Chapter-01

1.1

a) $2 \times 2 \times 2 \times 2 = 16 = 2^4$

b) $2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

1.2 a) ~~for~~ $6^4 = 1296$

b) $6^5 = 7776$

1.3 a) 1.

1.4

a) 5 coins. $[\because 2^5 = 32]$

b) 2 dice. $[\because 6^2 = 36]$.

1.5

a) 10111

$1 \times 2^0 = 1$
$1 \times 2^1 = 2$
$1 \times 2^2 = 4$
$0 \times 2^3 = 0$
$1 \times 2^4 = 16$

23

$(10111)_2 = (23)_{10}$

b) 11001010

$0 \times 2^0 = 0$
$1 \times 2^1 = 2$
$0 \times 2^2 = 0$
$1 \times 2^3 = 8$
$0 \times 2^4 = 0$
$0 \times 2^5 = 0$
$1 \times 2^6 = 64$
$1 \times 2^7 = 128$

202

$\therefore (11001010)_2 = (202)_{10}$

1.6.

a)

$2 \,|\, 42$
$2 \,|\, 21 - 0$
$2 \,|\, 10 - 1$
$2 \,|\, 5 - 0$
$2 \,|\, 2 - 1$
$2 \,|\, 1 - 0$
$0 - 1$

$(42)_{10} = (101010)_2$

b)

$2 \,|\, 495$
$2 \,|\, 247 - 1$
$2 \,|\, 123 - 1$
$2 \,|\, 61 - 1$
$2 \,|\, 30 - 1$
$2 \,|\, 15 - 0$
$2 \,|\, 7 - 1$
$2 \,|\, 3 - 1$
$2 \,|\, 1 - 1$
$0 - 1$

$\therefore (495)_{10} = (111101111)_2$

1.7.

a) 3B7C

$12 \times 16^0 = 12$

$7 \times 16^1 = 112$

$11 \times 16^2 = 2816$

$3 \times 16^3 = 12288$

15228

$(3B7C)_{16} = (15228)_{10}$

b) FF

1111 1111

$(FF)_{16} = (11111111)_2$

c) FA

$10 \times 16^0 = 10$

$15 \times 16^1 = 240$

250

$(FA)_{16} = (250)_{10}$

$\cancel{(10)_{16} =}$

10

$0 \times 16^0 = 0$

$1 \times 16^1 = 16$

16

$(10)_{16} = (16)_{10}$

E4

$4 \times 16^0 = 4$

$14 \times 16^1 = 224$

228

$(E4)_{16} = (228)_{10}$

1.8

Binary		Decimal
000	→	0
001	→	1
010	→	2
011	→	3
100	→	-4
101	→	-5
110	→	-6
111	→	-7

1.9

First Name: Naila

ASCII string:

1001110 1100001 1101001 1101100 110000

1.10

Quantum

1.11

a)

A	B	$\bar{A}+\bar{B}$
0	0	1
0	1	1
1	0	1
1	1	0

b) NAND gate.

1.12

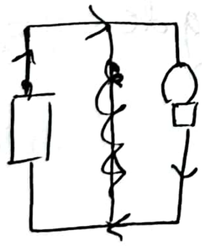
a)

A	B	$\bar{A}\bar{B}$
0	0	1
0	1	0
1	0	0
1	1	0

b.) NOR gate.

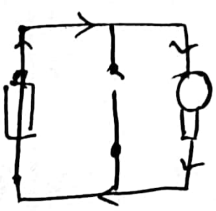
1.13

a.)



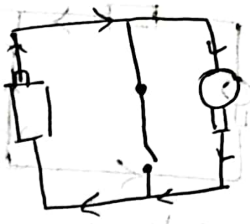
The bulb is on.

b.)



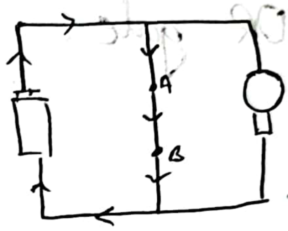
The bulb is on.

c.)



Bulb is on.

d)

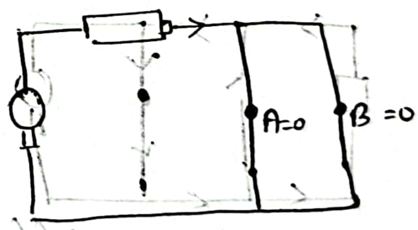


Bulb is off

e) AND gate.

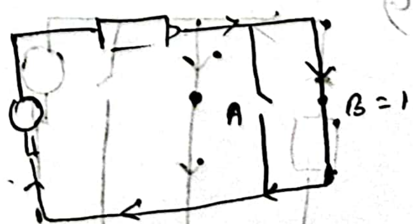
1.14

a)



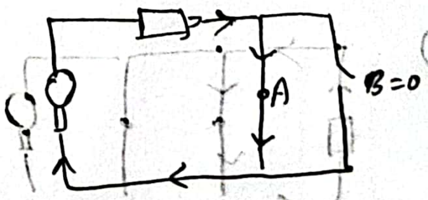
The bulb is off.

b.)

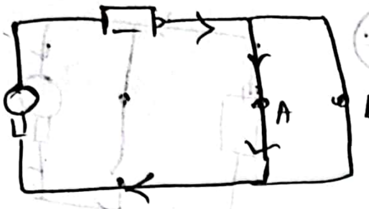


The bulb is on.

c.)



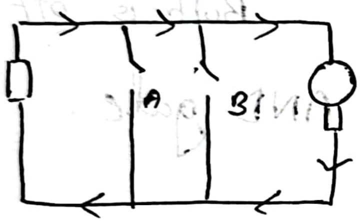
Bulb is on

d) 

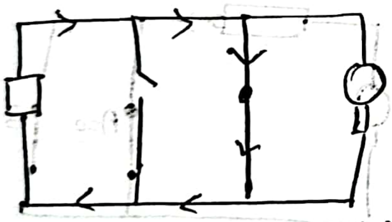
Bulb is on.

e) OR gate.

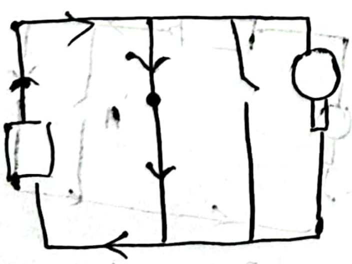
<u>1.15</u>

a) 

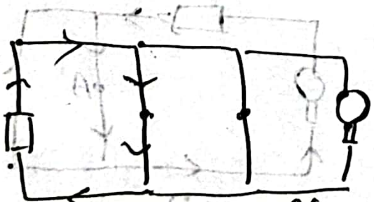
Bulb is on.

b) 

Bulb is off.

c) 

Bulb is off.

d) 

Bulb is off.

e) NAND gate.

<u>1.16</u>

a) Bulb is off.

b) On.

c) OFF

d) XOR gate.

1.17

a) Intel 8008 → 1972 → 3,500

b) RV32-WUJI → 2025 → 5931

1.18

a)

A	B	C	$A \oplus B \oplus C$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

b) output is 0.

c) Output is 1

1.19

A	B	C	A+B	$\overline{C}$	$(A+B).\overline{C}$	$\overline{(A+B).\overline{C}}$
0	0	0	0	1	0	1
0	0	1	0	0	0	1
0	1	0	1	1	1	0
0	1	1	1	0	0	1
1	0	0	1	1	1	0
1	0	1	1	0	0	1
1	1	0	1	1	1	0
1	1	1	[illegible]	0	0	1

1.20

a) $2^2 = 4$

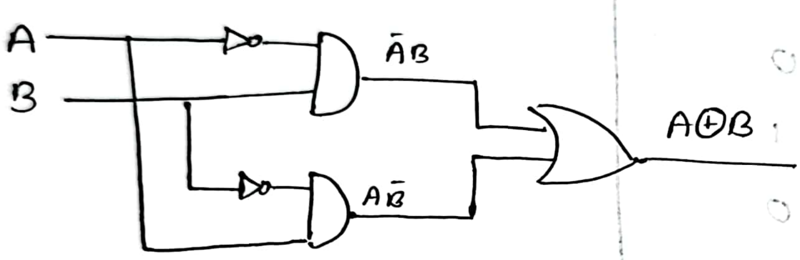
b) $2^4 = 16$

c) $2^8 = 256$

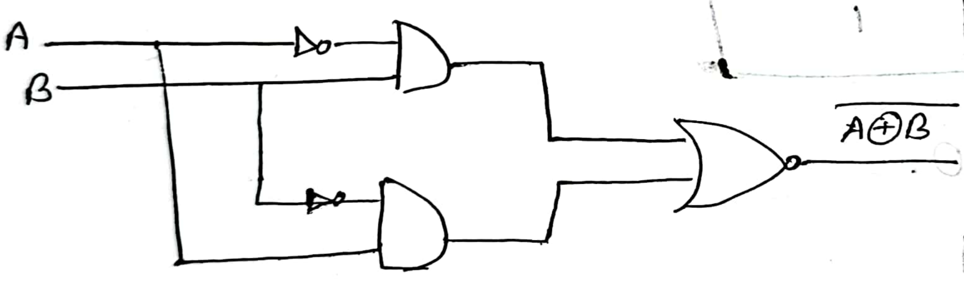
d) $2^{16} = 65536$.

e) 2^{2^n}

1.21

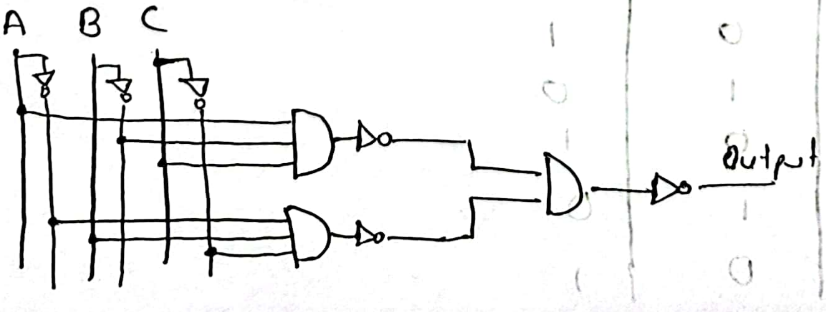


1.22



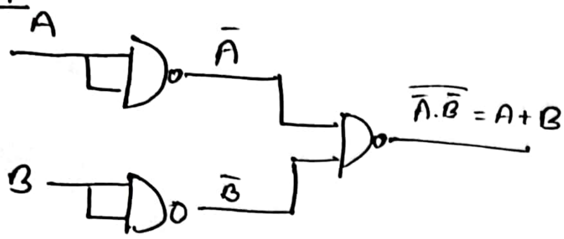
1.23

$$A'.B.C' + A.B'.C = \overline{\overline{A'.B.C'}\,.\,\overline{A.B'.C}}$$



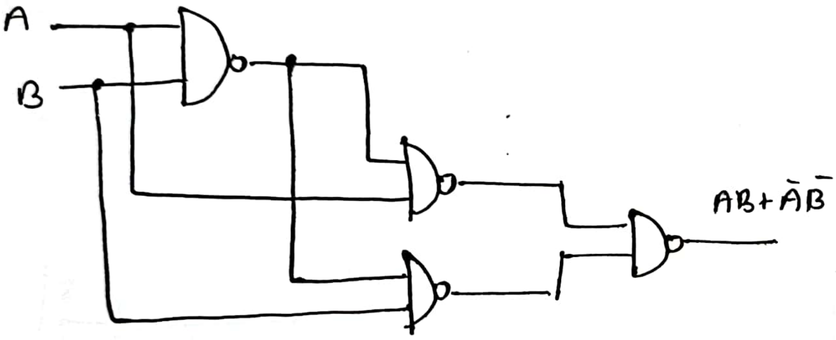
$\overline{\bar{A}B+A\bar{B}} = \overline{\bar{A}B}\cdot\overline{A\bar{B}}$
$= (A+\bar{B})\cdot(\bar{A}+B)$
$= AB+\bar{A}\bar{B}$

1.24



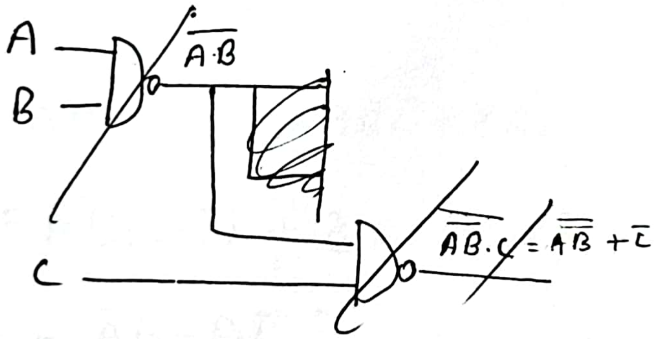
$\overline{\overline{ABC}\cdot\overline{ABC}}$
$= ABC + ABC$
$= ABC$

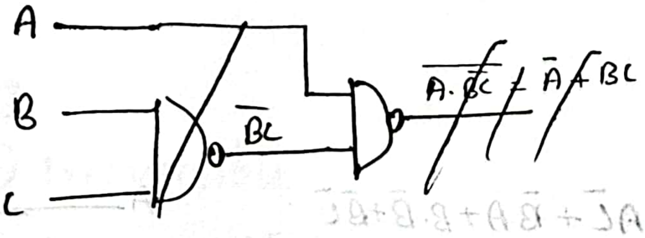
1.25.



1.26

~~(A·B·C)+C~~





1.27

A	B	$\bar{A}+\bar{B}$	Output
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

It is ~~sima~~ similar to the truth table of AND gate.

