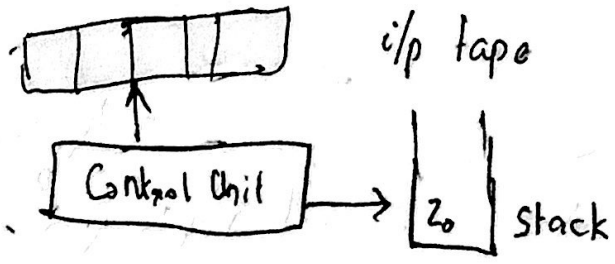
MODULE - IV

Push Down - Automata

→ NFA with stack

- CFG are recognized by PDA



7 tuple

$(Q, \Sigma, \delta, q_0, F, Z_0, N)$

Q – Set of states

Σ - Set of i/p symbols

δ - transition fn

Z_0 - initial symbol on stack

q_0 - Initial state

F - Set of final states

N → Elements of stack

Instantaneous Description (ID)

- Current configuration of PDA at any given time.
- 3 tuple (q, w, α)
- Move

$(q_0, abcba, Z_0) \vdash (q_0, bcba, aZ_0)$

Acceptance

By empty stack

$\{w \mid (q_0, w, z_0) \vdash (p, \varepsilon, \varepsilon)$ p not final

By final state

$\{w \mid (q_0, w, z_0) \vdash (p, \varepsilon, \alpha)$ $\alpha \in \gamma^*$

p - Final State.

Deterministic & Non-Deterministic PDA

PDA is deterministic if

$\delta(q, a, z)$ has only one element

If $\delta(q, \varepsilon, z)$ is not empty then $\delta(q, a, z)$ should be empty.

i.e, If there is a ε-transition, then, there should not be any transition from q, when top of stack is z

$\delta(q_0, \varepsilon, z_0) = q_1, z_0$

$\delta(q_0, a, z_0) = q_0, a z_0$

$\delta(q_0, b, z_0) = q_0, b z_0$

eg: Design a PDA to accept $L = \{a^n b^n \mid n \geq 1\}$

$\delta(q_0, a, z_0) = q_0, a z_0$

$\delta(q_0, a, a) = q_0, aa \rightarrow$ push

$\delta(q_0, b, a) = q_1, \varepsilon \rightarrow$ pop

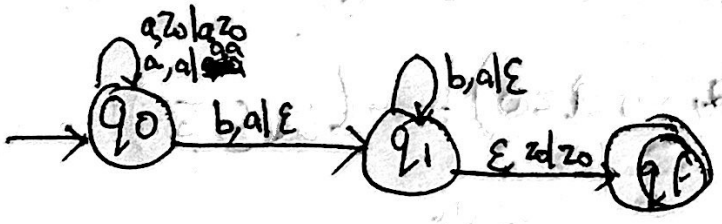
$\delta(q_1, b, a) = q_1, \varepsilon$

$\delta(q_1, \varepsilon, z_0) = q_f, z_0$ or $\delta(q_1, \varepsilon, z_0) = q_1, \varepsilon$

