

# Descriptive Statistics

Fall 2018

Instructor:

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# Topic Overview

- Some important terminology
- Methods of data representation: frequency tables, graphs, pie-charts, scatter-plots
- Data mean, median, mode, quantiles
- Chebyshev's inequality
- Correlation coefficient

# Terminology

- **Population:** The collection of all elements which we wish to study, example: data about occurrence of tuberculosis all over the world
- In this case, “population” refers to the set of people in the entire world.
- The population is often too large to examine/study.
- So we study a subset of the population – called as a **sample**.
- In an experiment, we basically collect **values** for **attributes** of each member of the sample – also called a **sample point**.
- Example of a relevant attribute in the tuberculosis study would be whether or not the patient yielded a positive result on the serum TB Gold test.
- See [http://www.who.int/tb/publications/global\\_report/en/](http://www.who.int/tb/publications/global_report/en/) for more information.

# Terminology

- **Discrete data:** Data whose values are restricted to a finite or countably infinite set. Eg: letter grades at IITB, genders, marital status (single, married, divorced), income brackets in India for tax purposes
- **Continuous data:** Data whose values belong to an uncountably infinite set (Eg: a person's height, temperature of a place, speed of a car at a time instant).

# Methods of Data Representation/Visualization

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# Frequency Tables

- For discrete data having a relatively small number of *values*, one can use a **frequency table**.
- Each row of the table lists the data value followed by the number of sample points with that value (*frequency* of that value).
- The values need not always be numeric!

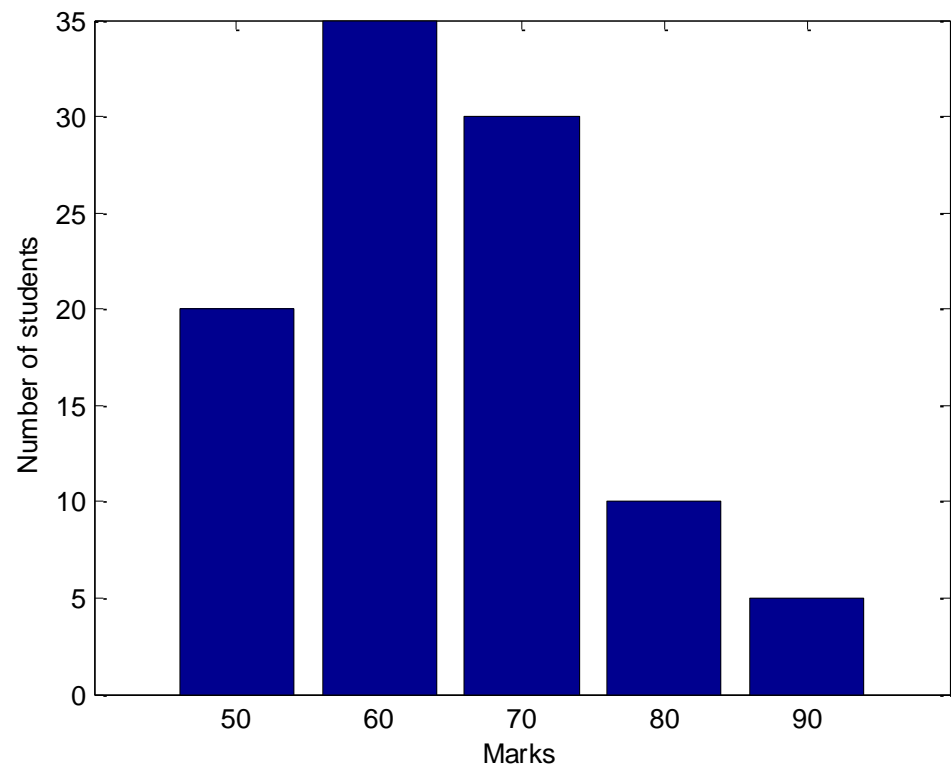
Grade	Number of students for that grade (total 100)
AA	100
AB	0
BB	0
BC	0
CC	0

The definition of an ideal course (per student perspective) at IITB ;-)

# Frequency Tables

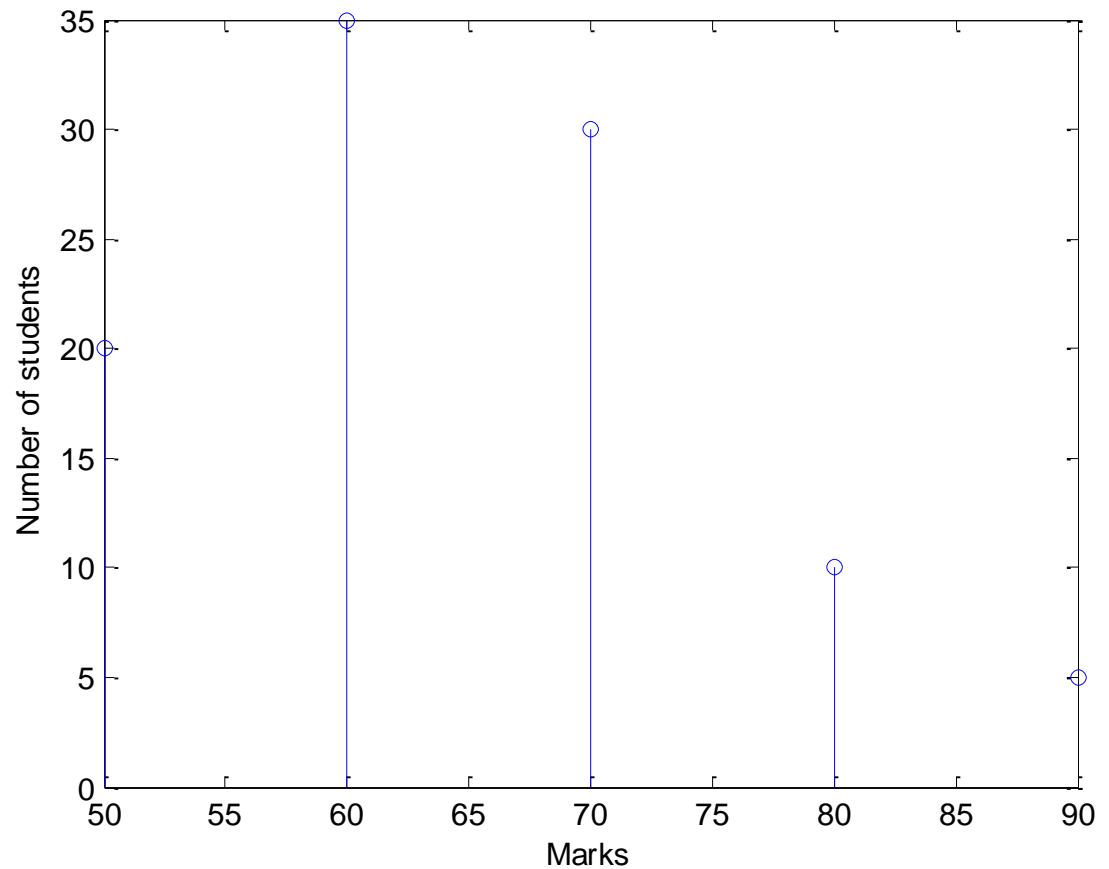
- The frequency table can be visualized using a **line graph** or a **bar graph** or a **frequency polygon**.

Grade	Number of students
AA	5
AB	10
BB	30
BC	35
CC	20



A **bar graph** plots the distinct data values on the X axis and their frequency on the Y axis by means of the height of a thick vertical bar!

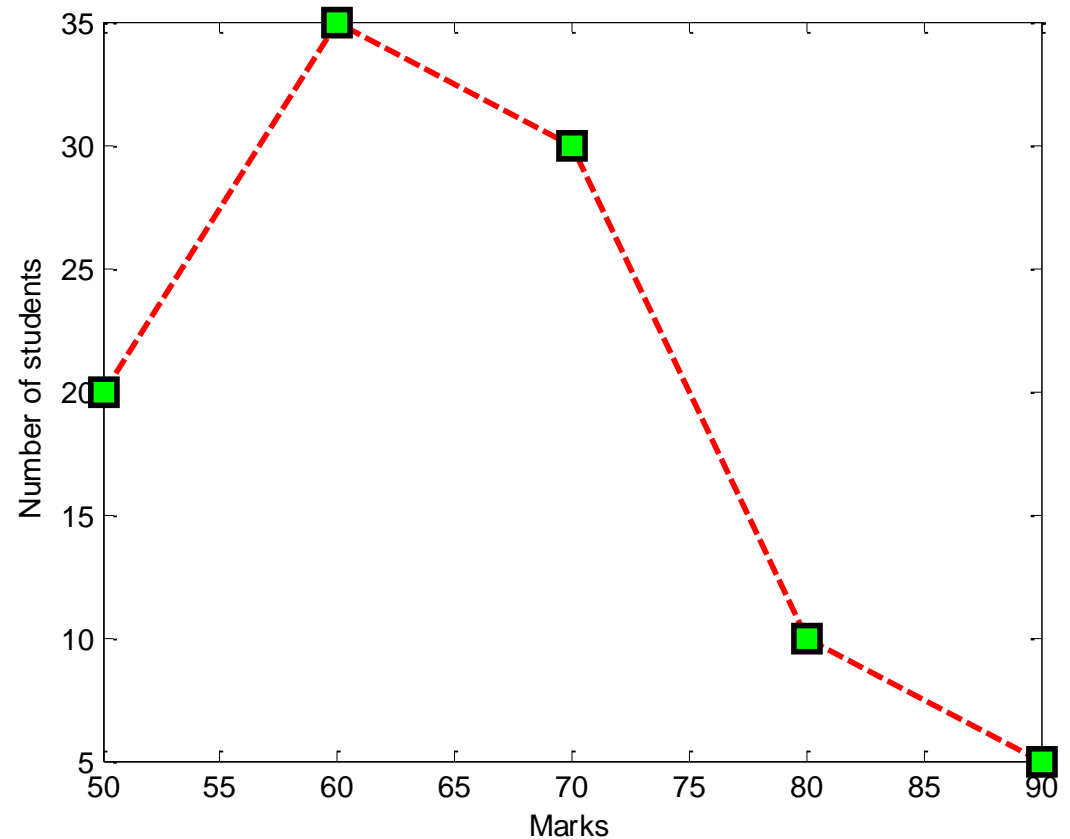
Grade	Number of students
AA	5
AB	10
BB	30
BC	35
CC	20



A **line diagram** plots the distinct data values on the X axis and their frequency on the Y axis by means of the height of a vertical line!



