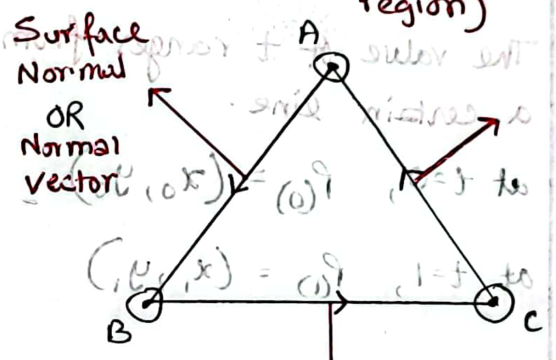
LINE CLIPPING ALGORITHMS:

CYRUS-BECK

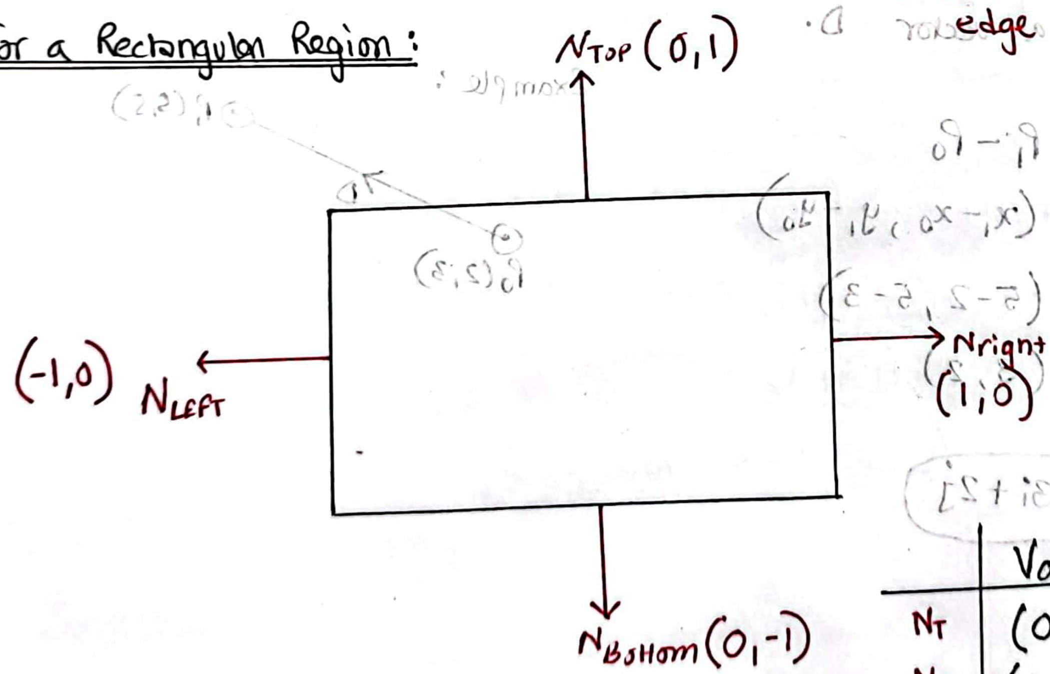
LECTURE 8

- The problem with Cohen-Sutherland was that the clipping region needed to be axis parallel. Which meant that we wouldn't be able to work on anything other than a regular rectangular clipping region.
- But with Cyrus-Beck we could potentially work on any polygon clipping region with n-sides. where ($n \geqslant 3$). (Althoug our main focus will still be on a rectangular clipping region)



(*) Considering the direction of the edge, anything on the left is inside & right is outside.

For a Rectangular Region:



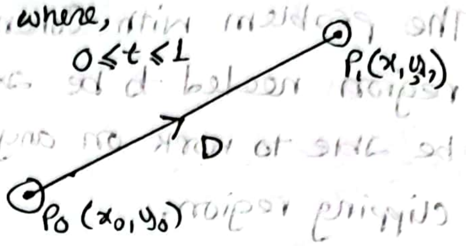
	Values	
N_T	(0,1)	Top
N_B	(0,-1)	Bottom
N_L	(-1,0)	Left
N_R	(1,0)	Right.

(*) N represents a perpendicular unit vector to each of the region.

Note : In Cohen-Sutherland we clipped the line based on the Outcode. Here we will do so based on the value of t.

