Tom Wang Ch-01

sat sun mon tue ~~wed~~ thu fri Date: 25 / 06 / 25

1.1: (a) $2^4 = 16$ for 4 coins (b) $2^5 = 32$ for 5 coins

1.2: (a) 6^4 for 4 dices (b) 6^5 for 5 dices

1.3: 1

1.4: (a) 5 coins needed for 26 letters

(b) 2 dices " " " "

~~(c)~~

1.5: (a) $10111_2 = 23_{10}$

(b) $11001010_2 = 203_{10}$

1.6: $42 = 101010_2$

$495 = 111101111$

1.7: a) $3B7C_{16} = 15228_{10}$

b) $FF = 1111\,1111_{(2)}$

c) FA,10,E4 = 250,10,228

1.8:
000 = 0
001 = 1
010 = 2
011 = 3
100 = -4
101 = -3
110 = -2
111 = -1

$-4+2+1$

1.9: $\underbrace{1001011}_{K}$ $\underbrace{1000001}_{A}$ $\underbrace{1011010}_{Z}$ $\underbrace{1001001}_{I}$

Quantum

1.10: 1010001 1110101 1100001 1101110 1110100 1111001 1101101
Q u a n t u m

1.11: (a)

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}+\bar{B}$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

(b) negative-OR = NAND

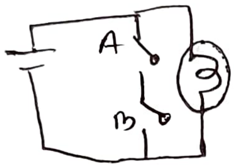
$\overline{AB}$

1.12: (a)

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}\bar{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

(b) negative-AND = NOR

1.13: the light bulb will always be on except if both switches are 1, or ON state. (a-d)



(e) The circuit correspond to NAND gate.

1.14: OR gate.

1.15: NOR gate

1.16: XOR gate

1.17: (a) MP944 (20 bit) 74,442 year 1970

(b) AMD Ryzen 7 (64 bit) 10,700,000,000 year 2021

1.18: XOR of 3 input:

A	B	C	$A \oplus B \oplus C$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

even parity = 0

odd parity = 1

1.19: $y = C + \overline{A+B}$

C	A	B	$\overline{A+B}$	$C+\overline{A+B}$
0	0	0	1	1
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

1.20: (a) 4

(b) $2^4 = 16$

(c) $2^8 = 256$

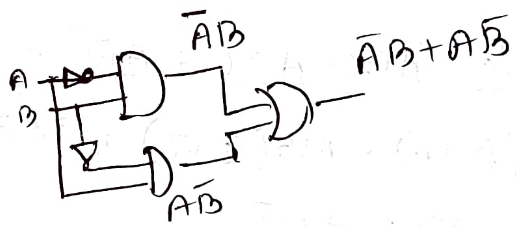
(d) $2^{2^4} = 2^{16}$

(e) 2^{2^n}

1.21: XOR

A	B	C
0	0	0
0	1	1
1	0	1
1	1	0

$\bar{A}B + A\bar{B}$



1.22: $\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}C + ABC$

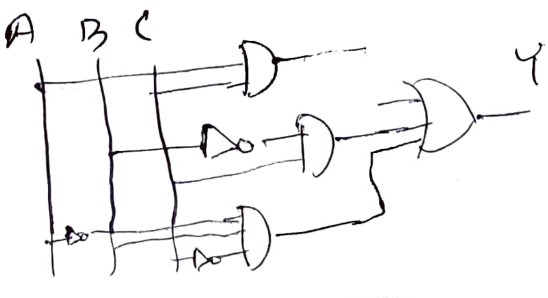
$\bar{A}\bar{C} + AC$

$= \bar{B}C(\bar{A}+A) + \cancel{B(\bar{A}\bar{C}+AC)}$ $\bar{A}B\bar{C} + \cancel{ABC}$

$= \cancel{\bar{B}C + B(\bar{A}\bar{C}+AC)}$ $+ AC(\bar{B}+B)$

$= \bar{A}(\bar{B}C+B\bar{C})$

$= \cancel{A} + AC + \bar{B}C + B\bar{A}\bar{C}$



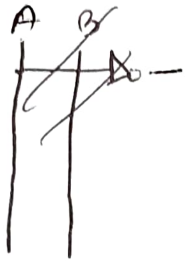
$Y = AC + \bar{B}C + B\bar{A}\bar{C}$

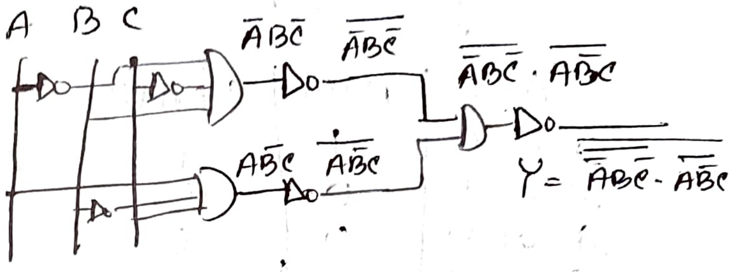
1.23:

$\bar{A}B\bar{C} + A\bar{B}C$

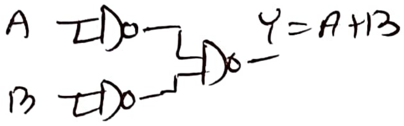
$= \overline{\overline{\bar{A}B\bar{C} + A\bar{B}C}}$

$= \overline{\overline{\bar{A}B\bar{C}} \cdot \overline{A\bar{B}C}}$

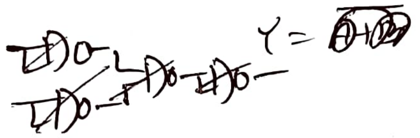




1.24:

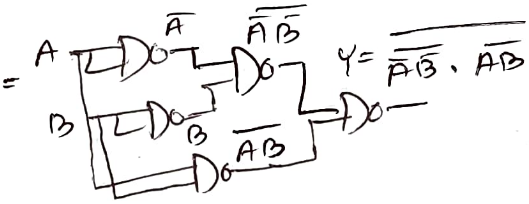


1.25: $\bar{A}\bar{B} + AB$



$= \overline{\overline{\bar{A}\bar{B} + AB}}$

$= \overline{\overline{\bar{A}\bar{B}} \cdot \overline{AB}}$ =



$\bar{A} + \bar{B} + \bar{C} = \overline{ABC}$

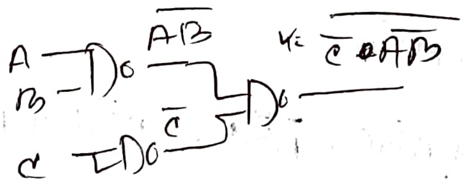
1.26: $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + ABC$

$= \bar{B}\bar{C}(\bar{A}+A) + AB(\bar{C}+C) + \bar{A}B\bar{C}$

$= \bar{B}\bar{C} + AB + \bar{A}B\bar{C}$

$= \bar{C}(\bar{A}B + \bar{B}) + AB$

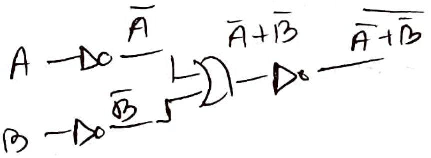
$= \bar{C} + AB$



$= \overline{\overline{\bar{C} + AB}}$

$= \overline{C \cdot \overline{AB}}$

1.27:



$\overline{\bar{A}+\bar{B}} = \bar{\bar{A}} \cdot \bar{\bar{B}} = AB$

1.28

A	A	$\overline{A+A}$
0	0	1
~~0~~	~~1~~	
~~1~~	~~0~~	
1	1	0

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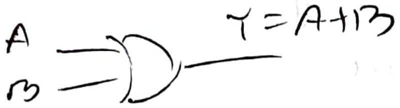
1.34 : $\overline{\bar{A}(A+\bar{B})} = \bar{\bar{A}} + \overline{A+\bar{B}} = A + \bar{A}\cdot B = A+B$

A	$\bar{A}$	B	$\bar{A}B$
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0



1.35 : $(A+\bar{B})(A+B+\bar{C})$

$= AA + AB + A\bar{C} + \bar{B}A + \bar{B}B + \bar{B}\bar{C}$

$= A + AB + A\bar{C} + A\bar{B} + \bar{B}\bar{C}$

$= A + A(B+\bar{B}) + \bar{C}(A+\bar{B})$

$= A + A + \bar{C}(A+\bar{B})$

$= A + \bar{C}(A+\bar{B})$

$= A + \bar{C}A + \bar{C}\bar{B}$

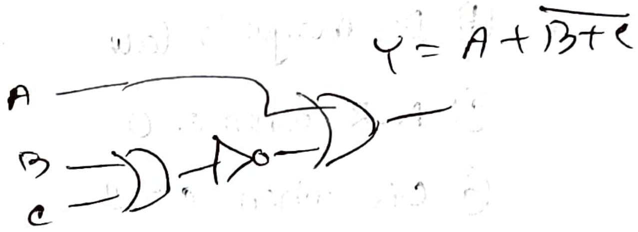
$= A(1+\bar{C}) + \bar{B}\bar{C}$

$= A + \bar{B}\bar{C}$

$= A + \overline{\overline{\bar{B}\bar{C}}}$

$= A + \overline{\bar{\bar{B}} + \bar{\bar{C}}}$

$= A + \overline{B+C}$



1.38 : (a), (b), (c), (d) = NO

1.39 : (a) Non-reversible
(b) Reversible.

1.40 : (a)

ABC	A'B'C'
000	000
001	001
010	010
011	011
100	100
101	110
110	101
111	111

(b) Reversible
every o/p is unique for the i/p.

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Ex. 1.41:

(a) Truth tables for Anti toffoli gate;

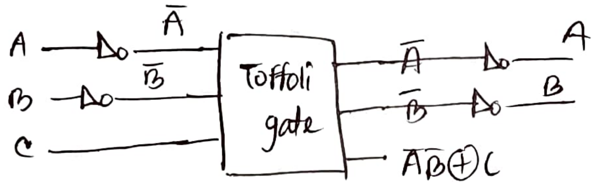
A B C	A	B	$\bar{A}\bar{B} \oplus C$
0 0 0	0	0	1
0 0 1	0	0	0
0 1 0	0	1	0
0 1 1	0	1	1
1 0 0	1	0	0
1 0 1	1	0	1
1 1 0	1	1	0
1 1 1	1	1	1

(b) De-morgan's law

(c) NOR when $c=0$

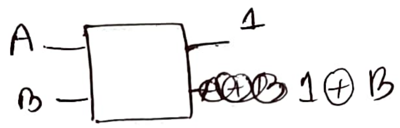
(d) OR when $c=1$.

(e)



1.42: (a) Reversible

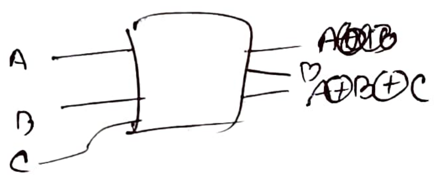
(b) irreversible.



1.43: Truth table for XOR;

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

irreversible



New truth table for reversible XOR;

A B C	A	B	$A \oplus B \oplus C$
0 0 0	0	0	0
0 0 1	0	0	1
0 1 0	0	1	1
0 1 1	0	1	0
1 0 0	1	0	1
1 0 1	1	0	0
1 1 0	1	1	0
1 1 1	1	1	1

unique o/p

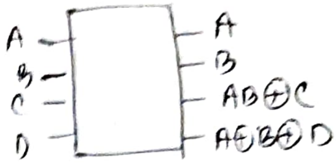
1.44 (a) Truth table:

A	B	B	A⊕B
0	0	0	0
0	1	1	1
1	0	0	1
1	1	1	0

reversible.

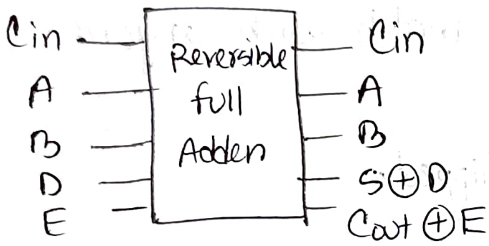
(b) Truth table.

A	B	AB	A⊕B
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0



not reversible

1.45



1.46: (a) 2^{14}

(b) spontaneously flip, radioactive atoms, presence absence, single event upset, ~~miniatured~~

(c) miniatured, (d) increased, greater, sky.

(e) cosmic rays, particles, black holes.

(f) cascade, transistor

(g) Error Correction Code

(h) month, cosmic rays

(i) 10 to 30

(j) 161

(k) flash, every star

1.47: (a) 0

(b) No

(c) Yes, second bit flipped.

(d) No, if two bits flip the parity will be same.

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1.48: (a) middle

(b) right

(c) decreases, $p < 0.5$ $P = 3p^2(1-p) + p^3 = 3(0.2)^2(1-0.2) + (0.2)^3 =$

(d) increases, $p > 0.5$ $P = 3(0.6)^2(1-0.6) + (0.6)^3 =$

1.49:

(a) $b_4 \oplus b_3 = 0 \therefore b_4 = b_3$
$b_3 \oplus b_2 = 0 \therefore b_3 = b_2$
$b_2 \oplus b_1 = 1 \therefore b_2 \neq b_1$
$b_1 \oplus b_0 = 1 \therefore b_1 \neq b_0$

Here, $b_4 = b_3 = b_2 = b_0$, b_1 does not match with anyone.

by majority vote, 1 bit b_1 was flipped.

