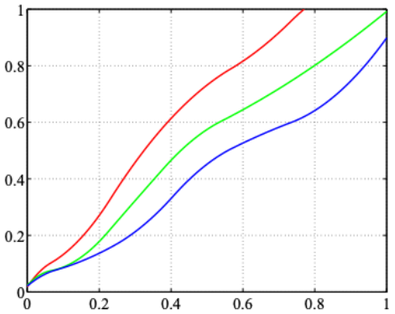


Point Processing and Image Filtering

Jun-Yan Zhu

16-726 Learning-based Image Synthesis, Spring 2021

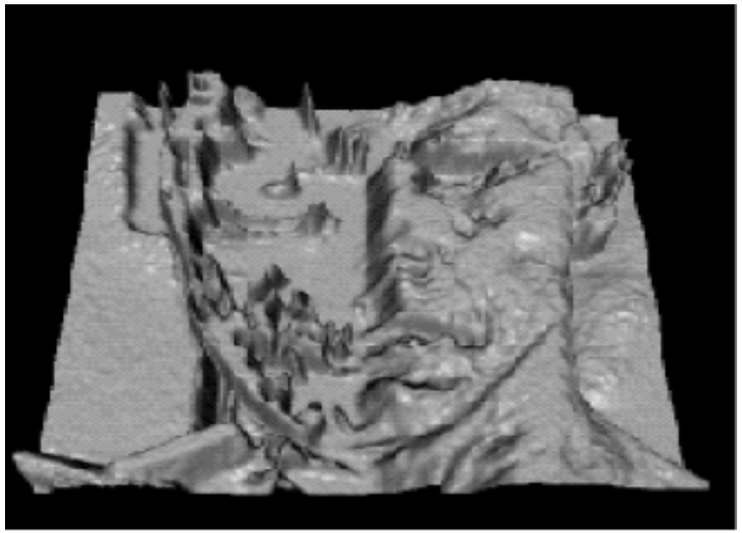
What is an Image?

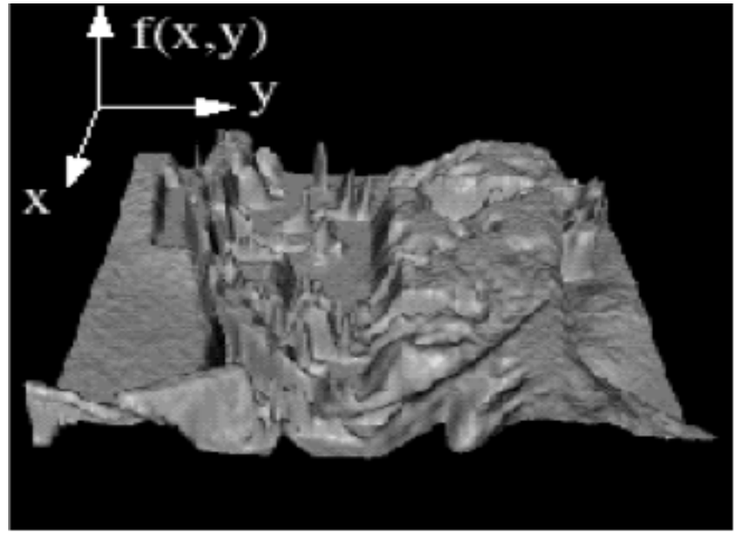
- We can think of an **image** as a function, *f*, from R^2 to R:
  - $f(x, y)$ gives the **intensity** at position (x, y)
  - Realistically, we expect the image only to be defined over a rectangle, with a finite range.
- A color image is just three functions pasted together. We can write this as a "vector-valued" function:

$$f(x,y) = \begin{bmatrix} r(x,y) \\ g(x,y) \\ b(x,y) \end{bmatrix}$$

Many slides are from Alyosha Efros, who also adopted some of them from other great people.

What is an Image?





Single channel grayscale image $f(x, y)$

What is an Image?

RGB image © A.A. Efros

BGR version

$$f(x,y) = \begin{bmatrix} r(x,y) \\ g(x,y) \\ b(x,y) \end{bmatrix}$$

- Common bugs: a function call requires BGR (RGB), you feed RGB (BGR)
- The same applies to display/visualization/save function.
- The crazy thing about deep network: the function calls still work. (e.g., recognize car and building in the image).

Problem: Dynamic Range

1

1500

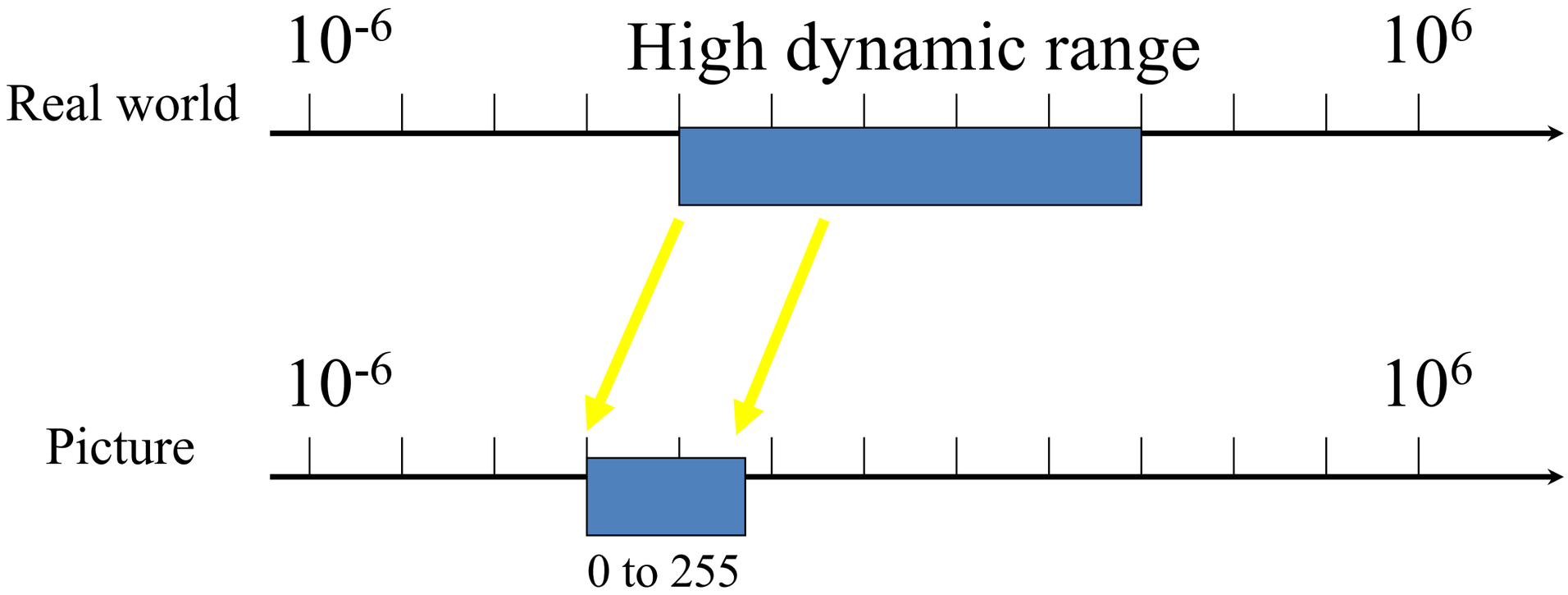
25,000

400,000

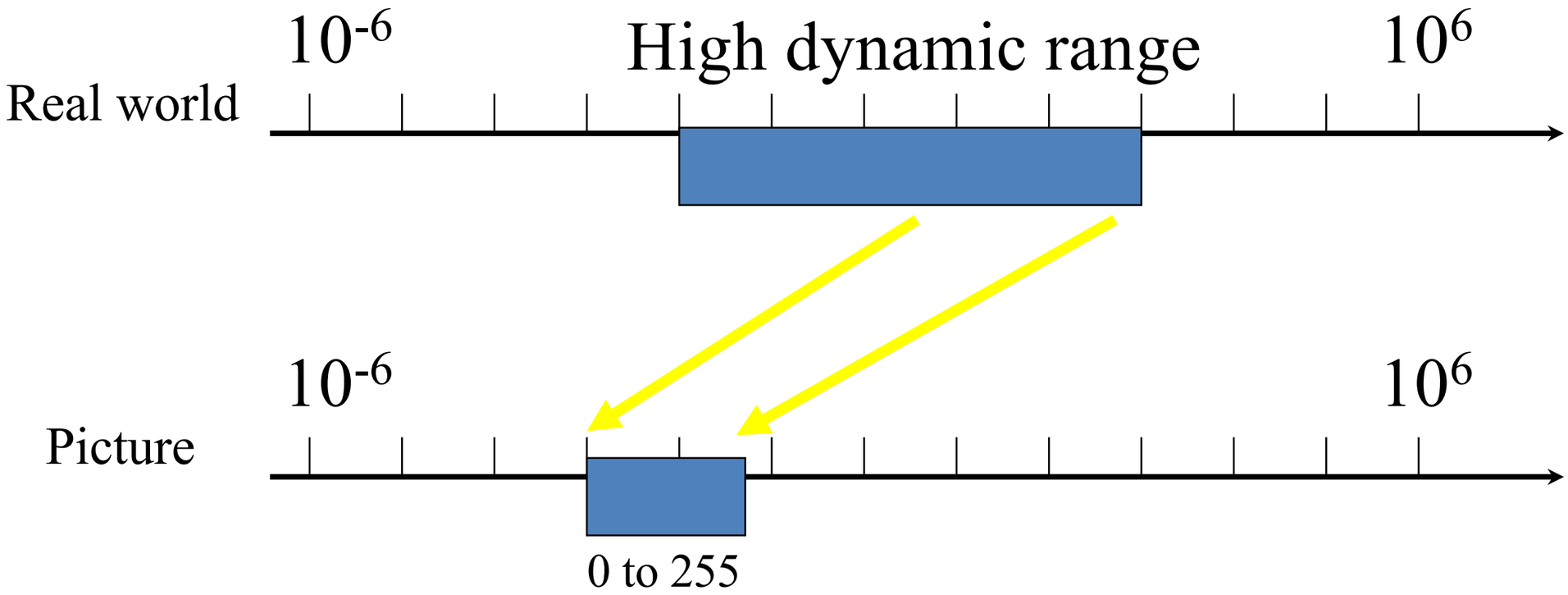
2,000,000,000

The real world is
High dynamic range

Long Exposure



Short Exposure



Basic Point Processing: Enhancement

a b
c d

FIGURE 3.9
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0$, and 5.0, respectively. (Original image for this example courtesy of NASA.)

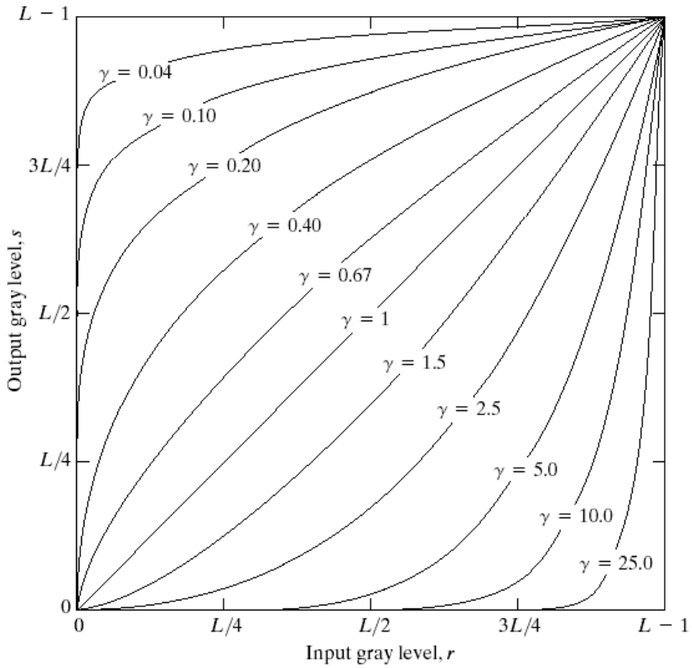


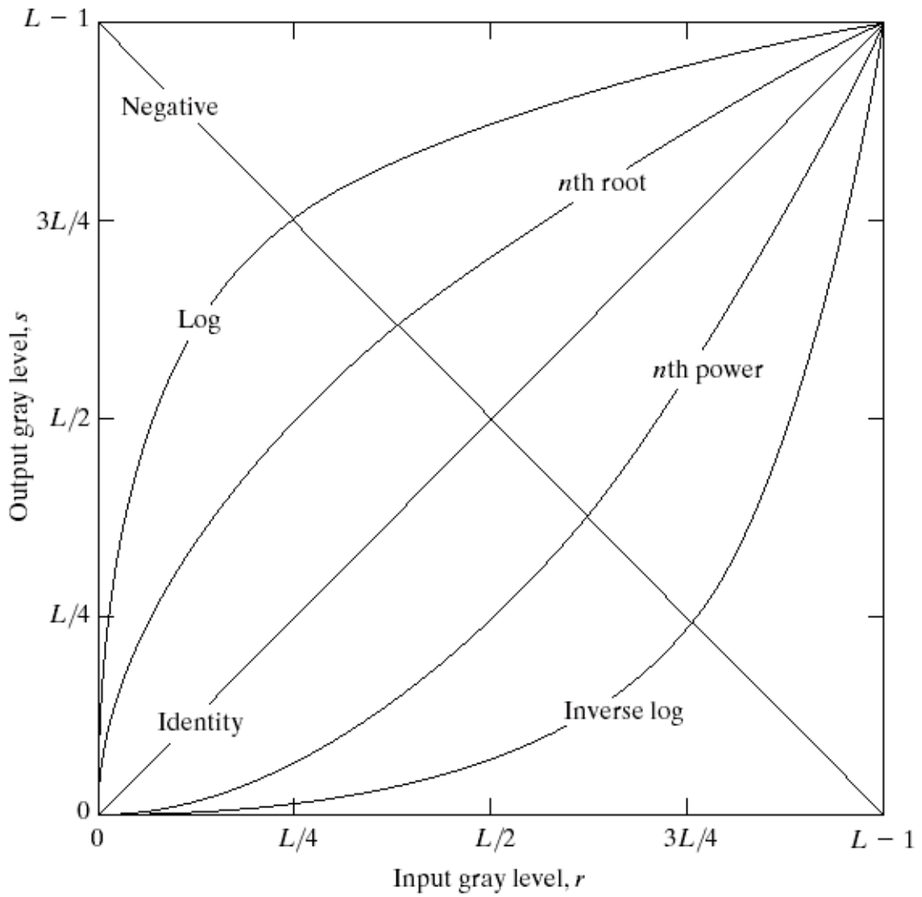
FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

$$c \cdot f(x, y)^{\gamma}$$

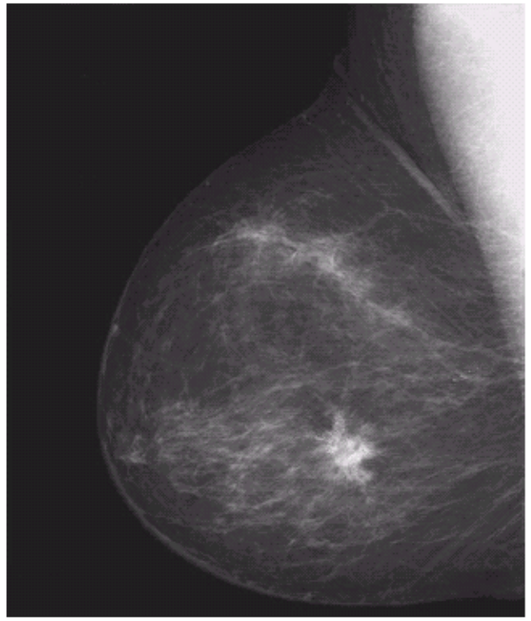
Power-law transformations

Basic Point Processing

FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.



Basic Point Processing



a b

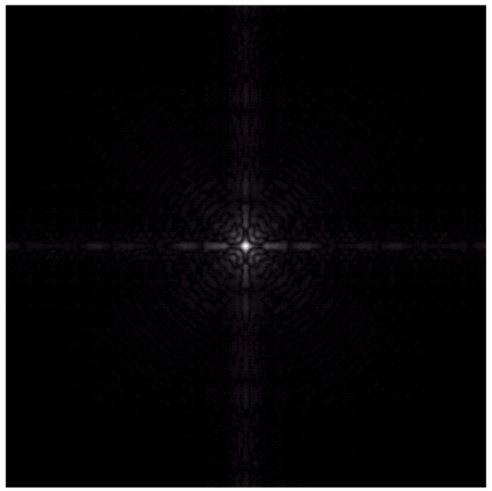
FIGURE 3.4
(a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

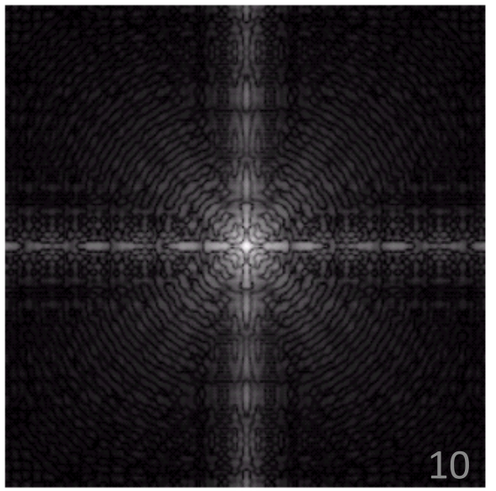
Negative

Log

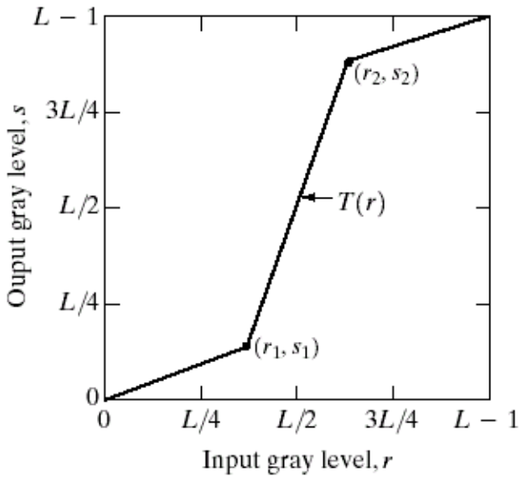
a b

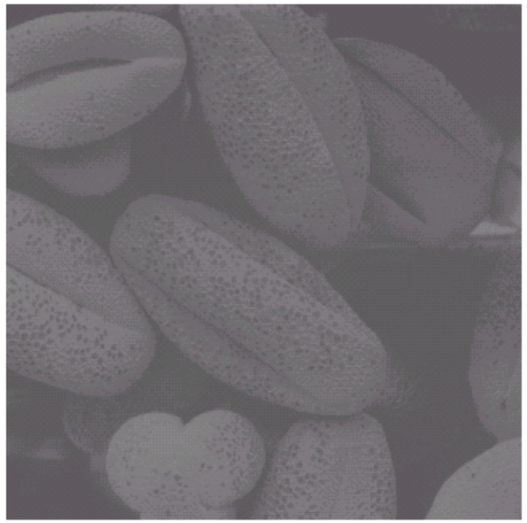
FIGURE 3.5
(a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.

