CSE427

Assignment-1

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1|

Head → 34
Tails → 36
Total → 70

∴ The value of $p = \frac{34}{70}$

≈ 0.485711

∴ The probability of having head is approximately ≈ 0.485711

2| Consider,

Head probability is p
tail probability is $1-p$
Number of Heads x
Number of tails y

According to Maximum log-likelihood →

$P(a_1) \rightarrow p^x \cdot (1-p)^y$ [1st attempt]
$P(a_2) \rightarrow p^x \cdot (1-p)^y$ [2nd attempt]

$$P(a_{10}) \rightarrow p^{x} \cdot (1-p)^{y} \quad [10^{th} \text{ attempt}]$$

$$\therefore P(a_i) \rightarrow p^{x_i} \cdot (1-p)^{y_i} \quad [i^{th} \text{ Attempt}]$$

As,

$$-\log P(A) = -\log P(a_1) - \log P(a_2) - \ldots - \log P(a_m)$$

$$= \sum_{i=1}^{m} -\log P(a_i)$$

$$= \sum_{i=1}^{m} -\log \left[p^{x_i} \cdot (1-p)^{y_i} \right]$$

$$= \sum_{i=1}^{m} x_i \log(p) - \sum_{i=1}^{m} y_i \cdot \log(1-p)$$

3] from question-2,

$$P(a_i) = p^{x_i} \cdot (1-p)^{y_i}$$

p is probability of head | x_i = Number of H
1-p is probability of tail | y_i = Number of T

Given,

$$q = \frac{\sum_{i=1}^{m} x_i}{mn}$$

$$\Rightarrow mn = \frac{\sum_{i=1}^{m} x_i}{q}$$

Here,

$$\frac{-\log P(A)}{mn} = -\frac{\sum_{i=1}^{m} x_i}{mn} \log p - \frac{\sum_{i=1}^{m} y_i}{mn} \log(1-p)$$

$$\Rightarrow -\frac{\log P(A)}{mn} = -q \log p - (1-q) \log(1-p)$$

$$\Rightarrow \frac{\partial}{\partial p}\left[-\frac{\log P(A)}{mn}\right] = \frac{\partial}{\partial p}\left[-q \log p - (1-q) \log(1-p)\right]$$

$$\Rightarrow 0 = -\frac{q}{p} - \frac{1-q}{1-p} \times -1$$

$$\Rightarrow \frac{q}{p} = \frac{1-q}{1-p}$$

$$\Rightarrow q - pq = p - pq$$

$$\therefore p = q$$

[Proved]

5)

From question - 01,

$$\log P(A) = \sum_{i=1}^{m} -x_i \cdot \log P - \sum_{i=1}^{m} y_i \cdot \log(1-P)$$

$$\Rightarrow \frac{\partial}{\partial P}\left[\log P(A)\right] = \frac{\partial}{\partial P}\left[-\sum_{i=1}^{m} x_i \cdot \log P - \sum_{i=1}^{m} y_i \cdot \log(1-P)\right]$$

$$\Rightarrow \quad 0 \quad = -\sum_{i=1}^{m} x_i \cdot \frac{1}{P} + \sum_{i=1}^{m} y_i \cdot \frac{1}{1-P}$$

x_i = total number of Head

y_i = total number of tails

$\therefore x_i = 34$ and $y_i = 36$

$$\Rightarrow -\frac{1}{P} \cdot 34 + \frac{1}{1-P} \cdot 36 = 0$$

$$\Rightarrow \frac{34}{P} = \frac{36}{1-P}$$

$$\Rightarrow 36P = 34 - 34P$$

$$\Rightarrow P = \frac{34}{70} \approx 0.478$$

6|

New 20 coin

number of
The coin will be head (0.478×20)

$= 9.56$

≈ 10

10 coins will head