

Chapter 1

Mathematical Problems

Fairoz Nower Khan

- For a specific machine, total instruction count = 2×10^9 , Avg CPI = 3 cycles/instruction, clock rate = 100MHz. We want to improve the compiler to have instruction count = 10^9 , new CPI = 4 cycles/instruction, clock rate = 250MHz. By what percentage is the new compiler improved?

$$\text{CPU Time} = \text{Instruction Count} \times \text{CPI} \times \text{Clock Cycle Time}$$

$$\text{CPU Time for Computer A} = (2 \times 10^9 \times 3) / 100 \text{ MHz}$$

$$\text{CPU Time for Computer B} = (10^9 \times 4) / 250 \text{ MHz}$$

$$\text{Percentage of Improvement} = ((\text{CPU Time for Computer A} - \text{CPU Time for Computer B}) / \text{CPU Time for Computer A}) \times 100$$

$$\text{Percentage of Improvement} = (44 \times 10^6) / 6 \times 10^7 \times 100\% = 73.33\%$$

- assume a new CPU has 6% less capacitive load compared to an existing CPU and has an 2% reduction in voltage. Now **calculate the reduction in overall power use.**

$$\text{Power} = \text{Capacitive load} \times \text{Voltage}^2 \times \text{Frequency}$$

$$P(\text{new}) = \text{Capacitive Load} (1-.06) \times (\text{Voltage} (1-.02))^2 \times F$$

$$P(\text{new}) = (0.94) \text{ Capacitive Load} \times ((.98)\text{Voltage})^2 \times F$$

$$P(\text{new})/P(\text{old}) = (0.94) \text{ Capacitive Load} \times ((.98)\text{Voltage})^2 \times F / \text{Capacitive Load} \times (\text{Voltage})^2 \times F$$

$$P(\text{new})/P(\text{old}) = 0.902$$

$$\text{Reduction in Power} = (1- 0.902) \times 100\%$$

SPEC Ratio

- SPEC Ratio for a specific program = $\text{Reference Time} / \text{Current CPU Time}$

- Given 2 Programs,
- Program A, Instruction Count = 10 , CPI=3, **Clock Period = 3 s**
Reference Time = 500s
- Program B , Instruction Count = 50 , CPI=7, **Clock Period = 3 s**
Reference Time = 2000s

CPU Time for A = 90s

SPEC Ratio= $500/90 = 5.55$

CPU Time for B = 1050s

SPEC Ratio= $2000/1050 = 1.9$

Geometric Mean = $(a_0 \times a_1 \times \dots \times a_n)^{1/n}$

Geometric Mean = $(5.55 \times 1.9)^{1/2} = 3.24$

- Now, assume that a particular operation takes **2.5X%** of the total execution time. What improvement is required if we want **2.5** times speedup in that operation, where **X** is equal to 10.

$$T_{\text{improved}} = \frac{T_{\text{affected}}}{\text{improvement factor}} + T_{\text{unaffected}}$$

$$100/2.5 = 25/n + 75$$

- Suppose you are training a face recognition model, which is heavily dependent on a process (**80%**). So, you installed a graphics card with to speed up that process. Now, you observe that it is taking only **3 days** to execute, as opposed to **6 days** before installing the card. What is the improvement?

$$T_{\text{improved}} = \frac{T_{\text{affected}}}{\text{improvement factor}} + T_{\text{unaffected}}$$

Improved time = 3 days, Affected Time = $6 \times 0.8 = 4.8$

$$3 = 4.8/n + 1.2$$

$$N = 2.67$$