- **Example:** Consider the function $f(x) = x^3 - 2x^2 - x + 2$.
    a. State the roots of the function $f(x)$.
    b. Construct three different fixed point function $g(x)$ such that $f(x) = 0$.
    c. Find the convergence rate for $g(x)$ constructed in the previous part, and which root it is converging to?

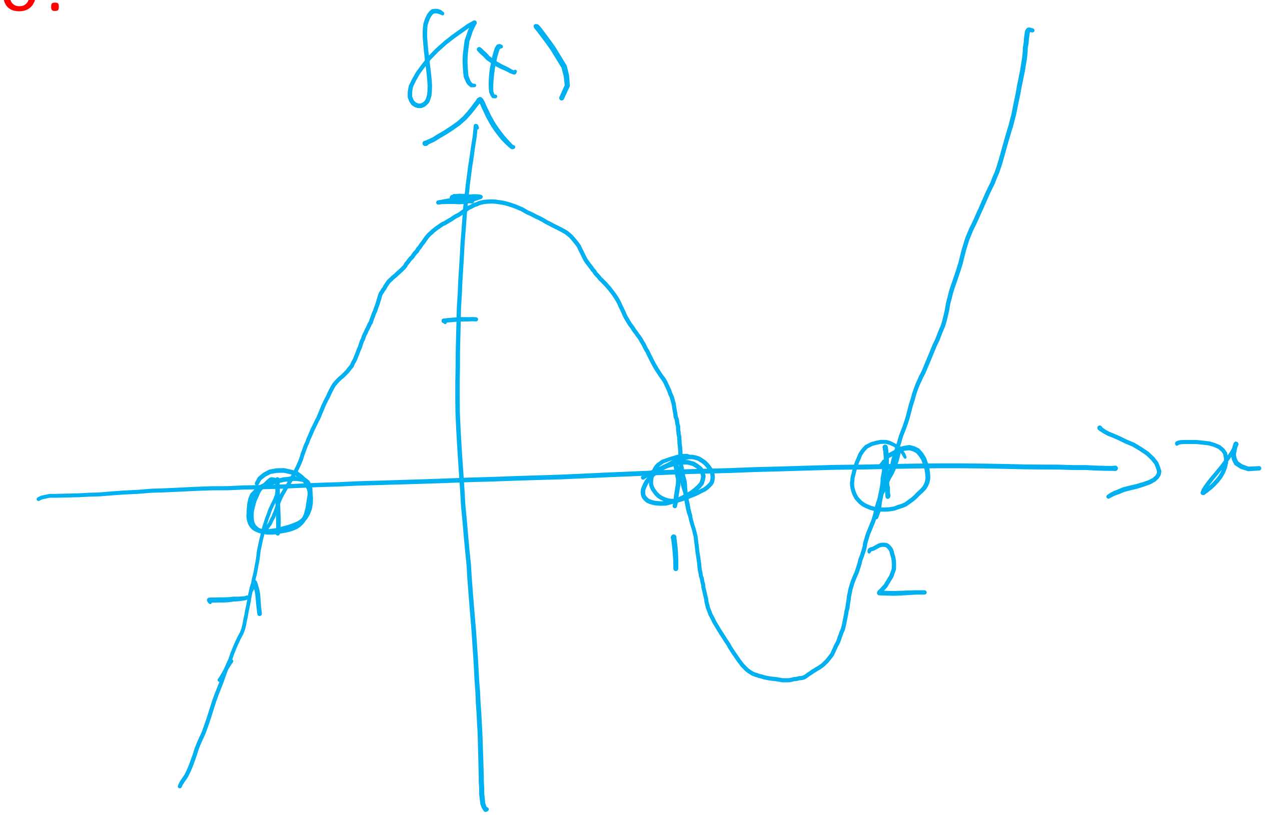
- **Solution:**

Roots : $f(x) = 0$

$x^2(x-2) - 1(x-2) = 0$

$\Rightarrow (x^2-1)(x-2) = 0$

$\Rightarrow \boxed{x_* = \pm 1, 2}$



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1(b) $x^3 - 2x^2 - x + 2 = 0$

$\Rightarrow \underbrace{x^3 - 2x^2 + 2}_{g(x)} = x \Rightarrow \boxed{g(x) = x} \longrightarrow (1)$

$x(x^2 - 2x - 1) = -2 \Rightarrow x = \dfrac{-2}{x^2 - 2x - 1} = g(x) \to (2)$

$x^3 - x + 2 = 2x^2 \Rightarrow x = \dfrac{1}{\sqrt{2}} \sqrt{x^3 - x + 2} \longrightarrow (3)$

(c) Convergence rate, $\lambda = g'(x_*) = \left| \frac{dg}{dx} \Big|_{x=x^*} \right|$

① $g(x) = x^3 - 2x^2 + 2$

$\Rightarrow \quad g'(x) = 3x^2 - 4x$

$\therefore \quad \lambda = |g'(x_*)| = \begin{cases} 7 > 1 \text{ for } x_* = -1 \Rightarrow \text{div} \\ |-1| = 1 \not< 1 \text{ for } x_* = 1 \Rightarrow \text{"} \\ 4 > 1 \text{ for } x_* = 2 \Rightarrow \text{div.} \end{cases}$

Need, $\lambda < 1$ for convergence

① $g(x) = -\dfrac{2}{x^2-2x-1}$

$\boxed{g(x) \text{ converging to } x_f = 1}$

$g'(x) = \dfrac{4(x-1)}{(x^2-2x-1)^2}$

$\therefore \lambda = |g'(x_f)| = \begin{cases} 2 > 1 & \text{for } x_f = -1 \Rightarrow \underline{\text{div}} \\ 0 & \text{for } x_f = 1 \Rightarrow \text{Superlinear} \\ 4 > 1 & \text{for } x_f = 2 \Rightarrow \text{div} \end{cases}$

③ $g(x) = \frac{1}{\sqrt{2}}\left(x^3 - x + 2\right)^{1/2}$

$$g'(x) = \frac{3x^2 - 1}{2\sqrt{2}\left(x^3 - x + 2\right)^{1/2}}$$

$$\lambda = |g'(x_x)| = \begin{cases} 0.5 < 1 \Rightarrow \text{it is linear convergence for } x_* = 1 \\ 0.5 < 1 \Rightarrow \text{linear convergen and converges to } x_* = -1 \\ 1.375 > 0 \text{ } \cancel{\text{for}} \text{ for } x = 2 \Rightarrow \text{div} \end{cases}$$