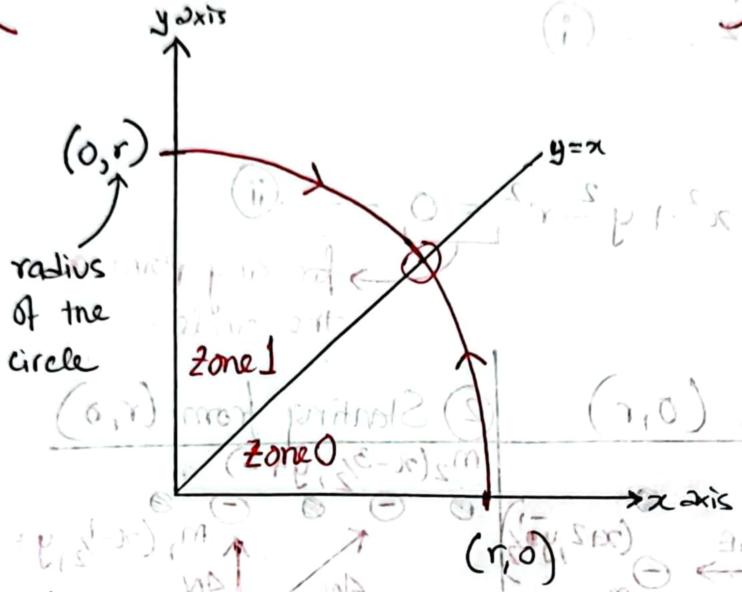
MID POINT CIRCLE ALGORITHM



∅ In midpoint circle algorithm we can follow two approaches

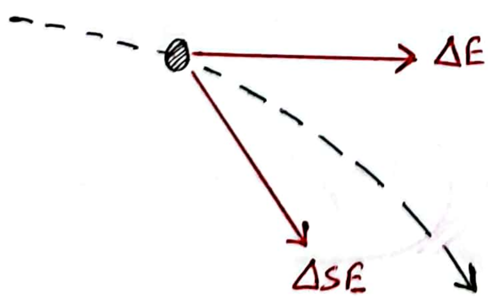
1. Starting from (0,r)
2. Starting from (r,0)

Concept:

1. Starting from (0,r)

We calculate the pixels belonging to zone 1 segment of the circle by starting from (0,r), and then map them onto the other zones using 8-way symmetry

For Zone 1 (0,r)

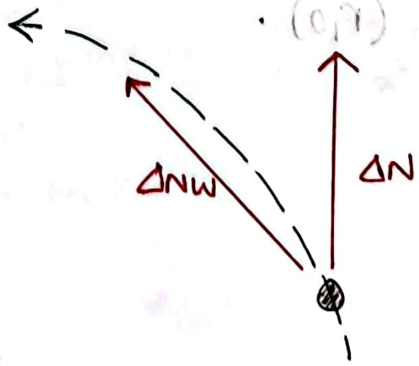


∅ There can be two movements, ΔE & ΔSE

2. Starting from (r,0)

We calculate the pixels belonging to zone 0 segment of the circle by starting from (r,0), and then map them onto the other zones using 8-way symmetry

For Zone 0 (r,0)



∅ ΔN & ΔNW movements

The general equation of a circle:

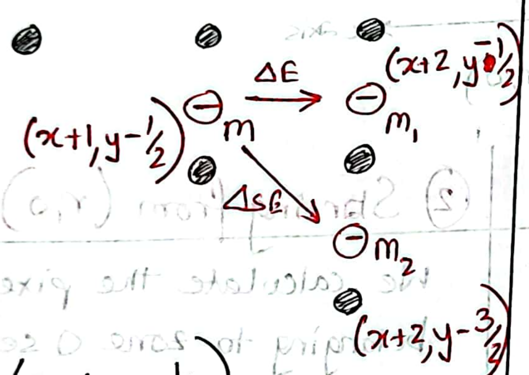
$$x^2+y^2=r^2 \quad \text{------ (i)}$$

Explicit form:

$$f(x,y) = x^2+y^2-r^2 = 0 \quad \text{------ (ii)}$$

→ for any point on the circle.

(1) Starting from $(0,r)$

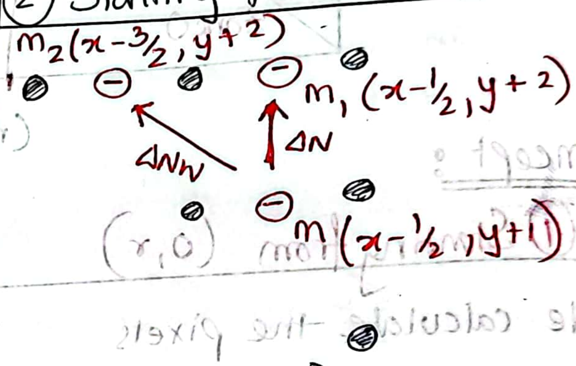


$m = (x+1, y-\frac{1}{2})$

$m_1 = (x+2, y-\frac{1}{2})$

$m_2 = (x+2, y-\frac{3}{2})$

(2) Starting from $(r,0)$



$m = (x-\frac{1}{2}, y+1)$

$m_1 = (x-\frac{1}{2}, y+2)$

$m_2 = (x-\frac{3}{2}, y+2)$

(*) We only need the mapping algorithm for zone 0 to all other zone, if we start at $(r,0)$.