(b) $b_4 \oplus b_3 = 0 \therefore b_4 = b_3$
$b_3 \oplus b_2 = 1 \therefore b_3 \neq b_2$
$b_2 \oplus b_1 = 0 \therefore b_2 = b_1$
$b_1 \oplus b_0 = 1 \therefore b_1 \neq b_0$

Here, $b_4 = b_3 = b_0$; $b_2 = b_1$

Two bits (b_2, b_1) were flipped.

(c) For a 5 bit repitition code, uncorrectable error occurs if 3, 4 or 5 bits are flipped.

$\therefore$ P(uncorrectable error) $: \binom{5}{3}p^3(1-p)^2 + \binom{5}{4}p^4(1-p) + \binom{5}{5}p^5$

$= 10p^3(1-p)^2 + 5p^4(1-p) + p^5$

(d) $10p^3(1-p)^2 + 5p^4(1-p) + p^5 < p$

Or, $10p^2(1-p)^2 + 5p^3(1-p) + p^4 < 1$

~~Or, $10p^3 - 15p^4 + 6p^5$~~

Or, $10p^2 - 15p^3 + 6p^4 < 1$

Or, $p^2(10 - 15p + 6p^2) < 1$

this inequality holds for $p < 0.5$

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(e) $p = 0.1$ for 5-bit repitition:

$10p^3(1-p)^2 + 5p^4(1-p) + p^5$

$= 10(0.1)^3(0.9)^2 + 5(0.1)^4(0.9) + (0.1)^5$

$= 10(0.001)(0.81) + 5(0.0001)(0.9) + (0.00001)$

$= 0.0081 + 0.00045 + 0.00001$

$= 0.00856$

(f) $p = 0.1$ for 3-bit repitition:

$3p^2(1-p) + p^3$

$= 3(0.1)^2(0.9) + (0.1)^3 = 3(0.01)(0.9) + (0.001) = 0.027 + 0.001$

$= 0.028$

Here, 5-bit repitition performs better as $0.00856 < 0.028$.

it reduces error by $\left(1 - \frac{0.00856 \times 100}{0.028}\right) = 69.5\%$ appx.

1.50:

(a) $f(n) = 5247n^2$ $g(n) = 11n^3$

at $n = 100$, $f(n)$ is bigger.

(b) at $n = 500$ $g(n)$ is bigger.

(c) $5247n^2 = 11n^3$

$n = \frac{5247}{11} = 477$

at $n = 477$ $f(n) = g(n)$, so at $n = 478$ $g(n)$ gets bigger

(d) 1. $f(n) = O(g(n))$ = true

2. $f(n) = o(g(n))$ = false

3. $f(n) = \Theta(g(n))$ =

4. $f(n) = \Omega(g(n))$ = true

5. $f(n) = \omega(g(n))$ = false

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1.51:

(a) $3n^2+7n+9$ – (iii) $\Theta(n^2)$

(b) $\log n$ – (i) $o(\sqrt{n})$

(c) $5e^n$ – (ii) $\Omega(2^n)$

(d) $\pi\sqrt{n}$ – (iv) $O(1)$

(e) π – $\cancel{(v)\ \omega(1)}$ $\omega(1)$

1.52

(a) $\log^2(n)$ – efficient

(b) n^3 – efficient

(c) $n^5\log^4(n)$ – efficient

(d) $2^{\sqrt{n}}$ – inefficient

(e) $2^{n/2}$ – inefficient

(f) $n!$ – inefficient

1.53:

(a) BPP – Bounded-Error Probabilistic Polynomial-Time

(b) Often identified as the class of feasible problems for a computer with access to a genuine random-number source

1.54:

Traveling salesman problem

1.55:

(a) 1. Birch and Swinnerton-Dyer Conjecture

2. Hodge Conjecture

3. Navier-Stokes Equation

4. P vs. NP

5. Poincaré conjecture

6. Riemann Hypothesis

7. Yang-Mills & the Mass Gap

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(b) stephen cook & Leonid Levin in 1971

1.56

1 planer graph: To test whether a graph is 1 planer or not.

2 Bottleneck Travelling Salesman: The problem is to find the Hamiltonian cycle in a weighted graph which minimizes the weight of the highest-weight edge of the cycle.

Eight Queens Puzzle: The problem of placing 8 chess queens of an 8X8 chessboard so that no 2 queens attack each other.

1.57:

pushdown automata: A type of automaton that employs a stack more capable than finite automata but less capable than the turing machine

Decision Tree model: A model of computation in which an algorithm can be considered to be a decision tree, i.e a sequence of queries or tests that are done adaptively, so the outcomes of previous tests can influence the test performed next.

External memory algorithm: Algorithms that are designed to process data that are too large to fit in the computer's memory at once.

1.58:

(a) ∆001
(b) ∆ ~~000~~ 011
(c) ∆ ~~000~~ 101
(d) ∆110
(e) here,

input: 00
01
10
11

output: 001
011
101
110

implements the NAND gate

(f) it calculates the NAND of all bits.

1.59:

q_{even} = ~~start~~ even stat q_{odd} = odd state q_s = start state

Current state	Read	Write	Move	Next State
q_s	∆	∆	→	q_{even}
q_{even}	0	0	→	q_{even}
q_{even}	1	1	→	q_{odd}
q_{even}	B	0	N	HALT
q_{odd}	0	0	→	q_{odd}
q_{odd}	1	1	→	q_{even}
q_{odd}	B	1	N	HALT

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1-60:

(a) Runs forever
(b) halts
(c) if H(Z) returns true, it means H halts, but Z responds by running forever. Z can't halt and run forever at the same time.
(d) if H(Z) returns false, it means H runs forever, but Z responds to it by halting. Z can't run forever and halt at the same time.

1-61:

(a) Complete, consistant, decidable
(b) programs, themselves
(c) Can every even number greater than 2 be written as sum of two primes?
(d) Runs forver
(e) Halts
(f) Runs forever
(g) it's a contradiction
(h) Runs forever
(i) contradiction
(k) undecidable, can't solve.

1-62:

Rice's theorem: For all non-trivial properties of a partial functions, it is undecidable whether a given machine computes a partial function with that property.

Hilbert's tenth problem: The problem of deciding whether a Diophantine equation (multivariable polynomial equation) has a solution in integers.

* The problem of determining if a given set of "Wang tiles" can tile the plane.

1-63:

(a) A million times (b) all known, quantum supremacy

1-64:

(a) Simulating Quantum physics

(b) Hardest

(c) Some

2.1: for position at 0:

(a) $X \to 1$ (b) $Y \to 1$ (c) $Z \to 0$ (d) $H \to +$ (e) $S \to 0$ (f) $\sqrt{X} \to -i$

(g) $I \to 0$ (h) nothing happens, 0

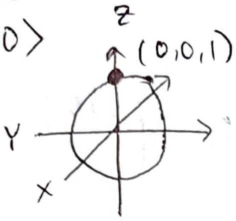
2.2: at position $= i$:

(a) $X \to -i$ (b) $Y \to {}^{i}\cancel{a}$ (c) $Z \to -i$ (d) $H \to -i$ (e) $S \to -$ (f) $\sqrt{X} \to 0$

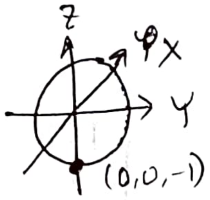
(g) $I \to i$ (h) 0 or 1 depending on the die roll.

2.3:

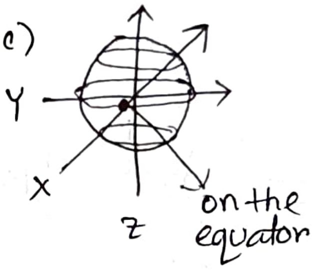
(a) $|0\rangle$



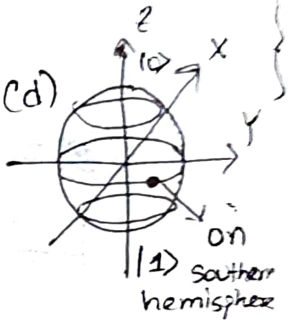
(b) $|1\rangle$



(c)

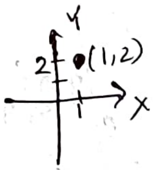


(d)



2.4: $z = 1+2i$

(a) $\Re(z) = 1$ (b) $\Im(z) = 2$ (c)



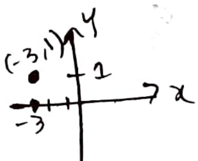
(d) $r = \sqrt{1+4} = \sqrt{5}$, $\theta = \cancel{2}\ \tan^{-1}(2)$

$\therefore \sqrt{5}\, e^{\cancel{2i}\tan^{-1}(2)}$

(e) $z^* = 1-2i$ (f) $|z| = |\sqrt{5}| = \sqrt{5}$ (g) $z^2 = 5$

2.5: $z = -3-i$

(a) $\Re(z) = -3$ (b) $\Im(z) = -i$ (c)



(d) $r = \sqrt{10}$ $\theta = \tan^{-1}\frac{1}{3}$

$\therefore \sqrt{10}\, e^{\tan^{-1}(\frac{1}{3})}$

(e) $z^* = -3+i$ (f) $|z| = \sqrt{10}$ (g) $z^2 = 10$

2.6:

~~(a) $\left(\frac{1+i\sqrt{3}}{3}\right)^2 = \frac{1+2i\sqrt{3}i+i^2 3}{9} = \frac{1+i2\sqrt{3}-3}{9} = \frac{-2+2\sqrt{3}i}{9}$~~

~~(b) $\left(\frac{2-i}{3}\right)^2 = \frac{1}{3} - \frac{4}{9}i$~~

~~(a) $\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)^2 = \frac{1}{9} + \frac{2i}{3\sqrt{3}} + \frac{i^2}{3}$~~ =

2.6:

(a) $P(|0\rangle) = \dfrac{|1+i\sqrt{3}|^2}{9} = \dfrac{(1+3)}{9} = \dfrac{4}{9}$

(b) $P(|1\rangle) = \dfrac{|2-i|^2}{9} = \dfrac{4+1}{9} = \dfrac{5}{9}$

2.7:

(a) $P(|0\rangle) = 1$

(b) $P(|1\rangle) = 0$

2.8:

$\cancel{\left|\left(\dfrac{e^{i\pi/8}}{\sqrt{5}}\right)^2\right| + |\beta|^2 = 1}$

Or, $\cancel{\dfrac{e^{\frac{2\pi i}{8}}}{5} + \beta^2 = 1}$

Or, $\cancel{\beta^2 = 1 - \dfrac{e^{\frac{2\pi i}{84}}}{5}}$

Or, $\cancel{\beta^2 = \dfrac{5 - e^{\frac{2\pi i}{84}}}{5}}$

Or, $\cancel{\beta =}$

for any real θ, $|e^{i\theta}| = 1$

$\therefore |\alpha|^2 + |\beta|^2 = 1$

Or, $\dfrac{|e^{\frac{i\pi}{8}}|^2}{5} + |\beta|^2 = 1$

Or, $\dfrac{1}{5} + |\beta|^2 = 1$

Or, $|\beta|^2 = \dfrac{4}{5}$

$\therefore \beta = \dfrac{2}{\sqrt{5}} e^{i\theta}$ for any real θ.

