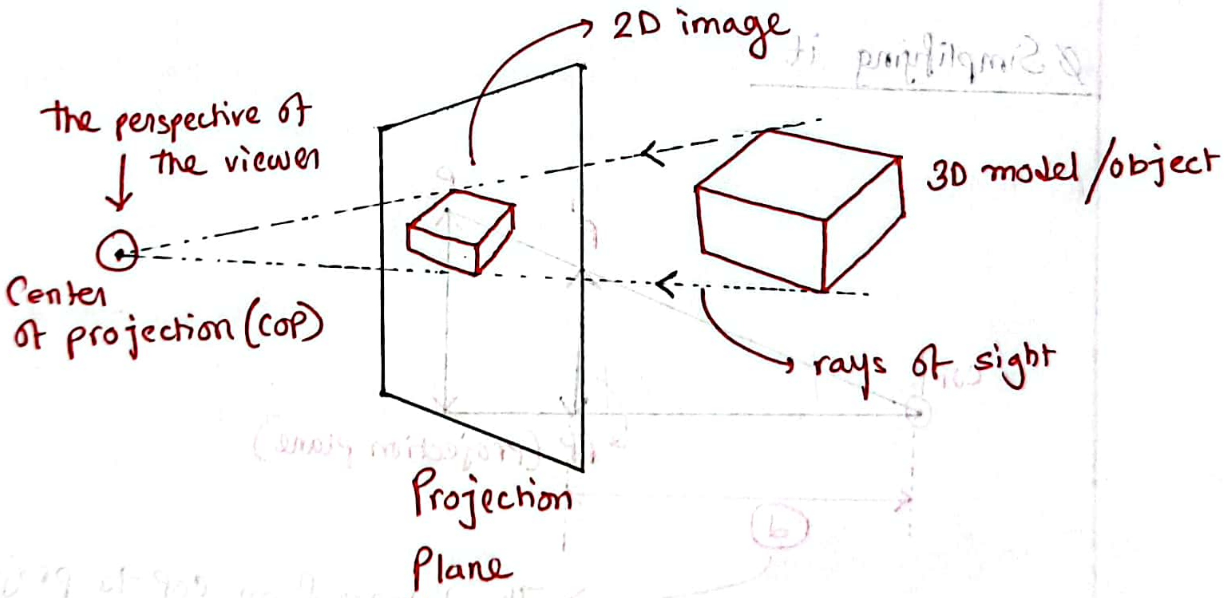
(PROJECTION)

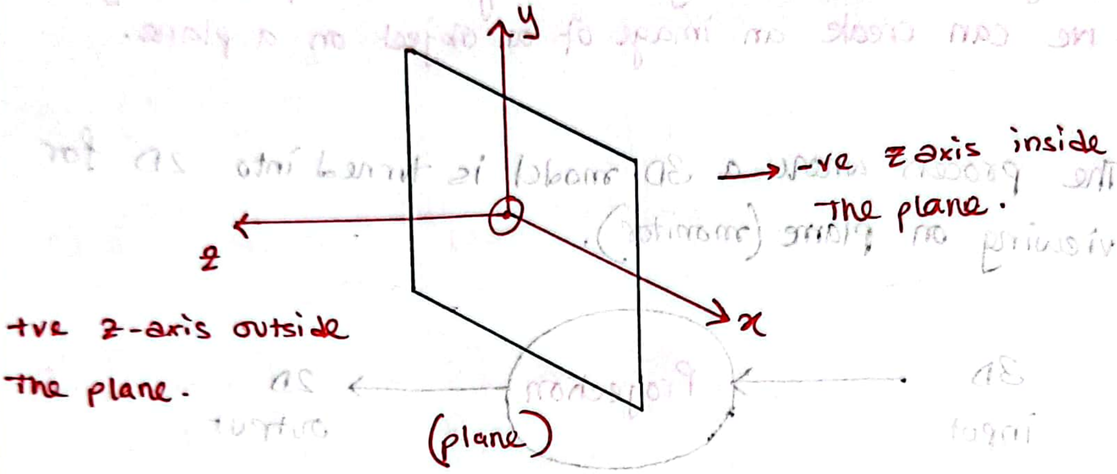
LECTURE 12

- Projection is basically an image or a process by which we can create an image of an object on a plane.

