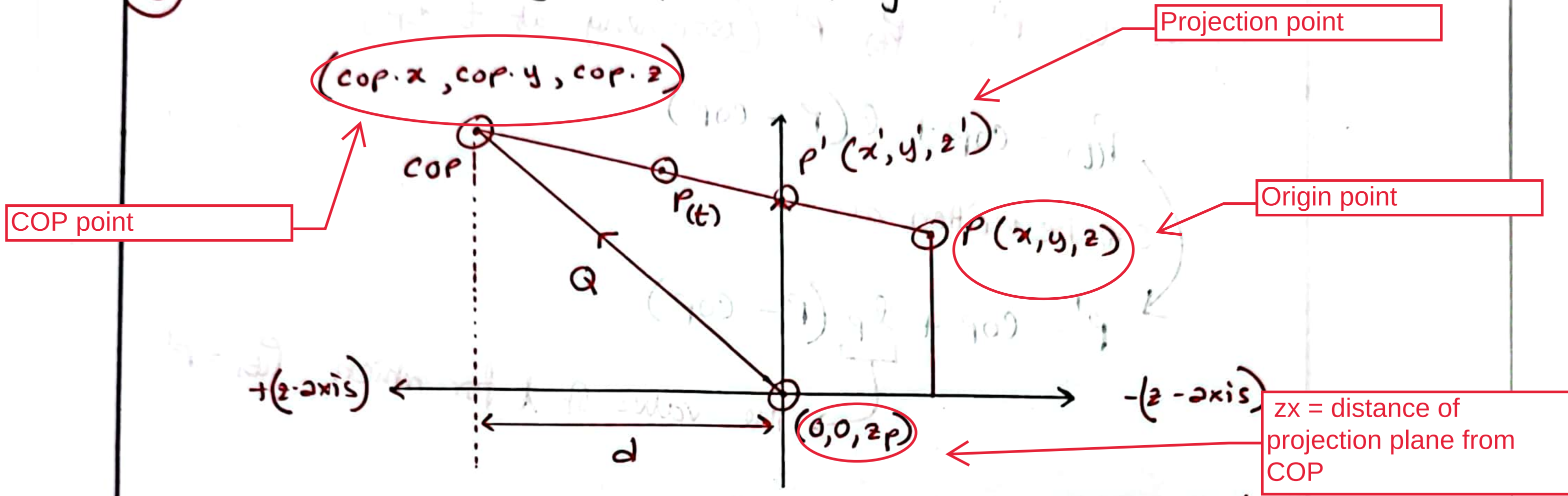
PROJECTION (CONTINUED)

LECTURE 13

(3) Derivation of a general purpose projection matrix :



∅ Representing the line that goes from the bottom of the projection plane to the COP as a vector Q.

$$Q :- (Q_{dx}, Q_{dy}, Q_{dz})$$

We can say,

$$Q = COP - (0,0,z_p) \quad \text{--- (i)}$$

$$\therefore \Rightarrow Q = (COP.x, COP.y, COP.z) - (0,0,z_p)$$

So,

(1) $Q_{dx} = COP.x$

(2) $Q_{dy} = COP.y$

(3) $Q_{dz} = COP.z - z_p$

OR

$$\left.\begin{aligned} COP.x &= Q_{dx} \\ COP.y &= Q_{dy} \\ COP.z &= z_p + Q_{dz} \end{aligned}\right] \text{--- (ii)}$$

∅ Suppose (P to COP, a parametric line)

→ then $P(t)$ is any point on the line.

$$\therefore \quad P(t) = COP + t(P - COP) \quad \text{--- (iii)}$$

can be written the other way around as well.

$P(t) = P + t(COP - P)$ → if P is considered the starting point.

Ø We can assume that for a certain value of t, $P_{(t)}$ will be equal to P', $P_{(t)} = P'$ (assuming at $t = t_p$)

$$P_{(t)} = cop + t(P - cop)$$

can be written as

$$P' = cop + \underbrace{t_p}_{\text{the value of } t \text{ for which } P_{(t)} = P'} (P - cop)$$

So,

$$P' = cop + t_p (P - cop) \quad \text{--- (iii)}$$

Ø substituting in the (ii) equation.

$$x' = cop.x + t_p (x - cop.x)$$

we know, $cop.x = Q d_x$

(1) $x' = Qd_x + t_p (x - Qd_x)$

(2) $y' = Qd_y + t_p (y - Qd_y)$

(3) $z' = Qd_z + z_p + t_p (z - Qd_z - z_p)$

--- (iv)

Ø we'll be getting an eqn for t_p from z' and substitute it in x' & y'

$$\Rightarrow t_p = \frac{z' - Qd_z - z_p}{z - Qd_z - z_p}$$

We know, $z' = z_p$ $\longrightarrow$ from the diagram, as z' is on the projection plane, it equals z_p.

$$t_p = \frac{\cancel{z'} - Qd_z - \cancel{z_p}}{z - Qd_z - z_p}$$

$$= \frac{-Qd_z}{z - Qd_z - z_p}$$ bring the numerator below

$$= \frac{1}{-\frac{z}{Qd_z} + 1 + \frac{z_p}{Qd_z}} \quad \text{--- (V)}$$

P.T.O $\rightarrow$

∅ Substituting eqn (v), tp in all of eqn (iv)

