(i) Let, $f(x) = e^x$

$\therefore f(x+h) = e^{x+h}$

From the definition of derivatives, $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$\begin{aligned}
\therefore f'(x) &= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} \\
&= \lim_{h \to 0} \frac{e^x(e^h - 1)}{h} \\
&= e^x \cdot \lim_{h \to 0} \frac{1}{h} \left\{ \left(1 + \frac{h}{1!} + \frac{h^2}{2!} + \frac{h^3}{3!} + \dots \right) - 1 \right\} \\
&= e^x \cdot \lim_{h \to 0} \left(1 + \frac{h}{2!} + \frac{h^2}{3!} + \dots \right) \\
&= e^x
\end{aligned}$$

(ii) Let, $f(x) = a^x$

$$\begin{aligned}
\therefore f'(x) &= \lim_{h \to 0} \frac{a^{x+h} - a^x}{h} \\
&= \lim_{h \to 0} \frac{a^x(a^h - 1)}{h} \\
&= a^x \cdot \lim_{h \to 0} \frac{1}{h} \left\{ \left(1 + \frac{h}{1!} \ln a + \frac{h^2}{2!} (\ln a)^2 + \dots \right) - 1 \right\} \\
&= a^x \cdot \lim_{h \to 0} \left(\ln a + \frac{h}{2!} (\ln a)^2 + \dots \right) \\
&= a^x \ln a
\end{aligned}$$

(iii) Let, $f(x) = \ln x$

$$\begin{aligned}
\therefore f'(x) &= \lim_{h \to 0} \frac{\ln(x+h) - \ln x}{h} \\
&= \lim_{h \to 0} \frac{\ln \frac{x+h}{x}}{h} \\
&= \lim_{h \to 0} \frac{1}{h} \ln \left(1 + \frac{h}{x} \right) \\
&= \lim_{h \to 0} \frac{1}{h} \left(\frac{h}{x} - \frac{1}{2} \cdot \frac{h^2}{x^2} + \frac{1}{3} \cdot \frac{h^3}{x^3} - \dots \right) \\
&= \lim_{h \to 0} \left(\frac{1}{x} - \frac{h}{2x^2} + \frac{h^2}{3x^3} - \dots \right) \\
&= \frac{1}{x}
\end{aligned}$$

(iv) Let, $f(x) = \sin x$

$$\begin{aligned}
\therefore f'(x) &= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} \\
&= \lim_{h \to 0} \frac{2 \cos \left(x + \frac{h}{2} \right) \sin \frac{h}{2}}{h} \\
&= \lim_{h \to 0} \left\{ \cos \left(x + \frac{h}{2} \right) \cdot \frac{\sin \frac{h}{2}}{\frac{h}{2}} \right\} \\
&= \lim_{h \to 0} \left\{ \cos \left(x + \frac{h}{2} \right) \right\} \cdot \lim_{\frac{h}{2} \to 0} \left\{ \frac{\sin \frac{h}{2}}{\frac{h}{2}} \right\} \\
&= \cos x \cdot 1 \\
&= \cos x
\end{aligned}$$