数

(1) $A \overset{def}{=} [a, b, c, d]$

$|A| = \begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 \end{vmatrix} \neq 0$ So, a, b, c, d linearly independent

$\therefore \{a, b, c, d\}$ is a basis of R^4

(2) $b \cdot c = -1 + 1 = 0$ so $b \perp c$

(3) $\cos\langle a, b\rangle = \dfrac{a \cdot b}{|a||b|} = \dfrac{2+1}{\sqrt{3} \cdot \sqrt{6}} = \dfrac{3}{\sqrt{18}} = \dfrac{1}{\sqrt{2}}$

$\therefore \langle a, b\rangle = \dfrac{\pi}{2}$

(4) $b = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}$ $c = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

Schmidt Orthogonalization

$$\beta_1 = a - \frac{a \cdot b}{b \cdot b} b - \frac{a \cdot c}{c \cdot c} c = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} - \frac{2}{3}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{6} \\ -\frac{1}{6} \\ \frac{1}{3} \end{bmatrix}$$

Let $\beta = 6 \cdot \beta_1 = \begin{bmatrix} 0 \\ -1 \\ -1 \\ 2 \end{bmatrix}$

$$\gamma = d - \frac{d \cdot b}{b \cdot b} b - \frac{d \cdot c}{c \cdot c} c - \frac{d \cdot \beta}{\beta \cdot \beta} \beta$$

$$= d - \frac{2}{3} b + \frac{1}{3} c + \frac{1}{3} \beta$$

$$= \begin{bmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ \frac{1}{3} \\ -\frac{4}{3} \end{bmatrix}$$

$\therefore$ Orthogonal basis: $\begin{array}{cccc} b & c & \beta & \gamma \\ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \\ -1 \\ 2 \end{bmatrix} & \begin{bmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ \frac{1}{3} \\ -\frac{4}{3} \end{bmatrix} \end{array}$

Orthonormal:
(标准正交)

$\begin{bmatrix} \frac{\sqrt{6}}{3} \\ -\frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{6} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix} \begin{bmatrix} 0 \\ -\frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{3} \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{19}}{19} \\ -\frac{\sqrt{19}}{19} \\ \frac{\sqrt{19}}{19} \\ -\frac{4\sqrt{19}}{19} \end{bmatrix}$

情2

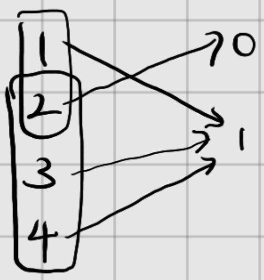
(1)(a) $\forall \alpha \in A_1 \cap A_2$, $\alpha \in A_1$, $\alpha \in A_2$

$\therefore\ f(\alpha) \in f(A_1)$ and $f(\alpha) \in f(A_2)$

i.e. $f(\alpha) \in f(A_1) \cap f(A_2)$

$\therefore\ f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$

(b)



$A_1 = \{1,2\}$ $A_2 = \{2,3,4\}$

$$f(x) = \begin{cases} 0 & x = 2 \\ 1 & x = 1,3,4 \end{cases}$$

$f(A_1) = \{0,1\}$ $f(A_2) = \{0,1\}$

$f(A_1 \cap A_2\} = f(\{2\}) = \{0\} \neq f(A_1) \cap f(A_2)$

(2)(a) $g \circ f(d_1) = g \circ f(d_2)$ i.e. $g(f(d_1)) = g(f(d_2))$

$\because g$ is one-to-one $\therefore f(d_1) = f(d_2)$

$\because f$ is one to one $\therefore d_1 = d_2$

$\therefore g \circ f$ is one to one

(b) $\forall a \in A$

$\because f$ is onto mapping $\therefore \exists\ b \in B$ s.t. $f(a) = b$

$\because g$ is onto mapping $\therefore \exists\ c \in C$ s.t. $g(b) = c$

$\therefore g \circ f(a) = c$ $\therefore g \circ f$ is onto mapping

计2

(1) $f(1,0) = f(0,1) = 2$

$f(1,1) = f(0, f(1,0)) = f(0,2) = 3$

$f(1,2) = f(0, f(1,1)) = f(0,3) = 4$

$\vdots$

$f(1,n) = n+2$

(2) $f(2,0) = f(1,1) = 3$

$f(2,1) = f(1, f(2,0)) = f(2,0)+2 = 5$

$f(2,2) = f(1, f(2,1)) = f(2,1)+2 = 7$

(3) (a) When calling a function, push current status into a stack, including parameters, return address, etc. Then proceed the function. After that, pop and return to the status before the function proceeded.

(b) 不会