2.9:

(a) $|A 2 e^{i\pi/6}|^2 + |-3A|^2 = 1$

Or, $4A^2 |e^{i\pi/6}|^2 + 9A^2 = 1$

Or, $4A^2 + 9A^2 = 1$

Or, $13A^2 = 1$

Or, $A^2 = \dfrac{1}{13}$

Or, $A = \dfrac{1}{\sqrt{13}} e^{i\theta}$ for any real θ

(b) $\left|\frac{e^{i\theta}}{\sqrt{13}} \cdot 2e^{i\pi/6}\right|^2 = \frac{|e^{i\theta}|^2 \cdot 4 |e^{i\pi/6}|^2}{13} = \frac{4}{13}$ $\left[\because |e^{i\theta}| = 1\right]$ for any real θ

(c) $\frac{|e^{i\theta}|^2 \cdot 9}{13} = \frac{9}{13}$

2.10:

(a) in z-basis $P(|0\rangle) = \frac{1}{4}$ $P(|1\rangle) = \frac{3}{4}$

(b) in x-basis $|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$ $|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$

$\therefore$ Qubit $= \frac{1}{2}\left(\frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle\right) - \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle\right)$

$= \frac{1}{2\sqrt{2}}|+\rangle + \frac{1}{2\sqrt{2}}|-\rangle - \frac{\sqrt{3}}{2\sqrt{2}}|+\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|-\rangle$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}|+\rangle + \frac{1+\sqrt{3}}{2\sqrt{2}}|-\rangle$

(c) $P(|+\rangle) = \left|\frac{1-\sqrt{3}}{2\sqrt{2}}\right|^2 = \frac{2-\sqrt{3}}{4}$, $P(|-\rangle) = \left|\frac{1+\sqrt{3}}{2\sqrt{2}}\right|^2 = \frac{2+\sqrt{3}}{4}$

2.11:

(a) Here, $|0\rangle = \frac{\sqrt{3}}{2}|a\rangle + \frac{-i}{2}|b\rangle = \frac{\sqrt{3}}{2}|a\rangle - \frac{i}{2}|b\rangle$

$|1\rangle = -\frac{i}{2}|a\rangle + \frac{\sqrt{3}}{2}|b\rangle$

$\therefore$ Qubit in $\{|a\rangle, |b\rangle\}$-basis: $\frac{1}{2}\left(\frac{\sqrt{3}}{2}|a\rangle - \frac{i}{2}|b\rangle\right) - \frac{\sqrt{3}}{2}\left(-\frac{i}{2}|a\rangle + \frac{\sqrt{3}}{2}|b\rangle\right)$

$= \frac{\sqrt{3}}{4}|a\rangle - \frac{i}{4}|b\rangle + \frac{i\sqrt{3}}{4}|a\rangle - \frac{3}{4}|b\rangle$

$= \frac{\sqrt{3}+\sqrt{3}i}{4}|a\rangle - \frac{3+i}{4}|b\rangle$

(b) $P(|a\rangle) = \left|\frac{\sqrt{3}(1+i)}{4}\right|^2 = \frac{3}{16} \times 2 = \frac{3}{8}$

$P(|b\rangle) = \frac{|3+i|^2}{16} = \frac{9+1}{16} = \frac{5}{8}$

2.12:

(a) $\frac{1}{2}$ (b) $\frac{1}{2}$

2.13:

(a) $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $e^{i\pi/8} = e^{i\pi/8}/\sqrt{2}\,(|0\rangle + |1\rangle)$

does not distinguish, global phase

(b) $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Yes, they'll vary in $X\{|+\rangle, |-\rangle\}$ basis.

(c) No, global phase

2.14:

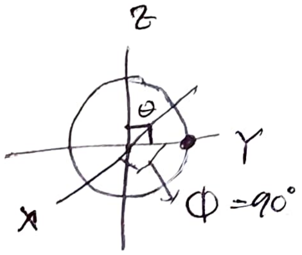
(a) $|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

$\cos\frac{\theta}{2} = \frac{1}{\sqrt{2}}$, $e^{i\phi}\sin\frac{\theta}{2} = e^{i\pi/2} \cdot \frac{1}{\sqrt{2}}$

$\frac{\theta}{2} = \pi/4$ $\phi = \pi/2$ $\theta = \pi/2$

$\theta = \pi/2$

(b) $\therefore$ $90°$ from North pole, $90°$ in xy plane.



2.15:

ⓐ Here, $\alpha = \frac{|1-i|}{2\sqrt{2}} = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2}$ ∴

$\tan\theta = \frac{-1}{2\sqrt{2}} \times 2\sqrt{2}$

$\therefore \theta = -\pi/4$

$e^{-\frac{i\pi}{4}}$

$\therefore \frac{1-i}{2\sqrt{2}} = \frac{1}{2}e^{-i\pi/4}$

$\therefore \frac{1}{2}e^{-i\pi/4}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

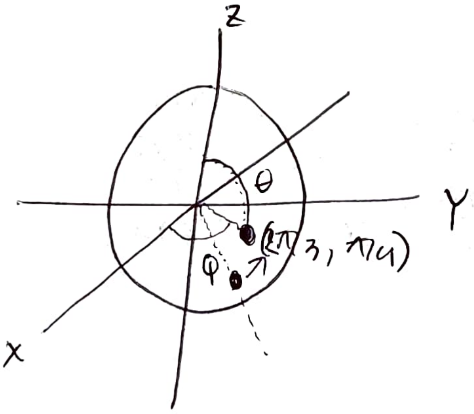
$= e^{-i\pi/4}\left(\frac{1}{2}|0\rangle + e^{i\pi/4}\frac{\sqrt{3}}{2}|1\rangle\right)$

$= \frac{1}{2}|0\rangle + e^{i\pi/4}\frac{\sqrt{3}}{2}|1\rangle$ [∵ Global phase]

$\therefore \theta = 2\times\cos^{-1}\left(\frac{1}{2}\right)$ $\phi = \pi/4$

$\theta = 2\pi/3$, $\phi = \pi/4$ $(\pi/3, \pi/4)$ (Ans)

ⓑ



2.16:

$|a\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle = \frac{\sqrt{3}}{2}|0\rangle + e^{i\pi/2}\cdot\frac{1}{2}|1\rangle$

$\therefore \theta_a = \pi/3$ $\phi_a = \pi/2$

$|b\rangle = \frac{i}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle = \frac{1}{2}e^{i\pi/2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle = e^{i\pi/2}\left(\frac{1}{2}|0\rangle + e^{-i\pi/2}\cdot\frac{\sqrt{3}}{2}|1\rangle\right)$

$= \frac{1}{2}|0\rangle + e^{-i\pi/2}\cdot\frac{\sqrt{3}}{2}|1\rangle$

$\therefore \theta_b = 2\pi/3 \quad \phi_b = 3\pi/2$

By comparing $|a\rangle = (\pi/6, \pi/2)$ and $|b\rangle = (\pi/3, -\pi/2)$, we can ~~say that these two points~~

$\theta_b = 2\pi/3 = \pi - \theta_a = \pi - \pi/3$

$\phi_b = \phi_a + \pi = \pi/2 + \pi = 3\pi/2$

$\therefore$ they lie on the opposite points.

2.17:

(a) $x = \sin\theta\cos\phi = \sin\pi/2 \cos\pi/2 = 0$

$y = \sin\theta \sin\phi = \sin\pi/2 \sin\pi/2 = 1$

$z = \cos\theta = \cos\pi/2 = 0$

$\therefore (x, y, z) = (0, 1, 0)$

(b) $x = \sin 2\pi/3 \cos\pi/4 = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}}$

$y = \sin 2\pi/3 \sin\pi/4 = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}}$

$z = \cos 2\pi/3 = -\frac{1}{2}$

2.18:

(a) polarization, decoherence

(b) electric fields

(c) laser beams

(d) molecules

(e) discrete energy levels, atomic nucleus

(f) Quantum information, hyperfine

(g) spin, microwave, optical

(h) charge, flux, phase

2-19:

(a) Photons: AEGIQ (2019)

(b) Trapped ions: eleQtrons (2020)

(c) Cold atoms: Atom Computing (2018)

(d) Nuclear Magnetic Resonance: SpinQ (2018)

(e) Quantum dots: Fujitsu (2015)

(f) Defect Qubits: Quantum Brilliance (2019)

(g) Superconductor: Google QUAIL (2013)

2-20:

(a) High-quality superconducting cavities coupled to non-linear oscillators

(b) Superconducting

(c) The rich state space of quantum harmonic oscillator and the desire to encode qubits in long-lifetime, error-correctable modes make this system a viable platform for quantum information processing.

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2.21:

(a) nucleus

(b) nuclear spin addressed through NMR

(c) spin

(d) up

(e) down

2.22

(a) $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$\therefore U|\psi\rangle = \alpha U|0\rangle + \beta U|1\rangle$

$= \alpha(|0\rangle + |1\rangle) + \beta(|0\rangle - |1\rangle)$

$= \alpha|0\rangle + \alpha|1\rangle + \beta|0\rangle - \beta|1\rangle$

$= (\alpha+\beta)|0\rangle + (\alpha-\beta)|1\rangle$

(b) $|\alpha+\beta|^2 + |\alpha-\beta|^2 = \alpha^2 + 2\alpha\beta + \beta^2 + \alpha^2 - 2\alpha\beta + \beta^2$

$= 2(\alpha^2+\beta^2) = 2\times 1 = 2$

Total $P(U|\psi\rangle) = 2$, so U is not a valid quantum gate.

2.23

(a) $U|\psi\rangle = \alpha U|0\rangle + \beta U|1\rangle$

$= \frac{\alpha\sqrt{3}}{2}|0\rangle + \frac{\alpha(\sqrt{3}+i)}{4}|1\rangle + \frac{\beta(\sqrt{3}+i)}{4}|0\rangle - \frac{\beta(\sqrt{3}+3i)}{4}|1\rangle$

$= \left(\frac{\alpha\sqrt{3}}{2} + \frac{\beta(\sqrt{3}+i)}{4}\right)|0\rangle + \left(\frac{\alpha(\sqrt{3}+i)}{4} - \frac{\beta(\sqrt{3}+3i)}{4}\right)|1\rangle$

(b) $P(U|\psi\rangle) = \left|\frac{\sqrt{3}}{2}\alpha + \frac{(\sqrt{3}+i)}{4}\beta\right|^2 + \left|\frac{\sqrt{3}+i}{4}\alpha - \frac{\sqrt{3}+3i}{4}\beta\right|^2$

$= \left(\frac{\sqrt{3}}{2}\alpha + \frac{\sqrt{3}+i}{4}\beta\right)\left(\frac{\sqrt{3}}{2}\alpha^* + \frac{\sqrt{3}-i}{4}\beta^*\right) + \left(\frac{\sqrt{3}+i}{4}\alpha - \frac{\sqrt{3}+3i}{4}\beta\right)\left(\frac{\sqrt{3}-i}{4}\alpha^* - \frac{\sqrt{3}-3i}{4}\beta^*\right)$

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$$= \frac{3}{4}|\alpha|^2 + \alpha\beta^* \frac{\sqrt{3}(\sqrt{3}-i)}{8} + \beta\alpha^* \frac{\sqrt{3}(\sqrt{3}+i)}{8} + |\beta|^2 \frac{(\sqrt{3}+i)(\sqrt{3}-i)}{16}$$

$$+ |\alpha|^2 \frac{(\sqrt{3}+i)(\sqrt{3}-i)}{16} - \alpha\beta^* \frac{(\sqrt{3}+i)(\sqrt{3}-3i)}{16} - \beta\alpha^* \frac{(\sqrt{3}+3i)(\sqrt{3}-i)}{16}$$

$$+ |\beta|^2 \frac{(\sqrt{3}+3i)(\sqrt{3}-3i)}{16}$$

$$= |\alpha|^2 \left\{ \frac{3}{4} + \frac{3+1}{16} \right\} + \alpha\beta^* \left\{ \frac{3-\sqrt{3}i}{8} - \frac{3-3\sqrt{3}i+\sqrt{3}i+3}{16} \right\}$$

$$+ \beta\alpha^* \left\{ \frac{3+\sqrt{3}i}{8} - \frac{3-\sqrt{3}i+3\sqrt{3}i+3}{16} \right\} + |\beta|^2 \left\{ \frac{\cancel{3+1}}{\cancel{16}\,4} + \frac{\cancel{3+9}}{16} \right\}$$

$$= |\alpha|^2 \frac{16}{16} + \alpha\beta^* \left\{ \frac{3-\sqrt{3}i}{8} - \frac{6-2\sqrt{3}i}{16} \right\} + \beta\alpha^* \left\{ \frac{3+\sqrt{3}i}{8} - \frac{6+2\sqrt{3}i}{16} \right\}$$

$$+ |\beta|^2 \frac{16}{16}$$

$$= \frac{16}{16} \left\{ |\alpha|^2 + |\beta|^2 \right\} + \cancel{\frac{9+3}{8}} \cancel{\frac{36+12}{16}} \left\{ \frac{6-2\sqrt{3}i-6+2\sqrt{3}i}{16} \right\} \alpha\beta^*$$

$$+ \beta\alpha^* \left\{ \frac{6+2\sqrt{3}i-6-2\sqrt{3}i}{16} \right\}$$

$$= \cancel{\frac{16}{16} + 0 + 0} \quad = 1+0+0 = 1$$

$\therefore$ Total probability $= P(|\psi\rangle) = 1$.

$\therefore$ The quantum gate is valid.

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2.24

(a) Yes, reversible (b) No, irreversible

2.25

(a) Yes, reversible (b) No, irreversible

2.26

$Z^{217} X^{101} Y^{50} (\alpha|0\rangle + \beta|1\rangle)$

$= (Z^2)^{108} \cdot Z (X^2)^{50} X (Y^2)^{25} (\alpha|0\rangle + \beta|1\rangle)$

$= ZX(\alpha|0\rangle + \beta|1\rangle)$

$= Z(\beta|0\rangle + \alpha|1\rangle)$ [$\because$ after applying Pauli X gate]

$= \beta|0\rangle - \alpha|1\rangle$

2.27

(a) $XZXZ(\alpha|0\rangle + \beta|1\rangle) = XZXZ\,\alpha|0\rangle + XZXZ(\beta|1\rangle)$

$\cancel{= XZX(\alpha|0\rangle - \beta|1\rangle)}$ $= XZX(\alpha|0\rangle) - XZX(\beta|1\rangle)$

$\cancel{= XZ(\beta|0\rangle - \alpha|1\rangle)}$ $= XZ(\alpha|1\rangle) - XZ(\beta|0\rangle)$

$\cancel{= X(\beta|0\rangle + \alpha|1\rangle)}$ $= -X(\alpha|1\rangle) - X(\beta|0\rangle)$

$= -\alpha|0\rangle - \beta|1\rangle$

$=$ $= -(\alpha|0\rangle + \beta|1\rangle)$ = R.H.S (Proved)

(b) $ZXZX(\alpha|0\rangle + \beta|1\rangle) = ZXZX\alpha|0\rangle + ZXZX\beta|1\rangle$

$= ZXZ(\alpha|1\rangle) + ZXZ\beta|0\rangle$

$= -ZX(\alpha|1\rangle) + ZX\beta|0\rangle$

$= -Z\alpha|0\rangle + Z\beta|1\rangle$

$= -\alpha|0\rangle - \beta|1\rangle = -(\alpha|0\rangle + \beta|1\rangle)$ = R.H.S (Proved)

2.28:

(a) $R_z(\theta)|\psi\rangle = \alpha R_z(\theta)|0\rangle + \beta R_z(\theta)|1\rangle$

$= \alpha|0\rangle + \beta e^{i\theta}|1\rangle$

(b) $|\alpha|^2 + |\beta|^2|e^{i\theta}|^2 = |\alpha|^2 + |\beta|^2 \; [\because |e^{i\theta}|^2 = 1]$

$= 1 \; [\because |\alpha|^2 + |\beta|^2 = 1]$

$\therefore R_z(\theta)$ is a valid quantum gate.