1.28

a)

A	Output
0	1
1	0

b)

A	B	output
0	0	0
0	1	1
1	0	1
1	1	1

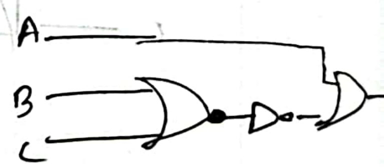
$$A + AB = A$$
$$A + \bar{A}B = A + B$$

1.34

$$\overline{\bar{A}(A+\bar{B})} = \bar{\bar{A}} + \overline{(A+\bar{B})}$$
$$= A + \bar{A}\cdot\bar{\bar{B}}$$
$$= A + \bar{A}B$$
$$= A + B$$

1.35

$$(A+\bar{B})(A+B+\bar{C}) = A\cdot A + AB + A\bar{C} + \bar{B}A + B\cdot\bar{B} + \bar{B}\bar{C}$$
$$= A + A(B+\bar{B}) + A\bar{C} + \bar{B}\bar{C}$$
$$= A + A + \bar{C}(A+\bar{B})$$
$$= A + \bar{C}(A+\bar{B})$$
$$= A + A\bar{C} + \bar{B}\bar{C} = A(1+\bar{C}) + \bar{B}\bar{C} = A + B'C'$$



1.36

$ABC + \bar{A}BC + AB\bar{C}$

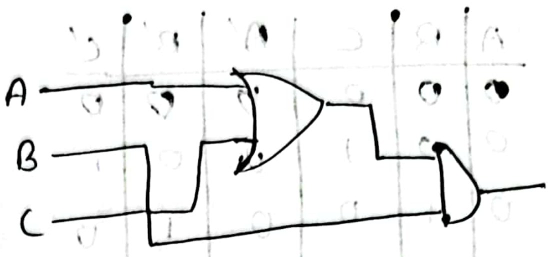
$= BC(A+\bar{A}) + AB\bar{C}$

$= BC + AB\bar{C}$

$= \cancel{B(C+\bar{C})}$

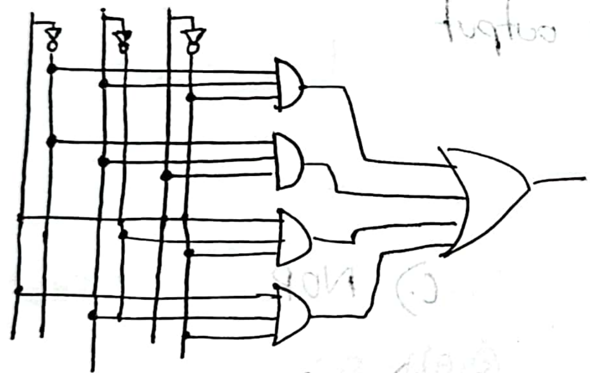
$= B(C+\bar{C}A)$

$= B(C+A)$



1.37

a) $\bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C}$



b.) $\bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C}$

$= \bar{A}B(C+\bar{C}) + A\bar{C}(B+\bar{B})$

$= \bar{A}B + A\bar{C}$

1.38

a.) Irreversible

b.) Irreversible

c.) Irreversible

d.) Irreversible.

1.39

b.) Reversible.

a.) Irreversible.

1.40

a)

A	B	C	A′	B′	C′
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	0
1	1	1	1	1	1

b) Reversible. Because the inputs can be determined from the output combinations.

1.41

a)

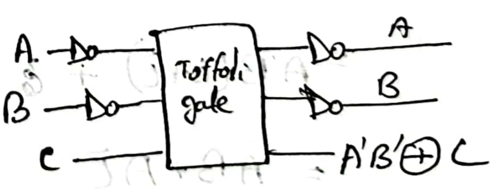
A	B	C	A	B	$\bar{A}\bar{B}\oplus C$
0	0	0	0	0	1
0	0	1	0	0	0
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	0
1	1	1	1	1	1

b) De Morgan's law.

c) NOR

d) OR

e)



1.42 a

1.42

a.) Reversible

b.) Irreversible.

A	B	
0	0	1
0	1	0
1	0	1
1	1	0

A	B	A	$B \oplus f(A)$
0	0	0	1
0	1	0	0
1	0	1	1
1	1	1	0

To make it reversible,
output the input ~~alo~~ also:

A	f(A)	f(A)'
0	1	0
1	1	1

1.43

XOR:

A	B	$A \oplus B = f(A,B)$
0	0	0
0	1	1
1	0	1
1	1	0

It is irreversible.

To make it reversible:

A	B	C	A	B	$C \oplus f(A,B)$
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	0	1	0
1	0	0	1	0	1
1	0	1	1	0	0
1	1	0	1	1	0

1.44

a)

A	B	B	$A \oplus B$
0	0	0	0
0	1	1	1
1	0	0	1
1	1	1	0

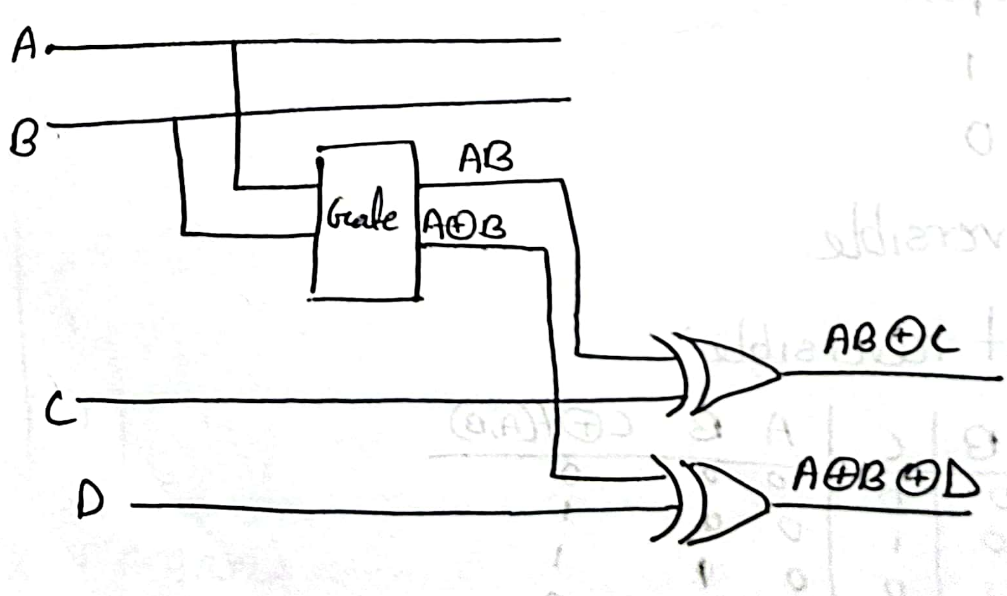
It is reversible.

b)

A	B	AB	$A \oplus B$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

The gate is irreversible.

To turn it into a reversible circuit:



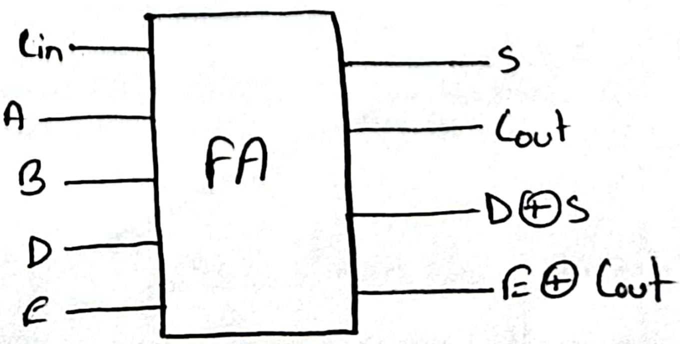
Truth table:

A	B	C	D	A	B	$C \oplus AB$	$D \oplus A \oplus B$
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	1
0	1	0	0	0	1	0	1
0	1	0	1	0	1	0	0
0	1	1	0	0	1	1	1
0	1	1	1	0	1	1	0
1	0	0	0	1	0	0	1
1	0	0	1	1	0	0	0
1	0	1	0	1	0	1	1
1	0	1	1	1	0	1	0
1	1	0	0	1	1	1	0
1	1	0	1	1	1	1	1
1	1	1	0	1	1	0	0
1	1	1	1	1	1	0	1

1.45

$S = A \oplus B \oplus C_{in}$

$C_{out} = AB + C_{in}(A \oplus B)$



1.46

a) 2^{13}

b) spontaneously flip, Radioactive atoms, presence, absence,

c) ~~Radioactive atoms~~ Single Event Upset (SEU)

d) ~~prese~~

c) miniaturized

d) increased, greater, sky.

e) cosmic rays, particles, black holes

f) cascade, transistor

g) Error Correction Code

h) month, cosmic rays.

i) 10 to 30

j) 161

k) flash, every star

1.47

a) First 7 bits: 1110001

The parity is even hence 0.

b) No.

c) Yes, because the parity bit does not match.

d) No.
Because, if another bit flipped, then the parity bit would be equal to that of the actual ~~d~~ string, even though the transmitted string is not correct.

1.48

a) Yes. middle bit.

b) Yes. Left bit.

c) $3p^2(1-p)+p^3 = 3\times(0.2)^2(1-0.2) + (0.2)^3$

$= 0.104$

It decreases the error probability.

d) $P=0.7$.

$\therefore\ 3\times(0.7)^2(1-0.7)+(0.7)^3 = 0.784$

It increase the error probability.

1.49

a) Yes. bit b_1 was flipped.

b) Yes. b_2 and b_1 were flipped.

c) $^5C_3\, p^3(1-p)^2 + {}^5C_4\, p^4(1-p) + {}^5C_5\, p^5$

$= 10p^3(1-p)^2 + 5p^4(1-p) + p^5$

d) $10p^3(1-p)^2 + 5p^4(1-p) + p^5 < p$

$\Rightarrow 10p^2(1-p)^2 + 5p^3(1-p) + p^4 < 1$

$\Rightarrow 10p^2 - 20p^3 + 10p^4 + 5p^3 - 5p^4 + p^4 < 1$

$\Rightarrow 6p^4 - 15p^3 + 10p^2 < 1$

$2p^2\left(3p^2 - \frac{15}{2}p + 5\right) < 1$

e) $P=0.1$

$10(0.1)^3(1-0.1)^2 + 5(0.1)^4(1-0.1) + (0.1)^5$

$= 0.00856$

It decreases the probability.

f) For 3-bit code repetition,

$P=0.1$

is $3\times(0.1)^2(1-0.1) + (0.1)^3$

$= 0.028 > 0.00856$

It is seen that 5-bit code performs better than 3 bit code.

1.50

a.) $n = 100$

$f(100) = 5247 n^2$

$= 5,24,7\,00,00$

$g(n) = 11 n^3$

$= 1,1\,0,00,000$

~~$f(n) > g(n$~~

$f(100) > g(100)$

b.) $n = 500$

$f(n) = 5247 \times (500)^2$

$= 1311750000.$

$g(n) = 11 (500)^3$

$= 1375000000$

~~$g(500) > f($~~

$g(500) > f(500)$

d.) $f(n) = O(g(n))$ - true

$f(n) = o(g(n))$ - true

$f(n) = \Theta(g(n))$ - true

$f(n) = \Omega(g(n))$ - False

$f(n) = \omega(g(n))$ - False.

c.) $11 n^3 > 5247 n^2$

$\Rightarrow n > 477.$

$\therefore n = 477.$

1.51

a) iii). $\Theta(n^2)$

b) $\omega(1)$

c.) ii. $\Omega(2^n)$

d.) $o(\sqrt{n})$

e.) $O(1)$

1.52

a) efficient

b) inefficient

c) inefficient

d) efficient

e) inefficient

f) inefficient

1.53

a) Bounded-Error Probabilistic Polynomial-Time

b) The class of feasible problems for a computer with access to a genuine random-number source.

1.54

Factoring.

1.55

a)

1. Birch and Swinnerton-Dyer Conjecture → Unsolved
2. Hodge Conjecture → unsolved
3. Navier-Stokes Equation → unsolved
4. P vs NP → Unsolved
5. Riemann Hypothesis → Unsolved
6. Yang-Mills & the Mass Gap → Unsolved
7. Poincaré Conjecture → Solved

b.)

Stephen Cook, 1971

1.56

1. Complete Coloring

In this problem, every pair of colors must appear on at least one pair of adjacent vertices. Determining if the achromatic number is greater than a given number is NP-complete.

2. 3-partition problem.

This problem challenges to give a decision whether a given multiset of integers can be partitioned into triplets that all have the same sum. The target sum T is computed by taking the sum of all elements in the set, then dividing by the number of triplets.

3. Hamiltonian path problem:
To decide whether a path exists in an undirected or directed graph that visits each vertex exactly once.

1.57

i. Finite-state machine

Also known as ~~Finit~~ finite-state automation is an abstract machine that can be only in one of the finite possible states at any given time. It changes from one state to another in response to given inputs and this process is called transition.

ii. Push-down automata

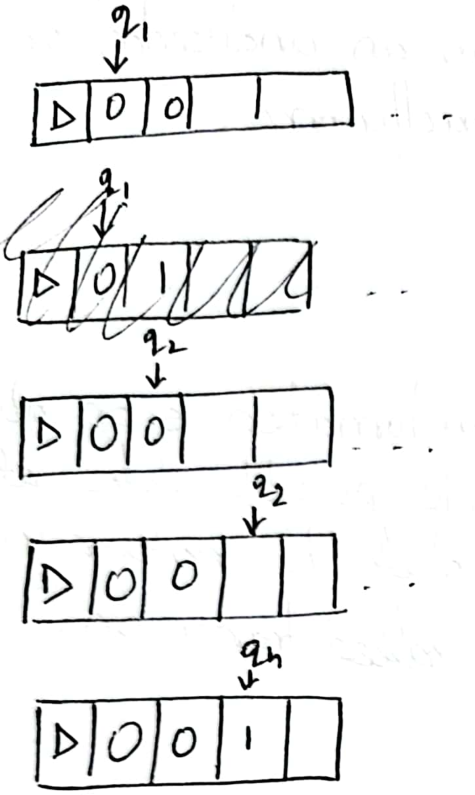
This computation employs a stack and are more capable than finite-state machines. A PDA uses the top of the stack to decide which transition to take and it can also manipulate the stack ~~at~~ as part of its transition.

iii. Combinatory logic

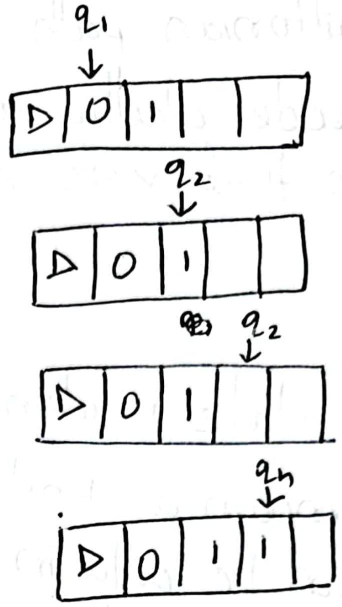
Eliminates the need for quantified variables in mathematical logic. It can be viewed as a variant of the lambda calculus in which lambda expressions are ~~r~~ replaced by a limited set of combinators, primitive functions without free variables.

