CSE330

Theory Assignment-3

Name: Kazi Md. Al-Wakil
ID: 19301051
Section: 10

Ans. to the ques. No:-1

(a)

$f(x) = x^3 - x^2 - 11x + 12$

interval $\rightarrow [-6.2, -2.8]$

$\varepsilon = 1 \times 10^{-3}$

Now,

$$n \geqslant \frac{\log|b-a| - \log(\varepsilon)}{\log(2)} - 1$$

$$\Rightarrow n \geqslant \frac{\log|-2.8+6.2| - \log(10^{-3})}{\log(2)} - 1$$

$\Rightarrow n \geqslant 10.7313$

$\Rightarrow n \geqslant 11$

So, minimum number of iterations required ~~11~~, $n \geqslant 11$.

(6)

Now, interval → [-6.2, -2.8]

Now,

$$x_m = \frac{x_l + x_u}{2} = \frac{-6.2 - 2.8}{2}$$

$$= -4.5$$

iteration: 1

$$f(x_l) \times f(x_m) = f(-6.2) \times f(-4.5)$$

$$= -196.568 \times (-49.875)$$

$$= 9803.829 > 0$$

$$= +ve$$

So, $x_l = x_m = -4.5,$

$x_u = -2.8$

$$x_m = \frac{-4.5 - 2.8}{2} = -3.65$$

iteration: 2

$$f(-4.5) \times f(-3.65) = (-49.875) \times (-9.7996)$$

$$= +ve$$

$x_l = x_m = -3.65$

$x_u = -2.8$

$$x_m = \frac{-2.8 - 3.65}{2} = -3.225$$

iteration:3

$f(-3.65) \times f(-3.225)$

$\Rightarrow (-9.7996) \times 3.53$

$\Rightarrow -ve$

$x_l = -3.65$

$x_u = -3.225$

$$x_m = \frac{-3.65 - 3.225}{2}$$

$= -3.4375$

iteration:4

$f(-3.65) \times f(-3.4375) = (-9.7996) \times (-2.623)$

$= +ve$

$x_l = x_m = -3.4375, \; x_u = -3.225$

$$x_m = \frac{-3.4375 - 3.225}{2} = -3.331$$

iteration:5

$f(-3.4375) \times f(-3.331) = -ve$

$x_l = -3.4375, \; x_u = x_m = -3.331$

$$x_m = \frac{-3.4375 - 3.331}{2} = -3.384$$

iteration: 6

$f(-3.4375) \times f(-3.384) = (-2.623) \times (-0.979)$
$= +ve$

$x_l = x_m = -3.384$, $x_u = -3.331$

$$x_m = \frac{-3.384 - 3.31}{2} = -3.3575$$

iteration: 7

$f(-3.384) \times f(-3.3575) = (-0.979) \times (-0.1888)$
$= +ve$

$x_l = x_m = -3.3575$, $x_u = -3.331$

$$x_m = \frac{-3.3575 - 3.331}{2} = -3.344$$

iteration: 8

$f(-3.3575) \times f(-3.344) = (-0.1888) \times 0.208$
$= -ve$

$x_l = -3.3575$, $x_u = x_m = -3.344$

$$x_m = \frac{-3.3575 - 3.344}{2} = -3.35075$$

iteration : 9

$$f(-3.3575) \times f(-3.35075)$$

$$= (-0.1888) \times (0.010) = -ve$$

$$x_l = -3.3575, \quad x_u = x_m = -3.35075$$

$$x_m = \frac{-3.3575 - 3.35075}{2} = -3.354$$

Iteration : 10

$$f(-3.3575) \times f(-3.354)$$

$$= (-0.1888) \times (-0.0855)$$

$$= +ve$$

$$x_l = x_m = -3.354, \quad x_u = -3.35075$$

$$x_m = \frac{x_l + x_u}{2} = \frac{-3.354 - 3.35075}{2} = -3.352$$

Iteration : 11

$$f(-3.354) \times f(-3.352) = (-0.0855) \times (-0.0267)$$

$$= +ve$$

$$x_l = x_m = -3.352, \quad x_u = -3.35075$$

$$x_m = \frac{-3.352 - 3.35075}{2} = -3.351$$

Iteration: 12

$$f(-3.352) \times f(-3.351) = (-0.0267) \times 2.746\times10^{-3}$$
$$= -ve$$

$x_l = -3.352$, $x_u = x_m = -3.351$

$$x_m = \frac{-3.352 - 3.351}{2} = -3.3515$$

Iteration: 13

$$f(-3.352) \times f(-3.3515) = (-0.0267) \times (-0.012)$$
$$= +ve$$

$x_l = x_m = -3.3515$, $x_u = -3.351$

$$x_m = \frac{-3.3515 - 3.351}{2} = -3.35125$$

Iteration: 14

$$f(-3.3515) \times f(-3.35125)$$
$$= (-0.012) \times (-4.602\times10^{-3}) = +ve$$

