log). We can see that its trajectory is completely different and steeper than the price dynamics of fundamental price or of the equilibrium price when the chartists do not have any psychological bias.

The periodogram (Fig. 7(d)) reveals the price cycle takes about 12 units of time, in average, which is smaller than when there is only one psychological characteristic, i.e., the price peak is reached more quickly compared to the other situations above. The trading volume continues low (13%) and the percentage of stocks in the chartists portfolio is similar to the situation of only loss aversion (52.5%) which is an influence of loss aversion predominating on confidence (for return autocorrelations of both simultaneous biases, see Table A.6).

Under the configuration of 95% fundamentalists and 5% different types of chartist investors with confidence and loss aversion, these last ones hold 24% of the total of stocks and fundamentalists, 76%. Ling et al. [27] estimated that the chartists would have about 17% of the shares and the fundamentalists, 83%. However, chartists would account for a significant proportion of the shares traded (80%). In our model, chartists are also responsible for most of the stocks traded, although with a smaller percentage (about 60%).

7. Concluding remarks

We have examined the effect of the psychological confidence and loss aversion variables on an artificial stock exchange. We first construct an optimized behavior for fundamentalist and chartist agents using a utility function that is compatible with the empirical evidence. We initially use experiments to determine whether the results are consistent. We find that when there are only fundamentalists in the stock exchange there is no autocorrelation of returns and no trading volume volatility or excessive price volatility in relation to fundamental prices. These are results usually found in models with homogeneous agents. When we introduce a small number (5%) of chartist agents (also called technical analysts), anomalies appear, i.e., a volume increase in stock trading and excess price volatility, among others. When there is only a chartist memory and equal to 10, the average rate of return and other data is higher than when the simulated market contains only fundamentalists. Including different memories, the data further increases, indicating that behavioral heterogeneity strongly affects the dynamics of financial variables. Incorporating confidence in only 5% of the chartists with heterogeneous memories increases the trading volume, both in relation to the case where there are fundamentalists and technical analysts without this bias, and these results are corroborated in the empirical literature. Adding loss aversion in only 5% of the chartist agents reduces trading volume significantly, but the chartist portfolio now has a much higher percentage of stocks, and a *buy and hold* strategy is adopted to mitigate losses. The introduction of confidence and loss aversion makes the price cycle become smaller than when the chartists with different memories have no psychological bias, i.e., the price peak is reached more quickly with the inclusion of psychological characteristics. Loss aversion seems to dominate confidence effects. Additionally, in order to illustrate how parameters affect our model, we vary the values of risk aversion from $\alpha = 1.01$ to $\alpha = 2$ under different configurations (5% chartists, 5% chartists with confidence and 5% chartists with loss aversion). All the curves shift vertically and their slopes are similar (for a discussion, see Appendix, Figs. A.1–A.3). Lastly, although our quantitative results may be larger than in real-world stock exchanges (see Cont [18]), the quality of our model data is similar to the empirical evidence regarding kurtosis (leptokurtic distribution), the absence of return autocorrelation, excess volatility, and high trading volume.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

Professor Bertella thanks the support of Sao Paulo Research Foundation (Fapesp) (Grant: 2018/22562-4) and also of National Council for Scientific and Technological Development (CNPQ) (Grants: 303986/2017-4 and 428433/2018-9).

Appendix

Parameters	Values used in simulations
Investor number	1000
Risk aversion α	1.5
Number of stocks	10 000
Risk-free interest rate r_f	0.01
Required rate of return on share k	0.04
Maximal one-period dividend decrease z_1	−0.07
Maximal one-period dividend growth z_2	0.1
Average dividend growth rate g	0.015
Wealth of each investor $t = 0$	$1000.00

Table A.1
Return autocorrelations (only fundamentalists).