1.58

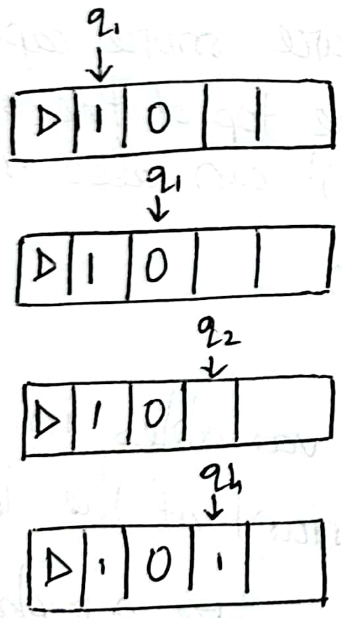
a)



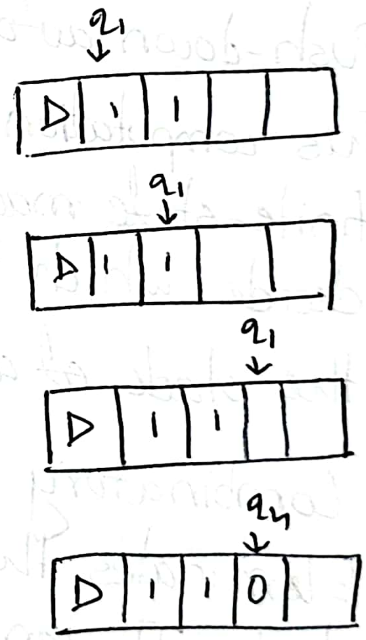
b.)



c)



d.)



e) If the input has at least ~~10, the~~ one 0, the machine appends a 1 at the end, ~~but~~ and appends a zero otherwise.

f.) Even in case of more than 2 bits, the machine is likely to perform in the similar manner and change states accordingly.

11101.

1.59.

Current state	Current tape	Write to tape	Move	Update state.
q_s	▷	▷	→	q_1
q_1	0	0	→	q_1
q_1	1	1	→	q_2
q_2	0	0	→	q_1
q_2		0	•	q_h.
q_2	1	1	→	q_1
q_1		1	•	q_h.

1.60

a) run forever

b) halt.

c) run

1.61

a.) complete, consistent, decidable ~~cor~~

b) programs, themselves

c.) "can every even number greater than 2 be written as the sum of two primes?"

d.) ~~runs forever halts~~ runs forever

e) ~~runs forever~~ halts

f.) By Berries definition, it should run forever but it contradicts Hal. Yes

g.) Berrie halts, ~~even~~ but Hal predicted Berries runs forever. Yes.

h) undecidable, cannot solve

1.62

i. Halting problem

From a description of an arbitrary computer program and an input, this problem determines whether the program will finish running or continue forever.

ii. Rice's theorem

This theorem states that all non-trivial semantic property of programs are undecidable.

iii. Hilbert's ~~Enst~~ Entscheidungsproblem asks for an algorithm that considers an inputted statement and answers "yes" or "no" according to whether it is universally valid.

1.63

a) a million times.

b) all known, quantum supremacy.

1.64

a.) Simulating quantum physics.

b.) hardest

c.) some

Chapter-2

2.1

a.) 1.　　e.) 0

b.) 1　　f.) $-i$

c.) 0　　g.) 0

d.) +　　h.) nothing happens.

2.2

a.) $-i$　　e.) +

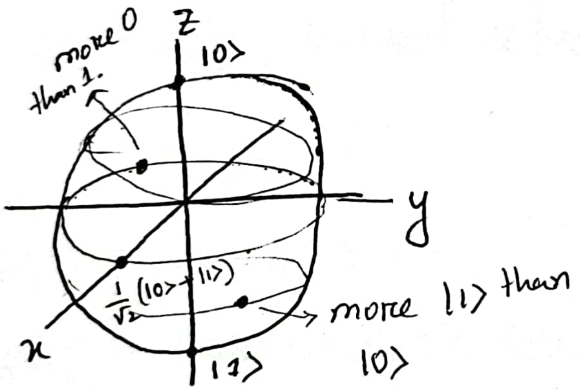
b.) $-i$　　f.) $-i$

c.) i　　g.) 0

d.) $-i$　　h.) nothing happens.

2.3

a.)

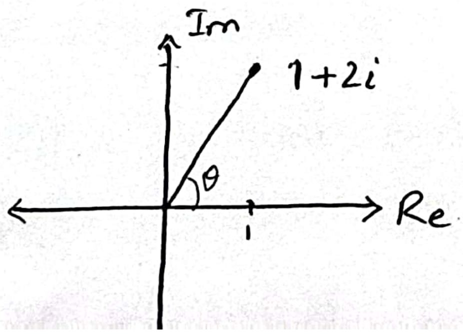


2.4

a.) $R(z)=1$.

b.) $I(z)=2$

c.)



d.) $r=\sqrt{1^2+2^2}=\sqrt{5}$

$\theta=\tan^{-1}\frac{2}{1}$

$re^{i\theta}=r(\cos\theta+i\sin\theta)$

$\therefore z=\sqrt{5}\left(\cos\tan^{-1}2+i\sin\left(\tan^{-1}2\right)\right)$.

~~d)~~

e) $z^* = 1 - 2i$

f) $|z| = \sqrt{1^2 + 2^2} = \sqrt{5}$.

g) $|z|^2 = 5$.

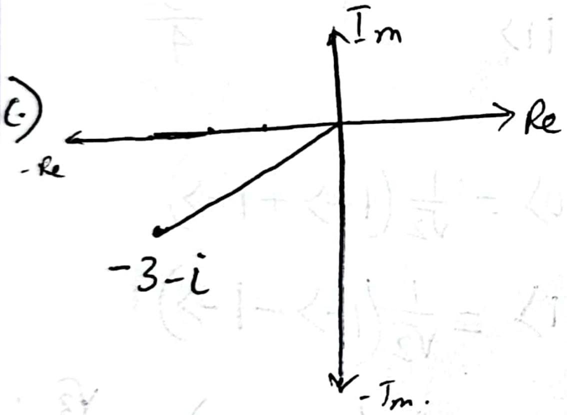
2.5

a) $z = -3 - i$

$R(z) = -3$

b) $I(z) = -1$

c)



d) $\theta = \tan^{-1}\frac{1}{3}$

$r = \sqrt{3^2 + 1^2} = \sqrt{10}$

e) $z^* = -3 + i$

f) $|z| = \sqrt{3^2 + 1^2} = \sqrt{10}$

g) 10.

2.6.

a) $\left|\frac{1+i\sqrt{3}}{3}\right|^2 = \frac{1+\cancel{2\sqrt{3}i}-\cancel{3}}{\cancel{9}} = \frac{-2}{\cancel{9}}(\cancel{1}-\sqrt{3}\cancel{i}) = \frac{4}{9}$

b) $\left|\frac{2-i}{3}\right|^2 = \frac{4-\cancel{4i}-1}{\cancel{9}} = \frac{3-\cancel{i4}}{\cancel{9}} = \frac{5}{9}$.

2.7

a) 1.

b) 0.

2.8

$$1 = \left|\frac{e^{i\pi/8}}{\sqrt{5}}\right|^2 + |\beta|^2$$

$$= \frac{1}{5} + |\beta|^2$$

$$\Rightarrow |\beta|^2 = \frac{4}{5}.$$

$$\therefore \beta = \frac{2}{\sqrt{5}}$$

2.9

a) $A(2e^{i\pi/6}|0\rangle - 3|1\rangle)$.

$$1 = |A.2e^{i\pi/6}|^2 + |A3|^2$$

$$= 4|A|^2 + 9|A|^2$$

$$= 13|A|^2$$

$$\Rightarrow |A|^2 = \frac{1}{13}$$

$$\therefore A = \frac{1}{\sqrt{13}}$$

b) $P(|0\rangle) = \left|\frac{2}{\sqrt{13}}\right|^2 = \frac{4}{13}$

c) $P(|1\rangle) = \left|\frac{-3}{\sqrt{13}}\right|^2 = \frac{9}{13}$

$$* \; |0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

$$|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$$

$$|0\rangle = \frac{1}{\sqrt{2}}(|i\rangle + |-i\rangle)$$

$$|1\rangle = \frac{-i}{\sqrt{2}}(|i\rangle - |-i\rangle)$$

2.10

a) ~~The state is~~ $\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle$.

$|0\rangle$ probability : $\frac{1}{4}$

$|1\rangle$ " : $\frac{3}{4}$

b) $|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$

$|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$

$$\therefore \frac{1}{2}\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) - \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$$

$$= \frac{1-\sqrt{3}}{2\sqrt{2}}|+\rangle + \frac{1+\sqrt{3}}{2\sqrt{2}}|-\rangle$$

c) The states achieved are $|+\rangle$ and $|-\rangle$.

$$P(|+\rangle) = \frac{(1-\sqrt{3})^2}{8} = 0.06$$

$$P(|-\rangle) = \frac{(1+\sqrt{3})^2}{8} = 0.93.$$

2.11

a.) $|0\rangle = \frac{\sqrt{3}}{2}|a\rangle - \frac{i}{2}|b\rangle$

$|1\rangle = -\frac{i}{2}|a\rangle + \frac{\sqrt{3}}{2}|b\rangle$

$\therefore \frac{1}{2}\left(\frac{\sqrt{3}}{2}|a\rangle - \frac{i}{2}|b\rangle\right) - \frac{\sqrt{3}}{2}\left(-\frac{i}{2}|a\rangle + \frac{\sqrt{3}}{2}|b\rangle\right)$

$= \frac{\sqrt{3}(1+i)}{4}|a\rangle - \frac{(i+3)}{4}|b\rangle$

b.) We get the states $|a\rangle$ and $|b\rangle$ withe the probabilities:

$P(|a\rangle) = \frac{3}{16}\cdot \cancel{4}2$

$\cancel{=0.265} = \frac{3}{8}$

$P(|b\rangle) = \frac{\cancel{9}10}{16} = \cancel{0.209}\ \frac{5}{8}$.

2.12

~~$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$~~

~~$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle$~~

a.) $\frac{1}{2}$

b.) $\frac{1}{2}$

2.13

a.) $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$e^{i\pi/8}|+\rangle = \frac{e^{i\pi/8}}{\sqrt{2}}(|0\rangle + |1\rangle)$

No, because $e^{i\pi/8}$ is a global state.

b.) $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ & $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

These two phases are different and can be measured in the x-basis to distinguish.

c.) $|0\rangle$ & $e^{i\pi/4}|0\rangle$ these two are only differed by a global variable

* $\alpha = \cos\frac{\theta}{2}$, $\beta = e^{i\varphi}\sin\left(\frac{\theta}{2}\right)$; $0 \le \theta \le \pi$
$0 \le \varphi < 2\pi$

$|\psi| = \alpha|0\rangle + \beta|1\rangle$

$\therefore \psi = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|1\rangle$

2.14

$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

a.) $\cos\frac{\theta}{2} = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$.

$\Rightarrow \theta = \frac{\pi}{2}$

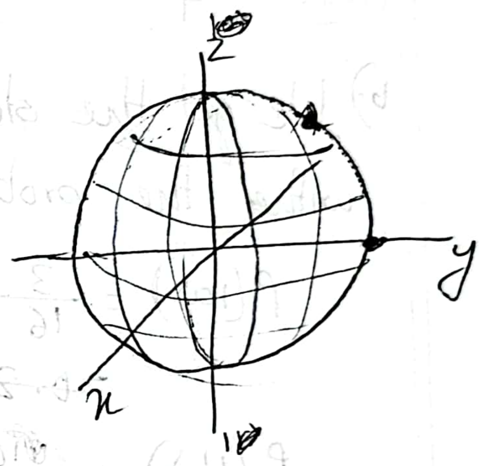
$e^{i\varphi}\sin\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{2}}\cdot i = \frac{1}{\sqrt{2}}\cdot e^{i\pi/2}$

$\Rightarrow e^{i\varphi}\sin\frac{\theta}{2} = e^{i\pi/2}\sin\frac{\pi}{4}$.

$\therefore \varphi = \frac{\pi}{2}$

$\therefore (\theta, \varphi) = \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

b.)



2.15

a.) $\frac{1-i}{2\sqrt{2}}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$.

$\frac{1-i}{2\sqrt{2}} = \frac{1}{2\sqrt{2}}\sqrt{2}\cdot e^{i\left(-\frac{\pi}{4}\right)} = \frac{1}{2}e^{-i\frac{\pi}{4}}$

$\therefore e^{-i\frac{\pi}{4}}\left(\frac{1}{2}|0\rangle + e^{i\pi/4}\frac{\sqrt{3}}{2}|1\rangle\right)$

$\therefore \cos\frac{\theta}{2} = \frac{1}{2} = \cos\frac{\pi}{3}$.

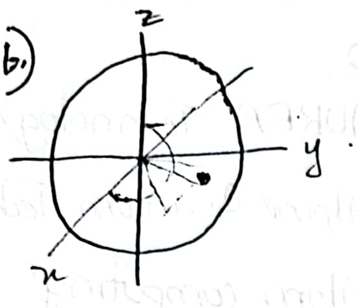
$\therefore \theta = \frac{2\pi}{3}$

$e^{i\varphi}\sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{3}}{2}e^{i\pi/4}$

$\therefore \varphi = \frac{\pi}{4}$.

$\therefore (\theta, \varphi) = \left(\frac{2\pi}{3}, \frac{\pi}{4}\right)$

b)



2.16

$$|a\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle$$

$$|b\rangle = \frac{i}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

$$\cos\frac{\theta_a}{2} = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$$

$$\therefore \theta_a = \frac{\pi}{3}$$

$$e^{i\varphi_a}\sin\frac{\theta_a}{2} = \frac{i}{2} = \frac{\pi}{6}e^{i\pi/2}$$

$$\therefore \varphi_a = \frac{\pi}{2}$$

$$\therefore (\theta_a, \varphi_a) = \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$$

~~$\cos\frac{\theta_b}{2} =$~~

Now, $|b\rangle = \frac{i}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

$$e^{i\pi/2}\left(\frac{1}{2}|0\rangle + e^{-i\pi/2}\frac{\sqrt{3}}{2}|1\rangle\right)$$

$$\cancel{= e^{i\pi/2}\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle}$$

$$\therefore \cos\frac{\theta_b}{2} = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\therefore \theta_b = \frac{2\pi}{3}$$

$$e^{i\varphi}\sin\left(\frac{\theta}{2}\right) = e^{-i\pi/2}\cdot\frac{\sqrt{3}}{2}$$

$$\therefore \varphi_b = -\frac{\pi}{2} = \frac{3\pi}{2}$$

$$\therefore (\theta_b, \varphi_b) = \left(\frac{2\pi}{3}, \frac{3\pi}{2}\right)$$

$$\theta_b = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\varphi_b = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$$

(Proved)

2.17

a.) $(\theta, \varphi) = (\frac{\pi}{2}, \frac{\pi}{2})$

$x = \sin\theta \cos\varphi$

$= \sin\frac{\pi}{2} \cos\frac{\pi}{2} = 0$

$y = \sin\theta \sin\varphi = 1$

$z = \cos\theta = 0$.

b.) $(\theta, \varphi) = (\frac{2\pi}{3}, \frac{\pi}{4})$

$x = \sin\frac{2\pi}{3} \cos\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}}{2\sqrt{2}}$.

