Machine Learning

CSE427

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Inspiring Excellence

Superhumungous Thanks

These slides were typeset by Syeda Ramisa Fariha.

Without her tremendous dedication, these slides would not exist.

Motivation of the No Free Lunch Theorem

- **Motivation:**

  a) Training data can mislead the trainer

  b) Restrict search to some $\mathcal{H}$

- How to choose $\mathcal{H}$?

  $\Rightarrow$ USE PRIOR KNOWLEDGE

- For example, an American baseball scout is scouting cricket players in Bangladesh

Motivation of the No Free Lunch Theorem

- S/he will choose a (*pace*, *accuracy*) rectangle because *pace* and *accuracy* are the most important qualities of a US baseball pitcher.
- S/he is using their prior knowledge about what makes a good pitcher.
- The scout knows from prior experience what features to emphasize.
- But baseball is not the same as cricket.
- In cricket the ball bounces off the ground – so the scout probably won't select the the right $\mathcal{H}$.

Prior Knowledge

- QUESTION: But, is prior knowledge absolutely necessary?
- Is there a *super learner* who can learn just by observing the data?
- Specifically, is there a *learning algorithm* A and *training set* of size m, such that for every distribution D, that is outputs a low risk h?
- If this were true, then a future quantum computer could analyze data and using its unique algorithm, find the right predictor for every problem.
- No specialized knowledge, or intuition would be required.

No Free Lunch Theorem

- The *No Free Lunch Theorem* states that no universal learner exists
- There is a distribution for which the learner fails.
- We will specialize to binary tasks.
- Failure means that: after receiving iid samples from the distribution

  $\implies$ The output hypothesis will have *large risk*.

  $\implies$ Also, there is another learner that will output a low risk hypothesis.
- Thus, we should generally use some *prior knowledge* when faced with a learning problem defined by a distribution D.

No Free Lunch Theorem

- One type of prior knowledge is
  - D comes from a specific parametric family of distributions
  - For example, suppose we want to predict the stock market return for Beximco.
  - Then prior knowledge tells us the that the distribution will be close to Lognormal.
  - This prior knowledge tells us not to consider for example flat distributions.

- Another type of prior knowledge is that there is a hypothesis $h \in \mathcal{H}$ such that, $L_D(h) = \text{small}$.
  - We should therefore try to mimic h.
  - For example, we know Warren Buffet is a good investor.
  - We should therefore try to copy some of his strategies.

The Bias-Complexity Tradeoff

- The error can be decomposed

$$\text{Total Error} = \text{Error in prior knowledge} + \text{Error from overfitting}$$

- Terminology

$$\begin{aligned} \text{Error in prior knowledge} &\equiv \text{approximation error} \\ &\equiv \text{bias} \\ \text{Error from overfitting} &\equiv \text{estimation error} \end{aligned}$$

- There is a tradeoff between *approximation error* and *estimation error*

- Approximation Error Up $\longrightarrow$ Estimation error Down

No Free Lunch Theorem

- **Basic theme:** No learner can succeed on all learning problems without knowing D

Theorem (No Free Lunch)

Let A be any learning problem for the task of binary classification,

Let m be any number smaller than $\frac{|\mathcal{X}|}{2}$. m is the training set size.

Then $\exists$ a distribution D over $X \times 0, 1$ such that,

1. $\exists\ f : X \to \{0, 1\}$ *with* $L_D(f) = 0$
2. *with probability of at least* $\frac{1}{7}$, *over the choice* $S \sim D^m$, *we have* $L_D(A(S)) \geq \frac{1}{8}$

Comments

- Every learner fails on some task that can be successfully learned by another learner

