Date:

TECHNO Printing & Packaging

CSE427

Assignment 2

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$$\bar{y}_i = w_1 x_{i,1} + w_2 x_{i,2} + \ldots + w_n x_{i,n} + b$$

$$w_i = w_i - \lambda \cdot \frac{\delta E}{\delta w_i}$$

$$b = b - \lambda \cdot \frac{\delta E}{\delta b}$$

1) Mean squared Error $\rightarrow$

$$E = \frac{1}{m} \sum_{i=1}^{m} (y_i - \bar{y}_i)^2$$

$$\therefore \frac{\delta E}{\delta w_i} = 2 \cdot \frac{1}{m} (y_i - \bar{y}_i) \cdot \frac{\delta}{\delta w_i} \cdot (y_i - \bar{y}_i)$$

$$= \frac{2}{m} \cdot (y_i - \bar{y}_i) \cdot \frac{\delta}{\delta w_i} \left(\sum^{n} w_i x_i + b - y_i \right)$$

$$= \frac{2}{m} (y_i - \bar{y}_i) \cdot x_i$$

$$\therefore \frac{\delta E}{\delta b} = 2 \cdot \frac{1}{m} (y_i - \bar{y}_i) \cdot \frac{\delta}{\delta b} \cdot (y_i - \bar{y}_i)$$

$$= \frac{2}{m} \cdot (y_i - \bar{y}_i) \cdot \frac{\delta}{\delta b} \left(\sum_{i=0}^{m} w_i x_i + b - y_i \right)$$

$$= \frac{2}{m} \cdot (y_i - \bar{y}_i)$$

2) Sum of squared Error:

$$E = \sum_{i=1}^{m} (y_i - \bar{y}_i)^2$$

$$\frac{\delta E}{\delta w_i} = 2 \cdot (y_i - \bar{y}_i) \cdot \frac{\delta}{\delta w_i} \cdot (y_i - \bar{y}_i)$$

$$= 2 \cdot (y_i - \bar{y}_i) \cdot \frac{\delta}{\delta w_i} \left(\sum_{i=0}^{n} w_i x_i + b - y_i \right)$$

$$= 2 \cdot (y_i - \bar{y}_i) \cdot x_i$$

$$\frac{\delta E}{\delta b} = 2 \cdot (y_i - \bar{y}_i) \cdot \frac{\delta}{\delta b} (y_i - \bar{y}_i)$$

$$= 2 \cdot (y_i - \bar{y}_i) \cdot \frac{\delta}{\delta b} \left(\sum_{i=0}^{n} w_i x_i + b - y_i \right)$$

$$= 2 \cdot (y_i - \bar{y}_i)$$

3) Mean squared logged Error

$$E = \frac{1}{m} \cdot \sum_{i=0}^{m} (\log \cdot y_i - \log \cdot \bar{y}_i)^2$$

$$\frac{\delta E}{\delta w_i} = 2 \cdot \frac{1}{m} \cdot (\log y_i - \log \bar{y}_i) \cdot \frac{\delta}{\delta w_i} (\log y_i - \log \bar{y}_i)$$

$$= \frac{2}{m} \cdot (\log y_i - \log \bar{y}_i) \cdot \frac{\delta}{\delta w_i} \left[\log \left(\sum_{i=0}^{n} w_i x_i + b \right) - \log y_i \right]$$

$$= \frac{2}{m} (\log y_i - \log \bar{y}_i) \cdot \frac{1}{w_i x_i} \cdot \frac{\delta}{\delta w_i} (w_i \cdot x_i)$$

$$= \frac{2}{m} (\log y_i - \log \bar{y}_i) \cdot \frac{1}{w_i x_i} \cdot x_i$$

$$= \frac{2}{m} (\log y_i - \log \bar{y}_i) \cdot \frac{1}{w_i}$$

$$\frac{\delta E}{\delta b} = \frac{2}{m} \cdot (\log y_i - \log \bar{y}_i) \cdot \frac{\delta}{\delta b} \left[\log \left(\sum w_i x_i + b \right) - \log y_i \right]$$

