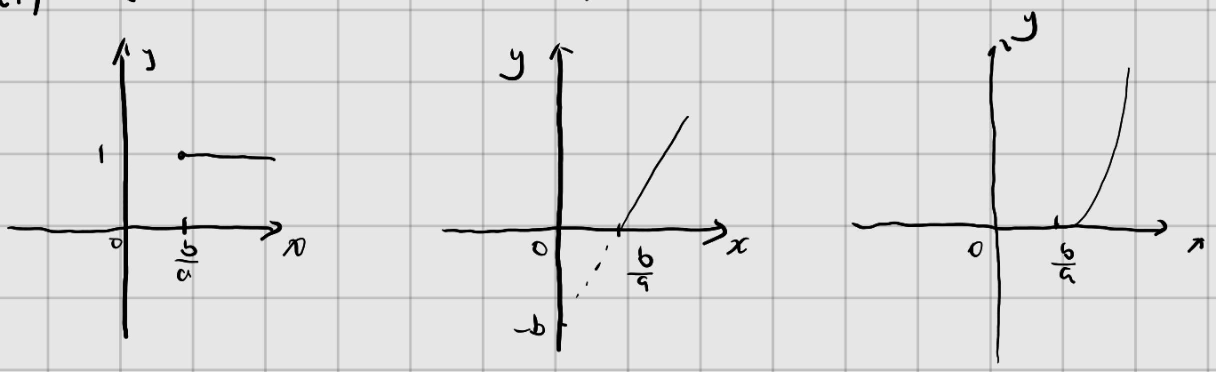
数

$$f(t)=\begin{cases}(at-b)^c & t\geqslant\frac{b}{a}\\ 0 & t<\frac{b}{a}\end{cases}$$

(1) $c=0$ $c=1$ $c=2$



(2) $c=0$:

$F(p)=\mathcal{L}[u(t-\frac{b}{a})]=e^{-\frac{b}{a}p}\cdot\frac{1}{p}$

$c=1$:

$F(p)=\mathcal{L}[(at-b)u(t-\frac{b}{a})]=\mathcal{L}[a(t-\frac{b}{a})u(t-\frac{b}{a})]$

$=a\mathcal{L}[(t-\frac{b}{a})u(t-\frac{b}{a})]$

$=ae^{-\frac{b}{a}p}\mathcal{L}[t]=ae^{-\frac{b}{a}p}\ \frac{1}{p^2}$

$c=2$:

$F(p)=\mathcal{L}[(at-b)^2u(t-\frac{b}{a})]=\mathcal{L}[a^2(t-\frac{b}{a})^2u(t-\frac{b}{a})]$

$=a^2e^{-\frac{b}{a}p}\mathcal{L}[t^2]=a^2e^{-\frac{b}{a}p}\ \frac{2}{p^3}$

(3) We know when $c=0$, $F(p)=\int_{\frac{b}{a}}^{+\infty}e^{-pt}dt=e^{-\frac{b}{a}p}\cdot\frac{1}{p}$

Suppose that when $c=k$, $F(p)=\int_{\frac{b}{a}}^{+\infty}(at-b)^ke^{-pt}dt=a^ke^{-\frac{b}{a}p}\cdot\frac{k!}{p^{k+1}}$

For $c=k+1$, $F(p)=\int_{\frac{b}{a}}^{+\infty}(at-b)^{k+1}e^{-pt}\,dt=-\frac{1}{p}\int_{\frac{b}{a}}^{+\infty}(at-b)^{k+1}\,de^{-pt}$

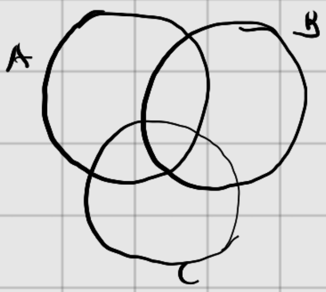
$=-\frac{1}{p}\left((at-b)^{k+1}e^{-pt}\Big|_{\frac{b}{a}}^{+\infty}-\int_{\frac{b}{a}}^{+\infty}e^{-pt}\,d(at-b)^{k+1}\right.$

$=\frac{k+1}{p}\cdot a\int_{\frac{b}{a}}^{+\infty}(at-b)^k\,de^{-pt}$

$=\frac{k+1}{p}\cdot a^{k+1}e^{-\frac{b}{a}p}\cdot\frac{k!}{p^{k+1}}=a^{k+1}e^{-\frac{b}{a}p}\cdot\frac{(k+1)!}{p^{k+2}}$

So we have proved that $\forall c \in \mathbb{N}$. $F(p) = a^c e^{-\frac{b}{a}p} \cdot \frac{c!}{p^{c+1}}$

情2



(1) Right.

For $A \times (B \cup C) = \{(a,b) \mid \forall \alpha \in A, \forall \beta \in B \cup C\}$

$\forall (\alpha, \beta) \in A \times (B \cup C)$,

$\beta \in B$ or $\beta \in C$

$\therefore (\alpha, \beta) \in (A \times B)$ or $(\alpha, \beta) \in (A \times C)$

$\therefore (\alpha, \beta) \in (A \times B) \cup (A \times C)$

$\therefore A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$

$\forall (\alpha, \beta) \in (A \times B) \cup (A \times C)$ i.e. $(\alpha, \beta) \in A \times B$ or $(\alpha, \beta) \in A \times C$

i.e. $\alpha \in A$, $\beta \in B$ or. $\beta \in C$ which is $\beta \in B \cup C$

$\therefore (\alpha, \beta) \in A \times (B \cup C)$

$\therefore (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$

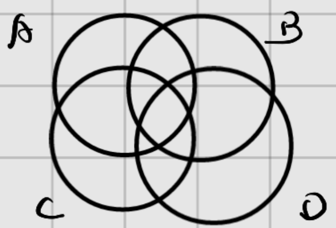
$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$

(2) Right. $(x, y) \in (A \times B) \cap (C \times D) \iff x \in A \cap C$ and $y \in B \cap D$

$\iff (x, y) \in (A \cap C) \times (B \cap D)$

$\therefore (A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

(3) Wrong.



For example. $\alpha \in A$ but $\alpha \notin C$, $\beta \in B$ but $\beta \notin D$

So $(\alpha, \beta) \in (A \cup C) \times (B \cup D)$

However $(\alpha, \beta) \notin (A \times B) \cup (C \times D)$

计2

(1) $f(0) = 0$

$f(1) = f(0) \times 2 + 1 = 1$

$f(2) = f(1) \times 2 + 2 = 4$

$f(3) = f(2) \times 2 + 3 = 11$

(2) 8. 14. pop r_1 ret

Line 6 to line 17 actually defines a recursive function. The function keep push its parameter and return address to the stack until a base case is reached. Then recursively compute the result from the base case and finally output.