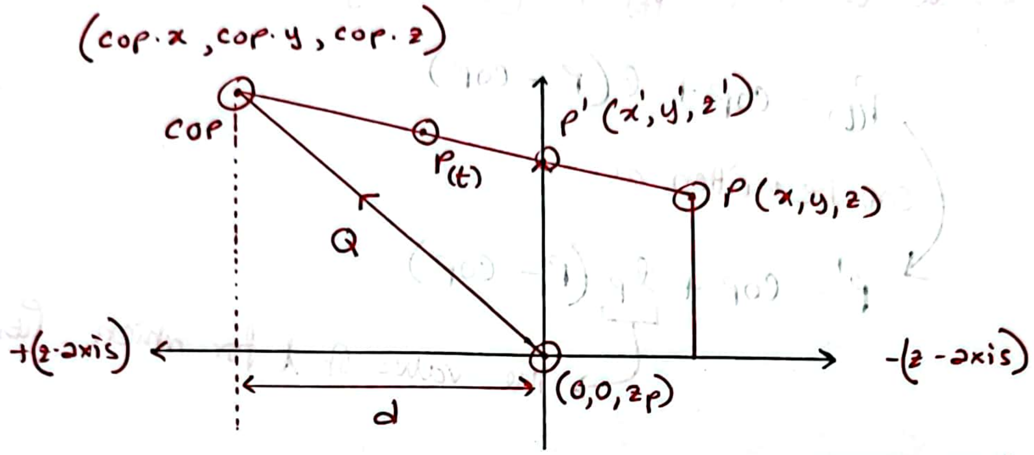
PROJECTION (CONTINUED)

LECTURE 13

3) Derivation of a general purpose projection matrix :



- Representing the line that goes from the bottom of the projection plane to the COP as a vector Q.

$$Q :- (Q_{dx}, Q_{dy}, Q_{dz})$$

We can say,

$$Q = COP - (0, 0, z_p) \text{ --- (i)}$$

$$\Rightarrow Q = (cop.x, cop.y, cop.z) - (0, 0, z_p)$$

So,

OR

(1) $Q_{dx} = cop.x$

(2) $Q_{dy} = cop.y$

(3) $Q_{dz} = cop.z - z_p$

$$\left.\begin{aligned} cop.x &= Q_{dx} \\ cop.y &= Q_{dy} \\ cop.z &= z_p + Q_{dz} \end{aligned}\right] \text{ --- (ii)}$$

- Suppose (P to COP, a parametric line)

→ then $P(t)$ is any point on the line.

$$\therefore P(t) = cop + t(P - cop) \text{ --- (iii)}$$

can be written the other way around as well.

$$P(t) = P + t(cop - P) \longrightarrow$$ if P is considered the starting point.

Ø We can assume that for acertain value of t, $P_{(t)}$ will be equal to P', $P_{(t)} = P'$ (assuming at $t = t_P$)

$$P_{(t)} = COP + t(P - COP)$$

can be written as

$$P' = COP + \underbrace{t_P}_{\text{the value of } t \text{ for which } P_{(t)} = P'}(P - COP)$$

So,

$$P' = COP + t_P (P - COP) \text{ --- (iii)}$$

Ø substituting in the (ii) equation.

$$x' = COP.x + t_P (x - COP.x)$$

we know, $COP.x = Q d_x$

(1) $x' = Q d_x + t_P (x - Q d_x)$

(2) $y' = Q d_y + t_P (y - Q d_y)$ --- (iv)

(3) $z' = Q d_z + z_P + t_P (z - Q d_z - z_P)$

Ø we'll be getting an eqn for t_P from z' and substitute it in x' & y'

$$\Rightarrow t_P = \frac{z' - Q d_z - z_P}{z - Q d_z - z_P}$$

We Know, $z' = z_p \longrightarrow$ from the diagram, as z' is on the projection plane, it equals z_p.

$$t_p = \frac{\cancel{z'} - Q d_z - \cancel{z_p}}{z - Q d_z - z_p}$$

$$= \frac{-Q d_z}{z - Q d_z - z_p}$$

bring the numerator below

$$= \frac{1}{-\frac{z}{Q d_z} + 1 + \frac{z_p}{Q d_z}} \quad \Big\} \text{---} \quad \text{V}$$

P.T.O $\rightarrow$

Substituting eqn (v), tp in all of eqn (iv)

$$x_1' = Q_dx + \left(\frac{1}{-z/Q_dz + 1 + zp/Q_dz}\right)(x - Q_dx)$$

$$= \frac{Q_dx}{1} + \left(\frac{x - Q_dx}{-z/Q_dz + 1 + zp/Q_dz}\right)$$

$$= \frac{-z/Q_dz \cdot Q_dx + \cancel{Q_dx} + zp.\frac{Q_dx}{Q_dz} + x - \cancel{Q_dx}}{-z/Q_dz + 1 + zp/Q_dz}$$

$$= \underbrace{\frac{x - z.\frac{Q_dx}{Q_dz} + zp.\frac{Q_dx}{Q_dz}}{-z/Q_dz + 1 + zp/Q_dz}}_{\text{vi}}$$

$$y' = \underbrace{\frac{y - z.Q_dy/Q_dz + zp.\frac{Q_dy}{Q_dz}}{-z/Q_dz + 1 + zp/Q_dz}}_{\text{vii}}$$

