数.

(1)
(a) $|\lambda I - A| = \begin{vmatrix} \lambda+1 & -1 & 0 \\ 1 & \lambda-2 & 1 \\ 0 & -1 & \lambda+1 \end{vmatrix} = \lambda^3 - 3\lambda - 2 + \lambda+1 + \lambda+1 = \lambda^3 - \lambda = \lambda(\lambda-1)(\lambda+1)$

$\lambda_1 = 0 \quad \lambda_2 = 1 \quad \lambda_3 = -1$

For $\lambda_1 = 0$, $(\lambda I - A)x = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Rightarrow x_1 = x_2 = x_3 \qquad u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

For $\lambda_2 = 1$, $(\lambda I - A)x = \begin{bmatrix} 2 & -1 & 0 \\ 1 & -1 & 1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Rightarrow \begin{cases} x_2 = 2x_3 \\ x_1 = x_3 \end{cases} \qquad u_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

For $\lambda_3 = -1$ $(\lambda I - A)x = \begin{bmatrix} 0 & -1 & 0 \\ 1 & -3 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Rightarrow \begin{cases} x_1 = -x_3 \\ x_2 = 0 \end{cases} \qquad u_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad P^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$

$P^{-1}AP = \mathrm{diag}\{\lambda_1, \lambda_2, \lambda_3\} = \begin{bmatrix} 0 & & \\ & 1 & \\ & & -1 \end{bmatrix}$

$(P^{-1}AP)^n = \begin{bmatrix} 0 & & \\ & 1^n & \\ & & (-1)^n \end{bmatrix}$

(c)

$A^n = P \cdot P^{-1} A^n P \cdot P^{-1} = P(P^{-1}AP)^n P^{-1}$

$= \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & & \\ & 1^n & \\ & & (-1)^n \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$

$= \sim$

(2) Fourier Series

(a) $a_0 = \frac{1}{\pi}\int_0^{2\pi} x\,dx = \frac{1}{2\pi} x^2 \Big|_0^{2\pi} = 2\pi$

(b) $a_n = \frac{1}{\pi}\int_0^{2\pi} x \cos nx\,dx = 0$

$b_n = \frac{1}{\pi}\int_0^{2\pi} x \sin nx\,dx$

$= -\frac{1}{\pi n}\int_0^{2\pi} x \; d\cos nx$

$= -\frac{1}{\pi n}\left(x\cos nx\Big|_0^{2\pi} - \int_0^{2\pi} \cos nx\,dx\right)$

$= -\frac{1}{\pi n} \cdot 2\pi = -\frac{2}{n}$

(c) $f(x) = x = \pi + \sum_{n=1}^{\infty} -\frac{2}{n}\sin nx$

$\therefore \sum_{n=1}^{\infty} \frac{\sin nx}{n} = \frac{\pi - x}{2}$

情2

(1) Number of elements before level k:

$$2^0 + 2^1 + 2^2 + \cdots + 2^{k-1} = 2^k - 1$$

Number of elements at level k is no greater than 2^k

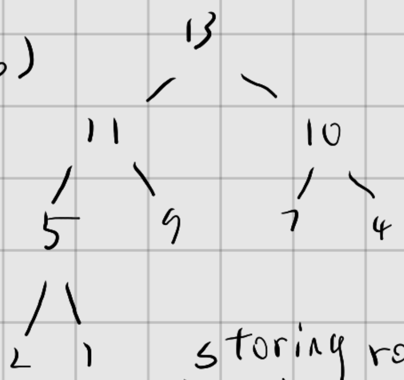
$$\therefore \begin{cases} n \geq 2^k - 1 + 1 \\ n \leq 2^k - 1 + 2^k \end{cases} \Rightarrow 2^k \leq n \leq 2^{k+1} - 1$$

(2) $O(\log n)$

Let $2^k \approx n \Rightarrow k = \log_2 n$

(3) (a) Replace root with the last element of the heap. Then keep comparing it with its children. If it's less than the greater child, swap them. Keep doing this until the tree becomes a heap again.

(b)



storing root into the head of an array and

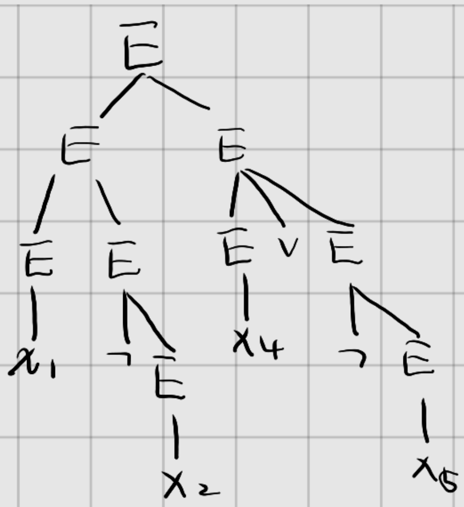
(4) (a) Keep replacing root with the last element, and using the method in (3) to make the heap still be heap, until all the elements are removed. And then all elements are storing in ascent order in the array.

(b) $O(n \log n)$

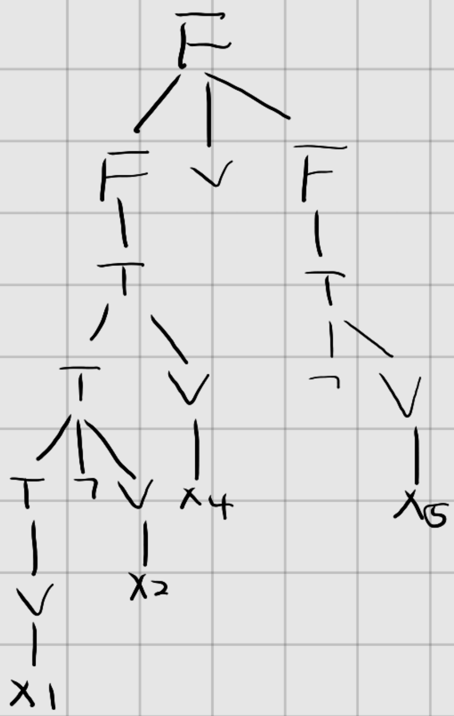
Replace n times. method in (3)

计2

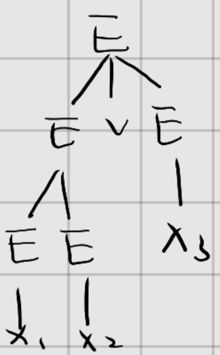
(1) μ: $\underbrace{x_1}_{E}\ \underbrace{\neg x_2}_{E}\ \underbrace{x_4}_{E} \vee \underbrace{\neg x_5}_{E}$



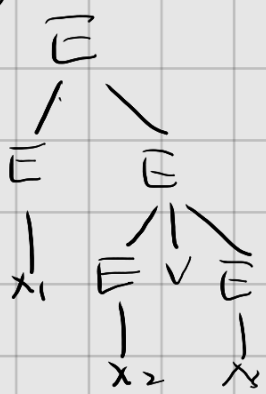
(2) $x_1 \neg x_2 x_4 \vee \neg x_5$



(3) $x_1 x_2 \vee x_3 \in L(G_1)$



or



ambiguous.

(4)(a)

$$F ::= T \mid F \vee F$$
$$T ::= UUU$$
$$U ::= V \mid \neg V$$
$$V ::= \sim$$

(b) Obviously for any sentence there is only one left most derivation.

$T ::= UUU$ ensure that each product has exactly 3 literals.