Parametric equation of a line :

where, $0 \leq t \leq 1$



$$P_{(t)} = P_0 + t(P_1 - P_0)$$

$$\Rightarrow P_{(t)} = (x_0, y_0) + t(x_1 - x_0, y_1 - y_0)$$

$$= (x_0, y_0) + t(x_1 - x_0, y_1 - y_0)$$

$$= \left(x_0 + t(x_1 - x_0)\right), \left(y_0 + t(y_1 - y_0)\right)$$

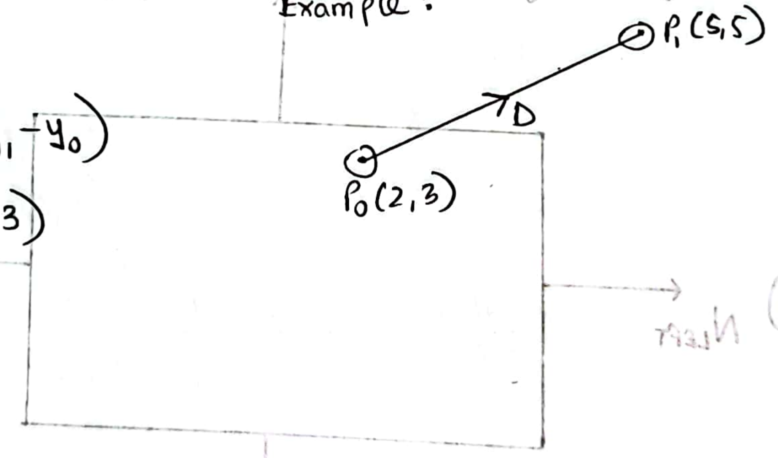
The value of t ranges from 0 to 1 for a certain line.

at $t=0$, $P_{(0)} = (x_0, y_0)$

at $t=1$, $P_{(1)} = (x_1, y_1)$

We can also represent the line P_0 to P_1 using a vector D.

Example :



$$D = P_1 - P_0$$

$$\Rightarrow D = (x_1 - x_0, y_1 - y_0)$$

$$\Rightarrow D = (5-2, 5-3)$$

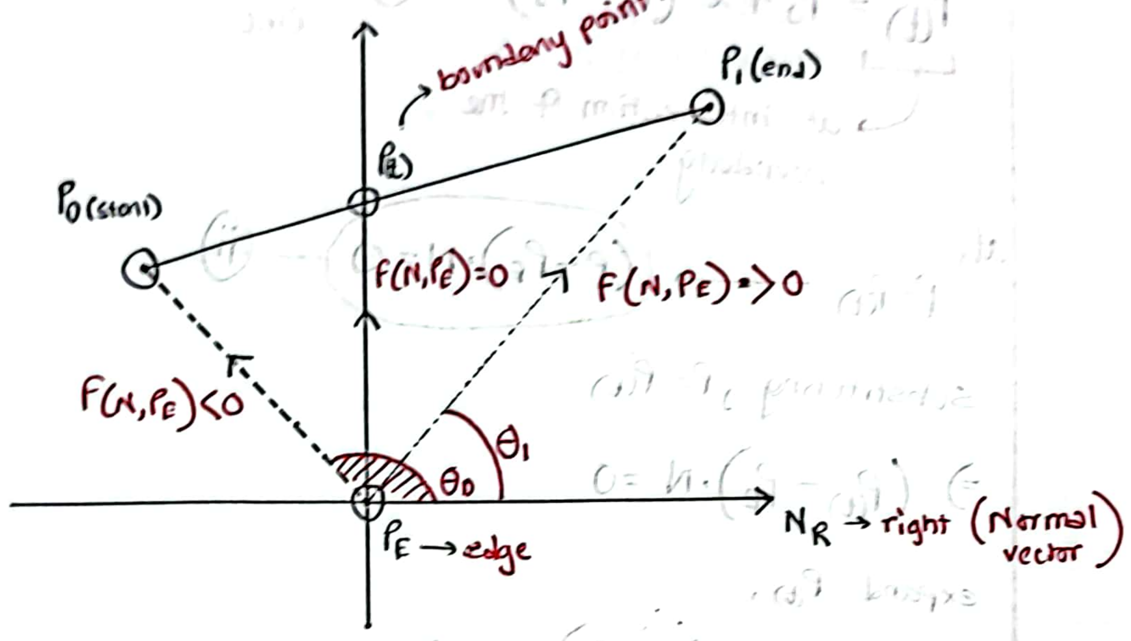
$$\Rightarrow D = (3, 2)$$

OR

$$\Rightarrow D = 3i + 2j$$

$F(N, P_E) = N \cdot (P - P_E)$ — dot product.