Derivation for $(0, r) \rightarrow$ zone 1

For ΔE movement:

$$f(m_1) \longrightarrow (x+2)^2 + (y-\tfrac{1}{2})^2 - r^2 = d_1$$

$$f(m) \longrightarrow \underset{(-)}{(x+1)^2} + \underset{(-)}{(y-\tfrac{1}{2})^2} \underset{(+)}{-r^2} = \underset{(-)}{d}$$

$$2x+3 \quad + \quad 0 \quad + 0 = d_1 - d = \Delta E$$

$$\therefore \Delta E = 2x+3 \rightarrow \text{(i)}$$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

For ΔSE movement:

$$f(m_2) \longrightarrow (x+2)^2 + (y-\tfrac{3}{2})^2 - r^2 = d_2$$

$$f(m) \longrightarrow \underset{(-)}{(x+1)^2} + \underset{(-)}{(y-\tfrac{1}{2})^2} \underset{(+)}{-r^2} = \underset{(-)}{d}$$

$$(2x+3) + (-2y+2) + 0 = d_2 - d = \Delta SE$$

$$\therefore \Delta SE = 2x - 2y + 5 \rightarrow \text{(ii)}$$

For dinit:

$$f(m) \longrightarrow (x_0+1)^2 + (y_0 - \tfrac{1}{2})^2 - r^2 = \text{dinit}$$

at x_0, y_0

$$\Rightarrow \underbrace{(x_0)^2} + 2x_0 + 1 + \underbrace{(y_0)^2} - y_0 + \tfrac{1}{4} \underbrace{-(r)^2} = \text{dinit}$$

These three terms accumulate to give 0, according to the general eqn of a circle: $x^2+y^2-r^2=0$

$$\Rightarrow \text{dinit} = 2x_0 - y_0 + \tfrac{5}{4}$$

$$\Rightarrow \text{dinit} = 2(0) - r + \tfrac{5}{4}$$

Since, the starting point is $(0, r)$

So we know,

$x_0 = 0,$

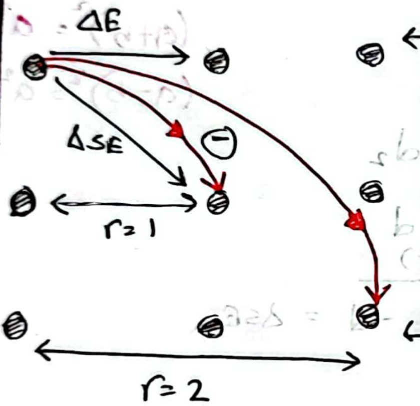
$y_0 = r,$

$$\therefore \text{dinit} = \tfrac{5}{4} - r \rightarrow \text{(iii)}$$

How to decide if ΔE/ΔSE ?

For that we need the value of dinit,

$d = \frac{5}{4} - r$



So,

for r=2, curve goes over the mid point → ΔE

for r=1, curve goes below the mid point. → ΔSE

(*) r=1 & r=2 are arbitrary values used just to understand the algorithm.

ΔE (when, r=2)	ΔSE (when, r=1)
$d = \frac{5}{4} - r$	$d = \frac{5}{4} - r$
$\Rightarrow d = \frac{5}{4} - 2 = -0.75$ (-ve)	$\Rightarrow d = \frac{5}{4} - 1 = 0.25$ (+ve)
if d is -ve, negative the movement is ΔE	if d is +ve, positive the movement is ΔSE

Pseudo Code:

```
Void drawCircle_zone1 (int r) {
        int d = 1-r          ──> since 1 & 1.25 are significantly close, we'll use 1.
    OR
        int d = 5-(4*r)      ──> multiplied with 4 to remove fraction, if we use this, we need to multiply ΔE & ΔSE with 4 as well.
                                 ΔE = 4x(2x+3)
                                 ΔSE = 4x(2x-2y+5)
        int x = 0
        int y = r
draw the pixel on all 8 zones <── draw8way (x,y)
        while (x≤y){
            if (d<0){ // ΔE since -ve, increase x by 1
                d = d+2x+3
                x = x+1
            }
            else { // ΔSE, increase x by 1, decrease y by 1
                d = d+2x-2y+5
                x = x+1
                y = y-1
            }
            draw8way (x,y);
        }
}
```

Q1) Draw a circle a Circle, with radius 10 and origin (0,0), slanting with (0,10)

$d = 1 - r$, $\longrightarrow (1 - 10 = -9)$

$\Delta E = 2x + 3$

$\Delta SE = 2x - 2y + 5$

x	y	d	$\Delta E / \Delta SE$	d update	PIXEL
0	10	-9	ΔE	$= -9 + (2\times 0 + 3) = -6$	(0,10)
1	10	-6	ΔE	$= -6 + (2\times 1 + 3) = -1$	(1,10)
2	10	-1	ΔE	$= -1 + (2\times 2 + 3) = 6$	(2,10)
3	10	6	ΔSE	$= 6 + (2\times 3 - 2\times 10 + 5) = -3$	(3,10)
4	9	-3	ΔE	$= -3 + (2\times 4 + 3) = 8$	(4,9)
5	9	8	ΔSE	$= 8 + (2\times 5 - 2\times 9 + 5) = 5$	(5,9)
6	8	5	ΔSE	$= 5 + (2\times 6 - 2\times 8 + 5) = 6$	(6,8)
7	7	6	~	~	(7,7)

answer.