- Trivial successful learner is an *ERM* learner with

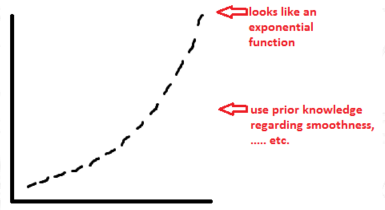
$$\mathcal{H} = \{f,\ \text{other hypotheses}\ \}$$

whose sample size satisfies

$$m \geq \left(\ln \frac{|\mathcal{H}|}{6/7}\right)\left(\frac{1}{1/8}\right)$$

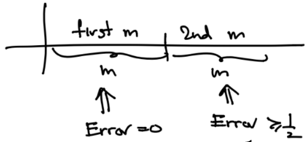
Proof:

- Intuition: any algorithm that observes $\frac{1}{2}$ of the instances in $C \subset X$ has no information on what the labels are for the rest of C.
- $C = subset\ of\ X$, $|C| = 2m$
- **Note:** *Don't assume prior knowledge*
- For example,



No Free Lunch Theorem

- Consider an appropriate example – Coin flipping.
  - ⇒ Flip a coin $2m$ times
  - ⇒ Know the result on the first m tosses
  - ⇒ Can we predict the results of the next m tosses?
- Expected minimal error



$$
\begin{aligned}
\mathbb{E}(\text{error}) &= \frac{1}{2} \times \text{error}(\text{first } m) + \frac{1}{2} \times \text{error}(\text{second } m) \\
&= \frac{1}{2} \times 0 + \frac{1}{2} \times \text{error}(\text{2nd } m) \\
&\geq 0 + \frac{1}{2} \times \frac{1}{2} \\
&= \frac{1}{4}
\end{aligned}
$$

No Free Lunch Theorem

- We can phrase the same argument more mathematically as follows,

$$
\begin{aligned}
\textbf{Error} &= \mathbb{E}_f\left[\underset{S\sim D^m}{\mathbb{E}}\left[\mathrm{Error}(A(S))\right]\right] \\
&= \mathbb{E}_f\left[\underset{S\sim D^m}{\mathbb{E}}\left[L_S(A(S))\right]\right] \\
&= \mathbb{E}_S\left[\mathbb{E}_f\left[\underset{x\sim X}{\mathbb{E}}\left[A(S)(x)\neq f(x)\right]\right]\right] \\
&= \underset{S,x}{\mathbb{E}}\ \underset{f}{\mathbb{E}}\left[A(S)(x)\neq f(x)\ \middle|\ x\in S\right]\mathbb{P}(x\in S) \\
&\quad + \underset{S,x}{\mathbb{E}}\ \underset{f}{\mathbb{E}}\left[A(S)(x)\neq f(x)\ \middle|\ x\notin S\right]\mathbb{P}(x\notin S) \\
&\geq 0+\frac{1}{2}\times\frac{1}{2} \\
&= \frac{1}{4}
\end{aligned}
$$

No Free Lunch Theorem

- Where we used $\mathbb{P}(\chi \notin S) = \frac{1}{2}$ and $\mathbb{E}\Big[A(S)(x) \neq f(x)\Big] \geq \frac{1}{2}$ for all $x \notin S$.

No Free Lunch Theorem

Back to proving NFL theorem:

- S is contained in C
- $|C| = 2m, \ |S| = m$
- There is a target function f that contradicts the labels that $A(S)$ predicts on the unobserved points in C
- f is a sequence since like $\underbrace{010101\ldots01}_{2m}$
- It assigns a 0 or a 1 to all of the $2m$ points of C.
- There are $T = 2^{2m}$ possible functions from $f : C \to \{0, 1\}$ and $\{f\} = \{f_1, f_2, \ldots, f_T\}$

No Free Lunch Theorem

- For each f_i, let D_i be the distribution over $C \times \{0, 1\}$ defined by

$$D_i(x, y) = \begin{cases} \frac{1}{|C|} & \textit{if } y = f(x_i), \quad i \in T, x \in X \\ 0 & \textit{otherwise} \end{cases}$$

