

MID-POINT LINE ALGORITHM (contd*)

LECTURE 4

Equation of a line :

Implicit form $\left\{ \begin{array}{l} y = mx + c \dots\dots (i) \end{array} \right.$, where $m = \frac{dy}{dx}$

↓

$$\Rightarrow y = \frac{dy}{dx} \cdot x + c \dots\dots \text{(multiply both sides with } dx \text{)}$$

$$\Rightarrow dx \cdot y = dy \cdot x + dx \cdot c \dots\dots \text{(take } dx \cdot y \text{ to the right hand side)}$$

$$\Rightarrow \underbrace{dy \cdot x}_{\downarrow} - \underbrace{dx \cdot y}_{\downarrow} + \underbrace{dx \cdot c}_{\downarrow} = 0 \dots\dots$$

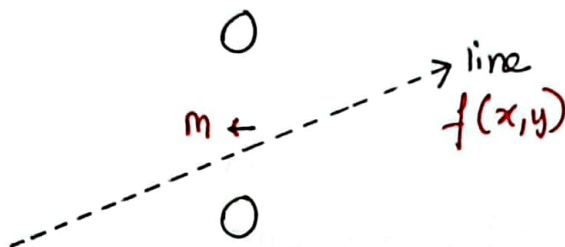
Explicit form $\left\{ \right.$

$$\boxed{Ax + By + C = 0} \dots\dots (ii)$$

replace,
✓ $dy = A$
✓ $-dx = B$
✓ $dx \cdot c = C$

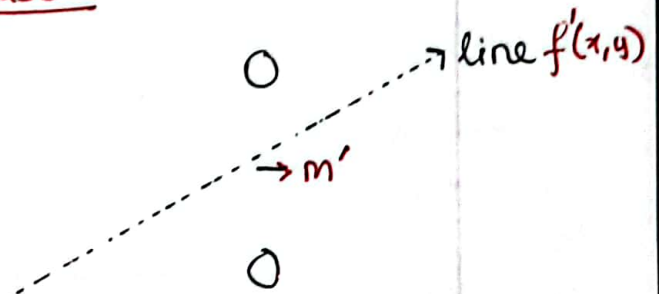
case 1

case 2



if we plug in the coordinates of M into the eqn of the line, the resulting value of the function will be (+ve)

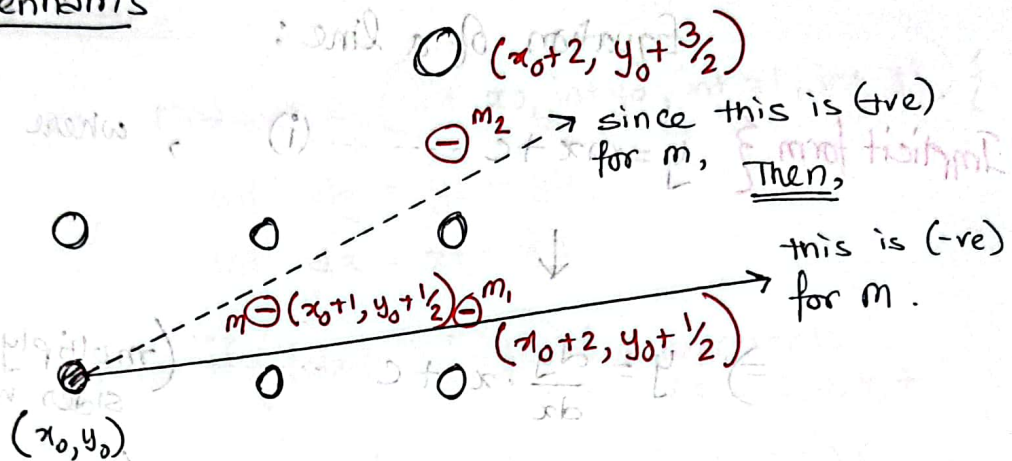
$$f(m) = +ve$$



if we plug in the coordinates of M into the eqn of the line, the resulting value of the function will be (-ve)

