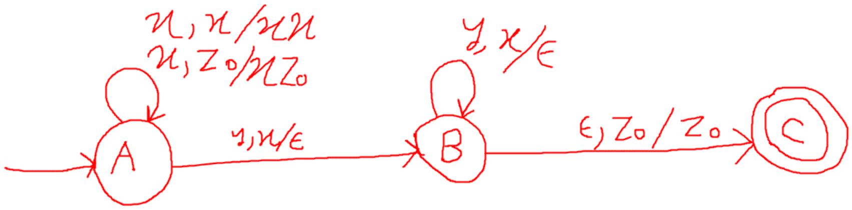
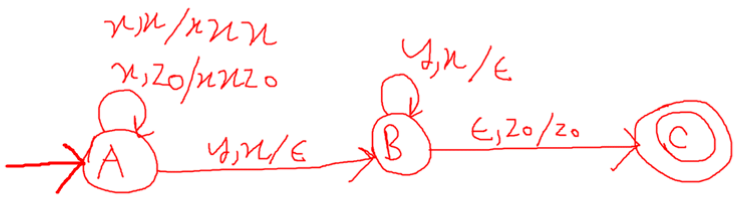
PDA Design

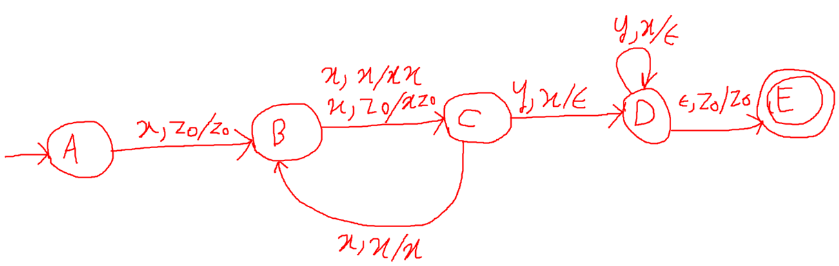
a) $\sum = \{x, y\}$, $L = \{ x^n y^n \mid n \geq 1 \}$ Hints: Push x's, Then pop x's for y's in input string



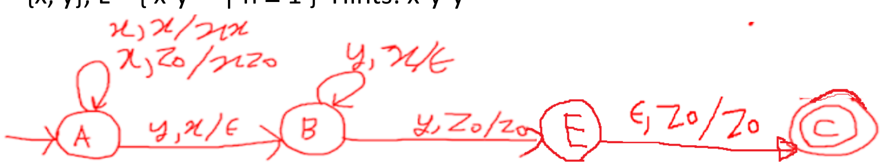
b) $\sum = \{x, y\}$, $L = \{ x^n y^{2n} \mid n \geq 1 \}$ Hints: for each x in input, push two 2 x's for having same no of x's & y's. Then pop x's for y's in input string



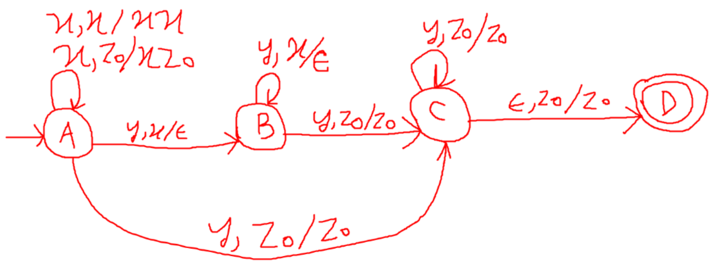
c) $\sum = \{x, y\}$, $L = \{ x^{2n} y^n \mid n \geq 1 \}$ Hints: for every 2 x's, push only 1 x