$y = \sin\frac{2\pi}{3} \sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}}{2\sqrt{2}}$

$z = \cos\frac{2\pi}{3} = -\frac{1}{2}$

2.19

a.) AUREA Technology

b.) Alpine Quantum Technologies

c.) Atom computing

d.) HP

e.) AEGIQ

f.) Quantum Brilliance.

g.) Archer Materials

~~2.20~~

2.18

a.) polarization, noise

b.) electric fields

c.) Lacer beams

d.) molecule, radio-frequency. one-cubit

e.) discrete energy levels, atomic nucleus.

f.) Quantum information,

g.)

h.) charge, flux, phase

2.20

a) Fluxonium

b) Superconducting

c) An attempt to combine the better features of the Transmon (intensivity to charge noise) and flux qubit (strong anharmonicity)

2.21

a) coherent state of light.

b) squeezed light

c) quadrature

d) amplitude-squeezed state

e) phase-squeezed state.

2.22.

a) $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$\therefore U|\psi\rangle = \alpha U|0\rangle + \beta U|1\rangle$

$= \alpha(|0\rangle + |1\rangle) + \beta(|0\rangle - |1\rangle)$

$= (\alpha+\beta)|0\rangle + (\alpha-\beta)|1\rangle.$

$\therefore |\alpha+\beta|^2 + |\alpha-\beta|^2$

$= |\alpha|^2 + |\beta|^2 + |\alpha|^2 + |\beta|^2$

$= 2|\alpha|^2 + 2|\beta|^2$

$= 2(|\alpha|^2 + |\beta|^2)$

$= 2$

b) No, it is not a valid quantum gate because the ~~so~~ sum of probability does not equal to 1.

2.23

a.) $U|\psi\rangle = \alpha U|0\rangle + \beta U|1\rangle$.

$$= \alpha\left(\frac{\sqrt{3}}{2}|0\rangle + \frac{\sqrt{3}+i}{4}|1\rangle\right) + \beta\left(\frac{\sqrt{3}+i}{4}|0\rangle - \frac{\sqrt{3}+3i}{4}|1\rangle\right)$$

$$= \left(\alpha\cdot\frac{\sqrt{3}}{2} + \beta\cdot\frac{\sqrt{3}+i}{4}\right)|0\rangle + \left(\alpha\frac{\sqrt{3}+i}{4} - \beta\frac{\sqrt{3}+3i}{4}\right)|1\rangle$$

Now,
$$\left|\alpha\frac{\sqrt{3}}{2} + \beta\frac{\sqrt{3}+i}{4}\right|^2 + \left|\alpha\frac{\sqrt{3}+i}{4} - \beta\frac{\sqrt{3}+3i}{4}\right|^2$$

$$\cancel{= \left|\alpha\frac{\sqrt{3}}{2} + \beta\frac{\sqrt{3}}{4}\right|^2 + \left|\frac{\beta}{16}\right| \left|\frac{\beta}{4}\right|^2 +}$$

$$= \frac{3}{4}|\alpha|^2 + \frac{1}{4}|\beta|^2 + \frac{1}{4}|\alpha|^2 + \frac{3}{4}|\beta|^2$$

$$= |\alpha|^2 + |\beta|^2$$

$$= 1$$

b.) Yes, U is a valid quantum gate because the sum of norm squares results to 1.

2.24

a.) Yes.
Because it ~~has un~~ is reversible.

b.) No.
Because it is irreversible.

2.25

a.) Yes. Reversible.

b.) No. Irreversible.

2.26

$$Z^{217} X^{101} Y^{50} (\alpha|0\rangle + \beta|1\rangle)$$
$$= (Z^2)^{108} . Z (X^2)^{50} . X (Y^2)^{25} (\alpha|0\rangle + \beta|1\rangle)$$
$$= I . Z . I . X . I (\alpha|0\rangle + \beta|1\rangle)$$
$$= ZX(\alpha|0\rangle + \beta|1\rangle)$$
$$= Z(\alpha|1\rangle + \beta|0\rangle)$$
$$= -\alpha|1\rangle + \beta|0\rangle$$
$$= \beta|0\rangle - \alpha|1\rangle$$

2.27

a) $L.H.S = XZXZ(\alpha|0\rangle + \beta|1\rangle)$
$$= XZX(\alpha|0\rangle - \beta|1\rangle)$$
$$= XZ(\alpha|1\rangle - \beta|0\rangle)$$
$$= X(-\alpha|1\rangle - \beta|0\rangle)$$
$$= -\alpha|0\rangle - \beta|1\rangle$$
$$= -(\alpha|0\rangle + \beta|1\rangle)$$
$= R.H.S.$

b) $L.H.S = ZXZX(\alpha|0\rangle + \beta|1\rangle)$
$$= ZXZ(\alpha|1\rangle + \beta|0\rangle)$$
$$= ZX(-\alpha|1\rangle + \beta|0\rangle)$$
$$= Z(-\alpha|0\rangle + \beta|1\rangle)$$
$$= -\alpha|0\rangle - \beta|1\rangle$$
$= -(\alpha|0\rangle + \beta|1\rangle) = R.H.S.$

2.28

a) $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
$$R_z(\theta)|\psi\rangle = \alpha|0\rangle + \beta e^{i\theta}|1\rangle$$

b) Total probability
$$|\alpha|^2 + |\beta e^{i\theta}|^2$$
$$= |\alpha|^2 + |\beta|^2$$
$$= 1.$$

Hence it is a valid quantum gate.

2.29

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

a) $H|\psi\rangle = \alpha H|0\rangle + \beta H|1\rangle$
$$= \alpha|+\rangle + \beta|-\rangle$$

b) Total probability $= |\alpha|^2 + |\beta$
$= 1$

So it is valid.

2.30

a) $H|-\rangle = \cancel{\text{the}}\ H\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(H|0\rangle - H|1\rangle\right)$

$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) - \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right]$

$= |1\rangle$

$= R.H.S$

b) $H|-i\rangle = \frac{1}{\sqrt{2}}\left(H|0\rangle - iH|1\rangle\right)$

$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) - i\cdot\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right]$

$= \frac{1}{\sqrt{2}}\left[\frac{1-i}{\sqrt{2}}|0\rangle + \frac{1+i}{\sqrt{2}}|1\rangle\right]$

$= \frac{1}{\sqrt{2}}\left(e^{-i\pi/4}|0\rangle + e^{i\pi/4}|1\rangle\right)$

$\frac{i\pi}{2} - \frac{i\pi}{4}$

$= \frac{\;}{4}$

$= e^{i\pi/4}\frac{1}{\sqrt{2}}\left(\cancel{e^{-i\pi/2}}|0\rangle + e^{i\frac{\pi}{2}}|1\rangle\right)$

$= e^{-i\pi/4}\frac{1}{\sqrt{2}}\cancel{(-i|0\rangle + |1\rangle)}(|0\rangle + i|1\rangle)$

$= e^{-i\pi/4}|i\rangle$

$\equiv |i\rangle$

~~Showed~~.

2.31

$Y^{51}H^{99}T^{36}Z^{25}|0\rangle$

$= YHZ|0\rangle$

$= YH|0\rangle$

$= Y\left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right]$

$= \frac{1}{\sqrt{2}}i|1\rangle - \frac{1}{\sqrt{2}}i|0\rangle$

2.32

$$
\begin{aligned}
HXH|0\rangle &= HX\left[\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\right] \\
&= H\left(\frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle\right) \\
&= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle) + \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\right] \\
&= \frac{1}{\sqrt{2}}\left[\sqrt{2}\,|0\rangle\right] \\
&= |0\rangle
\end{aligned}
$$

$Z|0\rangle = |0\rangle.$

$\cancel{\therefore HXH = Z}$

$$
\begin{aligned}
HXH|1\rangle &= HX\left[\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)\right] \\
&= H\left(\frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}|0\rangle\right) \\
&= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle) - \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\right] \\
&= \frac{1}{\sqrt{2}}\left[-\sqrt{2}(|1\rangle)\right] \\
&= -|1\rangle
\end{aligned}
$$

$Z|1\rangle = -|1\rangle$

$\cancel{\therefore HX}$

$\therefore HXH = Z$

2.33

a.)
$$
\begin{aligned}
HTHTH|0\rangle &= HTHT\left[\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\right] \\
&= HTH\left[\frac{1}{\sqrt{2}}|0\rangle + e^{i\pi/4}\frac{1}{\sqrt{2}}|1\rangle\right] \\
&= HT\left[\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\cdot(|0\rangle+|1\rangle) + e^{i\pi/4}\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right] \\
&= HT\left[\frac{1}{2}(|0\rangle+|1\rangle) + \frac{1}{2}\cdot\frac{(i+1)}{\sqrt{2}}(|0\rangle-|1\rangle)\right]
\end{aligned}
$$

$$= HT\left[\left(\frac{1}{2}+\frac{(i+1)}{2\sqrt{2}}\right)|0\rangle + \left(\frac{1}{2}-\frac{(i+1)}{2\sqrt{2}}\right)|1\rangle\right]$$

$$= HT\ \frac{1}{2}\left[(1+e^{i\pi/4})|0\rangle + (1-e^{i\pi/4})|1\rangle\right]$$

$$= H\ \frac{1}{2}\left[(1+e^{i\pi/4})|0\rangle + e^{i\pi/4}(1-e^{i\pi/4})|1\rangle\right]$$

$$= H\ \frac{1}{2}\left[(1+e^{i\pi/4})|0\rangle + (e^{i\pi/4}-i)|1\rangle\right]$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}}(1+e^{i\pi/4})(|0\rangle+|1\rangle) + \frac{1}{2}(e^{i\pi/4}-i)\cdot\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$$

$$= \frac{1}{2\sqrt{2}}\cdot(1+e^{i\pi/4}+e^{i\pi/4}-i)|0\rangle + \frac{1}{2\sqrt{2}}(1+e^{i\pi/4}-e^{i\pi/4}+i)|1\rangle$$

$$= \frac{1}{2\sqrt{2}}\left(1-i+\frac{2+2i}{\sqrt{2}}\right)|0\rangle + \frac{1}{2\sqrt{2}}(1+i)|1\rangle$$

$$= \frac{1}{2\sqrt{2}}\cdot\left[1+\sqrt{2}+(\sqrt{2}-1)i\right]|0\rangle + \frac{1}{2\sqrt{2}}(1+i)|1\rangle$$

b) $P(|0\rangle) = \left|\frac{1}{2\sqrt{2}}\left[1+\sqrt{2}+(\sqrt{2}-1)i\right]\right|^2$ $\quad P(|1\rangle) = \left|\frac{1}{2\sqrt{2}}(1+i)\right|^2$

$= \cancel{0.728}\ 0.75$ $\quad = \cancel{0.125}\ 0.25$

2.34

a) $\hat{n} = n_x\hat{x} + n_y\hat{y} + n_z\hat{z}$

$\therefore \hat{n} = (0,0,1)$

~~b)~~

b) $U = e^{i\gamma}\left[\cos\left(\frac{\theta}{2}\right)I - i\sin\left(\frac{\theta}{2}\right)(n_x X + n_y Y + n_z Z)\right]$

$= e^{i\gamma}\left[\cos\left(\frac{\pi}{8}\right)I - i\sin\left(\frac{\pi}{8}\right)(0 \cdot X + 0 \cdot Y + 1 \cdot Z)\right]$

$= e^{i\gamma}\left(\cos\frac{\pi}{8} I - i\sin\frac{\pi}{8} \cdot Z\right)$

c) $U|0\rangle = e^{i\gamma}\left(\cos\frac{\pi}{8} I - i\sin\frac{\pi}{8} \cdot Z\right)|0\rangle$

$= e^{i\gamma}\left(\cos\frac{\pi}{8} I - i\sin\frac{\pi}{8}\right)|0\rangle$

d) $U|1\rangle = e^{i\gamma}\left(\cos\frac{\pi}{8} I - i\sin\frac{\pi}{8} Z\right)|1\rangle$

$= e^{i\gamma}\left(\cos\frac{\pi}{8} I + i\sin\frac{\pi}{8}\right)|1\rangle$

e) From c and d, $U|0\rangle = e^{i\gamma} e^{-i\pi/8}|0\rangle$

$U|1\rangle = e^{i\gamma} e^{i\pi/8}|1\rangle = e^{i\gamma} \cdot e^{-i\pi/8} \cdot e^{i\pi/4}|1\rangle$

For T gate, $T|0\rangle = |0\rangle$

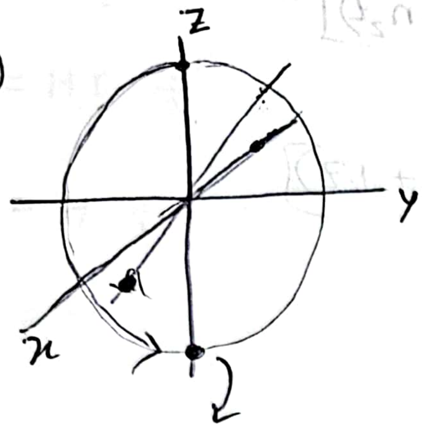
$\cancel{T|0\rangle = e}$

$T|1\rangle = e^{i\pi/4}|1\rangle$

$\therefore U = e^{i\gamma} e^{-i\frac{\pi}{8}} T.$

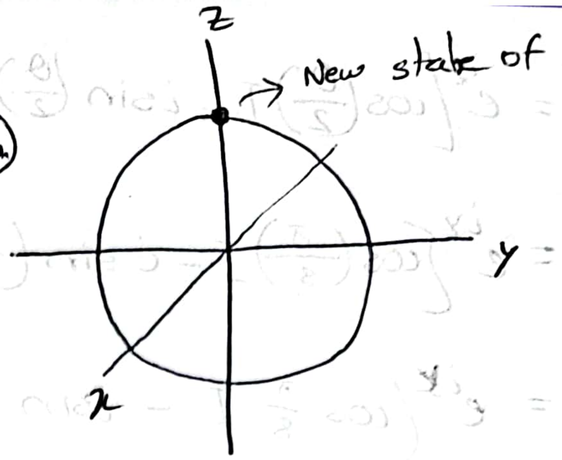
2.35

a)