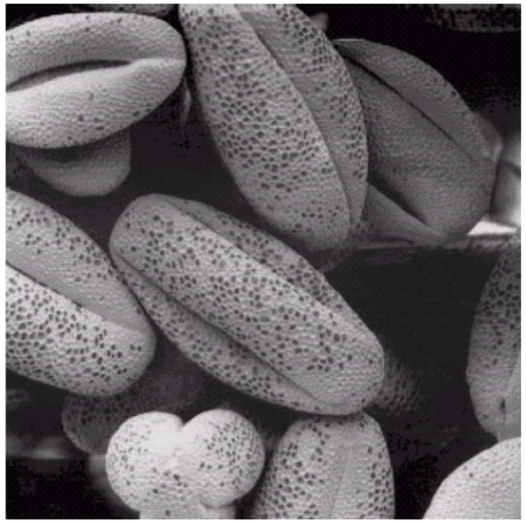




Contrast Stretching



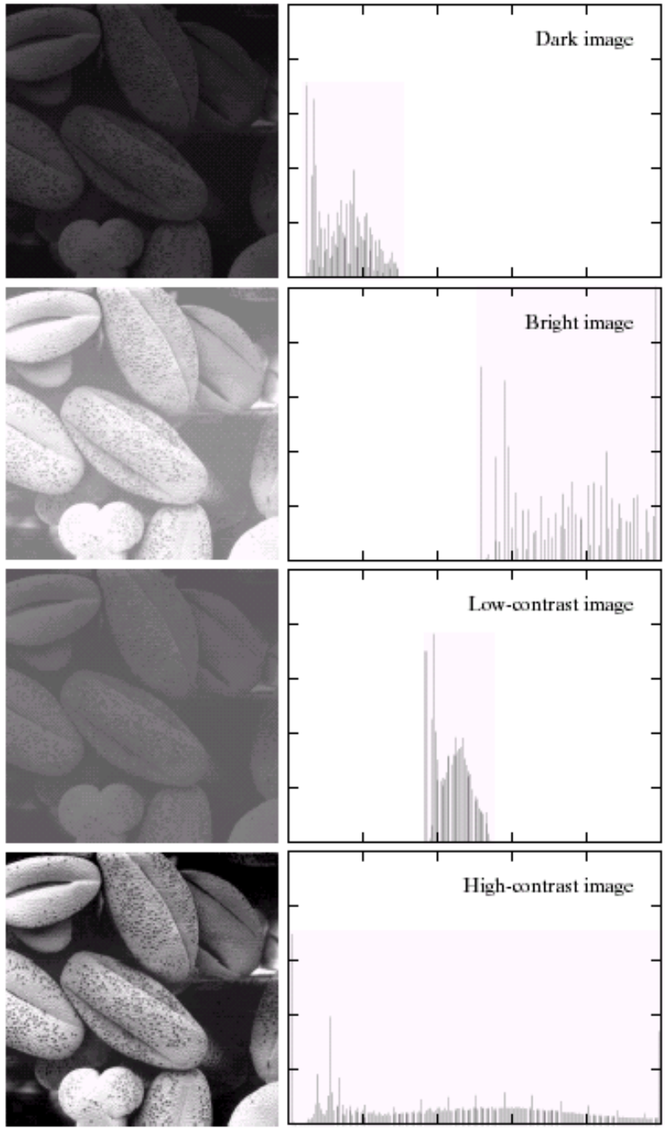


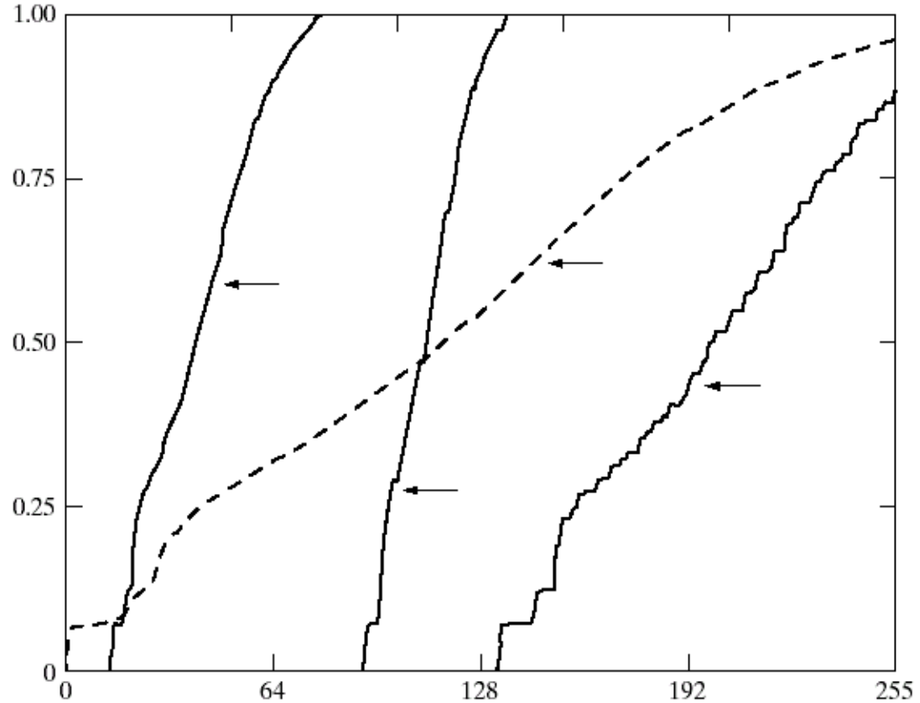


a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Image Histograms





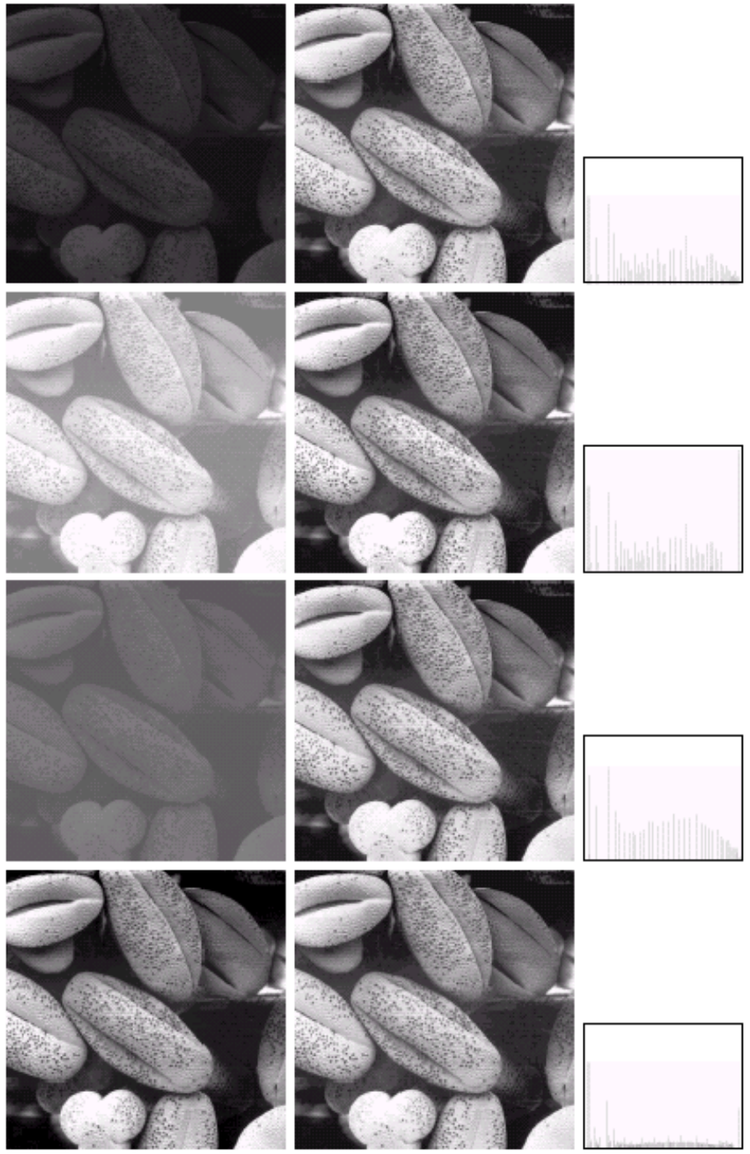
Cumulative Histograms

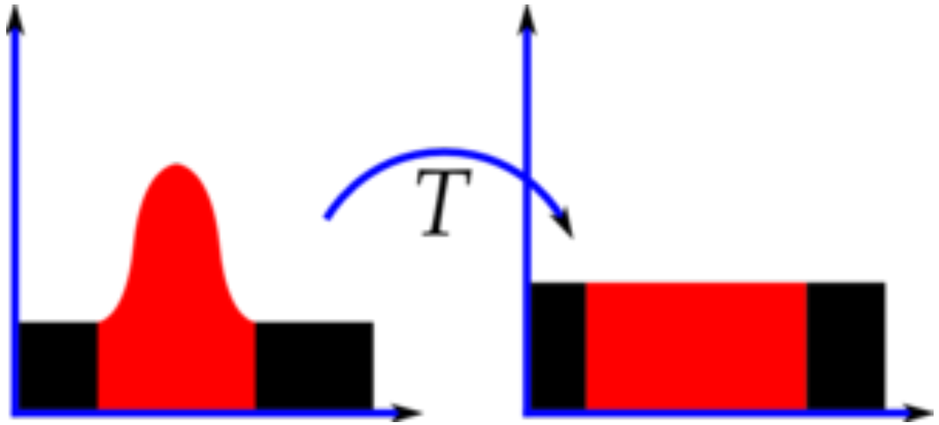
$$T(f(x,y))$$

a b

FIGURE 3.15 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Histogram Equalization





credit @ Zefram

a b c

FIGURE 3.17 (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.

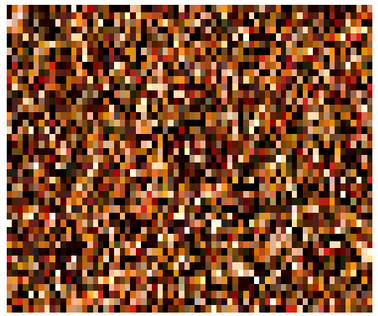
Color Transfer [Reinhard, et al, 2001]

Method: Adjust the mean and standard derivation for each channel

Erik Reinhard, Michael Ashikhmin, Bruce Gooch, Peter Shirley, **Color Transfer** between Images. *IEEE Computer Graphics and Applications*, 21(5), pp. 34–41. September 2001.

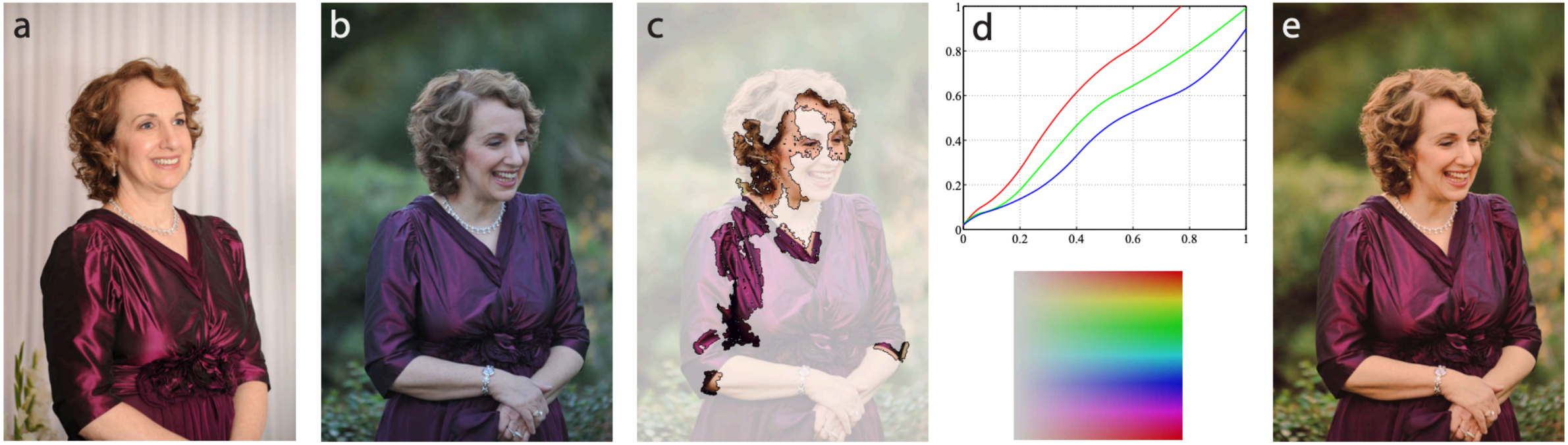
Limitations of Point Processing

Q: What happens if I reshuffle all pixels within the image?



A: It’s histogram won’t change. No point processing will be affected…

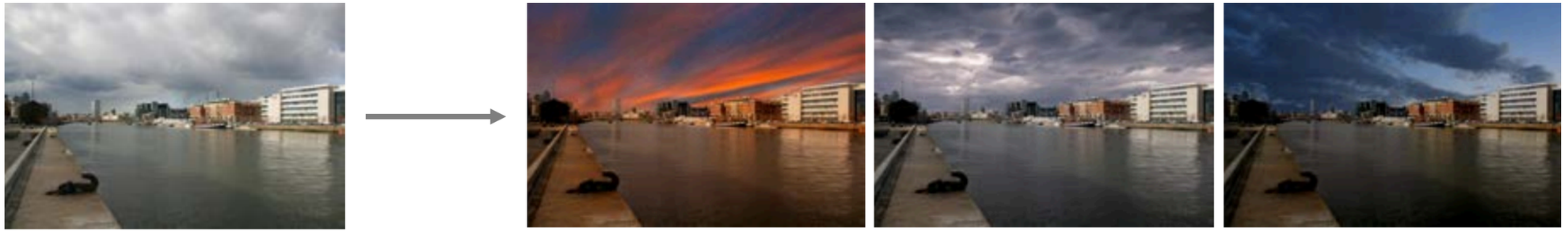
Solution: only transfer shared region?



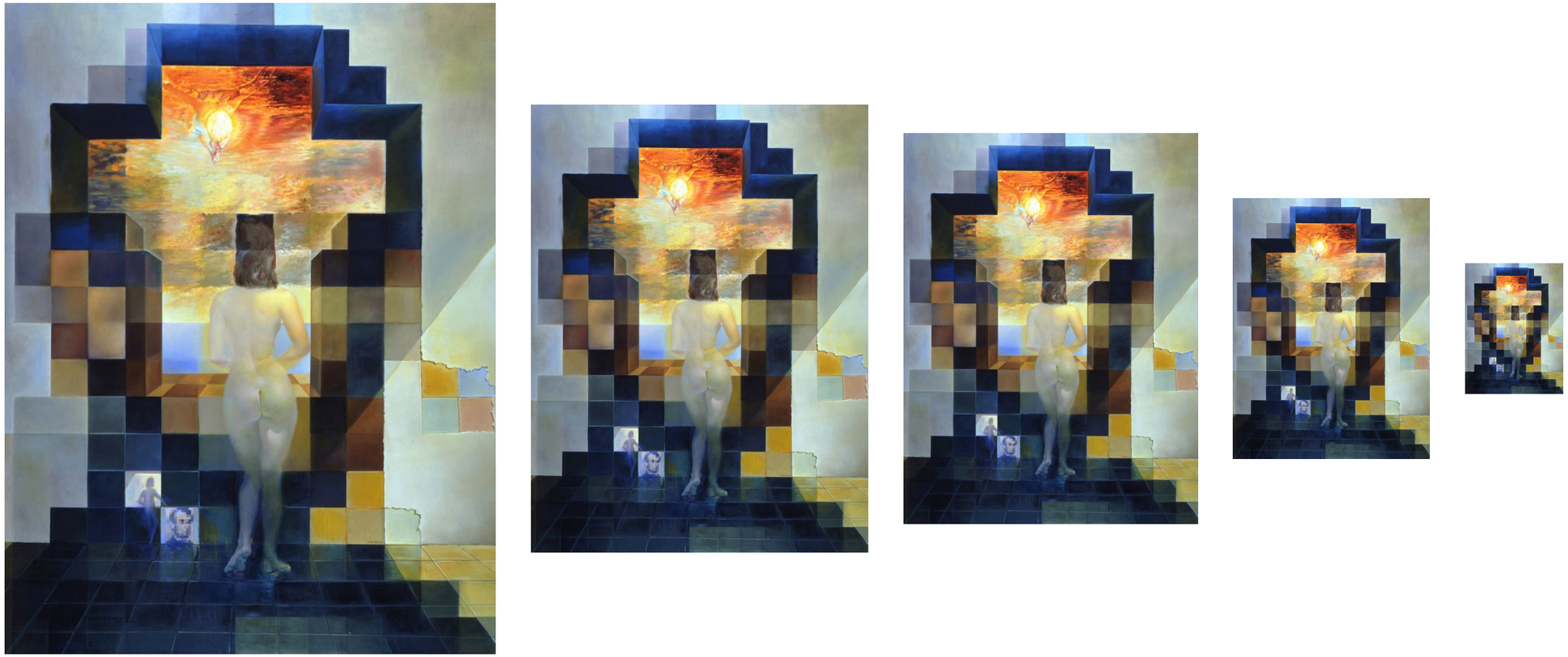
NRDC: Non-Rigid Dense Correspondence with Applications for Image Enhancement
HaCohen et al. SIGGRAPH 2011.

Limitations of Point Processing

- Q: What if I want to replace the sky in my image



- A: point processing cannot create new structure/texture/object. But Why?

