(PROJECTION)
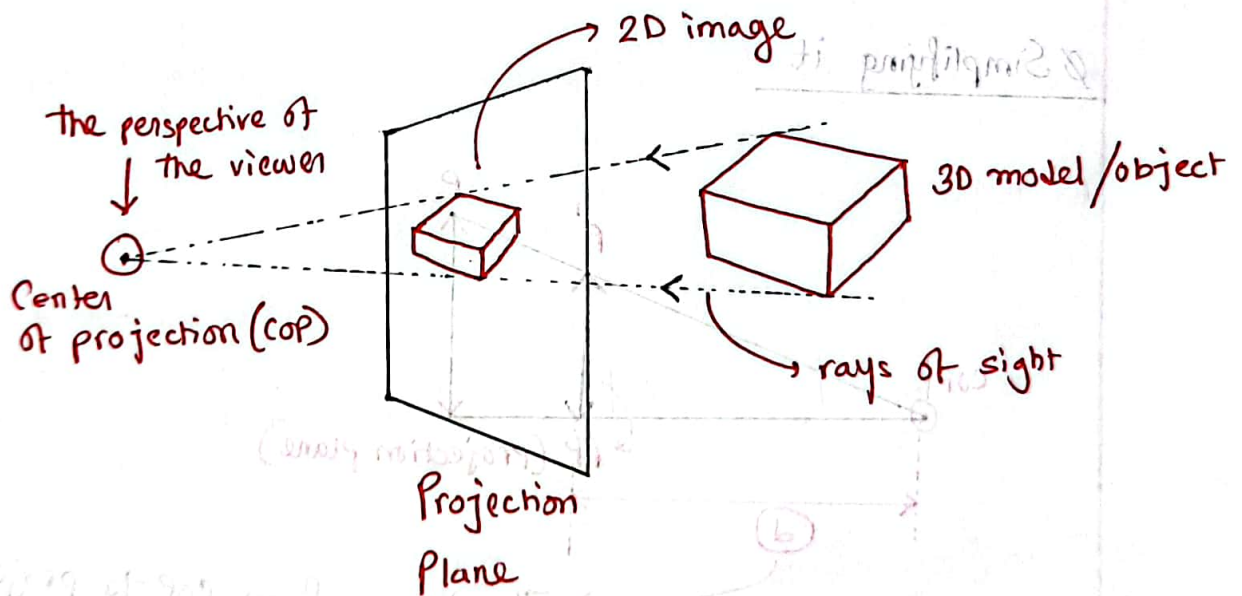
Ø Projection is basically an image or a process by which
we can create an image of an object on a plane.