Grade	Number of students
AA	5
AB	10
BB	30
BC	35
CC	20



A **frequency polygon** plots the frequency of each data value on the Y axis, and connects consecutive plotted points by means of a line.

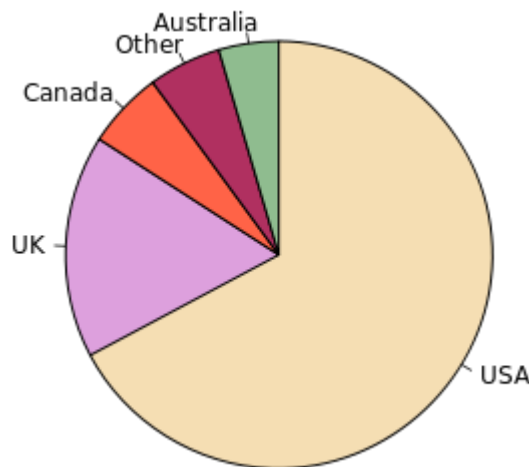
# Relative frequency tables

- Sometimes the actual frequencies are not important.
- We may be interested only in the *percentage* or *fraction* of those frequencies for each data value – i.e. *relative frequencies*.

Grade	Fraction of number of students
AA	0.05
AB	0.10
BB	0.30
BC	0.35
CC	0.20

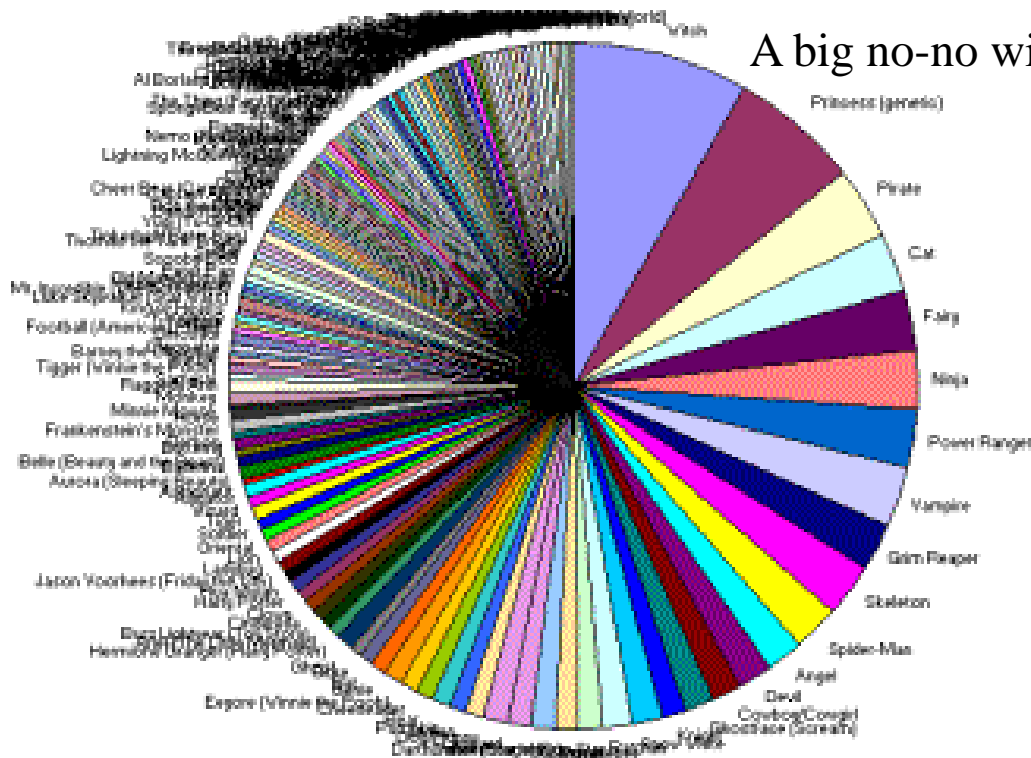
# Pie charts

- For a small number of distinct data values which are non-numerical, one can use a **pie-chart** (it can also be used for numerical values).
- It consists of a circle divided into sectors corresponding to each data value.
- The area of each sector = relative frequency for that data value.



Population of native English speakers:  
[https://en.wikipedia.org/wiki/Pie\\_chart](https://en.wikipedia.org/wiki/Pie_chart)

# Pie charts can be confusing



A big no-no with too many categories.



<http://stephenturbek.com/articles/2009/06/better-charts-from-simple-questions.html>

# Dealing with continuous data

- Many a time the data can acquire continuous values (eg: temperature of a place at a time instant, speed of a car at a given time instant, weight or height of an animal, etc.)
- In such cases, the data values are divided into intervals called as *bins*.
- The frequency now refers to the number of sample points falling into each bin.
- The bins are often taken to be of equal length, though that is not strictly necessary.

# Dealing with continuous data

- Let the sample points be  $\{x_i\}$ ,  $1 \leq i \leq N$ .
- Let there be some  $K$  ( $K \ll N$ ) bins, where the  $j^{\text{th}}$  bin has interval  $[a_j, b_j)$ .
- Thus frequency  $f_j$  for the  $j^{\text{th}}$  bin is defined as follows:

$$f_j = |\{x_i : a_j \leq x_i < b_j, 1 \leq i \leq N\}|$$

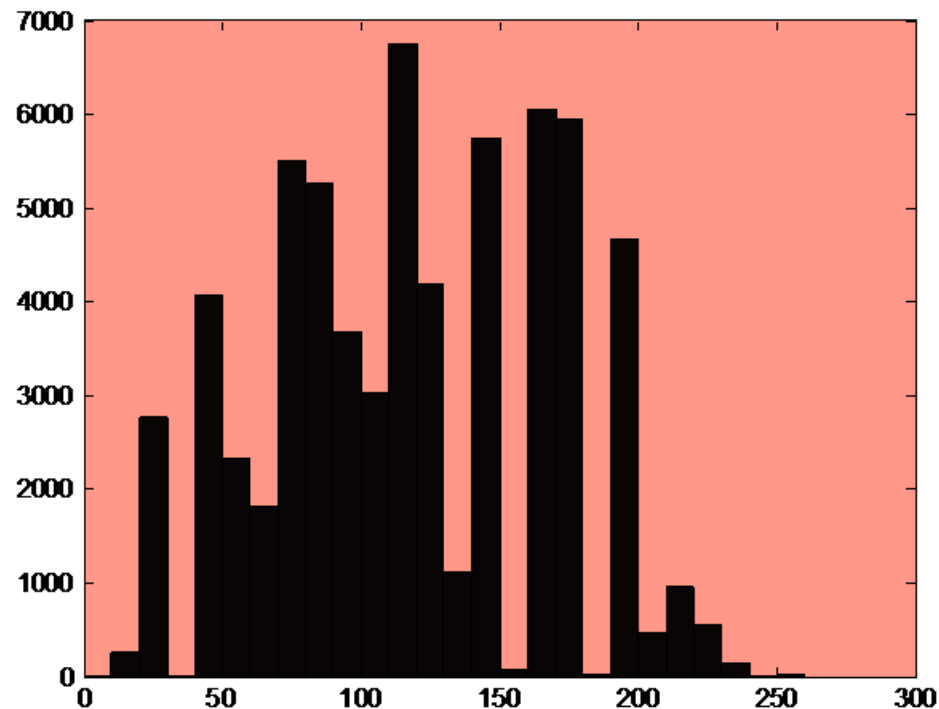
- Such frequency tables are also called **histograms** and they can also be used to store relative frequency instead of frequency.

# Example of a histogram: in image processing

- A grayscale image is a 2D array of size (say)  $H \times W$ .
- Each entry of this array is called a pixel and is indexed as  $(x,y)$  where  $x$  is the column index and  $y$  is the row index.
- At each pixel, we have an intensity value which tells us how bright the pixel is (smaller values = darker shades, larger value = brighter shades).
- Commonly, pixel values in grayscale photographic are 8 bit (ranging from 0 to 255).
- Histograms are widely used in image processing – in fact a histogram is often used in image retrieval (eg: finding images from the web that are most similar to a query image).



