

Question - 1:

Normalize the following numbers:

	Given Number	Normalized Number
i.	0.0000124678_{10}	1.24678×10^{-5}
ii.	$1584.234_{10} \times 10^5$	1.584234×10^8
iii.	4782.2354_{10}	4.7822354×10^3
iv.	110101.1111_2	1.10101111×2^5
v.	0.001100_2	1.100×2^{-3}
vi.	$1101.1111_2 \times 2^5$	1.101111×2^8

Question - 2:

Find the Biased Exponent of 1.1011×2^{34} in IEEE-754 single precision format.

$$\text{Bias} = 2^{8-1} = 127$$

$$\begin{aligned}\text{Biased exponent} &= 34 + 127 \\ &= 161 \\ &= 1010\ 0001 \ (\text{Ans})\end{aligned}$$

Question - 3:

Find the Biased Exponent of 1.1011×2^4 in 12-bit IEEE-754 format where the size of the exponent field is 4 bits.

$$\text{Bias} = 2^{4-1} - 1 = 7$$

$$\text{Biased exponent} = 4 + 7$$

$$= 11$$

$$= 1011 \text{ (Am)}$$

Question - 4:

Find the Biased Exponent of 1.1011×2^{34} in 64-bit IEEE-754 format.

$$\text{Bias} = 2^{11-1} - 1$$

$$= 1023$$

$$\begin{aligned}\text{Biased Exponent} &= (34 + 1023) = 1057 \\ &= 100\ 0010\ 0001 \text{ (Am)}\end{aligned}$$

Question - 5:

Find the Biased Exponent of 5678.898 in 34-bit IEEE-754 format where the size of the exponent field is 10 bits.

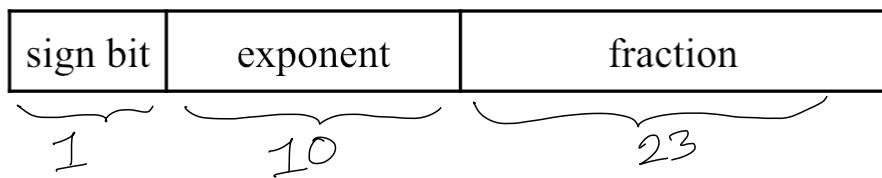
$$\begin{aligned}&5678.898 \\ &= 1.0110\ 0010\ 1110 \cdot 1110\ 0101\ 11\dots\dots \\ &= 1.0110\ 0010\ 1110\ 1110 \times 2^{12} \text{ actual exponent}\end{aligned}$$

$$\text{Bias} = 2^{10-1} - 1 = 511$$

$$\begin{aligned}\text{Biased exponent} &= 511 + 12 = 523 \\ &= 10\ 0000\ 1011 \text{ (Am)}\end{aligned}$$

Question - 6: $\xrightarrow{\text{sign bit} = 1}$

Convert -0.00987_{10} in 34-bit IEEE-754 floating point representation where the size of the fraction field is 23 bits.



$$-0.00987 = 0.0000\ 0010\ 0\dots$$

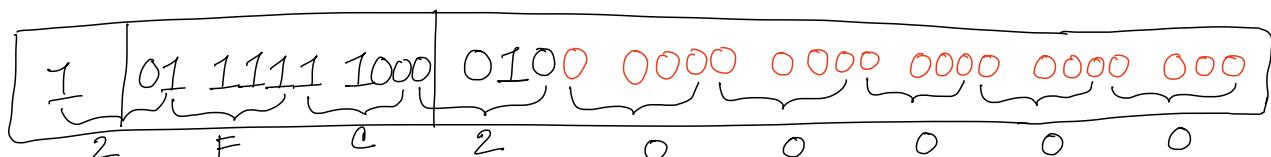
$$\begin{aligned} &= 1.010\dots \times 2^{-7} \\ &\quad \text{Fraction} \end{aligned}$$

-7 ← actual exponent

$$\text{Bias} = 2^{10-1} - 1 = 511$$

$$\text{Biased Exponent} = 511 + (-7) = 504 = 01\ 1111\ 1000$$

$$\text{Fraction} = 0100\ 0000\ 0000\ 0000\ 0000\ 000$$



$$= 0x2FC200000 (\text{Ans})$$

try Q7-Q8 yourself

Question - 9:

Convert the following IEEE-754 single-precision floating point numbers into decimal.

	Given Numbers	Decimal Representation
i.	0xFF1205BA	$-1.9409702 \times 10^{38}$
ii.	3457890989_{10}	Try next
iii.	23245613451_8	Try yourself

(i) $0xFF1205BA$

$= 1111\ 111\ 0001\ 0010\ 0000\ 0101\ 1011\ 1010$

Sign Bit
Biased Exp.
Fraction

$$\Rightarrow \text{Bias} = 2^{8-1} - 1 = 127$$

$$\text{Biased exponent} = 1111110 = 254$$

$$\therefore \text{Actual exp.} = 254 - 127 = 127$$

$$\Rightarrow \text{Fraction} = 0.001\ 0010\ 0000\ 0101\ 1011\ 1010$$

$$= 0.1407\ 9976\ 0818\ 4814\ 4531$$

$$\begin{aligned} \Rightarrow \text{Decimal value} &= (-1)^1 \times (1 + 0.1407\ 9976\ 0818\ 4814\ 4531) \times 2^{127} \\ &= -1.1407\ 9976\ 0818\ 4814\ 4531 \times 2^{127} \\ &= -1.9409702 \times 10^{38} \quad (\text{Ans}) \end{aligned}$$

Question - 10:

Multiply the given numbers using IEEE-754 single-precision floating-point representation. Check if the result has overflow or underflow. Show the result in decimal.

Note: Consider 10 decimal digits while converting from decimal to binary for the following questions.

	Given Numbers	Result	Overflow/ Underflow
i.	7.01_{10} and 0.71_{10}		
ii.	0.000101_2 and 10.1_2		
iii.	$0.000101_2 \times 2^{70}$ and $10010.000101_2 \times 2^{60}$		
iv.	1584.234_{10} and 1584.234_{10}		
v.	0.001100_2		
vi.	$1101.1111_2 \times 2^5$ and $110.000101_2 \times 2^6$		

$$(iii) \quad 0.000101 \times 2^{70} \times 10010.000101 \times 2^{60}$$

$$\Rightarrow 0.000101 \times 2^{70}$$

$$= 1.01 \times 2^{70-4}$$

$$= 1.01 \times 2^{66}$$

$$\Rightarrow 10010.000101 \times 2^{60}$$

$$\Rightarrow 1.0010\ 0001\ 01 \times 2^{60+9}$$

$$= 1.0010\ 0001\ 01 \times 2^{64}$$

$$\Rightarrow 1.01 \times 2^{66} \times 1.0010\ 0001\ 01 \times 2^{64}$$

$$\Rightarrow (1.01 \times 1.0010\ 0001\ 01) \times 2^{130}$$

$$\Rightarrow 1.01101 \times 2^{130} \leftarrow \begin{matrix} \text{actual} \\ \text{exponent} \end{matrix}$$

$$\text{Bias} = 2^{8-1} - 1 = 127$$

$$\text{Biased Exponent} = 127 + 130 \\ = 257$$

Range of Biased exponent = 0 to 2^{8-1}
 $= 0 \text{ to } 255$
 $= [1 \text{ to } 254] \quad [0 \& 255 \text{ reserved}]$

(Upper range) $\underline{254}$ is smaller than biased exponent ($\underline{257}$)

Hence, Overflow.

$$(iv) 1584.234 \times 1584.234$$

$$1584 = 110\ 0011\ 0000$$

$$.234 = .0011\ 1011\ 11\dots$$

$$1584.234 = 110\ 0011\ 0000. 0011\ 0111\dots$$

$$= 1.10\ 0011\ 0000\ 0011\ 0111\dots \times 2^{10}$$

$$\Rightarrow 1.10\ 0011\ 0000\ 0011\ 0111\dots \times 2^{10} \times 1.10\ 0011\ 0000\ 0011\ 0111\dots \times 2^{10}$$

$$= 10. 01100 \times 2^{20}$$

$$= 1. 0011\ 00 \times 2^{21}$$

$$\text{Bias} = 2^{8-1} - 1 = 127$$

$$\text{Biased Exponent} = 127 + 21 \\ = 148$$

Range of Biased exponent = 0 to 2^{8-1}
 $= 0 \text{ to } 255$
 $= [1 \text{ to } 254] \quad [0 \& 255 \text{ reserved}]$

$$1 < 148 < 254$$

Hence, No overflow / underflow

Question - 11:

Multiply the given numbers using IEEE-754 double-precision floating-point representation. Check if the result has overflow or underflow.

Note: Consider 10 decimal digits while converting from decimal to binary for the following questions.

	Given Numbers	Result	Overflow/ Underflow
i.	7.01_{10} and 0.71_{10}	<i>I try next</i>	
ii.	$0.000101_2 \times 2^{-850}$ and $10.1_2 \times 2^{-900}$	<i>I yourself</i>	
iii.	$0.0101_2 \times 2^{790}$ and $10010.0101_2 \times 2^{680}$		

$$(iii) 0.0101 \times 2^{790} \times 10010.0101 \times 2^{680}$$

$$\begin{aligned} &\Rightarrow 0.0101 \times 2^{790} && \Rightarrow 10010.0101 \times 2^{680} \\ &= 1.01 \times 2^{790-2} && = 1.00100101 \times 2^{680+4} \\ &= 1.01 \times 2^{788} && = 1.00100101 \times 2^{684} \end{aligned}$$

$$\Rightarrow 1.01 \times 2^{788} \times 1.00100101 \times 2^{684}$$

$$\begin{aligned} &= (1.01 \times 1.00100101) \times 2^{788+684} \\ &= 1.01101 \times 2^{1472} \quad \text{actual exponent.} \end{aligned}$$

$$\text{Bias}_0 = 2^{11-1} - 1 = 1023$$

$$\begin{aligned} \text{Biased Exponent} &= 1023 + 1472 \\ &= 2495 \end{aligned}$$

Range of Biased exponent = 0 to $2^{11}-1$
 = 0 to 2048
 = (1 to 2047) [0 & 2048 reserved]

$2495 > 2047$ (Upper range)

Hence, Overflow.

Question - 12:

Multiply the given numbers using 18bit IEEE-754 floating-point representation where the size of the fraction field is 12 bits. Check if the result has overflow or underflow.

Note: Consider 10 decimal digits while converting from decimal to binary for the following questions.



	Given Numbers	Result	Overflow/ Underflow
i.	7.01_{10} and 0.71_{10}		
ii.	$0.000101_2 \times 2^{-85}$ and $10.1_2 \times 2^{-90}$		
iii.	$0.0101_2 \times 2^{79}$ and $10010.0101_2 \times 2^{68}$		

$$(ii) 0.000101 \times 2^{-85} \times 10.1 \times 2^{-90}$$

$$\begin{aligned} &\Rightarrow 0.000101 \times 2^{-85} \\ &= 1.01 \times 2^{-85-4} \\ &= 1.01 \times 2^{-89} \end{aligned} \quad \left\{ \begin{aligned} &\Rightarrow 10.1 \times 2^{-90} \\ &= 1.01 \times 2^{-90+1} \\ &= 1.01 \times 2^{-89} \end{aligned} \right.$$

$$\Rightarrow 1.01 \times 2^{-89} \times 1.01 \times 2^{-89}$$

$$\begin{aligned} &= (1.01 \times 1.01) \times 2^{-89-89} \\ &= 1.1001 \times 2^{-178} \end{aligned}$$

actual exponent

$$\text{Bias} = 2^5 - 1 = 15$$

$$\begin{aligned} \text{Biased Exponent} &= 15 - 178 \\ &= -163 \end{aligned}$$

Range of Biased exponent = 0 to $2^5 - 1$
 $= 0$ to 31
 $= (1$ to 30) [0 & 31 reserved]
 $-163 < 1$ (lower range)

Hence, Underflow.