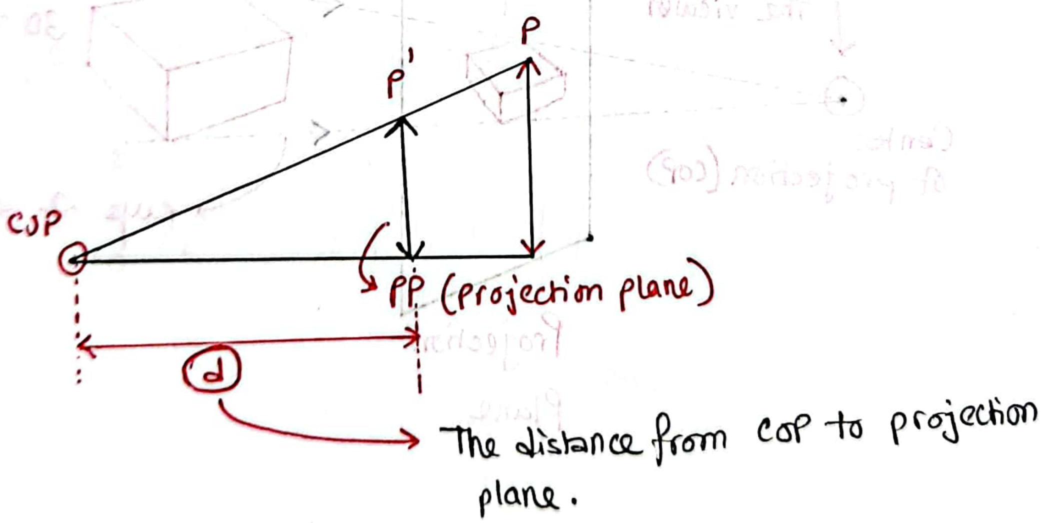
- The process where a 3D model is turned into 2D for viewing on plane (monitor).



Ø Placing the monitor in a 3D view, with all the axises.



Ø Simplifying it



$$|P'| = |P_r| \cdot |P|$$

projection

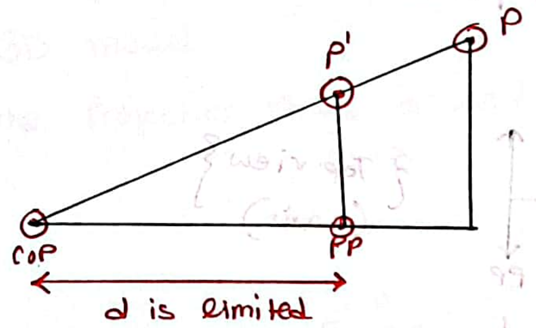
projection matrix.

object

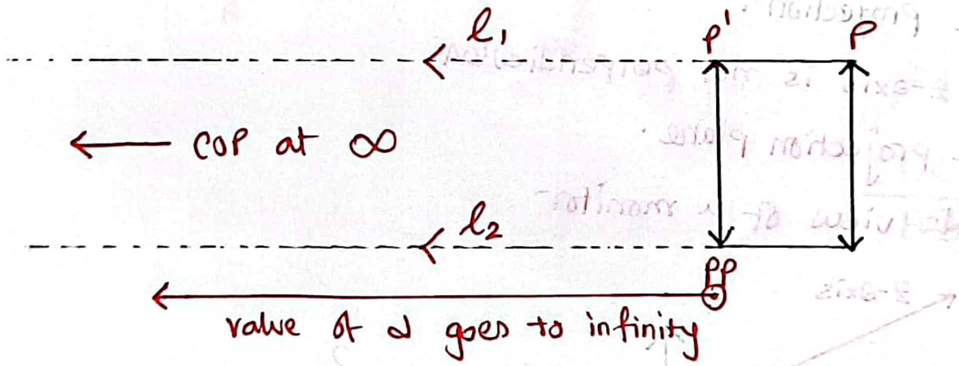
(Classification : Types of Projection)

∅ Based on the value of d.

(i) Perspective Projection (if the value of d is limited)



(ii) Parallel Projection (if the value of d is infinite)

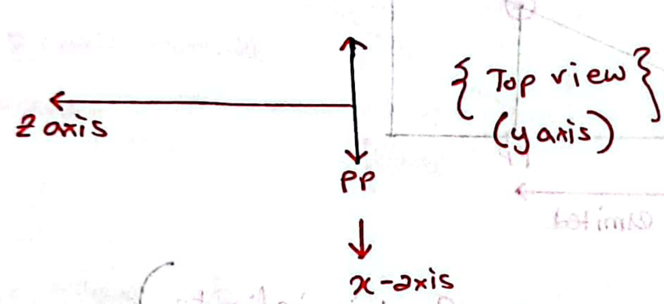


∅ if we represent the projection lines (rays of sight) as lines l_1 & l_2, and increase d to infinity, the lines l_1 & l_2 become parallel to each other. Hence, parallel projection.