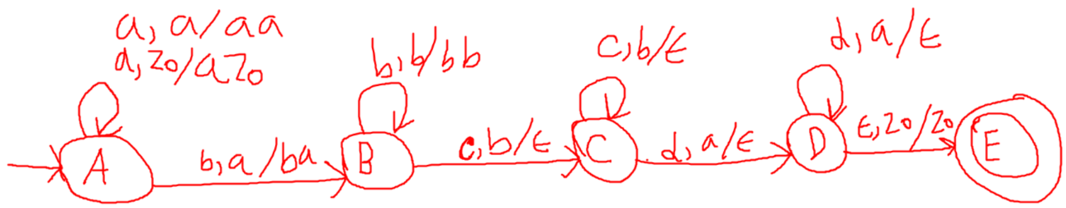
d) $\sum = \{x, y\}$, $L = \{ x^n y^{n+1} \mid n \geq 1 \}$ Hints: $x^n y^n y$



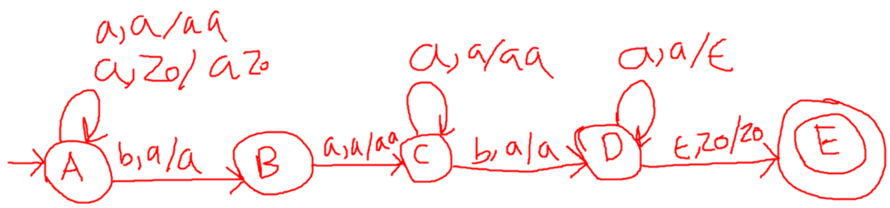
e) $\sum = \{x, y\}$, $L = \{ x^m y^n \mid n > m \geq 0\}$ Hints: m>=0, n>m, n>=m+1. So, $x^m y^n = x^m y^{m+1}$ y….. (at least one extra y at end of input string)



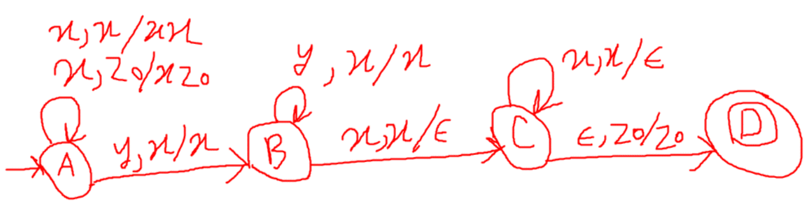
f) $\Sigma = \{a, b, c, d\}$, L={a^m b^n c^n d^m | m, n >=1 } Hints: a,d pair, b,c pair Push a's. Push b's. Pop b's for c's in input string. Pop a's for d'd in input string



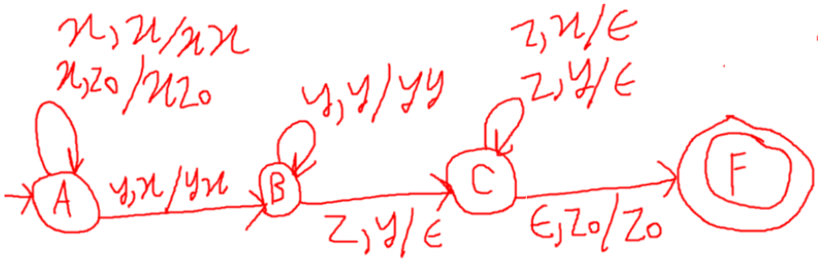
g) $\Sigma = \{a, b\}$, L={a^n b a^m b a^{m+n} | m, n >=1 } Hints: a^n b a^m b a^m a^n. Push all a's, ignore b. Push all a's, ignore b. pop a's for a's in input



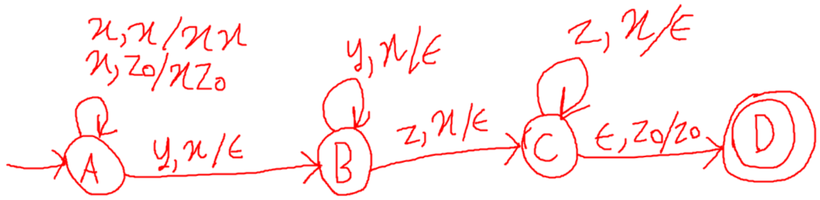
h) $\sum$ = {x, y}, L = { $x^n y^m x^n$ | m,n ≥ 1 } Hints: push all x's. Then transition for y. Ignore y's. pop x's for x's in input string



i) $\sum$ = {x, y, z}, L= { $x^n y^m z^{(n+m)}$ | n,m≥1} Hints: push all x's & y's. Then pop x's & y's for z's OR it can be solved with this logic x^n $x^m Z^m$ z^n

