Project title: Image noise reduction through the applications of linear and non-linear filters

Abstract:

In this project, linear and non-linear filtering is applied to different blurred image and performance is measured comparing their output image. First, target image is blurred using periodic noise. Then, image is de-blurred using optimal notch filtering using different size of neighborhood. Varying neighborhood size optimal notch filtering performance is improved. Then target image is distorted using motion blur and Gaussian additive noise. Then this noise is removed using Weiner filtering and constrained least square filtering (CLSF) technique. Their performance is improved varying parameters. Then out of focus blurring function point spread function is applied to the image. Then image de-blurring performance is measured with Weiner filtering and CLSF.

Technical Discussion:

Periodic noise filtering:

A periodic noise pattern equation is

$$r(x,y) = A\sin[2\pi u_0(x+B_x)/M + 2\pi v_0(y+B_y)/N] \tag{1}$$

Where A is the amplitude, u_0 and v_0 determines the sinusoidal frequencies with respect to x and y axis, respectively, B_x and B_y are phase displacement with respect to origin.

Optimum Notch filtering: The Fourier transform of the interference noise pattern ($N(u,v)$) is,

$$N(u,v) = H_{NP}(u,v)G(u,v) \tag{2}$$

Where $H_{NP}(u,v)$ and $G(u,v)$ are the notch filter and Fourier transform of corrupted image respectively. Corresponding spatial domain $\eta(u,v)$ is obtained as following

$$\eta(u,v) = F^{-1}\{H_{NP}(u,v)G(u,v)\} \tag{3}$$

The weighting or modulation function $w(x,y)$ can be obtained as

$$w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \overline{g}(x,y)\overline{\eta}(x,y)}{\eta^2(x,y) - \overline{\eta}^2(x,y)} \tag{4}$$

Estimation of the original image $f(x,y)$ can be found using following equation

$$f(x,y) = g(x,y) - w(x,y)\eta(x,y) \tag{5}$$

Blur and Additive noise filtering:

Blurring filter degradation function H(u,v) can be given as,

$$H(u,v) = \frac{T}{\pi(ua+vb)}\sin[\pi(ua+vb)]e^{-j\pi(ua+vb)} \tag{6}$$

Where a is the displaced distance, T is time and b is motion variance.

Minimum mean square error (Weiner) filtering can be performed using following equation,

$$F(u,v) = [\frac{1}{H(u,v)} * \frac{|H(u,v)|^2}{|H(u,v)|^2 + k}]G(u,v) \tag{7}$$

K is a specific constant added to the equation.

Frequency domain solution for constant least square filtering is following

$$F(u,v) = [\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)|^2}]G(u,v) \tag{8}$$

Where γ is a parameter which is adjusted to find a better filtering performance and P(u,v) is the Fourier transform of following function,

$$P(x,y)=\begin{pmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{pmatrix} \qquad (9)$$

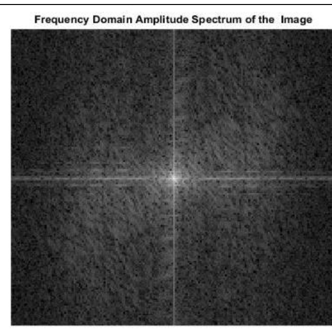
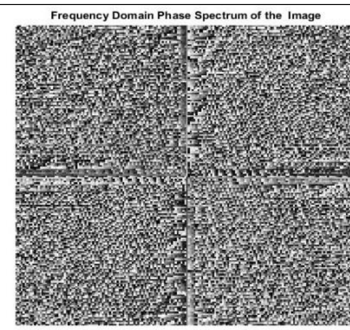
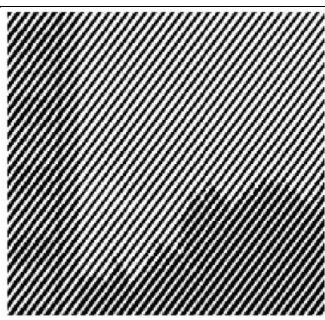
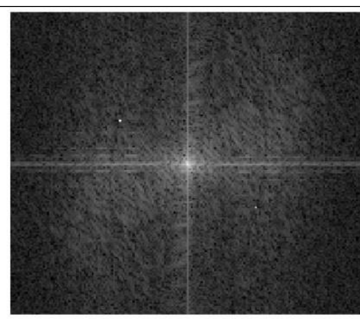
This is a Laplacian operator function.