Ø Based on the orientation of Projection Plane (PP)

(i) Orthographic Projection :

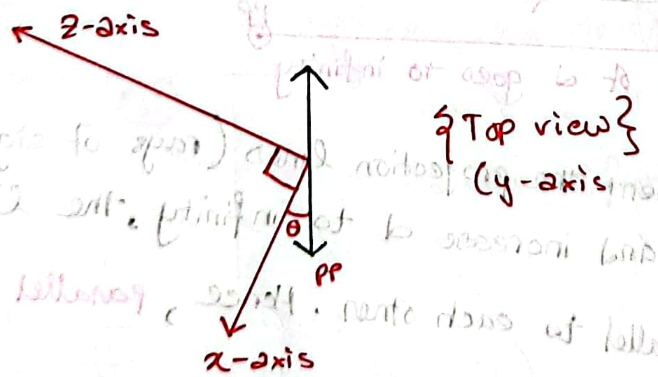
→ when the z-axis is perpendicular to the projection plane.



(ii) Oblique projection :

→ when z-axis is not perpendicular to the projection plane.

Ex: side view of a monitor

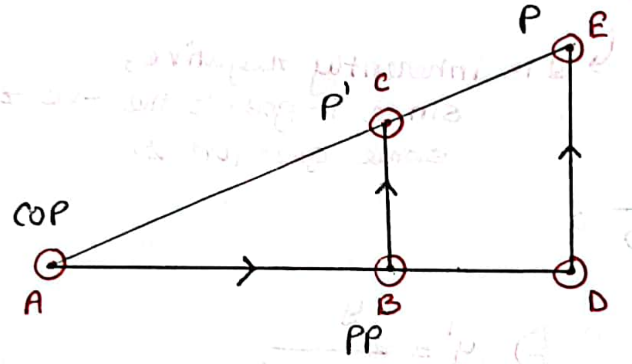


(Mathematics Of Projection)

Remember → P' is the final pixel/monitor coordinate.

Also, For an object the calculations and view are determined with 3D model.

And the Projection Plane is used to project the 2D pixel.



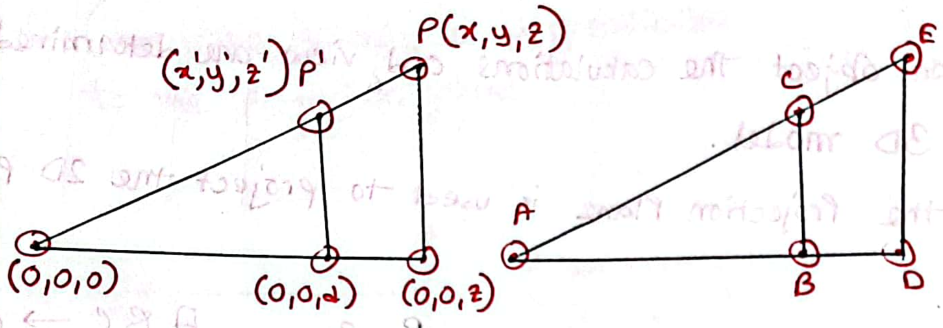
A, B, C → one triangle

A, D, E → another similar triangle.

So relation,

$$\frac{BC}{AB} = \frac{DE}{AD}$$

1 Derivation for the projection matrix when: (Origin is at COP)



↳ d is inherently negative, since it goes to the −ve z-axis. Same goes for z.

For y',

$$\frac{BC}{AB} = \frac{DE}{AD}$$

$$\Rightarrow \frac{y'}{d} = \frac{y}{z} \qquad \Rightarrow y' = \frac{y}{z/d}$$

For x',

$$\Rightarrow \frac{x'}{d} = \frac{x}{z} \qquad \Rightarrow x' = \frac{x}{z/d}$$

the denominator is the same for all.

For z',

$$\Rightarrow \frac{z'}{d} = \frac{z}{z/d} \qquad \Rightarrow z' = \frac{z}{z/d}$$

this is the projection matrix.

∅ The matrix :

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}}^{\text{this is the projection matrix.}} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

why ?

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

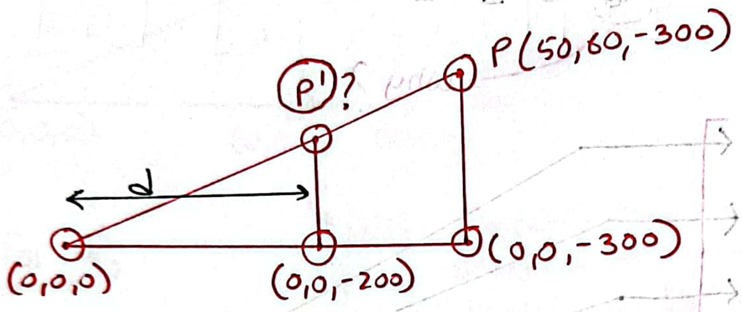
doesn't match, to make it match, we need to all the terms with z/d

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x/z/d \\ y/z/d \\ z/z/d \\ 1 \end{bmatrix}$$

done

Q) For the origin at COP, calculate the projected point coordinates for a point (50, 60, −300), where the projection plane is at a distance of 200 from the COP?