$|0\rangle$ goes to $|1\rangle$

b)



c) $\hat{n} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

d) $U = e^{i\gamma}\left[\cos\frac{\pi}{2} I - i\sin\frac{\pi}{2} \left(\frac{1}{\sqrt{3}} X + \frac{1}{\sqrt{3}} Y + \frac{1}{\sqrt{3}} Z\right)\right]$

$= -e^{i\gamma} i \cdot \frac{1}{\sqrt{3}}(X+Y+Z)$.

e) $U|0\rangle = -ie^{i\gamma} \cdot \frac{1}{\sqrt{3}}(X+Y+Z)|0\rangle$

$= -ie^{i\gamma}\frac{1}{\sqrt{3}}(X|0\rangle + Y|0\rangle + Z|0\rangle)$

$= -ie^{i\gamma}\frac{1}{\sqrt{3}}(|1\rangle + i|1\rangle + |0\rangle)$

$= -ie^{i\gamma}\frac{1}{\sqrt{3}}|0\rangle - (1+i)\, i\, e^{i\gamma}\frac{1}{\sqrt{3}}|1\rangle$

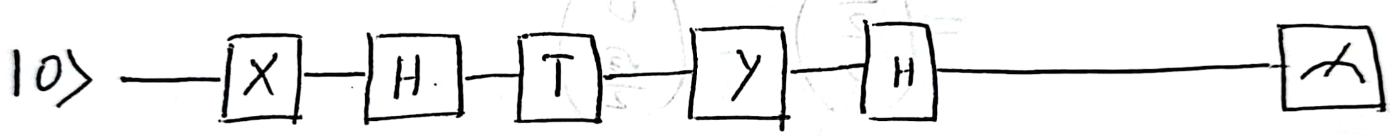
f.) $U|1\rangle = -ie^{i\gamma}\frac{1}{\sqrt{3}}\left(X|1\rangle + Y|1\rangle + Z|1\rangle\right)$

$= -ie^{i\gamma}\frac{1}{\sqrt{3}}\left(|0\rangle \cancel{+} -i|0\rangle - |1\rangle\right)$

$= -i(1-i)\, e^{i\gamma}\frac{1}{\sqrt{3}}\, |0\rangle + ie^{i\gamma}\frac{1}{\sqrt{3}}\, |1\rangle$.

2.36.

a)



b)

Chapter-3

*

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

3.1

$$\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{\sqrt{3}}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ \frac{\sqrt{3}}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$$

3.2

$$\begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix} = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle .$$

The states are $|0\rangle$ and $|1\rangle$ with probabilities: $P(|0\rangle) = \left|\frac{\sqrt{3}}{2}\right|^2 = \frac{3}{4}$

$$P(|1\rangle) = \left|\frac{1}{2}\right|^2 = \frac{1}{4}$$

3.3

a) $|a\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$

$\therefore \langle a| = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$

b) $\langle a| = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

c) $\langle b| = \frac{2}{3}|0\rangle + \frac{1+2i}{3}|1\rangle$

d) $\langle b| = \left(\frac{2}{3}, \frac{1+2i}{3}\right)$

3.4

a) $|a\rangle = \frac{3+i\sqrt{3}}{4}|0\rangle + \frac{1}{2}|1\rangle$

$|b\rangle = \frac{1}{4}|0\rangle + \frac{\sqrt{15}}{4}|1\rangle$

$\langle a| = \frac{3-i\sqrt{3}}{4}|0\rangle + \frac{1}{2}|1\rangle$

$\therefore \langle a|b\rangle = \left(\frac{3-i\sqrt{3}}{4} \quad \frac{1}{2}\right)\begin{pmatrix} \frac{1}{4} \\ \frac{\sqrt{15}}{4} \end{pmatrix}$

$= \frac{3-i\sqrt{3}}{16} + \frac{\sqrt{5}}{8}$

$= \frac{3-i\sqrt{3}+2\sqrt{5}}{16}$

b) $\langle b|a\rangle = \cancel{\frac{3+2\sqrt{5}+i\sqrt{3}}{16}} \left(\frac{1}{4} \quad \frac{\sqrt{15}}{4}\right)\begin{pmatrix} \frac{3+i\sqrt{3}}{4} \\ \frac{1}{2} \end{pmatrix}$

~~c) $\langle b|a\rangle = \langle a|b$~~

$= \frac{3+i\sqrt{3}}{16} + \frac{\sqrt{15}}{8}$

$= \frac{3+2\sqrt{5}+i\sqrt{3}}{16}.$

c) $\langle b|a\rangle = \langle a|b\rangle^*$

3.5

a) $|\psi\rangle = A(2|0\rangle + 3i|1\rangle)$

$$\therefore \langle\psi|\psi\rangle = A(2\ -3i)\,A\begin{pmatrix}2\\3i\end{pmatrix}$$

$$= 4A^2 + 9A^2$$

$$= 13|A|^2$$

b) To normalize $|\psi\rangle$, $13|A|^2 = 1$

$$\Rightarrow |A|^2 = \frac{1}{13}.$$

$$\therefore |A| = \frac{1}{\sqrt{13}}$$

3.6

a) $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$$\therefore \langle +|-\rangle = \frac{1}{\sqrt{2}}(1\ \ 1)\,\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$= \frac{1}{2} - \frac{1}{2}$$

$$= 0$$

$\therefore$ orthogonal.

b) $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$$\therefore \langle 0|+\rangle = (1\ \ 0)\,\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}} + 0 = \frac{1}{\sqrt{2}} \neq 0.$$

Not null ...

c) Let,

$$|A\rangle = \begin{pmatrix} \frac{1+\sqrt{3}i}{4} \\ \frac{\sqrt{2}-i}{2} \end{pmatrix}$$

$$|B\rangle = \begin{pmatrix} \frac{\sqrt{2}+i}{2} \\ \frac{-1+\sqrt{3}i}{4} \end{pmatrix}$$

$$\therefore \langle A|B\rangle = \begin{pmatrix} \frac{1-\sqrt{3}i}{4} & \frac{\sqrt{2}+i}{2} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}+i}{2} \\ \frac{-1+\sqrt{3}i}{4} \end{pmatrix}$$

$$= \frac{\sqrt{2}+i-\sqrt{6}i+\sqrt{3}}{8} + \frac{-\sqrt{2}+\sqrt{6}i-i-\sqrt{3}}{8}$$

$$= 0.$$

$\therefore$ orthogonal.

3.7

a.)
$$\langle a|b\rangle = 0.$$

$$\Rightarrow \begin{pmatrix} \frac{3-i\sqrt{3}}{4} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1/4 \\ x \end{pmatrix} = 0.$$

$$\Rightarrow \frac{3-i\sqrt{3}}{16} + \frac{x}{2} = 0.$$

$$\Rightarrow \frac{x}{2} = \frac{-3+i\sqrt{3}}{16}$$

$$\therefore x = \frac{-3+i\sqrt{3}}{8}.$$

b.)
$$\langle b|b\rangle = 1$$

$$\Rightarrow \begin{pmatrix} \frac{1}{4} & x \end{pmatrix} \begin{pmatrix} \frac{1}{4} \\ x \end{pmatrix} = 1$$

$$\Rightarrow \frac{1}{16} + |x|^2 = 1$$

$$\Rightarrow |x|^2 = \frac{15}{16}.$$

$$\therefore |x| = \frac{\sqrt{15}}{4}.$$

c.) From a, $x = \frac{-3+i\sqrt{3}}{8}$.

$$\therefore \langle b|b\rangle = \begin{pmatrix} \frac{1}{4} & \frac{-3-i\sqrt{3}}{8} \end{pmatrix} \begin{pmatrix} 1/4 \\ \frac{-3+i\sqrt{3}}{8} \end{pmatrix}$$

$$= \frac{1}{16} + \frac{9+3}{64}$$

$$= \frac{4+12}{64} = \frac{16}{64} = \frac{1}{4} \neq$$

~~Again, for~~ So, there is no value of x for which $|a\rangle$ and $|b\rangle$ are orthonormal.

3.8

$|a\rangle = \cancel{\begin{pmatrix} \cos\frac{\theta_a}{2} \\ \cos\frac{\theta_b}{2} \end{pmatrix}}$

$|b\rangle = \cancel{\cos\frac{\theta_b}{2}}$

$|a\rangle = \begin{pmatrix} \cos\frac{\theta_a}{2} \\ e^{i\varphi_a}\sin\frac{\theta_a}{2} \end{pmatrix}$

$|b\rangle = \begin{pmatrix} \cos\frac{\theta_b}{2} \\ e^{i\varphi_b}\sin\frac{\theta_b}{2} \end{pmatrix}$

$$\therefore \langle a|b\rangle = \begin{pmatrix} \cos\frac{\theta_a}{2} & e^{-i\varphi_a}\sin\frac{\theta_a}{2} \end{pmatrix} \begin{pmatrix} \cos\frac{\theta_b}{2} \\ e^{i\varphi_b}\sin\frac{\theta_b}{2} \end{pmatrix}$$

$$= \cos\frac{\theta_a}{2}\cos\frac{\theta_b}{2} + e^{-i\varphi_a + i\varphi_b}\sin\frac{\theta_a}{2}\sin\frac{\theta_b}{2}$$

$$= \cos\frac{\theta}{2}\cos\frac{\theta_b}{2} + \left[\cos(\varphi_b - \varphi_a) - i\sin(\varphi_b - \varphi_a)\right] \cdot \sin\frac{\theta_a}{2}\sin\frac{\theta_b}{2}$$

$$= \cos\frac{\theta}{2}\cos\frac{\theta_b}{2} + \left[\cos(\varphi_a + \pi - \varphi_a) - i\sin(\varphi_a + \pi - \varphi_a)\right]\sin\frac{\theta_a}{2}\sin\frac{\theta_b}{2}$$

$$= \cos\frac{\theta_a}{2}\cos\frac{\theta_b}{2} + (-1 - 0)\sin\frac{\theta_a}{2}\sin\frac{\theta_b}{2}$$

$$= \cos\frac{\theta_a}{2}\cos\frac{\theta_b}{2} - \sin\frac{\theta_a}{2}\sin\frac{\theta_b}{2}$$

$$= \cos\left(\frac{\theta_a}{2} + \frac{\theta_b}{2}\right) = \cos\left(\frac{\theta_a + \pi - \theta_a}{2}\right)$$

$$= \cos\frac{\pi}{2} = 0$$

$\therefore \langle a|b\rangle$ is orthogonal.

3.9

a) $$\langle i|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right)\left(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\langle 0|0\rangle + \frac{1}{2}\langle 0|1\rangle - \frac{\sqrt{3}}{2}i\langle 1|0\rangle - \frac{1}{2}i\langle 1|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \frac{\sqrt{3} - i}{2\sqrt{2}}$$

b) $\langle -i|\psi\rangle = \frac{1}{\sqrt{2}}(\langle 0| - i\langle 1|)\left(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\langle 0|0\rangle + \frac{1}{2}\langle 0|1\rangle + \frac{\sqrt{3}}{2}i\langle 1|0\rangle + \frac{1}{2}i\langle 1|1\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$= \frac{\sqrt{3}+i}{2\sqrt{2}}$

c.) The states are ~~either~~ $|i\rangle$ and $|-i\rangle$.

$P(|i\rangle) = \left|\frac{\sqrt{3}-i}{2\sqrt{2}}\right|^2 = \frac{4}{8} = \frac{1}{2}$

$P(|-i\rangle) = \left|\frac{\sqrt{3}+i}{2\sqrt{2}}\right|^2 = \frac{4}{8} = \frac{1}{2}$

d.)

$\langle a|\psi\rangle = \left(\frac{\sqrt{3}}{2}\langle 0| - \frac{i}{2}\langle 1|\right)\left(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle\right)$

$= \frac{3}{4}\langle 0|0\rangle + \frac{\sqrt{3}}{4}\langle 0|1\rangle + \frac{\sqrt{3}i}{4}\langle 1|0\rangle + \frac{i}{4}\langle 1|1\rangle$

$= \frac{3}{4} + \frac{i}{4} = \frac{3+i}{4}$.

e) $\langle b|\psi\rangle = \left(-\frac{i}{2}\langle 0| + \frac{\sqrt{3}}{2}\langle 1|\right)\left(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle\right)$

$= \frac{\sqrt{3}i}{4}\langle 0|0\rangle + \frac{i}{4}\langle 0|1\rangle + \frac{3}{4}\langle 1|0\rangle + \frac{\sqrt{3}}{4}\langle 1|1\rangle$

$= \frac{\sqrt{3}i}{4} + \frac{\sqrt{3}}{4}$

$= \frac{\sqrt{3}(i+1)}{4}$

f.) The states are $|a\rangle$ and $|b\rangle$.

$P(|a\rangle) = \left|\frac{3+i}{4}\right|^2 = \frac{10}{16} = \frac{5}{8}$ $\quad P(|b\rangle) = \left|\frac{\sqrt{3}i+\sqrt{3}}{4}\right|^2 = \frac{6}{16} = \frac{3}{8}$.

