数

(1) (a) $\lambda_1 = 2 \qquad u_1 = [1, 1, -1]^T$

$\lambda_2 = -1 \qquad u_2 = [1, 1, 2]^T$

$\lambda_3 = 1 \qquad u_3 = [1, -1, 0]^T$

(b) $P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ -1 & 2 & 0 \end{bmatrix}$

$P^{-1}AP = \begin{bmatrix} 2 & & \\ & -1 & \\ & & 1 \end{bmatrix}$

(2) (a) $\mathcal{L}\left[\frac{dx(t)}{dt}\right] = \int_0^\infty e^{-st}\frac{dx(t)}{dt}dt$

$= \int_0^\infty e^{-st}dx(t)$

$= e^{-st}x(t)\Big|_0^\infty - \int_0^\infty x(t)\,de^{-st}$

$= s\int_0^\infty x(t)e^{-st}dt$

$= s\mathcal{L}[x(t)]$

$\mathcal{L}\left[\frac{d^2x(t)}{dt^2}\right] = \mathcal{L}\left[\frac{d\left(\frac{dx(t)}{dt}\right)}{dt}\right] = s\mathcal{L}\left[\frac{dx(t)}{dt}\right] = s^2\mathcal{L}[x(t)]$

(b) $\mathcal{L}\left[\frac{d^2(x)}{dt^2} + 3\frac{dx(t)}{dt} + 2x(t)\right] = \mathcal{L}[\sin 2t]$

$\mathcal{L}[x(t)]\cdot(s^2+3s+2) = \mathcal{L}[\sin 2t]$

注：$\mathcal{L}[\sin wt] = \frac{w}{s^2+w^2} \qquad \mathcal{L}[\cos wt] = \frac{s}{s^2+w^2}$

$\mathcal{L}[\sin 2t] = \int_0^\infty e^{-st}\sin 2t\,dt = \frac{1}{2}\int_0^\infty e^{-s\cdot\frac{t}{2}}\sin t\,dt = -\frac{1}{2}\int_0^\infty e^{-\frac{1}{2}st}\,d\cos t$

$= -\frac{1}{2}\left(e^{-\frac{1}{2}st}\cos t\Big|_0^\infty - \int_0^\infty \cos t\,de^{-\frac{1}{2}st}\right)$

$= -\frac{1}{2}\left(-1 + \frac{1}{2}s\int_0^\infty e^{-\frac{1}{2}st}d\sin t\right)$

$= -\frac{1}{2}\left(-1+\frac{1}{2}s\left(\frac{1}{2}s\int_0^\infty e^{-\frac{1}{2}st}\sin t\,dt\right)\right)$

Let $I = \int_0^\infty e^{-\frac{1}{2}st}\, d\sin t$

Then $\frac{1}{2} I = -\frac{1}{2}\left(-1 + \frac{1}{2}s \cdot \left(\frac{1}{2}s \cdot I\right)\right)$

$-4I = -4 + s^2 \cdot I$

$I = \frac{4}{s^2+4}$ Therefore $\mathcal{L}[\sin 2t] = \frac{1}{2} I = \frac{2}{s^2+4}$

$$\mathcal{L}[x(t)] = \frac{2}{(s^2+4)(s^2+3s+2)} = \frac{2}{(s^2+4)(s+1)(s+2)}$$

$$\frac{As+B}{s^2+4} + \frac{C}{s+1} + \frac{D}{s+2} =$$

$$(As+B)(s^2+3s+2) = As^3 + 3As^2 + 2As + Bs^2 + 3Bs + 2B$$
$$= As^3 + (3A+B)s^2 + (2A+3B)s + 2B$$

$$C \cdot (s+2)(s^2+4) = C \cdot (s^3 + 2s^2 + 4s + 8)$$
$$= Cs^3 + 2Cs^2 + 4Cs + 8C$$

$$D \cdot (s+1)(s^2+4) = Ds^3 + Ds^2 + 4Ds + 4D$$

$$\begin{cases} 2B + 8C + 4D = 2 \\ 2A + 3B + 4C + 4D = 0 \\ 3A + B + 2C + D = 0 \\ A + C + D = 0 \end{cases} \quad \begin{cases} 2B + 8C + 4D = 2 \\ 3B + 2C + 2D = 0 \\ B - C - 2D = 0 \end{cases} \quad \begin{cases} 4B + 6C = 2 \\ 4B + C = 0 \end{cases}$$

$$C = \frac{2}{5}$$

$A = -\frac{3}{20}$ $\quad D = -\frac{1}{4}$ $\quad B = -\frac{1}{10}$

$$\mathcal{L}[x(t)] = -\frac{3}{20} \cdot \frac{s}{s^2+4} + -\frac{1}{20} \cdot \frac{2}{s^2+4} + \frac{2}{5} \cdot \frac{1}{s+1} - \frac{1}{4} \cdot \frac{1}{s+2}$$

