1 Risk Difference

The proportion mediated on the counterfactual risk difference scale is defined as the first line. Once expanded into the counterfactual risk expressions, it can be transformed into an expression solely in terms of the counterfactual risk ratios.

$$
\begin{aligned}
PM(RD) &= \frac{RD^{NIE}}{RD^{TE}} \\
&= \frac{E[Y_{a,M_a}] - E[Y_{a,M_{a*}}]}{E[Y_{a,M_a}] - E[Y_{a*,M_{a*}}]} \\
&\quad \text{Divide by } E[Y_{a*,M_{a*}}] \\
&= \frac{\frac{E[Y_{a,M_a}]}{E[Y_{a*,M_{a*}}]} - \frac{E[Y_{a,M_{a*}}]}{E[Y_{a*,M_{a*}}]}}{\frac{E[Y_{a,M_a}]}{E[Y_{a*,M_{a*}}]} - \frac{E[Y_{a*,M_{a*}}]}{E[Y_{a*,M_{a*}}]}} \\
&= \frac{RR^{TE} - RR^{NDE}}{RR^{TE} - 1} \\
&\quad \text{By multiplicative decomposition} \\
&= \frac{RR^{NDE}RR^{NIE} - RR^{NDE}}{RR^{NDE}RR^{NIE} - 1} \\
&= \frac{RR^{NDE}(RR^{NIE} - 1)}{RR^{NDE}RR^{NIE} - 1}
\end{aligned}
$$

2 Odds Ratio

Assuming a rare outcome that allows approximation of RR with OR, we can approximate the proportion mediated on the counterfactual *risk difference* scale as follows [Vanderweele and Vansteelandt, 2010].

$$
\begin{aligned}
PM(RD) &= \frac{RR^{NDE}(RR^{NIE} - 1)}{RR^{NDE}RR^{NIE} - 1} \\
&\approx \frac{OR^{NDE}(OR^{NIE} - 1)}{OR^{NDE}OR^{NIE} - 1}
\end{aligned}
$$

3 Bibliography

[Vanderweele and Vansteelandt, 2010] Vanderweele, T. J. and Vansteelandt, S. (2010). Odds ratios for mediation analysis for a dichotomous outcome. *Am. J. Epidemiol.*, 172(12):1339–1348.