3.10

a.) $\langle 0|\psi\rangle = \langle 0|\left(\frac{3+i\sqrt{3}}{4}|0\rangle - \frac{1}{2}|1\rangle\right)$

$= \frac{3+i\sqrt{3}}{4}\langle 0|0\rangle - \frac{1}{2}\langle 0|1\rangle$

$= \frac{3+i\sqrt{3}}{4}$

$\langle 1|\psi\rangle = \langle 1|\left(\frac{3+i\sqrt{3}}{4}|0\rangle - \frac{1}{2}|1\rangle\right)$

$= \frac{3+i\sqrt{3}}{4}\langle 1|0\rangle - \frac{1}{2}\langle 1|1\rangle$

$= -\frac{1}{2}$

States: $|0\rangle$ & $|1\rangle$

$P(|0\rangle) = \left|\frac{3+i\sqrt{3}}{4}\right|^2 = \frac{12}{16} = \frac{3}{4}$

$P(|1\rangle) = \left|-\frac{1}{2}\right|^2 = \frac{1}{4}$

b) $\langle +|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\left(\frac{3+i\sqrt{3}}{4}|0\rangle - \frac{1}{2}|1\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{3+i\sqrt{3}}{4}\langle 0|0\rangle - \frac{1}{2}\langle 0|1\rangle + \frac{3+i\sqrt{3}}{4}\langle 1|0\rangle - \frac{1}{2}\langle 1|1\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{3+i\sqrt{3}}{4} - \frac{1}{2}\right)$

$= \frac{1+i\sqrt{3}}{\sqrt{2}\,4}$

$\langle -|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\left(\frac{3+i\sqrt{3}}{4}|0\rangle - \frac{1}{2}|1\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{3+i\sqrt{3}}{4}\langle 0|0\rangle - \frac{1}{2}\langle 0|1\rangle - \frac{3+i\sqrt{3}}{4}\langle 1|0\rangle + \frac{1}{2}\langle 1|1\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{3+i\sqrt{3}}{4} + \frac{1}{2}\right) = \frac{5+i\sqrt{3}}{}$

The states are $|+\rangle$ & $|-\rangle$ and

$$P(|+\rangle) = \left|\frac{1+i\sqrt{3}}{4\sqrt{2}}\right|^2 = \frac{4}{32} = \frac{1}{8}$$

$$P(|-\rangle) = \left|\frac{\cancel{4}5+i\sqrt{3}}{4\sqrt{2}}\right|^2 = \frac{\cancel{1028}\,7}{\cancel{32}\,8} = \frac{7}{8}.$$

c) $$\langle i|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right)\left(\frac{3+i\sqrt{3}}{4}|0\rangle - \frac{1}{2}|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{3+i\sqrt{3}}{4}\langle 0|0\rangle - \frac{1}{2}\langle 0|1\rangle - \frac{3i-\sqrt{3}}{4}\langle 1|0\rangle + \frac{1}{2}i\langle 1|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{3+i\sqrt{3}}{4} + \frac{i}{2}\right)$$

$$= \frac{3+i(\sqrt{3}+2)}{4\sqrt{2}}$$

$$\langle -i|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - i|1\rangle\right)\left(\frac{3+i\sqrt{3}}{4}|0\rangle - \frac{1}{2}|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{3+i\sqrt{3}}{4}\langle 0|0\rangle - \frac{1}{2}\langle 0|1\rangle + \frac{3i-\sqrt{3}}{4}\langle 1|0\rangle - \frac{i}{2}\langle 1|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{3+i\sqrt{3}}{4} - \frac{i}{2}\right)$$

$$= \frac{3+i(\sqrt{3}-2)}{4\sqrt{2}}$$

d. The states are $|i\rangle$ and $|-i\rangle$

$$P(|i\rangle) = \left|\frac{3+i(\sqrt{3}+2)}{4\sqrt{2}}\right|^2 = \frac{16+4\sqrt{3}}{32} = 0.716$$

$$P(|-i\rangle) = \left|\frac{3+i(\sqrt{3}-2)}{4\sqrt{2}}\right|^2 = \frac{16-4\sqrt{3}}{32} = 0.2834$$

3.11

a.)

$$\langle +|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\frac{1}{\sqrt{6}}\left((1-2i)|0\rangle + |1\rangle\right)$$

$$= \frac{1}{\sqrt{12}}\left[(1-2i)\langle 0|0\rangle + 1\cdot\langle 0|1\rangle + (1-2i)\langle 1|0\rangle + 1\cdot\langle 1|1\rangle\right]$$

$$= \frac{1}{\sqrt{12}}(1-2i+1)$$

$$= \frac{2-2i}{\sqrt{12}} = \frac{2(1-i)}{2\sqrt{3}} = \frac{1-i}{\sqrt{3}}$$

$$\langle -|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\frac{1}{\sqrt{6}}\left[(1-2i)|0\rangle + |1\rangle\right]$$

$$= \frac{1}{\sqrt{12}}\left[(1-2i)\langle 0|0\rangle + 1\cdot\langle 0|1\rangle - (1-2i)\langle 1|0\rangle - 1\cdot\langle 1|1\rangle\right]$$

$$= \frac{1}{2\sqrt{3}}(1-2i-1)$$

$$= -\frac{i}{\sqrt{3}}$$

$$\therefore |\psi\rangle = \frac{1-i}{\sqrt{3}}|+\rangle - \frac{i}{\sqrt{3}}|-\rangle.$$

b.)

$$\langle i|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right)\frac{1}{\sqrt{6}}\left((1-2i)|0\rangle + |1\rangle\right)$$

$$= \frac{1}{\sqrt{12}}\left[(1-2i)\langle 0|0\rangle + 1\cdot\langle 0|1\rangle - i(1-2i)\langle 1|0\rangle - i\langle 1|1\rangle\right]$$

$$= \frac{1}{\sqrt{12}}(1-2i-i) = \frac{1-3i}{\sqrt{12}}$$

$$\langle -i|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - i|1\rangle\right)\frac{1}{\sqrt{6}}\left((1-2i)|0\rangle + |1\rangle\right)$$

$$= \frac{1}{\sqrt{12}}\left[(1-2i)\langle 0|0\rangle + 1\cdot\langle 0|1\rangle + i(1-2i)\langle 1|0\rangle + i\langle 1|1\rangle\right)$$

$$= \frac{1}{\sqrt{12}}(1-2i+i)$$

$$= \frac{1-i}{\sqrt{12}}$$

$$\therefore |\psi\rangle = \frac{1-3i}{\sqrt{12}}|\cancel{i}\rangle + \frac{1-i}{\sqrt{12}}|i\rangle$$

3.12

a) $$U = \frac{1}{\sqrt{2}}\cdot\cancel{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

b) $$U\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}\alpha - \beta i \\ -\alpha i + \beta\end{pmatrix}$$

c) $$\left|\frac{\alpha-\beta i}{\sqrt{2}}\right|^2 + \left|\frac{-\alpha i+\beta}{\sqrt{2}}\right|^2 = \frac{|\alpha|^2+|\beta|^2}{2} + \frac{|\alpha|^2+|\beta|^2}{2}$$

$$= \frac{2}{2} = 1$$

Since the sum of probabilities of $U\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$ is 1, it is a valid gate.

3.13

a) $U = \frac{1}{2\sqrt{3}} \begin{pmatrix} 3+i & 1-i \\ -(1+i) & 3-i \end{pmatrix}$

b)

3.14

$U = e^{i\gamma}\left[\cos\left(\frac{\theta}{2}\right) I - i\sin\left(\frac{\theta}{2}\right)(n_x X + n_y Y + n_z Z)\right]$

$n = (0, 1, 0)$

$\therefore U = e^{0}\left[\cos\left(\frac{\pi}{4}\right) I - i\sin\frac{\pi}{4}\, Y\right]$

$= \frac{1}{\sqrt{2}} I - i\frac{1}{\sqrt{2}} Y$

$= \frac{1}{\sqrt{2}}(I - iY)$

$= \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\right)$

$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

3.15

$$HTU|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{pmatrix}\begin{pmatrix}\frac{\sqrt{2}-i}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{2}+i}{2}\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & e^{i\pi/4}\\ 1 & -e^{i\pi/4}\end{pmatrix}\begin{pmatrix}\frac{\sqrt{2}-i}{2}\\ -\frac{1}{2}\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}\frac{\sqrt{2}-i}{2} - \frac{1}{2}e^{i\pi/4}\\ \frac{\sqrt{2}-i}{2} + \frac{1}{2}e^{i\pi/4}\end{pmatrix}$$

$$= \frac{1}{2\sqrt{2}}\begin{pmatrix}\sqrt{2}-i-e^{i\pi/4}\\ \sqrt{2}-i+e^{i\pi/4}\end{pmatrix}$$

3.16

a.) $$XY|0\rangle = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}$$

$$= \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0\\ i\end{pmatrix}$$

$$= \begin{pmatrix}i\\ 0\end{pmatrix}$$

$$\text{R.H.S} = iZ|0\rangle = i\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}$$

$$= i\begin{pmatrix}1\\ 0\end{pmatrix}$$

$$= \begin{pmatrix}i\\ 0\end{pmatrix}$$

= L.H.S

$$XY|1\rangle = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix}$$

$$= \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}-i\\ 0\end{pmatrix}$$

$$= \begin{pmatrix}0\\ -i\end{pmatrix}$$

$$iZ|1\rangle = i\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix}$$

$$= i\begin{pmatrix}0\\ -1\end{pmatrix}$$

$$= \begin{pmatrix}0\\ -i\end{pmatrix} = \text{L.H.S.}$$

b) $XY = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$= \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$

$iZ = i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$

$\therefore XY = iZ$

3.17

$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}$

$U^{\dagger} U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}$

$= \frac{1}{2} \begin{pmatrix} 1 - i^2 & i + i \\ -i - i & -i^2 + 1 \end{pmatrix}$

$= \frac{1}{2} \begin{pmatrix} 2 & 2i \\ -2i & 2 \end{pmatrix}$

$= \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \neq I$

$\therefore$ It is not a quantum gate.

3.18

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$$

$$\therefore U^{+}U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1-i^2 & 1+i^2 \\ 1+i^2 & 1-i^2 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \frac{2}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= I.$$

$\therefore U$ is a quantum gate.

$$\therefore U|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$U|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

3.19

a) $$U^{+} = \begin{pmatrix} \frac{1+\sqrt{3}}{2\sqrt{2}} - i\frac{1-\sqrt{3}}{2\sqrt{6}} & \frac{-1+\sqrt{3}}{2\sqrt{6}} - i\frac{1-\sqrt{3}}{2\sqrt{6}} \\ \frac{1-\sqrt{3}}{2\sqrt{6}} - i\frac{1-\sqrt{3}}{2\sqrt{6}} & \frac{1+\sqrt{3}}{2\sqrt{2}} - i\frac{-1+\sqrt{3}}{2\sqrt{6}} \end{pmatrix}$$

b) $U^{+}U|\psi\rangle = |\psi\rangle$

$$= \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle.$$

3.20

a) $$|i\rangle\langle -| = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & -1 \\ i & -i \end{pmatrix}$$

$$\cancel{\frac{1}{2}\begin{pmatrix} 1 & -1 \\ i & \end{pmatrix}}$$

b) $$\frac{1}{2}\begin{pmatrix} 1 & -i \\ -1 & i \end{pmatrix}\frac{1}{2}\begin{pmatrix} 1 & -1 \\ i & -i \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 1-i^2 & -1+i^2 \\ -1+i^2 & 1-i^2 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\neq I.$$

$\therefore$ It is not valid

3·21

a.) $\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1&0\end{pmatrix}+\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0&1\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1&0\end{pmatrix}\cancel{+}-\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0&1\end{pmatrix}\right]$

$$=\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1&0\\0&0\end{pmatrix}+\begin{pmatrix}0&1\\0&0\end{pmatrix}+\begin{pmatrix}0&0\\1&0\end{pmatrix}-\begin{pmatrix}0&0\\0&1\end{pmatrix}\right]$$

$$=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$$

b) $\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}=\frac{1}{2}\begin{pmatrix}1+1&1-1\\1-1&1+1\end{pmatrix}=\frac{2}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix}$

$=I.$

$\therefore$ It is a valid quantum gate.

3·22

$|+\rangle\langle+|+|-\rangle\langle-|=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\end{pmatrix}+\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1&-1\end{pmatrix}$

$$=\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}+\frac{1}{2}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}$$

$$=\frac{1}{2}\begin{pmatrix}2&0\\0&2\end{pmatrix}=\frac{2}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix}=I.$$

3.23

$$|0\rangle\langle 0| + |+\rangle\langle +| = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}$$

$$= \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$$

$$= \begin{pmatrix}1/2 & \frac{1}{2}\\ \frac{1}{2} & 1/2\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} \neq I.$$

Chapter-4

4.1

a.) When the other ship is also on one of the planets of centarious.

b.) Red moves to PHI PLUS.
Blue moves to PHI MINUS

c.) ~~They~~ The red player can move to Omega Two and then to Omega Three.
The blue player moves to Psi.

c.) Red player moves to Omega Zero and then to Omega Three.
Blue player moves directly to Omega Three using H card.

4.2

a.) 5

b.) conceptual

c.) 50 – 60%.

d.) entanglement.

4.3

a.) $\langle 10|11\rangle = \langle 1|1\rangle . \langle 0|1\rangle = 0$

b.) $\langle +-|01\rangle = \langle +|0\rangle\langle -|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$

$= \frac{1}{\sqrt{2}}(\langle 0|0\rangle + \langle 1|0\rangle)\frac{1}{\sqrt{2}}(\langle 0|1\rangle - \langle 1|1\rangle)$ $\cancel{\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\-1 & -1\end{pmatrix}}$

$= \frac{1}{\sqrt{2}} \cdot (-\frac{1}{\sqrt{2}})$ $\cancel{= \frac{1}{2}\begin{pmatrix}1-1 & 1-1\\1-1 & 1-1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0 & 0\\0 & 0\end{pmatrix} = 0}$

$= -\frac{1}{2}$

c) $\langle 1+0 | 1-0 \rangle = \langle 1|1\rangle \cdot \langle + | - \rangle \cdot \langle 0|0\rangle$

$$= 1 \cdot \frac{1}{\sqrt{2}}(1-1) \cdot 1$$

$$= 1 \cdot 0 \cdot 1$$

$$= 0.$$

4.4

$$|1\rangle \otimes |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\therefore |1\rangle \otimes |1\rangle \otimes |0\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

4.5

a) $$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\therefore |\psi\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \frac{i}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \frac{\sqrt{3}+i}{4}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

$$= \begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{i}{\sqrt{2}} \\ \frac{\sqrt{3}+i}{4} \end{pmatrix}$$

b) $\langle 00| = (1\ 0\ 0\ 0) \qquad \langle 10| = (0\ 0\ 1\ 0) \qquad \langle 11| = (0\ 0\ 0\ 1)$

$$\therefore \langle \psi | = \begin{pmatrix} \frac{1}{2} & 0 & \frac{-i}{\sqrt{2}} & \frac{\sqrt{3}-i}{4} \end{pmatrix}$$

4.6

$|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|$

$$= \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix} + \begin{pmatrix}0\\1\\0\\0\end{pmatrix}\begin{pmatrix}0 & 1 & 0 & 0\end{pmatrix} + \begin{pmatrix}0\\0\\1\\0\end{pmatrix}\begin{pmatrix}0 & 0 & 1 & 0\end{pmatrix} + \begin{pmatrix}0\\0\\0\\1\end{pmatrix}\begin{pmatrix}0 & 0 & 0 & 1\end{pmatrix}$$

$$= \begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\\0&0&0\end{pmatrix}$$

$$= \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} = I.$$

$x =$

4.7

$|00\rangle$ with probability $\left|\frac{i}{\sqrt{10}}\right|^2 = \frac{1}{10}$

$|01\rangle$ " " $\left|\frac{1-2i}{\sqrt{10}}\right|^2 = \frac{5}{10} = \frac{1}{2}$

$|10\rangle$ " " $\left|\frac{e^{i\pi/100}}{\sqrt{10}}\right|^2 = \frac{1}{10}$

$|11\rangle$ " " $\left|\frac{\sqrt{3}}{\sqrt{10}}\right|^2 = \frac{3}{10}$

4.8

$$A\left(\frac{1}{2}|00\rangle + i|01\rangle + \sqrt{2}\,|10\rangle - |11\rangle\right)$$

$$\therefore |A|^2 \cdot \left(\frac{1}{4} + 1 + 2 + 1\right) = 1$$

$$\Rightarrow |A|^2 = \frac{4}{17}.$$

$$\therefore A = \frac{2}{\sqrt{17}}.$$

$\therefore$ The normalized state is: $\frac{1}{\sqrt{17}}\left(|00\rangle + 2i|01\rangle + 2\sqrt{2}\,|10\rangle - 2|11\rangle\right)$.