$$= \frac{2}{m} (\log y_i - \log \bar{y}_i) \cdot \frac{\delta}{\delta b} (\log b)$$

$$= \frac{2}{m} (\log y_i - \log \bar{y}_i) \cdot \frac{1}{b}$$

4) Mean absolute Error :-

$$E = \frac{1}{m} \sum_{i=1}^{m} |y_i - \bar{y}_i|$$

$$\frac{\delta E}{\delta w_i} = \frac{1}{m} \cdot \frac{\delta}{\delta w_i} (y_i - \bar{y}_i)$$

$$= \frac{1}{m} \cdot \frac{\delta}{\delta w_i} \left(\sum_{i=0}^{n} w_i x_i + b - y_i \right)$$

$$= \frac{1}{m} \cdot x_i$$

$$= \frac{x_i}{m}$$

$$\frac{\delta E}{\delta b} = \frac{1}{m} \cdot \frac{\delta}{\delta b} (y_i - \bar{y}_i)$$

$$= \frac{1}{m} \cdot \frac{\delta}{\delta b} \left(\sum_{i=0}^{m} w_i x_i + b - y_i \right)$$

$$= \frac{1}{m} \cdot 1$$

$$= \frac{1}{m}$$

5. Huber Loss :

$$E = \frac{1}{m}\sum_{i=1}^{m}\begin{cases}\frac{1}{2}(y_i-\bar{y}_i)^2 & , |y_i-\bar{y}_i| \leq \delta \\ \delta(|y_i-\bar{y}_i|-\frac{1}{2}\delta) & , |y_i-\bar{y}_i| > \delta\end{cases}$$

If, $|y_i-\bar{y}_i| \leq \delta$

$$\frac{\delta E}{\delta \omega_i} = \frac{1}{m}\cdot 2\cdot(y_i-\bar{y}_i)\cdot\frac{\delta}{\delta\omega_i}(y_i-\bar{y}_i)$$

$$= \frac{2}{m}\cdot(y_i-\bar{y}_i)\cdot\frac{\delta}{\delta\omega_i}\left(\sum^{n} w_i x_i + b - y_i\right)$$

$$= \frac{2}{m}\cdot(y_i-\bar{y}_i)\cdot x_i$$

$$\frac{\delta E}{\delta b} = \frac{1}{m} \cdot 2 \cdot (y_i - \bar{y}_i) \cdot \frac{\delta}{\delta b} \left(\sum^{n} w_i x_i + b - y_i \right)$$

$$= \frac{2}{m} (y_i - \bar{y}_i) \cdot 1$$

$$= \frac{2}{m} (y_i - \bar{y}_i)$$

IF $|y_i - \bar{y}_i| > \delta$,

$$\frac{\delta E}{\delta w_i} = \frac{1}{m} \cdot \delta \cdot \frac{\delta}{\delta w_i} \left(|y_i - \bar{y}_i| - \frac{1}{2}\delta \right)$$

$$= \frac{\delta}{m} \cdot \frac{\delta}{\delta w_i} \left(\sum^{n} w_i x_i + b - y_i - \frac{1}{2}\delta \right)$$

$$= \frac{\delta}{m} \cdot x_i$$

$$\frac{\delta E}{\delta b} = \frac{1}{m} \cdot \delta \cdot \frac{\delta}{\delta b} \left(\sum^{m} w_i x_i + b - y_i - \frac{1}{2}\delta \right)$$

$$= \frac{\delta}{m}$$

So, if $|y_i - \bar{y}_i| > \delta$, $\quad \frac{\delta E}{\delta w_i} = \frac{\delta x_i}{m}$

$$\frac{\delta E}{\delta b} = \frac{\delta}{m}$$

If, $|y_i - \bar{y}_i| > \delta$, $\frac{\delta E}{\delta w_i} = \frac{2}{m}(y_i - \bar{y}_i)x_i$

$$\frac{\delta E}{\delta b} = \frac{2}{m}(y_i - \bar{y}_i)$$