Lag	Autocorrelation	Average t-value	Significant positive t-values*	Significant negative t-values*
1	0.000978	0.069323	2	0
2	−0.001286	−0.090957	0	1
3	0.000738	0.052118	0	1
4	0.004198	0.296837	1	0
5	0.003581	0.253296	3	0
6	−0.004235	−0.299538	0	0
7	0.00191	0.135035	2	3
8	−0.003555	−0.251456	1	3
9	−0.00065	−0.045901	2	0
10	−0.001487	−0.105224	1	3
11	−0.000698	−0.049344	1	2
12	0.003488	0.24674	3	1
13	−0.001286	−0.090978	0	1
14	−0.00107	−0.075736	0	1
15	0.006339	0.44836	3	0
16	0.00029	0.02053	1	2
17	−0.003705	−0.262052	1	1
18	−0.003654	−0.258404	1	1
19	−0.002587	−0.182944	0	2
20	−0.002013	−0.142368	0	1
21	0.001342	0.094928	0	0
22	−0.003046	−0.215394	0	1
23	−0.000484	−0.034254	1	1
24	−0.000199	−0.014071	0	0
25	0.000969	0.068482	1	3
26	−0.00068	−0.048081	1	3
27	0.000827	0.058437	1	3
28	0.006083	0.430313	4	1
29	0.001649	0.116596	0	1
30	0.001185	0.083844	2	2
31	0.001911	0.135149	1	2
32	−0.000797	−0.05636	0	1
33	−0.001626	−0.114984	1	1
34	0.002095	0.148125	2	2
35	−0.00323	−0.228395	0	0
36	−0.000699	−0.049412	3	1
37	−0.001168	−0.082658	2	3
38	−0.004267	−0.301761	0	2
39	0.003705	0.262055	3	0
40	0.003726	0.263476	1	1

*Significant at 95%.

Table A.2
Return autocorrelation (95% Fundamentalists + 5% Chartists, $m = 10$).

Lag	Autocorrelation	Average t-value	Significant positive t-values*	Significant negative t-values*
1	−0.079143	−5.630131	0	44
2	0.062636	4.456707	37	0
3	0.018784	1.353104	9	0
4	0.015946	1.136349	10	1
5	0.011286	0.799896	10	0
6	0.001066	0.075417	2	0
7	0.000613	0.050913	3	0
8	−0.001303	−0.090465	4	2
9	−0.008683	−0.614114	0	4
10	−0.002825	−0.199669	2	1
11	−0.004289	−0.302702	3	2
12	0.003588	0.254837	4	0
13	0.019741	1.399241	16	0
14	0.018997	1.345693	15	0
15	0.041592	2.956578	25	0
16	0.025376	1.796482	21	1
17	0.036006	2.559802	24	0
18	0.028329	2.012433	20	1
19	0.044202	3.150552	25	0
20	0.01888	1.339897	13	0
21	0.028361	2.010233	18	0
22	0.020635	1.465275	16	0
23	0.017738	1.256424	10	0
24	0.020852	1.490099	14	0
25	0.007422	0.52598	6	0
26	0.018203	1.298042	9	0
27	0.003416	0.241999	4	0
28	0.016463	1.167034	12	0
29	0.014761	1.04731	10	0
30	0.02235	1.654691	8	0
31	0.018538	1.312935	17	1
32	0.027676	1.963248	18	0
33	0.021172	1.50321	17	1
34	0.03709	2.649377	19	0
35	0.012487	0.885948	9	0
36	0.031812	2.258271	19	0
37	0.018127	1.287641	14	1
38	0.027655	1.968487	13	0
39	0.015035	1.065659	11	2
40	0.018586	1.319329	14	0

*Significant at 95%.

Table A.3
Return autocorrelation (95% Fundamentalists + 5% Chartists, $m = 5$, 10 and 20).