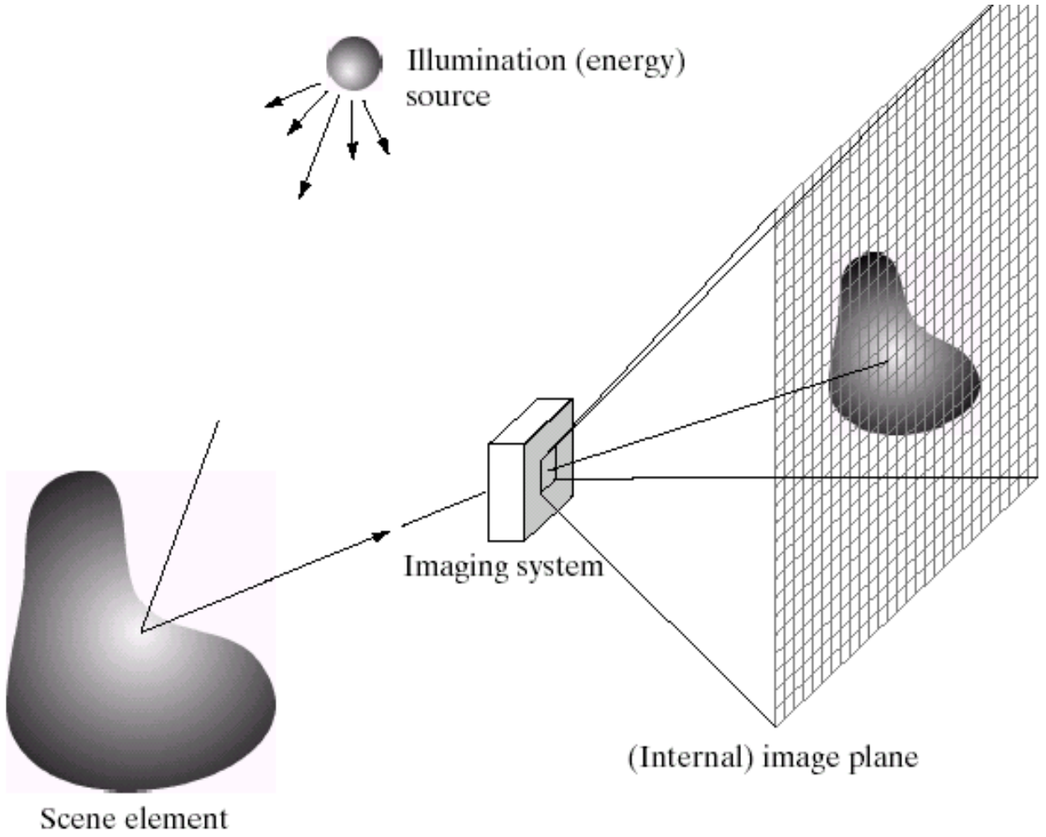


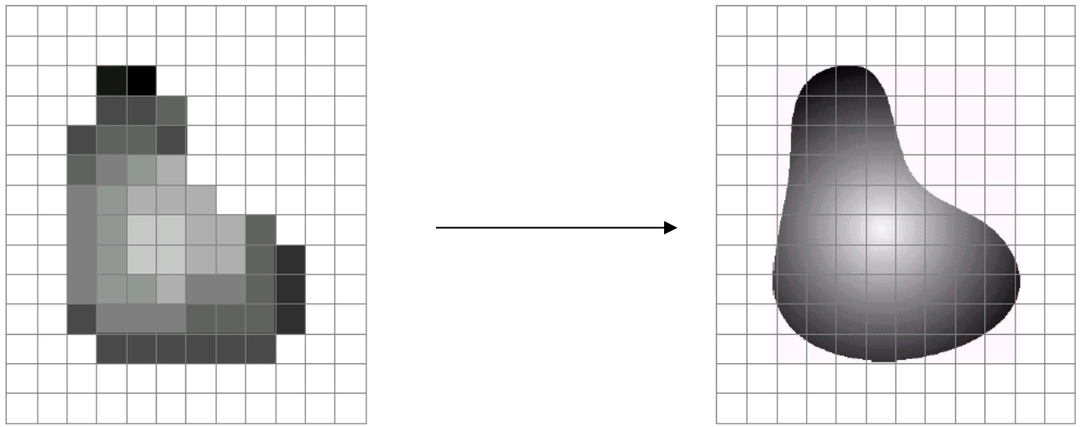
Point Processing and Image Filtering

Jun-Yan Zhu

16-726 Learning-based Image Synthesis, Spring 2021

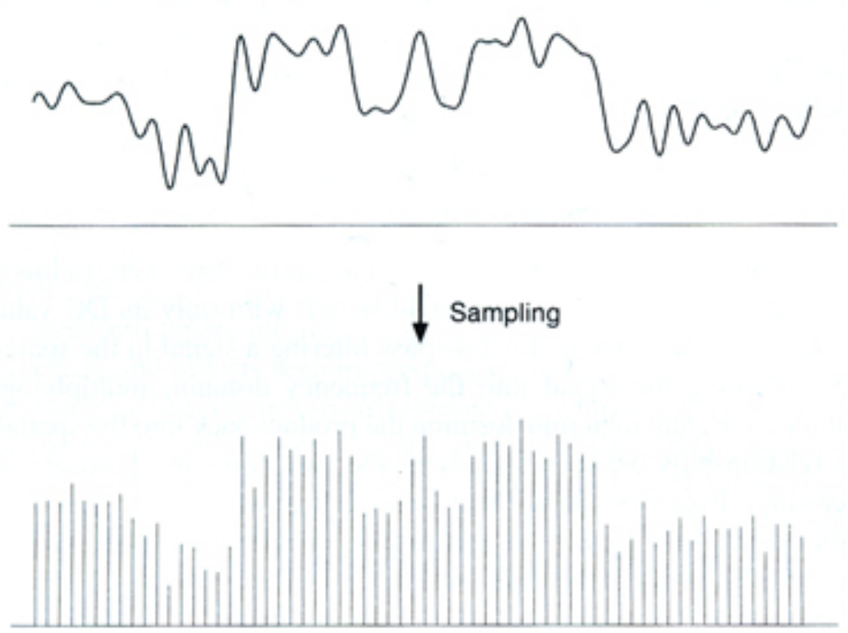
Sampling and Reconstruction





Sampled Representations

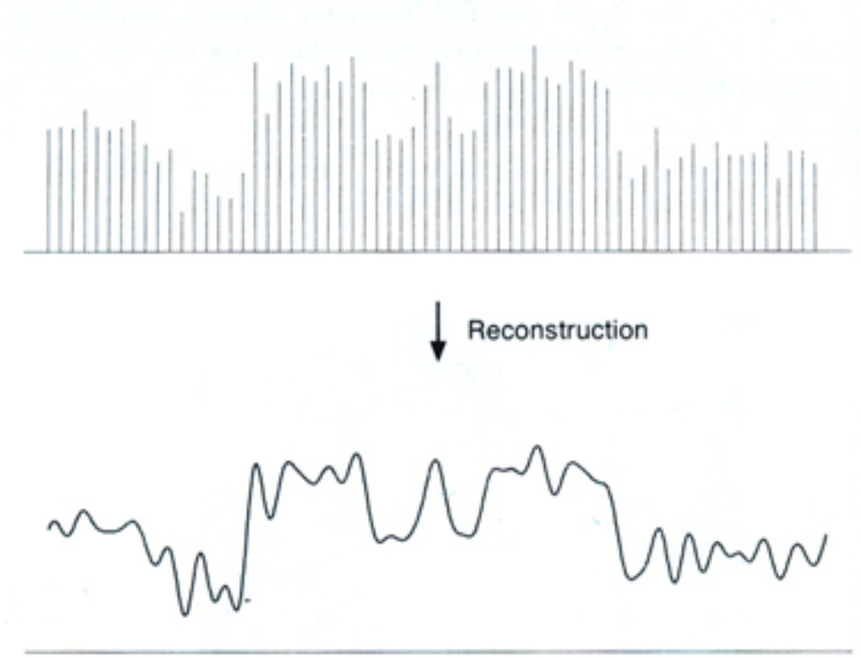
- How to store and compute with continuous functions?
- Common scheme for representation: samples
  - write down the function's values at many points



[FvDFH fig.14.14b / Wolberg]

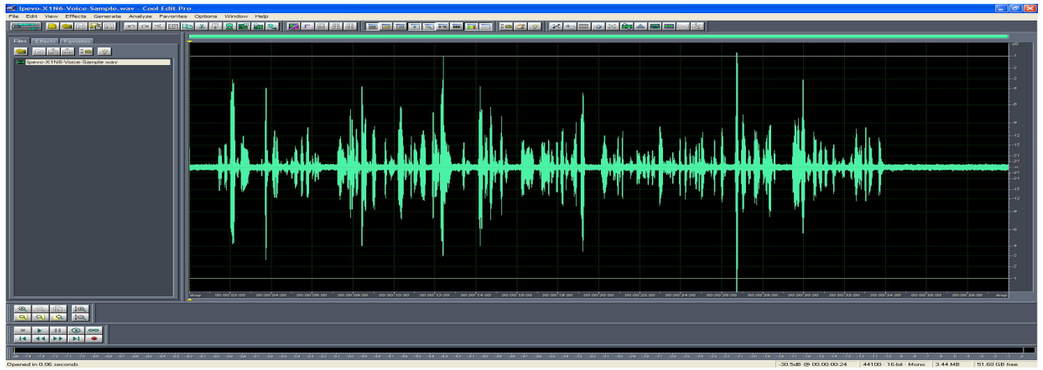
Reconstruction

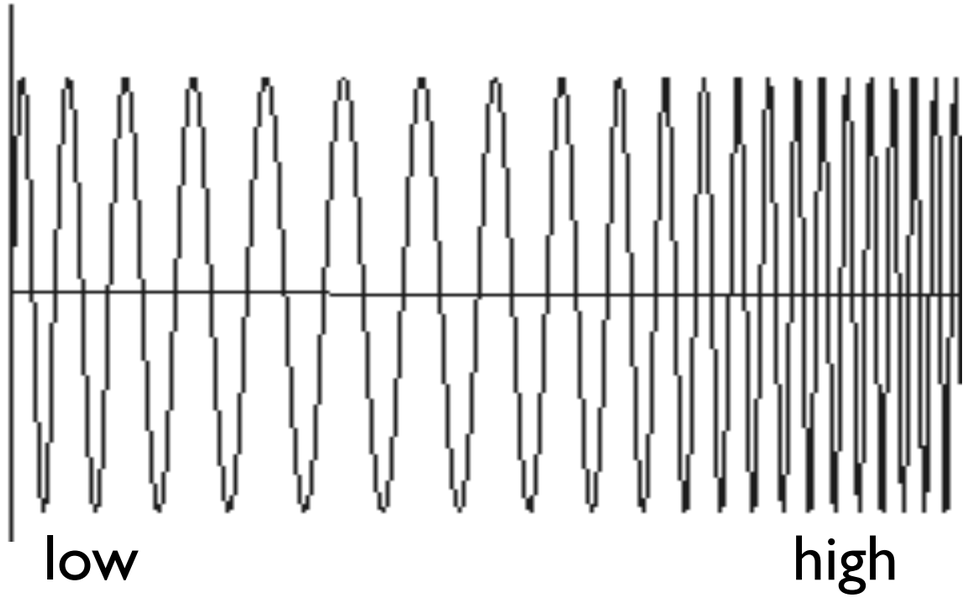
- Making samples back into a continuous function
  - for output (need realizable method)
  - for analysis or processing (need mathematical method)
  - amounts to “guessing” what the function did in between



[FvDFH fig.14.14b / Wolberg]

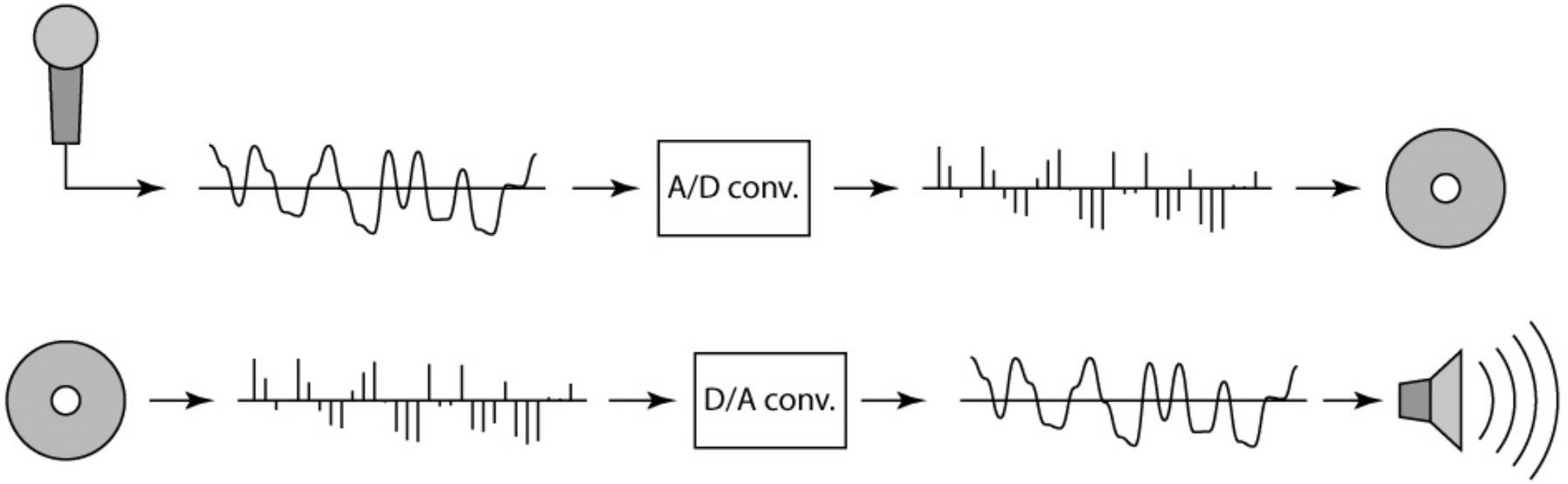
1D Example: Audio





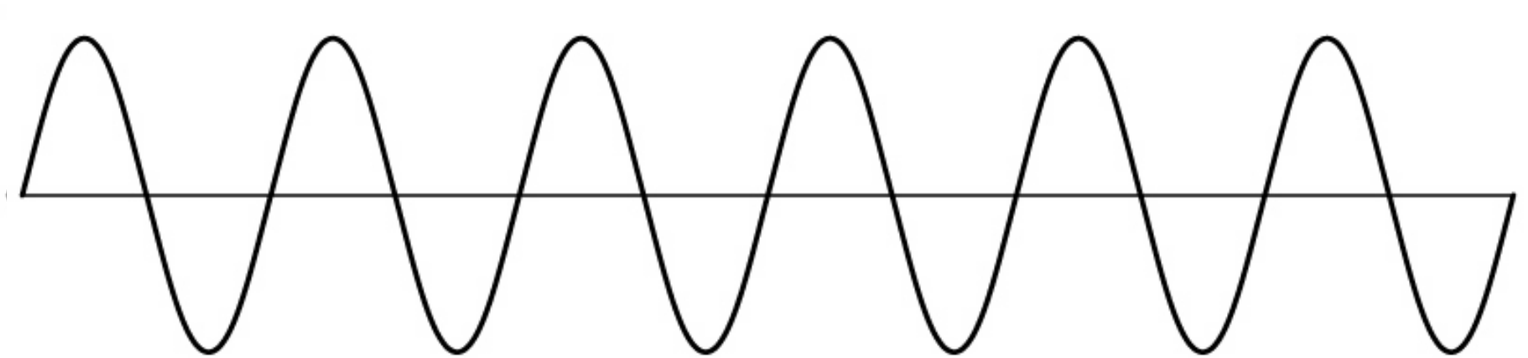
Sampling in Digital Audio

- Recording: sound to analog to samples to disc
- Playback: disc to samples to analog to sound again
  - how can we be sure we are filling in the gaps correctly?



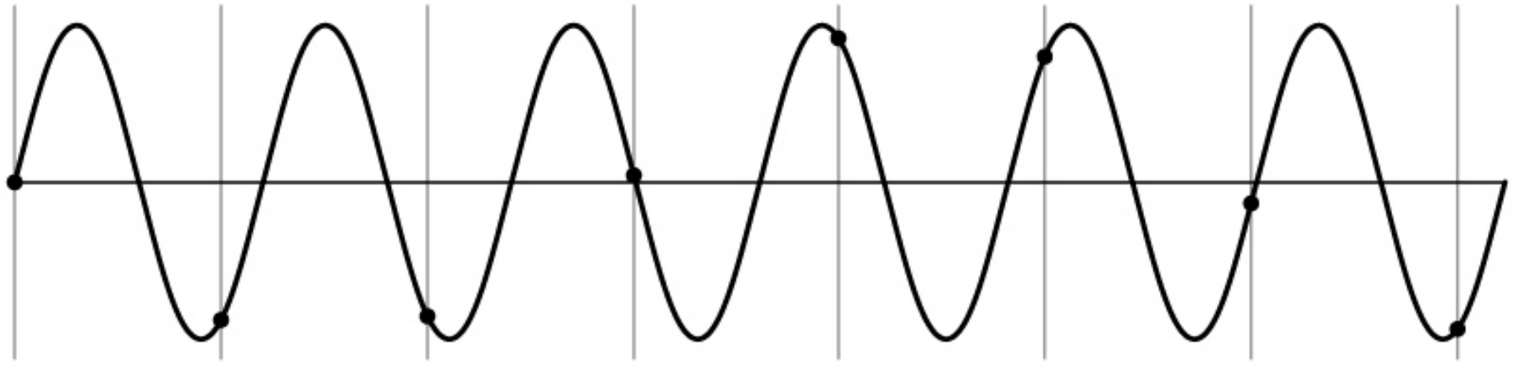
Sampling and Reconstruction

- Simple example: a sign wave



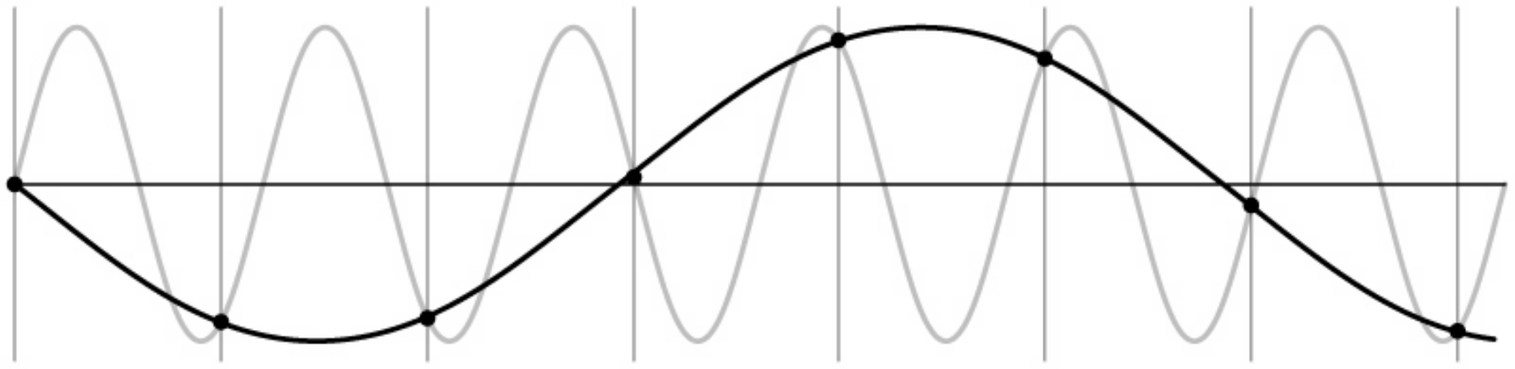
Undersampling

- What if we “missed” things between the samples?
- Simple example: undersampling a sine wave
  - unsurprising result: information is lost



