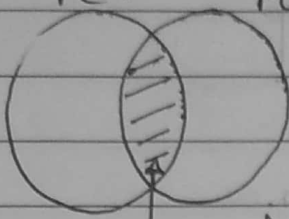
Subject: Probability Theory.

Date:

$P(A)$ $P(B)$



$P(A\cap B) = P(A \text{ and } B) = P(A, B)$

$P(A\cup B) = P(A \text{ or } B)$

$\cancel{P(A\cap B) = P(A) + P(B) - P(A\cup B)}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$P(A\mid B) = \dfrac{P(A\cap B)}{P(B)}$

$\cancel{P(A\cap B) = \dfrac{P(A\cap B)}{P(A)}}$

Joint Distribution Table:

	A	$\neg A$
B	$P(A\cap B)$	$P(\neg A\cap B)$
$\neg B$	$P(A\cap \neg B)$	$P(\neg A\cap \neg B)$

$P(A\cap B) + P(\neg A\cap B) + P(A\cap \neg B) + P(\neg A\cap \neg B) = 1$

$P(A) = P(A\cap B) + P(A\cap \neg B)$

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E.g.

	Alarm	¬Alarm
Burglary	0.09	0.01
¬Burglary	0.1	0.8

(a) Find the probability of alarm.

$P(\text{alarm}) = 0.09 + 0.1$

$= 0.19$

$P(\text{alarm}) = P(\text{alarm} \cap \text{Burglary}) + P(\text{alarm} \cap \neg\text{Burglary})$

(b) $P(\neg\text{alarm}) = 0.01 + 0.8$

$= 0.81$

(c) $P(\text{Burglary}) = P(\text{alarm} \cap \text{Burglary}) + P(\neg\text{alarm} \cap \text{Burglary})$

$= 0.09 + 0.01$

$= 0.1$

(d) $P(\neg\text{Burglary}) = P(\text{alarm} \cap \neg\text{Burglary}) + P(\neg\text{alarm} \cap \neg\text{Burglary})$

$= 0.1 + 0.8$

$= 0.9$

(e) $P(\text{alarm} \mid \text{Burglary}) = \dfrac{P(\text{alarm} \cap \text{Burglary})}{P(\text{Burglary})}$

$= \dfrac{0.09}{0.1}$

$= 0.9$

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(f) $P(\neg a \mid \neg b) = \dfrac{P(\neg \text{alarm} \cap \neg \text{Burglary})}{P(\neg \text{Burglary})}$

$= \dfrac{0.8}{0.9}$

$= 0.89$

Cheated on College Exam?

	Yes	No	Total
Male	0.32	0.22	0.54
Female	0.28	0.18	0.46
Total	0.60	0.40	1.00

(a) $P(\text{Male}) = P(\text{Male} \cap \text{Yes}) + P(\text{Male} \cap \text{No})$

$= 0.32 + 0.22$

$= 0.54$

(b) $P(\text{Female} \cap \text{No}) = 0.18$

(c) $P(\text{Male} \mid \text{Yes}) = \dfrac{P(\text{Male} \cap \text{Yes})}{P(\text{Yes})}$

$= \dfrac{0.32}{0.60}$

$= \cancel{0.92}\ 0.53$

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	t		$\neg t$	
	x c	$\neg x$ $\neg c$	x c	$\neg x$ $\neg c$
c	0.108	0.012	0.072	0.008
$\neg c$	0.016	0.064	0.144	0.576

$c \rightarrow$ cavity
$t \rightarrow$ toothache
$x \rightarrow$ x-ray
$c \rightarrow$ catch

(*) There are 5 variables and each of them have 2 outcomes. If a joint distribution table is formed using these 5 variables, then how many values will be there in the joint probability distribution table? $2^5 = 32$ values $\boxed{2^n \text{ values}}$

(a) $P(t \cap c) = P(t \cap c \cap x) + P(t \cap c \cap \neg x)$
$= 0.108 + 0.012$
$= 0.12$

$P(\sim) \leftarrow$ Marginal Probability

(b) $P(\neg c \cap x) = \cancel{P(\neg c \cap x) A}$
$= P(\neg c \cap x \cap t) + P(\neg c \cap x \cap \neg t)$
$= 0.016 + 0.144$
$= 0.16$

