Homework 5
(due Thursday, November 1st, 2018, 02:00)

1. What is the relationship between the $Z_k, k \in \{0, \dots, L-1\}$ and X_k and Y_k, the discrete Fourier Transforms of z, x and y respectively, where

$$z_l = \sum_{n=0}^{L-1} x_n y_{n \oplus l},\ l \in \{0, \dots, L-1\}$$

and $n \oplus l = n + l - L$ if $n + l \geq L$, $n + l$ otherwise. Prove your relationship.
2. (i) Rewrite circular convolution as matrix-vector computation. That is, we want you to write $z_l = \sum_{n=0}^{L-1} x_n y_{l \ominus n}$ as

$$z = Ab$$

where z is the vectorized version of $\{z_l\}_{l=0}^{L-1}$ and A (matrix) and b (vector) are your choices. (ii) Comment on the nature of A. (iii) After doing and documenting a google (or DuckDuckGo etc.) search, explicate the (particular and not generic) eigenvector-eigenvalue relationship in the eigen-decomposition of A.
3. Write a program to demonstrate (i) the speedup obtained by using FFTs in numerical convolution. The two sequences x and y must contain at least 1000 elements each. Make sure to zero-pad the sequences and show that you get the correct result. Write your own convolution code but you may use libraries for the FFT computation. Document the speedup using relative numbers of FLOPS. (ii) Then, document the errors between circular convolution using FFTs and linear convolution (direct computation) when you do not employ zero padding at all. In both (i) and (ii), use 5 sets of random x and y sequences.