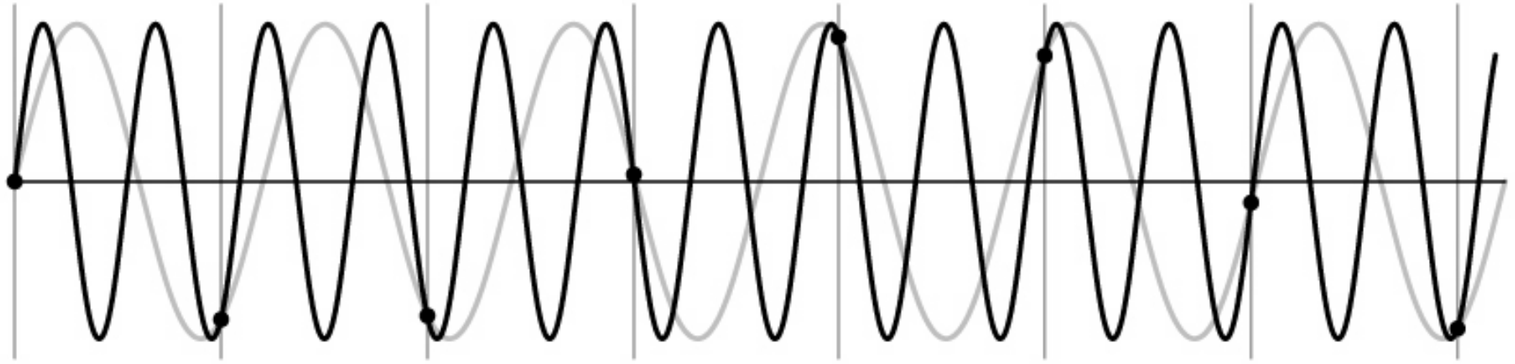
Undersampling

- What if we “missed” things between the samples?
- Simple example: undersampling a sine wave
  - unsurprising result: information is lost
  - surprising result: indistinguishable from lower frequency



Undersampling

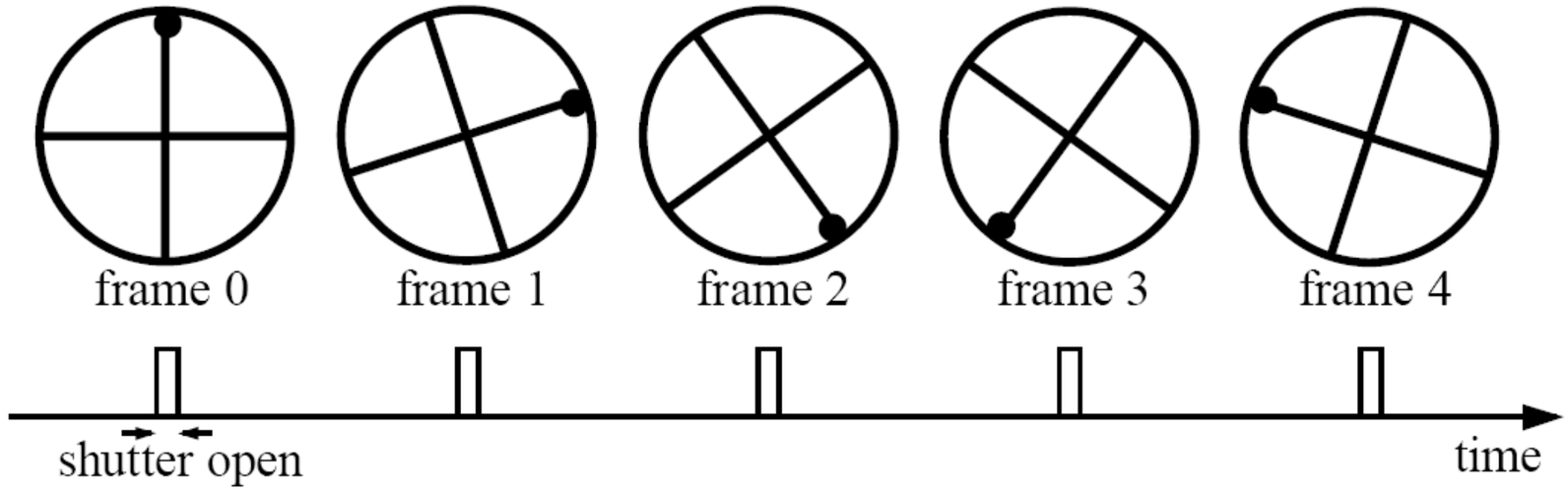
- What if we “missed” things between the samples?
- Simple example: undersampling a sine wave
  - unsurprising result: information is lost
  - surprising result: indistinguishable from lower frequency
  - also, was always indistinguishable from higher frequencies
  - *aliasing*: signals “traveling in disguise” as other frequencies



Aliasing in Videos

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

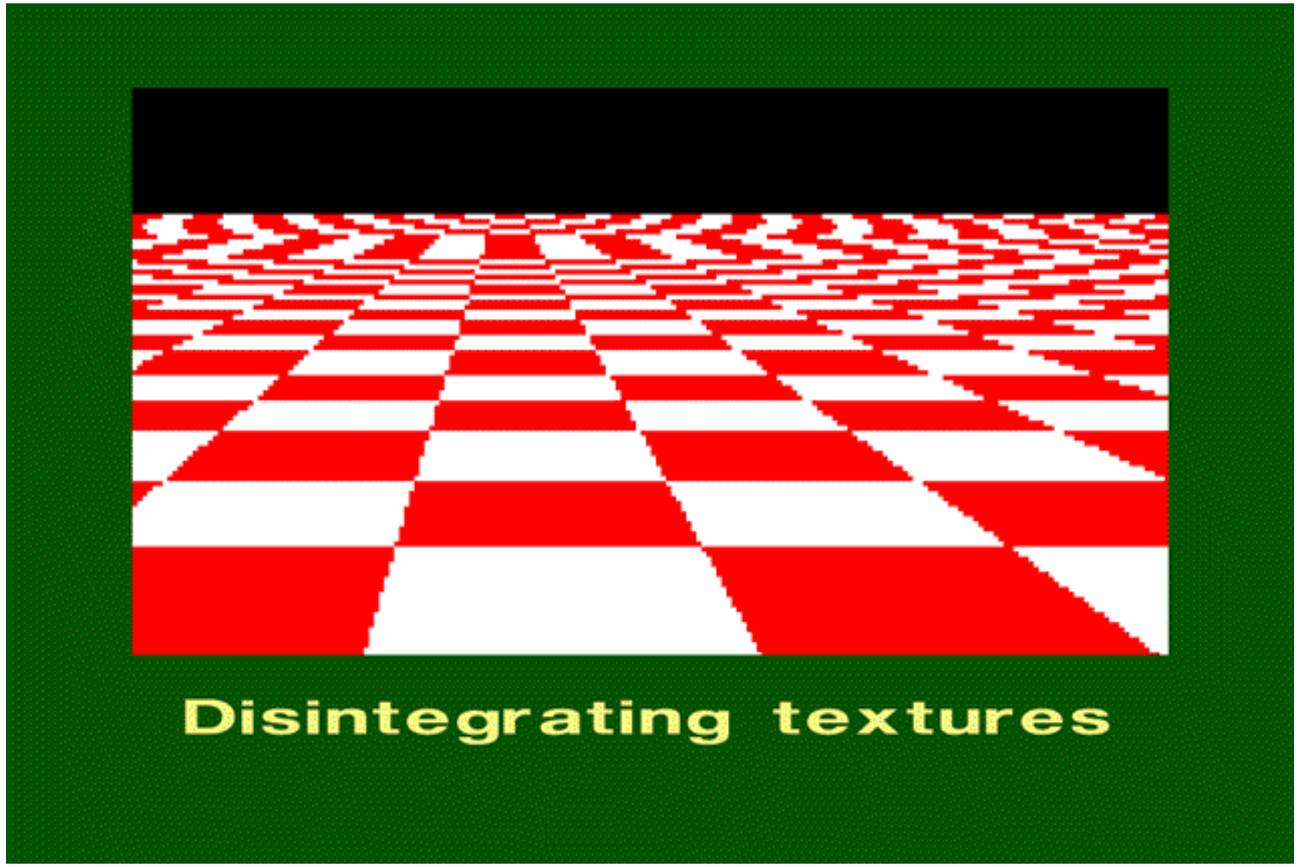
If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

Slide by Steve Seitz

Aliasing in Images



Antialiasing

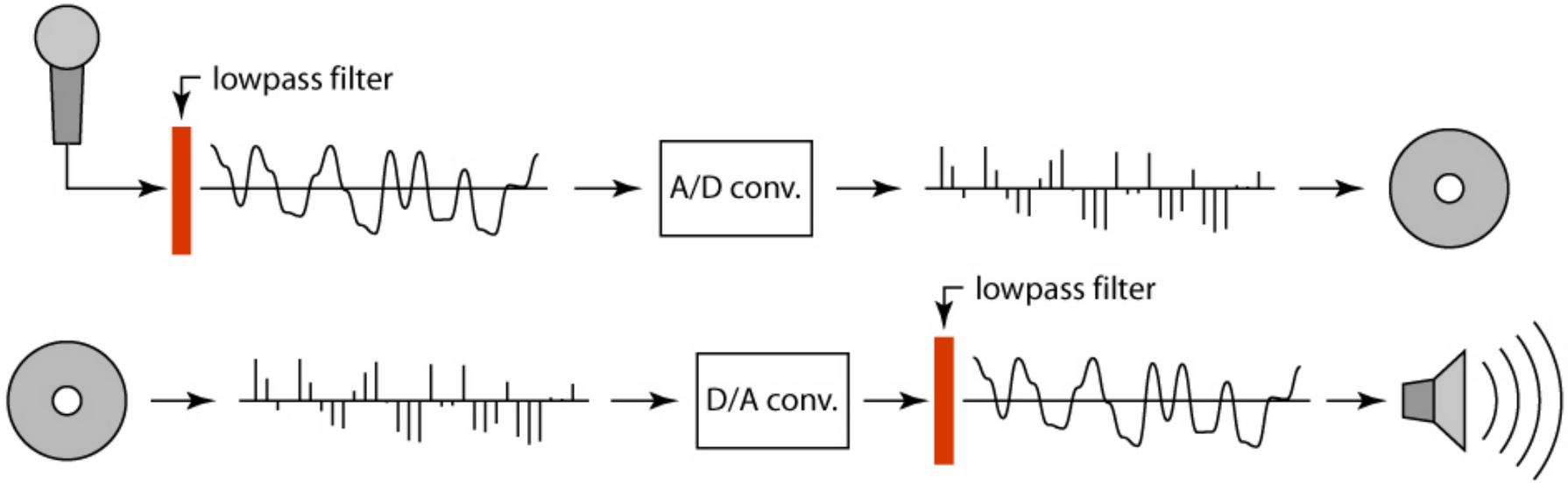
- What can we do about aliasing?

- Sample more often
  - Join the Mega-Pixel craze of the photo industry
  - But this can’t go on forever

- Make the signal less “wiggly”
  - Get rid of some high frequencies
  - Will loose information
  - But it’s better than aliasing

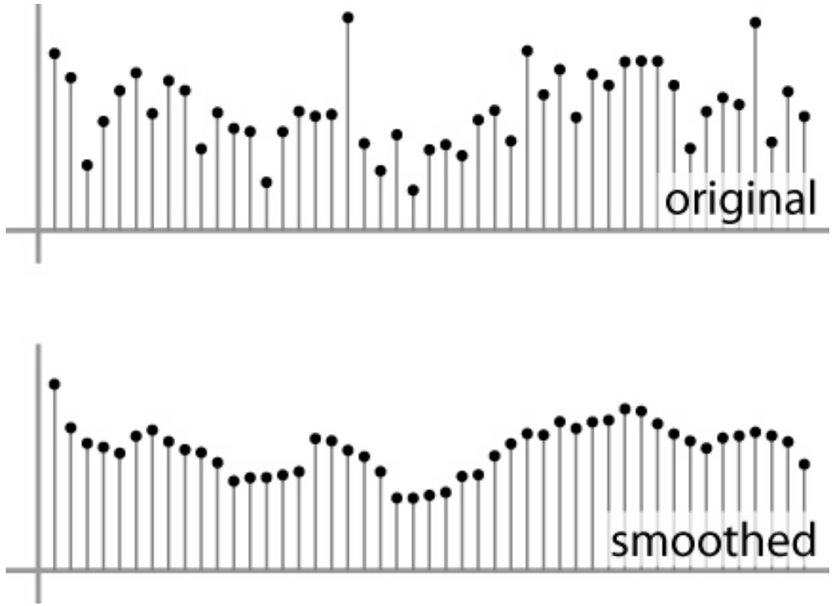
Preventing Aliasing

- Introduce lowpass filters:
  - remove high frequencies leaving only safe, low frequencies
  - choose lowest frequency in reconstruction (disambiguate)



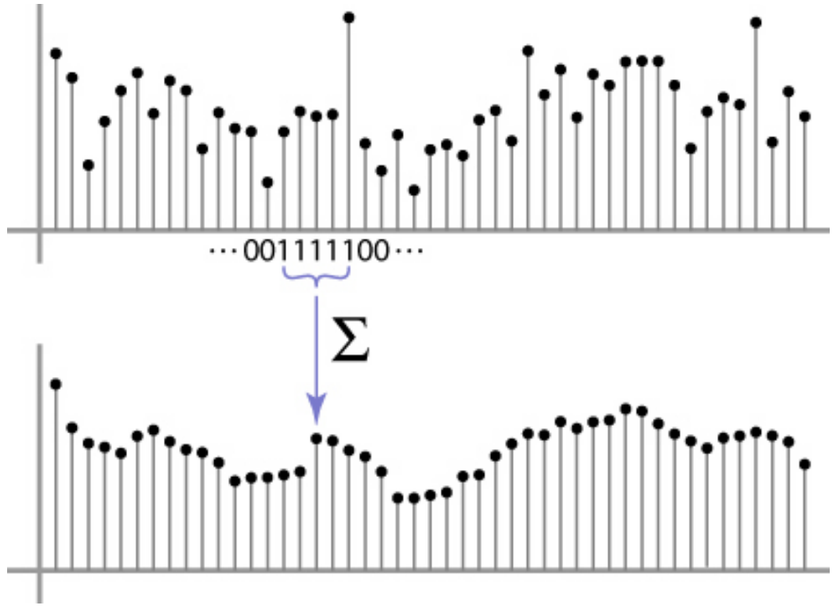
Moving Average

- basic idea: define a new function by averaging over a sliding window
- a simple example to start off: smoothing



Moving Average

- Can add weights to our moving average
- *Weights* […, 0, 1, 1, 1, 1, 1, 0, …] / 5



Cross-correlation

- Let F be the image, H be the kernel (of size 2*k*+1 x 2*k*+1), and G be the output image

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

- A “dot product” between local neighborhood and kernel for each pixel.

- This is called a **cross-correlation** operation:

$$G = H \otimes F$$

2D Box Filter

$$h[\cdot,\cdot]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

$$g[m,n] = \sum_{k,l} h[k,l]\, f[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10							

$$g[m,n] = \sum_{k,l} h[k,l]\, f[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20						

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30					

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30	30				

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30	30				
				?					

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30	30				
						?			
				50					

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$g[m,n] = \sum_{k,l} h[k,l]\, f[m+k, n+l]$$

Credit: S. Seitz

2D Box Filter

What does it do?

- Replaces each pixel with an average of its neighborhood

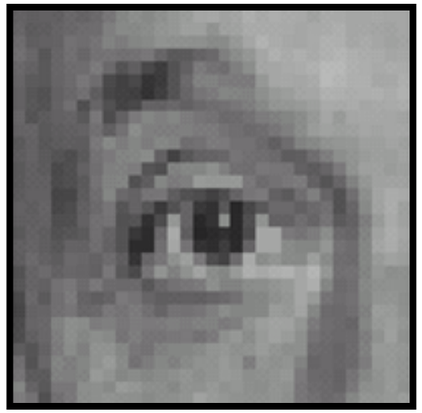
- Achieve smoothing effect (remove sharp features)

$$h[\cdot,\cdot]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

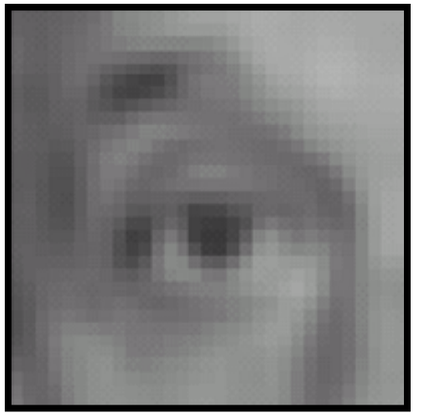
Slide credit: David Lowe (UBC)

Linear filters: Examples



Original

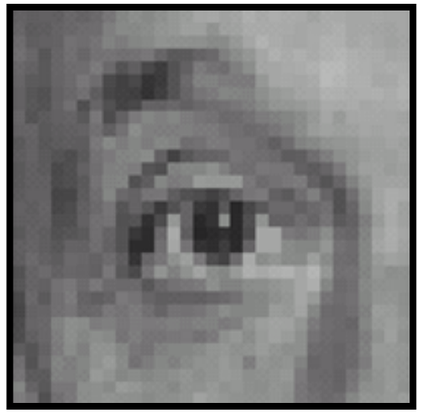
$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$



Blur (with a mean filter)

Source: D. Lowe

Practice with Linear Filters



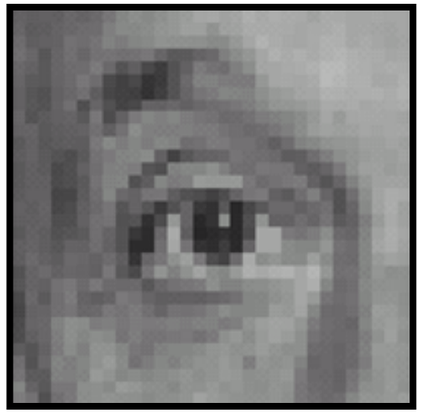
Original

0	0	0
0	1	0
0	0	0

?