4.9

The states are $|0\rangle$ and $|1\rangle$ when measuring only the left qubit.

$\therefore$ $|0\rangle$ with probability $\left|\frac{1}{4}\right|^2 + \left|\frac{1}{2}\right|^2 = \frac{5}{16}$ the state collapses to $A\left(\frac{1}{4}|00\rangle + \frac{1}{2}|01\rangle\right)$.

$|1\rangle$ " " $\left|\frac{1}{\sqrt{2}}\right|^2 + \left|\frac{\sqrt{3}}{4}\right|^2 = \frac{1}{2} + \frac{3}{16} = \frac{11}{16}$ & $B\left(\frac{1}{\sqrt{2}}|10\rangle + \frac{\sqrt{3}}{4}|$

$$A = \frac{4}{\sqrt{5}}, \quad B = \frac{4}{\sqrt{11}}$$

$\therefore$ The resulting states are $\frac{1}{\sqrt{5}}|00\rangle + \frac{2}{\sqrt{5}}|01\rangle$ for left qub[it] $|0\rangle$

and $\frac{2\sqrt{2}}{\sqrt{11}}|10\rangle + \frac{\sqrt{3}}{\sqrt{11}}|11\rangle$ for left qubit $|1\rangle$.

4.10

~~Prob($|00\rangle$) =~~

$|00\rangle$ with probability $\left|\frac{1}{6}\right|^2 + \left(\frac{1}{\sqrt{6}}\right)^2 = \frac{1}{36} + \frac{1}{6} = \frac{7}{36}$

$|01\rangle$ " " $\left|\frac{1}{3\sqrt{2}}\right|^2 + \left|\frac{1}{2}\right|^2 = \frac{1}{18} + \frac{1}{4} = \frac{11}{36}$

$|10\rangle$ " " $\left|\frac{1}{6}\right|^2 + \left|\frac{1}{6}\right|^2 = \frac{2}{36}$

$|11\rangle$ with probability $\left|\frac{1}{3}\right|^2 + \left|\frac{1}{\sqrt{3}}\right|^2 = \frac{1}{9} + \frac{1}{3} = \frac{4}{9} = \frac{16}{36}$

4.11

a) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.

$$|\psi_1\rangle|\psi_0\rangle = (\alpha_1|0\rangle + \beta_1|1\rangle)(\alpha_0|0\rangle + \beta_0|1\rangle)$$
$$= \alpha_1\alpha_0|00\rangle + \alpha_1\beta_0|01\rangle + \beta_1\alpha_0|10\rangle + \beta_1\beta_0|11\rangle$$

$\therefore \alpha_1\beta_0 = \frac{1}{\sqrt{2}} \quad \beta_1\alpha_0 = \frac{1}{\sqrt{2}} \quad \alpha_1\alpha_0 = 0 \quad \beta_1\beta_0 = 0$

If either α_1, α_0 or β_1, β_0 is equal to zero, then $\alpha_1\beta_0 = \frac{1}{\sqrt{2}}$ and $\beta_1\alpha_0 = \frac{1}{\sqrt{2}}$ will be false.

Hence it is an entangled state.

b) $\frac{1}{\sqrt{2}}(|10\rangle + i|11\rangle)$.

$\therefore \alpha_1\alpha_0 = 0, \ \alpha_1\beta_0 = 0, \ \beta_1\alpha_0 = \frac{1}{\sqrt{2}}, \ \beta_1\beta_0 = \frac{i}{\sqrt{2}}$

$\cancel{\alpha_0 = \frac{1}{\sqrt{2}\beta_1}} \quad \beta_1 = \frac{1}{\sqrt{2}\alpha_0} \quad \& \quad \frac{1}{\sqrt{2}\alpha_0}\beta_0 = \frac{i}{\sqrt{2}}$

$\Rightarrow \beta_0 = i\alpha_0$

And $\alpha_1 = 0$.

$$\therefore |\psi_1\rangle|\psi_0\rangle = \left(0.|0\rangle + \frac{1}{\sqrt{2}\alpha_0}|1\rangle\right)(\alpha_0|0\rangle + i\alpha_0|1\rangle)$$
$$= \frac{1}{\sqrt{2}}|1\rangle(|0\rangle + i|1\rangle)$$
$$= |1\rangle\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle\right)$$

$\therefore$ It is a product state.

4.12

a) $\frac{1}{4}\left(3|00\rangle - \sqrt{3}|01\rangle + \sqrt{3}|10\rangle - |11\rangle\right)$.

$\therefore \alpha_1\alpha_0 = \frac{3}{4} \quad \alpha_1\beta_0 = -\frac{\sqrt{3}}{4}, \quad \beta_1\alpha_0 = \frac{\sqrt{3}}{4}, \quad \beta_1\beta_0 = -\frac{1}{4}$

$\therefore \alpha_1 = \frac{3}{4\alpha_0}$

Putting α_1 in $\alpha_1\beta_0$, $\quad \frac{3}{4\alpha_0}\beta_0 = -\frac{\sqrt{3}}{4}$

$\Rightarrow \beta_0 = -\frac{1}{\sqrt{3}}\alpha_0$.

Again, $\beta_1 = \frac{\sqrt{3}}{4\alpha_0}$

Putting $\beta_1 = \frac{\sqrt{3}}{4\alpha_0}$ & $\beta_0 = -\frac{1}{\sqrt{3}}\alpha_0$ in $\beta_1\beta_0$,

$\frac{\sqrt{3}}{4\alpha_0}\cdot\left(-\frac{\alpha_0}{\sqrt{3}}\right) = -\frac{1}{4}$

which is true.

$\therefore$ The state is product state.

$\therefore |\psi_1\rangle|\psi_0\rangle = \left(\alpha_1|0\rangle + \beta_1|1\rangle\right)\left(\alpha_0|0\rangle + \beta_0|1\rangle\right)$

$= \left(\frac{3}{4\alpha_0}|0\rangle + \frac{\sqrt{3}}{4\alpha_0}|1\rangle\right)\left(\alpha_0|0\rangle - \frac{\alpha_0}{\sqrt{3}}|1\rangle\right)$.

$= \left(\frac{3}{4}|0\rangle + \frac{\sqrt{3}}{4}|1\rangle\right)\left(|0\rangle - \frac{1}{\sqrt{3}}|1\rangle\right)$

b) $\frac{1}{\sqrt{3}}|0\rangle|+\rangle + \sqrt{\frac{2}{3}}|1\rangle|-\rangle$

$= \frac{1}{\sqrt{3}}|0\rangle\,\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) + \sqrt{\frac{2}{3}}|1\rangle\cdot\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$

$= \frac{1}{\sqrt{6}}\left(|00\rangle + |01\rangle\right) + \frac{1}{\sqrt{3}}\left(|10\rangle - |11\rangle\right)$

Now, $\alpha_1\alpha_0 = \frac{1}{\sqrt{6}}$, $\alpha_1\beta_0 = \frac{1}{\sqrt{6}}$, $\beta_1\alpha_0 = \frac{1}{\sqrt{3}}$, $\beta_1\beta_0 = -\frac{1}{\sqrt{3}}$

Now, $\alpha_1 = \frac{1}{\sqrt{6}\,\alpha_0}$ — (i)

Putting (i) in $\alpha_1\beta_0 = \frac{1}{\sqrt{6}}$

$$\Rightarrow \frac{1}{\sqrt{6}\,\alpha_0}\beta_0 = \frac{1}{\sqrt{6}}.$$

$$\Rightarrow \beta_0 = \alpha_0.$$

Again, $\beta_1 = \frac{1}{\sqrt{3}\,\alpha_0}$.

Putting value of β_1 and β_0 in $\beta_1\beta_0 = -\frac{1}{\sqrt{3}}$,

$$\Rightarrow \frac{1}{\sqrt{3}\alpha_0}\cdot\alpha_0 \neq -\frac{1}{\sqrt{3}}$$

Since it is inconsistent so it is an entangled state.

4.13

a) $X \otimes I\,|\psi^+\rangle = \frac{1}{\sqrt{2}}(|11\rangle + |00\rangle)$ [applying to left bit only].

$$= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$= |\phi^+\rangle$$

b) $X \otimes I\,|\phi^+\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$

$$= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$= |\psi^+\rangle$$

c.) $X \otimes I\,|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|11\rangle - |00\rangle\right)$

$$= \frac{1}{\sqrt{2}}\left(|11\rangle \cancel{+e^{i\pi}} - |00\rangle\right)$$

$$\cancel{= e^{-i\pi}\frac{1}{\sqrt{2}}\left(e^{i\pi}|11\rangle + e^{i\pi}\right.}$$

$$= -\frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

$$= -|\phi^-\rangle$$

d.) $X \otimes I\,|\phi^-\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle - |01\rangle\right)$

$$= -\frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

$$= -|\psi^-\rangle$$

4.14

a.) $H \otimes X = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

b.) $|\psi\rangle = \frac{1}{4}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle + \frac{\sqrt{3}}{4}|11\rangle$

$\therefore (H \otimes X)|\psi\rangle = \frac{1}{4}|+\rangle|1\rangle + \frac{1}{2}|+\rangle|0\rangle + \frac{1}{\sqrt{2}}|-\rangle|1\rangle + \frac{\sqrt{3}}{4}|-\rangle|0\rangle$

$$= \frac{1}{4}\cdot\frac{1}{\sqrt2}(|0\rangle+|1\rangle)|1\rangle + \frac{1}{2\sqrt2}(|0\rangle+|1\rangle)|0\rangle + \frac{1}{\sqrt2\sqrt2}(|0\rangle-|1\rangle)|1\rangle + \frac{\sqrt3}{4\sqrt2}(|0\rangle-|1\rangle)|0\rangle$$

$$= \frac{1}{4\sqrt2}(|01\rangle+|11\rangle) + \frac{1}{2\sqrt2}(|00\rangle+|10\rangle) + \frac{1}{2}(|01\rangle-|11\rangle) + \frac{\sqrt3}{4\sqrt2}(|00\rangle-|10\rangle)$$

$$= \left(\frac{2\sqrt2+\sqrt3}{4\sqrt2}\right)|00\rangle + \left(\frac{1+2\sqrt2}{4\sqrt2}\right)|01\rangle + \frac{2-\sqrt3}{4\sqrt2}|10\rangle + \frac{1-2\sqrt2}{4\sqrt2}|11\rangle$$

4.15

a.) $$CNOT(X\otimes I) = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$= \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix}$$

$$= \begin{pmatrix}0&0&1&0\\0&0&0&1\\0&1&0&0\\1&0&0&0\end{pmatrix}$$

$$(X\otimes X)CNOT = \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

$$= \begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

$$= \begin{pmatrix}0&0&1&0\\0&0&0&1\\0&1&0&0\\ \dots\end{pmatrix}$$

$\therefore$ L.H.S = R.H.S

b) $CNOT(I \otimes X) = CNOT \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(I \otimes X)CNOT = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\therefore$ L.H.S = R.H.S

c) $CNOT(Z \otimes I) = CNOT \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(Z \otimes I)CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$\therefore$ L.H.S = R.H.S

d.) $CNOT(I\otimes Z) = CNOT \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$(Z\otimes Z)\,CNOT = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} CNOT$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$\therefore$ L.H.S = R.H.S.

4.16

a.) $A = A'$
$B = B'$

$C' = \cancel{A\oplus}(A\oplus C)\oplus B = A\oplus B\oplus C$

A	B	C	A'	B'	C'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	0	1	0
1	0	0	1	0	1
1	0	1	1	0	0
1	1	0	1	1	0
1	1	1	1	1	1

b) This circuit performs the same operation of XOR as in the reversible circuit in Ex. 1.43

4.17

let the ~~orig~~ left and right cubit be l, r.

$\therefore X \otimes I \,|l r\rangle = |l \oplus 1, r\rangle$

$CNOT\,|l\oplus 1, r\rangle = |l\oplus 1, r \oplus (l \oplus 1)\rangle$

Here, r is flipped if and only if $c=0$

Again, $X \otimes I\,|l\oplus 1, r\oplus(l\oplus 1)\rangle = |l, r\oplus(l\oplus 1)\rangle$

$\therefore$ The final state is $|l\; r\oplus l\oplus 1\rangle$ which is equivalent to a ACNOT gate.

