CSE331 Automata and Computability

Lecture 19, 20: Push Down Automata (PDA)

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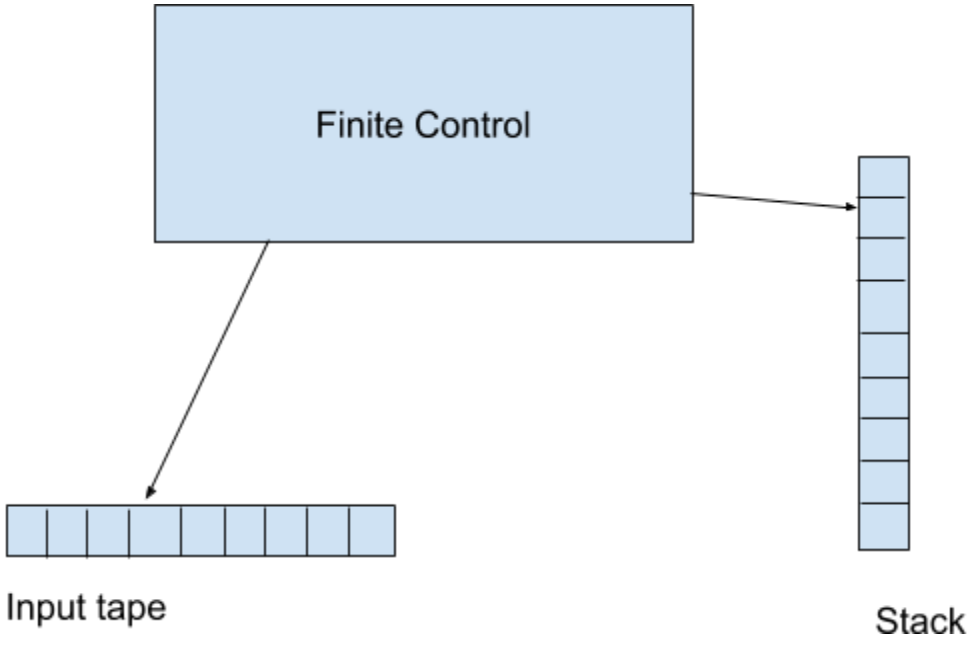
Introduction

A Push Down Automaton (PDA) is a non-deterministic Finite State Machine/Automaton with an additional memory in the form of a stack. They are more effective than a Finite State Machine/Automaton but less powerful than Turing Machine.

Components

A Push Down Automaton (PDA) has three components:

1. A finite control unit
2. Input tape
3. Stack



Formal Definition

A Push Down Automaton (PDA), M, can be defined as a 7 tuple:

$M = (Q, \Sigma, \Gamma, \delta, q0, z0, F)$

Q	A finite set of states
Σ	A finite input alphabet
Γ	A finite stack alphabet
q0	The initial/starting state, q0 is in Q
z0	A starting stack symbol, is in Γ
F	A set of final/accepting states, which is a subset of Q
δ	A transition function