2.29:

(a) $H|\psi\rangle = \alpha H|0\rangle + \beta H|1\rangle = \alpha|+\rangle + \beta|-\rangle$

$= \alpha\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + \beta\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$= \frac{\alpha}{\sqrt{2}}|0\rangle + \frac{\alpha}{\sqrt{2}}|1\rangle + \frac{\beta}{\sqrt{2}}|0\rangle - \frac{\beta}{\sqrt{2}}|1\rangle$

$= \frac{\alpha+\beta}{\sqrt{2}}|0\rangle + \frac{\alpha-\beta}{\sqrt{2}}|1\rangle$

(b) $\frac{|\alpha+\beta|^2}{2} + \frac{|\alpha-\beta|^2}{2} = \frac{|\alpha|^2 + 2\alpha\beta + |\beta|^2 + |\alpha|^2 - 2\alpha\beta + |\beta|^2}{2}$

$= \frac{2|\alpha|^2 + 2|\beta|^2}{2} = |\alpha|^2 + |\beta|^2 = 1$

$\therefore$ H is a valid quantum gate

2.30:

(a) $H|-\rangle = H\left(\frac{1}{\sqrt2}|0\rangle - \frac{1}{\sqrt2}|1\rangle\right) = \frac{1}{\sqrt2}H|0\rangle - \frac{1}{\sqrt2}H|1\rangle$

$= \frac{1}{\sqrt2}\times\frac{1}{\sqrt2}(|0\rangle+|1\rangle) - \frac{1}{\sqrt2}\times\frac{1}{\sqrt2}(|0\rangle-|1\rangle)$

$= \frac{|0\rangle}{2} + \frac{|1\rangle}{2} - \frac{|0\rangle}{2} + \frac{|1\rangle}{2} = |1\rangle$ = R.H.S (proved)

(b) $H|-i\rangle = H\left(\frac{1}{\sqrt2}|0\rangle - \frac{i}{\sqrt2}|1\rangle\right) = \frac{1}{\sqrt2}H|0\rangle - \frac{i}{\sqrt2}H|1\rangle$

$= \frac{1}{\sqrt2}\times\frac{1}{\sqrt2}(|0\rangle+|1\rangle) - \frac{i}{\sqrt2}\times\frac{1}{\sqrt2}(|0\rangle-|1\rangle)$

$= \frac{1}{2}(|0\rangle+|1\rangle) - \frac{i}{2}(|0\rangle-|1\rangle)$

$= \left(\frac{1}{2}-\frac{i}{2}\right)|0\rangle + \left(\frac{1}{2}+\frac{i}{2}\right)|1\rangle = \frac{1-i}{2}|0\rangle + \frac{1+i}{2}|1\rangle$

$= \frac{1}{\sqrt2}\left[e^{-i\pi/4}|0\rangle + e^{i\pi/4}|1\rangle\right] = \frac{1}{\sqrt2}\times e^{i\pi/4}\left[e^{-i\pi/2}|0\rangle + |1\rangle\right]$

$= \frac{e^{i\pi/4}}{\sqrt2}(i|0\rangle + |1\rangle) = e^{i\pi/4}|i\rangle = |i\rangle$ = R.H.S

2.31

$Y^{51}H^{99}T^{36}Z^{25}|0\rangle$

$= YHZ|0\rangle = YH|0\rangle = Y\frac{1}{\sqrt2}(|0\rangle+|1\rangle) = \frac{1}{\sqrt2}Y|0\rangle + \frac{1}{\sqrt2}Y|1\rangle$

$= \frac{1}{\sqrt2}\times i|1\rangle + \frac{1}{\sqrt2}\times -i|0\rangle = \frac{1}{\sqrt2}(i|1\rangle - i|0\rangle)$

$= -\frac{i}{\sqrt2}(|0\rangle - |1\rangle) = -i\times\frac{1}{\sqrt2}(|0\rangle-|1\rangle) = -i|-\rangle$ (Ans)

2.32

$L.H.S = HXH|0\rangle$

$= HX\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$= \frac{1}{\sqrt{2}}[H|0\rangle + H|1\rangle]$

$= \frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle] + \frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle]$

$= \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle + \frac{1}{2}|0\rangle - \frac{1}{2}|1\rangle$

$= |0\rangle$

$R.H.S = Z|0\rangle = |0\rangle$

$\therefore$ L.H.S = R.H.S (proved)

Again, $L.H.S = HXH|1\rangle$

$= HX(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle)$

$= H(\frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}|0\rangle)$

$= \frac{1}{\sqrt{2}}H|1\rangle - \frac{1}{\sqrt{2}}H|0\rangle$

$= \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) - \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$= \frac{1}{2}|0\rangle - \frac{1}{2}|1\rangle - \frac{1}{2}|0\rangle - \frac{1}{2}|1\rangle$

$= -|1\rangle$

$R.H.S = Z|1\rangle = -|1\rangle$ $\therefore$ L.H.S = R.H.S (proved)

$\therefore$ $HXH = Z$ (proved)

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2.33

(a) $HTHTH|0\rangle$

$$= HTHT\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$$

$$= \frac{1}{\sqrt{2}}HTH\left[T|0\rangle+T|1\rangle\right]$$

$$= \frac{1}{\sqrt{2}}HTH\left[|0\rangle+e^{i\pi/4}|1\rangle\right]$$

$$= \frac{1}{\sqrt{2}}HT\left[H|0\rangle+e^{i\pi/4}H|1\rangle\right]$$

$$= \frac{1}{\sqrt{2}}HT\left[\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)+\frac{e^{i\pi/4}}{\sqrt{2}}(|0\rangle-|1\rangle)\right]$$

$$= \frac{1}{\sqrt{2}}HT\left[\frac{|0\rangle}{\sqrt{2}}+\frac{e^{i\pi/4}}{\sqrt{2}}|0\rangle+\frac{|1\rangle}{\sqrt{2}}-\frac{e^{i\pi/4}}{\sqrt{2}}|1\rangle\right]$$

$$= \frac{1}{\sqrt{2}}HT\left[\frac{e^{i\pi/4}+1}{\sqrt{2}}|0\rangle+\frac{1-e^{i\pi/4}}{\sqrt{2}}|1\rangle\right]$$

$$= \frac{1}{\sqrt{2}}H\left[\frac{1+e^{i\pi/4}}{\sqrt{2}}|0\rangle+\frac{1-e^{i\pi/4}}{\sqrt{2}}\cdot e^{i\pi/4}|1\rangle\right]$$

$$= \frac{1+e^{i\pi/4}}{2}H|0\rangle+\frac{(1-e^{i\pi/4})e^{i\pi/4}}{2}H|1\rangle$$

$$= \frac{1+e^{i\pi/4}}{2\sqrt{2}}(|0\rangle+|1\rangle)+\frac{(1-e^{i\pi/4})e^{i\pi/4}}{2\sqrt{2}}(|0\rangle-|1\rangle)$$

$$= \frac{1+e^{i\pi/4}}{2\sqrt{2}}|0\rangle+\frac{1+e^{i\pi/4}}{2\sqrt{2}}|1\rangle+\frac{(1-e^{i\pi/4})(e^{i\pi/4})}{2\sqrt{2}}|0\rangle-\frac{(1-e^{i\pi/4})e^{i\pi/4}}{2\sqrt{2}}|1\rangle$$

$$= \frac{(1+e^{i\pi/4})+e^{i\pi/4}(1-e^{i\pi/4})}{2\sqrt{2}}|0\rangle+\frac{(1+e^{i\pi/4})-e^{i\pi/4}(1-e^{i\pi/4})}{2\sqrt{2}}|1\rangle$$

$$= \frac{1+e^{i\pi/4}+e^{i\pi/4}-e^{2i\pi/4}}{2\sqrt{2}}|0\rangle+\frac{1+e^{i\pi/4}-e^{i\pi/4}+e^{2i\pi/4}}{2\sqrt{2}}|1\rangle$$

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$$= \frac{1}{2\sqrt{2}}\left(1+2e^{i\pi/4} - (e^{i\pi/4})^2\right)|0\rangle + \frac{1}{2\sqrt{2}}\left(1+(e^{i\pi/4})^2\right)|1\rangle$$

$$= \frac{1}{2\sqrt{2}}\left(1+2e^{i\pi/4} - i\right)|0\rangle + \frac{1}{2\sqrt{2}}(1+i)|1\rangle$$

$$= \frac{1}{2\sqrt{2}}\left[(1-i+2e^{i\pi/4})|0\rangle + (1+i)|1\rangle\right]$$

(Ans)

(b) $\cancel{\frac{|1-i+2e^{i\pi/4}|^2}{8}}$ 4

$$\cancel{P|0\rangle = \frac{|1-i+2e^{i\pi/4}|^2}{8} = \frac{1-2i-1+2(1-i)\cdot 2e^{i\pi/4}+4e^{2i\pi/4}}{8}}$$

$$\cancel{= \frac{-2i+(2-2i)2e^{i\pi/4}+4}{}}$$

$$P|0\rangle = \frac{|1-i+2e^{i\pi/4}|^2}{8} = \frac{|1-i+2(\cos\pi/4+i\sin(\pi/4))|^2}{8}$$

$$= \frac{|1-i+\sqrt{2}+\sqrt{2}i|^2}{8} = \frac{|(1+\sqrt{2})+i(\sqrt{2}-1)|^2}{8}$$

$$= \frac{1+2\sqrt{2}+2+1-2\sqrt{2}+2}{8} = \frac{6}{8} = \frac{3}{4}$$

$$P|1\rangle = \frac{|1+i|^2}{8} = \frac{1+1}{8} = \frac{2}{8} = \frac{1}{4}$$

(Ans)

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2.34

(a) $\hat{n} = n_x \hat{X} + n_y \hat{Y} + n_z \hat{Z}$

$= 0 + 0 + \hat{Z}$

or, $\hat{n} = (n_x, n_y, n_z) = (0, 0, 1)$

(b) $U = e^{i\delta}\left[\cos(\theta/2) I - i\sin\left(\frac{\theta}{2}\right)(n_x \hat{X} + n_y \hat{Y} + n_z \hat{Z})\right]$

$= e^{i\delta}\left[\cos\pi/8\, I - i\sin\pi/8\,(0X + 0Y + Z)\right]$

$\cancel{= e^{i\delta}\left[\frac{1}{\sqrt{2}} I - i\sin \frac{1}{\sqrt{2}}\right.}$

$= e^{i\delta}\left[I\cos\pi/8 - i\sin\pi/8 \cdot Z\right]$

(c) $U|0\rangle = e^{i\delta}\left[\cos\pi/8\, I - i\sin\pi/8 \cdot Z\right]|0\rangle$

$\cancel{= e^{i\delta} I\cos\pi/8 |0\rangle - e^{i\delta} i\sin\pi/8 \cdot Z|0\rangle}$

$\cancel{= e^{i\delta}\cos\pi/8\, I|0\rangle - e}$

$= e^{i\delta} \cdot e^{-i\pi/8}|0\rangle$

(d) $U|1\rangle = e^{i\delta}\left[\cos\pi/8\, I - i\sin\pi/8\, Z\right]|1\rangle$

$= e^{i\delta}\left[\cos\pi/8\, I|1\rangle - i\sin\pi/8\, Z|1\rangle\right]$

$= e^{i\delta}\left[\cos\pi/8 |1\rangle + i\sin\pi/8 |1\rangle\right]$

$= e^{i\delta} \cdot \left[\cos\pi/8 + i\sin\pi/8\right]|1\rangle$

$= e^{i\delta} \cdot e^{i\pi/8}|1\rangle$

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(e) If $\gamma = \pi/8$ for global phase:

$$U|0\rangle = e^{i\pi/8} e^{-i\pi/8} |0\rangle = |0\rangle$$

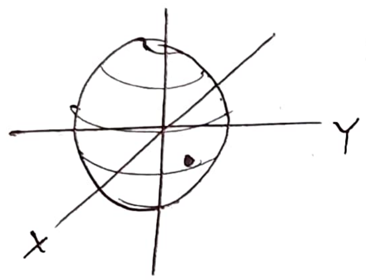
$$U|1\rangle = e^{i\pi/8} e^{i\pi/8} |1\rangle = e^{i(\pi/8+\pi/8)} |1\rangle = e^{i\pi/4} |1\rangle$$

we know, $T|0\rangle = |0\rangle$ and $T|1\rangle = e^{i\pi/4}|1\rangle$

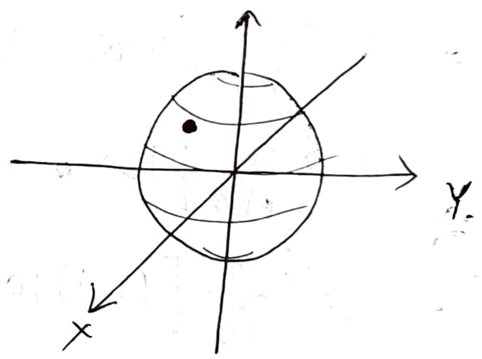
$\therefore$ for $\gamma = \pi/8$, U is T gate.

