MTH 3300
Fall 2022

Programming Problem 5: numericalintegration.py

Directions: Download the template file provided on Blackboard. Then open Spyder, load this template file, and write the following program. Submit your source code via Gradescope, in `.py` format; do NOT send any other files. READ THE INSTRUCTIONS on submitting your work in the Course Documents section of Blackboard.

Be sure to read the SPECIFICATIONS carefully! And write comments!

We discussed the Trapezoidal rule for approximating definite integrals:

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left(1f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + \ldots + 2f(x_{N-2}) + 2f(x_{N-1}) + 1f(x_N)\right)$$

where

$$\Delta x = \frac{b-a}{N}$$

and

$$x_i = a + i\Delta x$$

Here, N is the number of pieces you break the interval $[a, b]$ into; the more pieces, the better. The coefficients are 1 for the first term and the last term; all the rest are 2's: 1,2,2,2,2,..., 2,2,1.

So, for example: if we want to approximate $\int_1^2 x^4\,dx$ with $N = 6$ steps, I have:

$$\Delta x = \frac{2-1}{6} = \frac{1}{6}$$

$$x_0 = 1,\ x_1 = 1 + \frac{1}{6} = \frac{7}{6},\ x_2 = 1 + \frac{2}{6} = \frac{8}{6},\ x_3 = 1 + \frac{3}{6} = \frac{9}{6},\ x_4 = 1 + \frac{4}{6} = \frac{10}{6},\ x_5 = 1 + \frac{5}{6} = \frac{11}{6},\ x_6 = 1 + \frac{6}{6} = 2$$

$$\int_1^2 x^4\,dx \approx \frac{1/6}{2}\left(1(1)^4 + 2\left(\frac{7}{6}\right)^4 + 2\left(\frac{8}{6}\right)^4 + 2\left(\frac{9}{6}\right)^4 + 2\left(\frac{10}{6}\right)^4 + 2\left(\frac{11}{6}\right)^4 + 1(2)^4\right) \approx 6.26478909$$

The exact answer is $\frac{31}{5} = 6.2$, so we're pretty close.

Your code should additionally compute a left and right-hand Riemann sum, which uses *rectangles* rather than trapezoids under the curve to estimate the area. For these cases, the width of each rectangle would be Δx (same as above). The heights will be $f(x_0), f(x_1), \ldots, f(x_{N-1})$ (for the left-hand sum) or $f(x_1), f(x_2), \ldots, f(x_N)$ (for a right-hand sum).

So, for example if we want to approximate $\int_1^2 x^4\,dx$ with $N = 6$ steps,

$$\Delta x = \frac{1}{6} \text{ and } x_0 = 1,\ x_1 = 1 + \frac{1}{6} = \frac{7}{6},\ x_2 = 1 + \frac{2}{6} = \frac{8}{6},\ x_3 = 1 + \frac{3}{6} = \frac{9}{6},\ x_4 = 1 + \frac{4}{6} = \frac{10}{6},\ x_5 = 1 + \frac{5}{6} = \frac{11}{6},\ x_6 = 1 + \frac{6}{6} = 2$$

(same as in the example above), and with a *left-hand* sum, we get

$$\int_1^2 x^4\,dx \approx \frac{1}{6}\left(1(1)^4 + \left(\frac{7}{6}\right)^4 + \left(\frac{8}{6}\right)^4 + \left(\frac{9}{6}\right)^4 + \left(\frac{10}{6}\right)^4 + \left(\frac{11}{6}\right)^4\right).$$

With a *right-hand* sum, we get

$$\int_1^2 x^4\,dx \approx \frac{1}{6}\left(\left(\frac{7}{6}\right)^4 + \left(\frac{8}{6}\right)^4 + \left(\frac{9}{6}\right)^4 + \left(\frac{10}{6}\right)^4 + \left(\frac{11}{6}\right)^4 + 1(2)^4\right).$$

Create a program that asks the user to enter in A, B, and an integer N (this is set up for you in the provided template file) and uses the Trapezoidal rule, left-hand Riemann sum, and right-hand Riemann sum, with N intervals to approximate the value of

$$\int_A^B f(x)\,dx$$

where $f(x)$ is defined by

$$f(x) = \left(3 + e^{x^2}\right)^{1/10}.$$

Please follow the following additional guidelines:

1. Your code should contain no lists (other than a `range`, which technically isn't even a list!), and it should only have one `for` loop.
2. You should write a function called `f`, which receives one `float` named `x` as argument, and returns the value of $f(x) = \left(3 + e^{x^2}\right)^{1/10}$.

   In my template file, I have included test values for this function `f`. I have also provided the input prompts and final print statement.

Advice: remember that the Trapezoidal rule (and left and right-hand Riemann sums) can be written using Σ notation (plus a couple of additional terms and times a fraction). So this does not have to be complicated!

And finally, **make sure you understand the Trapezoidal rule (and left and right-hand Riemann sums) well enough to perform some calculations by hand**. It can be tedious, but if you can't perform the calculations by hand, you'll never be able to code it.

Test value: if $A = 2$, $B = 8$, and $N = 20$, then the output values for the Trapezoidal rule, left-hand, and right-hand Rieman sums (respectively) should be equal to 423.0154876, 332.9637058, and 513.0672694 to seven places after the decimal. If the last few digits don't match, perhaps you might have missed a term or two!

Additionally, there are three visible test cases and one hidden test case on Gradescope.

Specifications: your program must

- ask the user for values of A, B, and N, where N is an integer (you may assume the user complies), and A and B should be `floats`. (This code is already set up for you in the provided template file.)
- print an estimate for the value of $\int_A^B f(x)\,dx$ using Trapezoidal rule, a left-hand Riemann sum, and a right-hand Riemann sum, with N steps, where $f(x)$ is as given above. (The template file has print statements at the end.)
- use a Python function to represent the function $f(x)$.
- use one `for` loop only, and no lists (using a `range` in your loops is okay and is probably necessary).
- have no use of `numpy`