Transition Function

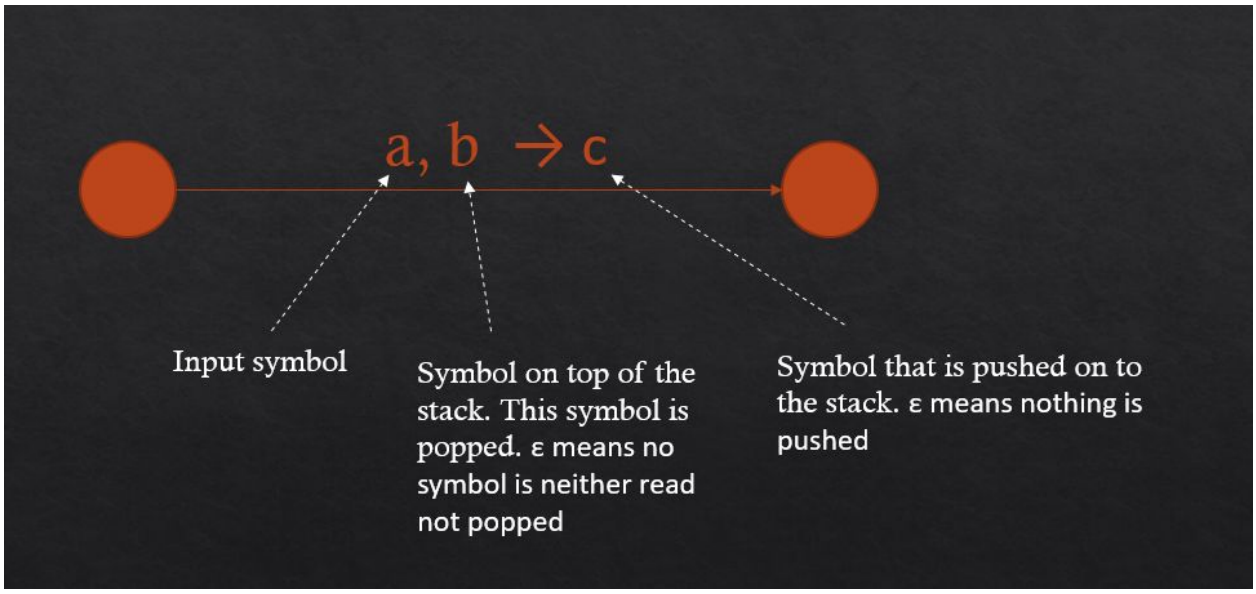
The transition function, δ, can be defined as $\delta: Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \rightarrow$ finite subsets of $Q \times \Gamma^*$. It takes in 3 parameters, **δ(q, a, X)**

- **q** is a state in Q
- **a** is a symbol in the alphabet Σ
- **X** is a symbol on top of the stack which is a member of Γ

The output of the transition function is a finite set of pairs (p, ϒ) where:

- p is a new state
- ϒ is a string of stack symbols that replaces the top of the stack

