CSE330

Theory Assignment - 4

Name: Kazi Md. Al-Wakil

ID: 19301051

Sec: 10

Answer to the ques. No: 1

(a)

Given that,

$$-x_1 + x_2 - x_3 = -1$$
$$2x_1 + 6x_2 - x_3 = 3$$
$$6x_1 + 5x_2 + 3x_3 = 8$$

The system has a unique solution. because from the system we can see that there are 3 unknowns and for the 3 unknowns we have 3 equations.

From the system we can see that we can have non-zero co-efficients and we can make a matrix. After solving the matrix we will have unique values of x_1, x_2 x_3.

Now, solving the system (with calculator):

$$x_1 = \frac{7}{9}$$
$$x_2 = \frac{1}{3}$$
$$x_3 = 5/9$$

So, we can conclude that the system has unique solution.

(b)

Augmented matrix:

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 6 & -1 \\ 6 & 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 8 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} -1 & 1 & -1 & -1 \\ 2 & 6 & -1 & 3 \\ 6 & 5 & 3 & 8 \end{array}\right]$$

(Ans)

(c)

Solving the augmented matrix, we found in question 1(b) using Gaussian Elimination method:

Date: ..../............/............

$$\left[\begin{array}{ccc|c} -1 & 1 & -1 & -1 \\ 2 & 6 & -1 & 3 \\ 6 & 5 & 3 & 8 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} -1 & 1 & -1 & -1 \\ 0 & 8 & -3 & 1 \\ 0 & 11 & -3 & 2 \end{array}\right] \quad \begin{array}{l} R_2 = R_2 + 2\ R_1 \\ R_3 = R_3 + 6\ R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} -1 & 1 & -1 & -1 \\ 0 & 1 & -\frac{3}{8} & \frac{1}{8} \\ 0 & 11 & -3 & 2 \end{array}\right] \quad R_2 = R_2/8$$

$$= \left[\begin{array}{ccc|c} -1 & 1 & -1 & -1 \\ 0 & 1 & -\frac{3}{8} & \frac{1}{8} \\ 0 & 0 & \frac{9}{8} & \frac{5}{8} \end{array}\right] \quad R_3 = R_3 - (11 \times R_2)$$

$$= \left[\begin{array}{ccc|c} -1 & 1 & -1 & -1 \\ 0 & 1 & -\frac{3}{8} & \frac{1}{8} \\ 0 & 0 & 1 & \frac{5}{9} \end{array}\right] \quad R_3 = R_3 \times \frac{8}{9}$$

Now,

$$x_3 = \frac{5}{9} \quad \text{--- (1)}$$

$$x_2 - \frac{3}{8}x_3 = \frac{1}{8}$$

$$\Rightarrow x_2 = \frac{1}{8} + \frac{3}{8}x_3$$

$$\Rightarrow x_2 = \frac{1}{8} + \frac{3}{8} \times \frac{5}{9}$$

$$\Rightarrow x_2 = \frac{1}{3} \quad \text{---- (11)}$$

And,

$$-x_1 + x_2 - x_3 = -1$$

$$\Rightarrow x_2 - x_3 + 1 = x_1$$

$$\Rightarrow x_1 = \frac{1}{3} - \frac{5}{9} + 1$$

$$\Rightarrow x_1 = \frac{7}{9}$$

So, $x_1 = \frac{7}{9}$, $x_2 = \frac{1}{3}$, $x_3 = \frac{5}{9}$

(Ans)

Ans to the ques. No: 2

Given that, (a)

x	y
0	3
-1	7
1	-2
3	-4

Here,

$f(0) = 3,$

$f(-1) = 7$

$f(1) = -2$

$f(3) = -4$

Now,

$P_2(x) = a_0 + a_1 x + a_2 x^2$

Now,

$P_2(0) = a_0 = 3$

$P_2(-1) = a_0 - a_1 + a_2 = 7$

$P_2(1) = a_0 + a_1 + a_2 = -2$

$P_2(3) = a_0 + 3a_1 + 9a_2 = -4$

Now,

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -2 \\ -4 \end{bmatrix}$$

$\Rightarrow \quad A \quad x = b$

Now,

~~$A^T A =$~~

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 3 & 9 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 1 & 1 & 9 \end{bmatrix}$$

$$b = \begin{bmatrix} 3 \\ 7 \\ -2 \\ -4 \end{bmatrix}$$

Now,

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 1 & 1 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 3 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & 11 \\ 3 & 11 & 27 \\ 11 & 27 & 83 \end{bmatrix}$$

Now,

$$A^T b = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 1 & 1 & 9 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \\ -2 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -21 \\ -31 \end{bmatrix}$$

(b)

Solving with LU decomposition:

Now,

$$V = A^T A$$

$$V = \begin{bmatrix} 4 & 3 & 11 \\ 3 & 11 & 27 \\ 11 & 27 & 83 \end{bmatrix} = A^{(1)} \quad \Rightarrow \quad \begin{aligned} R_2 &= R_2 - \left(\frac{3}{4}\right) R_1 \\ R_3 &= R_3 - \left(\frac{11}{4}\right) R_1 \end{aligned}$$

Now,

$$F^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 \\ -\frac{11}{4} & 0 & 1 \end{bmatrix}$$

$$A^2 = F^{(1)} \times A^{(1)}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 \\ -\frac{11}{4} & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & 11 \\ 3 & 11 & 27 \\ 11 & 27 & 83 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & 11 \\ 0 & 35/4 & 75/4 \\ 0 & 75/4 & 211/4 \end{bmatrix} \quad \Rightarrow \quad \begin{aligned} R_3 &= R_3 - \left(\frac{75}{4} \times \frac{4}{35}\right) R_2 \\ &= R_3 - \left(\frac{15}{7}\right) R_2 \end{aligned}$$

$$F^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{15}{7} & 1 \end{bmatrix}$$

Now,

$$U = A^{(3)} = F^{(2)} A^{(2)}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{15}{7} & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & 11 \\ 0 & \frac{35}{4} & \frac{75}{4} \\ 0 & \frac{75}{4} & \frac{211}{4} \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & 11 \\ 0 & \frac{35}{4} & \frac{75}{4} \\ 0 & 0 & 88/7 \end{bmatrix}$$

Now,

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{4} & 1 & 0 \\ \frac{11}{4} & \frac{15}{7} & 1 \end{bmatrix}$$

Now,

$$Lx = A^T b$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{4} & 1 & 0 \\ \frac{11}{4} & \frac{15}{7} & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -21 \\ -31 \end{bmatrix}$$

Here,

$\Rightarrow x_0 = 4$ --- (1)

$\frac{3}{4} x_0 + x_1 = -21$

$\Rightarrow x_1 = -24$

And,

$\frac{11}{4} x_0 + \frac{15}{7} x_1 + x_2 = -31$

$\Rightarrow x_2 = \frac{66}{7}$

Now,

~~Ha~~

$$\begin{bmatrix} 4 & 3 & 11 \\ 0 & \frac{35}{4} & \frac{75}{4} \\ 0 & 0 & \frac{88}{7} \end{bmatrix} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -24 \\ \frac{66}{7} \end{bmatrix}$$

Now,

$$\frac{88}{7} a_2 = \frac{66}{7}$$

$$\Rightarrow a_2 = \frac{3}{4} \quad \text{--- (i)}$$

Then,

$$\frac{35}{4} a_1 + \frac{75}{4} a_2 = -24$$

$$\Rightarrow \frac{35}{4} a_1 = -\frac{609}{16}$$

$$\Rightarrow a_1 = -\frac{87}{20} \quad \text{--- (ii)}$$

And,

$$4a_0 + 3a_1 + 11a_2 = 4$$

$$\Rightarrow 4a_0 = \frac{44}{5}$$

$$\Rightarrow a_0 = \frac{11}{5}$$

So, $a_0 = \frac{11}{5}$, $a_1 = -\frac{87}{20}$, $a_2 = \frac{3}{4}$

(Ans)