数

$$A=\frac{1}{3}\begin{bmatrix}1 & 2+2i\\ a+bi & 1\end{bmatrix}$$

注 { Adjoint Matrix 尤指 Hermite伴随，即共轭转置 A^* or A^H
中文"伴随矩阵"指 Adjugate Matrix $\mathrm{adj}(A)$

(1)(a) $A=A^H \Rightarrow 2+2i=a-bi \Rightarrow a=2\quad b=-2$

(b) $$A=\frac{1}{3}\begin{bmatrix}1 & 2+2i\\ 2-2i & 1\end{bmatrix}$$

$$|\lambda I-A|=\frac{1}{3}\begin{vmatrix}3\lambda-1 & -2-2i\\ -2+2i & 3\lambda-1\end{vmatrix}=\frac{1}{3}[(3\lambda-1)^2-(4+4)]=3\lambda^2-2\lambda-\frac{7}{3}$$

$$\lambda=\frac{1\pm2\sqrt{2}}{3}\in\mathbb{R}$$

(c) For $\lambda_1=\frac{1+2\sqrt{2}}{3}$

$(\lambda I-A)x=0$

$$\begin{bmatrix}2\sqrt{2} & -2-2i\\ -2+2i & 2\sqrt{2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=0 \Rightarrow x_1=\frac{1}{\sqrt{2}}(1+i)x_2 \Rightarrow u_1=\begin{bmatrix}\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\\ 1\end{bmatrix}$$

For $\lambda_2=\frac{1-2\sqrt{2}}{3}$

$(\lambda I-A)x=0$

$$\begin{bmatrix}-2\sqrt{2} & -2-2i\\ -2+2i & -2\sqrt{2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=0 \Rightarrow x_1=-\frac{1}{\sqrt{2}}(1+i)x_2 \Rightarrow u_2=\begin{bmatrix}-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\\ 1\end{bmatrix}$$

$u_1\cdot u_2=-1+1=0 \quad \therefore u_1\perp u_2$

(2)(a) (Unitary Matrix: $AA^H=I$) (酉矩阵)

$$AA^H=\frac{1}{9}\begin{bmatrix}1 & 2+2i\\ a+bi & 1\end{bmatrix}\begin{bmatrix}1 & a-bi\\ 2-2i & 1\end{bmatrix}=\frac{1}{9}\begin{bmatrix}1+a^2+b^2 & 2+2i+a-bi\\ 2-2i+a+bi & 8+1\end{bmatrix}=I$$

$$\Rightarrow\begin{cases}a=-2\\ b=2\end{cases}$$

(b) $$A=\frac{1}{3}\begin{bmatrix}1 & 2+2i\\ -2+2i & 1\end{bmatrix}$$ $(2i+2)(2i-2)$

$$|\lambda I-A|=\frac{1}{3}\begin{vmatrix}3\lambda-1 & -2-2i\\ 2-2i & 3\lambda-1\end{vmatrix}=\frac{1}{3}[(3\lambda-1)^2+8]=3\lambda^2-2\lambda+3$$

$$\lambda=\frac{2\pm\sqrt{4-36}}{2\cdot3}=\frac{1\pm2\sqrt{2}i}{3}$$

$$|\lambda|=\sqrt{\frac{1}{9}+\frac{8}{9}}=1$$

(3) (Orthogonal 正交: $A^T=A^{-1}$ / $AA^T=I$)

$$(B+iC)(B+iC)^H = (B+iC)(B^T-iC^T) = BB^T+CC^T+(CB^T-BC^T)i = I$$

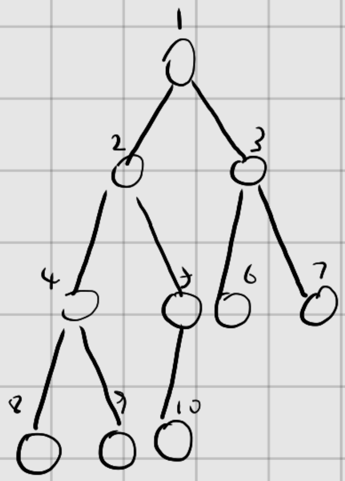
$\therefore$ $BB^T+CC^T=I$ $\quad CB^T-BC^T=0$

$$\begin{bmatrix} B & -C \\ C & B \end{bmatrix}\begin{bmatrix} B & -C \\ C & B \end{bmatrix}^T = \begin{bmatrix} B & -C \\ C & B \end{bmatrix}\begin{bmatrix} B^T & C^T \\ -C^T & B^T \end{bmatrix} = \begin{bmatrix} BB^T+CC^T & BC^T-CB^T \\ CB^T-BC^T & CC^T+BB^T \end{bmatrix} = I$$

Therefore $\begin{bmatrix} B & -C \\ C & B \end{bmatrix}$ is an orthogonal matrix

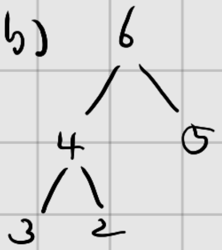
情2

(1) (a)



$P(V,W)$: $P(k, k+1)$

(b)



$\{6,4,5,3,2\}$

(2) For the worst situation, the search range is halved after each comparison, until only one element last. Then find the target or determine that is does not exist. This takes $O(\log N)$.

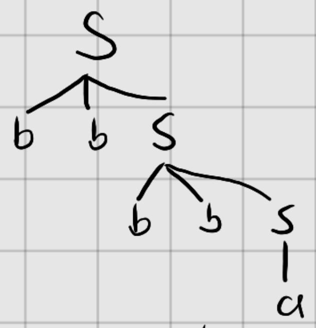
(3) (a)

```
Findpair(A, N, k):
    Heapsort(A, N)
    for i:=1 to N do
        j:= BinarySearch(A, N, k-A[i])
        if j != None then Return (i,j) endif
    endfor
    Return None
```

(b) $O(N\log N)$

计2

(1) (a) False. Only one tree

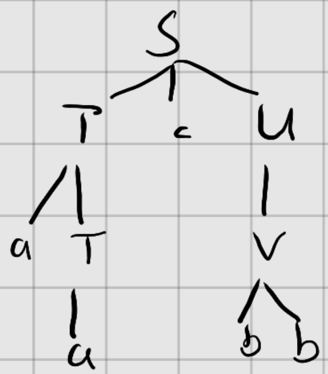


(b) $L(G_1) = \{ b^{2n} a^m \mid n = 0,1,2\cdots \quad m = 1,2\cdots \}$

$L(G_2) = \{ b^{2n} a^m \mid n = 0,1,2\cdots \quad m = 1,2\cdots \}$

True

(c) False. For example $aacbb \notin \{ a^i c b^{2i} \mid i \text{ is positive} \}$



Which is clearly from G_3.

(2) Postfix 后序 Infix 中序 Prefix 先序

(a) $\vee \underline{\wedge 0 1} \underline{\wedge 1 1}$

Postfix: $0 1 \wedge 1 1 \wedge \vee$

(b) $1 \xrightarrow{1} 11 \xrightarrow{0} 110 \xrightarrow{\vee} 11 \xrightarrow{\wedge} 1 \xrightarrow{0} 10 \xrightarrow{\vee} 1$

(c) $S ::= \vee SS \mid \wedge SS \mid 0 \mid 1$

There must be two numbers or expressions after each operator "$\wedge$" or "$\vee$".