CNF

Question -1) Remove unit productions from the CFG below

a) W' -> TWY | t | TW | WY
   W -> TWY | t | TW | WY
   T -> tTW | t | tW
   Y ->WyW | tTW | t | tW | yy

b) S' -> RSR | rT | RS | SR | r
   S -> RSR | rT | RS | SR | r
   R -> t | RSR | rT | RS | SR | r
   T -> t

c) S -> aAs | a | bb | AS | A | aa | SbS
   A -> aAs | a | bb | AS
   B -> A | aa | SbS
   C -> c

Question -2) Eliminate null productions from the Cfg below

a) W' -> W
   W -> TWY | t | TW | WY
   T -> tTW | t | tW
   Y ->WyW|T|yy

b) S -> A | B
   A -> aAs | a | bb | aS
   B -> A | aa | SBS | SS
   C -> c

c) S' -> S
   S -> RSR | rT | R
   R -> T
   T -> t

Question -3) Eliminate useless variables from the Cfg below

a) S -> aAa | bBb| aa
   A -> a
   B -> b
   C -> a | b | ab

b) W -> TW | t | TW
   T -> tTW | t | ε

c) S -> A | B
A -> aAs | a | bb | ε
B -> A | aa | SBS | ε

Question -4) Convert the following CFG to CNF (Chomsky Normal form).

a) Start symbol S

$$
\begin{aligned}
S &\rightarrow aAa \mid bBb \mid \epsilon \\
A &\rightarrow C \mid a \\
B &\rightarrow C \mid b \\
C &\rightarrow CDE \mid \epsilon \\
D &\rightarrow A \mid B \mid ab
\end{aligned}
$$

Step -1: Eliminate ε-productions

First, find the nullable variables. Here, C,B,A,S,D are nullable.

Let's remove nullability one by one.

First, removing nullability of C. (here in right side C, D both are nullable)

C CDE | ε

C εDE | CεE | εεE| CDE

C DE| CE| E |CDE

Now remove nullability of A

A C | a

Removing B's nullability

B C | b

Remove D's nullability

D A | B | ab

Remove S's nullability

S aAa | bBb | ε

S aAa | bBb | aεa | bεb

S aAa | bBb | aa | bb

After step 1, we get

S aAa | bBb | aa | bb

A C | a

B C | b

C CDE | DE |CE |E

D A | B | ab

Step -2: Eliminate unit production

S aAa | bBb| aa | bb

A CDE | DE |CE |a

B CDE | DE | CE | b

C CDE | DE |CE (since there is no production rule for E, eliminating variable E)

D CDE | DE | CE | a | b| ab

Step -3: eliminate variables that derives no terminal string :Variable-C

S aAa | bBb | aa | bb

A a

B b

D a | b| ab

Step -4: eliminate unreachable variables: D

S aAa | bBb | aa | bb

A a

B b

Step-5: convert to CNF: The grammar is clean now. Let's convert it to CNF now.

S XA | YB | AA | BB

A a

B b

X AA

Y BB

---

b) Start symbol S

$$
\begin{aligned}
S &\rightarrow AB \mid CA \\
A &\rightarrow a \\
B &\rightarrow BC \mid AB \\
C &\rightarrow aB \mid b
\end{aligned}
$$

Step -1: eliminate epsilon production :none

Step -2: Eliminate unit production.: none

Step -3: Eliminate variables that generates no terminal string: B, remove remove everything with B

S CA

A a

C b

Step -4: Eliminate unreachable variables:none

Step-5: convert to CNF: already in CNF form

S CA

A a

C b

---

c) Start symbol S

$$
\begin{aligned}
S &\rightarrow ASB \mid \epsilon \\
A &\rightarrow aAS \mid a \\
B &\rightarrow SbS \mid A \mid bb
\end{aligned}
$$

Step-0: Eliminate Start symbol from RHS: There is S in the right side of variables. So, add a new Start symbol.