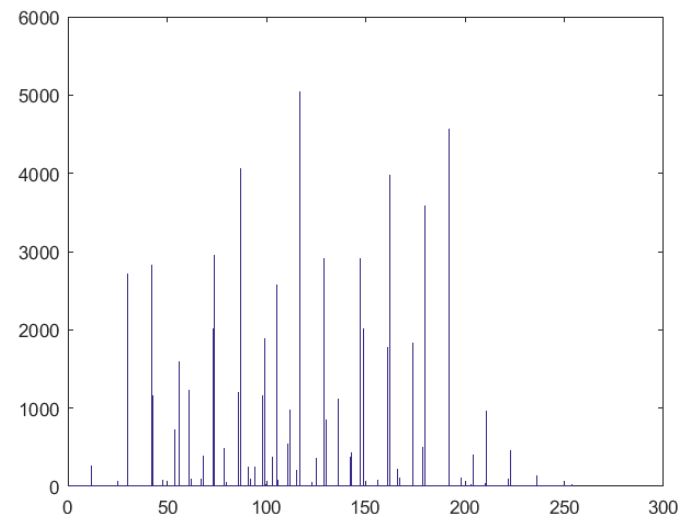
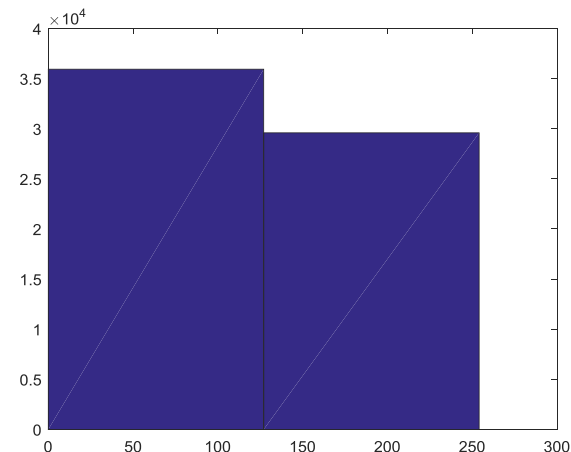
Example: histogram of the well-known “barbara image”, using bins of length 10. This image has values from 0 to 255 and hence there are 26 bins.





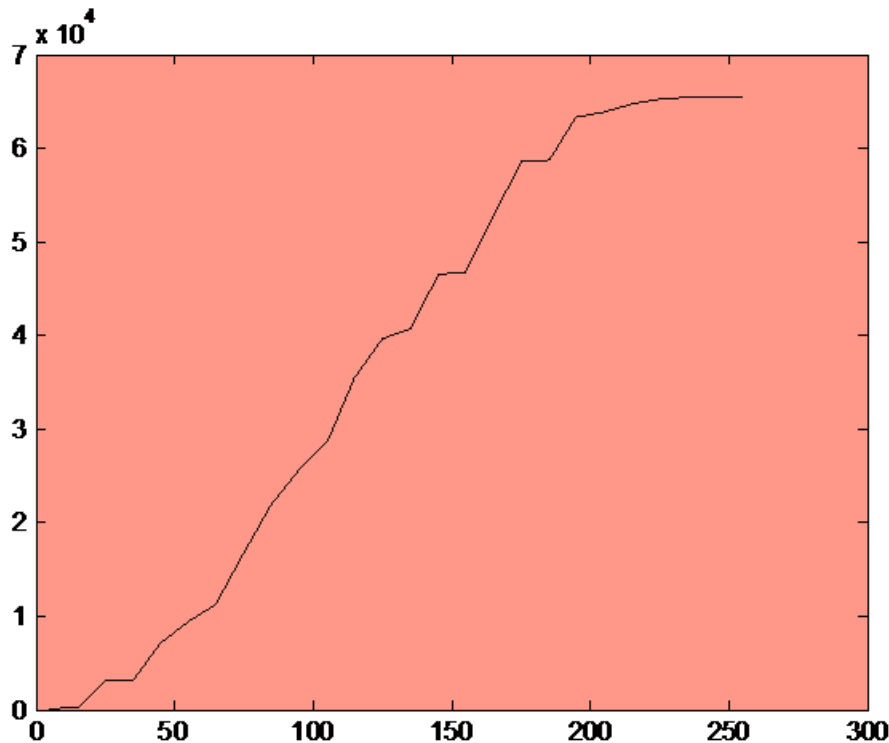
# The histogram binning problem

- If you have too few bins (each bin is very wide), there is very little idea you get about the data distribution from the histogram.
- Extreme: only one bin to represent all intensities in an image.
- If you have many bins (all will be narrow), then there are very points falling into each bin. Again there is very little idea you get about the data distribution from the histogram.
- Extreme: For intensities from a 512 x 512 image, if you had  $512^2$  histogram bins.



# Cumulative frequency plot

- The **cumulative** (relative) **frequency plot** (also called *ogive*) tells you the (proportion) number of sample points whose value is *less than or equal to* a given data value.



The cumulative frequency plot for the frequency plot from two slides back!

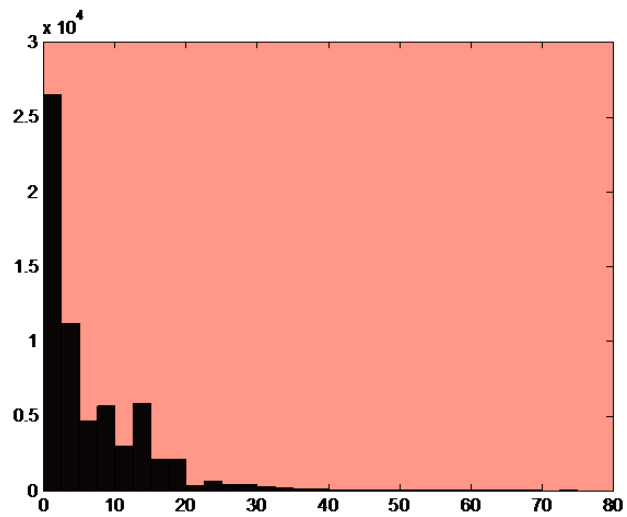
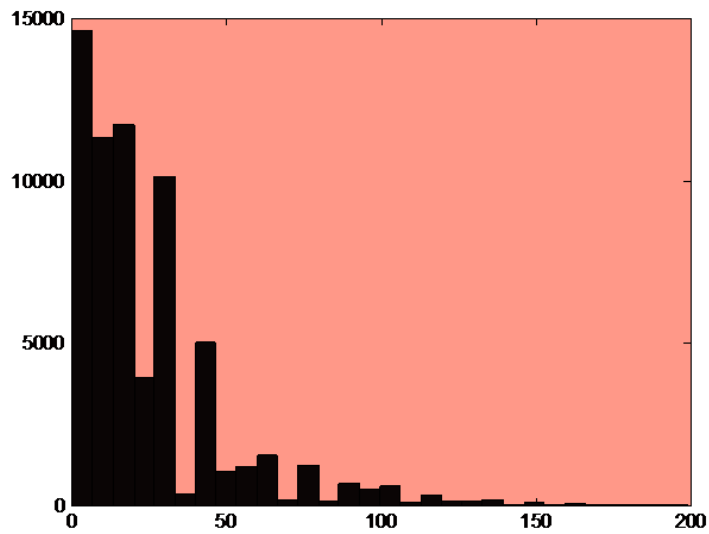
# Digression: A curious looking histogram in image processing

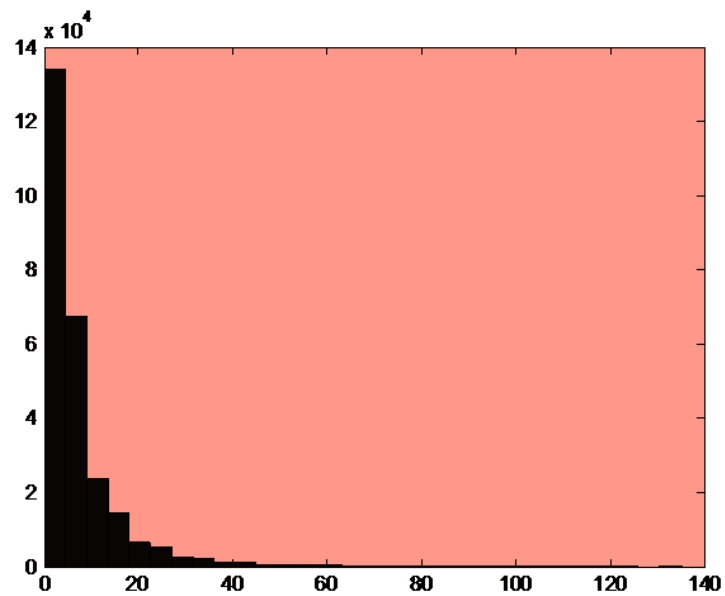
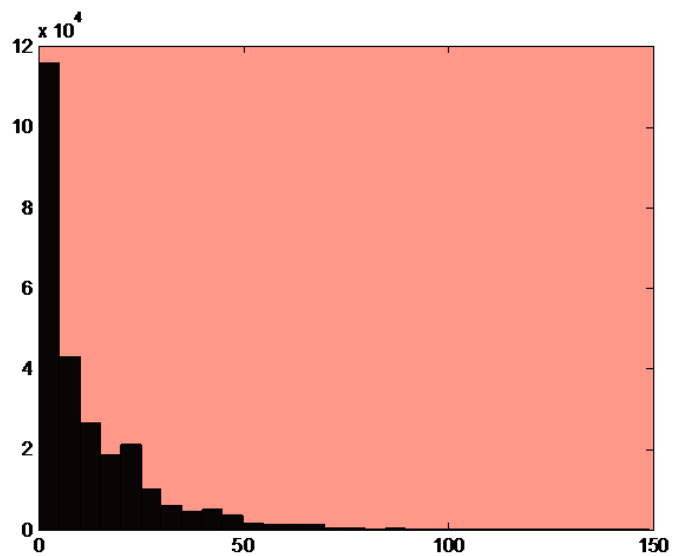
- Given the image  $I(x,y)$ , let's say we compute the x-gradient image in the following manner:

$$\forall x, y, 1 \leq x < W, 1 \leq y \leq H,$$

$$I_x(x, y) = I(x + 1, y) - I(x, y)$$

- And we plot the histogram of the **absolute** values of the x-gradient image.
- The next slide shows you how these histograms typically look! What do you observe?