Out of focus-blurring point spread function as,

$$h(x,y)=\begin{cases} 1/\pi r^2 when, x^2+y^2 \leq r^2 \\ 0, elsewhere \end{cases} \qquad (10)$$

Where r is the radius of the circle of confusion.

Discussion of esults:

 Figure 1.1 Input Image	 Figure 1.2: frequency spectrum of input image	 Figure 1.3: Phase Spectrum of input image
 Figure 1.3: Noisy image	 Figure 1.4: Frequency Spectrum of Noisy Image	 Figure 1.5: De-noised Image

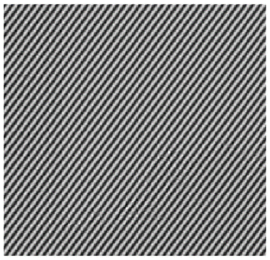


figure 1.7: Noise pattern

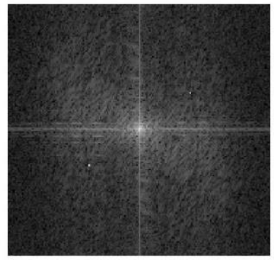


figure 1.8: nNoise Fourier spectrum

First the image was blurred using periodic noise. From the frequency spectrum we may find the noise spectrum. Then Butterworth notch reject filter is applied to the image. From the noise spectrum the noise location was determined and from that Butterworth notch filter is determined. From Notch reject filter and noisy image noise pattern is determined. For different neighborhood size, weight portion of the noise is determined. Weight is calculated based on average value of the neighborhood noise pattern and noisy image. For different neighborhood size the weight is calculated then at the end de-noised image is determined.

figure 2 1:original image

figure 2 2 Motion:blurred image

figure 2 3: Motion blur and additive blur image

figure 2 4: image restored using weiner filtering

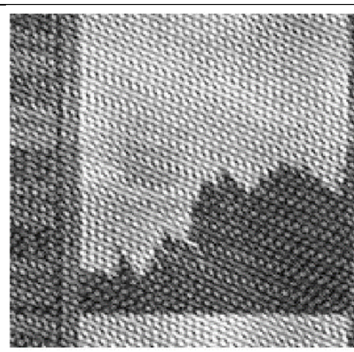


figure 2 5: Image restored using constrained least square filtering with gamma value of 0.0001

figure 2 6: Image restored using constrained least square filtering with gamma value of 0.0000009

First the image is blurred using motion blur using given equation. The similar a, b and T value is used to apply motion blur in the image. After applying motion blur Gaussian additive noise is applied to the image. Then image noise is minimized with applying Weiner filtering. Here, k value is varied to find better de-blurred image. Constrained least squares filtering algorithm is applied to find better de-blurred image according to specified equation. Gamma value is varied to find the optimum result. Here Newton-Raphson algorithm is used to find better possible value of gamma.

figure 3. 1:original Image

figure 3. 2: Out of Focus blurred image

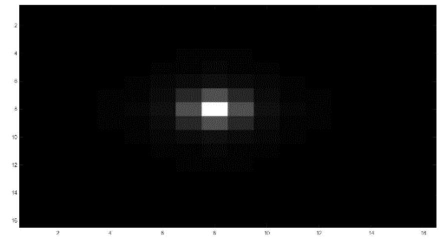


figure 3. 3: Blurred kernel

figure 3. 4: Gaussian additive noise

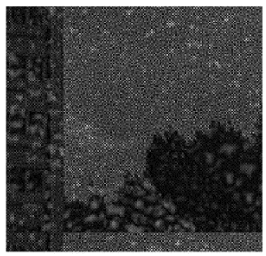


figure 3. 5: Corresponding Weiner filtering output

figure 3. 6: Constrained Least Square filtering

figure 3. 7: lessen Gaussian Additive Noise

figure 3. 8: Corresponding Weiner filtering output

figure 3. 9: Constrained Least Square filtering

Here, image is de-blurred using out of focus blurring function point spread function. Then additional Gaussian additive noise is added to image. To measure de-noising performance of the image with constrained least square filtering, first Weiner filtering is applied to the image. Then image is de-noised using constrained least square filtering (CLSF). It is found that CLSF performance is better than Weiner filter.