Lag	Autocorrelation	Average t-value	Significant positive t-values*	Significant negative t-values*
1	−0.045819	−3.261993	0	23
2	0.012234	0.866271	10	0
3	0.009521	0.674948	7	0
4	0.010207	0.687907	5	0
5	0.006299	0.447931	4	0
6	0.008574	0.610371	5	0
7	0.005203	0.368491	6	0
8	0.00856	0.606301	3	0
9	0.008116	0.573053	6	0
10	0.013567	0.939759	9	0
11	0.011416	0.808552	9	0
12	0.011119	0.787353	5	0
13	0.01164	0.824078	9	0
14	0.012438	0.845145	6	0
15	0.008626	0.610805	5	0
16	0.008695	0.616056	6	0
17	0.009232	0.653755	3	0
18	0.00652	0.461671	3	0
19	0.016235	1.194908	7	0
20	0.006471	0.458489	6	0
21	0.007061	0.499865	3	1
22	0.00939	0.654531	4	0
23	0.01064	0.754615	5	0
24	0.012401	0.882429	6	0
25	0.009243	0.654954	7	0
26	0.010537	0.745988	6	0
27	0.01282	0.894616	10	0
28	0.008892	0.629864	5	0
29	0.010182	0.722868	7	0
30	0.017539	1.252789	10	0
31	0.007157	0.507	4	0
32	0.014546	1.013946	9	0
33	0.005438	0.384728	5	0
34	0.008385	0.593596	6	0
35	0.008094	0.573117	6	0
36	0.008397	0.593857	4	0
37	0.008972	0.635238	7	0
38	0.00779	0.551315	5	0
39	0.012075	0.8556	9	0
40	0.008165	0.578378	5	0

*Significant at 95%.

Table A.4
Return autocorrelation with confidence (95% Fundamentalists + 5% Chartists) ($m = 5$, 10 and 20).

Lag	Autocorrelation	Average t-value	Significant positive t-values*	Significant negative t-values*
1	−0.04375	−3.115936	1	24
2	0.000009	0.000703	2	0
3	0.01113	0.787339	11	0
4	0.007645	0.541614	5	0
5	0.00237	0.167683	2	0
6	0.004466	0.315971	4	0
7	0.008285	0.586487	6	0
8	0.008074	0.574482	4	0
9	0.007169	0.507147	6	0
10	0.008261	0.58437	4	0
11	0.007576	0.535984	5	0
12	0.011265	0.798252	6	0
13	0.008887	0.629818	6	0
14	0.009223	0.652742	6	0
15	0.010838	0.767039	11	0
16	0.013153	0.937384	7	0
17	0.011105	0.786307	9	0
18	0.006348	0.449093	3	0
19	0.006281	0.444341	6	0
20	0.007509	0.531279	3	0
21	0.004817	0.340663	2	0
22	0.013826	0.979039	9	0
23	0.008464	0.598991	5	0
24	0.008108	0.573751	7	0
25	0.007925	0.560705	4	0
26	0.014343	1.016429	11	0
27	0.013538	0.971612	5	0
28	0.006519	0.461142	3	0
29	0.010806	0.765074	9	0
30	0.007654	0.541419	6	0
31	0.008683	0.614654	5	0
32	0.006516	0.461157	5	0
33	0.010583	0.74928	7	0
34	0.005453	0.385637	0	0
35	0.009144	0.647825	6	0
36	0.013427	0.950562	9	0
37	0.012665	0.905401	6	0
38	0.008123	0.574781	5	0
39	0.006837	0.483783	3	0
40	0.00739	0.522764	5	0

*Significant at 95%.

Table A.5
Return autocorrelation with loss aversion (95% Fundamentalists + 5% Chartists) ($m = 5$, 10 and 20).

Lag	Autocorrelation	Average t-value	Significant positive t-values*	Significant negative t-values*
1	−0.069998	−4.981831	0	33
2	0.019198	1.358516	19	0
3	0.019553	1.386092	14	0
4	0.012	0.856203	7	0
5	0.00621	0.439222	4	0
6	0.006427	0.454768	4	0
7	0.009989	0.707225	8	0
8	0.012353	0.875396	9	0
9	0.015603	1.1048	9	0
10	0.011613	0.821724	8	0
11	0.01582	1.12115	9	0
12	0.015728	1.113104	13	0
13	0.015488	1.095899	13	0
14	0.014444	1.022055	12	0
15	0.018954	1.342346	15	0
16	0.018978	1.343901	15	0
17	0.016464	1.165419	12	0
18	0.020358	1.50723	6	0
19	0.01177	0.83344	9	0
20	0.017352	1.232811	8	0
21	0.016604	1.182862	11	0
22	0.014301	1.012132	12	0
23	0.018171	1.288547	11	0
24	0.017229	1.22115	8	0
25	0.020374	1.44461	14	0
26	0.013796	0.976935	8	1
27	0.013327	0.943769	8	0
28	0.01554	1.10026	10	0
29	0.016785	1.187808	10	0
30	0.014534	1.028759	11	0
31	0.013258	0.938277	8	0
32	0.015894	1.125654	9	0
33	0.014089	0.996916	7	0
34	0.011112	0.788669	7	0
35	0.018215	1.290351	13	0
36	0.01628	1.152097	13	0
37	0.01538	1.089113	10	0
38	0.013566	0.960513	13	0
39	0.015663	1.109152	9	0
40	0.014446	1.022518	9	0

