20.3

(1)(a) $|\lambda I - A| = \begin{vmatrix} \lambda & 0 & -1 \\ 0 & \lambda-1 & 0 \\ 1 & 0 & \lambda \end{vmatrix} = \lambda^3 - \lambda^2 + \lambda - 1 = (\lambda-1)(\lambda^2+1) = (\lambda-i)(\lambda+i)(\lambda-1)$

$$\lambda_1 = i \qquad \lambda_2 = -i \qquad \lambda_3 = 1$$

$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ i \end{bmatrix} \quad u_2 = \begin{bmatrix} 1 \\ 0 \\ -i \end{bmatrix} \quad u_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

(b) $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ i & -i & 0 \end{bmatrix} \quad P^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & -\frac{1}{2}i \\ \frac{1}{2} & 0 & \frac{1}{2}i \\ 0 & 1 & 0 \end{bmatrix} \qquad P^{-1}AP = \begin{bmatrix} i & & \\ & -i & \\ & & 1 \end{bmatrix}$

$$A^n = P(P^{-1}AP)^P P^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ i & -i & 0 \end{bmatrix} \begin{bmatrix} i^n & & \\ & (-i)^n & \\ & & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & -\frac{1}{2}i \\ \frac{1}{2} & 0 & \frac{1}{2}i \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \sim$$

(c) $\exp(A) = P\exp(P^{-1}AP)P^{-1} = P \cdot \begin{bmatrix} e^i & & \\ & e^{-i} & \\ & & 1 \end{bmatrix} P^{-1} = \sim$

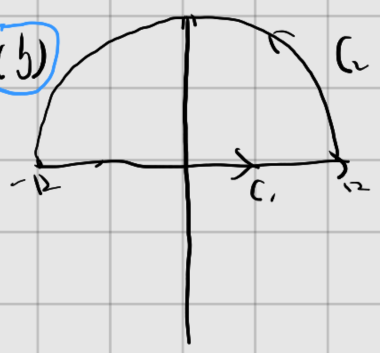
(2) $f(z) = \dfrac{z e^{iz}}{z^2 + a^2}$

(a) (Pole) $z = \pm a i$

$$\mathrm{Res}[f(z), ai] = \lim_{z \to ai} \frac{z e^{iz}}{z + ai} = \frac{ai\, e^{-a}}{2ai} = \frac{1}{2} e^{-a}$$

$$\mathrm{Res}[f(z), -ai] = \frac{1}{2} e^{a}$$

(b)



$C_2: z = Re^{it} \ (0 \le t \le \pi) \qquad dz = iRe^{it}$

$|e^{iz}| = |e^{i(x+iy)}| = |e^{-y} e^{ix}| = e^{-y}$

$\int_{C_2} f(z)\,dz = \int_0^{\pi} f(z)\, iRe^{it}\, dt = i\int_0^{\pi} f(z) z\, dt$

$\left| i\int_0^{\pi} f(z) z\, dt \right| \le \int_0^{\pi} |f(z) \cdot z|\, dt = \pi |f(z) \cdot z|$

$|f(z) z| = \dfrac{|z^2|\, e^{-y}}{|z^2 + a^2|} \to 0, \quad \therefore \left| \int_{C_2} f(z)\,dz \right| \le \pi |f(z) z| \to 0$

Therefore $\lim_{R\to\infty} \int_{C_2} f(z)\,dz = 0$

(c) $\lim_{R\to\infty} \int_{C_1+C_2} f(z)\,dz = 2\pi i \operatorname{Res}[f(z), ai] = e^{a}\pi i$

$\therefore \lim_{R\to\infty} \int_{C_1} f(z)\,dz = e^{a}\pi i - \lim_{R\to\infty} \int_{C_2} f(z)\,dz = e^{a}\pi i$

$$\int_{C_1} f(z)\,dz = \int_{-R}^{R} \frac{x e^{ix}}{x^2+a^2}\,dx$$

$$= \int_{-R}^{R} \frac{x(\cos x + i\sin x)}{x^2+a^2}\,dx$$

$$= \int_{-R}^{R} \frac{x\cos x}{x^2+a^2}\,dx + i\int_{-R}^{R} \frac{x\sin x}{x^2+a^2}\,dx$$

$$= i\int_{-R}^{R} \frac{x\sin x}{x^2+a^2}\,dx$$

$$\therefore \int_{-R}^{R} \frac{x\sin x}{x^2+a^2}\,dx = e^{a}\pi$$

18.8

(1) (a) 略

(b) $F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} dt$

$= \int_{-\infty}^{+\infty} f(t) \cos \omega t \, dt$

$= 2\int_{0}^{+\infty} f(t) \cos \omega t \, dt$

$= 2\int_{x_0-a}^{x_0+a} \cos \omega t \, dt$

$= \frac{2}{\omega} \sin \omega t \Big|_{x_0-a}^{x_0+a}$

$= \frac{2}{\omega} [\sin \omega (x_0+a) - \sin \omega (x_0-a)]$

$= \frac{1}{\omega} \cos x_0\omega \sin a\omega$

(c) $F(\omega) = \frac{1}{\omega} \cos x_0 \omega \sin \frac{x_0}{2} \omega$

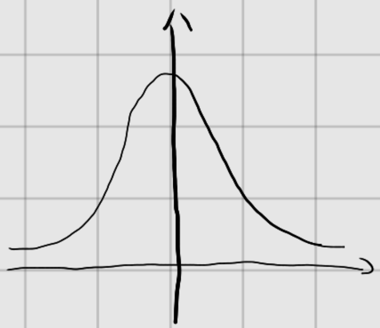
图略

(2) 略

19.3

(1) $f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{1}{2\sigma^2}x^2}$ $N(0,\sigma^2)$

(a)



(b) $F(w)=\frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{1}{2\sigma^2}w^2}$

标准高斯函数

注：$f=e^{-\pi t^2}$ 的正逆傅里叶变换都为其自身.