S_0 S

S ASB | ε

A aAS | a

B SbS | A | bb

Step -1: eliminate ε-production. : Nullable: S, S_0

Removing nullability of S

S ASB | ε

S AεB | ASB |ε

S ASB | AB

After ε-production removal:

S_0 S

S ASB | AB

A aAS | a | aA

B SbS | A | bb | Sb | bS | b

Step -2 : Eliminate unit production:

S_0 ASB | AB

S ASB | AB

A aAS | a | aA

B SbS | aAS | a | aA | bb | Sb | bS | b

Step -3: eliminate variables that derive no terminal string.: none

Step -4: Eliminate unreachable variables : none

Step-5: convert to CNF:

S_0 MB | AB

S MB | AB

A WS | a | XA

B ZS | WS | a | XA | YY | SY | YS | b

X a

Y b

Z SY

W XA

M AS

---

d) Start symbol S

$$
\begin{aligned}
S &\rightarrow 0A0 \mid 1B1 \mid BB \\
A &\rightarrow C \\
B &\rightarrow S \mid A \\
C &\rightarrow S \mid \epsilon
\end{aligned}
$$

Step-0: Eliminate Start symbol from RHS:

There is S in the right side of variables. So, Start new Start Symbol.

S_0 S

S 0A0 | 1B1 | BB

A C

B S|A

C S| ε

Step-1: Eliminate ε production.

Nullable: C,A,B,S,S0

Removing nullability of C

C S

Removing nullability of A

A C

Removing nullability of B

B S|A

Removing nullability of S. (here, A, B both nullable)

S 0A0 | 1B1 | BB

S 0ε0| 0A0 |1ε1 |1B1 | εB| Bε|BB

S 00|0A0| 11|1B1 | B| BB

Removing nullability of S_0

S_0 S

CFG after epsilon removal

S_0 S

S 0A0| 1B1 | BB| 00| 11 | B

A C

B S|A

C S

Step -2: Remove unit production

C 0A0 | 1B1 |BB | 00 | 11

B 0A0 | 1B1 |BB | 00 | 11

A 0A0 | 1B1|BB | 00 | 11

S 0A0 | 1B1 |BB | 00 | 11

S_0 0A0 | 1B1 | BB | 00|11

Step -3: eliminate variables that derive no terminal strings : none

S_0 0A0 | 1B1 | BB | 00|11

S 0A0 | 1B1 | BB | 00 | 11

A 0A0 | 1B1 |BB | 00 | 11

B 0A0 | 1B1 |BB | 00 | 11

C 0A0 | 1B1 |BB | 00 | 11

Step-4: Eliminate variables that are unreachable: A, C

S_0 0A0 | 1B1 | BB | 00|11

A 0A0 | 1B1 |BB | 00 | 11

B 0A0 | 1B1 |BB | 00 | 11

Step-5: convert to CNF: Now. The grammar is clean. Convert to CNF

S_0 MX | NY | BB | XX | YY

A MX | NY | BB | XX | YY

B MX | NY | BB | XX | YY

M XA

N YB

X 0

Y 1

---

e) Start symbol S

$$
\begin{aligned}
S &\rightarrow AAA \mid B \\
A &\rightarrow aA \mid B \\
B &\rightarrow \epsilon
\end{aligned}
$$

Step -1: Remove epsilon productions

Nullable: S,B,A

S AAA | AA | A| B

A aA | a | B

Eliminating B, since it only generates ε

Step-2: Remove unit production

S AAA | AA | aA | a

A aA | a

Eliminating B from RHS, Unit production B does not have any production rule

Step-3: Remove variables that generates no terminal: none

S AAA | AA | aA | a

A aA | a

Step-4: Removes unreachable variables:none

S AAA | AA | aA | a

A aA | a

Step-5: convert to CNF: The grammar is clean now. Convert to CNF

S YA | AA | XA | a

A XA | a

Y AA

X a

---

f) Start symbol S

$$S \rightarrow AB \mid CD$$
$$A \rightarrow BC \mid a$$
$$B \rightarrow AC \mid C$$
$$C \rightarrow AB \mid CD$$
$$D \rightarrow AC \mid d$$

Step -1: eliminate epsilon production: none

S AB | CD

A BC | a

B AC | C

C AB | CD

D AC | d

Step-2: eliminate unit production

S AB | CD

A BC | a

B AC | AB | CD

C AB | CD

D AC | d

Step -3: Eliminate variables that generate no terminal string: S, B, C

A a

D d

Eliminate unreachable variable:

No grammar.

Question -2) Convert the following CFG to CNF (Chomsky Normal form).

a) **Start symbol S**

Step -1: Eliminate ε-productions

First, find the nullable variables. Here, C,B,A,S,D are nullable.

Let's remove nullability one by one.

