CSE423 2/9/23

1) In 2D, when dealing with matrices, we use ~~3x3~~ 2x2 matrices normally. However, in homogenous co-ordinate system, each point has an extra value w, which is usually equal to 1. Homogenous co-ordinates allow for all three kinds of transformations to be expressed using multiplication. Multiplying a homogenous co-ordinate with a transformation matrix results in a successful transformation, while keeping w unchanged

Given

$$x' = ax + by + c$$
$$y' = dx + ey + f$$

Cartesian Co-ordinates

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ d & e \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ f \end{bmatrix}$$

$$p' = Mp + t$$

Homogenous co-ordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$p' = Mp$$

- Projective
  - Affine
    - Similitudes
      - Rigid/Euclidean
        - Translation
      - Rigid/Euclidean & Linear
        - Identity
        - Rotation
    - Linear
      - Scaling
      - Reflection
      - Shear
    - Similitudes & Linear
      - Isotropic Scaling

3. 3D point rotation across X-axis and the center of rotation ~~of~~ (a, b, c)

$$V = \begin{bmatrix} 1 \\ r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$V' = \begin{bmatrix} 1 \\ r\cos(\phi+\theta) \\ r\sin(\phi+\theta) \end{bmatrix}$$

$$x' = x$$

$$y' = r\cos\phi\cos\theta - r\sin\phi\sin\theta = y\cos\theta - z\sin\theta$$

$$z' = r\overset{\cos\phi}{\cancel{\cos\theta}}\sin\theta + r\sin\phi\cos\theta = y\sin\theta + z\cos\theta$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. 3D point rotation across Y-axis and the center of rotation (a, b, c)

$$V = \begin{bmatrix} r\sin\phi \\ \cancel{r\cos\phi}\ 1 \\ r\cos\phi \end{bmatrix}$$

$$V' = \begin{bmatrix} r\sin(\phi+\theta) \\ 1 \\ r\cos(\phi+\theta) \end{bmatrix}$$

$$x' = r\cos\phi\sin\theta + r\sin\phi\cos\theta = z\sin\theta + x\cos\theta$$

$$y' = y$$

$$z' = r\cos\phi\cos\theta - r\sin\phi\sin\theta = z\cos\theta - x\sin\theta$$

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. 3D point rotation across Z-axis and the centre of rotation (a, b, c)

$$V = \begin{bmatrix} r\cos\phi \\ r\sin\phi \\ 1 \end{bmatrix}$$

$$V' = \begin{bmatrix} r\cos(\phi+\theta) \\ r\sin(\phi+\theta) \\ 1 \end{bmatrix}$$

$$x' = r\cos\phi\cos\theta - r\sin\phi\sin\theta = x\cos\theta - y\sin\theta$$

$$\cancel{y' = r\sin\phi}$$

$$y' = r\cos\phi\sin\theta + r\sin\phi\cos\theta = x\sin\theta + y\cos\theta$$

$$z' = z$$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⑥ Given Line L, ~~with~~ where $L = y = mx + c$
where $\tan\theta = m$, and point $P(x, y)$

- Transform to origin: $T_{(0,-c)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}$
- Rotate clockwise about $-\theta$, $R_{-\theta} = \begin{bmatrix} \cos-\theta & -\sin-\theta & 0 \\ \sin-\theta & \cos-\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- Reflect about x-axis, $Refl_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- Rotate anti-clockwise, about θ, $R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- Return to original c, $T_{(0,+c)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$

~~Given $\theta = 45$ and $-\theta = -45$, $c = 2$~~

$$M_L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos-\theta & -\sin-\theta & 0 \\ \sin-\theta & \cos-\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_L = \begin{pmatrix} \cos\theta\cos(-\theta) + \sin\theta\sin(-\theta) & \sin\theta\cos(-\theta) - \cos\theta(\sin -\theta) & -c\left(\sin\theta\cos-\theta - \cos\theta\sin-\theta\right) \\ \sin\theta\cos-\theta - \cos\theta\sin(-\theta) & -\sin\theta\sin-\theta - \cos\theta\cos-\theta & c-c\left(-\sin\theta\sin-\theta - \cos\theta\cos-(\theta)\right) \\ 0 & 0 & 1 \end{pmatrix}$$