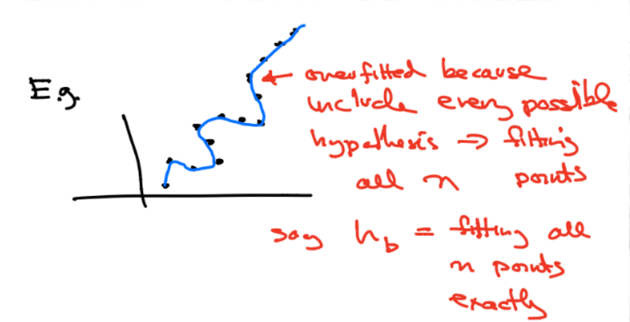
- Here, for each f_i, we are artificially creating a D_i for which f_i is the *true labeling function*
- Thus, the probability of choosing (x, y) is $\frac{1}{|C|}$ if y is the true label and 0 otherwise
- Thus, on C, $L_{D_i}(f_i) = 0$, since by construction f_i is the *true labeling function* for $x \sim D_i$
- Now, **NFL** basically says that, if you include every possible hypothesis (e.g. theory) you don't learn much

The Bias-Complexity Tradeoff

- Learnable problems are problems with good enough hypothesis to fit the problem, but restricted enough to not overfit the sample

- By including so many possible hypotheses → gets lots of *overfitting error*

⇒ That's why there is a *minimum expected error*



No Free Lunch Theorem

- We will show that for every algorithm A,
- receiving a training set of m examples from $C \times \{0,1\}$,
- that returns a function $A(S) : C \to \{0,1\}$
- (suppose that $h_a = A(S)$), it holds that

$$\max_{i \in [T]} \mathop{\mathbb{E}}_{S \in D_i^m} \left[\underbrace{L_{D_i}(A(S))}_{A(S)(x) \neq f_i(x)} \right] \geq \frac{1}{4}$$

- Here f_i is the *true labeling function* for some problem

No Free Lunch Theorem

- Using the fact that there is a problem for which the error is at least 25%, we can show that

$$\mathbb{P}\left[L_D(A'(S)) \geq \frac{1}{8}\right] \geq \frac{1}{7} \tag{1}$$

  This is the the second part of the NFL Theorem.

- This follows from *Markov's Inequality*.

- The basic message is that for every ML algorithm $A \in \mathcal{H}$, there is an ML problem which A does bad on.

Markov Inequality

Markov's inequality:

Suppose Z is a random variable in $[0,1]$ and $\mathbb{E}(Z) = \mu$. Then for any $a \in (0,1)$,

$$\mathbb{P}(Z > 1 - a) \geq \frac{\mu - (1 - a)}{a}$$

Or equivalently,

$$\mathbb{P}(Z > a) \geq \frac{\mu - a}{1 - a} \geq \mu - a$$

Applying the Markov Inequality

- Thus, if for some ML problem, $\mathop{\mathbb{E}}_{S \in D_i^m} \left[\underbrace{L_{D_i}(A(S))}_{A(S)(x) \neq f_i(x)} \right] \geq \frac{1}{4}$

$$
\begin{aligned}
\mathop{\mathbb{P}}_{S \sim D^m}\left[L_D(A(S)) \geq \frac{1}{8}\right] &= \mathop{\mathbb{P}}_{S \sim D^m}\left[L_D(A(S)) \geq (1 - \frac{7}{8})\right] \\
&= \frac{\mathbb{E}(L_D(A(S))) - (1 - \frac{7}{8})}{\frac{7}{8}} \\
&= \frac{\frac{1}{4} - \frac{1}{8}}{\frac{7}{8}} \\
&\geq \frac{\frac{1}{8}}{\frac{7}{8}} \\
&= \frac{1}{7}
\end{aligned}
$$