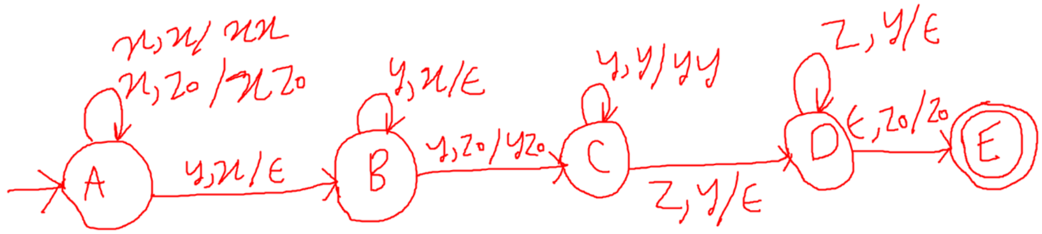


j) $\sum$ = {x, y, z}, L= { $x^{(n+m)}y^mz^n$ | n,m≥1} Hints: push all x's. Pop x's for y's & pop x's for z's **<u>sequentially.</u>**
OR it can be solved with this logic $x^nx^my^mz^n$

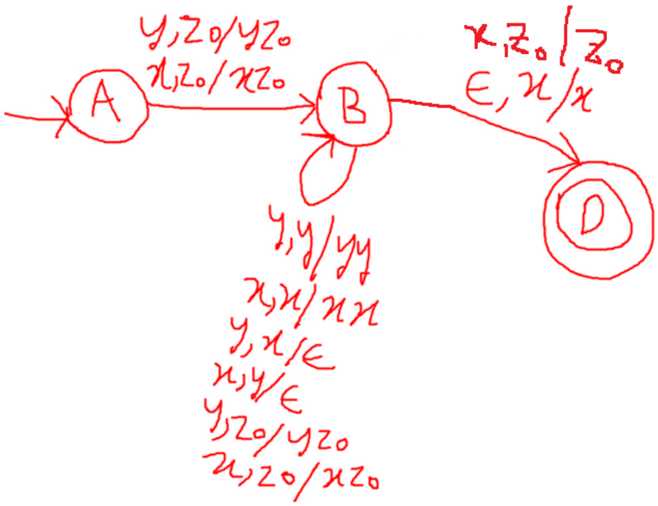


You can also follow the method of (i) for this.

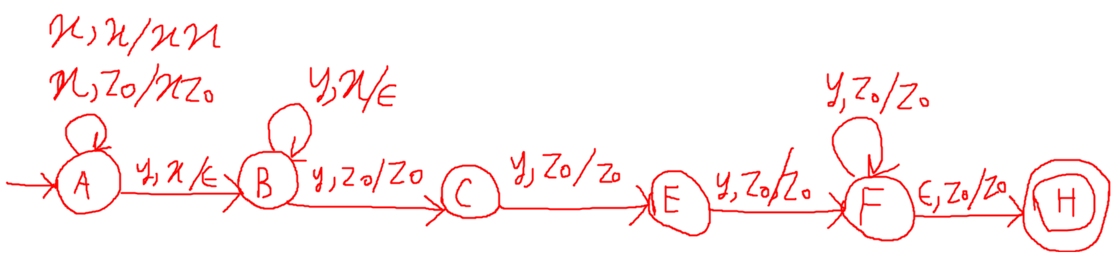
k) $\sum$ = {x, y, z}, L= { $x^ny^{(n+m)}z^m$ | n,m≥1} Hints: x^ny^n y^mz^m push all x's, then pop x's for y's. when stack empty, push all y's, then pop y's for z's.