Keeping the denominator same, so that it is easier to represent in a matrix.

$$z' = zp$$

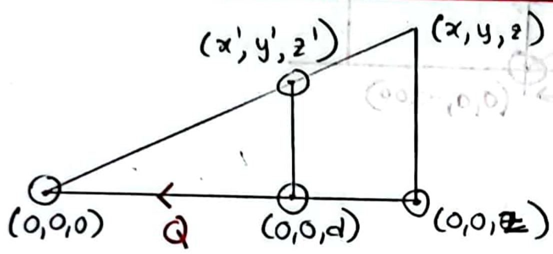
$$= zp\left(\frac{-z/Q_dz + 1 + zp/Q_dz}{-z/Q_dz + 1 + zp/Q_dz}\right)$$

$$= \underbrace{\frac{-z.\frac{zp}{Q_dz} + zp\left(1 + zp/Q_dz\right)}{-z/Q_dz + 1 + zp/Q_dz}}_{\text{viii}}$$

The Matrix:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{Q d_x}{Q d_z} & z_p \cdot \frac{Q d_x}{Q d_z} \\ 0 & 1 & -\frac{Q d_y}{Q d_z} & z_p \cdot \frac{Q d_y}{Q d_x} \\ 0 & 0 & -\frac{z_p}{Q d_z} & z_p \left(1 + \frac{z_p}{Q d_z}\right) \\ 0 & 0 & -\frac{1}{Q d_z} & 1 + \frac{z_p}{Q d_z} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Derivation of origin at COP (from general Matrix)



$Q d_x = 0$

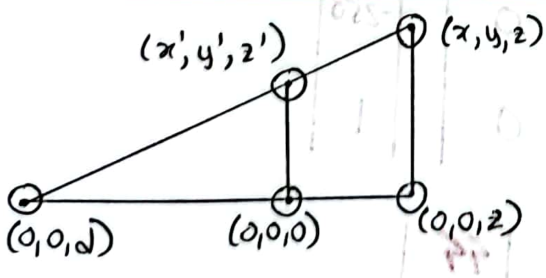
$Q d_y = 0$

$Q d_z = -d$

$z_p = d$

$$= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{vmatrix}$$

Derivation of origin at PP (from general matrix)



$Q d_x = 0$

$Q d_y = 0$

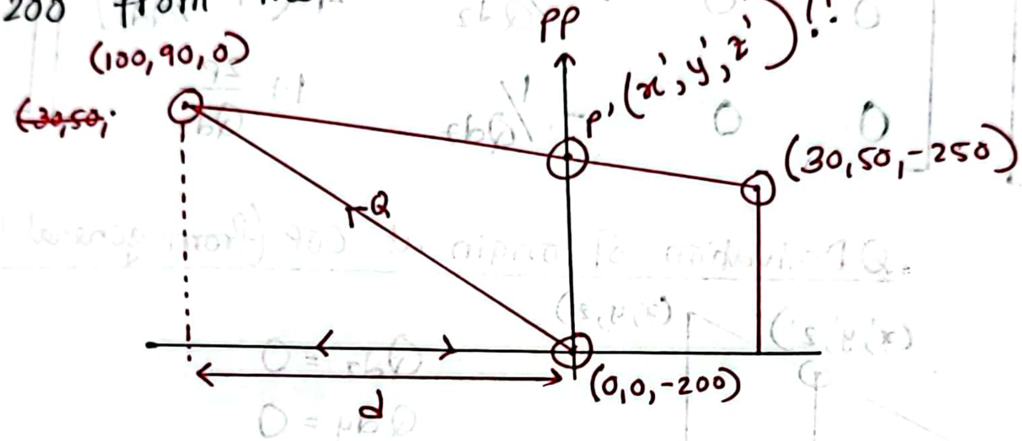
$Q d_z = d$

$z_p = 0$

$$= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/d & 1 \end{vmatrix}$$

Q) Calculate the projected point coordinates for a point (30, 50, −250), where COP is at (100, 90, 0) and the projectiong plane is at a distance of 200 from the CoP.

Ans :



So,

$Q_{dx} = 100$

$Q_{dy} = 90$

$Q_{dz} = 200$

$z_p = -200$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1/2 & -100 \\ 0 & 1 & -9/20 & -90 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/200 & 0 \end{bmatrix} \cdot \begin{bmatrix} 30 \\ 50 \\ -250 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 30 + 125 - 100 \\ 50 + 112.5 - 90 \\ -250 \\ 5/4 \end{bmatrix} = \begin{bmatrix} 44 \\ 58 \\ -200 \\ 1 \end{bmatrix}$$

try to keep fractions to decrease the error.

$\therefore p' = (44, 58, -200)$ Ans//