(7) P (100, -60, 80) c = (50, 20, 45) Y-axis 60°

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 60^\circ & 0 & \sin 60 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 60 & 0 & \cos 60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -50 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x=100 \\ y=-60 \\ z=80 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{3}/2 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ -80 \\ 35 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 55.3 \\ -80 \\ -25.8 \\ 1 \end{bmatrix} = \begin{bmatrix} 105.3 \\ -60 \\ 19.19 \\ 1 \end{bmatrix}$$

⑧ $P = \begin{bmatrix} 100 \\ -60 \\ 80 \end{bmatrix}$ $C = \begin{bmatrix} 50 \\ 20 \\ 45 \end{bmatrix}$ x-axis 60°

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 60 & -\sin 60 & 0 \\ 0 & \sin 60 & \cos 60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -50 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cancel{\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}} \begin{bmatrix} 100 \\ -60 \\ 80 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 60 & -\sin 60 & 0 \\ 0 & \sin 60 & \cos 60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ -80 \\ 35 \\ 1 \end{bmatrix}$$

$$= \begin{pmatrix} 100 \\ \cancel{50} \\ -50.3 \\ -6.7 \\ 1 \end{pmatrix}$$

//

⑨ $P = \begin{bmatrix} 100 \\ -60 \\ 80 \end{bmatrix}$ $C = \begin{bmatrix} 50 \\ 20 \\ 45 \end{bmatrix}$ Z-axis 60°

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 60 & -\sin 60 & 0 & 0 \\ \sin 60 & \cos 60 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -50 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ -60 \\ 45 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 144.3 \\ 23.3 \\ 80 \\ 1 \end{bmatrix}$$

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(a) 3D rotation, 90° clk, y axis, about (a,b,c), translation $(a\,b\,c)$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos -90 & 0 & \sin -90 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin -90 & 0 & \cos -90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -a \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(b) $ax + by + c = 0$ $\quad y = -\frac{a}{b}x - \frac{c}{b}$ $\quad \theta = \tan^{-1}\left(-\frac{a}{b}\right)$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -a \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{c}{b} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\tan^{-1}(a/b) & -\sin(\tan^{-1}(a/b)) & 0 \\ \sin(\tan^{-1}(a/b)) & \cos(\tan^{-1}(a/b)) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\tan^{-1}(a/b)) & -\sin(\tan^{-1}(a/b)) & 0 & 0 \\ \sin(\tan^{-1}(a/b)) & \cos(\tan^{-1}(a/b)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{c}{b} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(c) $$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & e \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -d \\ 0 & 1 & 0 & -e \\ 0 & 0 & 1 & -f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & e \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos 45 & -\sin 45 & 0 & 0 \\ \sin 45 & \cos 45 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -d \\ 0 & 1 & 0 & -e \\ 0 & 0 & 1 & -f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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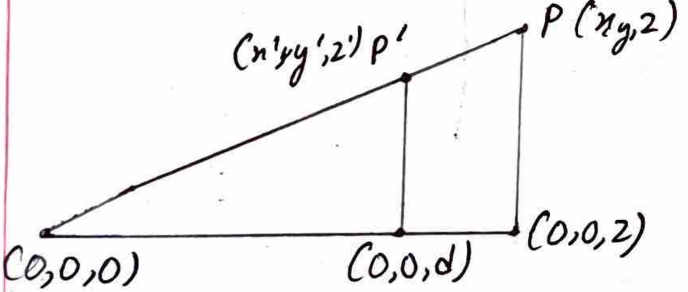
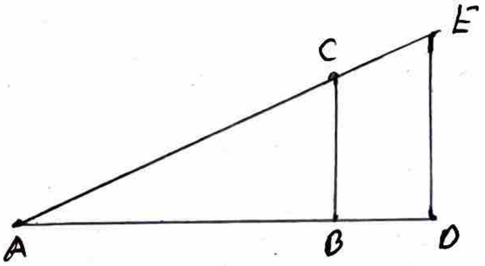
11. Classification Tree:

- Projections
  - Parallel
    - Orthographic
      - Multiview
      - Axonometric
        - Isometric
        - Dimetric
        - Trimetric
    - Oblique
      - General
        - Cavalier
        - Cabinet.
  - Perspective:
    - One point
    - Two Point
    - Three Point

12. Derivation for Origin at COP:





y':

$$\frac{BC}{AB} = \frac{DE}{AD}$$

$$\Rightarrow \frac{y'}{d} = \frac{y}{z} \Rightarrow y' = \frac{y}{z/d}$$

x':

$$\Rightarrow \frac{x'}{d} = \frac{x}{z} \Rightarrow x' = \frac{x}{z/d}$$

z':

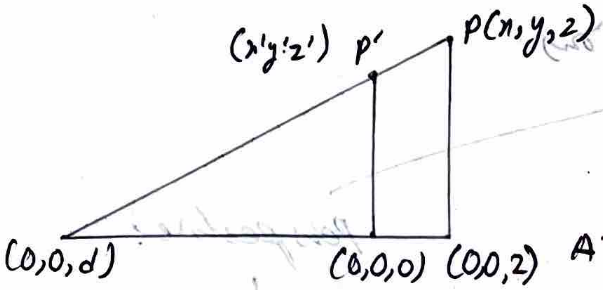
$$\Rightarrow \frac{z'}{d} = \frac{z}{z/d} \Rightarrow z' = \frac{z}{z/d}$$

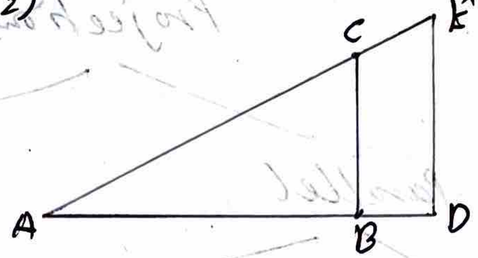
$$\therefore \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x/zd \\ y/zd \\ z/zd \\ 1 \end{bmatrix}$$

Derivation for origin at P.P:





y':

$$\frac{BC}{AB} = \frac{DE}{AD}$$

$$\Rightarrow \frac{y'}{d} = \frac{y}{d-z} \Rightarrow y' = \frac{y}{\frac{d-z}{d}} = \frac{y}{1-\frac{z}{d}}$$

x':

$$\Rightarrow x' = \frac{x}{1-\frac{z}{d}}$$

z':

$z' = 0$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{d} & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \\ 1-\frac{z}{d} \end{bmatrix} = \begin{bmatrix} \frac{x}{1-\frac{z}{d}} \\ \frac{y}{1-\frac{z}{d}} \\ 0 \\ 1 \end{bmatrix}$$

14. $\begin{bmatrix} 423 \\ -423 \\ 423 \\ 1 \end{bmatrix}$ $\quad COP = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$

$d = 426$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{-426} & 0 \end{bmatrix} \begin{bmatrix} 50 \\ 60 \\ -300 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 423 \\ -423 \\ 423 \\ \cancel{-0.992}\ \frac{141}{142} \end{bmatrix} = \begin{bmatrix} -420.02 \\ 420.02 \\ -420.02 \\ 1 \end{bmatrix}$$

15. $$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0.1 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 40 \\ 30 \\ 20 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 39 \\ 32 \\ 16 \\ 20 \end{bmatrix} = \begin{bmatrix} 1.95 \\ 1.6 \\ 0.8 \\ 1 \end{bmatrix}$$

16. A global lighting model refers to light independent of a light source and local lighting models are contributed by specific light sources.

17. The gourad model performs phong lighting at the vertices and linearly interpolates the resulting color over faces. Phong's model interpolates normal vectors of face vertices at each pixel and then performs phong lighting on each pixel.

18. I = Ambient Component + Diffuse Component + Specular Component.

$$\cancel{= I_a k_a + I_p k_d (L \cdot N) + I_p k_s (R \cdot V)^k}$$

$$= I_a k_a + I_p f_{att} \left[k_d \cdot \max\{(L \cdot N), 0\} + k_s \cdot \max\{(R \cdot V)^k, 0\} \right]$$

19. Diffuse reflection model is when light is reflected equally in all directions ~~form~~ from a point light source.
Example: Reflections off rough surface such as paper.