Move

$(q_0, aabb, z_0) \vdash (q_0, abb, a z_0) \vdash (q_0, bb, aa z_0) \vdash$

$(q_1, b, a z_0) \vdash (q_1, \varepsilon, z_0) \vdash (q_f, \varepsilon, z_0)$



read $\delta(q_0, b, a) = q_1, a$

Q. Design a PDA for $a^n b c^n$

$$\delta(q_0, a, z_0) = q_0, a z_0$$

$$\delta(q_0, a, a) = q_0, aa$$

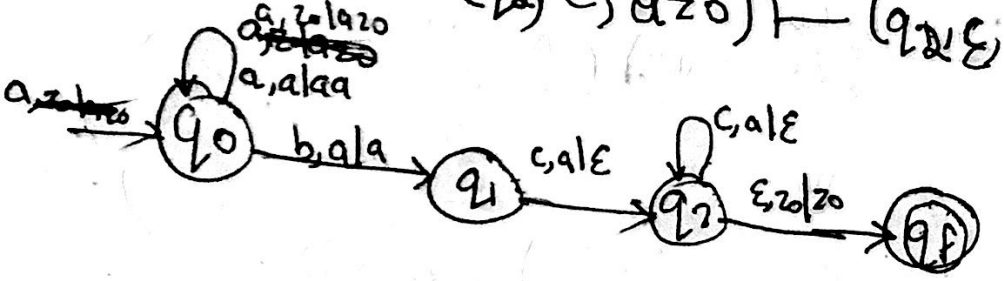
$$\delta(q_0, b, a) = q_1, a$$

$$\delta(q_1, c, a) = q_2, \varepsilon$$

$$\delta(q_2, c, a) = q_2, \varepsilon$$

$$\delta(q_2, \varepsilon, z_0) = q_f, z_0$$

$(q_0, aabcc, z_0) \vdash (q_0, abcc, a z_0) \vdash (q_0, bcc, aa z_0) \vdash$
$(q_1, cc, aa z_0) \vdash (q_2, c, a z_0) \vdash (q_2, \varepsilon, z_0) \vdash (q_f, \varepsilon, z_0)$



Q.

Dec 2017

Q. Design a PDA for $L = \{a^n b^{2n} \mid n > 0\}$

abb
aabbbb

$\delta(q_0, a, z_0) = q_0, a z_0$

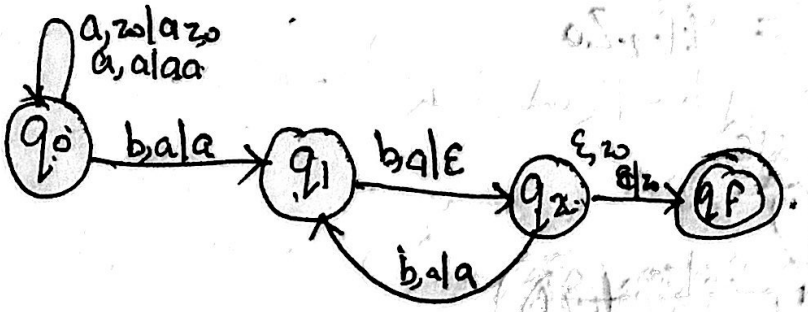
$\delta(q_0, a, a) = q_0, aa$

$\delta(q_0, b, a) = q_1, a$

$\delta(q_1, b, a) = q_2, \varepsilon$

$\delta(q_2, b, a) = q_1, a$

$\delta(q_2, \varepsilon, z_0) = q_f, z_0$



$(q_0, aabbbb, z_0) \vdash (q_0, abbbb, az_0) \vdash (q_0, bbbb, aaz_0) \vdash$
$(q_1, bbb, aaz_0) \vdash (q_2, bb, az_0) \vdash (q_1, b, az_0) \vdash (q_2, \varepsilon, z_0)$
$\vdash (q_f, \varepsilon, z_0)$

