数

(1)(a) $|\lambda I - A| = \begin{vmatrix} \lambda-1 & -2 & 0 \\ 1 & \lambda+2 & -1 \\ 0 & 0 & \lambda-1 \end{vmatrix} = \lambda(\lambda-1)(\lambda+1)$

$\lambda_1 = 0 \quad \lambda_2 = 1 \quad \lambda_3 = -1$

For $\lambda_1 = 0 \quad (\lambda I - A)x = 0$

$\Rightarrow \begin{bmatrix} -2 & -2 & 0 \\ 1 & 1 & -1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Rightarrow \begin{cases} x_1 = -x_2 \\ x_3 = 0 \end{cases} \xRightarrow{\text{let } x_2 = 1} u_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

For $\lambda_2 = 1 \quad \cdots \quad u_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ For $\lambda_3 = 0 \quad \cdots \quad u_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(b) $P = (u_1, u_2, u_3) = \begin{bmatrix} -1 & -2 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$P^{-1}AP = \begin{bmatrix} 0 & & \\ & 1 & \\ & & -1 \end{bmatrix}$

$A^3 + aA^2 + bA + cE = 0$

$(P^{-1}AP)^3 + a(P^{-1}AP)^2 + b(P^{-1}AP) + cE = 0$

$\begin{bmatrix} 0 & & \\ & 1 & \\ & & -1 \end{bmatrix} + \begin{bmatrix} 0 & & \\ & a & \\ & & a \end{bmatrix} + \begin{bmatrix} 0 & & \\ & b & \\ & & -b \end{bmatrix} + \begin{bmatrix} c & & \\ & c & \\ & & c \end{bmatrix} = 0$

$\begin{cases} c = 0 \\ 1 + a + b + c = 0 \\ -1 + a - b + c = 0 \end{cases} \Rightarrow \begin{cases} a = 0 \\ b = -1 \\ c = 0 \end{cases}$

(c) $P^{-1}(A^4 + A^3 - A^2 + 4A - 5E)P$

$= (P^{-1}AP)^4 + (P^{-1}AP)^3 - (P^{-1}AP)^2 + 4P^{-1}AP - 5E$

$= \begin{bmatrix} -8 & & \\ & -5 & \\ & & 0 \end{bmatrix}$

$A^4 + A^3 - A^2 + 4A - 5E = P \begin{bmatrix} -8 & & \\ & -5 & \\ & & 0 \end{bmatrix} P^{-1} = \begin{bmatrix} -1 & -2 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -8 & & \\ & -5 & \\ & & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ -1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 10 & 0 \\ -8 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ -1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 6 & 2 \\ -3 & -11 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

(2) $F(s) = \mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$

(a) $\mathcal{L}[f(at+b)] = \int_0^\infty e^{-st} f(at+b)\,dt$

Let $u = at+b$, $u \in [b, +\infty]$, $t = \frac{u-b}{a}$

$\therefore \mathcal{L}[f(at+b)] = \frac{1}{a}\int_b^\infty e^{-s\cdot\frac{u-b}{a}} f(u)\,du$

$$= \frac{e^{\frac{bs}{a}}}{a} \int_b^\infty e^{-\frac{s}{a}u} f(u)\,du$$

$$= \frac{e^{\frac{bs}{a}}}{a} \left[F\left(\frac{s}{a}\right) - \int_0^b e^{-\frac{st}{a}} f(t)\,dt\right]$$

(b) $\mathcal{L}[f(t+2)] - 3\mathcal{L}[f(t+1)] + 2\mathcal{L}[f(t)] = \mathcal{L}[t] = \frac{1}{s^2}$ $(t \geq 0)$

$$e^{2s}\Big[F(s) - \underbrace{\int_0^2 e^{-st} f(t)\,dt}_{0}\Big] - 3e^s\Big[F(s) - \underbrace{\int_0^1 e^{-st} f(t)\,dt}_{0}\Big] + 2F(s) = \frac{1}{s^2}$$

$$F(s) = \frac{1}{s^2} \cdot \frac{1}{e^{2s} - 3e^s + 2}$$

情2

(归并排序)

(1) Comapare the first element in A_1 with the first element in A_2. Select the smaller one and append it to B. Keep doing this until all elements is appended to B.

(2) Recursively split array C into to halves until only one element lefts. Using the procedure in (1), merge two arrays into one in assending order.

The split takes $O(\log N)$, and the merging in every recursion takes $O(N)$. Therefore the whole algorithm takes $O(N\log N)$

(3) (基数排序)

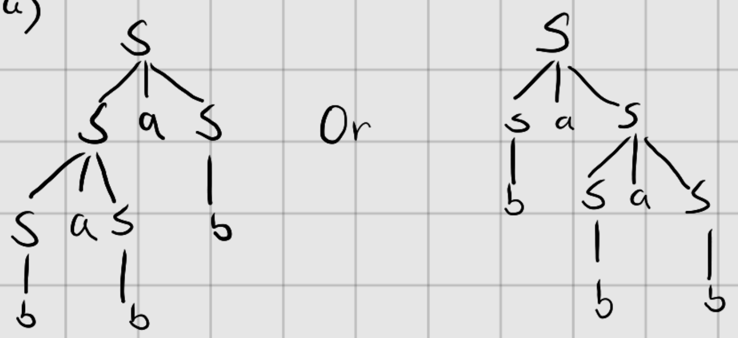
In these k numbers, for any given digit position, there is only one element whose value in the position is non-zero.

For example: 5, 40, 300, 6000, 10000 ...

it2.

(1) True. babab:

(a)



(b) Ture. babab can not be generated from G_2

(c) False.

$S ::= aaaU \mid baU \mid bbaU$

$U ::= b \mid bU$

(2) (Postfix)

(a)

$3 \xrightarrow{5} 3\ 5 \xrightarrow{7} 3\ 5\ 7 \xrightarrow{+} 3\ 2 \xrightarrow{*} 6 \xrightarrow{2} 6\ 2 \xrightarrow{+} 8$

(b)

$S ::= SS+ \mid SS* \mid A$

$A ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$