4.18

$CNOT\,|+\rangle|-\rangle = (H\otimes H)\,CNOT_{01}\,(H\otimes H)\,|+\rangle|-\rangle$

$= (H\otimes H)\,CNOT_{01}\,|0\rangle|1\rangle$

$= (H\otimes H)\,|0\rangle|1\rangle$

$= |-\rangle|-\rangle$

$CNOT\,|-\rangle|+\rangle = (H\otimes H)\,CNOT_{01}\,(H\otimes H)\,|-\rangle|+\rangle$

$= (H\otimes H)\,CNOT_{01}\,|1\rangle|0\rangle = (H\otimes H)\,|1\rangle|0\rangle$

$= |-\rangle|+\rangle$

$$\mathrm{CNOT}\,|-\rangle|-\rangle = (H\otimes H)\,\mathrm{CNOT}_{01}\,(H\otimes H)\,|-\rangle|-\rangle$$
$$= (H\otimes H)\,\mathrm{CNOT}_{01}\,|1\rangle|1\rangle$$
$$= (H\otimes H)\,|0\rangle|1\rangle = |+\rangle|-\rangle.$$

$\square$

4.19

$$|\phi^+\rangle\langle\phi^+| + |\phi^-\rangle\langle\phi^-| + |\psi^+\rangle\langle\psi^+| + |\psi^-\rangle\langle\psi^-|$$
$$= \tfrac{1}{2}(|00\rangle+|11\rangle)(\langle 00|+\langle 11|) + \tfrac{1}{2}(|00\rangle-|11\rangle)(\langle 00|-\langle 11|) + \tfrac{1}{2}(|01\rangle+|10\rangle)(\langle 01|+\langle 10|)$$
$$+ \tfrac{1}{2}(|01\rangle-|10\rangle)(\langle 01|-\langle 10|)$$

$$= |00\rangle\langle 00| + |11\rangle\langle 11| + |01\rangle\langle 01| + |10\rangle\langle 10|$$

$$= \begin{pmatrix}1\\0\\0\\0\end{pmatrix}(1\ 0\ 0\ 0) + \begin{pmatrix}0\\0\\0\\1\end{pmatrix}(0\ 0\ 0\ 1) + \begin{pmatrix}0\\1\\0\\0\end{pmatrix}(0\ 1\ 0\ 0) + \begin{pmatrix}0\\0\\1\\0\end{pmatrix}(0\ 0\ 1\ 0)$$

$$= \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} = I_4.$$

4.20

$$Z = \begin{pmatrix}1&0\\0&-1\end{pmatrix} \qquad a=1,\ b=0 \qquad c=0,\ d=-1$$

$$\therefore CZ = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}$$

$$CZ|00\rangle = |00\rangle$$
$$CZ|01\rangle = |01\rangle$$
$$CZ|10\rangle = |1\rangle\otimes|0\rangle = |10\rangle$$
$$CZ|11\rangle = |1\rangle\otimes(-|1\rangle) = -|11\rangle$$

4.21

a) $SWAP|w_0\rangle = \frac{1}{2}(|00\rangle - |10\rangle + |01\rangle + |11\rangle)$

$= \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle + |11\rangle) = w_3$

b) $(X \otimes I)|w_1\rangle = \frac{1}{2}(-|10\rangle + |11\rangle + |00\rangle + |01\rangle)$

$= \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle + |11\rangle) = w_3$

c) $CNOT_{01}|w_2\rangle = \frac{1}{2}(|00\rangle + |11\rangle + |10\rangle - |01\rangle)$

$= \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle) = w_0$.

d) $CNOT_{10}|w_3\rangle = \cancel{\frac{1}{2}(|10\rangle + |01\rangle - |00\rangle + |11\rangle)}\ \frac{1}{2}(|00\rangle + |01\rangle - |11\rangle + |10\rangle)$

$\cancel{= \frac{1}{2}(|00\rangle + |01\rangle - |11\rangle}$

$= \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle) = w_2$

4.22

$|00\rangle \to \frac{1}{\sqrt{2}}(|00\rangle + i|11\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\0\\i\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\i\end{pmatrix}$

$|01\rangle \to \frac{1}{\sqrt{2}}(|01\rangle - i|10\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\-i\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-i\\0\end{pmatrix}$

$|10\rangle \to \frac{1}{\sqrt{2}}(|10\rangle - i|01\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\-i\\0\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\-i\\1\\0\end{pmatrix}$

$|11\rangle \to \frac{1}{\sqrt{2}}(|11\rangle + i|00\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}i\\0\\0\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}i\\0\\0\\1\end{pmatrix}$

$$\therefore MS = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & -i & 0 \\ 0 & -i & 1 & 0 \\ i & 0 & 0 & 1 \end{pmatrix}$$

b) Done using calculator. $MS^8 = I$.

4.23

Let, the qubits be a (control 1), b (control 2), c (target)

I am actually confused on how to prove this.

4.24

a) The anti-Toffoli gate flips the third or right bit only if the first and second (left & middle) bits are 0. So the transformation will be:

$$\text{Toffoli}\,|000\rangle = |001\rangle$$
$$\text{Toffoli}\,|001\rangle = |000\rangle$$
$$\text{Toffoli}\,|010\rangle = |010\rangle$$
$$\vdots$$
$$\text{Toffoli}\,|111\rangle = |111\rangle.$$

b.) anti-Toffoli = $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

4.25

a.) $|+\rangle$ and $|-\rangle$ are orthogonal, so yes.

b) $|i\rangle$ and $|-i\rangle$ are orthogonal. so yes.

c.) $\langle 0|+\rangle = \langle 0| \cdot \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$= \frac{1}{\sqrt{2}}\langle 0|0\rangle + \frac{1}{\sqrt{2}} \cdot \langle 0|1\rangle$

$= \frac{1}{\sqrt{2}}$

$\neq 0$

$\therefore$ No, these cannot be cloned.

4.28

a.) The CNOT between a_3 and b_3 and the bottommost s resulted in $a_3 \oplus (a_3 \oplus b_3) = b_3$ and $s_3 = a_3 \oplus b_3 \oplus c_3'$

Now replacing these two by a single CNOT between c_3 and b_3 gives $c_3' \oplus a_3 \oplus b_3 = s_3$. Hence it is correct

b) A carry is computed with 1 Toffoli & 2 CNOTs.

A sum " " " 2 CNOTS.

∴ Toffoli gates = $4n-2$

CNOT gates = $4n$

4.30

a) Toffoli gates: $4\times4-2 = 14$

CNOT gates: $4\times4 = 16$

b) Toffoli: $4\times8-2 = 30$

CNOT: $4\times8 = 32$

4.31

$3\times4+1 = 13$.

∴ To add numbers of length n.: $3n+1$

4.32

a) Tom Wong. They are the same person.

b) "hands", quantum, some, no faster.

4.33

a) ~~Lack~~ {Toffoli, H, Z} lacks complex amplitudes & generation of more than the Clifford group.

b) {H, X, Y, Z, S, T} lacks 'generate more than the Clifford group'

c) {SWAP, H, S, T} lacks entanglement & 'more than Clifford group'.

4.34.

The 3 ways are to replace either of CNOT, H and S with Toffoli, $R_{\pi/8}$ and T gates respectively.

4.35

For first CNOT, control qubit is 1, target is qubit 2.

$\therefore |000\rangle \rightarrow |000\rangle$

$|100\rangle \rightarrow |110\rangle$.

For second CNOT, control qubit is 1, ~~targ~~ target is qubit 3.

$\therefore |000\rangle \rightarrow |000\rangle$

$|110\rangle \rightarrow |111\rangle$.

$\therefore$ The final state is $\alpha|000\rangle + \beta|111\rangle = \alpha|0_L\rangle + \beta|1_L\rangle$

4.36.

a.) No error.

b) q_0 flipped. Apply X gate to q_0.

c) q_2 " . Apply X gate to q_2

d) q_1 " . Apply X " " q_1

4.37

After first CNOT, $|000\rangle \rightarrow |000\rangle$
$|100\rangle \rightarrow |110\rangle$

After 2^{nd} CNOT, $|000\rangle \rightarrow |000\rangle$
$|110\rangle \rightarrow |111\rangle$

After applyin H gate,

$|000\rangle \rightarrow |+++\rangle$

$|111\rangle \rightarrow |---\rangle$.

$\therefore \alpha|+++\rangle + |---\rangle = \alpha|0_L\rangle +$

4.38.

a) After applying H to $|a\rangle$ & $|b\rangle$,

$|a\rangle \rightarrow |0\rangle$

$|b\rangle \rightarrow |0\rangle$

~~The~~ Output of CNOT gates $\rightarrow |0 \oplus 0\rangle = |0\rangle$

Final states $\rightarrow$ ~~$|a\rangle = |+\rangle$~~ $= |0\rangle \rightarrow |+\rangle$

$|b\rangle = |0\rangle \rightarrow |+\rangle$

$\therefore$ ~~$|00\rangle$~~

$\therefore |++\rangle$

b) Resulting state:

$|a\rangle = |+\rangle$

$|b\rangle = |-\rangle$.

$|0\rangle \rightarrow |1\rangle$.

$\therefore |-+\rangle$

c) $|a\rangle = |-\rangle$

$|b\rangle = |+\rangle$

$|0\rangle \rightarrow |1\rangle$

$\therefore |+-\rangle$

d) $|a\rangle = |-\rangle$, $|b\rangle = |-\rangle$

$|0\rangle \rightarrow |0\rangle$

$\therefore |--\rangle$

e) It does so because the Hadamard gate changes the phases to X-basis and thus the $|0\rangle$ state is used to calculate the parity ~~etc~~ which results in $|0\rangle$ if the number of minus is even and in $|1\rangle$ otherwise.

4.39

This is equivalent to the circuit from the text because ~~in~~ for $|b\rangle$, there ~~has been used~~ is the use of Hadamard gate two times which results to the same output as the input. In case of $|C\rangle$ as well, the H gate has been used just before it was used as the control bit for the NOT gate; which is equivalent to the structure of the circuit in text.

4.40

$$\alpha\left(i\sqrt{1-\epsilon^2}\,|{+}{+}{+}\rangle + \epsilon|{-}{+}{+}\rangle\right) + \beta\left(\epsilon|{+}{-}{-}\rangle + i\sqrt{1-\epsilon^2}\,|{-}{-}{-}\rangle\right)$$

$$= \alpha i\sqrt{1-\epsilon^2}\,|{+}{+}{+}\rangle + \alpha\epsilon|{-}{+}{+}\rangle + \beta\epsilon|{+}{-}{-}\rangle + \beta i\sqrt{1-\epsilon^2}\,|{-}{-}{-}\rangle$$

a) $$\left|\alpha i\sqrt{1-\epsilon^2}\right|^2 + \left|\beta i\sqrt{1-\epsilon^2}\right|^2 = |\alpha|^2(1-\epsilon^2) + |\beta|^2(1-\epsilon^2)$$
$$= (|\alpha|^2+|\beta|^2)(1-\epsilon^2) = 1-\epsilon^2$$

The state collapses to $A\left(\alpha i\sqrt{1-\epsilon^2}\,|{+}{+}{+}\rangle + \beta i\sqrt{1-\epsilon^2}\,|{-}{-}{-}\rangle\right)$
$= \alpha|{+}{+}{+}\rangle + \beta|{-}{-}{-}\rangle$; $A = \frac{1}{i\sqrt{1-\epsilon^2}}$.

$\therefore$ No gate required.

b) $$\therefore |\beta\epsilon|^2 + |\alpha\epsilon|^2 = |\alpha|^2|\epsilon|^2 + |\beta|^2|\epsilon|^2$$
$$= (|\alpha|^2+|\beta|^2)|\epsilon|^2 = |\epsilon|^2.$$

The state collapses to $B\left(\beta\epsilon|{+}{-}{-}\rangle + \alpha\epsilon|{-}{+}{+}\rangle\right) = \beta|{+}{-}{-}\rangle + \alpha|{-}{+}{+}\rangle$
; $B = \frac{1}{\epsilon}$.

To correct the state, we apply $Z \otimes I \otimes I$.

c) The probability is 0.

d) Probability is 0.

4.41

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.

$\therefore |\psi 00000000\rangle = \alpha|000000000\rangle + \beta|100000000\rangle$

a) $|000\rangle \to |000\rangle$

$|100\rangle \to |111\rangle$.

$\therefore$ Final state becomes $\alpha|000\rangle|000\rangle|000\rangle + \beta|100\rangle|100\rangle|100\rangle$

b) $H|000\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |100\rangle)$.

$\therefore \alpha|000\rangle|000\rangle|000\rangle \to \frac{\alpha}{\sqrt{2}}(|000\rangle + |100\rangle)(|000\rangle + |100\rangle)(|000\rangle + |100\rangle)$

again, $\beta|100\rangle|100\rangle|100\rangle \to \frac{\beta}{\sqrt{2}}(|000\rangle - |100\rangle)(|000\rangle - |100\rangle)(|000\rangle - |100\rangle)$

$\therefore \frac{\alpha}{\sqrt{2}}(|000\rangle + |100\rangle)(|000\rangle + |100\rangle)(|000\rangle + |100\rangle) + \frac{\beta}{\sqrt{2}}(|000\rangle - |100\rangle)(|000\rangle - |100\rangle)(|000\rangle - |100\rangle)$

c) Here, $|000\rangle \to |000\rangle$

$|100\rangle \to |111\rangle$.

Again, $\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2^{3/2}}$.

$\therefore \frac{\alpha}{2^{3/2}}(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle) + \frac{\beta}{2^{3/2}}(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$

4·42

Yes there are.

q_6, q_5, q_1 bits flipped.

To correct them, apply X gates to those bits.

4·43

a) $(|000\rangle + |111\rangle \to |000\rangle + |100\rangle$

$$|000\rangle \to H|0\rangle \otimes |00\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)(|00\rangle)$$
$$= \frac{1}{\sqrt{2}}(|000\rangle + |100\rangle).$$

$$|100\rangle \to H|1\rangle \otimes |00\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|00\rangle$$
$$= \frac{1}{\sqrt{2}}(|000\rangle - |100\rangle)$$

$$\therefore |000\rangle + |100\rangle \to \sqrt{2}|000\rangle$$

Again, $|000\rangle - |111\rangle \to |000\rangle - |100\rangle$.

$$|000\rangle \to H|0\rangle \otimes |00\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |100\rangle)$$
$$|100\rangle \to H|1\rangle \otimes |00\rangle = \frac{1}{\sqrt{2}}(|000\rangle - |100\rangle).$$

$$\therefore |000\rangle - |100\rangle \to \sqrt{2}|100\rangle$$

$$\therefore \alpha(|000\rangle + |111\rangle)(|000\rangle - |111\rangle)(|000\rangle + |111\rangle) \to 2|000\rangle|100\rangle|000\rangle$$
$$= \alpha|000100000\rangle.$$

$$\beta(|000\rangle - |111\rangle)(|000\rangle + |111\rangle)(|000\rangle - |111\rangle) = 2|100\rangle\cdot|000\rangle\cdot|100\rangle.$$
$$= \beta|100000100\rangle.$$

$$\therefore \alpha|000100000\rangle + \beta|100000100\rangle$$

b) For $\alpha|000100000\rangle$, $q_2=0$, $q_5=1$.

$\therefore$ The phase parity of two adjacent triplets is 1 after the first two CNOTs.

~~For $\beta|100000100\rangle$, q~~

similarly, it can be shown that it stores the phase parity for the adjacent triplets in case of $\beta|100000100\rangle$

c) Since the H gate is applied two times each time to q_2, q_5 and q_8, hence the output remains the same with another CNOT gate.

d) The Z gate may be applied to q_5, ~~q_4 i.e. any one of the middle triplets~~.

4.46

a) Yes.

q_7, q_3 and q_2 flipped.

Apply X gate to these qubits.

b) Yes.

Triplet_0 flipped.

Apply Z gate to any one of q_2, q_1 or q_0.