$x_l = x_m = -3.35125$, $x_u = -3.351$

$$x_m = \frac{-3.35125 - 3.351}{2} = -3.351125$$

Now,

$$f(x_m) = f(-3.351125)$$
$$= -9.3 \times 10^{-4} < 1 \times 10^{-3}$$

So, $x_m = x_* = -3.351$

The root, $x_* = -3.351$ (Ans)

Ans. to the ques. No: 2

Given,

$f(x) = x^2 e^{-x} - 0.5$

$\epsilon = 1 \times 10^{-4} = 0.0001$

$x_0 = 0.2$

Now, $f'(x) = x^2 e^{-x}(-1) + e^{-x} 2x$

$= e^{-x} 2x - e^{-x} x^2$

K	x_K	$f(x_K)$	is $f(x_K) < \epsilon$
0	0.2	-0.46725	No
1	1.7853	0.03466	No
2	1.2462	-0.05396	No
3	1.4437	-0.008	No
4	1.4859	-0.00035	No
5	1.488	6.527×10^{-6}	Yes

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.2 - \frac{-0.46725}{0.29474}$$

$$= 1.7853$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.7853 - \frac{0.03466}{0.064298}$$

$$= 1.2462$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.2462 - \frac{-0.05336}{0.27016}$$

$$= 1.4437$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.4437 - \frac{-0.008}{0.18958}$$

$$= 1.4859$$

$$x_5 = 1.4859 - \frac{-0.00036}{0.17287}$$

$$= 1.488$$

Date: ..../

So, $x_* = 1.488$, the root of the equation.

(Ans)

Ans. to the ques. No:-3

(a)

$f(x) = x^6 - x^3 - 1$

Let,

$x^3 = u$

So,

$x^6 - x^3 - 1 = 0$

$\Rightarrow (x^3)^2 - x^3 - 1 = 0$

$\Rightarrow u^2 - u - 1 = 0$

$\Rightarrow u = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times (-1)}}{2}$

$= \frac{1 \pm \sqrt{5}}{2}$

$u = \frac{1+\sqrt{5}}{2}$, $u = \frac{1-\sqrt{5}}{2}$

$\Rightarrow x^3 = \frac{1+\sqrt{5}}{2}$, $x^3 = \frac{1-\sqrt{5}}{2}$

$\Rightarrow x_1 = \sqrt[3]{\frac{1+\sqrt{5}}{2}}$, $x_2 = \sqrt[3]{\frac{1-\sqrt{5}}{2}}$

$= 1.174$ $\qquad = -0.852$

(b)

$f(x) = x^6 - x^3 - 1 = 0$

Now,

$x^6 - x^3 - 1 = 0$

$\Rightarrow \cancel{x^3 \neq 1 - x^6}$

$\Rightarrow x^3 = x^6 - 1$

$\Rightarrow x = \sqrt[3]{x^6 - 1}$

$\Rightarrow g_1(x) = \sqrt[3]{x^6 - 1}$

And,

$x^6 - x^3 - 1 = 0$

$\Rightarrow x^6 = x^3 + 1$

$\Rightarrow x = \sqrt[6]{x^3 + 1}$

$\Rightarrow g_2(x) = \sqrt[6]{x^3 + 1}$

Now,

$$g_1'(x) = \frac{d}{dx}(x^6 - 1)^{\frac{1}{3}}$$

$$= \frac{2x^5}{(x^6 - 1)^{2/3}}$$

Now, For, $x_1 = 1.174$

$$\lambda = |g_1'(x_1)|$$

$$\cancel{= \left|\frac{2x^5}{\;}\right|}$$

$$= \left|\frac{2 \times (1.174)^5}{((1.174)^6 - 1)^{2/3}}\right| = 3.236 > 1,$$

$g_1(x)$ is divergent for x_1.

For, $x_2 = -0.852$

$$\lambda = \left| \frac{2 \times (-0.852)^5}{\left((-0.852)^6 - 1\right)^{2/3}} \right|$$

$= 1.238 > 1$

$g_1(x)$ is also divergent for x_2

Now,

$$g_2'(x) = \frac{d}{dx} \sqrt[6]{x^3+1}$$

$$= \frac{x^2}{2(x^3+1)^{5/6}}$$

Now, For, $x_1 = 1.174$

$$\lambda = \left| g_2'(x_1) \right|$$

$$= \left| \frac{(1.174)^2}{2\left[(1.174)^3+1\right]^{5/6}} \right|$$

$= 0.309$

Here, $0 < \lambda < 1$; linear convergent

So, $g_2(x)$ is convergent for x_1

For, $x_2 = -0.852$

$$\lambda = \left| g_2'(x_2) \right|$$

$$= \left| \frac{(-0.852)^2}{2\left[(-0.852)^3 + 1\right]^{5/6}} \right|$$

$$= 0.8102$$

Here, $0 < \lambda < 1$; linear convergent.

