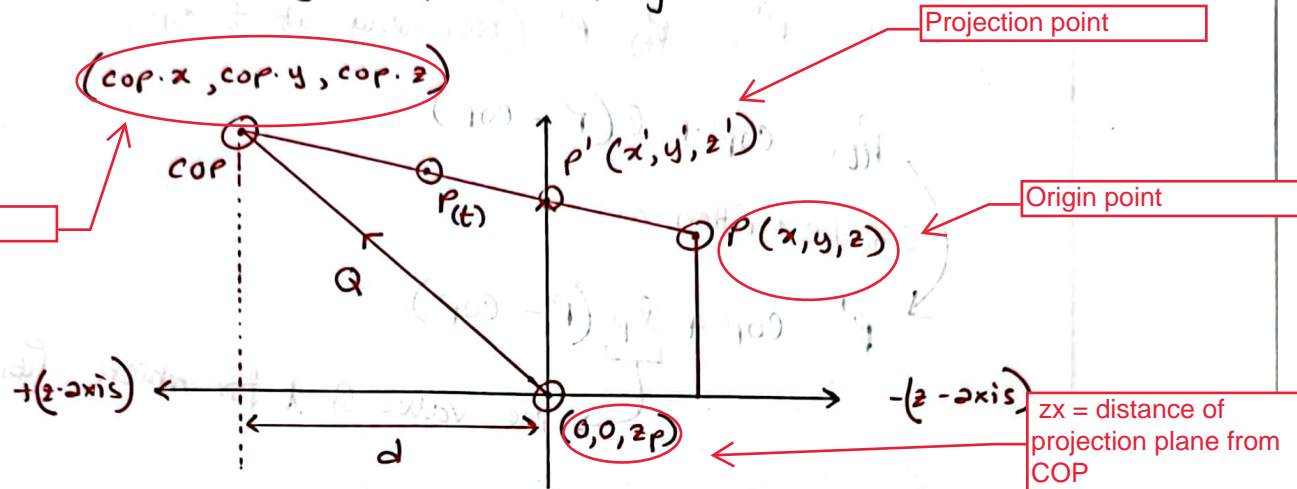


3) Derivation of a general purpose projection matrix:



Representing the line that goes from the bottom of the projection plane to the COP as a vector Q .

$$Q := (Q_{dx}, Q_{dy}, Q_{dz})$$

We can say,

$$Q = COP - (0, 0, z_p) \quad \text{--- (i)}$$

$$\therefore Q = (cop.x, cop.y, cop.z) - (0, 0, z_p)$$

So,

$$(1) \quad Q_{dx} = cop.x$$

$$(2) \quad Q_{dy} = cop.y$$

$$(3) \quad Q_{dz} = cop.z - z_p$$

OR

$$cop.x = Q_{dx}$$

$$cop.y = Q_{dy}$$

$$cop.z = z_p + Q_{dz} \quad \text{--- (ii)}$$

Suppose P to COP, a parametric line

→ then $P(t)$ is any point on the line.

$$\therefore P(t) = cop + t(P - cop) \quad \text{--- (iii)}$$

can be written the other way around as well.

$$P(t) = P + t(cop - P) \rightarrow \text{if } P \text{ is considered the starting point.}$$

∅ We can assume that for a certain value of t , P_t will be equal to P' , $P_t = P'$ (assuming at $t = t_p$)

$$P_t = \text{cop} + t(P - \text{cop})$$

can be written as

$$P' = \text{cop} + t_p(P - \text{cop})$$

the value of t for which $P_t = P'$

So,

$$P' = \text{cop} + t_p(P - \text{cop}) \quad \text{--- (iii)}$$

∅ substituting in the (ii) equation.

$$x' = \text{cop} \cdot x + t_p(x - \text{cop} \cdot x)$$

we know, $\text{cop} \cdot x = Qdx$

$$(1) \quad x' = Qdx + t_p(x - Qdx)$$

$$(2) \quad y' = Qdy + t_p(y - Qdy)$$

$$(3) \quad z' = Qdz + z_p + t_p(z - Qdz - z_p)$$

∅ we'll be getting an eqn for t_p from z' and substitute it in x' & y'

$$\Rightarrow t_p = \frac{z' - Qdz - z_p}{z - Qdz - z_p}$$

We know, $z' = z_p \rightarrow$ From the diagram, as z' is on one projection plane, it equals z_p .

$$l_p = \frac{z' - Q_{dz} - z_p}{z - Q_{dz} - z_p}$$

$$= \frac{-Q_{dz}}{z - Q_{dz} - z_p} \quad \leftarrow \text{bring the numerator below}$$

$$= \frac{1}{-\frac{z}{Q_{dz}} + 1 + \frac{z_p}{Q_{dz}}} \quad \text{--- (V) ---}$$

$$\text{(iv)} \rightarrow \left\{ \frac{s/b}{s/b} \cdot s + \frac{s/b}{s/b} \cdot s - s \right. =$$

$$\left[\frac{s/b}{s/b} + 1 + \frac{s}{s/b} - \frac{s}{s/b} \right] \quad \text{--- (V) ---}$$

P.T.O. \rightarrow

$$\left(\frac{s/b}{s/b} + 1 + \frac{s}{s/b} - \frac{s}{s/b} \right) \quad \text{--- (V) ---}$$

$$\text{(iv)} \rightarrow \left\{ \frac{s/b}{s/b} \cdot s + \frac{s/b}{s/b} \cdot s - s \right. =$$

Substituting eqn (v), z_p in all of eqn (iv)

$$x'_1 = Q dx + \left(\frac{1}{-\frac{z}{Q_{d2}} + 1 + \frac{z_p}{Q_{d2}}} \right) (x - Q dx)$$

$$= \frac{Q dx}{1} + \left(\frac{x - Q dx}{-\frac{z}{Q_{d2}} + 1 + \frac{z_p}{Q_{d2}}} \right)$$

$$= \frac{\cancel{\frac{z}{Q_{d2}}} \cdot Q dx + \cancel{Q} x + z_p \cdot \frac{Q dx}{Q_{d2}} + x - \cancel{Q} dx}{-\frac{z}{Q_{d2}} + 1 + \frac{z_p}{Q_{d2}}}$$

$$= \frac{x - z \cdot \frac{Q dx}{Q_{d2}} + z_p \cdot \frac{Q dx}{Q_{d2}}}{-\frac{z}{Q_{d2}} + 1 + \frac{z_p}{Q_{d2}}} \quad \text{--- (vi)}$$

$$y' = \frac{y - z \cdot \frac{Q dy}{Q_{d2}} + z_p \cdot \frac{Q dy}{Q_{d2}}}{-\frac{z}{Q_{d2}} + 1 + \frac{z_p}{Q_{d2}}} \quad \text{--- (vii)}$$

$$z' = z_p$$

$$= z_p \left(\frac{-\frac{z}{Q_{d2}} + 1 + \frac{z_p}{Q_{d2}}}{-\frac{z}{Q_{d2}} + 1 + \frac{z_p}{Q_{d2}}} \right)$$

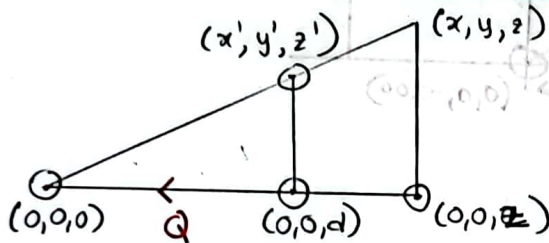
$$= \frac{-z \cdot \frac{z_p}{Q_{d2}} + z_p \left(1 + \frac{z_p}{Q_{d2}} \right)}{-\frac{z}{Q_{d2}} + 1 + \frac{z_p}{Q_{d2}}} \quad \text{--- (viii)}$$

Keeping the denominator same, so that it is easier to represent in a matrix.

Ø The Matrix:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -\frac{Qdx}{Qdz} \\ 0 & 1 & 0 & -\frac{Qdy}{Qdz} \\ 0 & 0 & 1 & -\frac{z_p}{Qdz} \\ 0 & 0 & 0 & 1 + \frac{z_p}{Qdz} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Ø Derivation of origin at COP (from general matrix)



$$Qdx = 0$$

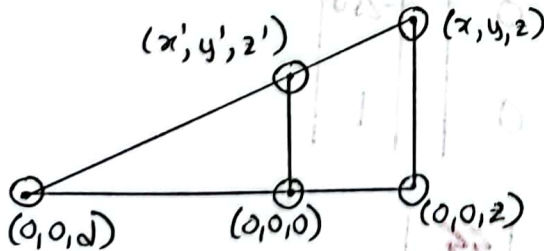
$$Qdy = 0$$

$$Qdz = -d$$

$$z_p = d$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

Ø Derivation of origin at PP (from general matrix)



$$Qdx = 0$$

$$Qdy = 0$$

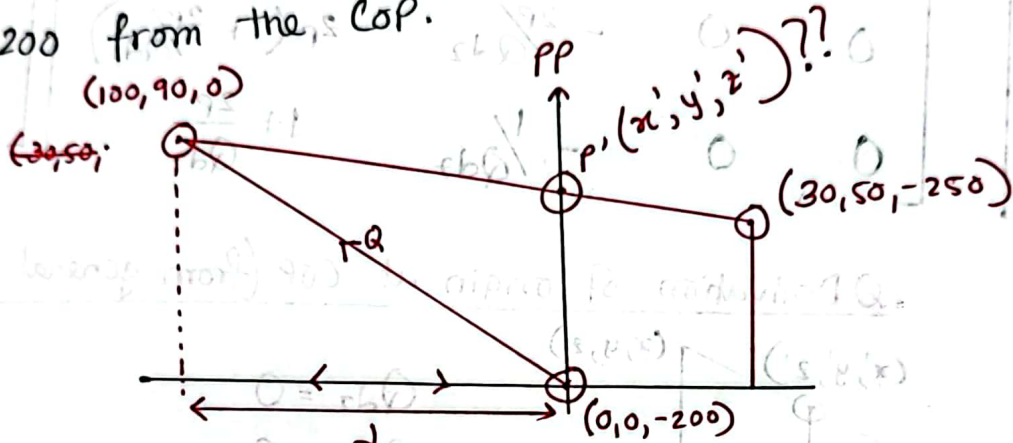
$$Qdz = d$$

$$z_p = 0$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/d & 1 \end{bmatrix}$$

Q) Calculate the projected point coordinates for a point $(30, 50, -250)$, where COP is at $(100, 90, 0)$ and the projecting plane is at a distance of 200 from the COP.

Ans:



So,

$$Qdx = 100$$

$$Qdy = 90$$

$$Qdz = 200$$

$$z_p = -200$$

$$\begin{vmatrix} x' \\ y' \\ z' \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & -1/2 & -100 \\ 0 & 1 & -9/20 & -90 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/200 & 0 \end{vmatrix} \cdot \begin{vmatrix} 30 \\ 50 \\ -250 \\ 1 \end{vmatrix}$$

$$= \begin{vmatrix} 30 + 125 & -100 \\ 50 + 112.5 & -90 \\ -250 & 5/4 \end{vmatrix} = \begin{vmatrix} 44 \\ 58 \\ -200 \end{vmatrix}$$

try to keep fractions to decrease the error.

$$\therefore P' = (44, 58, -200)$$

Ans//