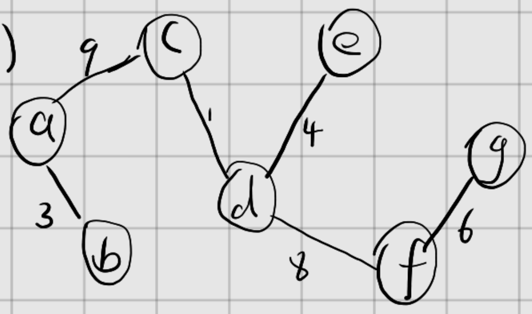
$$\underline{x(t) = -\frac{3}{20}\cos(2t) - \frac{1}{20}\sin(2t) + \frac{2}{5}e^{-t} - \frac{1}{4}e^{-2t}}$$

情2

(1)(a) $\lim_{n\to\infty} \frac{f(x)}{n^2} = \frac{1}{5}$ ✓

(b) $\lim_{n\to\infty} \frac{f(x)}{n^2} = \frac{3n\log n^2 + 4n^2}{n^2} = \lim_{n\to\infty}\left(4 + \frac{6n\log n}{n^2}\right) = 4$ ✓

(2).(3)



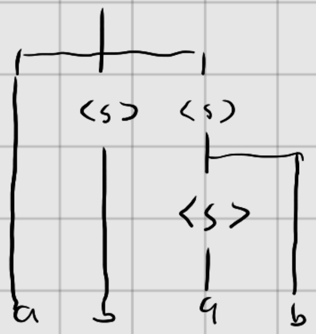
(Kruskal Algorithm)

(4) Sort the edges in ascending order. Select the smallest edge and we call the set of it "A". Then for the next edge, merge it into A if no cycle would appear. Keep doing this until all of the edges are checked.

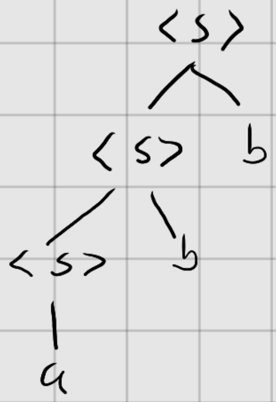
The sort algorithm is $O(E\log E)$. The checking and merging is $O(E)$. $O(E\log E + E) = O(E\log E)$. Therefore, the time complexity of the whole algorithm is $O(E\log E)$

it2

(1) (a)



(b) $abb \Leftarrow \langle s \rangle bb \Leftarrow \langle s \rangle b \Leftarrow \langle s \rangle$



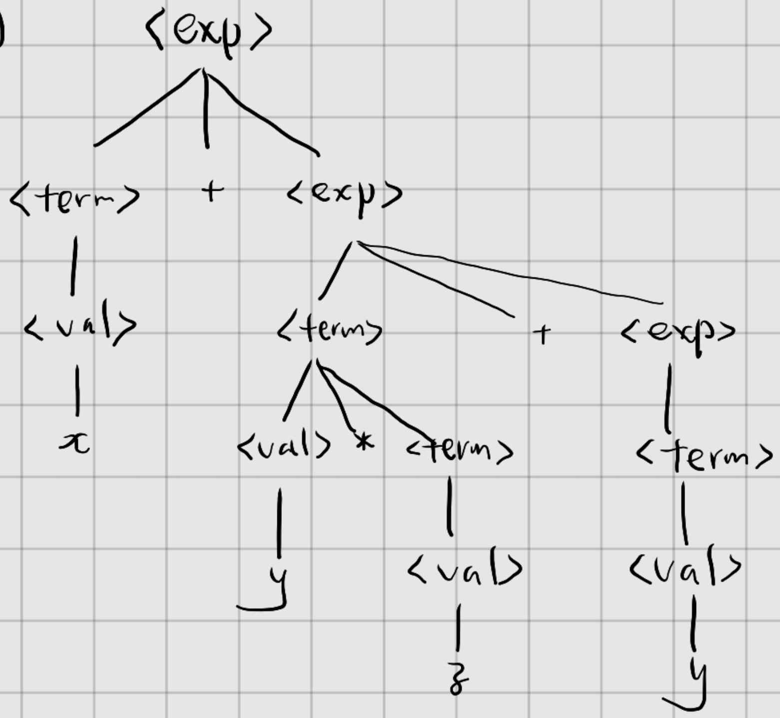
$abb \Leftarrow a\langle s \rangle\langle s \rangle \Leftarrow \langle s \rangle$

(2)(a) $\langle exp \rangle ::= \langle term \rangle \mid \langle term \rangle + \langle exp \rangle$

$\langle term \rangle ::= \langle val \rangle \mid \langle val \rangle * \langle term \rangle$

$\langle val \rangle ::= x \mid y \mid z$

(b)



(c) <exp> ::= <term> | <term> + <exp> | <term> - <exp>

<term> ::= <factor> | <factor> * <term> | <factor> / <term>

<factor> ::= <val> | (<exp>)

<val> ::= x | y | z