20. Specular Reflection model is where light is reflected in a single ray from a point light source off a surface.
Example: Reflection of objects on metal surfaces.

21. It is the phenomena of light becoming weaker the further it gets from the source.

22. $L = (50, 70, 1500)$

$I_p = 0.95$

$n = \hat{k}$

$L = 50 - 0, 70 - 0, 1500 - 5.5$

$= \dfrac{80\hat{i} + 70\hat{j} + 1494.5\hat{k}}{1496.97}$

$L \cdot n = \dfrac{1494.5}{1496.97} = 0.9983$

$D = 0.95 \times 0.8 \times 0.9983$

$= 0.7587$

23. $L = (50, 70, 1500)$ $I_p = 0.95$

$n = \hat{i} + \hat{j} + \hat{k}$

$L = 50 - 20, 70 - 10, 1500 - 120$

$= \dfrac{30i + 60j + 1380k}{1381.63}$

$L \cdot n = \dfrac{1470}{1381.63} = 1.064$

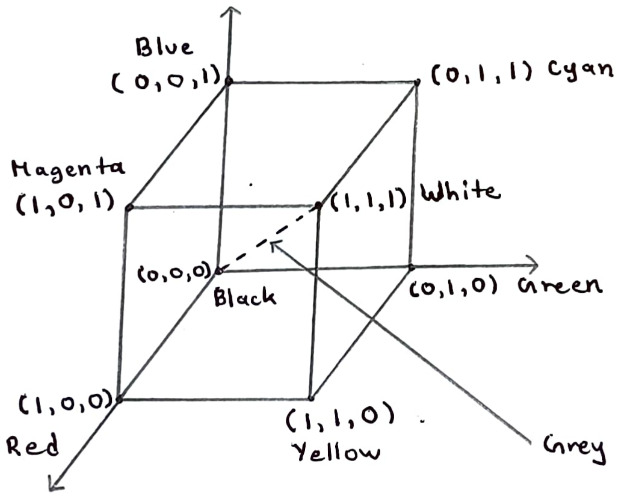
$D = 0.95 \times 0.8 \times \cancel{0.}\ 1.064$

$= 0.80864$

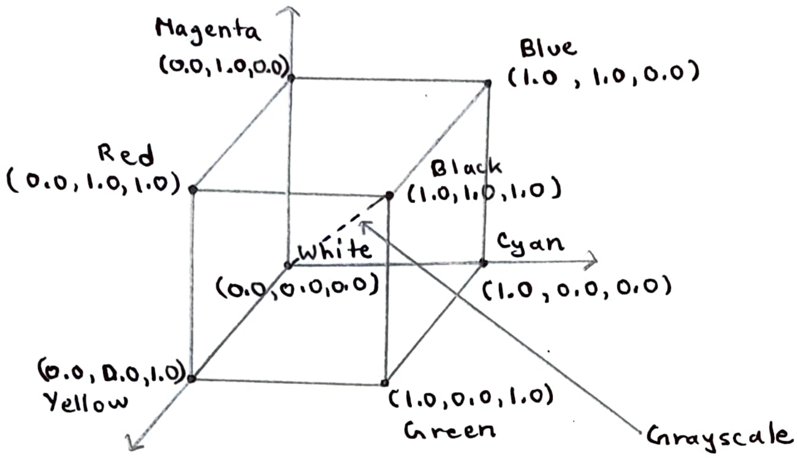
24. If we are working on a computer, the colours we see on screen are created with light using the additive colour method. Additive colour mixing ~~beings~~ begins with black and ends with white. As more colour is added, the result is lighter and tends to white.

On the other hand, when we mix colours using paint, or through the printing process, we are using the subtractive colour method. Subtractive colour mixing ~~being~~ begins with white and ends with black. As more colour is added, the result is ~~lighter~~ darker and tends to black.