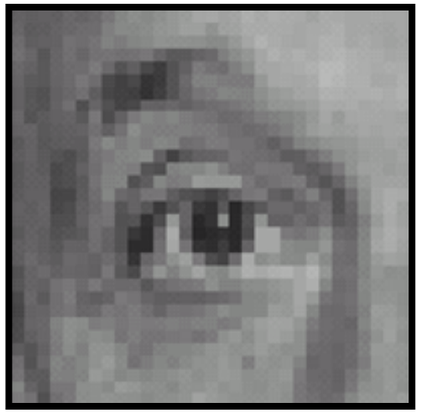
Source: D. Lowe

Practice with Linear Filters



Original

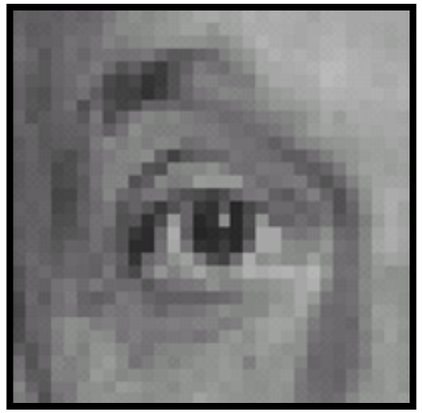
0	0	0
0	1	0
0	0	0



Filtered
(no change)

Source: D. Lowe

Practice with Linear Filters



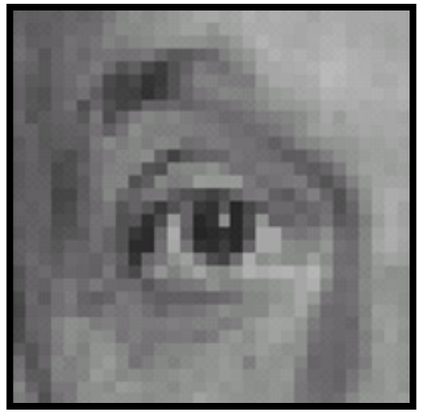
Original

0	0	0
0	0	1
0	0	0

?

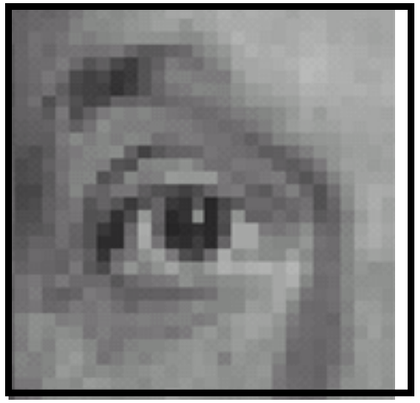
Source: D. Lowe

Practice with Linear Filters



Original

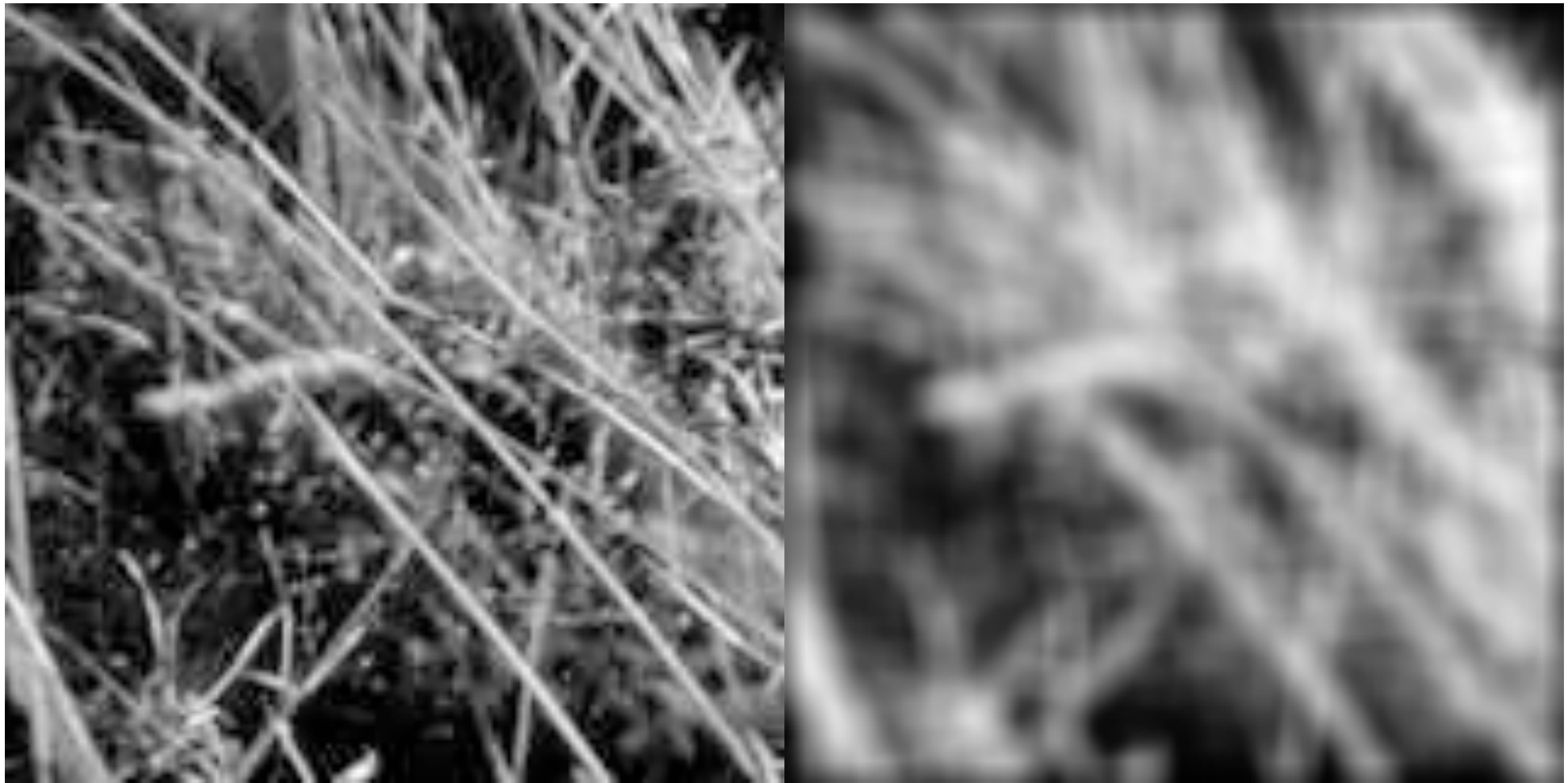
0	0	0
0	0	1
0	0	0



Shifted left
By 1 pixel

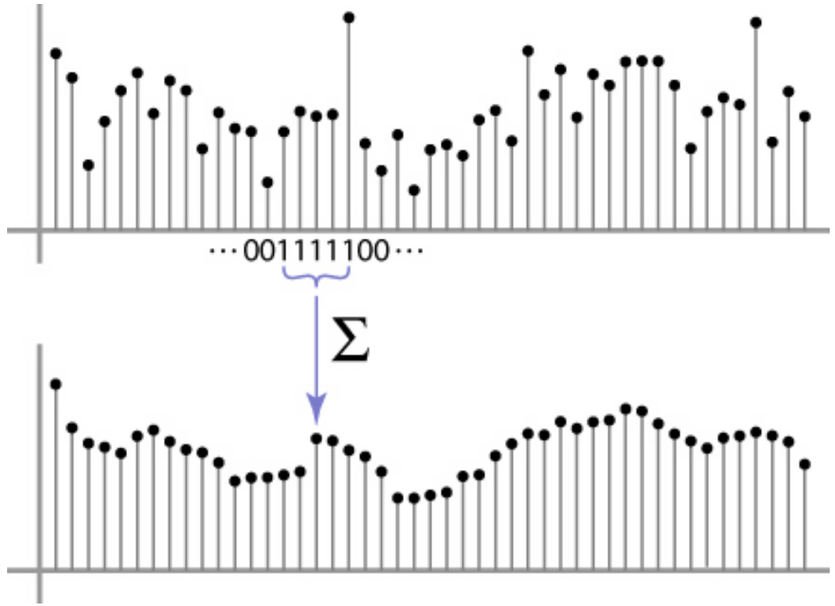
Source: D. Lowe

Back to 2D Box Filter



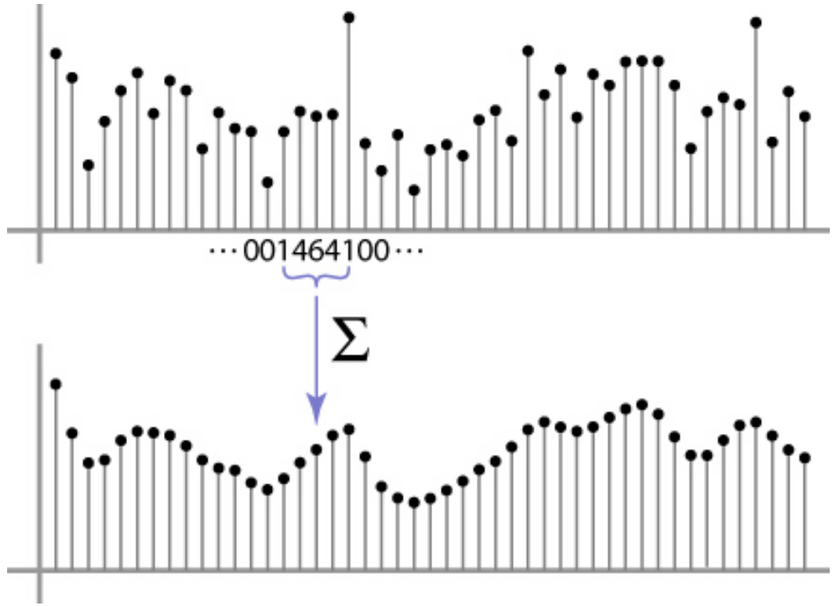
Moving Average

- Can add weights to our moving average
- *Weights* […, 0, 1, 1, 1, 1, 1, 0, …] / 5



Weighted Moving Average

- bell curve (gaussian-like) weights […, 1, 4, 6, 4, 1, …]



Gaussian filtering

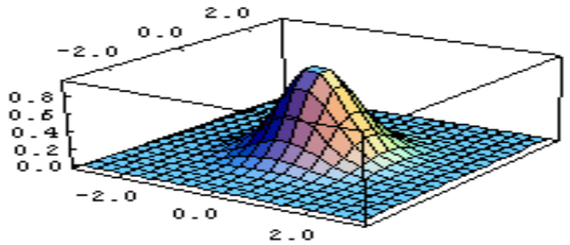
•A Gaussian kernel gives less weight to pixels further from the center of the window

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

$$\frac{1}{16}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline\end{array}$$

$H[u, v]$

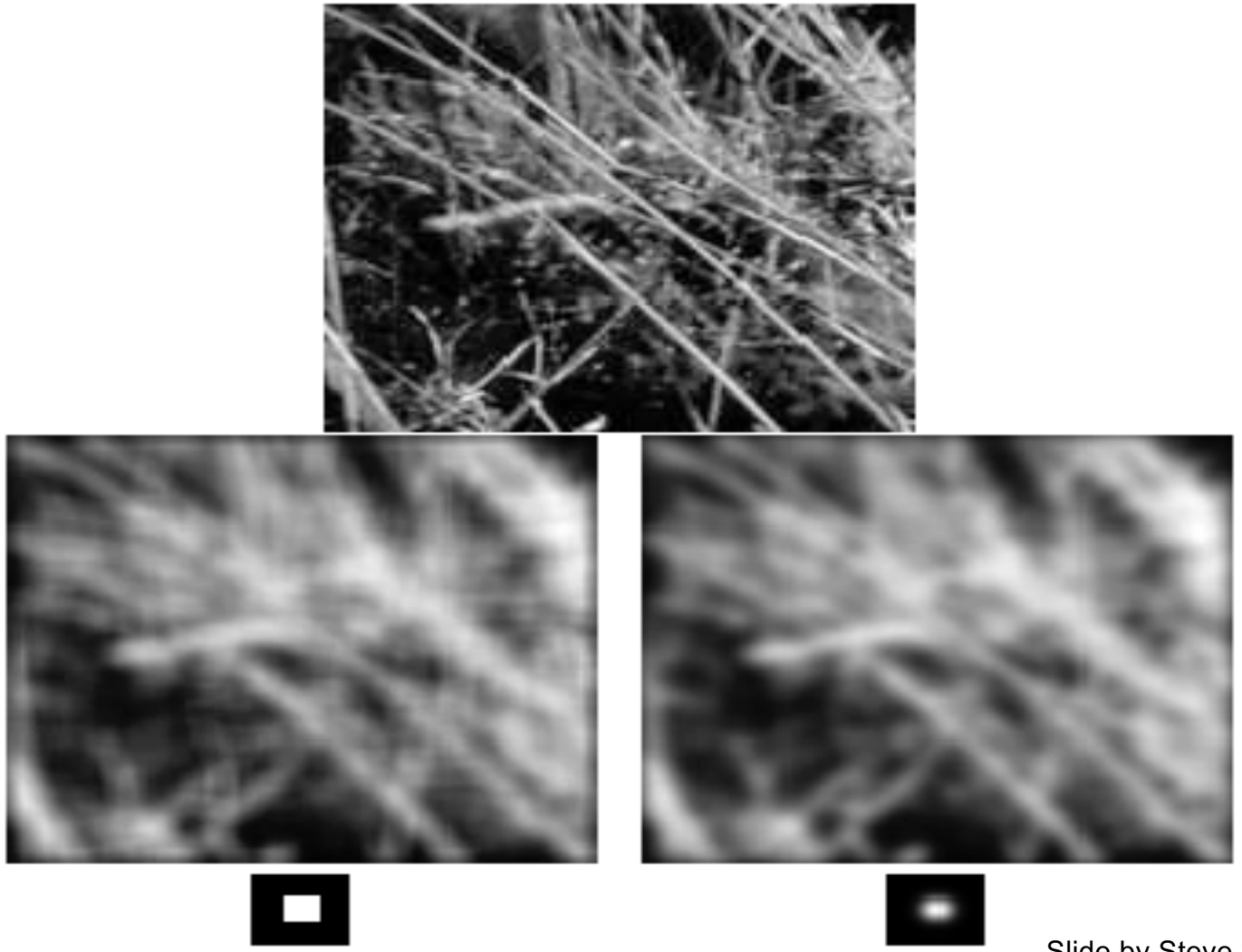


•This kernel is an approximation of a Gaussian function:

$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

Slide by Steve Seitz

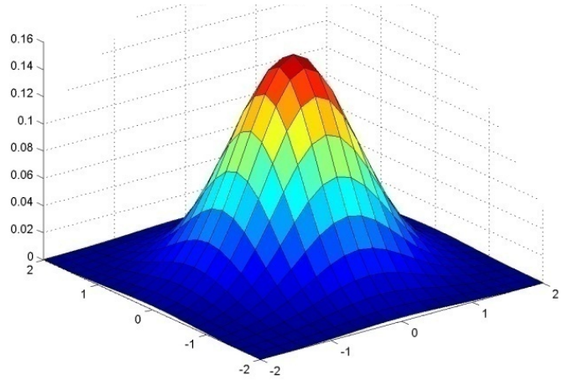
Mean vs. Gaussian Filtering

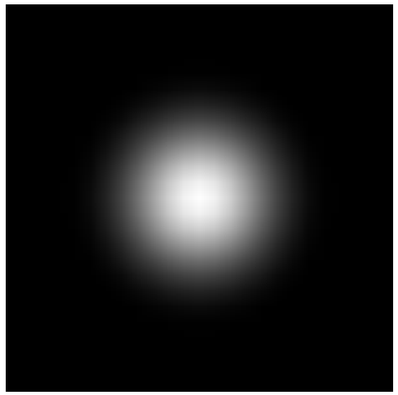


Slide by Steve Seitz

Gaussian filtering

- Weight contributions of neighboring pixels by nearness





0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

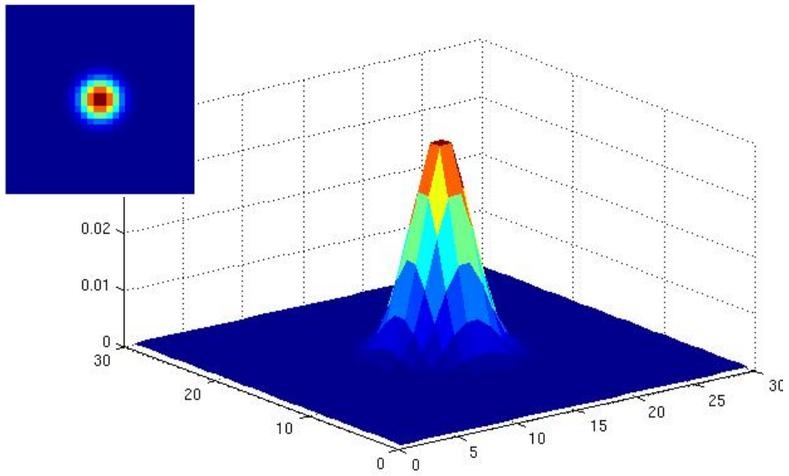
5 x 5, $\sigma = 1$

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

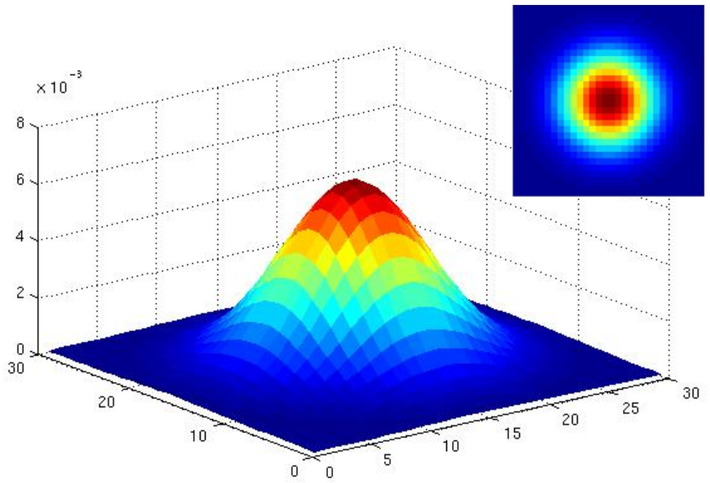
Slide credit: Christopher Rasmussen

Gaussian Kernel

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$



σ = 2 with 30 x 30 kernel

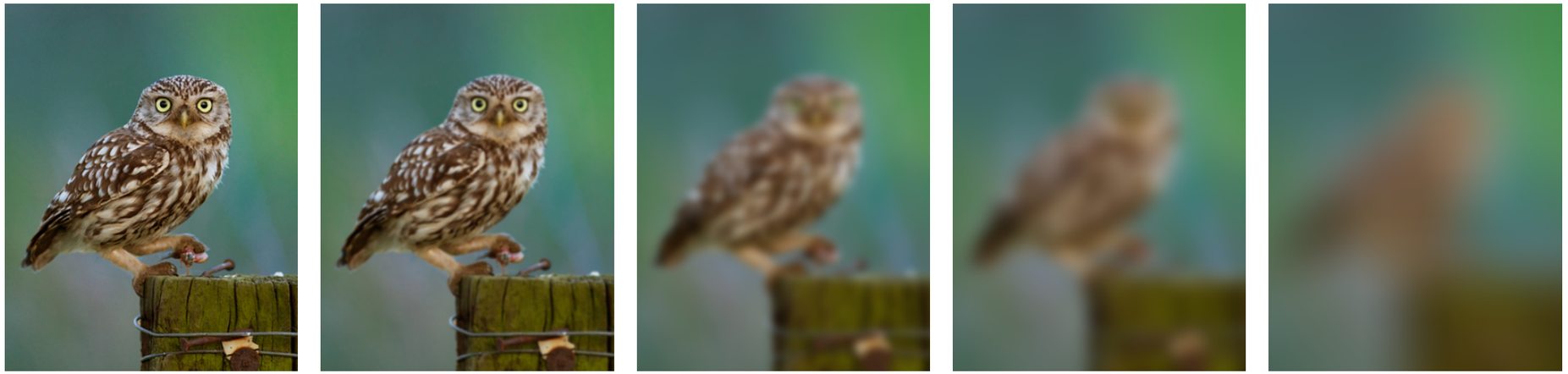


σ = 5 with 30 x 30 kernel

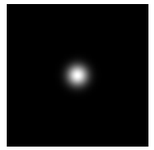
- Standard deviation σ: determines extent of smoothing