2.35:

(a)



(b)



(c) $\hat{n} = n_x \hat{X} + n_y \hat{Y} + n_z \hat{Z}$

$$= \frac{1}{\sqrt{3}} X + \frac{1}{\sqrt{3}} Y + \frac{1}{\sqrt{3}} Z$$

$\therefore \hat{n} = (n_x, n_y, n_z) = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(d) $U = e^{i\gamma}\left[\cos(\pi/2) I - i\sin(\pi/2)\left(\frac{1}{\sqrt{3}}X + \frac{1}{\sqrt{3}}Y + \frac{1}{\sqrt{3}}Z\right)\right]$

$$= e^{i\gamma}\left[0 - i\left(\frac{1}{\sqrt{3}}X + \frac{1}{\sqrt{3}}Y + \frac{1}{\sqrt{3}}Z\right)\right]$$

$$= -ie^{i\gamma}\frac{1}{\sqrt{3}}(X+Y+Z)$$

(e) $U|0\rangle = -ie^{i\theta}\frac{1}{\sqrt{3}}(X+Y+Z)|0\rangle$

$= -ie^{i\theta}\frac{1}{\sqrt{3}}\left(X|0\rangle + Y|0\rangle + Z|0\rangle\right)$

$= -ie^{i\theta}\frac{1}{\sqrt{3}}\left(|1\rangle + i|1\rangle + |0\rangle\right)$

$= -ie^{i\theta}\frac{1}{\sqrt{3}}\left[(1+i)|1\rangle + |0\rangle\right]$

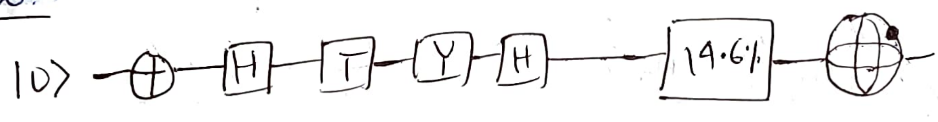
(f) $U|1\rangle = -ie^{i\theta}\frac{1}{\sqrt{3}}(X+Y+Z)|1\rangle$

$= -ie^{i\theta}\frac{1}{\sqrt{3}}\left(X|1\rangle + Y|1\rangle + Z|1\rangle\right)$

$= -ie^{i\theta}\frac{1}{\sqrt{3}}\left[|0\rangle - i|0\rangle - |1\rangle\right]$

$= -ie^{i\theta}\frac{1}{\sqrt{3}}\left[(1-i)|0\rangle - |1\rangle\right]$

2.36:

(a) 

$HYTHX|0\rangle = HYTH|1\rangle = HYT\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}HY\left[|0\rangle - e^{i\pi/4}|1\rangle\right]$

$= \frac{1}{\sqrt{2}}H\left[i|1\rangle - e^{i\pi/4}i|0\rangle\right] = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}i(|0\rangle - |1\rangle) - ie^{i\pi/4}\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right]$

$= \frac{1}{2}i(|0\rangle - |1\rangle) - \frac{1}{2}ie^{i\pi/4}(|0\rangle + |1\rangle)$

$= \frac{i}{2}|0\rangle - \frac{i}{2}|1\rangle - \frac{ie^{i\pi/4}}{2}|0\rangle - \frac{ie^{i\pi/4}}{2}|1\rangle$

$= \frac{i(1+e^{i\pi/4})}{2}|0\rangle - \frac{i(1-e^{i\pi/4})}{2}|1\rangle$

$P(U|1\rangle) = \frac{|i + ie^{i\pi/4}|^2}{4} = \frac{|i + i(\cos\pi/4 + i\sin\pi/4)|^2}{4} = \frac{|i + i\cdot\frac{1}{\sqrt{2}} + i^2\frac{1}{\sqrt{2}}|^2}{4}$

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$$= \frac{\left|i - \frac{i}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right|^2}{4} = \frac{\frac{1}{2} + \left(1 - \frac{1}{\sqrt{2}}\right)^2}{4} = \frac{2 - \sqrt{2}}{4} = 0.14644$$

$$= 14.6\%$$

(c) from (b) $U|1\rangle = 14.6\%$

$$U|0\rangle = \frac{|i + ie^{i\pi/4}|^2}{4} = \frac{|i + i(\cos\pi/4 + i\sin\pi/4)|^2}{4}$$

$$= \frac{\left|i + i\cdot\frac{1}{\sqrt{2}} + i^2\cdot\frac{1}{\sqrt{2}}\right|^2}{4} = \frac{\left|i\left(1+\frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\right|^2}{4} = \frac{\left(-\frac{1}{\sqrt{2}}\right)^2 + \left(1+\frac{1}{\sqrt{2}}\right)^2}{4}$$

$$= \frac{2+\sqrt{2}}{4} = 0.8535 = 85.4\%$$

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3.1:

$$\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle = \frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix} - \frac{\sqrt{3}}{2}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1/2\\0\end{pmatrix} + \begin{pmatrix}0\\-\sqrt{3}/2\end{pmatrix}$$

$$= \begin{pmatrix}1/2\\-\sqrt{3}/2\end{pmatrix}$$

3.2:

$$\begin{pmatrix}\sqrt{3}/2\\1/2\end{pmatrix} = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$$

$$P|0\rangle = \frac{3}{4}, \quad P|1\rangle = \frac{1}{4}$$

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3.3:

(a) $|a\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle \Rightarrow |a\rangle = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}$

$\therefore \langle a| = \left(\sqrt{3}/2 \quad 1/2 \right) \Rightarrow \langle a| = \sqrt{3}/2 \langle 0| + \frac{1}{2}\langle 1|$

(b) $\langle a| = \left(\frac{\sqrt{3}}{2} \quad \frac{1}{2} \right)$

(c) $\langle b| = \frac{2}{3}\langle 0| + \frac{1+2i}{3}\langle 1|$

(d) $\langle b| = \left(\frac{2}{3} \quad \frac{1+2i}{3} \right)$

3.4:

(a) ~~$\langle b| = \frac{1}{4}\langle 0| + \frac{\sqrt{15}}{4}\langle 1|$~~ $\langle a| = \frac{3-i\sqrt{3}}{4}\langle 0| + \frac{1}{2}\langle 1|$

$\langle a|b\rangle = \left(\frac{3-i\sqrt{3}}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{15}}{4}\right) = \frac{3+2\sqrt{15}-i\sqrt{3}}{16} = \frac{1}{16}\left[3+2\sqrt{15}-i\sqrt{3}\right]$

(b) $\langle b|a\rangle = \left(\frac{1}{4}\right)\left(\frac{3+i\sqrt{3}}{4}\right) + \left(\frac{\sqrt{15}}{4}\right)\left(\frac{1}{2}\right)$

$= \frac{3+i\sqrt{3}}{16} + \frac{\sqrt{15}}{8} = \frac{3+2\sqrt{15}+i\sqrt{3}}{16} = \frac{1}{16}\left(3+2\sqrt{15}+i\sqrt{3}\right)$

(c) $\langle a|b\rangle = \frac{1}{16}\left[3+2\sqrt{15}-i\sqrt{3}\right]$

$\langle b|a\rangle = \frac{1}{16}\left[3+2\sqrt{15}+i\sqrt{3}\right]$

Here, $\langle a|b\rangle = \langle b|a\rangle^*$, they are complex conjugates

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3.5:

(a) $|\psi\rangle = A\left(2|0\rangle + 3i|1\rangle\right) = A2|0\rangle + A3i|1\rangle$

$\langle\psi| = A2\langle 0| - A3i\langle 1|$

$\langle\psi|\psi\rangle = |A|^2 4 - |A|^2 9i^2 = |A|^2 4 + |A|^2 9 = 13|A|^2$

(b) $13|A|^2 = 1 \quad \therefore \quad |A|^2 = \frac{1}{13} \quad \therefore \quad A = \frac{1}{\sqrt{13}}$

3.6:

(a) $|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \quad \therefore \quad \langle +| = \frac{1}{\sqrt{2}}\langle 0| + \frac{1}{\sqrt{2}}\langle 1|$

$\langle +|-\rangle = \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right)\right) = \frac{1}{2} - \frac{1}{2} = 0$

$\therefore$ $|+\rangle$ & $|-\rangle$ are orthogonal

(b) $\langle 0|+\rangle = 1 \times \frac{1}{\sqrt{2}} + 0 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \neq 0$

$\therefore$ Not orthogonal

(c) $|P\rangle = \frac{1+\sqrt{3}i}{4}|0\rangle + \frac{\sqrt{2}-i}{2}|1\rangle$ and $Q \; \frac{\sqrt{2}+i}{2}|0\rangle + \frac{-1+\sqrt{3}i}{4}|1\rangle$

$\langle P| = \frac{1-\sqrt{3}i}{4}\langle 0| + \frac{\sqrt{2}+i}{2}\langle 1|$

$\langle P|Q\rangle = \left(\frac{1-\sqrt{3}i}{4} \times \frac{\sqrt{2}+i}{2}\right) + \left(\frac{\sqrt{2}+i}{2} \times \frac{-1+\sqrt{3}i}{4}\right)$

$= \left(\frac{\sqrt{2}+i - \sqrt{3}\cdot\sqrt{2}\cdot i - \sqrt{3}i^2}{8}\right) + \left(\frac{-\sqrt{2}-i + \sqrt{2}\sqrt{3}i + \sqrt{3}i^2}{8}\right) = 0$

$\therefore$ Orthogonal.

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3.7:

(a) $\langle a| = \frac{3-i\sqrt{3}}{4}\langle 0| + \frac{1}{2}\langle 1|$

$$\langle a|b\rangle = \left(\frac{3-i\sqrt{3}}{4}\times\frac{1}{4}\right) + \left(\frac{1}{2}x\right) = 0$$

$$\text{or, } \frac{3-i\sqrt{3}}{16} + \frac{x}{2} = 0$$

$$\text{or, } \frac{x}{2} = \frac{-3+i\sqrt{3}}{16}$$

$$\text{or, } x = \frac{-3+i\sqrt{3}}{8}$$

(b) $\cancel{\frac{1}{8}+x^2}$ $\langle b| = \frac{1}{4}\langle 0| + x\langle 1|$

$$\langle b|b\rangle \Rightarrow \frac{1}{\cancel{8}\,16} + x^2 = 1$$

$$\text{Or, } x^2 = -\frac{15}{\cancel{8}16} \quad \text{Or, } x = \sqrt{-\frac{15}{\cancel{8}16}} = \frac{i\sqrt{15}}{4} = e^{i\theta}\frac{\sqrt{15}}{4}$$

(c) $|a\rangle$ is not normalized, so even if $|b\rangle$ is normalized ~~the~~ at (b), the ~~...~~ $\langle a|b\rangle$ will not be 0.

$\therefore$ none.