25. RGB colour cube:



CMY colour cube:



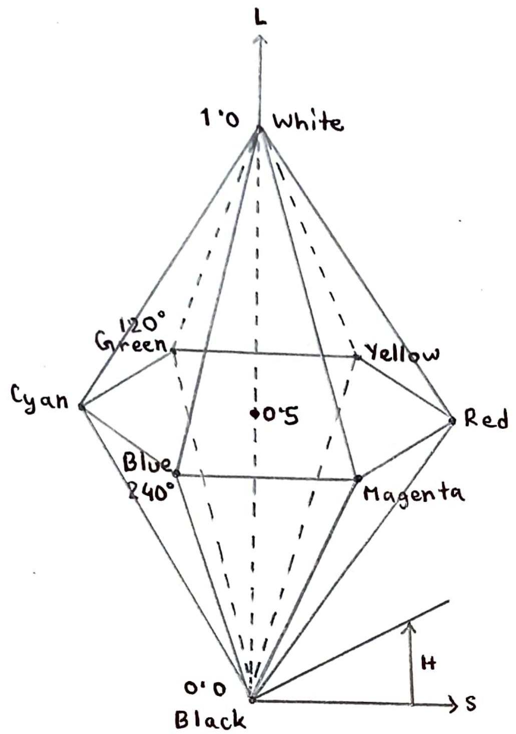
26. ('RGB colour model' and 'CMY colour model' same as question no. 25)

CMY is a subtractive colour model. Its primary pigments are Cyan, Magenta and Yellow.

Equal amounts of the primary pigments should produce Black. But in practice, a "Muddy-Black" colour is produced.

Hence, in order to produce "True-Black", a fourth colour "Black" is added giving rise to CMYK model.

27. HLS colour model :



28. Algorithm for converting RGB colour values into HLS values:

```
max = max (R, G, B)
min = min (R, G, B)

l = (max + min) / 2

if max = min:
    s = 0
    h = undefined
else:
    if l ≤ 0'5 :
        s = (max - min) / (max + min)
    else:
        s = (max - min) / (2 - max - min)
```

```
Δ = max - min
if R==max:
    h = (G-B) / Δ
else if G==max:
    h = 2+[(B-R) /Δ]
else if B==max:
    h = 4+[(R-G) /Δ]
h=h*60
if h<0 :
    h=h+360
```

29. Algorithm for converting RGB colour values into HSV colour values:

```
max= max (R,G,B)
min = min (R,G,B)
Δ = max - min

V= (max * 100)

if max!=0 :
    s = (Δ / max) * 100
else:
    s = 0
    h = -1
    v = undefined
    return

if R==max:
    h=(G-B) / Δ
else if G==max:
    h = 2+[(B-R) /Δ]
else if B==max:
    h = 4+[(R-G) /Δ]
h = h*60
if h<0:
    h = h+360
```

30. RGB → HLS

i) (0'25, 0'3, 1'0)

R = 0'25 , G = 0'3 , B = 1'0

max = 1'0

min = 0'25

l = (1 + 0'25) / 2 = 0'625

max != min :

l > 0'5 :

s = (1 - 0'25) / (2 - 1 - 0'25)

= 1

Δ = 1 - 0'25 = 0'75

B == max :

h = 4 + [(0'25 - 0'3) / 0'75]

= ~~5'275~~ ~~5'267~~ 3'933

h = h × 60

= ~~5'267 × 60~~ 3'933 × 60

= ~~316°~~ ~~2360°~~ 236°

RGB → HLS

(0'25, 0'3, ~~0'4~~ 1'0) ⟶ h = 236° , l = 0'625 , s = 1

l ≃ 63% , s = 100%

ii) (0'01, ~~1'07~~ 1'0, 0'09) ⟶ h = 125° , l = 0'505 , s = 1

l ≃ 51% , s = 100%

R = 0'01 , G = 1'0 , B = 0'09

~~min = 0~~

max = 1'0

min = 0'01

l = 0'505

s = (1 - 0'01) / (2 - 1 - 0'01) = 1

Δ = 0'99 , G == max

h = 2 + [(0'09 - 0'01) / 0'99] = ~~2'101~~ 2'081

h = ~~2'101 × 60 = 126°~~ 2'081 × 60 = ~~124°~~ 125°

iii) $(0'8, 0'8, 0'35) \longrightarrow h = 60°$, ~~l=57~~ $l = 0'575$, $s = 0'529$