$$f'(m') = -ve$$

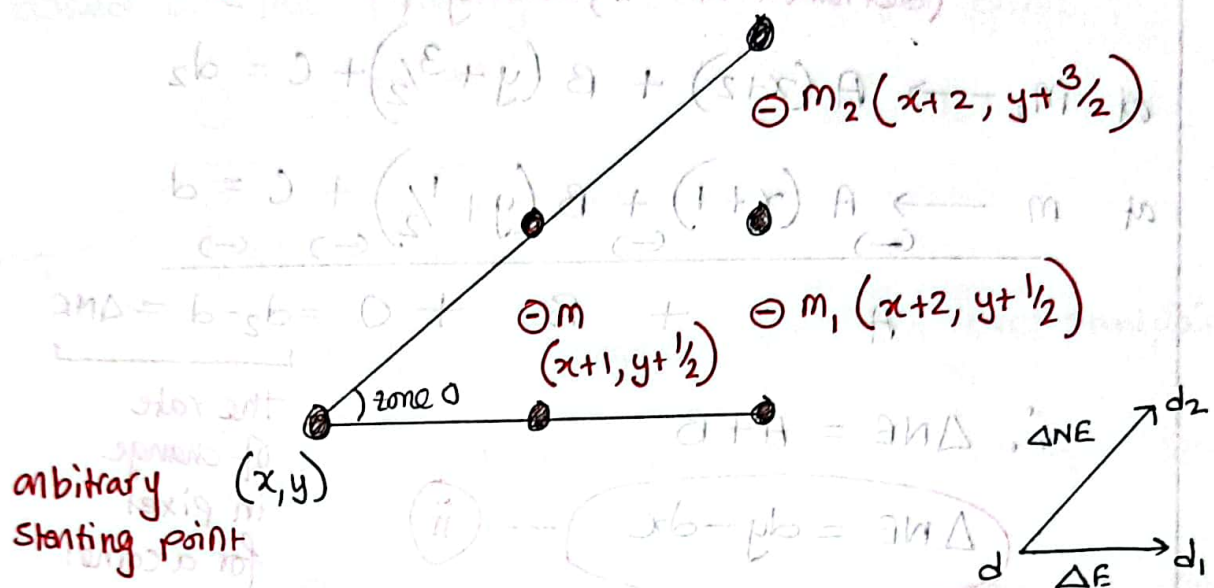
Bresenham's



we will be considering the rate of change (in the deviation from the midpoint)

(ii) to further locate the pixels in the path.

Designing the Algo for Zone 0.



(deviation at m_1 , solving for ΔE)

$$\text{at } m_1, \rightarrow A(x+2) + B(y+\frac{1}{2}) + C = d_1$$

$$\text{at } m, \rightarrow A(x+1) + B(y+\frac{1}{2}) + C = d$$

$$\lim_{\Delta x \rightarrow 0} (A(x+2) + B(y+\frac{1}{2}) + C) - (A(x+1) + B(y+\frac{1}{2}) + C) = d_1 - d \approx \Delta E$$

the rate of change of pixel for a horizontal movement.

$$\therefore A = \Delta E$$

meaning, $\Delta E = dy$

(i)

(deviation at m_2 , solving for ΔNE)

$$\text{at } m_2 \rightarrow A(x+2) + B(y+3/2) + C = d_2$$

$$\text{at } m \rightarrow A(x+1) + B(y+1/2) + C = d$$

$$A + B + 0 = d_2 - d = \Delta NE$$

$$\therefore \Delta NE = A + B$$

$$\Delta NE = dy - dx$$

ii

the rate of change in pixel for a corner movement.

we still need to solve for the initial deviation / d_{init} at m , for starting points (x_0, y_0)

$$\text{at } m, A(x_0+1) + B(y_0+1/2) + C = d_{init}$$

$$\Rightarrow Ax_0 + By_0 + C + A + B/2 = d_{init}$$

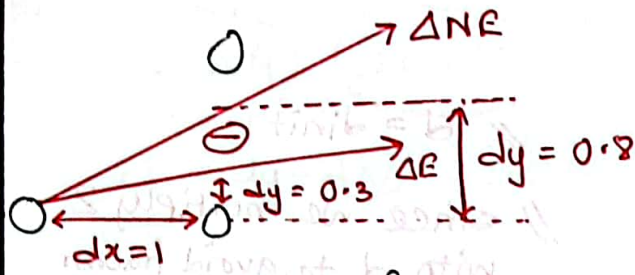
$$\Rightarrow d_{init} = A + B/2$$

since (x_0, y_0) is a coordinate from the line $Ax_0 + By_0 + C = 0$

$$\Rightarrow d_{init} = dy - dx/2$$

iii

Ø Now, we need figure out if we'll move to $\Delta E / \Delta NE$ based on the polarity of the value for d_{init} :



Ø for ΔNE , where line goes over midpoint.

$$d_{init} = dy - \frac{dx}{2} = 0.8 - \frac{1}{2} = 0.3 (+ve)$$

Ø for ΔE , where line goes below midpoint.

$$d_{init} = 0.3 - \frac{1}{2} = -0.2 (-ve)$$

*

∴ We can conclude that for a +ve value for d the movement will be ΔNE ,

AND,

for a -ve value for d the movement will be ΔE .