Applying the Markov Inequality

- This shows that

$$\mathbb{P}\left[L_D(A(S)) \geq \frac{1}{8}\right] \geq \frac{1}{7} \tag{2}$$

Aside: Markov's Inequality

- The expected value of the non-negative random variable Z can be written as,

$$\begin{aligned}
\mathbb{E}(Z) &= \int_0^1 z\,\mathbb{P}(Z=z)dz \\
&= \int_0^a z\,\mathbb{P}(Z=z)dz + \int_a^1 z\,\mathbb{P}(Z=z)dz \\
&\geq \int_a^1 z\,\mathbb{P}(Z=z)dz \\
&\geq a\int_a^1 \mathbb{P}(Z=z)dz \\
&= a\mathbb{P}(Z\geq a)
\end{aligned}$$

- Therefore

$$\mathbb{P}(Z\geq a)\leq \frac{1}{a}\,\mathbb{E}(Z)$$

Aside: Reverse Markov Inequality

Reverse Markov Inequality:

Let $Y = 1 - Z$. Since $0 \leq Z \leq 1$, we have $0 \leq Y \leq 1$.

Then,

$$\mathbb{E}(Y) = \mathbb{E}(1 - Z) = 1 - \mathbb{E}(Z) = 1 - \mu$$

Applying Markov's inequality to Y.

$$\mathbb{P}(Z \leq 1 - a) = \mathbb{P}(1 - Z \geq a) = \mathbb{P}(Y \geq a) \leq \frac{\mathbb{E}(y)}{a} = \frac{1 - \mu}{a}$$

Thus,

$$\mathbb{P}(Z > 1 - a) > 1 - \frac{1 - \mu}{a} = \frac{\mu - (1 - a)}{a}$$

Back to the No Free Lunch Theorem

- Recall that f_i is the true labeling function given that the distribution is D_i
- If the distribution is D_i, then the possible training sets that can be given to the algorithm are

$$\{S_1^i, S_2^i, \ldots, S_k^i\}$$

- How many training sets of size m are there?

$$\text{Number of training sets} = (2m)^m$$

Back to the No Free Lunch Theorem

- A training set where the true labeling function is f_i is denoted by

$$S_j^i = \{x_1, \ldots, x_m\}$$

- For example, suppose x_i is the result of the ith coin toss.

- For example, suppose the 12th training set can, where the true label is f_i, might be denoted by

$$S_{12}^i = \{x_1, x_{16}, x_{22}, \ldots, x_{64}\}$$

- Thus S_{12}^i consists of the first coin toss, the 16th coin toss, the 22nd coin toss, ..., the 64th coin toss.

Back to the No Free Lunch Theorem

- All of the training sets have the same probability of being sampled.

$$\mathop{\mathbb{E}}_{S \sim D_i^m}\left[L_{D_i}(A(S))\right] = \frac{1}{k}\sum_{i=1}^{k} L_{D_i}(A(S_j^i))$$

- Thus

$$\max_{i \in [T]}\left[\frac{1}{k}\sum_{j=1}^{k} L_{D_i}(A(S_j^i))\right] = \max_{i \in [T]}\left[\mathop{\mathbb{E}}_{S \sim D^m}\left(L_{D_i}(A(S^i))\right)\right]$$

- Why are we taking $\max_{i \in [T]}$?

Back to the No Free Lunch Theorem

- The idea is:

- We are looking for a distribution D_i for which $\mathbb{E}[L_{D_i}(A(S))]$ is largest.

- I.e, we are looking for a machine learning problem for which our predictor $A(S)$ gives the largest expected error.