First, removing nullability of C. (here in right side C, D both are nullable)

C CDE | ε

C εDE | CεE | εεE| CDE

C DE| CE| E |CDE

Now remove nullability of A

A C | a

Removing B's nullability

B C | b

Remove D's nullability

D A | B | ab

Remove S's nullability

S aAa | bBb | Ɛ

S aAa | bBb | aƐa | bƐb

S aAa | bBb | aa | bb

After step 1, we get

S aAa | bBb | aa | bb

A C | a

B C | b

C CDE | DE |CE |E

D A | B | ab

Step -2: Eliminate unit production

S aAa | bBb| aa | bb

A CDE | DE |CE |a

B CDE | DE | CE | b

C CDE | DE |CE (since there is no production rule for E, eliminating variable E)

D CDE | DE | CE | a | b| ab

Step -3: eliminate variables that derives no terminal string :Variable-C

S aAa | bBb | aa | bb

A a

B b

D a | b| ab

Step -4: eliminate unreachable variables: none

S aAa | bBb | aa | bb

A a

B b

Step-5: convert to CNF: The grammar is clean now. Let's convert it to CNF now.

S XA | YB | AA | BB

A a

B b

X AA

Y BB

---

b) Start symbol S

Step -1: eliminate epsilon production :none

Step -2: Eliminate unit production.: none

Step -3: Eliminate variables that generates no terminal string: B, remove remove everything with B

S CA

A a

C b

Step -4: Eliminate unreachable variables:none

Step-5: convert to CNF: already in CNF form

S CA

A a

C b

---

c) Start symbol S

Step-0: Eliminate Start symbol from RHS: There is S in the right side of variables. So, add a new Start symbol.

S_0 S

S ASB | ε

A aAS | a

B SbS | A | bb

Step -1: eliminate ε-production. : Nullable: S, S_0

Removing nullability of S

S ASB | ε

S AεB | ASB |ε

S ASB | AB

After ε-production removal:

S_0 S

S ASB | AB

A aAS | a | aA

B SbS | A | bb | Sb | bS | b

Step -2 : Eliminate unit production:

S_0 ASB | AB

S ASB | AB

A aAS | a | aA

B SbS | aAS | a | aA | bb | Sb | bS | b

Step -3: eliminate variables that derive no terminal string.: none

Step -4: Eliminate unreachable variables : none

Step-5: convert to CNF:

S_0 MB | AB

S MB | AB

A WS | a | XA

B ZS | WS | a | XA | YY | SY | YS | b

X a

Y b

Z SY

W XA

M AS

---

d) Start symbol S

Step-0: Eliminate Start symbol from RHS:

There is S in the right side of variables. So, Start new Start Symbol.

S_0 S

S 0A0 | 1B1 | BB

A C

B S|A

C S| ε

Step-1: Eliminate ε production.

Nullable: C,A,B,S,S0

Removing nullability of C

C S

Removing nullability of A

A C

Removing nullability of B

B S|A

Removing nullability of S. (here, A, B both nullable)

S 0A0 | 1B1 | BB

S 0ε0| 0A0 |1ε1 |1B1 | εB| Bε|BB

S 00|0A0| 11|1B1 | B| BB

Removing nullability of S_0

S_0 S

CFG after epsilon removal

S_0 S

S 0A0| 1B1 | BB| 00| 11 | B

A C

B S|A

C S

Step -2: Remove unit production

C 0A0 | 1B1 |BB | 00 | 11

B 0A0 | 1B1 |BB | 00 | 11

A 0A0 | 1B1|BB | 00 | 11

S 0A0 | 1B1 |BB | 00 | 11

S_0 0A0 | 1B1 | BB | 00|11

Step -3: eliminate variables that derive no terminal strings : none

S_0 0A0 | 1B1 | BB | 00|11

S 0A0 | 1B1 | BB | 00 | 11

A 0A0 | 1B1 |BB | 00 | 11

B 0A0 | 1B1 |BB | 00 | 11

C 0A0 | 1B1 |BB | 00 | 11

Step-4: Eliminate variables that are unreachable: A, C

S_0 0A0 | 1B1 | BB | 00|11

A 0A0 | 1B1 |BB | 00 | 11

B 0A0 | 1B1 |BB | 00 | 11

Step-5: convert to CNF: Now. The grammar is clean. Convert to CNF

S_0 MX | NY | BB | XX | YY

A MX | NY | BB | XX | YY

B MX | NY | BB | XX | YY

M XA

N YB

X 0

Y 1

---

e) **Start symbol S**

Step -1: Remove epsilon productions

Nullable: S,B,A

S AAA | AA | A| B

A aA | a | B

Eliminating B, since it only generates Ɛ

Step-2: Remove unit production

S AAA | AA | aA | a

A aA | a

Eliminating B from RHS, Unit production B does not have any production rule

Step-3: Remove variables that generates no terminal: none

S AAA | AA | aA | a

A aA | a

Step-4: Removes unreachable variables:none

S AAA | AA | aA | a

A aA | a

Step-5: convert to CNF: The grammar is clean now. Convert to CNF

S YA | AA | XA | a

A XA | a

Y AA

X a

---

f) Start symbol S

Step -1: eliminate epsilon production: none

S AB | CD

A BC | a

B AC | C

C AB | CD

D AC | d

Step-2: eliminate unit production

S AB | CD

A BC | a

B AC | AB | CD

C AB | CD

D AC | d

Step -3: Eliminate variables that generate no terminal string: S, B, C

A a

D d

Eliminate unreachable variable:

No grammar.

G) S → aSb|b

Answer

S_0 WB |b

S WB |b

A a

B b

W AS

CYK

Question -5)

Grammar	Strings to check
S → AB | BC A → BA | a B → CC | b C → AB | a	a) w = ababb b) w = baaba c) w = ababa d) w = baaab e) w = aabab f) w = ababaa

a) ababb

{} 15		
{B} 14	{} 25	
{B} 13	{S,C} 24	{} 35

{S,C} 12	{S,A} 23	{S,C} 34	{} 45	
{A,C} 11	{B} 22	{A,C} 33	{B} 44	{B} 55
a	b	a	b	

b

cell 15 does not contain S. so the string is not in the language

b) baaba

{S,A,C} 15				
{} 14	{S,A,C} 25			
{} 13	{B} 24	{B} 35		
{S,A} 12	{B} 23	{S,C} 34	{S,A} 45	
{B} 11	{A,C} 22	{A,C} 33	{B} 44	{A,C} 55
b		a	a	b

a

cell 15 contains S. so the string is in the language

c) ababa

{S,A,C} 15				
{B} 14	{B} 25			
{B} 13	{S,C} 24	{B} 35		
{S,C} 12	{S,A} 23	{S,C} 34	{S,A} 45	
{A,C} 11	{B} 22	{A,C} 33	{B} 44	{A,C} 55
a		b	a	b

a

cell 15 contains S. so the string is in the language

d) baaab

{S,C} 15				
{S,A,C} 14	{S,C} 25			
{} 13	{S,A,C} 24	{B} 35		
{S,A} 12	{B} 23	{B} 34	{S,C} 45	
{B} 11	{A,C} 22	{A,C} 33	{A,C} 44	{B} 55

b a a a

b

cell 15 contains S. so the string is in the language

e) aabab

{S,C} 15				
{S,A,C} 14	{B} 25			
{B} 13	{B} 24	{S,C} 35		
{B} 12	{S,C} 23	{S,A} 34	{S,C} 45	
{A,C} 11	{A,C} 22	{B} 33	{A,C} 44	{B} 55

a a b a

b

cell 15 contains S. so the string is in the language

f) ababaa

{B} 16					
{S,A,C} 15	{S,A} 26				
{B} 14	{B} 25	{S,A} 36			
{B} 13	{S,C} 24	{B} 35	{} 46		
{S,C} 12	{S,A} 23	{S,C} 34	{S,A} 45	{B} 56	
{A,C} 11	{B} 22	{A,C} 33	{B} 44	{A,C} 55	{A,C} 66

a b a b a

cell 16 does not contain S. so the string is not in the language

Grammar	Strings to check	Solutions
S → AB | BC A → BA | a B → CC | b C → AB | a	a) w = ababb b) w = baaba c) w = ababa d) w = baaab e) w = aabab f) w = ababaa	a) w = ababb - NO b) w = baaba- YES c) w = ababa -YES d) w = baaab -YES e) w = aabab -YES f) w = ababaa- NO

==========
==

Question-6

aabbbb

{} 16					
{ S_0 , S} 15	{ } 26				
{W} 14	{} 25	{} 36			
{ } 13	{ S_0 , S } 24	{} 35	{} 46		
{ } 12	{W} 23	{} 34	{} 45	{} 56	
{A } 11	{A} 22	{ S_0,S,B} 33	{ S_0,S,B } 44	{ S_0,S,B } 55	{ S_0,S,B } 66
a	a	b	b	b	b

cell 16 does not contain S_0. so the string is not in the language