$l = 58\%$ $s = 53\%$

$R = 0'8$, $G = 0'8$, $B = 0'35$

$max = 0'8$ (R)

$min = 0'35$

$l = (0'8 + 0'35)/2 = 0'575$

~~s = (1 - 0'~~

$s = (0'8 - 0'35)/(2 - 0'8 - 0'35)$

$= 0'529$

$\Delta = 0'45$, $R == max$

$h = (0'8 - 0'35)/\ 0'45$

$= 1$

$h = 1 \times 60 = 60°$

iv) $(0'0, 0'4, 0'4) \longrightarrow h = 180°$, $l = 0'2$, $s = 1$

$l = 20\%$ $s = 100\%$

$R = 0'0$, $G = 0'4$, $B = 0'4$

$max = 0'4$ (G)

$min = 0'0$

$l = (0'4 + 0'0)/2 = 0'2$

$s = (0'4 - 0'0)/(0'4 + 0'0)$

$= 1$

$\Delta = 0'4$, $G == max$

$h = 2 + [(0'4 - 0'0)\ /0'4]$

$= \cancel{6}\ 3$

~~h = 6 × 60 = 360°~~

$h = 3 \times 60 = 180°$

v) $(1.0, 1.0, 0.5) \longrightarrow h = 60°, \; L = 0.75, \; S = 1$

$L = 75\%$ $\quad S = 100\%$

$R = 1.0, \quad G = 1.0, \quad B = 0.5$

$max = 1.0 \, (R)$

$min = 0.5$

$L = (1 + 0.5)/2 = 0.75$

$S = (1 - 0.5)/(2 - 1 - 0.5)$

$= 1$

$\cancel{h =} \; \Delta = 0.5, \quad R = max$

$h = (1.0 - 0.5)/0.5$

$= 1$

$h = 1 \times 60 = 60°$

vi) $(0.7, 0.71, 0.7) \longrightarrow h = 120°, \; L = 0.705, \; S = 0.017$

$L \simeq 71\%$ $\quad S = \cancel{1\%} \; 2\%$

$R = 0.7, \quad G = 0.71, \quad B = 0.7$

$max = 0.71$

$min = 0.7$

$L = (0.71 + 0.7)/2 = 0.705$

$S = (0.71 - 0.7)/(2 - 0.71 - 0.7)$

$= 0.017$

$\Delta = 0.01, \quad G = max$

~~h = [2 + (0.7 - 0.7)] / 0.01~~

~~= 200~~

~~h = 200 × 60 =~~

$h = 2 + \left(\frac{0.7 - 0.7}{0.01} \right)$

$= 2$

$h = 2 \times 60 = 120°$

vii) $(0'5, 0'5, 0'5) \longrightarrow h = 0°$, $l = 0'5$, $s = 0\%$

$l = 50\%$

$l = (0'5 + 0'5) / 2 = 0'5$

viii) $(1'0, 1'0, 1'0) \longrightarrow h = 0°$, $l = 1$, $s = 0\%$

$l = 100\%$

$l = (1+1) / 2 = 1$

31. a) If ID = 15101208 ,

A = 15 , B = 10 , C = 12 , D = 08

CMY $(0'A, 0'C, 0'D) \longrightarrow (0'15, 0'12, 0'80)$

RGB $(0'85, 0'88, 0'2)$

$max = 0'88$

$min = 0'2$

$\Delta = 0'68$

$v = 0'88 \times 100 = 88\%$

$$s = \frac{0'68}{0'88} \times 100 = 77\%$$

G = = max :

$h = 2 + [(B-R) / \Delta]$

$= 2 + [(0'2 - 0'85) / 0'68]$

$= 1'044$

$h = 1'044 \times 60 = 63°$

$h = 63°$, $s = 77\%$, $v = 88\%$ (Ans.)