Q. $a^n b^m c^{(n+m)}$ abbccc

$\delta(q_0, a, z_0) = q_0, a z_0$

$\delta(q_0, a, a) = q_0, aa$

$\delta(q_0, b, a) = q_0, ba$

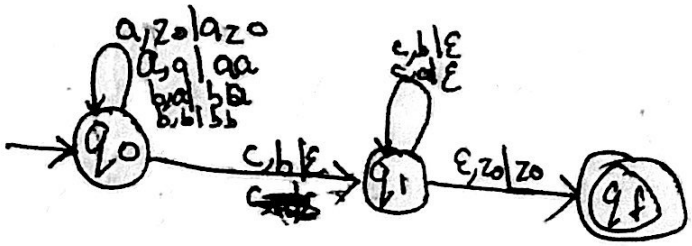
$\delta(q_0, b, b) = q_0, bb$

$\delta(q_0, c, b) = q_1, \varepsilon$

$\delta(q_1, c, b) = q_1, \varepsilon$

$\delta(q_1, c, a) = q_1, \varepsilon$

$\delta(q_1, \varepsilon, z_0) = q_f, z_0$



$(q_0, abbccc, z_0) \vdash (q_0, bbccc, az_0) \vdash (q_0, bccc, baz_0) \vdash$

$(q_0, ccc, bbaz_0) \vdash (q_1, cc, baz_0) \vdash (q_1, c, az_0) \vdash$

$(q_1, \varepsilon, z_0) \vdash (q_f, \varepsilon, z_0)$

DEC 2018

Q. Construct PDA for $(0^n1^n)^*$

$\delta(q_0, 0, Z_0) = q_0, 0Z_0$

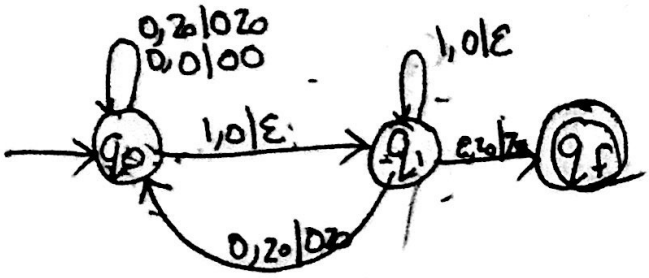
$\delta(q_0, 0, 0) = q_0, 00$

$\delta(q_0, 1, 0) = q_1, \varepsilon$

$\delta(q_1, 1, 0) = q_1, \varepsilon$

$\delta(q_1, 0, Z_0) = q_0, 0Z_0$

$\delta(q_1, \varepsilon, Z_0) = q_f, Z_0$



$(q_0, 0101, Z_0) \vdash (q_0, 101, 0Z_0) \vdash (q_1, 01, Z_0) \vdash (q_0, 1, 0Z_0) \vdash$