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3.8:

$$\langle a| = \cos\left(\frac{\theta_a}{2}\right)\langle 0| - e^{i\phi_a}\sin\left(\frac{\theta_a}{2}\right)\langle 1|$$

$$\langle a|b\rangle = \cos\left(\frac{\theta_a}{2}\right)\cos\left(\frac{\theta_b}{2}\right) - \left(e^{i\phi_a}\sin\left(\frac{\theta_a}{2}\right)e^{i\phi_b}\sin\left(\frac{\theta_b}{2}\right)\right)$$

$$= \cos\left(\frac{\theta_a}{2}+\frac{\theta_b}{2}\right) = \cos\left(\frac{\theta_a+\theta_b}{2}\right) = \cos(\pi/2) \quad [\because \theta_a+\theta_b=\pi]$$

opposite points

$$= 0 = R.H.S$$

(shown)

3.9:

(a) $|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \therefore \langle i| = \frac{1}{\sqrt{2}}(\langle 0| - i\langle 1|)$

$$\langle i|\psi\rangle = \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} + \left(-\frac{i}{\sqrt{2}}\times\frac{1}{2}\right) = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{i}{2\sqrt{2}} = \frac{\sqrt{3}-i}{2\sqrt{2}}$$

(b) $|-i\rangle = \frac{1}{\sqrt{2}}|0\rangle - i|1\rangle \therefore \langle -i| = \frac{1}{\sqrt{2}}\langle 0| + \frac{i}{\sqrt{2}}\langle 1|$

$$\langle -i|\psi\rangle = \left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}\right) + \left(\frac{i}{\sqrt{2}}\times\frac{1}{2}\right) = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{i}{2\sqrt{2}} = \frac{\sqrt{3}+i}{2\sqrt{2}}$$

(c) $|\psi\rangle = \langle i|\psi\rangle|i\rangle + \langle -i|\psi\rangle|-i\rangle$

$$= \frac{\sqrt{3}-i}{2\sqrt{2}}|i\rangle + \frac{\sqrt{3}+i}{2\sqrt{2}}|-i\rangle$$

$$P(|i\rangle)\ \frac{3+1}{8} = \frac{4}{8} = \frac{1}{2} \qquad P(|-i\rangle) = \frac{3+1}{8} = \frac{4}{8} = \frac{1}{2}$$

(d) $\langle a| = \frac{\sqrt{3}}{2}\langle 0| - \frac{i}{2}\langle 1|$

$\langle a|\psi\rangle = \frac{3}{4} - \frac{i}{4} = \frac{3-i}{4}$

(e) $\langle b| = \frac{-i}{2}\langle 0| + \frac{\sqrt{3}}{2}\langle 1|$

$\langle b|\psi\rangle = \frac{-i\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}(1-i)}{4}$

(f) $|\psi\rangle = \langle a|\psi\rangle|a\rangle + \langle b|\psi\rangle|b\rangle$

$= \frac{3-i}{4}|a\rangle + \frac{\sqrt{3}(1-i)}{4}|b\rangle$

$P(|a\rangle) = \frac{9+1}{16} = \frac{5}{8}$ $P(|b\rangle) = \frac{3+3}{16} = \frac{3}{8}$

3-10:

(a) $P(|0\rangle) = \frac{9+3}{16} = \frac{3}{4}$ $P(|1\rangle) = \frac{1}{4}$

(b) $\langle +|\psi\rangle = \frac{1}{\sqrt{2}}\left(\frac{3+i\sqrt{3}}{4}\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right) = \frac{3+i\sqrt{3}}{4\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{3+i\sqrt{3}-2}{4\sqrt{2}}$

$= \frac{1+i\sqrt{3}}{4\sqrt{2}}$

$\langle -|\psi\rangle = \frac{1}{\sqrt{2}}\left(\frac{3+i\sqrt{3}}{4}\right) + \frac{1}{2\sqrt{2}} = \frac{3+i\sqrt{3}}{4\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{3+i\sqrt{3}+2}{4\sqrt{2}}$

$= \frac{5+i\sqrt{3}}{4\sqrt{2}}$

$\therefore |\psi\rangle = \langle +|\psi\rangle|+\rangle + \langle -|\psi\rangle|-\rangle$

$= \frac{1+i\sqrt{3}}{4\sqrt{2}}|+\rangle + \frac{5+i\sqrt{3}}{4\sqrt{2}}|-\rangle$

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$$P(|+\rangle) = \frac{1+3}{32} = \frac{4}{32} = \frac{1}{8}, \quad P(|-\rangle) = \frac{25+3}{32} = \frac{28}{32} = \frac{7}{8}$$

(c) $$\langle i|\psi\rangle = \frac{1}{\sqrt{2}}\left(\frac{3+i\sqrt{3}}{4}\right) + \frac{i}{\sqrt{2}}\left(\frac{1}{2}\right) = \frac{3+i\sqrt{3}}{4\sqrt{2}} + \frac{i}{2\sqrt{2}} = \frac{3+i\sqrt{3}+2i}{4\sqrt{2}}$$

$$\langle -i|\psi\rangle = \frac{1}{\sqrt{2}}\left(\frac{3+i\sqrt{3}}{4}\right) - \frac{i}{\sqrt{2}}\left(\frac{1}{2}\right) = \frac{3+i\sqrt{3}}{4\sqrt{2}} - \frac{i}{2\sqrt{2}} = \frac{3+i\sqrt{3}-2i}{4\sqrt{2}}$$

$$\therefore |\psi\rangle = \langle i|\psi\rangle |i\rangle + \langle -i|\psi\rangle |-i\rangle$$

$$= \frac{3+i\sqrt{3}+2i}{4\sqrt{2}} |i\rangle + \frac{3+i\sqrt{3}-2i}{4\sqrt{2}} |-i\rangle$$

$$= \frac{3+(\sqrt{3}+2)i}{4\sqrt{2}} |i\rangle + \frac{3-i(2-\sqrt{3})}{4\sqrt{2}} |-i\rangle$$

$$P(|i\rangle) = \frac{9+7+4\sqrt{3}}{32} = \frac{16+4\sqrt{3}}{32} = \frac{4+\sqrt{3}}{8} = 0.717$$

$$P(|-i\rangle) = \frac{9+7-4\sqrt{3}}{32} = \frac{16-4\sqrt{3}}{32} = \frac{4-\sqrt{3}}{8} = 0.283$$

3.11:

$$\langle +|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}\frac{1-2i}{\sqrt{6}} \\ \frac{1}{\sqrt{6}}\end{pmatrix} = \frac{1}{\sqrt{2}}\left(\frac{1-2i}{\sqrt{6}} + \frac{1}{\sqrt{6}}\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1-2i+1}{\sqrt{6}}\right) = \frac{2(1-i)}{2\sqrt{3}} = \frac{1-i}{\sqrt{3}}$$

$$\langle -|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\end{pmatrix}\begin{pmatrix}\frac{1-2i}{\sqrt{6}} \\ \frac{1}{\sqrt{6}}\end{pmatrix} = \frac{1}{\sqrt{2}}\left(\frac{1-2i}{\sqrt{6}} - \frac{1}{\sqrt{6}}\right) = \frac{1}{\sqrt{2}}\left(\frac{-2i}{\sqrt{6}}\right)$$

$$= \frac{-2i}{2\sqrt{3}} = -\frac{i}{\sqrt{3}}$$

$$\therefore |\psi\rangle = \frac{1-i}{\sqrt{3}}|+\rangle - \frac{i}{\sqrt{3}}|-\rangle$$

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(b) $\langle i|\psi\rangle = \frac{1}{2\sqrt{3}}(1\ -i)\begin{pmatrix}1-2i\\1\end{pmatrix} = \frac{1}{2\sqrt{3}}(1-2i-i) = \frac{1-3i}{2\sqrt{3}}$

$\langle -i|\psi\rangle = \frac{1}{2\sqrt{3}}(1\ i)\begin{pmatrix}1-2i\\1\end{pmatrix} = \frac{1}{2\sqrt{3}}(1-2i+i) = \frac{1-i}{2\sqrt{3}}$

$\therefore |\psi\rangle = \frac{1-3i}{2\sqrt{3}}|i\rangle + \frac{1-i}{2\sqrt{3}}|-i\rangle$

3.12:

(a) $U = \begin{pmatrix}1/\sqrt{2} & \frac{-i}{\sqrt{2}}\\ \frac{-i}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}$

(b) $U\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \begin{pmatrix}1/\sqrt{2} & -i/\sqrt{2}\\ -i/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\-i & 1\end{pmatrix}\begin{pmatrix}\alpha\\\beta\end{pmatrix}$

$= \frac{1}{\sqrt{2}}\begin{pmatrix}\alpha - i\beta\\ -i\alpha+\beta\end{pmatrix}$

(c) $\cancel{\left|\frac{\alpha - i\beta}{\sqrt{2}}\right|^2 = \frac{\alpha^2+\beta^2}{2} = \frac{1}{2}}$ $\left|\frac{\alpha-i\beta}{\sqrt{2}}\right|^2 + \left|\frac{-i\alpha+\beta}{\sqrt{2}}\right|^2$

$= \frac{\alpha^2+\beta^2}{2} + \frac{\alpha^2+\beta^2}{2} = \frac{1}{2}+\frac{1}{2} = 1$

$\therefore$ it is a valid quantum gate.

3.13:

(a) $\cancel{U = \frac{1}{2\sqrt{3}}\begin{pmatrix}3+i & 1-i\\ -1-i & -(1+i)\end{pmatrix}}$ $U = \frac{1}{2\sqrt{3}}\begin{pmatrix}3+i & 1-i\\ -(1+i) & 3-i\end{pmatrix}$

(b) $HUH|0\rangle = HU\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \cancel{\frac{1}{2\sqrt{3}\sqrt{2}}H(} \frac{1}{2\sqrt{6}}H\begin{pmatrix}3+i & 1-i\\ -(1+i) & 3-i\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}$

$$= \frac{1}{2\sqrt{6}} H \begin{pmatrix} 3+i+1-i \\ -1-i+3-i \end{pmatrix} = \frac{1}{2\sqrt{6}} H \begin{pmatrix} 4 \\ 2-2i \end{pmatrix} = \frac{1}{\sqrt{6}} H \begin{pmatrix} 2 \\ 1-i \end{pmatrix}$$

$$= \frac{1}{\sqrt{6}}\left[2H|0\rangle + (1-i)H|1\rangle\right] = \frac{1}{\sqrt{6}}\left[2\cdot\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) + \frac{1-i}{\sqrt{2}}(|0\rangle-|1\rangle)\right]$$

$$= \frac{\sqrt{2}}{\sqrt{6}}(|0\rangle+|1\rangle) + \frac{1-i}{2\sqrt{3}}(|0\rangle-|1\rangle) = \frac{1}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle + \frac{1-i}{2\sqrt{3}}|0\rangle - \frac{1-i}{2\sqrt{3}}|1\rangle$$

$$= \left(\frac{1}{\sqrt{3}} + \frac{1-i}{2\sqrt{3}}\right)|0\rangle + \left(\frac{1}{\sqrt{3}} - \frac{1-i}{2\sqrt{3}}\right)|1\rangle$$

$$= \frac{2+1-i}{2\sqrt{3}}|0\rangle + \frac{2-1+i}{2\sqrt{3}}|1\rangle = \frac{3-i}{2\sqrt{3}}|0\rangle + \frac{1+i}{2\sqrt{3}}|1\rangle$$

$$P(|0\rangle) = \frac{9+1}{12} = \frac{5}{6} = 0.833 = 83.3\%$$

$$P(|1\rangle) = \frac{1+1}{12} = \frac{\cancel{2}\,1}{6} = 0.167 = 16.7\%$$

3.14:

$$e^{i\gamma}\left[\cos(\pi/4)\, I - i\sin(\pi/4)\,(Y)\right]$$

$$= \cos\frac{1}{\sqrt{2}} I - i\cdot\frac{1}{\sqrt{2}}\cdot Y = \frac{1}{\sqrt{2}} I - \frac{i}{\sqrt{2}} Y = \frac{1}{\sqrt{2}}\begin{pmatrix}1&0\\0&1\end{pmatrix} - \frac{i}{\sqrt{2}}\begin{pmatrix}0&-i\\i&0\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1&0\\0&1\end{pmatrix} - \begin{pmatrix}0&1\\-1&0\end{pmatrix}\right]$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1&-1\\1&1\end{pmatrix}$$

(Ans)

$$U = \frac{1}{\sqrt{2}} I|0\rangle - \frac{i}{\sqrt{2}} Y|0\rangle + \frac{1}{\sqrt{2}} I|1\rangle - \frac{i}{\sqrt{2}} Y|1\rangle$$

$$= \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}\cdot i|1\rangle + \frac{1}{\sqrt{2}}|1\rangle - \frac{i}{\sqrt{2}}(-i|0\rangle)$$

$$= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|1\rangle + \frac{i^2}{\sqrt{2}}|0\rangle$$

$$= \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)|0\rangle + \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)|1\rangle \quad \cancel{\sqrt{2}|1\rangle}$$

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3.15:

$$HTU|0\rangle = HT\frac{1}{2}\begin{pmatrix}\sqrt{2}-i & 1\\ -1 & \sqrt{2}+i\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{2}HT\begin{pmatrix}\sqrt{2}-i+0\\ -1+0\end{pmatrix}$$

$$= \frac{1}{2}HT\begin{pmatrix}\sqrt{2}-i\\ -1\end{pmatrix} = \frac{1}{2}H\begin{pmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{pmatrix}\begin{pmatrix}\sqrt{2}-i\\ -1\end{pmatrix} = \frac{1}{2}H\begin{pmatrix}\sqrt{2}-i+0\\ 0-e^{i\pi/4}\end{pmatrix}$$

$$= \frac{1}{2}H\begin{pmatrix}\sqrt{2}-i\\ -e^{i\pi/4}\end{pmatrix} = \frac{1}{2}\times\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}\sqrt{2}-i\\ -e^{i\pi/4}\end{pmatrix}$$

$$= \frac{1}{2\sqrt{2}}\begin{pmatrix}\sqrt{2}-i-e^{i\pi/4}\\ \sqrt{2}-i+e^{i\pi/4}\end{pmatrix}$$

(Ans)

3.16:

(a) $XY|0\rangle = X\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = X\begin{pmatrix}0+0\\ i\end{pmatrix} = X\begin{pmatrix}0\\ i\end{pmatrix} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0\\ i\end{pmatrix} = \begin{pmatrix}0+i\\ 0+0\end{pmatrix} = i\begin{pmatrix}1\\0\end{pmatrix}$

$iZ|0\rangle = i\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = i\begin{pmatrix}1\\0\end{pmatrix}$ $\therefore$ L.H.S = R.H.S

(b) $XY|1\rangle = X\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = X\begin{pmatrix}-i\\0\end{pmatrix} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}-i\\0\end{pmatrix} = \begin{pmatrix}0\\ -i\end{pmatrix}$

$iZ|1\rangle = i\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = i\begin{pmatrix}0\\-1\end{pmatrix} = \begin{pmatrix}0\\-i\end{pmatrix}$ $\therefore$ L.H.S = R.H.S

(b) $XY = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$ ~~$= \begin{pmatrix}0(0)+1(i)\\ 1(0)+0(0)\end{pmatrix} = (0+i$~~

$= \begin{pmatrix}0+i & 0\\ 0 & -i\end{pmatrix} = \begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix} = i\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = iZ$ = R.H.S

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3.17:

$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}$ $U^{\dagger} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & -1 \end{pmatrix}$

$$U^{\dagger}U = \frac{1}{2}\begin{pmatrix} 1 & -i \\ -i & -1 \end{pmatrix}\begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1(1)-i(i) & 1(i)-i(-1) \\ -i(1)-1(i) & -i(i)-1(-1) \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1-i^2 & i+i \\ -i-i & -i^2+1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2 & 2i \\ -2i & 2 \end{pmatrix} = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \neq I$$

$\therefore$ Not a quantum gate.