Back to the No Free Lunch Theorem

- All we are trying to show is that there exists at least 1 problem for which our universal algorithm $A(S)$ fails.
- Note: MAX OF SETS $\geq$ AVERAGE OF SETS $\geq$ MINIMUM OF SETS
- Therefore,

$$\begin{aligned}
\max_{i\in[T]} \frac{1}{k}\sum_{j=1}^{k} L_{D_i}(A(S_j^i)) &\geq \underbrace{\frac{1}{T}\sum_{i=1}^{T}}_{\text{average}} \frac{1}{k}\sum_{j=1}^{k} L_{D_i}(A(S_j^i)) \\
&= \frac{1}{k}\sum_{j=1}^{k}\frac{1}{T}\sum_{i=1}^{T} L_{D_i}(A(S_j^i)) \\
&\geq \min_{j\in[k]} \frac{1}{T}\sum_{i=1}^{T} L_{D_i}(A(S_j^i))
\end{aligned}$$

Back to the No Free Lunch Theorem

- Here, we have fixed the $j \in [k]$ that gives the minimum of $\mathbb{E}_f(L_D(A(S))$
- Call $S_j = (x_1, \ldots, x_m)$
- $v_1, \ldots, v_p$ are examples/instances in C that don't appear in S_j
- $p \geq m$, since $|C| = 2m, |S| = m$.
- For example, if there are repetitions in S_j, such that $S_j = (x_1, \ldots, x_1)$, then $p > m$

Back to the No Free Lunch Theorem

- For every $h \in \mathcal{H}$

$$
\begin{aligned}
L_{D_i}(h) &= \frac{1}{2m} \sum_{x \in C} \mathbb{1}\Big(h(x) \neq f_i(x)\Big) && \mathbb{1} = \text{Indicator function} \\
&\geq \frac{1}{2m} \sum_{r=1}^{p} \mathbb{1}\Big(h(v_r) \neq f_i(v_r)\Big) && \text{Because less points in } v_1, \ldots, v_p \\
&\geq \frac{1}{2p} \sum_{r=1}^{p} \mathbb{1}\Big(h(v_r) \neq f_i(v_r)\Big) && \text{since } p \geq m
\end{aligned}
$$

Back to the No Free Lunch Theorem

- Thus,

$$
\begin{aligned}
&\frac{1}{T}\sum_{i=1}^{T} L_{D_i}(A(S_j^i)) \\
&\geq \frac{1}{T}\sum_{i=1}^{T}\frac{1}{2p}\sum_{r=1}^{p}\mathbb{1}\Big(A(S_j^i)(v_r) \neq f_i(v_r)\Big), \text{ where } h(v_r) \equiv A(S_j^i)(v_r) \\
&= \frac{1}{2p}\sum_{r=1}^{p}\frac{1}{T}\sum_{i=1}^{T}\mathbb{1}\Big(A(S_j^i)(v_r) \neq f_i(v_r)\Big) \\
&\geq \frac{1}{2}\min_{r\in[p]}\frac{1}{T}\sum_{i=1}^{T}\mathbb{1}\Big(A(S_j^i)(v_r) \neq f_i(v_r)\Big) \text{ choosing the minimum term}
\end{aligned}
$$

Back to the No Free Lunch Theorem

- As before, fix the r that gives minimum contribution

- Now, partition the different hypothesis functions f_i into disjoint pairs $(f_i, f_{i'})$.

- Choose the partitioning to satisfy,

  1. for every $c \in C, f_i(c)$ and $f_i(C)$ are different if and only if c is outside of the training set.

  2. This means the f_i and $f_{i'}$ agree on the training set. Thus, $S_j^i = S_j^{i'}$ (i.e. the y values for $(x_1, \ldots, x_m)$ are the same. For example, $f_i(x_1) = f_{i'}(x_2), f_i(x_2) = f_{i'}(x_2), \ldots$ etc.