Source: K. Grauman

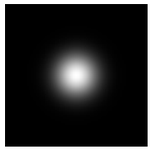
Gaussian filters



σ = 1 pixel



σ = 5 pixels

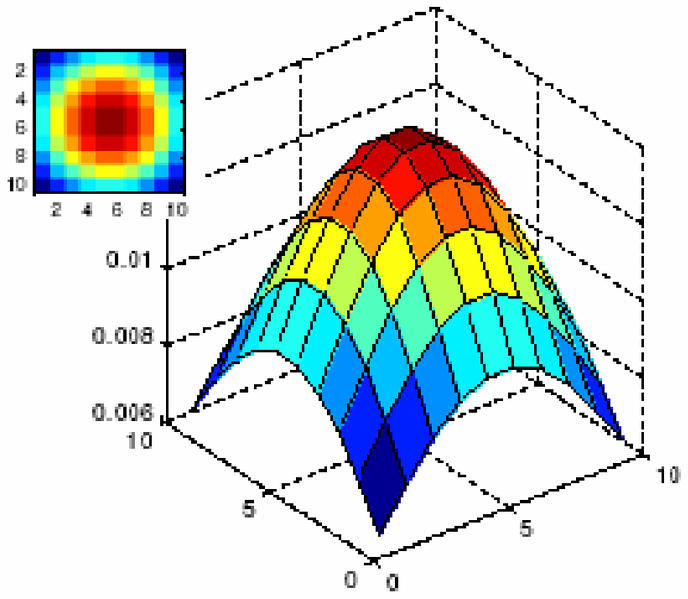


σ = 10 pixels

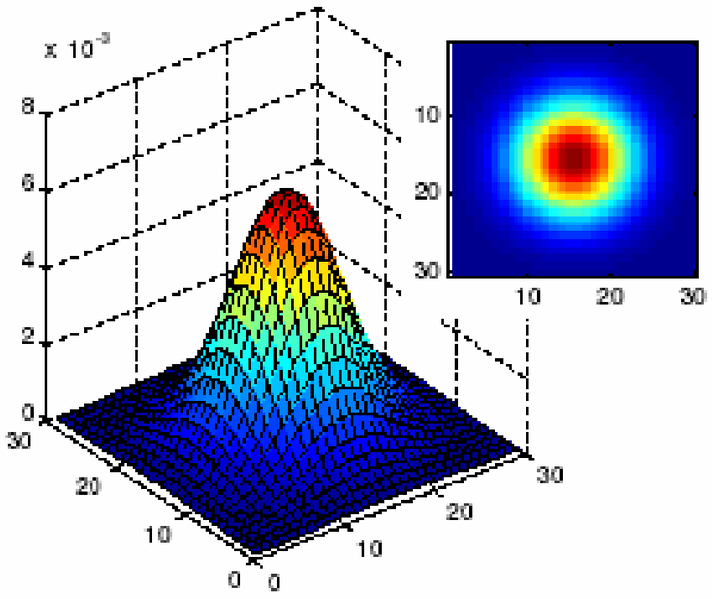
σ = 30 pixels

Choosing kernel width

- Gaussian function has infinite support, but discrete filters use finite kernels



σ = 5 with 10x10 kernel



σ = 5 with 30x30 kernel

Source: K. Grauman

Cross-correlation vs. Convolution

- **cross-correlation**: $$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

$$G = H \otimes F$$

- A **convolution** operation is a cross-correlation where the filter is flipped both horizontally and vertically before being applied to the image:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

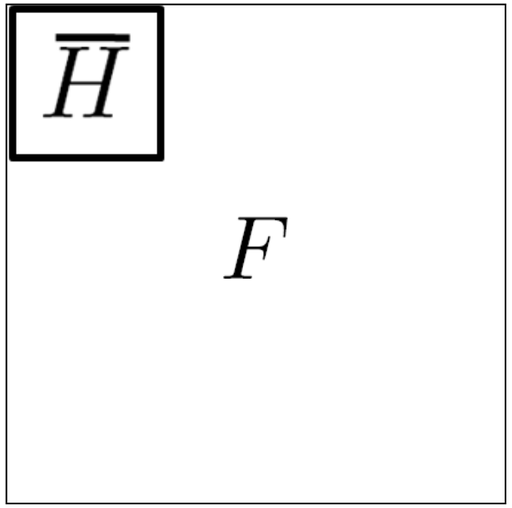
- It is written:

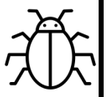
$$G = H \star F$$

- Convolution is **commutative** and **associative**

Slide by Steve Seitz

Convolution





Different DL libraries handle it differently. Some flips; some not.
Be careful when you port Conv weights from one library to another.

Computer vision 101: most students fail to flip the kernel.

Adapted from F. Durand

Convolution is nice!

- Notation: $b = c * a$
- Convolution is a multiplication-like operation
  - Commutative: $a * b = b * a$
  - Associative: $a * (b * c) = (a * b) * c$
  - distributes over addition: $a * (b + c) = a * b + a * c$
  - scalars factor out: $\alpha a * b = a * \alpha b = \alpha(a * b)$
  - identity: unit impulse *e* = […, 0, 0, 1, 0, 0, …]: $a * e = a$
- Conceptually no distinction between filter and signal
- Usefulness of associativity
  - often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
  - this is equivalent to applying one filter: $a * (b_1 * b_2 * b_3)$

Gaussian and Convolution

- Removes “high-frequency” components from the image (low-pass filter)
- Convolution with self is another Gaussian

  - Convolving twice with Gaussian kernel of width

    = convolving once with kernel of width σ

$$\sigma\sqrt{2}$$

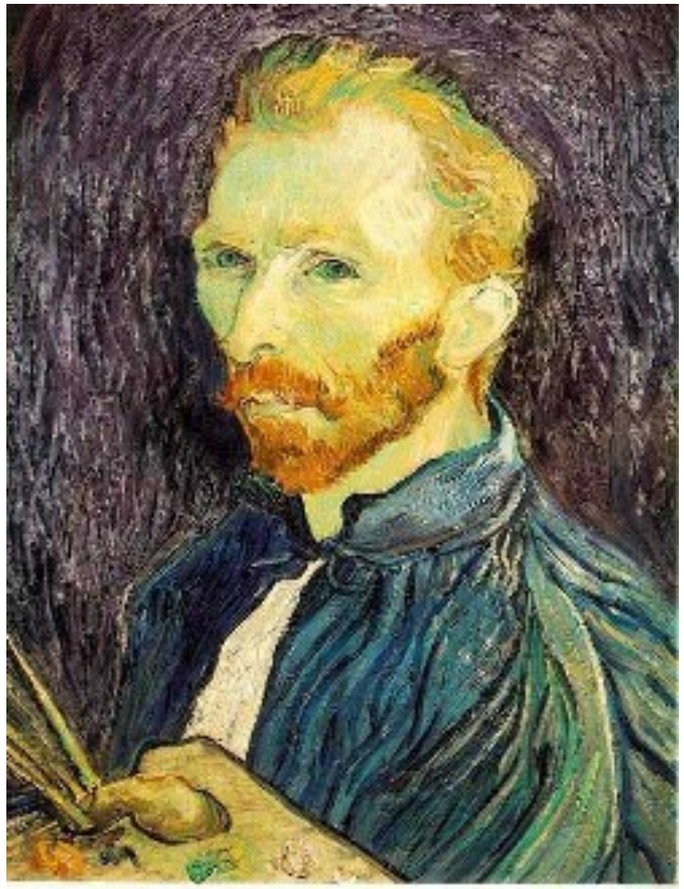
Source: K. Grauman

Image h

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?

Image sub-sampling



1/4

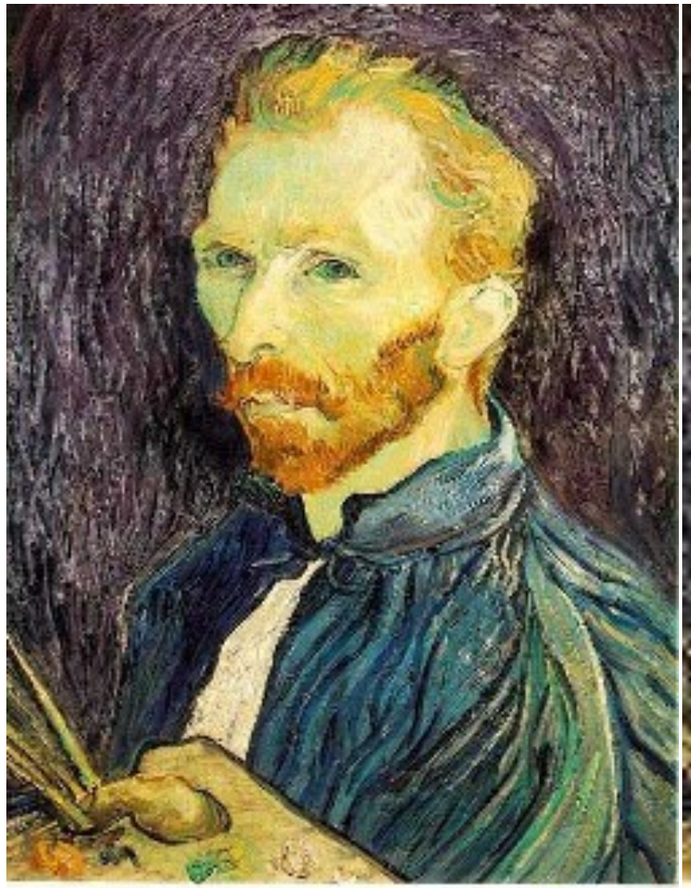
1/8

Throw away every other row and column to create a 1/2 size image

- called *image sub-sampling*

Slide by Steve Seitz

Image sub-sampling



1/2

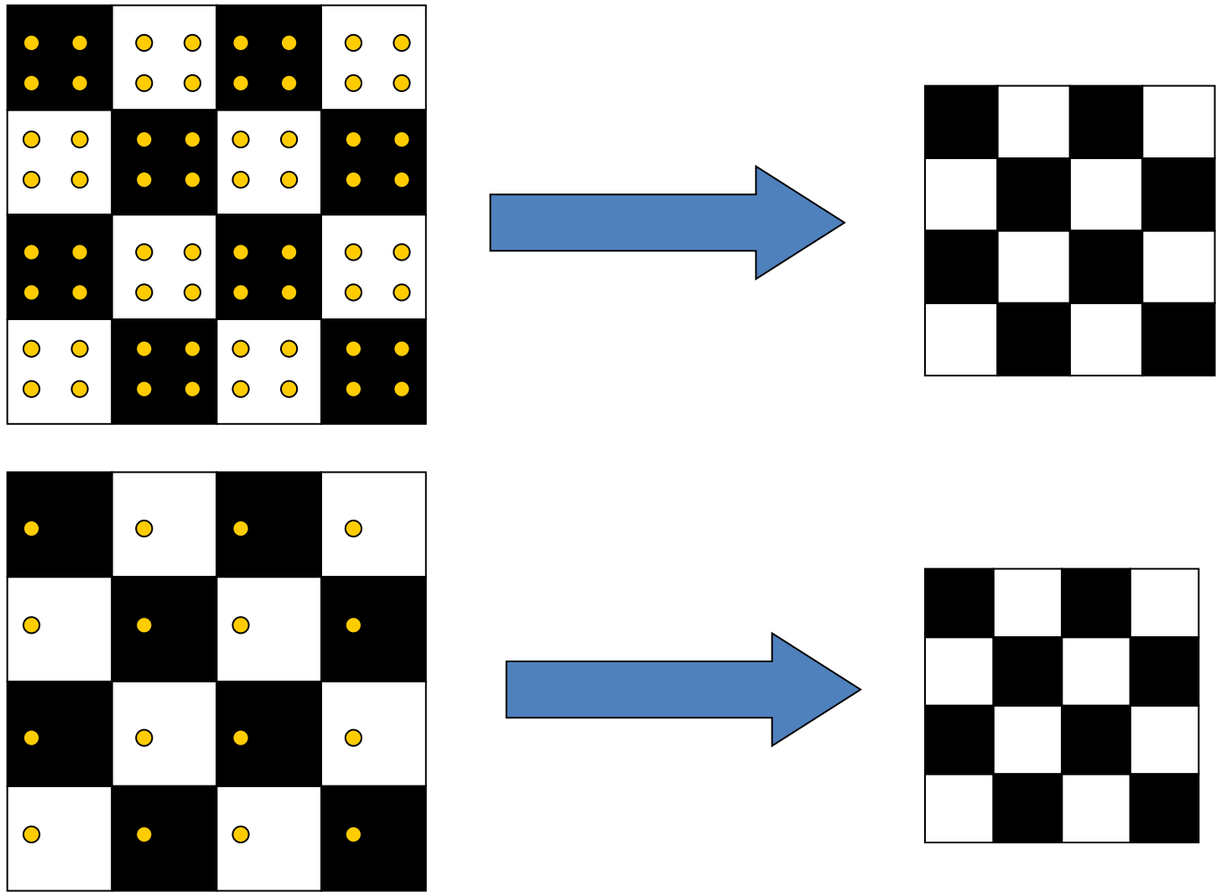
1/4 (2x zoom)

1/8 (4x zoom)

Aliasing! What do we do?