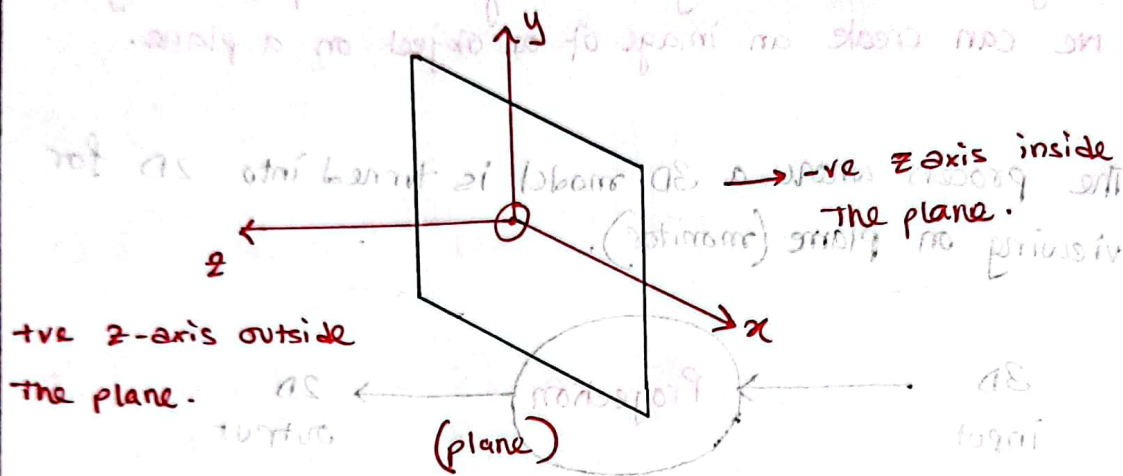Ø The process where a 3D model is turned into 2D for
viewing on plane. (monitor).
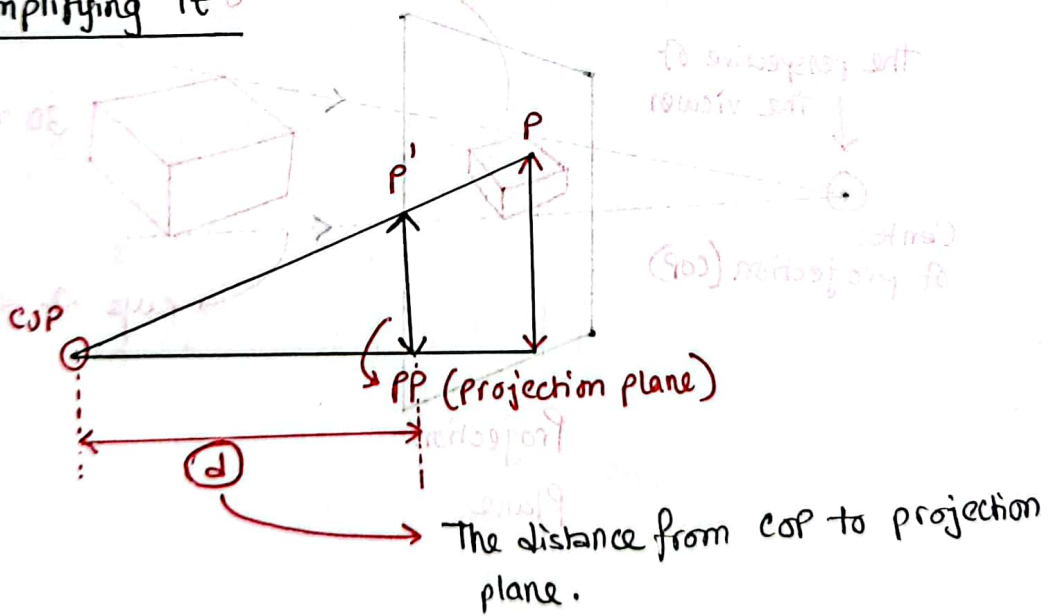
$$3D \text{ input} \longrightarrow \text{Projection} \longrightarrow 2D \text{ output}$$

- the perspective of the viewer
- Center of projection (cop)
- 2D image
- 3D model /object
- rays of sight
- Projection Plane

$$|r'| = |c_1 \cdot r_1|$$

∅ Placing the monitor in a 3D view, with all the axises.

→ -ve z-axis inside the plane.

+ve z-axis outside the plane.

(plane)

∅ Simplifying it

COP

P'

P

PP (projection plane)

ⓓ

→ The distance from COP to projection plane.
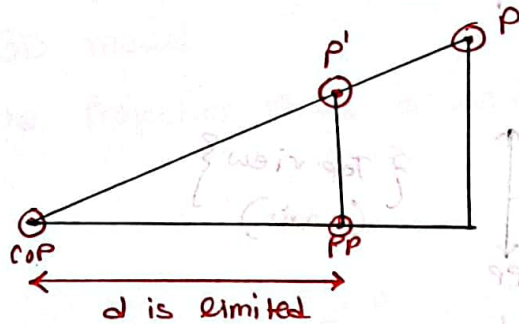
$$|P'| = |P_r| \cdot |P|$$

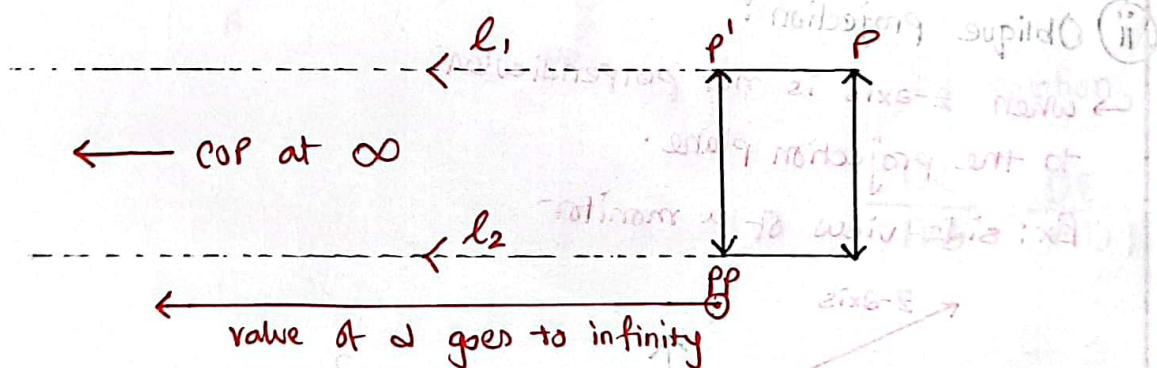projection

projection matrix.

→ object

(Classification : Types of Projection)

Ø Based on the value of d.

① Perspective Projection (if the value of d is limited)

d is limited

⑪ Parallel Projection (if the value of d is infinite)

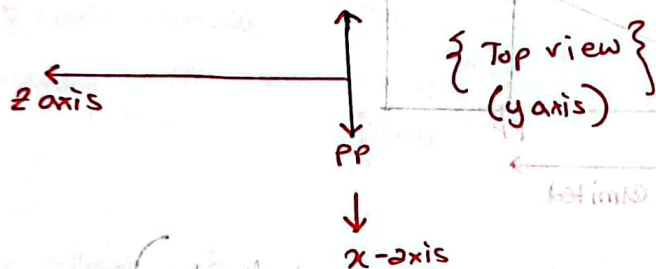COP at ∞

value of d goes to infinity

Ø if we represent the projection lines (rays of sight) as lines l_1 & l_2, and increase d to infinity, the lines l_1 & l_2 become parallel to each other. Hence, parallel projection.

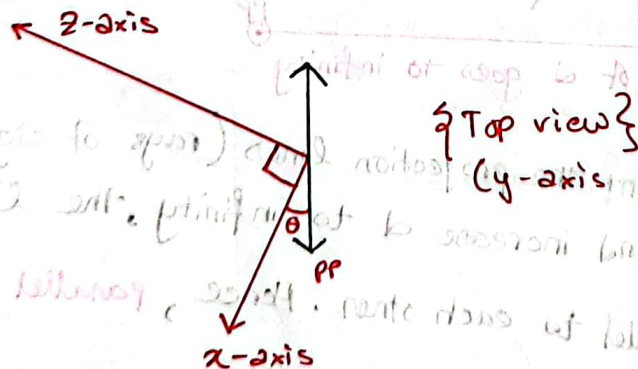∅ Based on the orientation of Projection Plane (PP)

(i) Orthographic Projection:

→ when the z-axis is perpendicular to the projection plane.

{ Top view }
(y axis)

z axis

PP

x-axis

(ii) Oblique projection:

→ when z-axis is not perpendicular to the projection plane.

Ex: side view of a monitor

z-axis

{ Top view }
(y-axis)

θ

PP

x-axis

(Mathematics of Projection)

Remember → P' is the final pixel/monitor coordinate.

Also, for an object the calculations and view are determined with 3D model.
And the Projection Plane is used to project the 2D pixel.

A, B, C → one triangle

A, D, E → another similar triangle.

