Higher Mathematics Model Test

Time: 2 hours 35 minutes **Marks: 50**

Instructions:

- Answer **five** questions from the Creative Essay-Type section, taking at least **one** from each part.

- Answer **any five** questions from the Short Answer section.

Creative Essay-Type Questions

Part A: Algebra

1. Let $A = (1 - 2x + x^2)^3$ and $B = \left(k + \dfrac{3}{x}\right)^5$.

   (a) Expand $(2 - 3x)^4$ using Pascal's triangle.

   (b) Expand A using the binomial theorem.

   (c) Given that the coefficient of $\dfrac{1}{x^4}$ in B is -810, find the value of k.

2. Let $P(x) = x^4 + 7x^3 + 17x^2 + 17x + 6$ and $Q(x, y, z) = \dfrac{1}{x^3} + \dfrac{1}{8y^3} + \dfrac{1}{64z^3}$.

   (a) Show that $(x + 1)$ is a factor of $F(x) = x^3 - x^2 + 5x + 7$.

   (b) Factorize $P(x)$.

   (c) If $Q(x, y, z) = \dfrac{3}{8xyz}$, show that $4yz + 2zx + xy = 0$ or $x = 2y = 4z$.

3. Let $P = a^3(b - c) + b^3(c - a) + c^3(a - b)$ and $Q = \dfrac{x}{(x - 1)(x^2 + 4)}$.

   (a) Show that $F(x, y, z) = xy + yz + zx$ is a symmetric expression.

   (b) Factorize P.

   (c) Decompose Q into partial fractions.

Part B: Geometry

4. Consider the lines:

$$
\begin{array}{lll}
\text{(i)} & y = 3x - 10, & \text{intersects the } x\text{-axis at point } A; \\
\text{(ii)} & 2x - y = 4, & \text{intersects the } y\text{-axis at point } B; \\
\text{(iii)} & x - 2y + 10 = 0. &
\end{array}
$$

Lines (i) and (ii) intersect at point C.

   (a) Find the slope of line (iii).

   (b) Show that lines (i), (ii), and (iii) are concurrent.

(c) Calculate the area of triangle $\triangle ABC$.

5. Let $A = (-4, 13)$, $B = (8, 8)$, $C = (13, -4)$, and $D = (1, 1)$ be the vertices of quadrilateral $ABCD$.

   (a) Find the angle between line BD and the x-axis.
   (b) Determine the nature of the quadrilateral $ABCD$.
   (c) Find the area of the portion of quadrilateral $ABCD$ that forms a triangle with the x-axis.

Part C: Trigonometry and Probability

6. Let $a = \cot\theta$ and $b = \operatorname{cosec}\theta$.

   (a) If $4\theta = \pi$, find the value of $a^2 - b$.
   (b) Given $a + b = x$, show that $\cos\theta = \dfrac{x^2 - 1}{x^2 + 1}$.
   (c) If $3(a^2 + b^2) = 5$, find the values of θ where $0 < \theta < 2\pi$.

7. The probability of Rahim traveling from Dhaka to Chattogram by bus is $\dfrac{3}{5}$, and by train is $\dfrac{1}{5}$. The probability of traveling from Chattogram to Cox's Bazar by bus is $\dfrac{6}{13}$, and by airplane is $\dfrac{3}{26}$.

   (a) What is the probability of getting a prime number when rolling a standard dice?
   (b) Draw a probability tree diagram for the above travel scenario.
   (c) Find the probability that Rahim travels to Chattogram **not** by train and then travels to Cox's Bazar by **neither** bus nor airplane.

Short Answer Questions

Answer any five questions.

1. Find the coefficient of x^2 in the expansion of $(1 - x)^5$.

2. Find the remainder when $x^3 - 2x^2 + x - 7$ is divided by $x + 3$.

3. Decompose $\dfrac{1}{x^2 + 5x + 6}$ into partial fractions.

4. Given a point $P(x, y)$, the distance to the y-axis is equal to the distance from P to the point $Q(3, 2)$. Prove that $y^2 - 4y - 6x + 13 = 0$.

5. Show that if $A(a, b)$, $B(b, a)$, and $C\left(\dfrac{1}{a}, \dfrac{1}{b}\right)$ are collinear, then $a + b = 0$.

6. Prove that
$$\cos\left(\frac{17\pi}{10}\right) + \cos\left(\frac{13\pi}{10}\right) + \cos\left(\frac{9\pi}{10}\right) + \cos\left(\frac{\pi}{10}\right) = 0.$$

7. In a lottery of 500 tickets with 30 prizes, all tickets are sold. Rahim owns 5 tickets. Prizes are drawn in reverse order—from the 30th to the 1st. What is the probability that Rahim wins the 3rd prize, given that none of his tickets won any of the 27 previous prizes?