*Significant at 95%.

Table A.6
Return autocorrelation with loss aversion and confidence (95% Fundamentalists + 5% Chartists) ($m = 5$, 10 and 20).

Lag	Autocorrelation	Average t-value	Significant positive t-values*	Significant negative t-values*
1	−0.040609	−2.815227	0	25
2	0.001462	0.095204	2	0
3	0.013855	0.949516	6	0
4	0.003107	0.20344	1	0
5	0.007607	0.532808	4	0
6	0.009634	0.674748	5	0
7	0.011014	0.738829	5	0
8	0.009284	0.66676	7	0
9	0.011408	0.811896	9	0
10	0.01194	0.808187	8	0
11	0.006645	0.476215	5	0
12	0.011841	0.830393	9	0
13	0.017428	1.232802	8	0
14	0.007766	0.540687	4	0
15	0.007593	0.53917	3	0
16	0.007493	0.530585	6	0
17	0.009962	0.675926	5	0
18	0.009263	0.664404	5	0
19	0.008113	0.585061	5	0
20	0.008684	0.603086	4	0
21	0.009704	0.678603	6	0
22	0.006341	0.465285	5	0
23	0.008909	0.62463	5	0
24	0.010333	0.712049	7	0
25	0.01115	0.779921	6	0
26	0.012331	0.895768	4	0
27	0.007742	0.560767	7	0
28	0.007806	0.547621	3	0
29	0.011778	0.842516	6	0
30	0.010141	0.698911	7	0
31	0.010217	0.711423	5	0
32	0.008688	0.623864	6	0
33	0.009623	0.671135	6	0
34	0.00803	0.572332	6	0
35	0.006751	0.493065	7	0
36	0.008495	0.59338	5	0
37	0.012799	0.898199	6	0
38	0.012058	0.852521	6	0
39	0.007883	0.553922	4	0
40	0.005737	0.4034	0	0

*Significant at 95%.

In order to ilustrate and see how the parameters affect our model, we vary the values of the degree of the risk aversion (α) from 1.01 to 2 for several configurations (5% chartists - Fig. A.1, 5% chartists with confidence - Fig. A.2, and 5% chartists with loss aversion - Fig. A.3). Note that if the population is more (less) risk averse, the expected return on the risky asset must increase (decrease) (or, equivalently, the price of the risky asset must decline (rise)). The curves are almost identical in shape. However, including loss aversion (Fig. A.3) makes the green line (low risk aversion) move substantially downward indicating that the cumulative distribution of the return rate is smaller than in the other two cases. This is a plausible result since the return rate of a stock should decrease when the population is less risk averse and, simultaneously, part of that population is also loss averse. In this case, investing in stocks follows the principle of buy and hold.

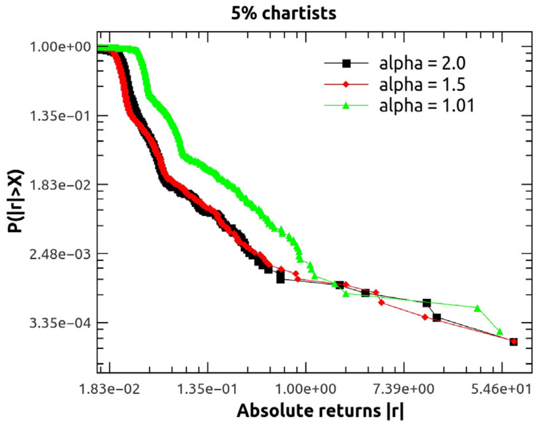

Fig. A.1. Cumulative distribution of returns for different risk aversion parameters ($\alpha = 1.01$, 1.5 and 2).