So relation,

$$\frac{BC}{AB} = \frac{DE}{AD}$$

Centre of Projection

① Ø Derivation for the projection matrix
when : (Origin is at COP)

$P(x,y,z)$

$(x',y',z')\, P'$

$(0,0,0)$ $(0,0,d)$ $(0,0,z)$ A   B   D   E   C

↳ d is inherently negative,
since it goes to the $-ve$ z-axis.
same goes for z.
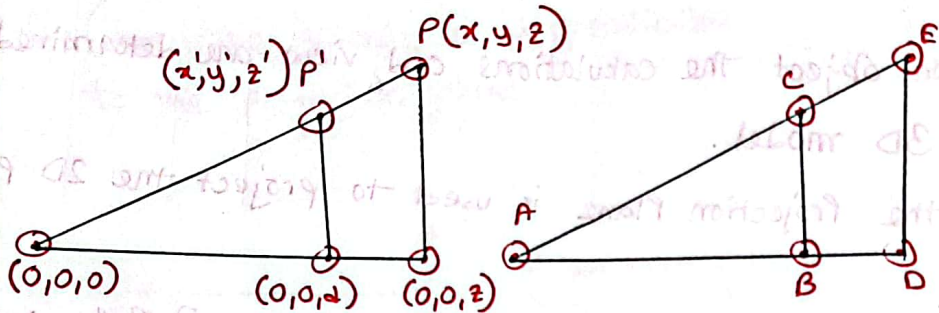
⊞ for y',

$$\frac{BC}{AB} = \frac{DE}{AD} \;?$$

$$\Rightarrow \frac{y'}{d} = \frac{y}{z} \qquad \Rightarrow y' = \frac{y}{z/d}$$

for x',

$$\Rightarrow \frac{x'}{d} = \frac{x}{z} \qquad \Rightarrow x' = \frac{x}{z/d}$$

for z',

$$\Rightarrow \frac{z'}{d} = \frac{z}{z/d} \qquad \Rightarrow z' = \frac{z}{z/d}$$

the denominator
is the same
for all.

→ this is the projection matrix.

∅ The matrix:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

→ why?

$$\Rightarrow \quad \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

doesn't match, to make it match, we need to all the terms with z/d

$$\Rightarrow \quad \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x/z/d \\ y/z/d \\ z/z/d \\ 1 \end{bmatrix}$$

→ done ←

Q) For the origin at COP, calculate the projected point coordinates for a point $(50, 60, -300)$, where the projection plane is at a distance of 200 from the COP?

Ans

P'?   P(50, 60, -300)

d

(0,0,0)   (0,0,-200)   (0,0,-300)
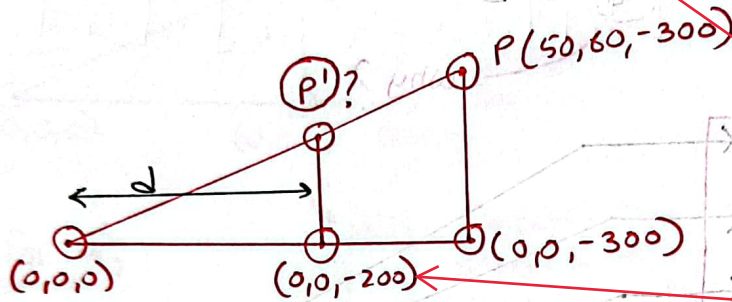
The value become negative

$\Rightarrow$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{-200} \end{bmatrix} \begin{bmatrix} 50 \\ 60 \\ -300 \\ 1 \end{bmatrix}$$

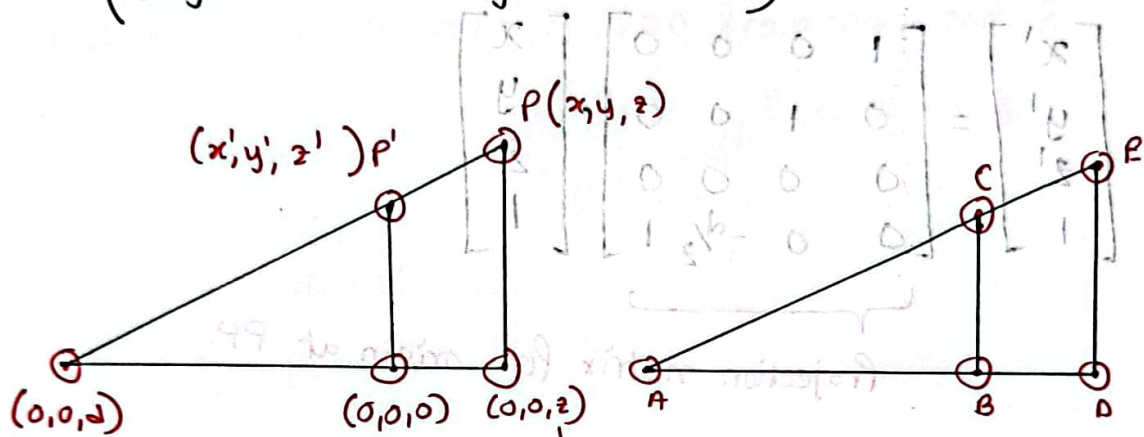$$= \begin{bmatrix} 50 \\ 60 \\ -300 \\ \frac{3}{2} \end{bmatrix}$$