θ = Angle between the vectors & N_R



So, $N \cdot (P - P_E)$ has 3 possible values; +ve → leaving the frame

-ve → entering the frame

0 → on the intersection/boundary.

↓ dot product between two vectors can be represented with "cos" of the angle between the vectors.

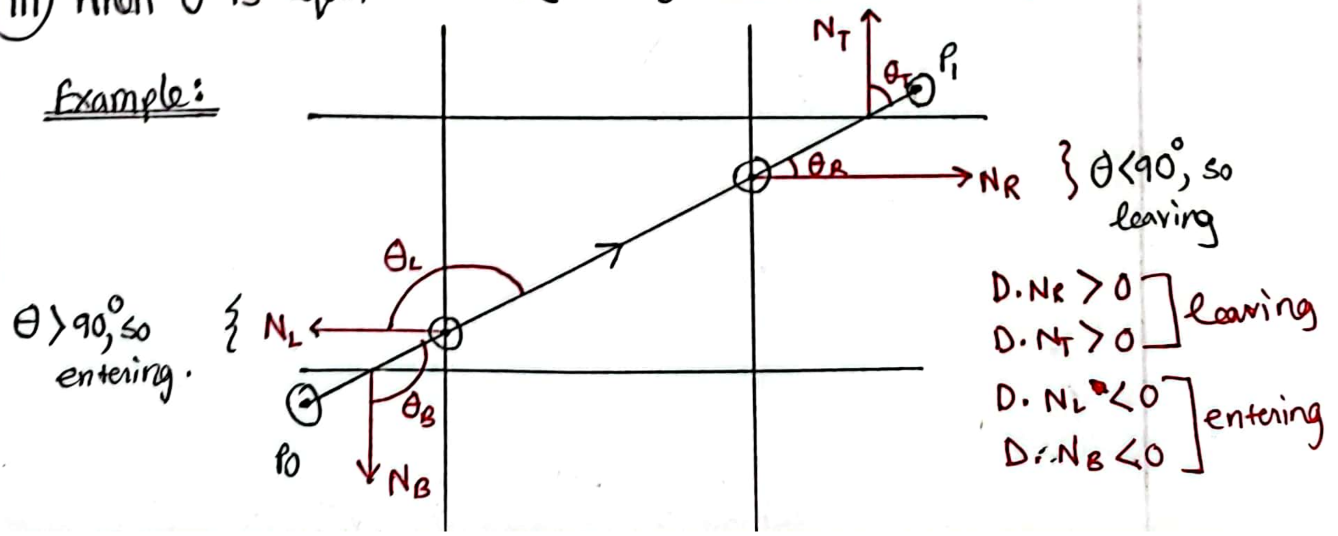
Considerations:

(i) When θ is less than 90° $(\theta < 90^\circ)$, $(P - P_E) \cdot N > 0$, {Leaving the frame}

(ii) When θ is greater than 90° $(\theta > 90^\circ)$, $(P - P_E) \cdot N < 0$, {Entering the frame}

(iii) When θ is equal to 90° $(\theta = 90^\circ)$, $(P - P_E) \cdot N = 0$, {On the boundary}

Example:



We know,

$$P(t) = P_0 + t\,(P_1 - P_0) \quad \cdots \text{(i)}$$ Parametric eqn of a line.

$\underbrace{P(t)}$ → at intersection of the boundary

at, $P = P(t) \longrightarrow (P - P_E)\cdot N = 0 \quad \cdots \text{(ii)}$

Substituting, $P = P(t)$

$\Rightarrow (P(t) - P_E)\cdot N = 0$

expand $P(t)$,

$\Rightarrow \left(P_0 + t(P_1 - P_0) - P_E\right)\cdot N = 0$

$\Rightarrow \underbrace{N\cdot P_0} + N\cdot t\cdot P_1 - N\cdot t\cdot P_0 - \underbrace{N\cdot P_E} = 0$

take this term to the RHS

$\Rightarrow N\cdot t\cdot P_1 - N\cdot t\cdot P_0 = N\cdot P_E - N\cdot P_0$

$\Rightarrow t\,(N\cdot P_1 - N\cdot P_0) = N\cdot(P_E - P_0)$

$$\Rightarrow t = \frac{N\cdot(P_E - P_0)}{N\cdot(P_1 - P_0)}$$

→ This equation will be used to find the of line & the edge.