Pseudo Code (MPL Zone 0):

drawline (x_0, y_0, x_1, y_1) {

$$dy = y_1 - y_0$$

$$dx = x_1 - x_0$$

$$d = 2dy - dx$$

// $d = d_{init}$.

$$\Delta E = 2dy$$

// since we multiply 2 with d to avoid fraction

$$\Delta NE = 2(dy - dx)$$

// we need to multiply

ΔE & ΔNE with 2 as well.

$$x = x_0$$

$$y = y_0$$

draw (x, y);

while ($x \leq x_2$) {

if ($d \leq 0$) {

// for ΔE movement only x increases.

$x++$

$$d += \Delta E$$

// add ΔE to current value of d

} else {

// for ΔNE movement x, y both increase by 1.

$x++$

$y++$

$$d += \Delta NE$$

// add ΔNE to current value of d .

}

draw (x, y)

}

}

Q1)

Draw $(30, 50)$ to $(40, 54)$ using MPL

$\Delta y = 4$, $\Delta x = 10$

$d = \Delta y - \Delta x / 2 = 2(\Delta y) - \Delta x = 8 - 10 = -2$

use this to avoid fraction.

$\Delta E = 2 \cdot \Delta y = +8$

$\Delta NE = 2(\Delta y - \Delta x) = 2(4 - 10) = -12$

x	y	d	$\Delta E / \Delta NE$	(PIXEL)
30	50	-2	ΔE	(30, 50)
31	50	6	ΔNE	(31, 50)
32	51	-6	ΔE	(32, 51)
33	51	2	ΔNE	(33, 51)
34	52	-10	ΔE	(34, 52)
35	52	-2	ΔE	(35, 52)
36	52	6	ΔNE	(36, 52)
37	53	-6	ΔE	(37, 53)
38	53	2	ΔNE	(38, 53)
39	54	-10	ΔE	(39, 54)
40	54	-2	ΔE	(40, 54)

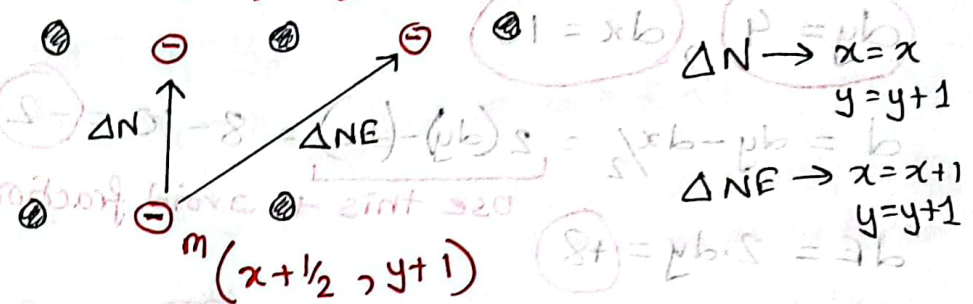
done

$x_b - y_b = \text{limit} \therefore$

Ø Home Assignment Design for Zone 1 & Zone 5

Zone 1

$$m_1(x+\frac{1}{2}, y+2) \quad m_2(x+\frac{3}{2}, y+2)$$



Ø ΔN

at m_1
at m

$$A(x+\frac{1}{2}) + B(y+2) + C = d_1$$

$$\hookrightarrow A(x+\frac{1}{2}) + B(y+1) + C = d$$

$$0 + B + 0 = \Delta N$$

$$\therefore \Delta N = -dx \rightarrow \text{we'll use } [-2dx]$$

Ø ΔNE

at m_2
at m

$$A(x+\frac{3}{2}) + B(y+2) + C = d_2$$

$$\hookrightarrow A(x+\frac{1}{2}) + B(y+1) + C = d$$

$$A + B = \Delta NE$$

$$\therefore \Delta NE = dy - dx \rightarrow \text{we'll use } [2(dy - dx)]$$

Ø d_{init}

at m
 (x_0, y_0)

$$A(x_0 + \frac{1}{2}) + B(y_0 + 1) + C = d_{init}$$

$$A/\frac{1}{2} + B = d_{init}$$

$$\therefore d_{init} = dy/\frac{1}{2} - dx \rightarrow \text{we'll use } [dy - 2dx]$$