

Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Write all answers sequentially.
- Start answering a question (not the part of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file and rename it "ID#_FirstName.TheorySection#.pdf".
- Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.

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1. Consider a set of data points $(0, 1)$, $(0.5, 1.6487)$ and $(1, 2.7183)$. Now answer the following:
 - (a) (6 marks) Compute the Lagrange bases: $l_0(x)$, $l_1(x)$ and $l_2(x)$.
 - (b) (3 marks) Write the interpolating polynomial and find the value of the interpolating polynomial at $x = 0.2$.
 - (c) (3 marks) If the data are given by the function $f(x) = e^x$, what will be the maximum error of the Lagrange polynomial function at 0.2? (use Weierstrass Approximation)
 - (d) (2 marks) Why is the Lagrange method better than the Vandermonde matrix method to find an interpolating polynomial?
 2. Consider the data points $(-1, 8)$, $(0, 4)$ and $(1, 16)$ and . Using these data and Newton's divided/difference method, answer the following:
 - (a) (6 marks) Compute the coefficients a_k for $k = 0, 1, 2$. Also write the interpolating polynomial.
 - (b) (2 marks) Using the polynomial you found in the previous part, find $p_n(0.5)$ and $p_n(-0.9)$.
 3.
 - (a) (6 marks) Compute the upper bound of the error using Cauchy's Theorem for $f(x) = \sin^2(x/2)$ with nodes $\{-\pi/3, 0, \pi/3\}$ within the interval $[-1.2, 1.2]$.
 - (b) (2 marks) Why are Chebyshev nodes an optimal choice in Interpolation?
 4. Consider the following data set:

x	$f(x)$	$f'(x)$
0.1	-0.62050	3.58502
0.2	-0.28340	3.14033

Answer the following based on the above data:

- (a) (8 marks) Compute the Hermite bases: $h_0(x)$, $h_1(x)$, $\hat{h}_0(x)$ and $\hat{h}_1(x)$.
- (b) (2 marks) Write the Hermite polynomial and find the value at $x = 0.15$.