* List of All possible boundaries:

	Boundary	N	$N\cdot(P_E - P_0)$	$N\cdot(P_1 - P_0)$	t
1	Top where, $y_E = y_{max}$	$(0,1)$	$= (0,1)\cdot((x_E - x_0),(y_E - y_0))$ $= (x_E - x_0)\cdot 0 + (y_E - y_0)\cdot 1$ $= (y_{max} - y_0)$	$= (x_1 - x_0)\cdot 0 + (y_1 - y_0)\cdot 1$ $= (y_1 - y_0)$	$= \dfrac{y_{max} - y_0}{y_1 - y_0}$
2	RIGHT $x_E = x_{max}$	$(1,0)$	$= (x_{max} - x_0)\cdot 1 + (y_E - y_0)\cdot 0$ $= (x_{max} - x_0)$	$= (x_1 - x_0)$	$= \dfrac{x_{max} - x_0}{x_1 - x_0}$
3	BOTTOM $y_E = y_{min}$	$(0,-1)$	$= \underbrace{-1}_{\text{creates no effect}}(y_{min} - y_0)$	$= -1(y_1 - y_0)$	$= \dfrac{y_{min} - y_0}{y_1 - y_0}$
4	LEFT $x_E = x_{min}$	$(-1,0)$	$= \underbrace{-1}_{\text{creates no effect}}(x_{min} - x_0)$	$= -1(x_1 - x_0)$	$= \dfrac{x_{min} - x_0}{x_1 - x_0}$

* $(P_E - P_0)\cdot N = (x_E - x_0)\cdot N_x + (y_E - y_0)\cdot N_y$

* $(P_1 - P_E)\cdot N = (x_1 - x_0)\cdot N_x + (y_1 - y_0)\cdot N_y$

(*) Explanation for the derivation of t

$$(P_E - P_0) \cdot N,$$

for Top, $N = (0,1)$ & $P_E = (x_E, y_{max})$

$$\left[(x_E, y_{max}) - (x_0, y_0)\right] \cdot (0,1)$$

$$\Rightarrow (x_E - x_0) \cdot 0 + (y_{max} - y_0) \cdot 1$$

$$\Rightarrow (y_{max} - y_0)$$

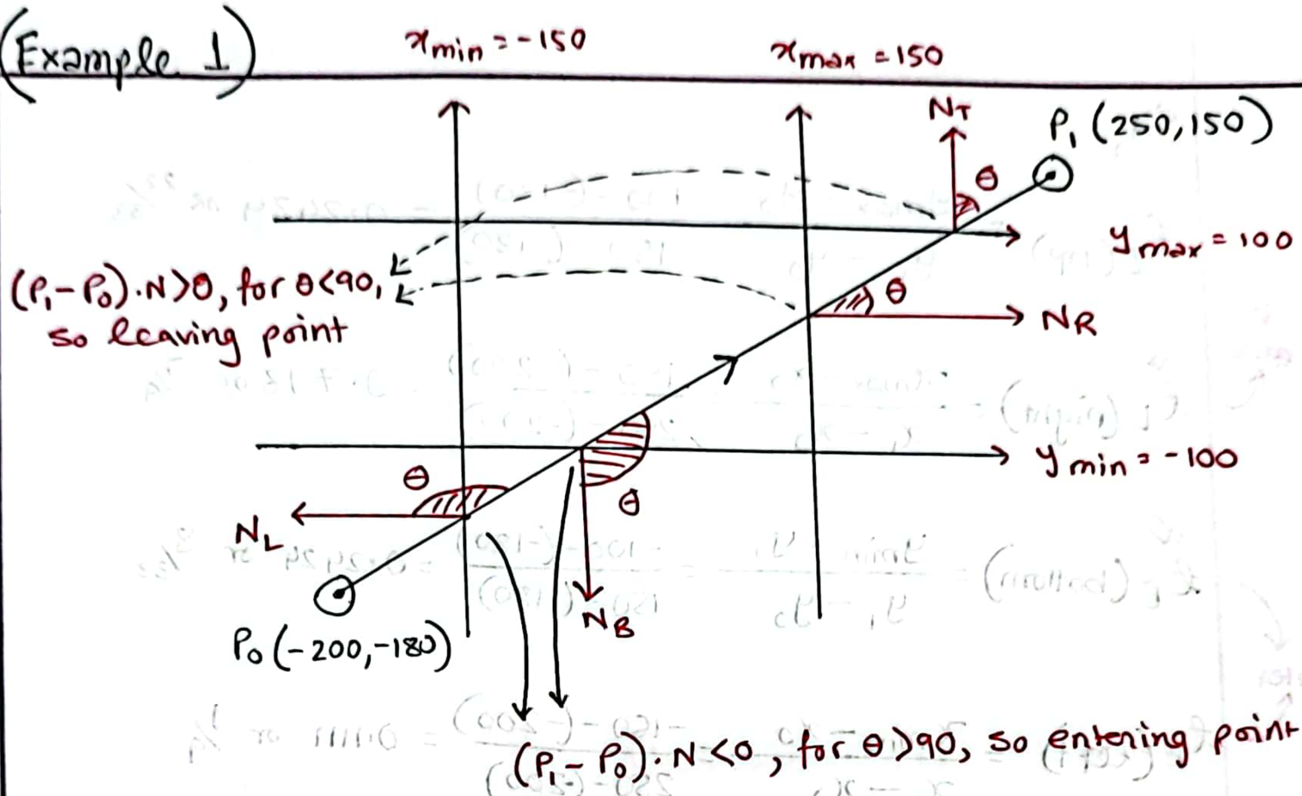
Also, $(P_1 - P_0) \cdot N \Rightarrow (x_1 - x_0) \cdot 0 + (y_1 - y_0) \cdot 1$

$$\Rightarrow (y_1 - y_0)$$

Hence,

$$t = \frac{y_{max} - y_0}{y_1 - y_0}$$

(Example 1)



Here,

(t_E) the entering t's are $\rightarrow$ $\{t_{left}$ & $t_{bottom}\}$

(t_L) the leaving t's are $\rightarrow$ $\{t_{right}$ & $t_{top}\}$

Imp (✱) Cyrus beck says that the line has to be drawn from $t_E(\max)$ to $t_L(\min)$.

So,

$$t_L(\text{top}) = \frac{y_{max} - y_0}{y_1 - y_0} = \frac{100-(-180)}{150-(-180)} = 0.8484 \text{ OR } 28/33$$

t leave

$$t_L(\text{right}) = \frac{x_{max} - x_0}{x_1 - x_0} = \frac{150-(-200)}{250-(-200)} = 0.778 \text{ or } 7/9$$

$$t_E(\text{bottom}) = \frac{y_{min} - y_0}{y_1 - y_0} = \frac{-100-(-180)}{150-(-180)} = 0.2424 \text{ or } 8/33$$

t enter

$$t_E(\text{left}) = \frac{x_{min} - x_0}{x_1 - x_0} = \frac{-150-(-200)}{250-(-200)} = 0.1111 \text{ or } 1/9$$

(*) drawn from $t_E(max)$ to $t_L(min)$

$t = 8/33$, $t_E(max) \longrightarrow (-90.91, -100)$ drawn

to

$t = 7/9$, $t_L(min) \longrightarrow (150, 76.67)$

Example 2 (Type of sum to expect in exam)

Q) Clip Region $(-100,-120)$ to $(150,200)$

Draw the line (clipped) for $P_0(-125,260)$ to $(195,-140)$

ANS: Region $\longrightarrow (-100,-120)$ to $(150,200)$

$x_{min} = -100$ | $y_{min} = -120$

$x_{max} = 150$ | $y_{max} = 200$

$$\text{line}\left[\begin{array}{c|c} x_0 = -125 & x_1 = +195 \\ y_0 = 260 & y_1 = -140 \end{array}\right]$$

for Vector D,

$$D = P_1 - P_0$$
$$= (x_1 - x_0, y_1 - y_0)$$
$$= (320, -400)$$

① Solving for left :

$N_L \cdot D = (-1,0)\cdot(320,-400) = -320 < 0$, Entering

$$\therefore t_{left} = \frac{x_{min} - x_0}{x_1 - x_0} = \frac{-100-(-125)}{195-(-125)} = 0.078 \text{ OR } 5/64$$

② Solving for right :

$N_r \cdot D = (1,0)\cdot(320,-400) = 320 > 0$, Leaving

$$\therefore t_{Right} = \frac{x_{max} - x_0}{x_1 - x_0} = \frac{150+125}{195+125} = 0.8594 \text{ OR } 55/64$$

③ Solving for top :

$$N_T \cdot D = (0,1) \cdot (320,-400) = -400 < 0, \text{ Entering}$$

$$t_{top} = \frac{y_{max} - y_0}{y_1 - y_0} = \frac{200 - 260}{-140 - 260} = 0.15 \text{ OR } 3/20$$

④ Solving for Bottom :

$$N_B \cdot D = (0,-1) \cdot (320,-400) = 400 > 0, \text{ Leaving}$$

$$t_{Bottom} = \frac{y_{min} - y_0}{y_1 - y_0} = \frac{-120 - 260}{-140 - 260} = 0.95 \text{ OR } 19/20$$

⑤ Now,

$t_{E(max)} = t_{top} = 0.15 = 3/20$

$t_{L}(min) = t_{right} = 55/64$

Draw $(t_{top} \longrightarrow t_{right})$, $(t_{E(max)} \longrightarrow t_L(min))$

So, at $t = t_E(max)$

$$x = -125 + \left(3/20 \cdot (195+125)\right) = -77$$

$$y = y_{max} = 200$$

So, $(x_0, y_0) = (-77, 200) \rightarrow$ new

at $t = t_L(min)$

$$x = x_{max} = 150$$

$$y = 260 + \left(55/64 \cdot (-140-260)\right)$$

$$= -83.75$$

ANS

So, $(x_1, y_1) = (150, -83.75)$

Solve at Home:

Q 1) Clip Region $(-60,-50)$ to $(60,50)$

Line $(-100,60)$ to $(110,-70)$

—x—

Q 2) Clip Region $(-120,-100)$ to $(120,100)$

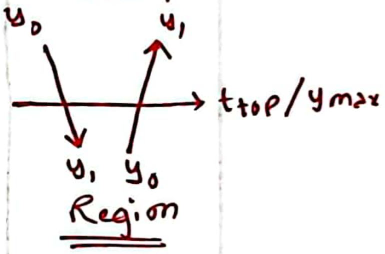
Line $(-135,-200)$ to $(140,200)$

—x—

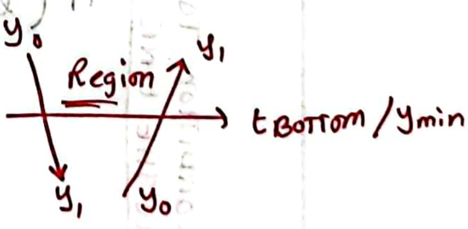
Pseudo Code

```
void Cyrus_Beck (int x0, int y0, int x1, int y1)
    float tEmax = 0.0 , tLmin = 1.0 , t ;
    ymax = ... , ymin = ... , xmax = ... , xmin = ...

For TOP
    t = (ymax - y0)/(y1 - y0)
    if (y1 > y0){ // Leaving
        if (t < tLmin){
            tLmin = t
        }
    }
    else { // Entering
        if (t > tEmax){
            tEmax = t
        }
    }

FOR BOTTOM
    t = (ymin - y0)/(y1 - y0)
    if (y1 > y0){ // entering
        if (t > tEMAX){
            tEMAX = t
        }
    }
    else { // leaving
        if (t < tLMIN){
            tLMIN = t
        }
    }
```

Example:



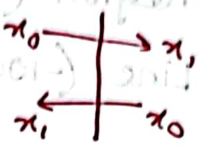
Example:



FOR RIGHT

```
t = (xmax - x0) / (x1 - x0)
if (x1 > x0) {   // leaving
    if (t < tLMIN) {
        tLMIN = t
    }
}
else {   // entering
    if (t > tEMAX) {
        tEMAX = t
    }
}
```

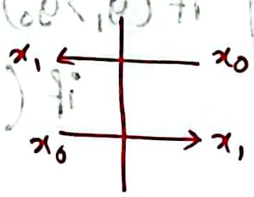
tRIGHT / xmax



FOR LEFT

```
t = (xmin - x0) / (x1 - x0)
if (x1 > x0) {   // entering
    if (t > tEMAX) {
        tEMAX = t
    }
}
else {   // leaving
    if (t < tLMIN) {
        tLMIN = t
    }
}
```

tLEFT / xmin



FINAL CONDITION TO DRAW THE LINE

```
if (tLMIN >= tEMAX) {   // only acceptable condition

    new P0 = Point (tEMAX)
    new P1 = Point (tLMIN)
    drawline (new P0.x, new P0.y, new P1.x, new P1.y)
}
```

(*)

(*)

POI new P_0, new P_1

POI Point(float t)

$POI.x = P_0.x + t\,(P_1.x - P_0.x)$

$POI.y = P_0.y + t\,(P_1.y - P_0.y)$

return POI

}