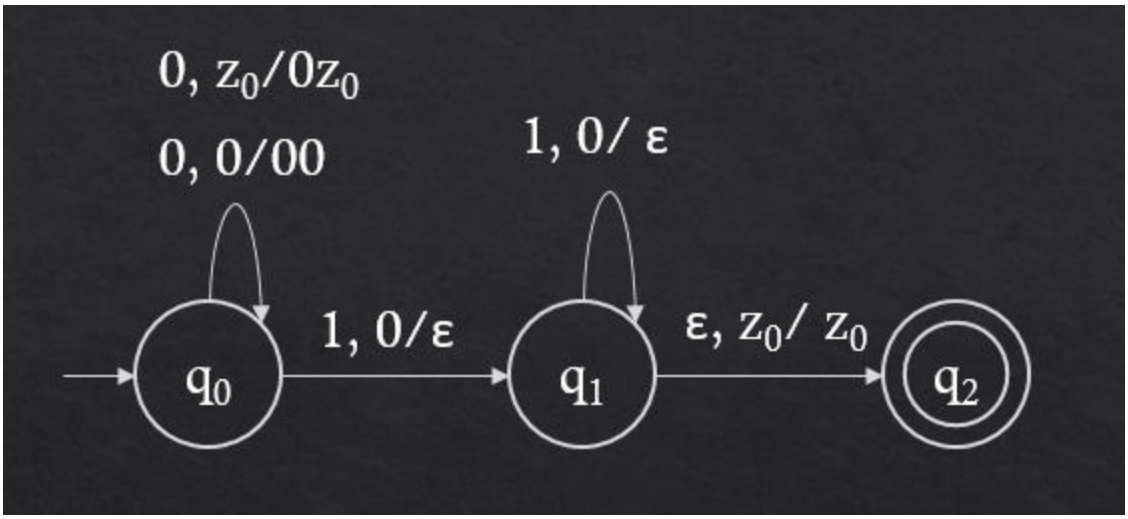
Graphical Representation



Examples

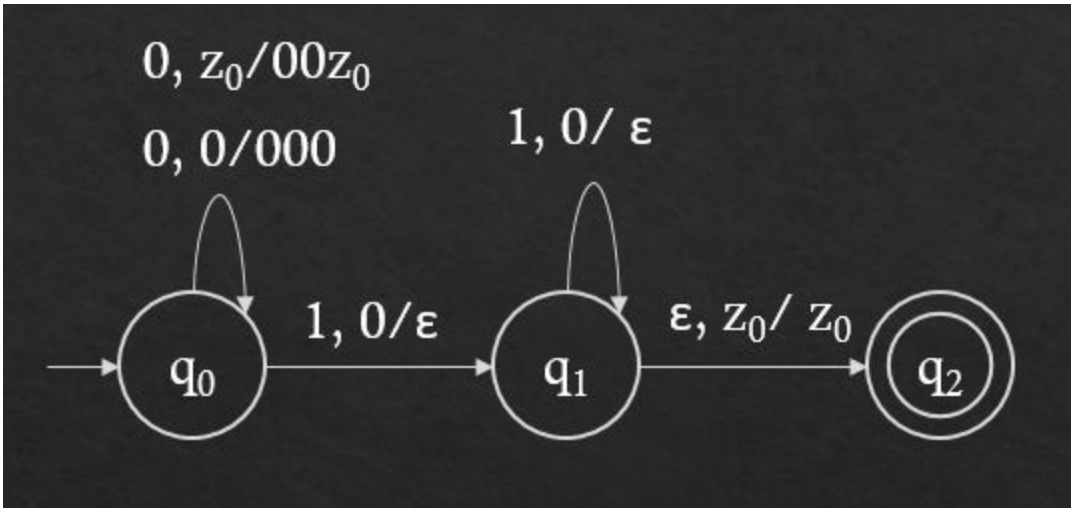
L = {0^n1^n, n > 0}

Solution: Push down one 0 for each 0 in the input string onto the stack. Pop one 0 for each 1 in the input. If it reaches the end of the string and if the top of the stack is z_0, i.e. all the 0's have been popped off, it accepts.



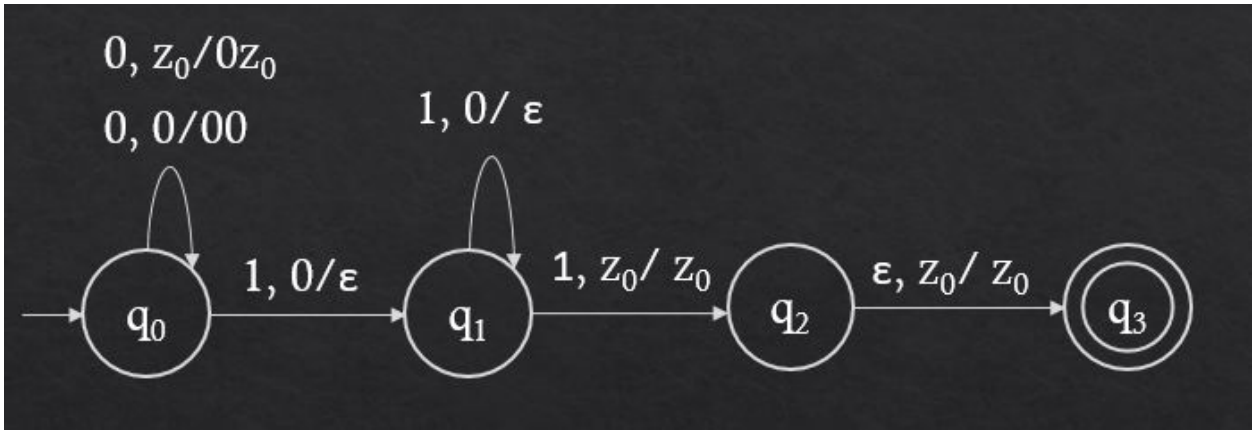
L = {0^n1^{2n}, n > 0}

Solution: Push down two 0's for each 0 in the input string onto the stack. Pop one 0 for each 1 in the input. If it reaches the end of the string and if the top of the stack is z_0, i.e. all the 0's have been popped off, it accepts.



L = {0^n1^{n+1}, n > 0}

Solution: Push down one 0 for each 0 in the input string onto the stack. Pop one 0 for each 1 in the input. Once all the 0's have been popped off of the stack and the top of the stack is z_0, it reads one more 1 from the input string. If it reaches the end of the string then, it accepts the string.



$L = \{ww^R, w = (a+b)^+\}$ Which is an even length palindrome

Solution: The state q_1 pushes everything onto the stack assuming it's not the middle of the string. One copy of the Automaton and the stack moves to q_2 on epsilon transition (like an NFA) assuming that every point is the middle of the string. In this state, the automaton matches the input symbol with the top of the stack and if it matches, it pops off the symbol.