$(q_1, \varepsilon, Z_0) \vdash (q_f, \varepsilon, Z_0)$

Q. Construct PDA wcw^R, consisting 0's & 1's.

01C10

0011C1100

$\delta(q_0, 0, z_0) = q_0, 0z_0$

$\delta(q_0, 1, z_0) = q_0, 1z_0$

$\delta(q_0, 0, 0) = q_0, 00$

$\delta(q_0, 1, 0) = q_0, 10$

$\delta(q_0, 0, 1) = q_0, 01$

$\delta(q_0, 1, 1) = q_0, 11$

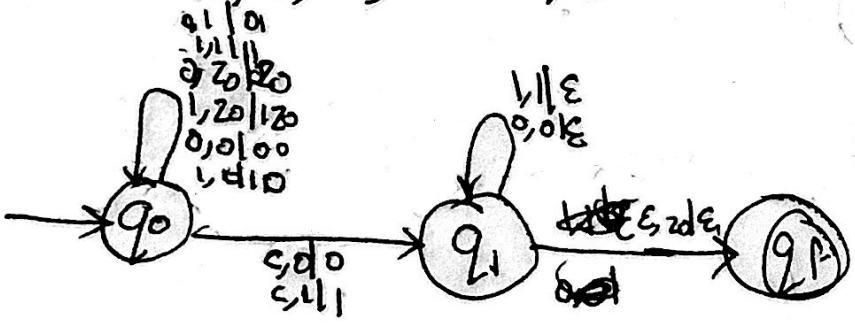
$\delta(q_0, c, 0) = q_1, 0$

$\delta(q_0, c, 1) = q_1, 1$

$\delta(q_1, 1, 1) = q_1, \varepsilon$

$\delta(q_1, 0, 0) = q_1, \varepsilon$

$\delta(q_1, \varepsilon, z_0) = q_f, \varepsilon$



Q. $N_a(\omega) = N_b(\omega)$

$\delta(q_0, a, z_0) = q_0, az_0$

$\delta(q_0, b, z_0) = q_0, bz_0$

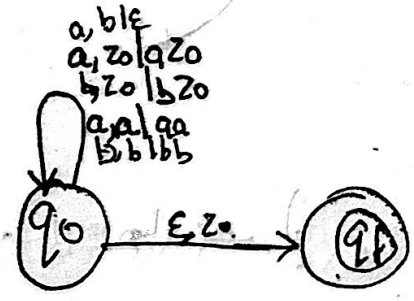
$\delta(q_0, a, a) = q_0, aa$

$\delta(q_0, b, a) = q_0, \varepsilon$

$\delta(q_0, b, b) = q_0, bb$

$\delta(q_0, a, b) = q_0, \varepsilon$

$\delta(q_0, \varepsilon, z_0) = q_f, z_0$



Q. $0^n 1^n 0^m 1^n$

$\delta(q_0, 0, z_0) = q_0, 0z_0$

$\delta(q_0, 0, 0) = q_0, 00$

$\delta(q_0, 1, 0) = q_0, 10$

$\delta(q_0, 1, 1) = q_0, 11$

$\delta(q_0, 0, 1) = q_1, \varepsilon$

~~$\delta(q_1, 0, 1) = q_1, \varepsilon$~~

~~$\delta(q_1, 0, 0) = q_1, \varepsilon$~~

~~$\delta(q_0, 1, 0) =$~~

$\delta(q_1, 1, 0) = q_1, \varepsilon$

$\delta(q_1, 1, 1) = q_1, \varepsilon$

$\delta(q_1, \varepsilon, z_0) = q_f, z_0$

CFG to PDA

Q.

(i) Convert grammar to GNF

(ii) Let q_0 be start state & z_0 initial symbol on stack without consuming i/p, push S on to stack & state change to q_1.

$$\delta(q_0, \varepsilon, z_0) = q_1, S z_0$$

(iii) For each production of the form

$$A \to a\alpha$$

Introduce transition

$$\delta(q_1, a, A) = q_1, \alpha$$

(iv) Finally in state q_1, without consuming input change state to q_f.

$$\delta(q_1, \varepsilon, z_0) = q_f, z$$

Design PDA

Q. $S \to cABC$

$A \to aB \mid a$

$B \to bA \mid b$

$C \to a$

$B \to b$

$\delta(q_0, \varepsilon, z_0) = q_1, S z_0$

$\delta(q_1, c, S) = q_1, ABC$

~~Design PDA~~ $\delta(q_1, a, A) = q_1, B$

$\delta(q_1, a, A) = q_1, \varepsilon$

$\delta(q_1, b, B) = q_1, A$

$\delta(q_1, b, B) = q_1, \varepsilon$

$\delta(q_1, \varepsilon, z_0) = q_f$

$\delta(q_1, a, C) = q_1, \varepsilon$

$\delta(q_1, b, D) = q_1, \varepsilon$

Move

$S \to aABC$
$aaBC$
$aabC$
$aaba$

$(q_0, aaba, z_0) \vdash (q_1, aaba, Sz_0) \vdash (q_1, aba, ABCz_0) \vdash$
$(q_1, ba, BCz_0) \vdash (q_1, a, Cz_0) \vdash (q_1, \varepsilon, z_0) \vdash$
(q_f, ε, z_0)

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a. $S \to aABB | aAA$
$A \to aBB | a$
$B \to bBB | aA$
$bBB | aBB | a$

$\delta(q_0, \varepsilon, z_0) = q_1, Sz_0$
$\delta(q_1, a, S) = q_1, ABB$
$\delta(q_1, a, S) = q_1, AA$
$\delta(q_1, a, A) = q_1, BB$
$\delta(q_1, a, A) = q_1, \varepsilon$
$\delta(q_1, b, B) = q_1, BB$
$\delta(q_1, a, B) = q_1, BB$
$\delta(q_1, a, B) = q_1, \varepsilon$

$\delta(q_1, \varepsilon, z_0) = q_f, z_0$