So, $g_2(x)$ is also convergent for x_2

So, $g_1(x)$ converges to ~~only~~ no roots and

$g_2(x)$ " " both the roots.

(Ans)

(c)

$x_0 = 60$

$g(x) = \sqrt[6]{x^3+1}$

$g(60) = 7.746$

$g(7.746) = 2.784$

$g(2.784) = 1.681$

$g(1.681) = 1.338$

$g(1.338) = 1.226$

$g(1.226) = 1.190$

$g(1.190) = 1.179$

$g(1.179) = 1.176$

$g(1.176) = 1.175$

$g(1.175) = 1.174$

$g(1.174) = 1.174$

$g(x) = x$

So, $x_* = 1.174$

So, The root $x_* = 1.174$

(Ans)

Ans. to the ques. No:- 4

$f(x) = 2x^3 + 7x^2 - 14x + 5$

$x_0 = -5.5, \; x_1 = -4.5$

$$x_{k+1} = x_k - \frac{f(x_k) \times (x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}$$

Now,

$x_2 = x_{k+1} = x_{1+1} = x_2 \quad [k=1]$

$$x_2 = x_1 - \frac{f(x_1) \times (x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$= -4.5 - \frac{27.5 \times (-4.5 + 5.5)}{27.5 - (-39)}$$

$= -4.9135 \quad ; \; f(x_2) \neq 0$

$$x_3 = x_2 - \frac{f(x_2) \times (x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$= -4.9135 - \frac{5.5382 \times (-0.4135)}{5.5382 - 27.5}$$

$\cancel{= -5.02} \quad ; \; f(x_3) \neq 0$

$= -5.0178$

$$x_4 = x_3 - \frac{f(x_3) \times (x_3 - x_2)}{f(x_3) - f(x_2)}$$

$$= -5.0178 - \frac{-1.1821 \times -0.1043}{-1.1821 - 5.5382}$$

$$= -4.9995 \quad ; \quad f(x_4) \neq 0$$

$$x_5 = x_4 - \frac{f(x_4) \times (x_4 - x_3)}{f(x_4) - f(x_3)}$$

$$= -4.9995 - \frac{0.03299 \times 0.0183}{0.03299 + 5.0178}$$

$$= -4.9996 \quad ; \quad f(x_5) \neq 0$$

$$x_6 = x_5 - \frac{f(x_5) \times (x_5 - x_4)}{f(x_5) - f(x_4)}$$

$$= -4.9996 - \frac{0.0264 \times (-10^{-4})}{0.0264 - 0.03299}$$

$$= -5.0000 \; ; \quad f(x_6) = 0$$

So, x_6 is a root of the function.

$x_* = -5.0000$.

(Ans)

Ans. to the ques. No. 5

$f(x) = xe^{x} - 1$

$\epsilon = 1 \times 10^{-5} = 0.00001$

$g(x) = e^{-x}$ $[x_0 = 0]$

$\Rightarrow g(0) = 1$; $x_0 = 0$

$\Rightarrow g(1) = 0.3679$; $x_1 = 1$

$\Rightarrow g(0.3679) = 0.6922$; $x_2 = 0.3679$

$$\hat{x}_2 = x_0 - \frac{(x_1 - x_0)^2}{x_2 - 2x_1 + x_0}$$

$$= 0 - \frac{(1-0)^2}{0.3679 - 2}$$

$$= 0.6127$$

$g(0.6127) = 0.5419$; $\hat{x}_2 = 0.6127$

$g(0.5419) = 0.5816$; $x_3 = 0.5419$

$g(0.5816) = 0.559$; $x_4 = 0.5816$

$$\hat{x}_4 = \hat{x}_2 - \frac{(x_3 - \hat{x}_2)^2}{x_4 - 2x_3 + \hat{x}_2}$$

$$= 0.6127 - \frac{(0.5419 - 0.6127)^2}{0.5816 - (2 \times 0.5419) + 0.6127}$$

$$= 0.5673$$

$g(0.5673) = 0.5671$; $x_5 = 0.5671$

$g(0.5671) = 0.5671$; $x_6 = 0.5671$

$\Rightarrow g(x) = x$

So, root of the equation, $x_* = 0.567$

(Ans)