Q2) Calculate the points for zone 1 of a circle with radius = 6 and origin centered at (10,10)?

↳ we'll carry out the normal calculations but since the origin is at (10,10) we need to add +10 to the x coordinate & +10 to the y coordinate of each pixel to translate the circle.

$d = 1 - r$

$\Delta E = 2x + 3$

$\Delta SE = 2x - 2y + 5$

x	y	d	ΔE/ΔSE	d update	Actual Pixel (x+10, y+10)
0	6	-5	ΔE	$= -5 + (2\times0+3) = -2$	$(0+10, 6+10) = (10,16)$
1	6	-2	ΔE	$= -2 + (2\times1+3) = 3$	(11,16)
2	6	3	ΔSE	$= 3 + (2\times2 - 2\times6 + 5) = 0$	(12,16)
3	5	0	ΔSE	$= 0 + (2\times3 - 2\times5 + 5) = 1$	(13,15)
4	4	1	~	~	(14,14)

X

Derivation for $(r,0) \rightarrow$ zone 0

ΔN

$f(m_1) \rightarrow (x-\frac{1}{2})^2 + (y+2)^2 - r^2 = d_1$

$f(m) \rightarrow (x-\frac{1}{2})^2 + (y+1)^2 - r^2 = d$

$0 + (2y+3) + 0 = d_1 - d = \Delta N$

$\therefore \Delta N = 2y+3$

ΔNW

$f(m_2) \rightarrow (x-\frac{3}{2})^2 + (y+2)^2 - r^2 = d_2$

$f(m) \rightarrow (x-\frac{1}{2})^2 + (y+1)^2 - r^2 = d$

$(-2x+2) + (2y+3) = d_2 - d = \Delta NW$

$\therefore \Delta NW = 2y - 2x + 5$

dinit

$f(m) \longrightarrow (x_0 - \frac{1}{2})^2 + (y_0+1)^2 - r^2 = dinit$

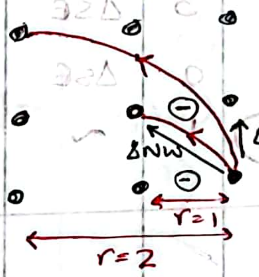
at (x_0, y_0) $\quad dinit = \frac{5}{4} - r$

$x_0 = r,$

$y_0 = 0,$

So, $d > 0$, ΔNW

$d < 0$, ΔN



Pseudo

```
void drawCircle_zone0 (int r) {
    int d = 1 - r
    x = r
    y = 0
    draw 8way (x, y)
    while (x >= y) {
        if (d > 0) { // ΔNW
            d += 2y - 2x + 5
            x--
            y++
        }
        else { // ΔN
            d += 2y+3
            y++
        }
        draw 8way (x, y)
    }
}
```

Q3) For a circle with radius = 20, and origin (0,0). Draw the circle using MPC algo for zone 0, starting at (20,0). [For 12 pixels]

$d = -19$

$dN = 2y+3 \longrightarrow$ for $d<0$

$dNW = 2y - 2x + 5 \longrightarrow$ for $d>0$

SL	x	y	d	$\Delta N / \Delta NW$	d update	PIXEL
1.	20	0	-19	ΔN	$= -19 + (2\times 0 + 3) = -16$	(20,0)
2.	20	1	-16	ΔN	$= -16 + (2\times 1 + 3) = -11$	(20,1)
3.	20	2	-11	ΔN	$= -11 + (2\times 2 + 3) = -4$	(20,2)
4.	20	3	-4	ΔN	$= -4 + (2\times 3 + 3) = 5$	(20,3)
5.	20	4	5	ΔNW	$= 5 + (2\times 4 - 2\times 20 + 5) = -22$	(20,4)
6.	19	5	-22	ΔN	$= -22 + (2\times 5 + 3) = -9$	(19,5)
7.	19	6	-9	ΔN	$= -9 + (2\times 6 + 3) = 6$	(19,6)
8.	19	7	6	ΔNW	$= 6 + (2\times 7 - 2\times 19 + 5) = -13$	(19,7)
9.	18	8	-13	ΔN	$= -13 + (2\times 8 + 3) = 6$	(18,8)
10.	18	9	6	ΔNW	$= 6 + (2\times 9 - 2\times 18 + 5) = -7$	(18,9)
11.	17	10	-7	ΔN	$= -7 + (2\times 10 + 3) = 16$	(17,10)
12.	17	11	16	—	—	(17,11)