考数

(1)

$$|\lambda I-A|=\begin{vmatrix}\lambda-3 & -1 & -1\\ 2 & \lambda-4 & 1\\ 2 & 1 & \lambda\end{vmatrix}=\lambda^3-7\lambda^2+12\lambda-2-2+2(\lambda-4)-(\lambda-3)+2\lambda$$

$$=\lambda^3-7\lambda^2+15\lambda-9$$

$$=(\lambda-1)(\lambda-3)^2$$

$\lambda_1=1 \quad \lambda_2=3$ (二重)

For $\lambda_1=1$:

$(\lambda I-A)x=0$

$$\begin{bmatrix}-2 & -1 & -1\\ 2 & -3 & 1\\ 2 & 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=0 \Rightarrow \begin{cases}2x_1+x_2+x_3=0\\ x_2=0\end{cases} \Rightarrow \begin{cases}x_3=-2x_1\\ x_2=0\end{cases}$$

Let $x_1=1$ then $u_1=(1,0,-2)^T$

For $\lambda_2=3$:

$(\lambda I-A)x=0$

$$\begin{bmatrix}0 & -1 & -1\\ 2 & -1 & 1\\ 2 & 1 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=0 \quad \begin{cases}x_2+x_3=0\\ 2x_1+x_2+3x_3=0\end{cases} \Rightarrow \begin{cases}x_2=-x_3\\ x_1=-x_3\end{cases}$$

Let $x_3=1$ then $u_2=(-1,-1,1)^T$

(2) $B^0=I \quad B^1=\begin{bmatrix}a & & \\ & b & 1\\ & & b\end{bmatrix}$ Suppose that $B^k=\begin{bmatrix}a^k & & \\ & b^k & kb^{k-1}\\ & & b^k\end{bmatrix}$

$$B^{k+1}=B^k\cdot B=\cdots=\begin{bmatrix}a^{k+1} & & \\ & b^{k+1} & (k+1)b^k\\ & & b^{k+1}\end{bmatrix}$$

$$\therefore B^n=\begin{bmatrix}a^n & & \\ & b^n & nb^{n-1}\\ & & b^n\end{bmatrix}$$

Jordan

(3) For $\lambda_1=1$, $u_1=(1,0,-2)^T$

For $\lambda_2=2$, whose algebraic multiplicity is 2

(代数重数)

$$(\lambda I - A)^2 x = 0$$

$$\begin{bmatrix} 0 & -1 & -1 \\ 2 & -1 & 1 \\ & 2 & -1\,3 \end{bmatrix}^2 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -4 & 0 & -4 \\ 0 & 0 & 2 \\ 8 & 0 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Rightarrow x_1 = -x_3, \ \forall x_2$$

Generalized eigenvectors: $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Therefore $P = (u_1, v_1, v_2) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$

$$P^{-1} = \begin{bmatrix} -1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & & \\ & 3 & 1 \\ & & 3 \end{bmatrix} = P^{-1}AP$$

$$B^n = (P^{-1} A P)^n = P^{-1} A^n P = \begin{bmatrix} 1 & & \\ & 3^n & n \cdot 3^{n-1} \\ & & 3^n \end{bmatrix}$$

$$A^n = P B^n P^{-1} = \begin{bmatrix} -1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3^n & n \cdot 3^{n-1} \\ 0 & 0 & 3^n \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$$

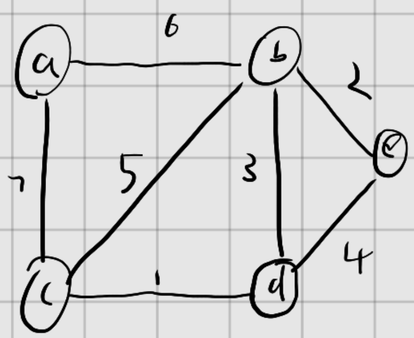
$$= \begin{bmatrix} -1 & 0 & -3^n \\ 2 & 0 & 3^n \\ 0 & 3^n & n \cdot 3^{n-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 + 3^n & 0 \\ 2 - 2 \cdot 3^n & 2 - 3^n & 0 \\ -2n \cdot 3^{n-1} & -n 3^{n-1} & 3^n \end{bmatrix}$$

情2

(1) (a)

$$
\begin{array}{c|ccccc}
 & a & b & c & d & e \\
\hline
a & 0 & 6 & 7 & 0 & 0 \\
b & 6 & 0 & 5 & 3 & 2 \\
c & 7 & 5 & 0 & 1 & 0 \\
d & 0 & 3 & 1 & 0 & 4 \\
e & 0 & 2 & 0 & 4 & 0
\end{array}
$$



a → b 6 → c 7 X

b → a 6 → c 5 → d 3 → e 2 X

c → a 7 → b 5 → d 1 X

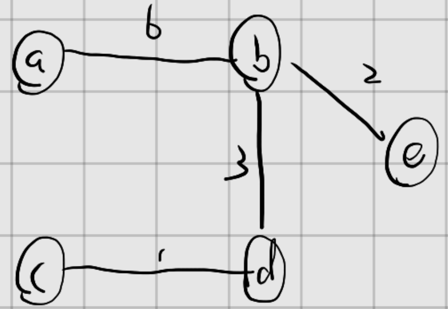
d → c 1 → b 3 → e 4 X

e → b 2 → d 4 X

(b) adjacent matrix : $O(n^2)$

adjacent list : $O(m+n)$

(c)



(Kruscal / Prim)

(2) 1. Initialize pointer $i=0, j=m-1$, start from $(0, m-1)$

2. Repeat
- If $x = M[i][j]$, output "YES"
- If $x < M[i][j]$, $j--$
- If $x > M[i][j]$, $i++$

3. If pointer reaches the boundary ($j<0$, $i>n-1$)
break the loop and output "NO"

itL

(1)(a) $S ::= aS \mid ab \mid ba \mid ccc$

False. $S \to aS \to a \cdots aS \to a^nS \to \begin{cases} a^n ab \\ a^n ba \\ a^n ccc \end{cases}$ $n = 0, 1, \cdots$

For every string in L_1, there only one syntax tree exists.

(b) $L_2 = a^n b a^m$ $m, n = 1, 2, \cdots$

False. $S ::= aS \mid aU$
$U ::= ba \mid Ua$

Only need 2 nonterminal symbols.

(c) False
$L_1 \cap L_2 = \{aba\}$

(2)
(a) $3 \xrightarrow{1} 31 \xrightarrow{2} 312 \xrightarrow{+} 33 \xrightarrow{*} 9 \xrightarrow{9} 99 \xrightarrow{4} 994 \xrightarrow{\nabla} 4 \xrightarrow{5} 45 \xrightarrow{+} 9$

(b) (i) $\underbrace{1\,1+1+\cdots 1+}_{n}$ depth: 2

(ii) $\underbrace{11 \cdots 1}_{2k+1}\underbrace{\nabla \cdots \nabla}_{k}$ depth: $2k+1 = \frac{2n+1}{3}$

$2k+1+k = n$

$k = \frac{n-1}{3}$