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$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6a^3 & 3a^2 & 2a \end{bmatrix}$$

$$|A| = -6a^3$$

$$A^{-1} = \begin{bmatrix} \frac{5}{6a} & \frac{1}{3a^2} & -\frac{1}{6a} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$(b) |\lambda I - A| = (\lambda - a)(\lambda + 2a)(\lambda - 3a)$$

$$\lambda_1 = -2a \quad \lambda_2 = a \quad \lambda_3 = 3a$$

$$\Downarrow \quad \Downarrow \quad \Downarrow$$

$$u_1 = \begin{bmatrix} 1 \\ -2a \\ 4a^2 \end{bmatrix} \quad u_2 = \begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix} \quad u_3 = \begin{bmatrix} 1 \\ 3a \\ 9a^2 \end{bmatrix}$$

$$(c) \lim_{n \rightarrow \infty} \begin{bmatrix} x_n \\ x_{n+1} \\ x_{n+2} \end{bmatrix} = \lim_{n \rightarrow \infty} A^n \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = 0$$

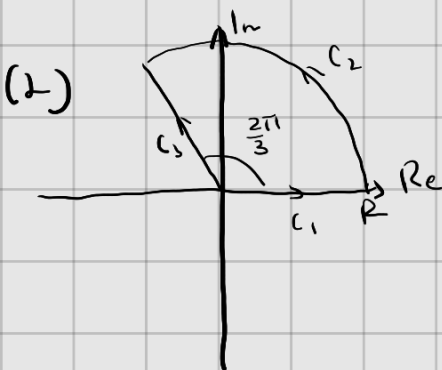
$$\therefore \lim_{n \rightarrow \infty} A^n = 0$$

$$P^{-1}AP = \text{diag} \{ -2a, a, 3a \}$$

$$\therefore \lim_{n \rightarrow \infty} (P^{-1}AP)^n = \lim_{n \rightarrow \infty} \begin{bmatrix} (-2a)^n & & \\ & a^n & \\ & & (3a)^n \end{bmatrix} = 0$$

$$\therefore | -2a | < 1, |a| < 1, |3a| < 1$$

$$\therefore a \in (0, \frac{1}{3})$$



$$(a) \text{ Let } z^3 + 1 = 0 \Rightarrow z = e^{\pi i}, e^{\frac{\pi}{3}i}, e^{-\frac{\pi}{3}i}$$

The region enclosed by C_1, C_2, C_3 contains only the pole $z_0 = e^{\frac{\pi}{3}i}$, Let K be $C_1 + C_2 - C_3$,

$$I_1 + I_2 - I_3 = \oint_K \frac{z}{z^3 + 1} dz = \oint_K \frac{z}{(z - e^{\pi i})(z - e^{\frac{\pi}{3}i})(z - e^{-\frac{\pi}{3}i})} dz$$

$$= 2\pi i \frac{e^{\frac{\pi}{3}i}}{(e^{\frac{\pi}{3}i} - e^{\pi i})(e^{\frac{\pi}{3}i} - e^{-\frac{\pi}{3}i})}$$

$$(b) \quad C_1: z=t \quad (0 \leq t \leq R), \quad C_3: z=t \cdot e^{\frac{2\pi i}{3}} \quad (0 \leq t \leq R)$$

$$\text{Let } u = t \cdot e^{\frac{2\pi i}{3}} \quad du = e^{\frac{2\pi i}{3}} dt$$

$$\begin{aligned} \int_{C_3} \frac{z}{z^3+1} dz &= \int_0^R \frac{e^{\frac{2\pi i}{3}}}{u^3+1} du = \int_0^R \frac{e^{\frac{2\pi i}{3}} t}{t^3+1} \cdot e^{\frac{2\pi i}{3}} dt \\ &= e^{\frac{4\pi i}{3}} \int_0^R \frac{t}{t^3+1} dt = e^{\frac{4\pi i}{3}} \int_{C_1} \frac{z}{z^3+1} dz \end{aligned}$$

$$I_3 = e^{\frac{4\pi i}{3}} I_1$$

$$(c) \quad C_2: z = R e^{i\theta} \quad (0 \leq \theta \leq \frac{2\pi}{3}) \quad dz = i R e^{i\theta} d\theta$$

$$\begin{aligned} I_2 &= \int_{C_2} \frac{z}{z^3+1} dz = \int_0^{\frac{2\pi}{3}} \frac{R e^{i\theta}}{R^3 e^{i3\theta} + 1} \cdot i R e^{i\theta} d\theta \\ &= \int_0^{\frac{2\pi}{3}} \frac{i R^2 e^{i2\theta}}{R^3 e^{i3\theta} + 1} d\theta \end{aligned}$$

$$|I_2| \leq \int_0^{\frac{2\pi}{3}} \frac{R^2}{R^3+1} d\theta \quad \lim_{R \rightarrow \infty} |I_2| = 0$$

$$\therefore \lim_{R \rightarrow \infty} I_2 = 0$$

$$(d) \quad I = \int_0^\infty f(x) dx = \lim_{R \rightarrow \infty} I_1$$

From (a)(b)(c) we know:

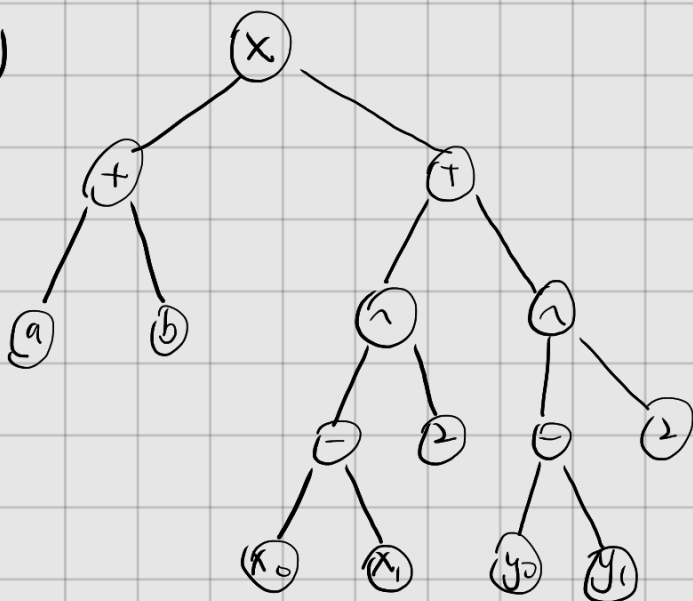
$$I_1 + I_2 - I_3 = I_1 - e^{\frac{4\pi i}{3}} I_1 + I_2 = \frac{2\pi i}{(e^{\frac{4\pi i}{3}} - e^{\pi i})(e^{\frac{\pi i}{3}} - e^{\frac{2\pi i}{3}})}$$

$$\lim_{R \rightarrow \infty} (1 - e^{\frac{4\pi i}{3}}) I_1 = \frac{2\pi i}{(e^{\frac{4\pi i}{3}} - e^{\pi i})(e^{\frac{\pi i}{3}} - e^{\frac{2\pi i}{3}})}$$

$$\therefore I = \frac{2\pi i}{(1 - e^{\frac{4\pi i}{3}})(e^{\frac{\pi i}{3}} - e^{\pi i})(e^{\frac{\pi i}{3}} - e^{\frac{2\pi i}{3}})}$$

情 2.

(1)



(2) void Traverse (T) {

if (T → left = Null or T → right = Null) {

output (T.value);

return;

}

Traverse (T → left);

Traverse (T → right);

output (T.value);

return;

}

(3) $ab + x_0 x_1 - 2 \wedge y_0 y_1 - 2 \wedge + x$

(4) If a number or a variable is encountered, push it into the stack. If an operator is encountered, pop the stack twice. then push an arbitrary number or variable into the stack. However if there is no element in stack to pop, stop the scanning process. The expression is right only if there only one element left in the stack after the scanning process ends.

(5) There will be nothing left in the stack after scanning. Therefore it is wrong.

$$ab + a$$

There will be more than one element left in the stack after scanning. Therefore there's error.

if 2

(1) Call-By-Value:

$$\begin{aligned} f_1(2, 4) &\rightarrow IFEQ(2, 4, 1, 4 \times f_1(2+1, 4)) \\ &\rightarrow 4 \times f_1(2+1, 4) \\ &\rightarrow 4 \times f_1(3, 4) \\ &\rightarrow 4 \times IFEQ(3, 4, 1, 4 \times f_1(3+1, 4)) \\ &\rightarrow 4 \times 4 \times f_1(3+1, 4) \\ &\rightarrow 4 \times 4 \times f_1(4, 4) \\ &\rightarrow 4 \times 4 \times IFEQ(4, 4, 1, 4 \times f_1(4+1, 4)) \\ &\rightarrow 4 \times 4 \times 1 \\ &\rightarrow 16 \end{aligned}$$

Call-By-Name:

$$\begin{aligned} f_1(2, 4) &\rightarrow IFEQ(2, 4, 1, 4 \times f_1(2+1, 4)) \rightarrow 4 \times f_1(2+1, 4) \\ &\rightarrow 4 \times IFEQ(2+1, 4, 1, 4 \times f_1((2+1)+1, 4)) \\ &\rightarrow 4 \times IFEQ(3, 4, 1, 4 \times f_1((2+1)+1, 4)) \rightarrow 4 \times 4 \times f_1((2+1)+1, 4) \\ &\rightarrow 4 \times 4 \times IFEQ((2+1)+1, 4, 1, 4 \times f_1(((2+1)+1)+1, 4)) \\ &\rightarrow 4 \times 4 \times IFEQ(4, 4, 1, 4 \times f_1(((2+1)+1)+1, 4)) \\ &\rightarrow 4 \times 4 \times 1 \rightarrow 16 \end{aligned}$$

| | | |
|-------|-------|---|
| (2) | CBV | CBN |
| Add | $n-m$ | $1+2+\dots+n-m = \frac{1}{2}(n-m)(1+n-m)$ |
| Multi | $n-m$ | $n-m$ |

(3) dummy (recur(x))