# Summarizing the Data

```
08 02 22 97 38 15 00 40 00 75 04 05 07 78 52 12 50 77 91 08
49 49 99 40 17 81 18 57 60 87 17 40 98 43 69 48 04 56 62 00
81 49 31 73 55 79 14 29 93 71 40 67 53 88 30 03 49 13 36 65
52 70 95 23 04 60 11 42 69 24 68 56 01 32 56 71 37 02 36 91
22 31 16 71 51 67 63 89 41 92 36 54 22 40 40 28 66 33 13 80
24 47 32 60 99 03 45 02 44 75 33 53 78 36 84 20 35 17 12 50
32 98 81 28 64 23 67 10 26 38 40 67 59 54 70 66 18 38 64 70
67 26 20 68 02 62 12 20 95 63 94 39 63 08 40 91 66 49 94 21
24 55 58 05 66 73 99 26 97 17 78 78 96 83 14 88 34 89 63 72
21 36 23 09 75 00 76 44 20 45 35 14 00 61 33 97 34 31 33 95
78 17 53 28 22 75 31 67 15 94 03 80 04 62 16 14 09 53 56 92
16 39 05 42 96 35 31 47 55 58 88 24 00 17 54 24 36 29 85 57
86 56 00 48 35 71 89 07 05 44 44 37 44 60 21 58 51 54 17 58
19 80 81 68 05 94 47 69 28 73 92 13 86 52 17 77 04 89 55 40
04 52 08 83 97 35 99 16 07 97 57 32 16 26 26 79 33 27 98 66
88 36 68 87 57 62 20 72 03 46 33 67 46 55 12 32 63 93 53 69
04 42 16 73 38 25 39 11 24 94 72 18 08 46 29 32 40 62 76 36
20 69 36 41 72 30 23 88 34 62 99 69 82 67 59 85 74 04 36 16
20 73 35 29 78 31 90 01 74 31 49 71 48 86 81 16 23 57 05 54
01 70 54 71 83 51 54 69 16 92 33 48 61 43 52 01 89 19 67 48
```

# Summarizing a sample-set

- There are some values that can be considered “representative” of the entire sample-set. Such values are called as a “statistic”.
- The most common statistic is the sample (arithmetic) **mean**:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

- It is basically what is commonly regarded as “average value”.

# Summarizing a sample-set

- Another common statistic is the sample **median**, which is the “middle value”.
- We sort the data array **A** from smallest to largest. If  $N$  is odd, then the median is the value at the  $(N+1)/2$  position in the sorted array.
- If  $N$  is even, the median can take any value in the interval  $(A[N/2], A[N/2+1])$  – why?



# Properties of the mean and median

- Consider each sample point  $x_i$  were replaced by  $ax_i + b$  for some constants  $a$  and  $b$ .
- What happens to the mean? What happens to the median?
- Consider each sample point  $x_i$  were replaced by its square.
- What happens to the mean? What happens to the median?

# Properties of the mean and median

- **Question:** Consider a set of sample points  $x_1, x_2, \dots, x_N$ . For what value  $y$ , is the sum total of the **squared** difference with every sample point, the least? That is, what is:

$$\arg \min_y \sum_{i=1}^N (y - x_i)^2$$

Total squared deviation  
(or total squared loss)

Answer: mean  
(proof done in  
class)

- **Question:** For what value  $y$ , is the sum total of the **absolute** difference with every sample point, the least? That is, what is:

$$\arg \min_y \sum_{i=1}^N |y - x_i|$$

Total absolute deviation  
(or total absolute loss)

Answer: median  
(two proofs done  
in class – with  
and without  
calculus)

# Properties of the mean and median

- The mean need not be a member of the original sample-set.
- The median is always a member of the original sample-set if  $N$  is odd.
- The median is not unique and will not be a member of the set if  $N$  is even.

# Properties of the mean and median

- Consider a set of sample points  $x_1, x_2, \dots, x_N$ . Let us say that some of these values get grossly corrupted.
- What happens to the mean?
- What happens to the median?

# Example

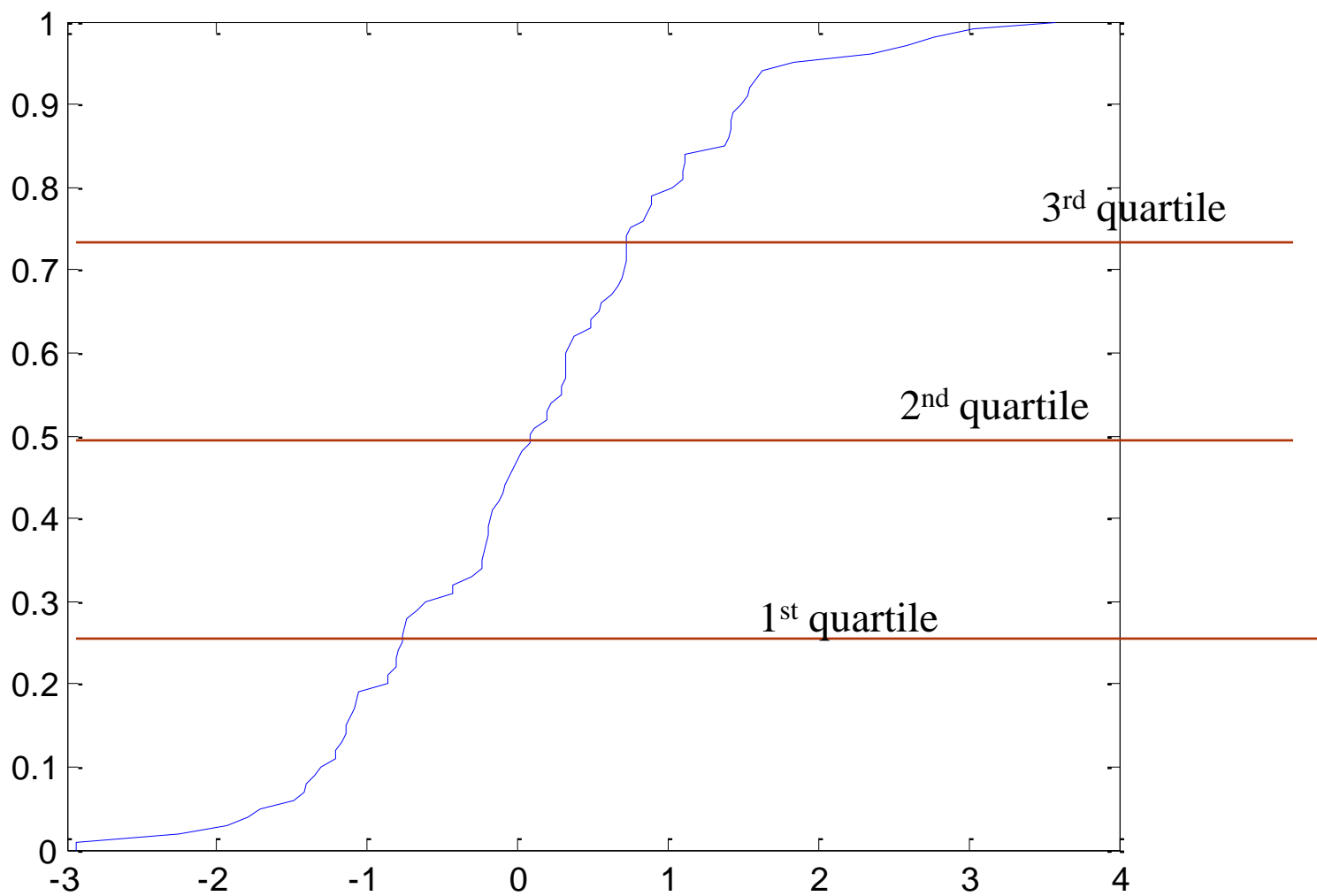
- Let  $A = \{1, 2, 3, 4, 6\}$
- $\text{Mean}(A) = 3.2$ ,  $\text{median}(A) = 3$
- Now consider  $A = \{1, 2, 3, 4, 20\}$
- $\text{Mean}(A) = 6$ ,  $\text{median}(A) = 3$ .

# Concept of quantiles

- The sample  $100p$  percentile ( $0 \leq p \leq 1$ ) is defined as the data value  $y$  such that  $100p\%$  of the data have a value less than or equal to  $y$ , and  $100(1-p)\%$  of the data have a larger value.
- For a data set with  $n$  sample points, the sample  $100p$  percentile is that value such that at least  $np$  of the values are less than or equal to it. And at least  $n(1-p)$  of the values are greater than it.

# Concept of quantiles

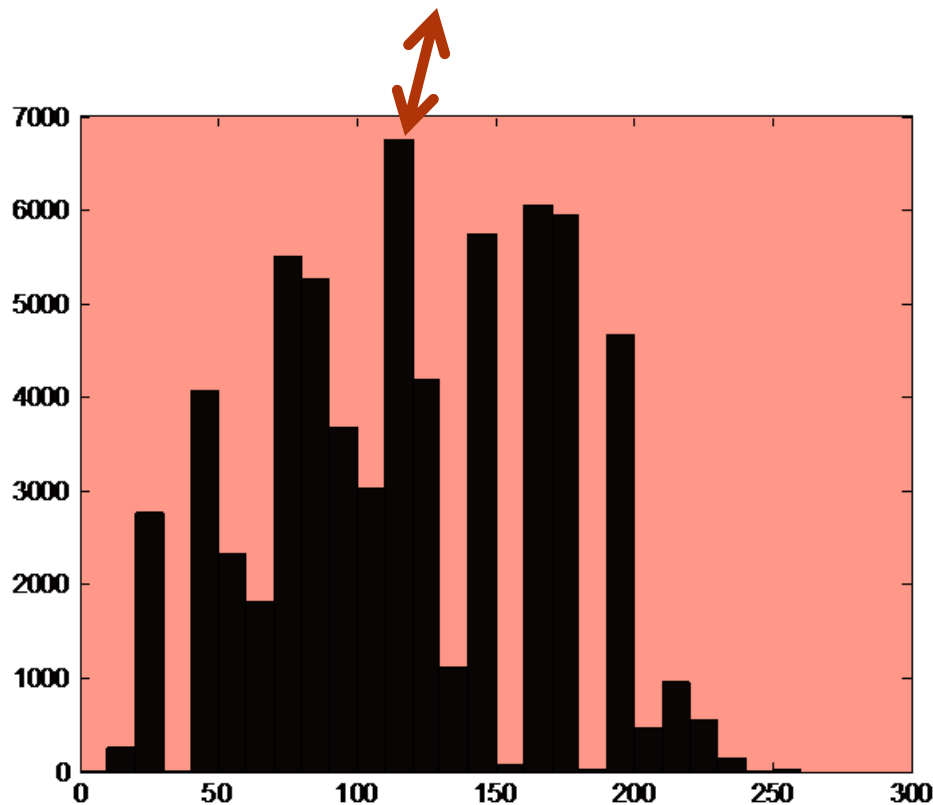
- The sample 25 percentile = first quartile.
- The sample 50 percentile = second quartile.
- The sample 75 percentile = third quartile.
- Quantiles can be inferred from the cumulative relative frequency plot (how?).
- Or by sorting the data values (how?).





# Concept of mode

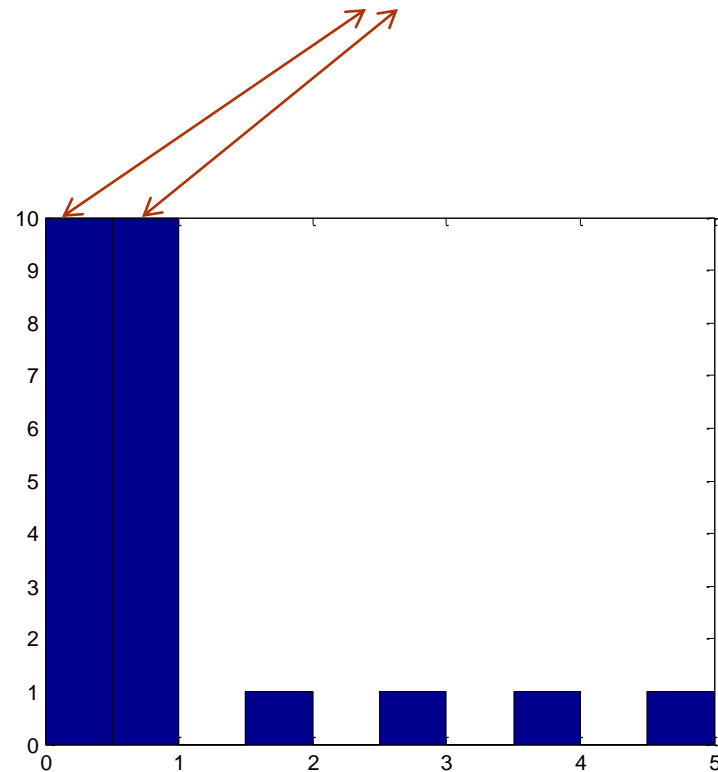
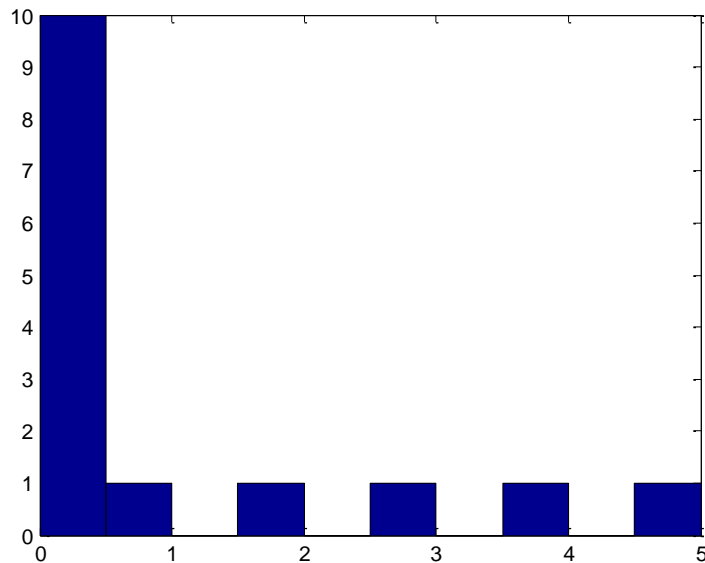
- The value that occurs with the highest frequency is called the mode.



# Concept of mode

- The mode may not be unique, in which case all the highest frequency values are called **modal values**.

Mode at 0



# Histogram for finding mean

- Given the histogram, the mean of a sample can be approximated as follows:

$$\bar{x} \approx \frac{\sum_{j=1}^K f_j (a_j + b_j) / 2}{N}$$

# Histogram for finding median

- Given the histogram, the median of a sample is the value at which you can split the histogram into two regions of equal areas.
- Keep adding areas from the leftmost bins till you reach more than  $N/2$  – now you know the bin in which the median will lie – the median is the midpoint of the bin.
- More useful for histograms whose “bins” contain single values.

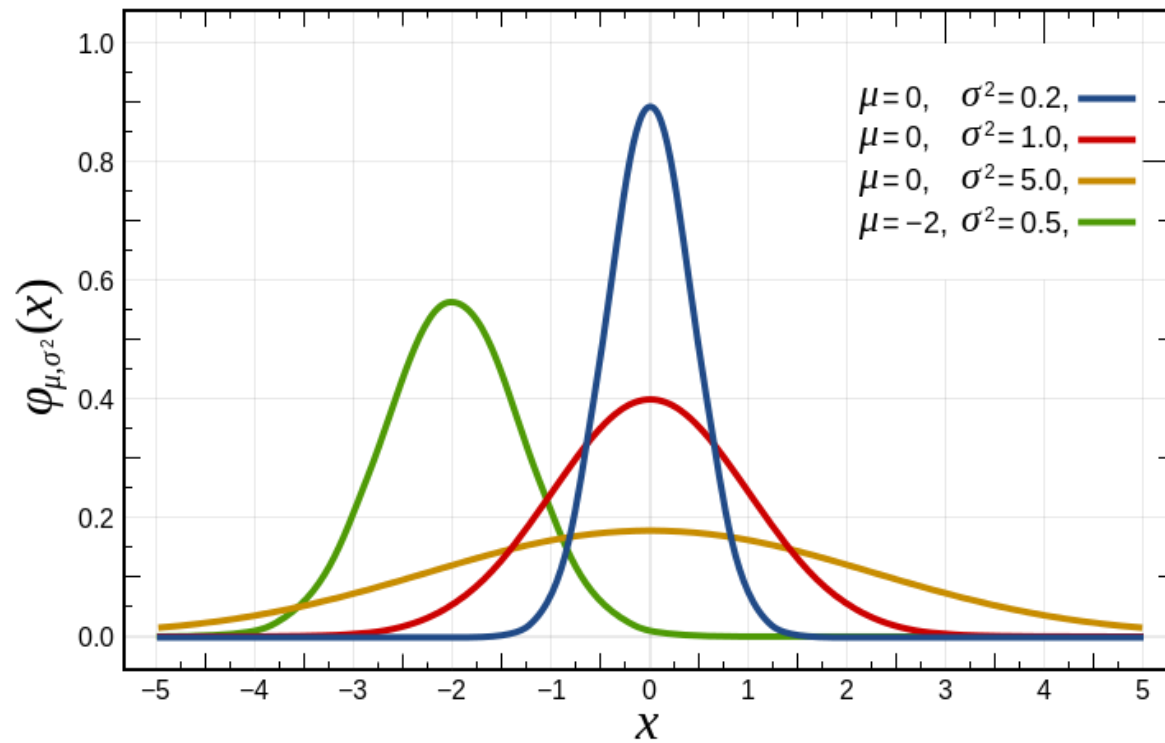
# Variance and Standard deviation

- The **variance** is (approximately) the average value of the squared distance between the sample points and the sample mean. The formula is:

$$\text{variance} = s^2 = \frac{1}{N-1} \sum_{i=1}^N (\bar{x} - x_i)^2$$

The division by  $N-1$  instead of  $N$  is for a very technical reason. As such, the variance is computed usually when  $N$  is large so the numerical difference is not much.

- The variance measures the “spread of the data around the sample mean”.
- Its positive square-root is called as the **standard deviation**.



[Image source](#)

# Variance and Standard deviation: Properties

- Consider each sample point  $x_i$  were replaced by  $ax_i + b$  for some constants  $a$  and  $b$ . What happens to the standard deviation?

# Standard deviation: practical application 1

- Let us say a factory manufactures a product which is required to have a certain weight  $w$ .
- In practice, the weight of each instance of the product will deviate from  $w$ .
- In such a case, we need to see whether the average weight is close to (or equal to  $w$ ).
- But we also need to see that the standard deviation is small.
- In fact, the standard deviation can be used to predict how likely it is that the product weight will deviate significantly from the mean.



# Standard deviation: practical application 2

- In the definition of disease such as osteoporosis (low bone density)
- A person whose bone density is less than  $2.5\sigma$  below the average bone density for that age-group, gender and geographical region, is said to be suffering from osteoporosis. Here  $\sigma$  is the standard deviation of the bone density of that particular population.



[Image source](#)

# Chebyshev's inequality

- Suppose I told you that the average marks for this course was 75 (out of 100). And that the variance of the marks was 25.
- Can you say something about how many students secured marks from 65 to 85?
- You obviously cannot predict the exact number – but you can say **something** about this number.
- That something is given by Chebyshev's inequality.

# Chebyshev's inequality: and Chebyshev



[https://en.wikipedia.org/wiki/Pafnuty\\_Chebyshev](https://en.wikipedia.org/wiki/Pafnuty_Chebyshev)

Russian mathematician:  
Stellar contributions in probability and statistics,  
geometry, mechanics

Two-sided Chebyshev's inequality:

The proportion of sample points  $k$  or more than  $k$  ( $k > 0$ ) standard deviations away from the sample mean is less than or equal to  $1/k^2$ .

# Chebyshev's inequality: and Chebyshev

Two-sided Chebyshev's inequality:

The proportion of sample points  $k$  or more than  $k$  ( $k > 0$ ) standard deviations away from the sample mean is less than or equal to  $1/k^2$ .



$$S_k = \{x_i : |x_i - \bar{x}| \geq k\sigma\}$$

$$\frac{|S_k|}{N} \leq \frac{1}{k^2}$$

Proof: on the board!  
And in the book.

# Chebyshev's inequality

- Applying this inequality to the previous problem, we see that the fraction of students who got less than 65 or more than 85 marks is as follows:

$$S_k = \{x_i : |x_i - \bar{x}| \geq k\sigma\}$$

$$\frac{|S_k|}{N} \leq \frac{1}{k^2}$$

$$\bar{x} = 75$$

$$\sigma = 5$$

$$k = 2$$

$$\frac{|S_k|}{N} \leq \frac{1}{4}$$

- So the fraction of students who got from 65 to 85 is more than  $1 - 0.25 = 0.75$ .

# Chebyshev's inequality

1	Kerala	93.91
2	Lakshadweep	92.28
3	Mizoram	91.58
4	Tripura	87.75
5	Goa	87.40
6	Daman & Diu	87.07
7	Puducherry	86.55
8	Chandigarh	86.43
9	Delhi	86.34
10	Andaman & Nicobar Islands	86.27
11	Himachal Pradesh	83.78
12	Maharashtra	82.91

Mean = 87.69

Std. dev. = 3.306

Fraction of states with literacy rate  
in the range  
( $\mu - 1.5\sigma$ ,  $\mu + 1.5\sigma$ ) is  $11/12 \approx 91\%$

As predicted by Chebyshev's  
inequality, it is **at least**  
 $1 - 1/(1.5^2) \approx 0.55$

The bounds predicted by this  
inequality are loose – but they  
are correct!

[https://en.wikipedia.org/wiki/Indian\\_states\\_ranking\\_by\\_literacy\\_rate](https://en.wikipedia.org/wiki/Indian_states_ranking_by_literacy_rate)

# One-sided Chebyshev's inequality

- Also called the Chebyshev-Cantelli inequality.

The proportion of sample points  $k$  or more than  $k$  ( $k > 0$ ) standard deviations away from the sample mean **and greater than the sample mean** is less than or equal to  $1/(1+k^2)$ .



$$S_k = \{x_i : x_i - \bar{x} \geq k\sigma\}$$

$$\frac{|S_k|}{N} \leq \frac{1}{1+k^2}$$

Notice: no absolute value!

Proof: on the board!  
And in the book.

# One-sided Chebyshev's inequality (Another form)

- Also called the Chebyshev-Cantelli inequality.

The proportion of sample points  $k$  or more than  $k$  ( $k > 0$ ) standard deviations away from the sample mean **and less than the sample mean** is less than or equal to  $1/(1+k^2)$ .



$$S_k = \{x_i : x_i - \bar{x} \leq -k\sigma\}$$

$$\frac{|S_k|}{N} \leq \frac{1}{1+k^2}$$

Notice: no absolute value!

Proof: on the board!  
And in the book.



# Correlation between different data values

- Sometimes each sample-point can have a pair of attributes.
- And it may so happen that large values of the first attribute are accompanied with large (or small) values of the second attribute for a large number of sample-points.

# Correlation between different data values

- Example 1: Populations with higher levels of fat intake show higher incidence of heart disease.
- Example 2: People with higher levels of education often have higher incomes.
- Example 3: Literacy Rate in India as a function of time?

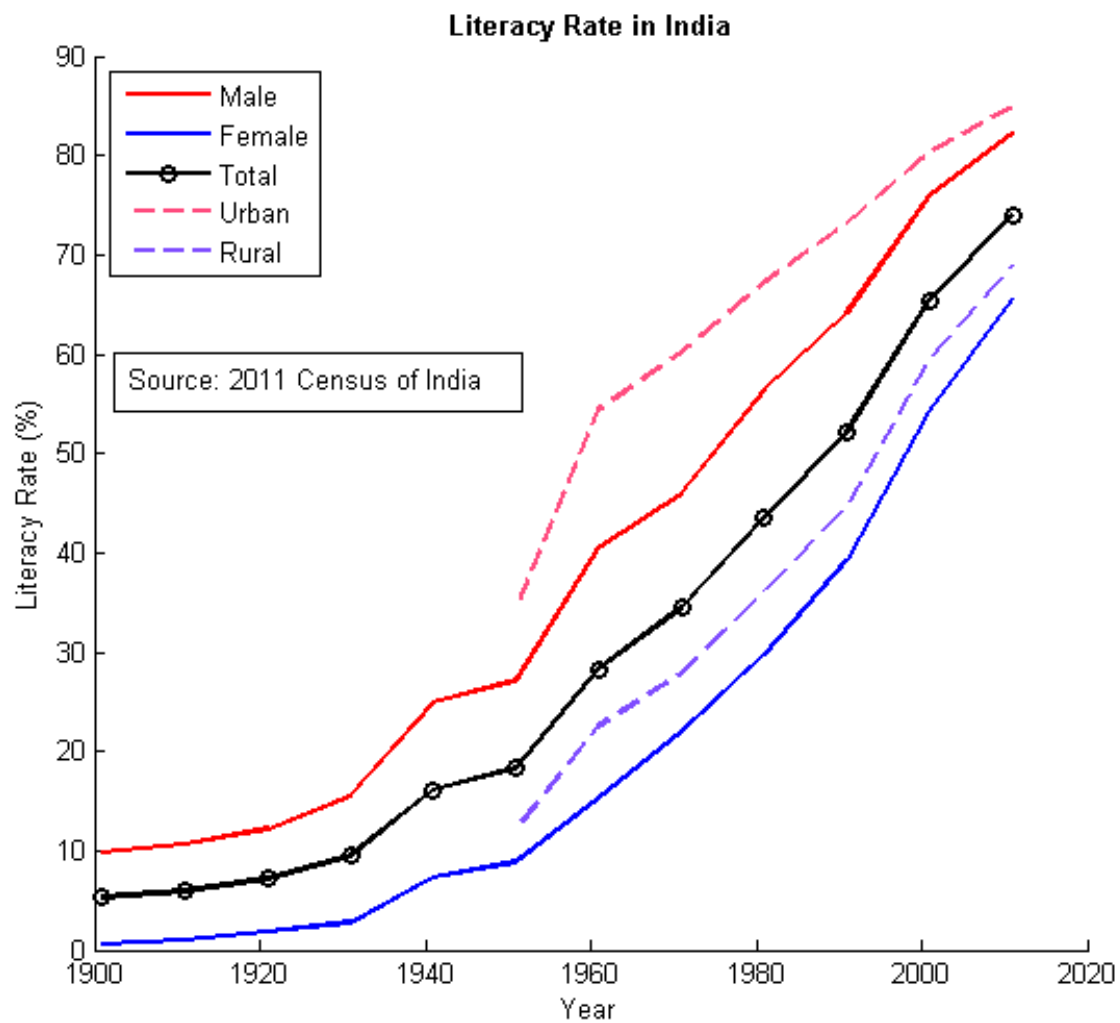


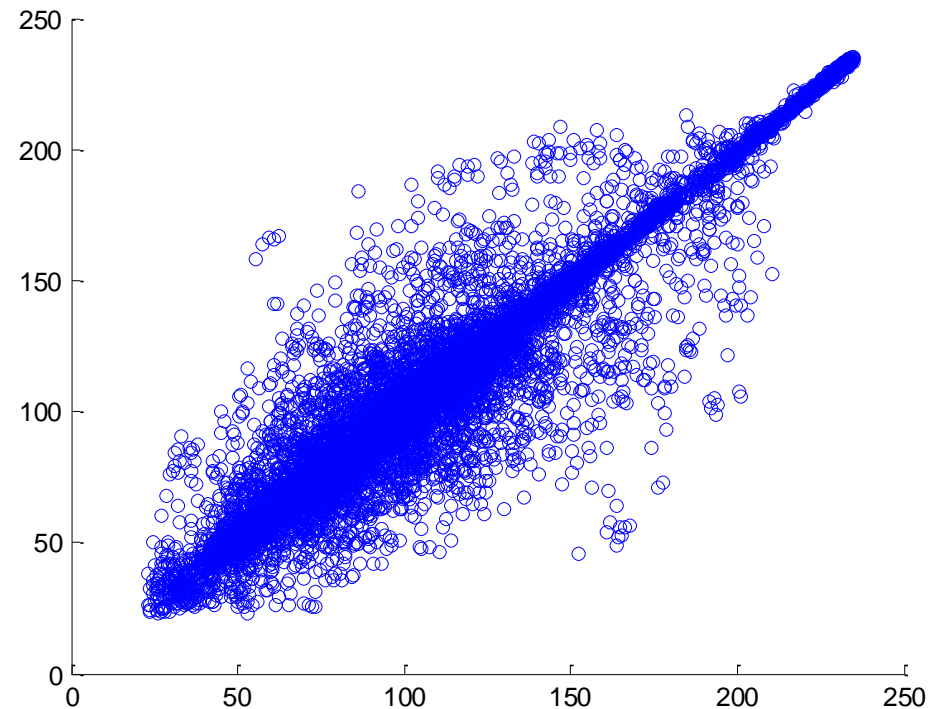
Image source

# Visualizing such relationships?

- Can be done by means of a scatter plot
- X axis: values of attribute 1, Y axis: values of attribute 2
- Plot a marker at each such data point. The marker may be a small circle, a +, a \*, and so on.

# Visualizing such relationships?

- Image processing example: pixel intensity value and intensity value of the pixel right neighbor



# Correlation coefficient

- Let the sample-points be given as  $(x_i, y_i)$ ,  $1 \leq i \leq N$ .
- Let the sample standard deviations be  $\sigma_x$  and  $\sigma_y$ , and the sample means be  $\mu_x$  and  $\mu_y$ .
- The **correlation-coefficient** is given as:

$$r(x, y) = \frac{\sum_{i=1}^N (x_i - \mu_x)(y_i - \mu_y)}{\sqrt{\sum_{i=1}^N (x_i - \mu_x)^2 \sum_{i=1}^N (y_i - \mu_y)^2}} = \frac{\sum_{i=1}^N (x_i - \mu_x)(y_i - \mu_y)}{(N-1)\sigma_x\sigma_y}$$

# Correlation coefficient

- The correlation-coefficient is given as:

$$r(x, y) = \frac{\sum_{i=1}^N (x_i - \mu_x)(y_i - \mu_y)}{\sqrt{\sum_{i=1}^N (x_i - \mu_x)^2 \sum_{i=1}^N (y_i - \mu_y)^2}} = \frac{\sum_{i=1}^N (x_i - \mu_x)(y_i - \mu_y)}{(N-1)\sigma_x\sigma_y}$$

- $r > 0$  means the data are **positively correlated** (one attribute being higher implies the other is higher)
- $r < 0$  means the data are **negatively correlated** (one attribute being higher implies the other is lower)
- $r = 0$  means the data are **uncorrelated** (there is no such relationship!)
- $r$  is **undefined** if the standard deviation of either  $x$  or  $y$  is 0.

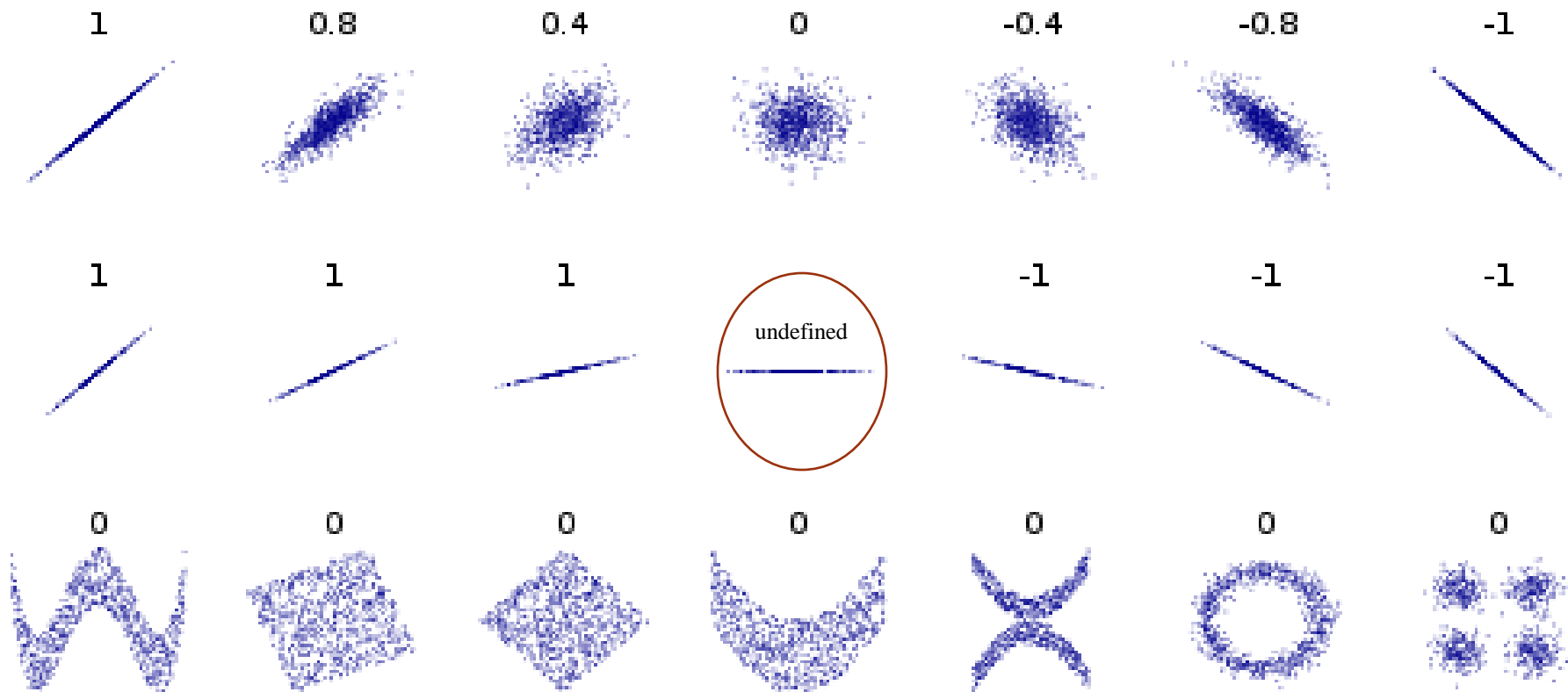
# Correlation coefficient: Properties

- The correlation-coefficient is given as:

$$r(x, y) = \frac{\sum_{i=1}^N (x_i - \mu_x)(y_i - \mu_y)}{\sqrt{\sum_{i=1}^N (x_i - \mu_x)^2 \sum_{i=1}^N (y_i - \mu_y)^2}} = \frac{\sum_{i=1}^N (x_i - \mu_x)(y_i - \mu_y)}{(N-1)\sigma_x\sigma_y}$$

- $-1 \leq r \leq 1$  always!





Correlation coefficient values for various toy datasets in 2D:  
for each dataset, a scatter plot is provided

[https://en.wikipedia.org/wiki/Correlation\\_and\\_dependence](https://en.wikipedia.org/wiki/Correlation_and_dependence)

# Correlation coefficient: geometric interpretation

- Consider the  $N$  values  $x_1, x_2, \dots, x_N$ . We will assemble them into a vector  $\mathbf{x}$  (1D array) of  $N$  elements.
- We will also create vector  $\mathbf{y}$  from  $y_1, y_2, \dots, y_N$ .
- Now create vectors  $\mathbf{x} - \mu_x$  and  $\mathbf{y} - \mu_y$  – by deducting  $\mu_x$  from each element of  $\mathbf{x}$ , and  $\mu_y$  from each element of  $\mathbf{y}$ .
- Note that you may be used to vectors in 2D or 3D, but in statistics or machine learning, we frequently use vectors in  $N$ -D!

# Correlation coefficient: geometric interpretation

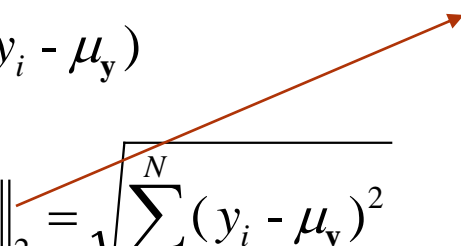
- Then  $r(\mathbf{x}, \mathbf{y})$  is basically the cosine of the angle between  $\mathbf{x} - \mu_x$  and  $\mathbf{y} - \mu_y$ !

$$r(\mathbf{x} - \mu_x, \mathbf{y} - \mu_y) = \cos \theta = \frac{(\mathbf{x} - \mu_x) \bullet (\mathbf{y} - \mu_y)}{\|\mathbf{x} - \mu_x\|_2 \|\mathbf{y} - \mu_y\|_2}$$

$$(\mathbf{x} - \mu_x) \bullet (\mathbf{y} - \mu_y) = \sum_{i=1}^N (x_i - \mu_x)(y_i - \mu_y)$$

$$\|\mathbf{x} - \mu_x\|_2 = \sqrt{\sum_{i=1}^N (x_i - \mu_x)^2}, \|\mathbf{y} - \mu_y\|_2 = \sqrt{\sum_{i=1}^N (y_i - \mu_y)^2}$$

Vector magnitude -  
also called the L2-  
norm of the vector.



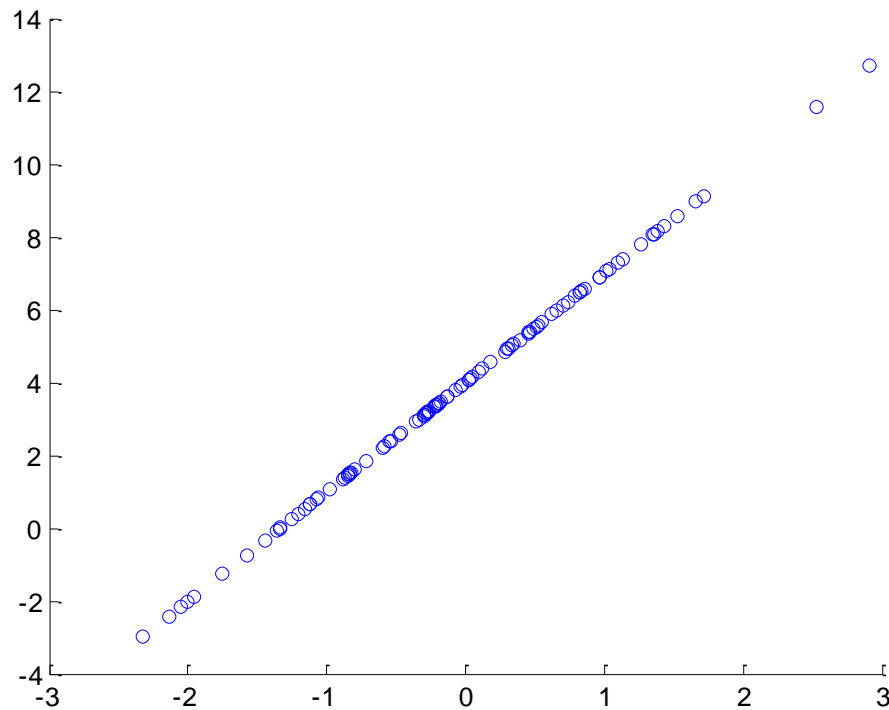
- Note that the cosine of an angle has a value from -1 to +1.

# Correlation coefficient: Properties

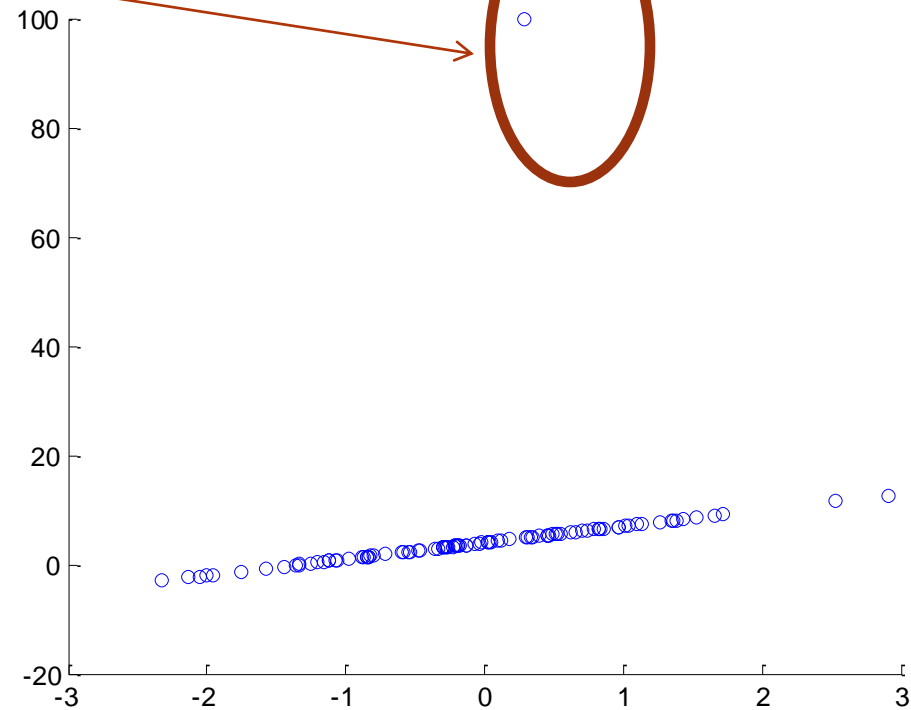
- In the following, we have  $a, b, c, d$  constant.
- If  $y_i = a + bx_i$  where  $b > 0$ , then  $r(x, y) = 1$ .
- If  $y_i = a + bx_i$  where  $b < 0$ , then  $r(x, y) = -1$ .
- If  $r$  is the correlation coefficient of data pairs as  $(x_i, y_i)$ ,  $1 \leq i \leq N$ , then it is also the correlation coefficient of data pairs  $(b + ax_i, d + cy_i)$  when  $a$  and  $c$  have the same sign.

# Correlation coefficient: a word of caution

- Sensitive to outliers!



$r = 1$

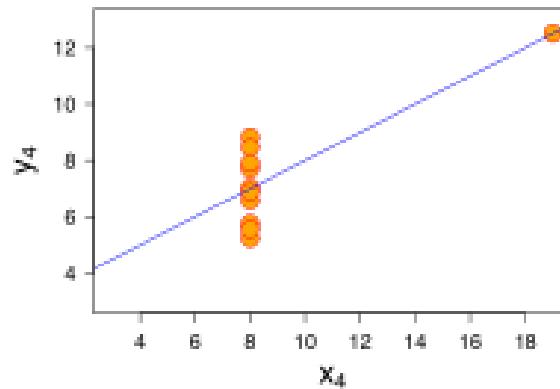
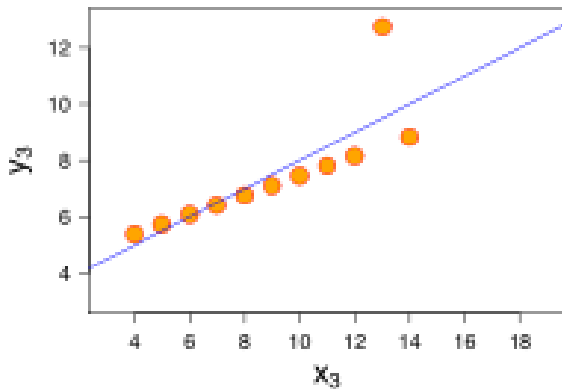
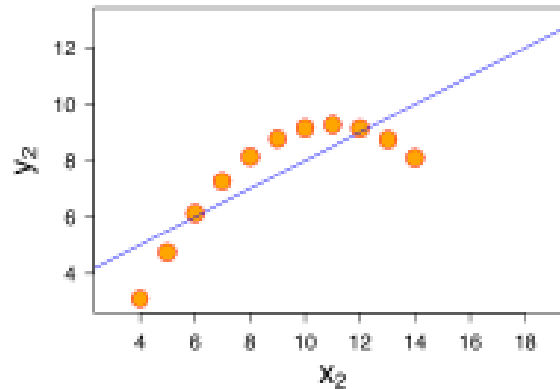
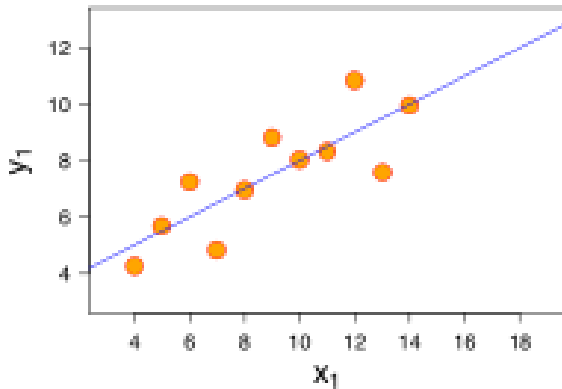


$r = 0.33$

# Caution with correlation: Anscombe's quartet

- The correlation coefficient can be a misleading value, and graphical examination of the data is important.
- This was illustrated beautifully by a British statistician named Frank Anscombe – by showing four examples that graphically appear very different – even though they produce identical correlation coefficients.
- These examples are famously called Anscombe's quartet.

# Caution with correlation: Anscombe's quartet



In each of these examples, the following quantities were the same:

- Mean and variance of  $x$
- Mean and variance of  $y$
- Correlation coefficient  $r(x,y)$

But the data are graphically very different!

# Reflective (or Uncentered) correlation coefficient

- A version of the correlation coefficient in which you do not deduct the mean values from the vectors!

$$r(\mathbf{x}, \mathbf{y}) = \frac{\sum_{i=1}^N (x_i - \mu_x)(y_i - \mu_y)}{\sqrt{\sum_{i=1}^N (x_i - \mu_x)^2 \sum_{i=1}^N (y_i - \mu_y)^2}} \neq r_{\text{uncentered}}(\mathbf{x}, \mathbf{y}) = \frac{\sum_{i=1}^N x_i y_i}{\sqrt{\sum_{i=1}^N x_i^2 \sum_{i=1}^N y_i^2}}$$

- Uncentered c.c. is not “translation invariant”:

$$r(\mathbf{x}, \mathbf{y}) = r(\mathbf{x} + \mathbf{a}, \mathbf{y} + \mathbf{b})$$

$$r_{\text{uncentered}}(\mathbf{x}, \mathbf{y}) \neq r_{\text{uncentered}}(\mathbf{x} + \mathbf{a}, \mathbf{y} + \mathbf{b})$$



# Correlation does not **necessarily** imply causation

- A high correlation between two attributes does not mean that one causes the other.
- Example 1: Fast rotating windmills are observed when the wind speed is high. Hence can one say that the windmill rotation produces speedy wind? (a wind**mill** in the literal sense 😊)

# Correlation does not **necessarily** imply causation

- In example 1, the cause and effect were swapped. High wind speed leads to fast rotation and not vice-versa.
- Example 2: High sale of ice-cream is correlated with larger occurrence of drowning. Hence can one say that ice-cream causes drowning?
- In this case, there is a third factor that is highly correlated with both – ice-cream sales, as well as drowning. Ice-cream sales and swimming activities are on the rise in the summer!

# Correlation does not **necessarily** imply causation

- The above statement does not mean that correlation is *never* associated with causation (example: increase in age does cause increase in height in children or adolescents) – just that it is not *sufficient* to establish causation.
- Consider the argument: “High correlation between tobacco usage and lung cancer occurrence does **not** imply that smoking causes lung cancer.”

# Correlation does not necessarily imply causation – but it **may**!

- However multiple observational studies that eliminate other possible causes do lead to the conclusion that smoking causes cancer!
  - ❑ higher tobacco dosage associated with higher occurrence of cancer
  - ❑ stopping smoking associated with lower occurrence of cancer
  - ❑ higher duration of smoking associated with higher occurrence of cancer
  - ❑ unfiltered (as opposed to filtered) cigarettes associated with higher occurrence of cancer
- See <https://www.sciencebasedmedicine.org/evidence-in-medicine-correlation-and-causation/> and <http://www.americanscientist.org/issues/pub/what-everyone-should-know-about-statistical-correlation> for more details.