// dividing all terms with $\frac{3}{2}$ OR multiply with $\frac{2}{3}$

$$= \begin{bmatrix} 33.33 \\ 40 \\ -200 \\ 1 \end{bmatrix}$$

Ans: $P'\left(33.33, 40, -200\right)$

Multiply with 2/3

2) Derivation for the projection matrix
When: (origin is at Projection Plane)

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & -1/d & 0 & 0 \end{bmatrix} = \begin{bmatrix} x' \\ y \\ z' \\ 1 \end{bmatrix}$$

→ Here d is inherently positive, since it is towards the +ve z-axis.

For y',

$$\frac{BC}{AB} = \frac{DE}{AD}$$

$$\Rightarrow \frac{y'}{d} = \frac{y}{d-z} \qquad \Rightarrow y' = \frac{y}{\frac{d-z}{d}} = \frac{y}{1 - z/d}$$

↳ since z is inherently -ve, this actually an addition.

→ while z is inherently negative, as it is in the -ve z-axis.

For x'

$$\Rightarrow x' = \frac{x}{1 - z/d}$$

for z'

$$\Rightarrow z' = 0 \quad [\text{as origin is at PP}]$$

Origin at Projection Plane

∅ **The matrix :**

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{d} & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

<u>Projection matrix for origin at PP.</u>

Step1

$$\Rightarrow) \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \\ 1 - \frac{z}{d} \end{bmatrix} = \begin{bmatrix} \frac{x}{1 - \frac{z}{d}} \\ \frac{y}{1 - \frac{z}{d}} \\ 0 \\ 1 \end{bmatrix}$$

Step2 : Divide by 1 - (z/d)

Q) For origin at Projection plane, calculate the projected point coordinates for a point $(50, 60, -100)$, where the projection plane is at a distance of 200 from the CoP?

Ans:
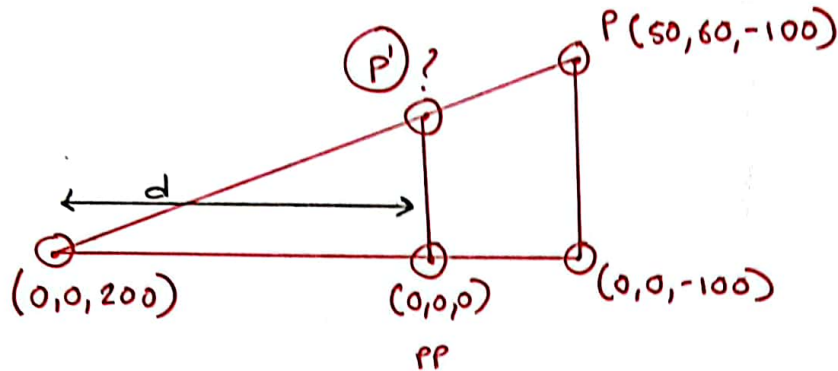
$$\Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{200} & 1 \end{bmatrix} \cdot \begin{bmatrix} 50 \\ 60 \\ -100 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 50 \\ 60 \\ 0 \\ 1 + \frac{1}{2} = 1.5 \end{bmatrix} = \begin{bmatrix} 50/1.5 \\ 60/1.5 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 33.33 \\ 40 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore P' = (33.33, 40, 0) \quad \underline{Ans:}$$