1 Midterm exam practice problem 4

$$\begin{aligned}
0 &= E\left[g(t)\right] \\
0 &= \int_{\theta}^{\infty} g(t) n e^{-n(t-\theta)} \mathrm{d}t \\
\frac{\partial}{\partial \theta} 0 &= \frac{\partial}{\partial \theta} \int_{\theta}^{\infty} g(t) n e^{-n(t-\theta)} \mathrm{d}t \\
&\text{By Differentiation under the integral sign} \\
0 &= -g(\theta) n e^{-n(\ \theta\ -\theta)} + \int_{\theta}^{\infty} \frac{\partial}{\partial \theta} g(t) n e^{-n(t-\theta)} \mathrm{d}t \\
0 &= -g(\theta) n e^{-n(\ \theta\ -\theta)} + \int_{\theta}^{\infty} g(t) n^2 e^{-n(t-\theta)} \mathrm{d}t \\
&\text{All } t \text{ in the first term should be replaced with } \theta
\end{aligned}$$

Differentiation under the integral sign
https://en.wikipedia.org/wiki/Differentiation_under_the_integral_sign

2 Midterm exam practice problem 6

$$\begin{aligned}
&\text{From Lab 6 Problem 1, posterior for } theta \text{ is} \\
\pi(\theta|x) &= \frac{xI(0 \le x < \theta < 1)}{\theta^2(1-x)} \\
&\text{Bayes estimator using squared error loss is posterior mean} \\
E[\theta^3|x] &= \int_x^1 \theta^3 \pi(\theta|x) \mathrm{d}\theta \\
&= \int_x^1 \theta^3 \frac{xI(0 \le x < \theta < 1)}{\theta^2(1-x)} \mathrm{d}\theta \\
&= \int_x^1 \theta^3 \frac{xI(0 \le x < \theta < 1)}{\theta^2(1-x)} \mathrm{d}\theta \\
&= \int_x^1 \theta^3 \frac{x}{\theta^2(1-x)} \mathrm{d}\theta \\
&= \int_x^1 \theta \frac{x}{(1-x)} \mathrm{d}\theta \\
&= \frac{x}{1-x} \int_x^1 \theta \mathrm{d}\theta \\
&= \frac{x}{1-x} \left[\frac{1}{2}\theta^2\right]_x^1 \\
&= \frac{x}{2(1-x)} \left[\theta^2\right]_x^1 \\
&= \frac{x}{2(1-x)} \left[1-x^2\right] \\
&= \frac{x}{2(1-x)} \left[(1+x)(1-x)\right] \\
&= \frac{x\ (1+x)}{2} \\
&\text{The first } x \text{ was missed in the solution}
\end{aligned}$$

3 Lab 6 practice problem 2(3)

$$\begin{aligned}
\pi(\sigma^2|\mathbf{x}) &\propto \left[\frac{(n-1)S^2}{\sigma^2}\right]^{\left(\frac{n+1}{2}+1-1\right)} \exp\left[-\frac{(n-1)S^2}{2\sigma^2}\right] \\
&= \left[\frac{(n-1)S^2}{\sigma^2}\right]^{\left(\frac{n+3}{2}-1\right)} \exp\left[-\frac{(n-1)S^2}{2\sigma^2}\right] \\
&\chi^2 \text{ with } n+3 \text{ degrees of freedom}
\end{aligned}$$