Back to the No Free Lunch Theorem

- This implies that if the condition $A(S_j^i)(v_r) \neq f_i(v_r)$ holds,

  then, if $A(S_j^i)(v_r) = 1, \implies f_i(v_r) = 0,$

  and the condition $A(S_j^i)(v_r) \neq f_i(v_r)$ will hold, since, then $f_i(v_r) = 0.$

- If $A(S_j^i)(v_r) \neq f_i(v_r)$ doesn't hold, then $A(S_j^i)(v_r) \neq f_{i'}(v_r)$

- Thus,

$$\mathbb{1}\Big(A(S_j^i)(v_r) \neq f_i(v_r)\Big) + \mathbb{1}\Big(A(S_j^i)(v_r) \neq f_{i'}(v_r)\Big) = 1$$

Back to the No Free Lunch Theorem

- Since, $S_j^i = S_j^{i'}$

$$\mathbb{1}\left(A(S_j^i)(v_r) \neq f_i(v_r)\right) + \mathbb{1}\left(A(S_j^{i'})(v_r) \neq f_{i'}(v_r)\right) = 1$$

- Averaging over all the functions $f \in \mathcal{H}$

$$\frac{1}{T}\sum_{i} \mathbb{1}\left(A(S_j^i)(v_r) \neq f_i(v_r)\right) = \frac{1}{T}\sum_{i'} \mathbb{1}\left(A(S_j^i)(v_r) \neq f_{i'}(v_r)\right)$$

- Thus,

$$\frac{1}{T}\sum_{i=1}^{T} \mathbb{1}\left(A(S_j^i)(v_r) \neq f_i(v_r)\right) = \frac{1}{2}$$

Back to the No Free Lunch Theorem

- Substituting everything in,

$$\begin{aligned}
\max_{i\in[T]} \left[\mathop{\mathbb{E}}_{S\sim D^m} L_{D_i}(A(S)) \right] &\geq \min_{j\in[k]} \frac{1}{T} \sum_{i=1}^{T} L_{D_i}(A(S_j^i)) \\
&\geq \frac{1}{2} \min_{r\in[p]} \frac{1}{T} \sum_{i=1}^{T} \mathbb{1}\Big(A(S_j^i)(v_r) \neq f_i(v_r) \Big) \\
&\geq \frac{1}{2} \times \frac{1}{2} \\
&= \frac{1}{4}
\end{aligned}$$

- Thus our predictor $h_a = A(S_j^i)$ which does well on D_a fails on D_i.

Consequences of the NFL Theorem

- Suppose, we have no *prior knowledge* for a binary prediction problem.
- Then, we should consider all possible hypotheses $h_i, \ i \in [T]$ where $T = 2^{\text{number of points}}$
- Every possible $h_i \in \mathcal{H}$ is then a possible best predictor for our problem
- A procedure such as ERM will output one of the h in $\mathcal{H}$ as our predictor.
- The NFL theorem says that, our hypothesis (such as the ERM predictor) will fail on some machine learning task
- Thus, $\mathcal{H}$ is not PAC Learnable.

Infinite Domain Sets

Corollary: Let X be an infinite domain set and let $\mathcal{H}$ be the set of all functions from X to $\{0,1\}$. Then $\mathcal{H}$ is not PAC learnable.

Proof:

- Use proof by contradiction. Assume that $\mathcal{H}$ is learnable
- Choose for example, $\epsilon = \frac{1}{8}$ and $\delta = \frac{1}{7}$.
- By definition of PAC learnability, $\exists$ an algorithm A and $m(\epsilon, \delta)$ such that for D over $X \times \{0,1\}$, we have $L_D(A(S)) \leq \epsilon$ with probability $1 - \delta$.

  (Suppose, we assume realizability, $\exists f : X \to \{0,1\}$ such that $L_D(f) = 0$)

Infinite Domain Sets

- NFL states that since $|X| \geq 2m$, for every algorithm A, there is a distribution D, such that with probability more than $\delta = \frac{1}{7}$ that, $L_D(A(S)) > \frac{1}{8} = \epsilon$.

- CONTRADICTION: $\mathcal{H}$ is not PAC Learnable.

- To prevent this failure,

  $\Rightarrow$ Use a prior in terms of conditional probability.

  $\Rightarrow$ Use prior knowledge.