(c) $P(\neg x) = P(t \cap c \cap \neg x) + P(\neg t \cap c \cap \neg x) + P(t \cap \neg c \cap \neg x)$
$+ P(\neg t \cap \neg c \cap \neg x)$
$= 0.012 + 0.008 + 0.064 + 0.576$
$= 0.66$

$P(t \cap \neg x) = 0.076$

(d) $P(t \mid \neg x) = \dfrac{P(t \cap \neg x)}{P(\neg x)} = \dfrac{0.012 + 0.064}{0.66} = 0.12$

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(c) $P(\text{toothach} \mid \text{cavity} \cap \text{catch}) = \dfrac{P(\text{toothache} \cap \text{cavity} \cap \text{catch})}{P(\text{cavity} \cap \text{catch})}$

$$= \frac{0.108}{0.108 + 0.072}$$

$$= 0.6$$

if event A does not affect event B then A and B are independent

$P(A \cap B) = P(A) * P(B)$

H = Head
T = Tail

$H \cap H = 0.25$

~~$H \cap T = 0.25$~~

$T \cap H = 0.25$

$T \cap T = 0.25$

$P(H \cap T) = P(H) * P(T)$

$\Rightarrow 0.25 = 0.5 * 0.5$

$\Rightarrow 0.25 = 0.25$

$\therefore LHS = RHS$

So, Coin Tosses are independent.

$P(A \cap B \mid C) = P(A \mid C) * P(B \mid C)$

If A and B are fully independent, it doesn't necessarily imply that they are also conditionally independent.

$$\Rightarrow \frac{P(A \cap B \cap C)}{P(C)} = P(A \mid C) * P(B \mid C)$$

$$\Rightarrow P(A \cap B \cap C) = P(A \mid C) \times P(B \mid C) \times P(C)$$

$P(A \mid B) = P(A)$ if conditionally independent.

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$P(smart \cap study \cap prep)$	smart		$\neg$smart	
	study	$\neg$study	study	$\neg$study
prepared	0.432	0.16	0.084	0.008
$\neg$prepared	0.048	0.16	0.036	0.072

(a) Is smart independent of study?

$P(smart \cap study) = P(smart) * P(study)$

$\Rightarrow 0.432 + 0.048 = (0.432 + 0.16 + 0.048 + 0.16) * (0.432 + 0.048 + 0.084 + 0.036)$

$\Rightarrow 0.48 = 0.48$

$\therefore$ smart is independent of study.

(b) Is prepared independent of study?

$P(prepared \cap study) = P(prepared) * P(study)$ NO!

$\Rightarrow 0.432 + 0.084 = (0.432 + 0.16 + 0.084 + 0.008) * (0.432 + 0.048 + 0.084 + 0.036)$

$\Rightarrow 0.516 \neq 0.4104$

(c) Is smart conditionally independent of prepared, given study?

$P(smart \cap prepared \mid study) = P(smart \mid study) * P(prepared \mid study)$

$\Rightarrow \frac{P(smart \cap prepared \cap study)}{P(study)} = P(smart \mid study) * P(prepared \mid study)$

$\Rightarrow \frac{0.432}{0.6} = \cancel{(0.432+0.048) * P(0.432+0.084)}$

$= \frac{P(smart \cap study)}{P(study)} * \frac{P(prepared \cap study)}{P(study)}$

$\Rightarrow 0.72 = \frac{0.432+0.048}{0.6} * \frac{0.432+0.048}{0.6}$

$\Rightarrow 0.72 \neq 0.638$

So, smart and prep aren't conditionally independent given study.

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(2) Is study conditionally independent of prepared, given smart?

$P(\text{study} \cap \text{prep} \mid \text{smart}) = P(\text{study} \mid \text{smart}) * P(\text{prep} \mid \text{smart})$

$\Rightarrow \dfrac{P(\text{study} \cap \text{prep} \cap \text{smart})}{P(\text{smart})} = \dfrac{P(\text{study} \cap \text{smart})}{P(\text{smart})} * \dfrac{P(\text{prep} \cap \text{smart})}{P(\text{smart})}$

$\Rightarrow 0.432 = (0.432 + 0.048) * (0.432 + 0.16)$

$\Rightarrow 0.432 \neq 0.28416$

Subject : ~~Naive Bayes.~~

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Bayes Theorem.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$\Rightarrow P(A \cap B) = P(A|B) \cdot P(B)$ $\quad$ $\cancel{\Rightarrow P(A \cap B) =}$