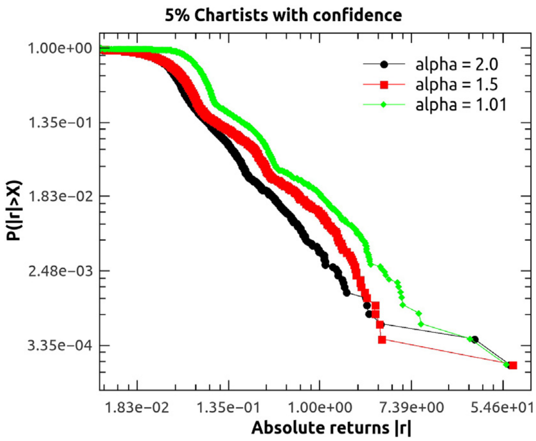

Fig. A.2. Cumulative distribution of returns for different risk aversion parameters ($\alpha = 1.01$, 1.5 and 2).

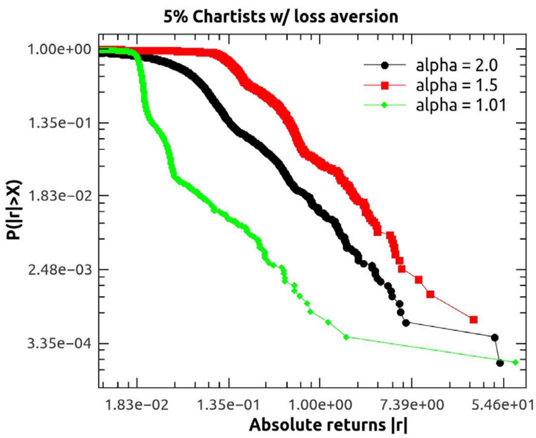

Fig. A.3. Cumulative distribution of returns for different risk aversion parameters ($\alpha = 1.01$, 1.5 and 2).

References

[1] H.A. Simon, A behavioral model of rational choice, Q. J. Econ. 69 (1) (1955) 99–118, http://dx.doi.org/10.2307/1884852, arXiv:/oup/backfile/content_public/journal/qje/69/1/10.2307/1884852/2/69-1-99.pdf.

[2] H.A. Simon, Models of bounded rationality. volume 2: Behavioural economics and business organization, Organ. Stud. 6 (3) (1982) 505, http://dx.doi.org/10.1177/017084068500600320, arXiv:https://doi.org/10.1177/017084068500600320.

[3] D. Kahneman, A. Tversky, Judgment under uncertainty: Heuristics and biases, Science 185 (4157) (1974) 1124–1131, http://dx.doi.org/10.1126/science.185.4157.1124, arXiv:http://science.sciencemag.org/content/185/4157/1124.full.pdf URL http://science.sciencemag.org/content/185/4157/1124.

[4] D. Kahneman, A. Tversky, Prospect theory: An analysis of decision under risk, Econometrica 47 (2) (1979) 263–291, URL https://EconPapers.repec.org/RePEc:ecm:emetrp:v:47:y:1979:i:2:p:263-91.

[5] N. Barberis, R.H. Thaler, A survey of behavioral finance, in: Advances in Behavioral Finance, Vol. II, Princeton University Press, Princeton, 2005, p. 712.

[6] H. Takahashi, T. Terano, Agent-based approach to investors? Behavior and asset price fluctuation in financial markets, J. Artif. Soc. Soc. Simul. 6 (3) (2003).

[7] M. Lovric, Behavioral Finance and Agent-Based Artificial Markets, Erasmus University Rotterdam, 2011.

[8] H. Levy, M. Levy, S. Solomon, Microscopic Simulation of Financial Markets: From Investor Behavior to Market Phenomena, Academic Press, 2000, p. 300.

[9] M.A. Bertella, F.R. Pires, L. Feng, H.E. Stanley, Confidence and the stock market: An agent-based approach, PLoS One 9 (1) (2014) 1–9.