3.18:

$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$ $U^{\dagger} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$

$$U^{\dagger}U = \frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1(1)-i(i) & 1(1)-i(-i) \\ 1(1)+i(i) & 1(1)+i(-i) \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1-i^2 & 1+i^2 \\ 1+i^2 & 1-i^2 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$ $\therefore$ it's a quantum gate.

$U|0\rangle = \cancel{\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)}\ \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) = |i\rangle$

$U|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle) = |-i\rangle$

3.19:

(a) $$U^{-1} = U^{\dagger} = \begin{pmatrix} \frac{1+\sqrt{3}}{2\sqrt{2}} - i\frac{1-\sqrt{3}}{2\sqrt{6}} & \frac{1-\sqrt{3}}{2\sqrt{6}} - i\frac{1-\sqrt{3}}{2\sqrt{6}} \\ \frac{-1+\sqrt{3}}{2\sqrt{6}} - i\frac{1-\sqrt{3}}{2\sqrt{6}} & \frac{1+\sqrt{3}}{2\sqrt{2}} - i\frac{-1+\sqrt{3}}{2\sqrt{6}} \end{pmatrix}$$

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(b) $|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$

$U^\dagger U|\psi\rangle = |\psi\rangle$

3.20:

(a) $|i\rangle\langle -|,\ |i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

$$\therefore |i\rangle\langle -| = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{pmatrix}\left(\frac{1}{\sqrt{2}}\right) & \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{pmatrix}\left(-\frac{1}{\sqrt{2}}\right) \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ i \end{pmatrix} \frac{1}{2}\begin{pmatrix} -1 \\ -i \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ i & -i \end{pmatrix}$$

(b) $U = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ i & -i \end{pmatrix}$, $U^\dagger = \frac{1}{2}\begin{pmatrix} 1 & -i \\ -1 & i \end{pmatrix}$

$$UU^\dagger = \frac{1}{4}\begin{pmatrix} 1 & -1 \\ i & -i \end{pmatrix}\begin{pmatrix} 1 & -i \\ -1 & i \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1(1)-1(-1) & 1(-i)-1(i) \\ i(1)-i(-1) & i(-i)-i(i) \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1+1 & -i-i \\ i+i & -i^2-i^2 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 2 & -2i \\ 2i & 2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix} \neq I$$ Not a quantum gate.

3.21:

$\frac{1}{\sqrt{2}}|0\rangle\langle 0| + \frac{1}{\sqrt{2}}|0\rangle\langle 1| + \frac{1}{\sqrt{2}}|1\rangle\langle 0| - \frac{1}{\sqrt{2}}|1\rangle\langle 1|$

$$= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}(1\ 0) + \begin{pmatrix} 1 \\ 0 \end{pmatrix}(0\ 1) + \begin{pmatrix} 0 \\ 1 \end{pmatrix}(1\ 0) - \begin{pmatrix} 0 \\ 1 \end{pmatrix}(0\ 1)\right]$$

$$\cancel{= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{bmatrix} 2 \\ 0 \end{bmatrix}} =$$

$$\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1&0\\0&0\end{pmatrix}+\begin{pmatrix}0&1\\0&0\end{pmatrix}+\begin{pmatrix}0&0\\1&0\end{pmatrix}-\begin{pmatrix}0&0\\0&1\end{pmatrix}\right]$$

$$=\frac{1}{\sqrt{2}}\begin{bmatrix}1+0+0\mp 0 & 0+1+0\mp 0\\ 0+0+1\pm 0 & 0+0+0-1\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$$

(b) $U=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$ $U^{\dagger}=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$

$$UU^{\dagger}=\frac{1}{2}\begin{pmatrix}1(1)+1(1) & 1(1)+1(-1)\\ 1(1)-1(1) & 1(1)-1(-1)\end{pmatrix}=\frac{1}{2}\begin{pmatrix}2&0\\0&2\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}=I$$

It's a quantum gate.

3.22:

L.H.S $=|+\rangle\langle+|+|-\rangle\langle-|$ $= \cancel{\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}(0}$

$$=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\end{pmatrix}+\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1&-1\end{pmatrix}$$

$$=\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1&1\end{pmatrix}+\frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1&-1\end{pmatrix}$$

$$=\frac{1}{2}\left[\begin{pmatrix}1&1\\1&1\end{pmatrix}+\begin{pmatrix}1&-1\\-1&1\end{pmatrix}\right]=\frac{1}{2}\begin{bmatrix}2&0\\0&2\end{bmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}=I$$

= R.H.S

3.23:

L.H.S $=|0\rangle\langle 0|+|+\rangle\langle+|=\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1&0\end{pmatrix}+\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1&1\end{pmatrix}$

$$=\begin{pmatrix}1&0\\0&0\end{pmatrix}+\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}=\begin{pmatrix}1+\frac{1}{2}&1/2\\1/2&1/2\end{pmatrix}=\begin{pmatrix}3/2&1/2\\1/2&1/2\end{pmatrix}=\frac{1}{2}\begin{pmatrix}3&1\\1&1\end{pmatrix}\neq I$$

4.1:

(a) when the other player is on ONE.

(b) To phi minus

(c) when the blue player uses an H engine card.

4.2:

(a) 5 (b) conceptual. (c) 50–60%. (d) Entanglement.

4.3:

(a) $\langle 10|11\rangle = \langle 1|1\rangle \cdot \langle 0|1\rangle = 0$

(b) $\langle +-|01\rangle = \langle +|0\rangle \cdot \langle -|1\rangle = \frac{1}{\sqrt{2}} \times -\frac{1}{\sqrt{2}} = -\frac{1}{2}$

(c) $\langle 1+0|1-0\rangle = \langle 1|1\rangle \cdot \langle +|-\rangle \cdot \langle 0|0\rangle = 1 \times 0 \times 1 = 0$

4.4:

$$|1\rangle \otimes |1\rangle \otimes |0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ 1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = R.H.S$$

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4.5:

$$|\psi\rangle = \frac{1}{2}|00\rangle + \frac{i}{\sqrt{2}}|10\rangle + \frac{\sqrt{3}+i}{4}|11\rangle$$

(a) $|\psi\rangle$ as column vector: $\begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{i}{\sqrt{2}} \\ \frac{\sqrt{3}+i}{4} \end{pmatrix}$

(b) $\langle\psi|$ as row vector: $\begin{pmatrix} \frac{1}{2} & 0 & \frac{i}{\sqrt{2}} & \frac{\sqrt{3}+i}{4} \end{pmatrix}$

4.6:

$$L.H.S = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|$$

$$= \begin{pmatrix}1\\0\\0\\0\end{pmatrix}(1000) + \begin{pmatrix}0\\1\\0\\0\end{pmatrix}(0100) + \begin{pmatrix}0\\0\\1\\0\end{pmatrix}(0010) + \begin{pmatrix}0\\0\\0\\1\end{pmatrix}(0001)$$

$$= \begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} = R.H.S$$

4.7:

$$P(|00\rangle) = \left|\frac{i}{\sqrt{10}}\right|^2 = \frac{1}{10}$$

$$P(|01\rangle) = \frac{|1-2i|}{10} = \frac{1+4}{10} = \frac{1}{2}$$

$$P(|10\rangle) = \frac{|e^{i\pi/100}|^2}{10} = \frac{1}{10}$$

$$P(|11\rangle) = \frac{3}{10}$$

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4.8:

$$|A|^2\left(\frac{1}{4}+1+2+1\right)=1$$

$$|A|^2\left(\frac{1+4+8+4}{4}\right)=1$$

$$|A|^2\left(\frac{17}{4}\right)=1$$

$$|A|^2=\frac{4}{17}$$

$$A=\frac{2}{\sqrt{17}}$$

$$\therefore A=2/\sqrt{17}$$

4.9:

for $|0\rangle$:

~~for with~~ probability $\frac{1}{16}+\frac{1}{4}=\frac{1+4}{16}=\frac{5}{16}$

Resulting state: $|A|^2\left(\left|\frac{1}{4}\right|+\left|\frac{1}{2}\right|\right)^2=|A|^2\left(\frac{1}{16}+\frac{1}{4}\right)=1$ Or, $|A|^2=\frac{16}{5}$

$$A=\frac{4}{\sqrt{5}}$$

$$\Rightarrow\ =\frac{4}{\sqrt{5}}\cdot\frac{1}{4}|00\rangle+\frac{4}{\sqrt{5}}\cdot\frac{1}{2}|01\rangle$$

$$=\frac{1}{\sqrt{5}}|00\rangle+\frac{2}{\sqrt{5}}|01\rangle$$

for $|1\rangle$: probability $\frac{1}{2}+\frac{3}{16}=\frac{8+3}{16}=\frac{11}{16}$

Resulting state: $\sqrt{\left(\frac{1}{2}+\frac{3}{16}\right)^{-1}}\left(\frac{1}{\sqrt{2}}|10\rangle+\frac{\sqrt{3}}{4}|11\rangle\right)$

$$=\sqrt{\frac{16}{11}}\times\frac{1}{\sqrt{2}}|10\rangle+\frac{4}{\sqrt{11}}\times\frac{\sqrt{3}}{4}|11\rangle$$

$$=\frac{2\sqrt{2}}{\sqrt{11}}|10\rangle+\frac{\sqrt{3}}{\sqrt{11}}|11\rangle$$

4.10:

for $|00\rangle$: probability: $\left(\frac{1}{36}+\frac{1}{6}\right) = \frac{7}{36}$

resulting state: $\dfrac{\frac{1}{6}|000\rangle + \frac{1}{\sqrt{6}}|010\rangle}{\sqrt{7}/6}$

$$= \frac{1}{6}\times\frac{6}{\sqrt{7}}|000\rangle + \frac{1}{\sqrt{6}}\times\frac{6}{\sqrt{7}}|010\rangle$$

$$= \frac{1}{\sqrt{7}}|000\rangle + \frac{\sqrt{6}}{\sqrt{7}}|010\rangle$$

for $|01\rangle$: probability: $\left|\frac{1}{3\sqrt{2}}\right|^2 + \left|\frac{1}{2}\right|^2 = \frac{1}{18} + \frac{1}{4} = \frac{11}{36}$

Resulting state: $\dfrac{\frac{1}{3\sqrt{2}}|001\rangle + \frac{1}{2}|011\rangle}{\sqrt{11}/6}$

$$= \frac{1}{3\sqrt{2}}\times\frac{6}{\sqrt{11}}|001\rangle + \frac{1}{2}\times\frac{6}{\sqrt{11}}|011\rangle$$

$$= \frac{\sqrt{2}}{\sqrt{11}}|001\rangle + \frac{3}{\sqrt{11}}|011\rangle$$

for $|10\rangle$: probability: $\left|\frac{1}{6}\right|^2 + \left|\frac{1}{6}\right|^2 = \frac{1}{18}$

resulting state: $\dfrac{\frac{1}{6}|100\rangle + \frac{1}{6}|110\rangle}{1/3\sqrt{2}}$

$$= \frac{3\sqrt{2}}{6}|100\rangle + \frac{3\sqrt{2}}{6}|110\rangle$$

$$= \frac{1}{\sqrt{2}}|100\rangle + \frac{1}{\sqrt{2}}|110\rangle$$

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for $|11\rangle$: probability: $\frac{1}{9} + \frac{1}{3} = \frac{4}{9}$

resulting state $\frac{3}{2} \times \frac{1}{3} |101\rangle + \frac{3}{2} \times \frac{1}{\sqrt{3}} |111\rangle$

$= \frac{1}{2} |101\rangle + \frac{\sqrt{3}}{2} |111\rangle$

4.11:

(a) $\frac{1}{\sqrt{2}} |01\rangle + \frac{1}{\sqrt{2}} |10\rangle$

matching with $\alpha_1\alpha_0 |00\rangle + \alpha_1\beta_0 |01\rangle + \alpha_0\beta_1 |10\rangle + \beta_1\beta_0 |11\rangle$

$\alpha_1\alpha_0 = \frac{1}{\sqrt{2}} \cdot \alpha_1\beta_0 = 0 \quad \alpha_0\beta_1 = 0 \quad \beta_1\beta_0 = \frac{1}{\sqrt{2}}$

The qubits can't be factorized, so it's an entangled state.