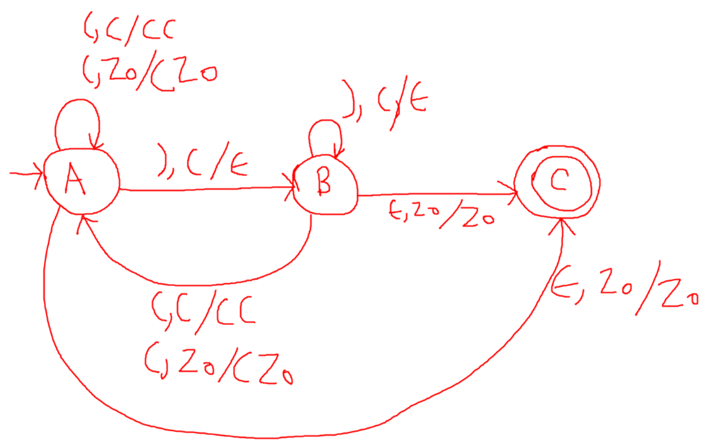
l) $\sum$ = {x, y}, L = {no of x's are greater than the no of y's} Hints: push all x's and y's. Pop x's for y's (at least one x extra at end)



m) $\sum$ = {x, y}, L = { $x^n y^m$ | m,n ≥ 1 & m > n+2}
Hints: minimum value of n is 1. So, m >n+2, m>1+2, m>3. minimum no of x is 1 & minimum no of y is 4.
Again, m>n+2 , m>=n+3
Push all x's in stack, then pop x's for y's. Then you should have at least 3 extra y's in the input String.



n) Design a PDA to match parenthesis. Hints: Push all ('s. Pop '(' for ')' in the string.



Conceptual Question:

1.The Pushdown automation is essentially an ε-NFA with the addition of a stack.

2. A PDA chooses its next move based on its current state, the next input symbol, and the symbol at the top of its stack. It may also choose to make a move independent of the input symbol and without consuming that symbol from the input. Being nondeterministic, the PDA may have some finite number of choices of move; each is a new state and a string of stack symbols with which to replace the symbol currently on top of the stack.

3. There are two ways in which we may allow the PDA to signal acceptance. One is by entering an accepting state; the other by emptying its stack. These methods are equivalent in the sense that any language accepted by one method is accepted by the other method.

4. The transition function of PDA takes three arguments.

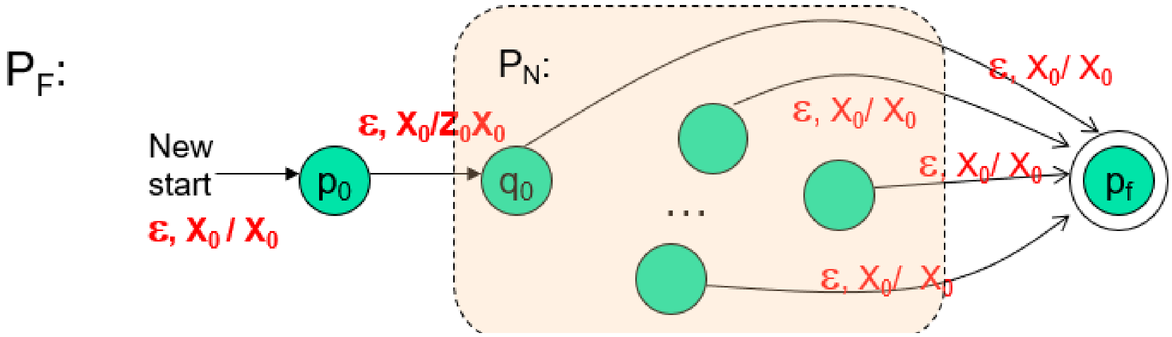
a) A state, in Q.

b) An input, which is either a symbol in $\sum$ or ε.

c) A stack symbol in Γ.

5. Here P_N = empty stack PDA and P_F = Final state PDA

- Whenever P_N's stack becomes empty, make P_F go to a final state without consuming any addition symbol
- To detect empty stack in P_N: P_F pushes a new stack symbol X_0 (not in Γ of P_N) initially before simulating P_N



6. Here P_N = empty stack PDA and P_F = Final state PDA

- Whenever P_F reaches a final state, just make an ε -transition into a new end state, clear out the stack and accept
- Danger: What if P_F design is such that it clears the stack midway *without* entering a final state?

    to address this, add a new start symbol X_0 (not in Γ of P_F)