[10] M.A. Bertella, F.R. Pires, H.H.A. Rego, J.N. Silva, I. Vodenska, H.E. Stanley, Confidence and self-attribution bias in an artificial stock market, PLoS One 12 (2) (2017) 1–20, http://dx.doi.org/10.1371/journal.pone.0172258.

[11] M.J. Gordon, G. Paradis, C. Rorke, Experimental evidence on alternative portfolio decision rules, Amer. Econ. Rev. 62 (1) (1972) 107–118, URL https://EconPapers.repec.org/RePEc:aea:aecrev:v:62:y:1972:i:1:p:107-18.

[12] I. Friend, M.E. Blume, The demand for risky assets, Amer. Econ. Rev. 65 (5) (1975) 900–922, URL https://EconPapers.repec.org/RePEc:aea:aecrev:v:65:y:1975:i:5:p:900-922.

[13] Y. Kroll, H. Levy, A. Rapoport, Experimental tests of the separation theorem and the Capital asset pricing model, Amer. Econ. Rev. 78 (3) (1988) 500–519, URL https://EconPapers.repec.org/RePEc:aea:aecrev:v:78:y:1988:i:3:p:500-519.

[14] H. Levy, Absolute and relative risk aversion: An experimental study, J. Risk Uncertain. 8 (3) (1994) 289–307, http://dx.doi.org/10.1007/BF01064046.

[15] R. Shiller, Do stock prices move too much to be justified by subsequent changes in dividends?, Amer. Econ. Rev. 71 (3) (1981) 421–436, URL https://EconPapers.repec.org/RePEc:aea:aecrev:v:71:y:1981:i:3:p:421-36.

[16] J. Karpoff, The relation between price changes and trading volume: A survey, J. Financ. Quant. Anal. 22 (01) (1987) 109–126.

[17] A. Admati, P. Pfleiderer, A theory of intraday patterns: Volume and price variability, Rev. Financ. Stud. 1 (1) (1988) 3–40.

[18] R. Cont, Empirical properties of asset returns: Stylized facts and statistical issues, Quant. Finance 1 (2) (2001) 223–236, arXiv:https://doi.org/10.1080/713665670.

[19] J.Y. Campbell, A.W. Lo, A.C. MacKinlay, The Econometrics of Financial Markets, Princeton University Press, Princeton, NJ, 1997.

[20] M. Alpert, H. Haiffa, A progress report on the training of probability assessors, in: D. Kahneman, P. Slovic, A. Tversky (Eds.), Judgment under Uncertainty: Heuristics and Biases, Cambridge University Press, 1982, pp. 294–305.

[21] B. Fischhoff, P. Slovic, S. Lichtenstein, Knowing with certainty: The appropriateness of extreme confidence, J. Exp. Psychol.: Hum. Percept. Perform. 3 (4) (1977) 552–564.

[22] S. Gervais, T. Odean, Learning to be overconfident, Rev. Financ. Stud. 14 (1) (2001) 1–27.

[23] T. Odean, Volume, volatility, price, and profit when all traders are above average, J. Finance 53 (6) (1998) 1887–1934.

[24] T. Odean, Do investors trade too much?, Amer. Econ. Rev. 89 (5) (1999) 1279–1298, http://dx.doi.org/10.1257/aer.89.5.1279, URL http://www.aeaweb.org/articles?id=10.1257/aer.89.5.1279.

[25] A. Tversky, D. Kahneman, The framing of decisions and the psychology of choice, Science 211 (4481) (1981) 453–458, http://dx.doi.org/10.1126/science.7455683, arXiv:http://science.sciencemag.org/content/211/4481/453.full.pdf URL http://science.sciencemag.org/content/211/4481/453.

[26] D. Kahneman, A. Tversky, Advances in prospect theory: Cumulative representation of uncertainty, J. Risk Uncertain. 5 (4) (1992) 297–323, http://dx.doi.org/10.1007/BF00122574.

[27] F. Ling, L. Baowen, B. Podobnik, T. Preis, H.E. Stanley, Linking agent-based models and stochastic models of financial markets, Proc. Natl. Acad. Sci. 109 (22) (2012) 8388–8393.