∅ $$x_1' = Q_{dx} + \left(\frac{1}{-\frac{z}{Q_{dz}} + 1 + \frac{zp}{Q_{dz}}} \right)(x - Q_{dx})$$

$$= \frac{Q_{dx}}{1} + \left(\frac{x - Q_{dx}}{-\frac{z}{Q_{dz}} + 1 + \frac{zp}{Q_{dz}}} \right)$$

$$= \frac{-\frac{z}{Q_{dz}} \cdot Q_{dx} + \cancel{Q_{dx}} + zp \cdot \frac{Q_{dx}}{Q_{dz}} + x - \cancel{Q_{dx}}}{-\frac{z}{Q_{dz}} + 1 + \frac{zp}{Q_{dz}}}$$

$$= \frac{x - z \cdot \frac{Q_{dx}}{Q_{dz}} + zp \cdot \frac{Q_{dx}}{Q_{dz}}}{-\frac{z}{Q_{dz}} + 1 + \frac{zp}{Q_{dz}}} \tag{vi}$$

∅ $$y' = \frac{y - z \cdot \frac{Q_{dy}}{Q_{dz}} + zp \cdot \frac{Q_{dy}}{Q_{dz}}}{-\frac{z}{Q_{dz}} + 1 + \frac{zp}{Q_{dz}}} \tag{vii}$$

Keeping the denominator same, so that it is easier to represent in a matrix.

∅ $$z' = zp$$

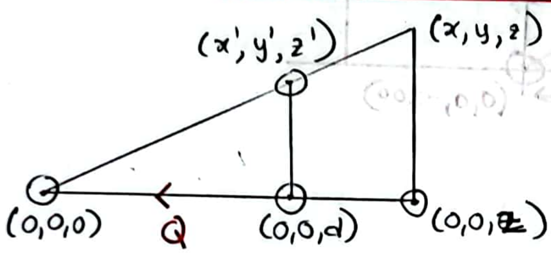
$$= zp \left(\frac{-\frac{z}{Q_{dz}} + 1 + \frac{zp}{Q_{dz}}}{-\frac{z}{Q_{dz}} + 1 + \frac{zp}{Q_{dz}}} \right)$$

$$= \frac{-z \cdot \frac{zp}{Q_{dz}} + zp\left(1 + \frac{zp}{Q_{dz}}\right)}{-\frac{z}{Q_{dz}} + 1 + \frac{zp}{Q_{dz}}} \tag{viii}$$

The Matrix:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\dfrac{Q d_x}{Q d_z} & z_p \cdot \dfrac{Q d_x}{Q d_z} \\ 0 & 1 & -\dfrac{Q d_y}{Q d_z} & z_p \cdot \dfrac{Q d_y}{Q d_x} \\ 0 & 0 & -z_p / Q d_z & z_p \left(1 + z_p / Q d_z\right) \\ 0 & 0 & -1/Q d_z & 1 + \dfrac{z_p}{Q d_z} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Derivation of origin at COP (from general Matrix)



$Q d_x = 0$

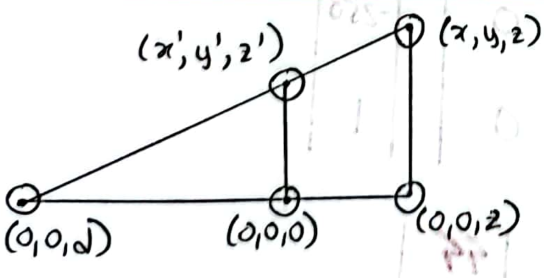
$Q d_y = 0$

$Q d_z = -d$

$z_p = d$

$$= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{vmatrix}$$

Derivation of origin at PP (from general matrix)



$Q d_x = 0$

$Q d_y = 0$

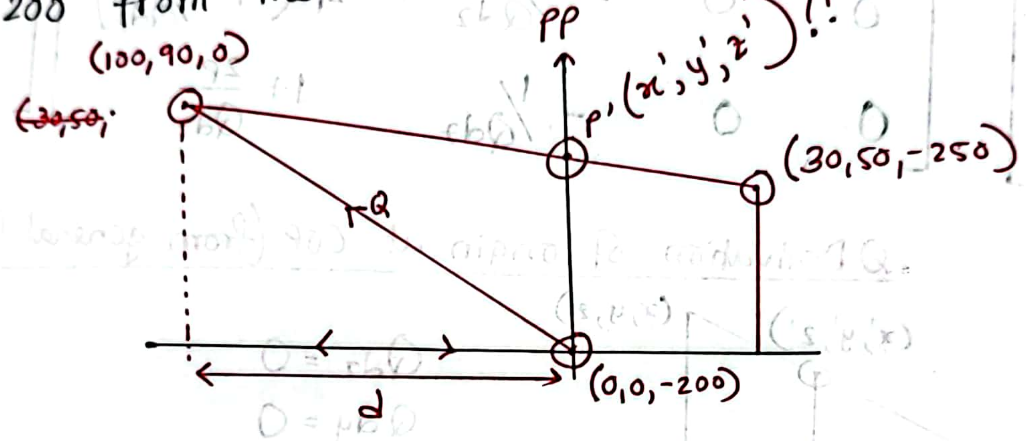
$Q d_z = d$

$z_p = 0$

$$= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/d & 1 \end{vmatrix}$$

Q) Calculate the projected point coordinates for a point $(30, 50, -250)$, where COP is at $(100, 90, 0)$ and the projectiong plane is at a distance of 200 from the CoP.

Ans :



So,

$Qdx = 100$

$Qdy = 90$

$Qdz = 200$

$z_p = -200$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1/2 & -100 \\ 0 & 1 & -9/20 & -90 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/200 & 0 \end{bmatrix} \cdot \begin{bmatrix} 30 \\ 50 \\ -250 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 30 + 125 - 100 \\ 50 + 112.5 - 90 \\ -250 \\ 5/4 \end{bmatrix} = \begin{bmatrix} 44 \\ 58 \\ -200 \\ 1 \end{bmatrix}$$

$\therefore p' = (44, 58, -200)$ Ans//

try to keep fractions to decrease the error.