$\cancel{\Rightarrow}$ $\quad$ $\Rightarrow P(B \cap A) = P(B|A) \cdot P(A)$

$\Rightarrow P(A \cap B) = P(B|A) \cdot P(A)$

$\therefore P(A \cap B) = P(A \cap B)$

$\Rightarrow P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$

$$\Rightarrow \boxed{P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}}$$

Bayes Theorem

$P(A) + P(\neg A) = 1$

$\Rightarrow P(\neg A) = 1 - P(A)$

$P(A|B) + P(\neg A|B) = 1$

$\Rightarrow P(\neg A|B) = 1 - P(A|B)$

$P(A) = P(A \cap B) + P(A \cap \neg B)$ $\leftarrow$ Calculating Marginal probability.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$\Rightarrow P(A \cap B) = P(A|B) * P(B)$

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E.g. HIV Global Prevalence = 0.008

Test with 95% specificity and sensitivity.

$P(T \mid HIV) = 95\%$ T = True

$P(\neg T \mid \neg HIV) = 95\%$

(a) After performing a test, the result came positive. What is the likelihood of the person being affected with HIV?

$P(HIV) = 0.008$

$P(\neg HIV) = 1 - 0.008 = 0.992$

$$P(HIV \mid T) = \frac{P(T \mid HIV) \cdot P(HIV)}{P(T)}$$

$$P(\neg HIV \mid T) = \frac{P(T \mid \neg HIV) \cdot P(\neg HIV)}{P(T)}$$

$P(T \mid HIV) = 0.95$

$P(\neg T \mid HIV) = 1 - 0.95 = 0.05$

$\therefore P(HIV \mid T) = P(T \mid HIV) \cdot P(HIV) = 0.95 * 0.008 = 0.0076$

$P(\neg T \mid \neg HIV) = 0.95$

$P(T \mid \neg HIV) = 1 - 0.95 = 0.05$

$P(\neg HIV \mid T) = P(T \mid \neg HIV) \cdot P(\neg HIV) = 0.05 * 0.992 = 0.0496$

$$P(HIV \mid T) = \frac{P(HIV \cap T)}{P(T)}$$

So, the person is less likely to have HIV.

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(b) Perform a test and the result is positive. What is the probability of having HIV ?

$$P(HIV|T) = \frac{P(T|HIV) \cdot P(HIV)}{P(T)}$$

$$= \frac{0.95 * 0.008}{P(T \cap HIV) + P(T \cap \neg HIV)}$$

$$= \frac{0.95 * 0.008}{}$$

$$= \frac{0.0076}{P(T|HIV) * P(HIV) + P(T|\neg HIV) * P(\neg HIV)}$$

$$= \frac{0.0076}{(0.95 * 0.008) + (0.05 \times 0.992)}$$

$$= 0.133$$

$$P(T) = P(T \cap HIV) + P(T \cap \neg HIV)$$

$$= P(T|HIV) * P(HIV) + P(T|\neg HIV) * P(\neg HIV)$$

$$P(A|B) = P(A \cap B) / P(B)$$

$$\Rightarrow P(A \cap B) = P(A|B) * P(B)$$

Subject :   Date :

Q Guilty or not?

A person is put infront of a Jury. The Jury finds the defendant guilty in 98% of the cases in which the defendant has commited a crime, and it ~~dis~~ finds the defendant not guilty in only 97% of the cases in which the defendant has not commited a crime.
Furthermore, only 0.008 of the entire population has committed a crime.

If a random person is found guilty by the jury, what's more likely: criminal or not criminal?

$P(Guilty | criminal) = 0.98$
$P(\neg Guilty | criminal) = 1 - 0.98$
$= 0.02$

$P(\neg Guilty | \neg criminal) = 0.97$
$\Rightarrow P(Guilty | \neg criminal) = 0.03$

$P(criminal) = 0.008$
$P(\neg criminal) = 1 - 0.008$
$= 0.992$

$P(\neg Criminal | Guilty)$
$= P(Guilty | \neg Criminal) * P(\neg Criminal)$
$= \overset{0.03}{\cancel{0.03}} * 0.992$
$= \cancel{0.2976}\ 0.02976$

~~Give~~
The person is more likely to be not a criminal.

~~P(Guilty)~~
$P(Criminal | Guilty) = P(Guilty | Criminal) * P(Criminal)$
$= 0.98 * 0.008$
$= 0.00784$