Ans



$$\Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/_{-200} & \end{bmatrix} \cdot \begin{bmatrix} 50 \\ 60 \\ -300 \\ 1 \end{bmatrix}$$

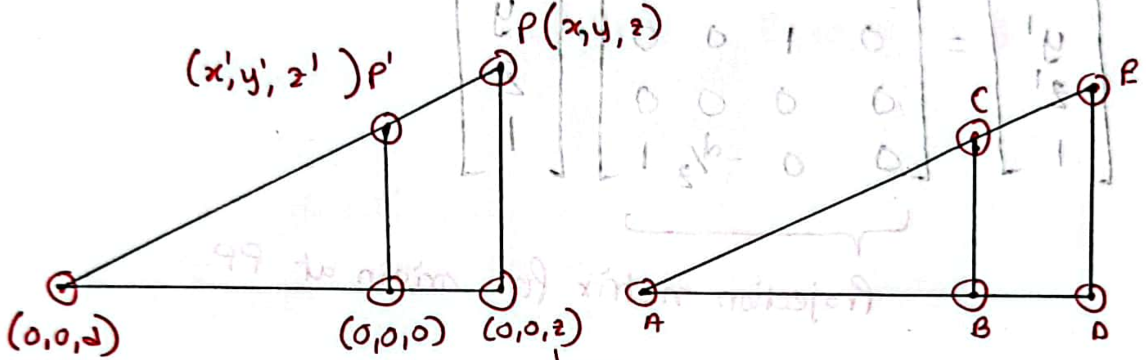
$$= \begin{bmatrix} 50 \\ 60 \\ -300 \\ 3/2 \end{bmatrix}$$

// dividing all terms with $3/2$ OR multiply with $2/3$

$$= \begin{bmatrix} 33.33 \\ 40 \\ -200 \\ 1 \end{bmatrix}$$

Ans: P'(33.33, 40, −200)

2) Derivation for the projection matrix When : (origin is at Projection Plane)



Here d is inherently positive, since it is towards the +ve z-axis.

while z is inherently negative, as it is in the -ve z-axis.

For y',

$$\frac{BC}{AB} = \frac{DE}{AD}$$

$$\Rightarrow \frac{y'}{d} = \frac{y}{d-z} \qquad \Rightarrow y' = \frac{y}{\frac{d-z}{d}} = \frac{y}{1 - z/d}$$

since z is inherently -ve, this actually an addition.

For x'

$$\Rightarrow x' = \frac{x}{1 - z/d}$$

For z'

$\Rightarrow z' = 0$ [as origin is at PP]

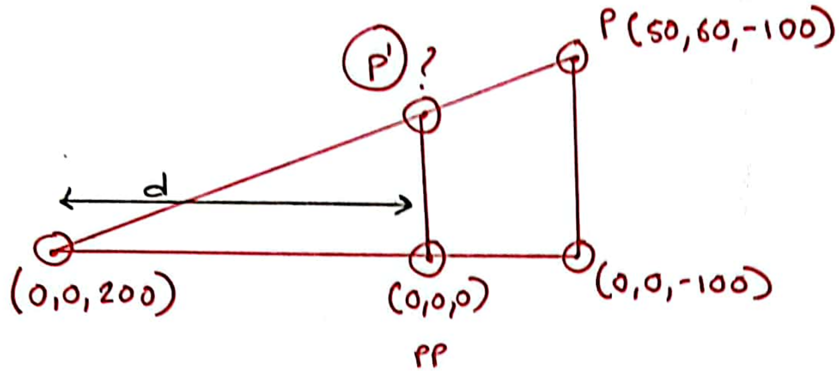
The matrix :

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/d & 1 \end{bmatrix}}_{\text{Projection matrix for origin at PP.}} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \\ 1 - z/d \end{bmatrix} = \begin{bmatrix} x/_{1-z/d} \\ y/_{1-z/d} \\ 0 \\ 1 \end{bmatrix}$$

Q) For origin at Projection plane, calculate the projected point coordinates for a point (50, 60, -100), where the projection plane is at a distance of 200 from the COP?

Ans:



$$\Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/200 & 1 \end{bmatrix} \cdot \begin{bmatrix} 50 \\ 60 \\ -100 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 50 \\ 60 \\ 0 \\ 1 + 1/2 = 1.5 \end{bmatrix} = \begin{bmatrix} 50/1.5 \\ 60/1.5 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 33.33 \\ 40 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore P' = (33.33, 40, 0)$ Ans: