Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Write all answers sequentially.
- Start answering a question (not the pat of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file.
- Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.

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1. In the classes, there are three forms of floating number representation,

$$\text{Lecture Note Form} \quad : \quad F = \pm(0.d_1 d_2 d_3 \cdots d_m)_\beta \, \beta^e \ , \tag{1}$$

$$\text{Normalized Form} \quad : \quad F = \pm(1.d_1 d_2 d_3 \cdots d_m)_\beta \, \beta^e \ , \tag{2}$$

$$\text{Denormalized Form} \quad : \quad F = \pm(0.1 d_1 d_2 d_3 \cdots d_m)_\beta \, \beta^e \ , , \tag{3}$$

   where $d_i, \beta, e \in \mathbb{Z}$, $0 \leq d_i \leq \beta - 1$ and $e_{\min} \leq e \leq e_{\max}$. Now, let's take, $\beta = 2$, $m = 4$ and $-3 \leq e \leq 6$. Based on these, answer the following:

   (a) (3 marks) What are the maximum numbers that can be stored in the system by the three forms defined above?

   (b) (3 marks) What are the non-negative minimum numbers that can be stored in the system by the three forms defined above?

   (c) (4 marks) Using Eq.(1), find all the decimal numbers for $e = -1$, plot them on a real line and show if the number line is equally spaced or not.

2. Let $\beta = 2$, $m = 4$, $e_{\min} = -1$ and $e_{\max} = 2$. Answer the following questions:

   (a) (2 marks) Compute the minimum of $|x|$ for normalized form.

   (b) (2 marks) Compute the Machine Epsilon value for the normalized form.

   (c) (2 marks) State what you can see about the relation between Machine Epsilon value and the exponent.

   (d) (2 marks) Compute the Machine Epsilon value for the denormalized form.

   (e) (2 marks) Compute the maximum delta value for the form given in Eq.(1).

3. Consider the function $f(x) = xe^x$. In the following, the interpolating polynomial, $p_3(x)$, is computed by using Taylor expansion. To do so, do the following tasks:

   (a) (2 marks) Using Taylor expansion write $f(x)$ as an infinite series.

   (b) (2 marks) Find the values of a_0, a_1, a_2 and a_3 if the function is interpolated by a degree three polynomial $p_3(x)$.

   (c) (2 marks) Compute $f(0.1)$ and $p_3(0.1)$ up to seven significant figures.

   (d) (2 marks) Find the percent error for interpolating $f(x)$ by $p_3(x)$.

4. Consider the same function $f(x) = xe^x$ again. Now, we are going to find the interpolating polynomial by using the Vandermonde matrix method:

   (a) (1 mark) Construct the Vandermonde matrix V if $f(x)$ passes through the nodes -1, 0 and 1.

   (b) (1 mark) Compute: $\det(V)$.

   (c) (4 marks) Find the inverse matrix V^{-1}.

   (d) (2 marks) The coefficients a_0, a_1, a_2 of the interpolating polynomial $p_2(x)$ can be computed by using the equation $A = V^{-1}F$, where the symbols have their usual meanings. Using V^{-1} and the function values at the nodal points, find the values of A up to five decimal places.

   (e) (2 marks) Write down the expression for $p_2(x)$. Also compute $p_2(0.25)$ and $f(0.25)$ up to five decimal places.

   (f) (2 marks) Find the percent error of interpolating $f(0.25)$ by $p_2(0.25)$.

☺☺☺ Yes!!!!! Let the PHUN begin!!! ☺☺☺