- This will help us avoid distributions that will cause us to fail

- Impose prior knowledge by restricting the hypothesis class $\mathcal{H}$

$$\mathbb{P}(h \mid \text{prior knowledge}) = \frac{\mathbb{P}(h \cap \text{prior knowledge})}{\mathbb{P}(\text{prior knowledge})}$$

How To Choose A Good Hypothesis Class?

- To choose a good hypothesis class: include enough hypothesis such that it includes the hypothesis with no error (in *PAC* context) or small error (in *agnostic PAC* context)

- But including the richest $\mathcal{H}$, which contains all possible hypothesis leads to failure $\rightarrow$ as just seen

- This leads to a trade-off (bigger/smaller $\mathcal{H}$).

The Bias-Complexity Tradeoff

- Decompose the error of an $ERM_{\mathcal{H}}$ predictor as follows

$$L_D(h_S) = \epsilon_{app} + \epsilon_{est}$$

where

$$\begin{aligned}
\epsilon_{app} &= \text{approximation error} \\
\epsilon_{est} &= \text{estimation error}
\end{aligned}$$

and

$$\begin{aligned}
\epsilon_{app} &= \min_{h \in \mathcal{H}} L_D(h) \\
\epsilon_{est} &= L_D(h_S) - \epsilon_{app}
\end{aligned}$$

- Recall for the agnostic case,

$$L_D(h) \leq \min_{h \in \mathcal{H}} L_D(h) \; + \; \epsilon$$

Here,

$$\begin{aligned} \min_{h \in \mathcal{H}} L_D(h) &= \epsilon_{app} \\ \epsilon &= \epsilon_{est} \end{aligned}$$

Approximation Error (Inductive bias)

- Risk comes from restricting to $\mathcal{H}$

- Doesn't depend on sample size, it is determined by $\mathcal{H}$

- Making $\mathcal{H}$ bigger can make ϵ_{app} smaller

- Under *Realizability*, $\epsilon_{app} = 0$

Estimation Error

- This is also known as *Empirical Error* or *Training Error*.

- Amount of ϵ_{est} depends on $|S|$

$$\epsilon_{est} \sim \frac{1}{m}$$

- For finite $\mathcal{H}$, ϵ_{est} depends on $|\mathcal{H}|$'s complexity

$$\epsilon_{est} \sim \log |\mathcal{H}|$$

Tradeoff between Estimation and Approximation Error

- Since,

$$\epsilon_{est} \sim -\epsilon_{app}$$

and

$$\begin{aligned} \epsilon_{est} &= \text{complexity} \\ \epsilon_{app} &= \text{bias} \end{aligned}$$

$\implies$ This gives us a

$$\boxed{\textit{Bias-Complexity Tradeoff Issue}}$$

Tradeoff between Estimation and Approximation Error

- $|\mathcal{H}|$ large $\rightarrow \epsilon_{app} \downarrow$, but $\epsilon_{est} \uparrow$, because of *overfitting*

- $|\mathcal{H}|$ small $\rightarrow \epsilon_{app} \uparrow$, but $\epsilon_{est} \downarrow$, because of *underfitting*

- Suppose, $\mathcal{H} = \{h\}$, h $=$ *Bayes' optimal classifier*

  This is a good choice, but this classifier depends on knowing the unknown distribution D

- Goal of *learning Theory:* Make $\mathcal{H}$ as rich as possible while keeping *estimation error* small

$\Rightarrow$ Design good hypothesis classes for which ϵ_{app} not large

Tackling unfamiliar problems

- When facing an unfamiliar problem
  - ⇒ Don't know the optimal classifier or how to construct it
  - ⇒ Do have some prior knowledge
  - ⇒ This enables us to design hypothesis classes with ϵ_{app} and ϵ_{est} not too high
- For example, when looking for life in the universe
  - ⇒ No idea what aliens are made up of, or what kind of environments they might live in
  - ⇒ Use *prior knowledge* that all known life needs *oxygen* and *water*
  - ⇒ Look for water/oxygen rich planets