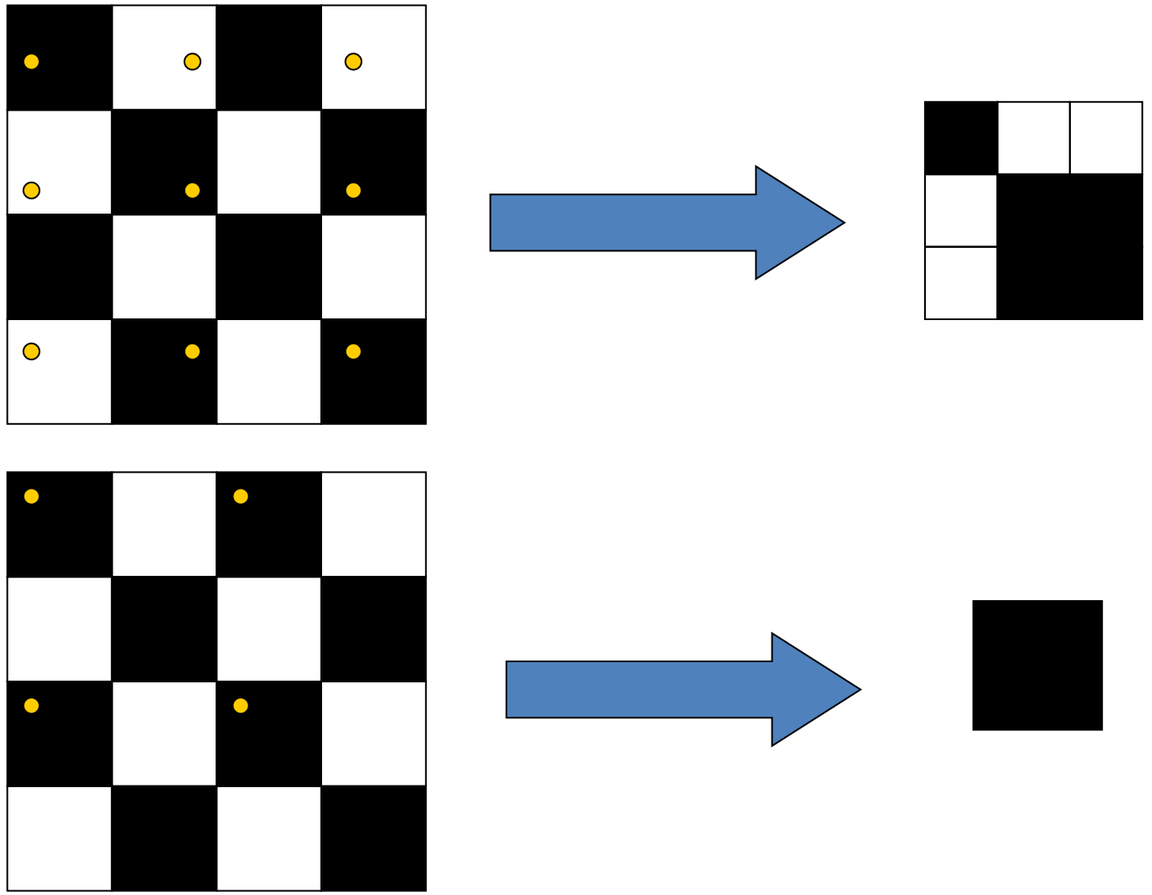
Slide by Steve Seitz

Sampling an image



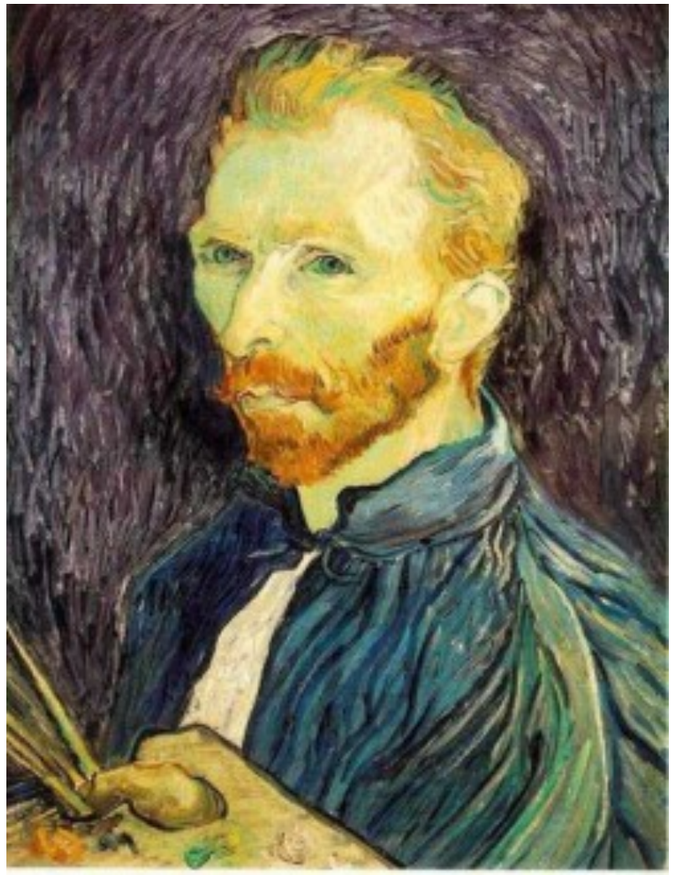
Examples of GOOD sampling

Undersampling



Examples of BAD sampling -> Aliasing

Gaussian (lowpass) pre-filtering



Gaussian 1/2

G 1/4

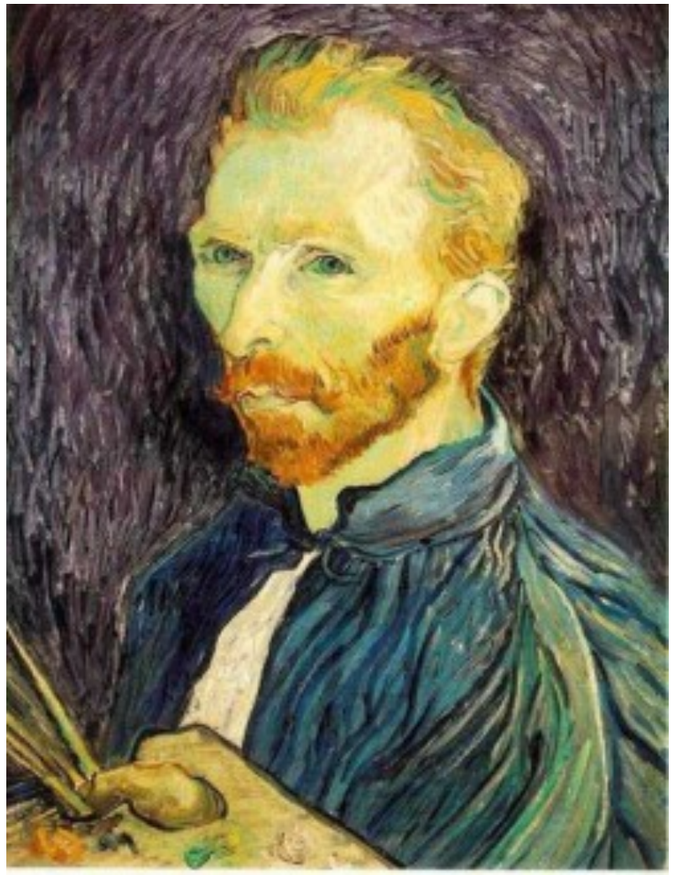
G 1/8

Solution: filter the image, *then* subsample

- Filter size should double for each ½ size reduction. Why?

Slide by Steve Seitz

Subsampling with Gaussian pre-filtering



Gaussian 1/2

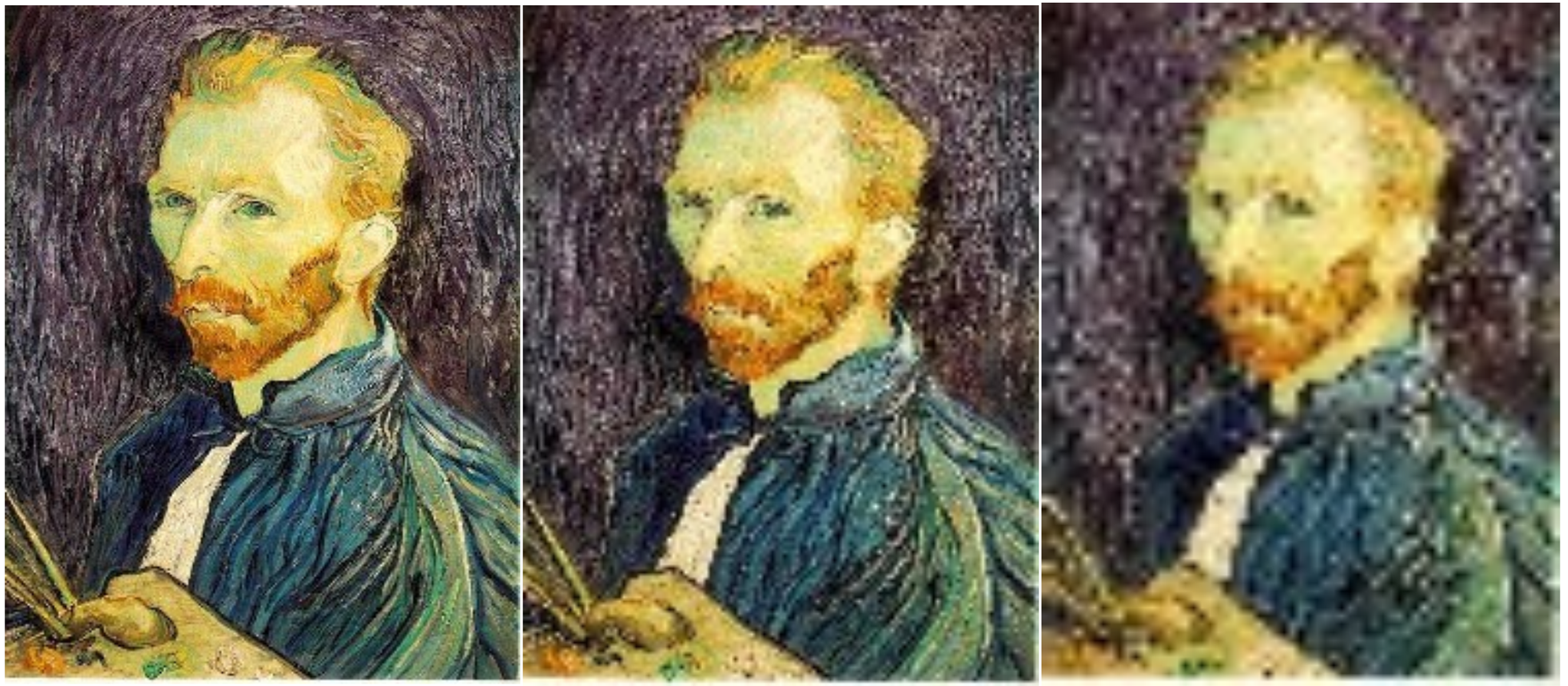
G 1/4

G 1/8

More and more neural networks (e.g., StyleGAN2) are using anti-aliasing conv layers
See Zhang. ICML 2019. “Making Convolutional Networks Shift-Invariant Again”

Slide by Steve Seitz

Compare with...

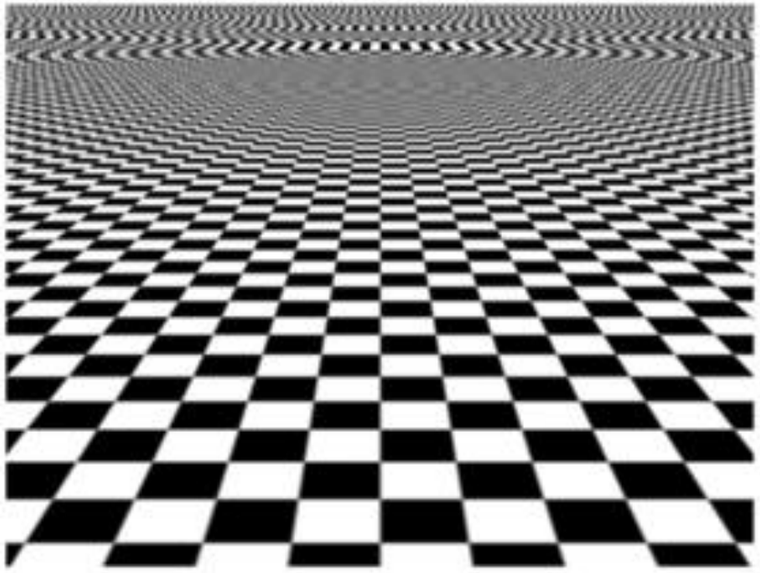


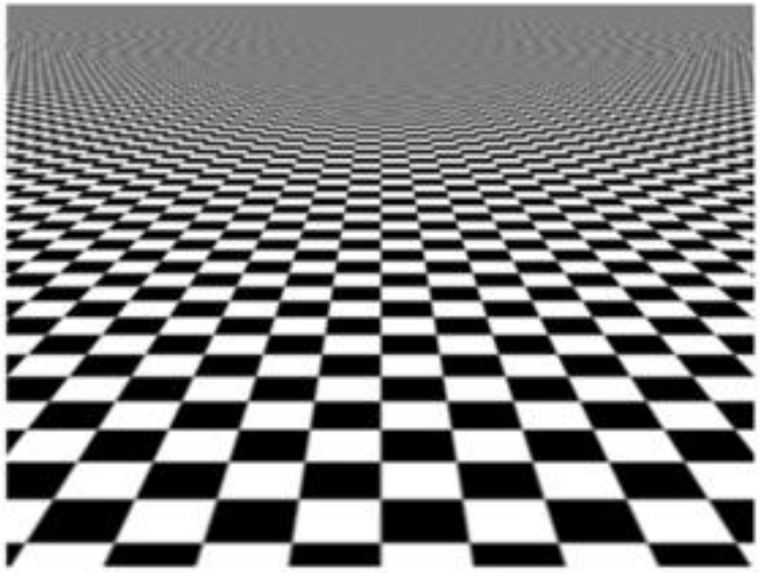
1/2 1/4 (2x zoom) 1/8 (4x zoom)

Several computer vision/deep learning libraries do not pre-fliter.
Be careful when you use them for pre-processing your images.
https://twitter.com/jaakkolehtinen/status/1258102168176951299

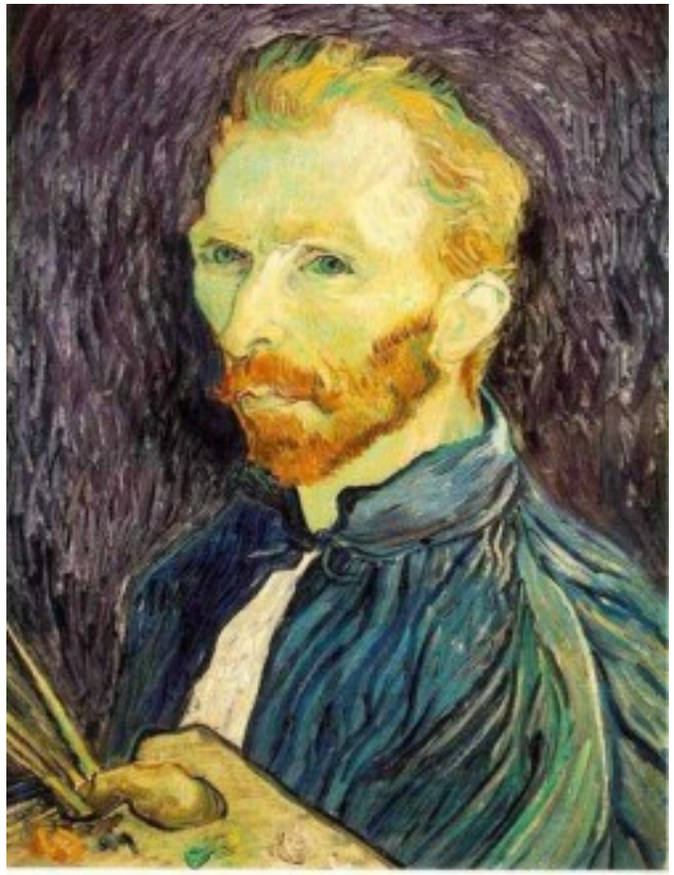
Slide by Steve Seitz

More Gaussian pre-filtering





Iterative Gaussian (lowpass) pre-filtering



Gaussian 1/2

G 1/4

G 1/8

filter the image, *then* subsample

- Filter size should double for each ½ size reduction. Why?
- How can we speed this up?

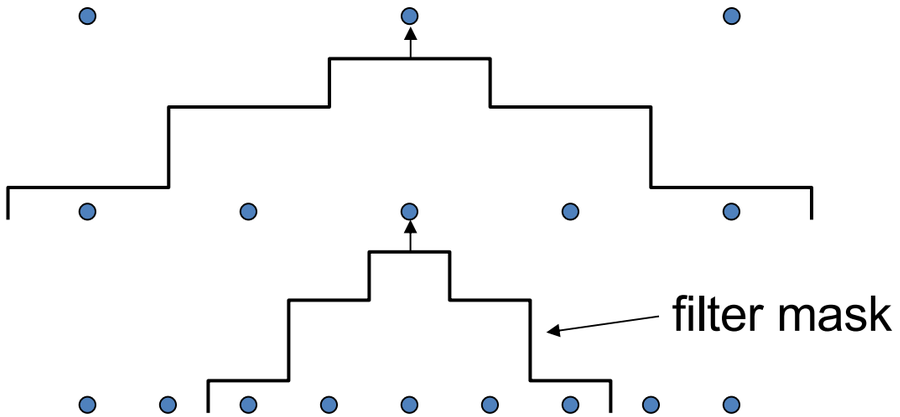
Slide by Steve Seitz

512 256 128 64 32 16 8

A bar in the big images is a hair on the zebra's nose; in smaller images, a stripe; in the smallest, the animal's nose

Figure from David Forsyth

Gaussian pyramid construction



Repeat

- Filter
- Subsample

Until minimum resolution reached

- can specify desired number of levels (e.g., 3-level pyramid)

The whole pyramid is only 4/3 the size of the original image!

Slide by Steve Seitz

What are they good for?

- Improve Search
  - Search over translations
    - Classic coarse-to-fine strategy
  - Search over scale
    - E.g., find a face at different scales

Partial derivatives with convolution

•For 2D function f(x,y), the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

•For discrete data, we can approximate using finite differences:

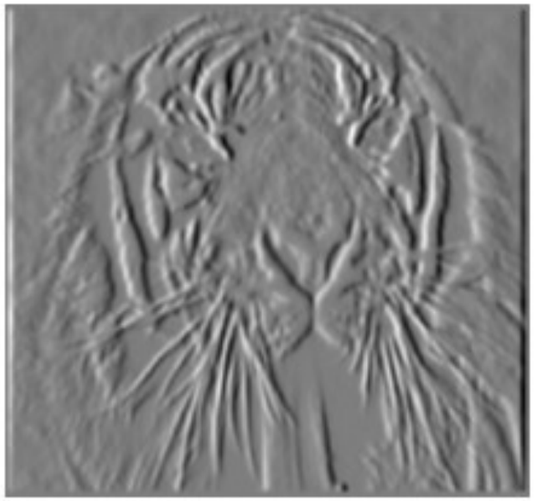
$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

•To implement above as convolution, what would be the associated filter?

Source: K. Grauman

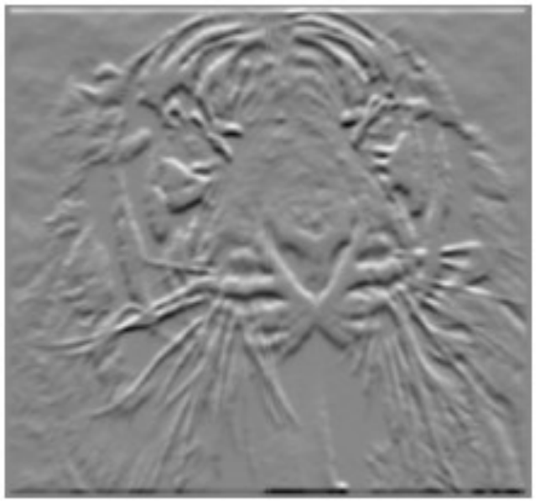
Partial derivatives of an image

$$\frac{\partial f(x,y)}{\partial x}$$



-1	1

$$\frac{\partial f(x,y)}{\partial y}$$



-1
1

or

1
-1

Which shows changes with respect to x?

Finite difference filters

- Other approximations of derivative filters exist:

Prewitt: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

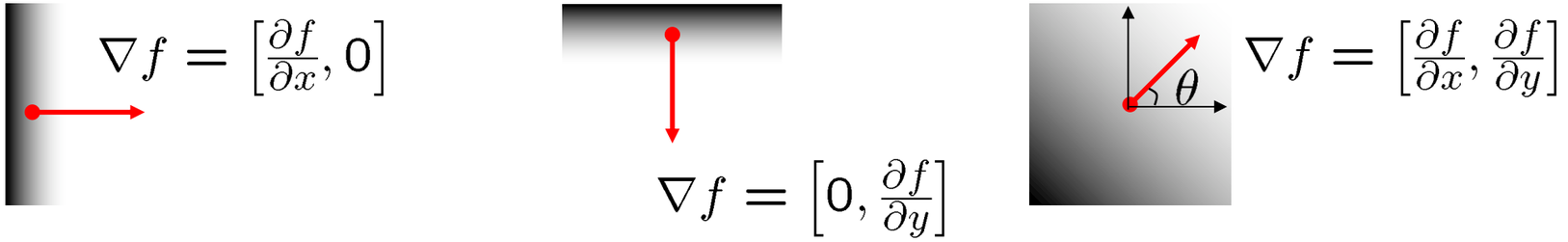
Sobel: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

Roberts: $M_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Source: K. Grauman

Image gradient

- The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$



The gradient points in the direction of most rapid increase in intensity

- How does this direction relate to the direction of the edge?

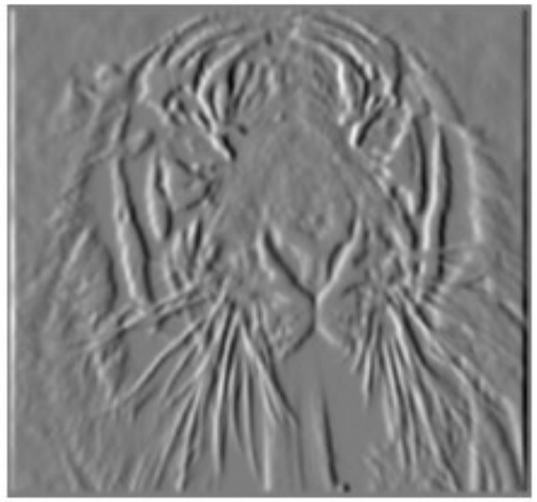
The gradient direction is given by $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

The *edge strength* is given by the gradient magnitude

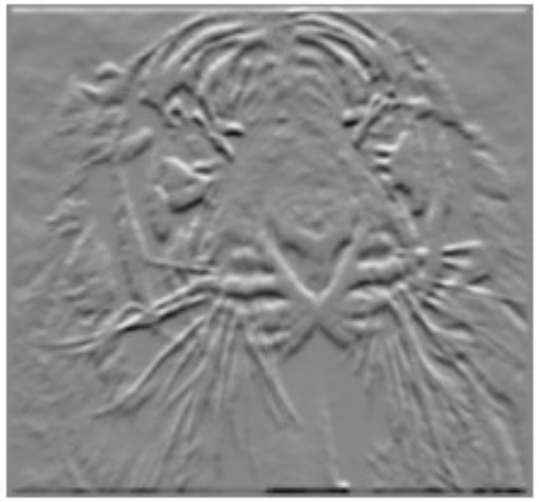
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Source: Steve Seitz

Image Gradient



$$\frac{\partial f(x,y)}{\partial x}$$

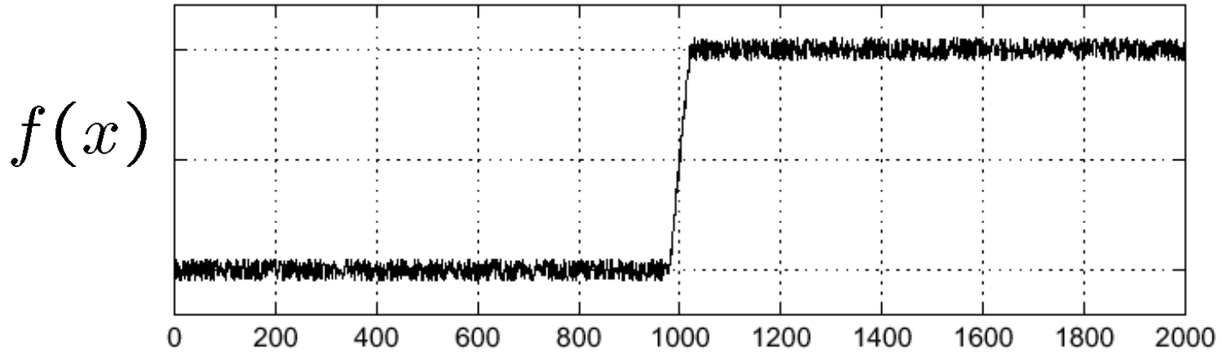


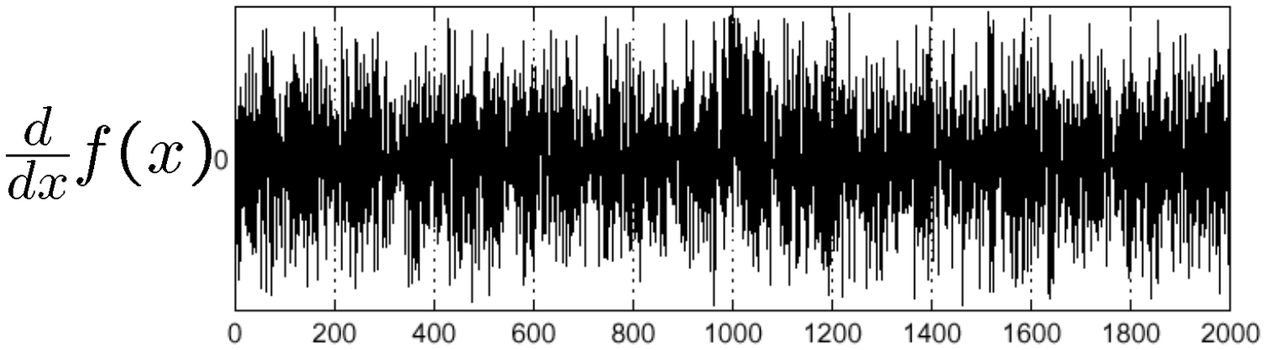
$$\frac{\partial f(x,y)}{\partial y}$$

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Effects of noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal

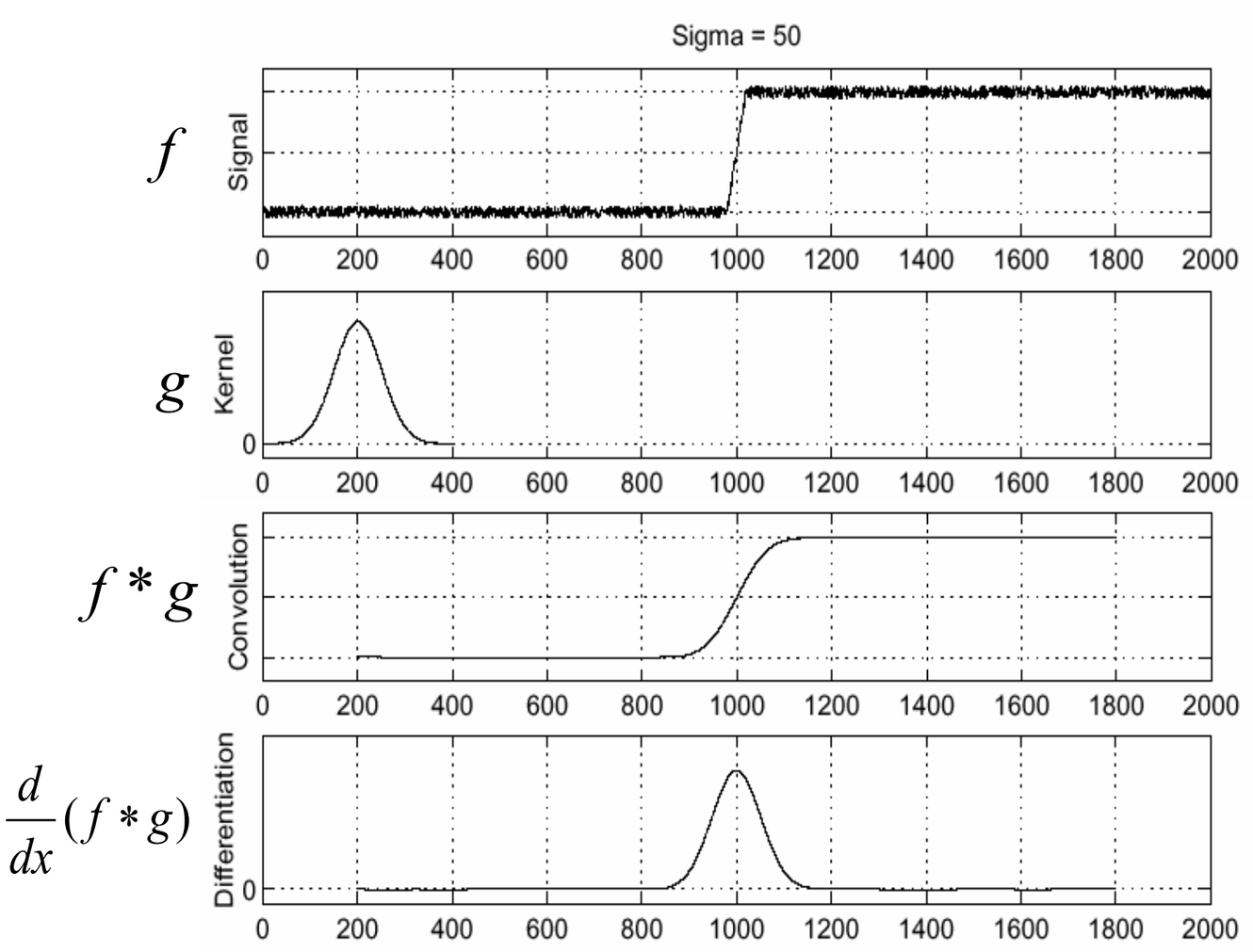




Where is the edge?

Source: S. Seitz

Solution: smooth first



- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

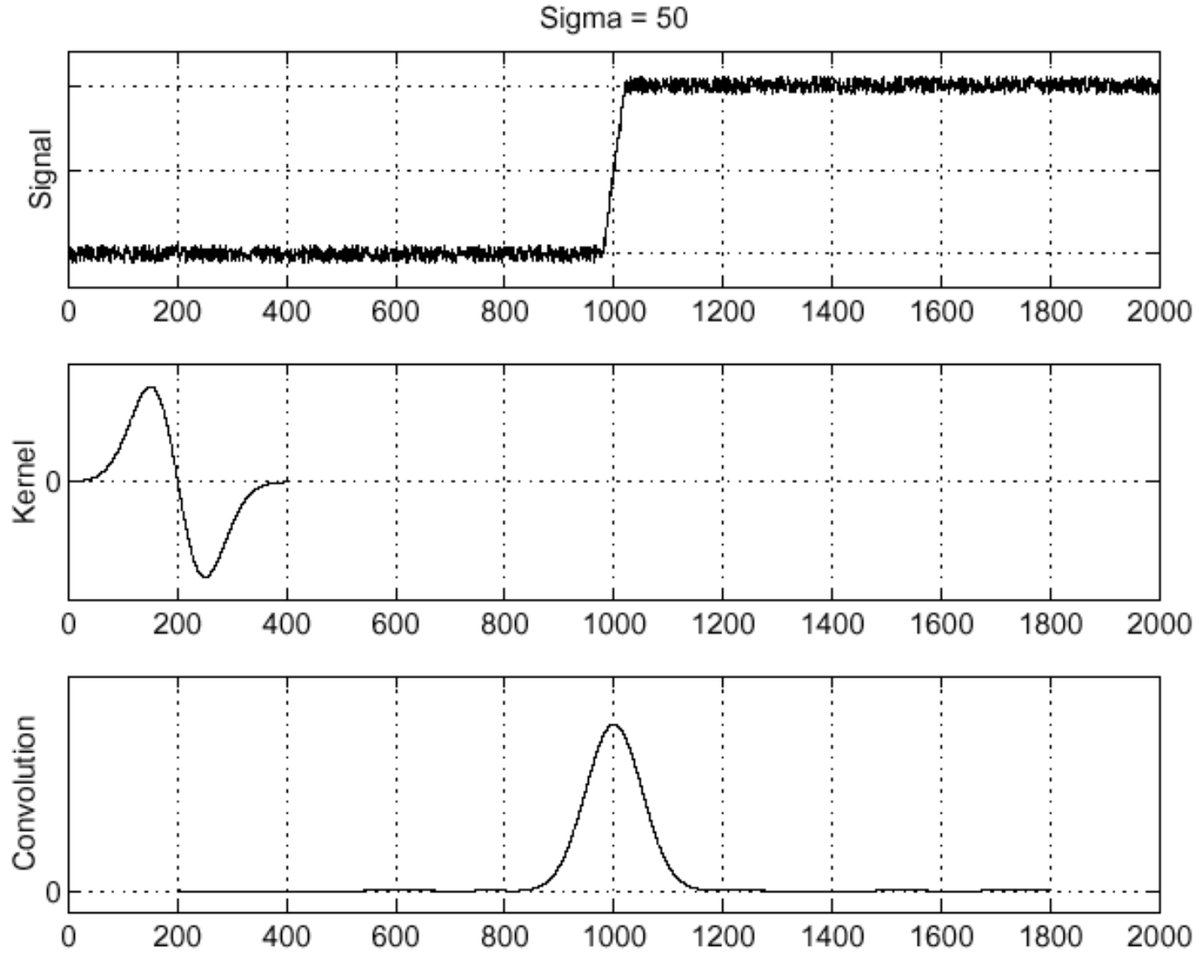
Source: S. Seitz

Derivative theorem of convolution

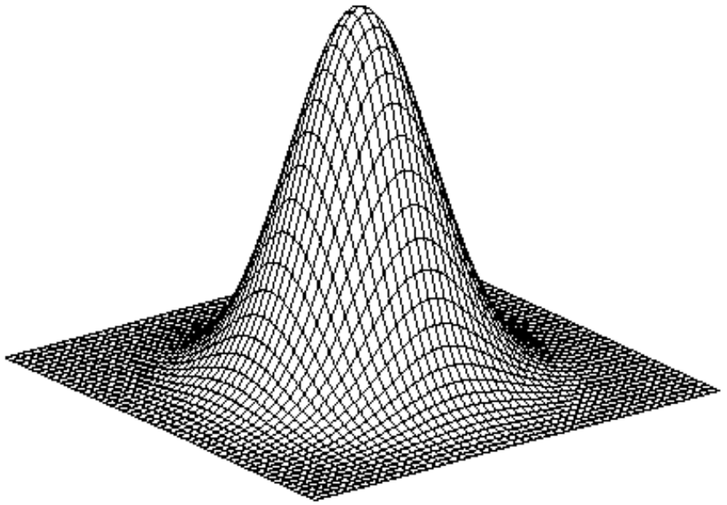
- This saves us one operation: $\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$

$\frac{\partial}{\partial x}h$

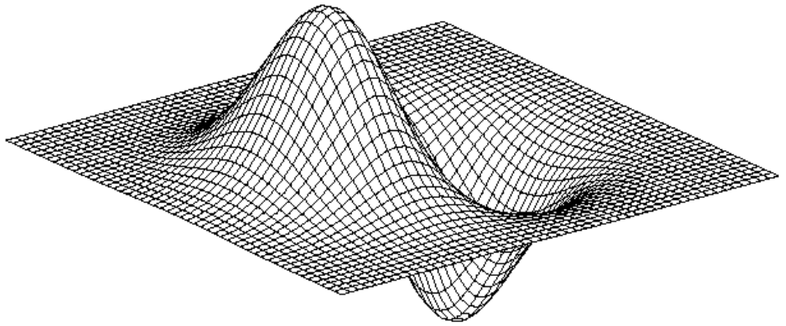
$(\frac{\partial}{\partial x}h) \star f$



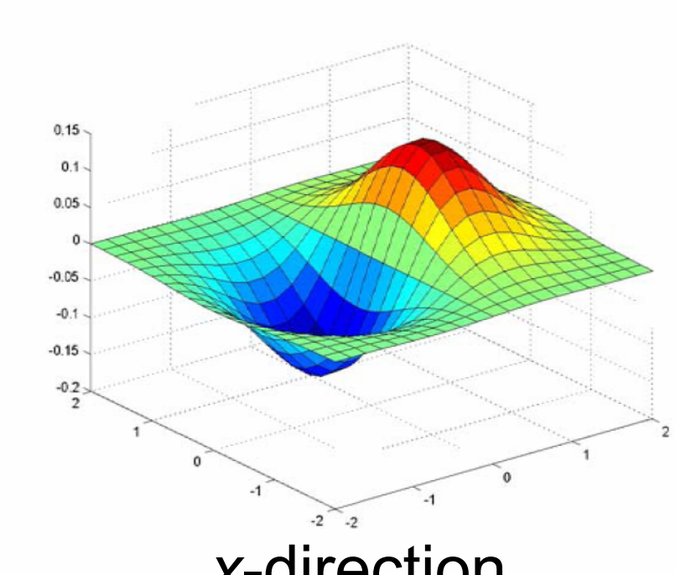
Derivative of Gaussian filter



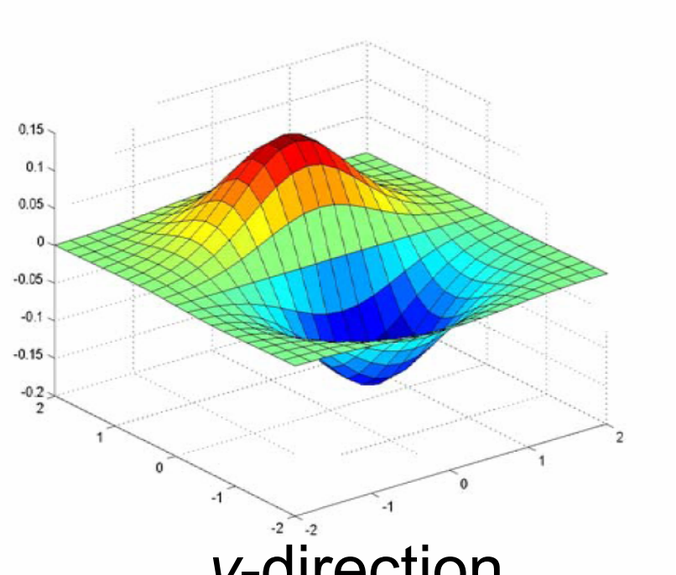
* [1 -1] =



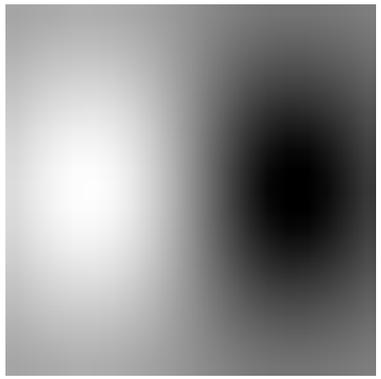
Derivative of Gaussian filter

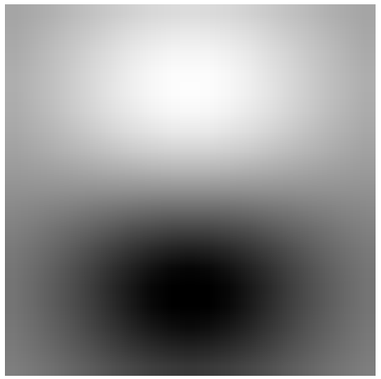


x-direction



y-direction





Practical matters

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods:
    - clip filter (black)
    - wrap around
    - copy edge
    - reflect across edge

Source: S. Marschner

Boundary artifacts in deep image synthesis
Try different padding choices.
Try reflected padding first.

Thank You!

16-726, Spring 2021

https://learning-image-synthesis.github.io/