(b) $\frac{1}{\sqrt{2}} |01\rangle + \frac{i}{\sqrt{2}} |11\rangle$

comparing with $\alpha_1\alpha_0 = 0 \quad \alpha_1\beta_0 = \frac{1}{\sqrt{2}} \quad \alpha_0\beta_1 = 0 \, , \beta_1\beta_0 = \frac{i}{\sqrt{2}}$

Now, ~~$\alpha_1\alpha_0 = \frac{1}{\sqrt{2}}$~~ ~~$\alpha_1\alpha_0 = 0$~~ ~~$\alpha_1\beta_0 = \frac{1}{\sqrt{2}}$~~ ~~$\frac{\beta_1}{\alpha_1\sqrt{2}} = \frac{i}{\sqrt{2}}$~~

~~$\alpha_1 = \frac{1}{\alpha_0\sqrt{2}}$~~ ~~$\alpha_0\beta_1 = 0$~~ ~~$\beta_0 = \frac{1}{\alpha_0\sqrt{2}}$~~ ~~$\frac{\beta_1}{\alpha_1} = i$~~

~~$\frac{1}{\alpha_0\sqrt{2}} \cdot \beta_0 = 0$~~ ~~$\therefore \alpha_0 = 0$~~ ~~$\beta_1 = \alpha_1 i$~~

Here, $\alpha_1\alpha_0 = 0$, $\alpha_0\beta_1 = 0$, either $\alpha_0 = 0$, or $\alpha_1 = \beta_1 = 0$

if $\alpha_0 = 0$ and $\beta_0 = 1$, $\therefore \alpha_1 = \frac{1}{\sqrt{2}} \quad \beta_1 = \frac{i}{\sqrt{2}}$

$\therefore$ The factorization is $|\psi\rangle = (0|0\rangle + 1|1\rangle) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle\right)$

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$= |1\rangle \otimes |i\rangle$

Ans: Product state, $|1\rangle \otimes |i\rangle$

4.12:

ⓐ $\frac{3}{4}|00\rangle - \frac{\sqrt{3}}{4}|01\rangle + \frac{\sqrt{3}}{4}|10\rangle - \frac{1}{4}|11\rangle$

Here, $\alpha_1\alpha_0 = \frac{3}{4}$, $\alpha_1\beta_0 = \frac{-\sqrt{3}}{4}$, $\alpha_0\beta_1 = \frac{\sqrt{3}}{4}$, $\beta_1\beta_0 = -\frac{1}{4}$

$\alpha_1 = \frac{3}{4\alpha_0}$ $\therefore \frac{3}{4\alpha_0}\cdot\beta_0 = \frac{-\sqrt{3}}{4}$

Or, $\frac{\sqrt{3}\beta_0}{\alpha_0} = -1$

or, $\sqrt{3}\beta_0 = -\alpha_0$

Or, $\alpha_0 = -\sqrt{3}\beta_0$

or, $\beta_0 = \frac{-\alpha_0}{\sqrt{3}}$

$\cancel{\therefore -\sqrt{3}\beta_0\beta_1 = \frac{\sqrt{3}}{4}}$

$\cancel{\text{or, } \beta_1 = \frac{\sqrt{3}}{4}\times\frac{1}{-\sqrt{3}}\times\frac{1}{\beta_0}}$

~~or, β_1~~

$\beta_1 \cdot \frac{-\alpha_0}{\sqrt{3}} = -\frac{1}{4}$

or, $\beta_1 = \frac{-1}{4}\times\frac{\sqrt{3}}{-\alpha_0}$

Or, $\beta_1 = \frac{\sqrt{3}}{4\alpha_0}$

Now, plugging into the product: $\cancel{\left(\frac{3}{4\alpha_0}|0\rangle - \frac{\alpha_0}{\sqrt{3}}|1\rangle\right)(}$

$$(\alpha_1|0\rangle + \beta_1|1\rangle)(\alpha_0|0\rangle + \beta_0|1\rangle)$$

$$= \left(\frac{3}{4\alpha_0}|0\rangle + \frac{\sqrt{3}}{4\alpha_0}|1\rangle\right)\left(\alpha_0|0\rangle - \frac{\alpha_0}{\sqrt{3}}|1\rangle\right)$$

$$= \frac{\sqrt{3}}{4\alpha_0}\left(\sqrt{3}|0\rangle + |1\rangle\right)\left(|0\rangle - \frac{1}{\sqrt{3}}|1\rangle\right)\alpha_0$$

$$= \frac{\sqrt{3}}{4}\left(\sqrt{3}|0\rangle + |1\rangle\right)\left(|0\rangle - \frac{1}{\sqrt{3}}|1\rangle\right)$$

$$= \frac{1}{2}\left(\sqrt{3}|0\rangle + |1\rangle\right)\cdot\frac{\sqrt{3}}{2}\left(|0\rangle - \frac{1}{\sqrt{3}}|1\rangle\right)$$

$$= \left(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle\right)\left(\frac{\sqrt{3}}{2}|0\rangle - \frac{1}{2}|1\rangle\right)$$

$\therefore$ Product state.

(b) $\frac{1}{\sqrt{3}}|0\rangle|+\rangle + \sqrt{\frac{2}{3}}|1\rangle|-\rangle$

$= \frac{1}{\sqrt{3}}|0\rangle \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) + \sqrt{\frac{2}{3}} \cdot |1\rangle \cdot \frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$

$= \frac{1}{\sqrt{6}}|0\rangle(|0\rangle+|1\rangle) + \frac{1}{\sqrt{3}}|1\rangle(|0\rangle-|1\rangle)$

$= \frac{1}{\sqrt{6}}|00\rangle + \frac{1}{\sqrt{6}}|01\rangle + \frac{1}{\sqrt{3}}|10\rangle - \frac{1}{\sqrt{3}}|11\rangle$

$\alpha_1\alpha_0 = \frac{1}{\sqrt{6}}$, $\alpha_1\beta_0 = \frac{1}{\sqrt{6}}$, $\alpha_0\beta_1 = \frac{1}{\sqrt{3}}$, $\beta_1\beta_0 = -\frac{1}{\sqrt{3}}$

$\alpha_1 = \frac{1}{\alpha_0\sqrt{6}}$, $\frac{\beta_0}{\alpha_0\sqrt{6}} = \frac{1}{\sqrt{6}}$, $\beta_1 = \frac{1}{\sqrt{3}\alpha_0}$

$\frac{\beta_0}{\alpha_0} = 1$

$\beta_0 = \alpha_0$

Here, $\beta_1\beta_0 = \frac{1}{\sqrt{3}\alpha_0} \times \alpha_0 = \frac{1}{\sqrt{3}}$ doesn't match with initial $\beta_1\beta_0$

again, $\alpha_1\beta_0 = \frac{1}{\alpha_0\sqrt{6}} \times \alpha_0 = \frac{1}{\sqrt{6}} = \alpha_1\beta_0$, match.

again, $\alpha_0\beta_1 = \alpha_0 \times \frac{1}{\sqrt{3}\alpha_0} = \frac{1}{\sqrt{3}} = \alpha_0\beta_1$ match

every corresponding values do not match, so product state is not possible.

∴ Entangled state.

4.13

(a) $|\psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

Applying X gate on left qubit : $(X \otimes I)|\psi^+\rangle = \frac{1}{\sqrt{2}}(|11\rangle + |00\rangle)$

" " " " right " : $(I \otimes X)|\psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$\therefore (X \otimes I)|\psi^+\rangle = (I \otimes X)|\psi^+\rangle = |\phi^+\rangle$

(b) $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Applying X gate on left qubit : $(X \oplus I)|\phi^+\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$

" " " " right " : $(I \otimes X)|\phi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

$\therefore (X \oplus I)|\phi^+\rangle = (I \otimes X)|\phi^+\rangle = |\psi^+\rangle$

(c) $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

X on left : $(X \oplus I)|\psi^-\rangle = \frac{1}{\sqrt{2}}(|11\rangle - |00\rangle) = -\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) = -|\phi^-$

X on right : $(I \otimes X)|\psi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) = |\phi^-\rangle$

(d) $|\phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

X on left : $(X \otimes I)|\phi^-\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle) = -|\psi^-\rangle$

X " right : $(I \otimes X)|\phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) = |\psi^-\rangle$

4.14:

(a) $H \otimes X = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & 1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ 1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & -1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{pmatrix}$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

(b) $|\psi\rangle = \frac{1}{4}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle + \frac{\sqrt{3}}{4}|11\rangle$

$$\cancel{(H \otimes X)|\psi\rangle = \frac{1}{\sqrt{2}} \times \frac{1}{4}\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \times \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}}$$

$$\cancel{+ \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{4}\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}}$$

$$\cancel{= \frac{1}{4\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} +}$$

representing $|\psi\rangle$ as a column vector: $\begin{pmatrix} 1/4 \\ 1/2 \\ 1/\sqrt{2} \\ \sqrt{3}/4 \end{pmatrix}$

Now, $(H \otimes X)|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 1/4 \\ 1/2 \\ 1/\sqrt{2} \\ \sqrt{3}/4 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1/2 + \frac{\sqrt{3}}{4} \\ 1/4 + 1/\sqrt{2} \\ 1/2 - \sqrt{3}/4 \\ 1/4 - 1/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{2+\sqrt{3}}{4} \\ \frac{1+2\sqrt{2}}{4} \\ \frac{2-\sqrt{3}}{4} \\ \frac{1-2\sqrt{2}}{4} \end{pmatrix}$

$$= \frac{1}{\sqrt{2}}\left(\frac{2+\sqrt{3}}{4}\right)|00\rangle + \frac{1}{\sqrt{2}}\left(\frac{1+2\sqrt{2}}{4}\right)|01\rangle + \frac{1}{\sqrt{2}}\left(\frac{2-\sqrt{3}}{4}\right)|10\rangle + \frac{1}{\sqrt{2}}\left(\frac{1-2\sqrt{2}}{4}\right)|11\rangle$$

$$= \frac{2+\sqrt{3}}{4\sqrt{2}}|00\rangle + \frac{1+2\sqrt{2}}{4\sqrt{2}}|01\rangle + \frac{2-\sqrt{3}}{4\sqrt{2}}|10\rangle + \frac{1-2\sqrt{2}}{4\sqrt{2}}|11\rangle$$

$$\approx \frac{2+\sqrt{3}}{4\sqrt{2}}|00\rangle + \frac{4+\sqrt{2}}{8}|01\rangle + \frac{2-\sqrt{3}}{4\sqrt{2}}|10\rangle + \frac{4-\sqrt{2}}{8}|11\rangle$$

4.15:

(a) $CNOT(X \otimes I) = CNOT \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = CNOT \begin{pmatrix} 0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & 1\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ 1\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & 0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix}$

$= CNOT \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

$(X \otimes X)\,CNOT = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} CNOT = \begin{pmatrix} 0\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & 1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ 1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & 0\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

$\therefore$ L.H.S. = R.H.S

(b) $CNOT(I \otimes X) = CNOT \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = CNOT \begin{pmatrix} 1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & 0\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ 0\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & 1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$(I \otimes X)\,CNOT = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} CNOT = \begin{pmatrix} 1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & 0\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ 0\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & 1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$\therefore$ L.H.S = R.H.S

(c) $$\text{CNOT}(Z\otimes I) = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}1\begin{pmatrix}1&0\\0&1\end{pmatrix} & 0\begin{pmatrix}1&0\\0&1\end{pmatrix}\\ 0\begin{pmatrix}1&0\\0&1\end{pmatrix} & -1\begin{pmatrix}1&0\\0&1\end{pmatrix}\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$$

$$= \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&-1\\0&0&-1&0\end{pmatrix}$$

$$(Z\otimes I)\,\text{CNOT} = \begin{pmatrix}1\begin{pmatrix}1&0\\0&1\end{pmatrix} & 0\begin{pmatrix}1&0\\0&1\end{pmatrix}\\ 0\begin{pmatrix}1&0\\0&1\end{pmatrix} & -1\begin{pmatrix}1&0\\0&1\end{pmatrix}\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

$$= \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&-1\\0&0&-1&0\end{pmatrix}$$

$\therefore$ L.H.S = R.H.S

(d) $$\text{CNOT}(1\otimes Z) = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}1\begin{pmatrix}1&0\\0&-1\end{pmatrix} & 0\begin{pmatrix}1&0\\0&-1\end{pmatrix}\\ 0\begin{pmatrix}1&0\\0&-1\end{pmatrix} & 1\begin{pmatrix}1&0\\0&-1\end{pmatrix}\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix}$$

$$(Z\otimes Z)\,\text{CNOT} = \begin{pmatrix}1\begin{pmatrix}1&0\\0&-1\end{pmatrix} & 0\begin{pmatrix}1&0\\0&-1\end{pmatrix}\\ 0\begin{pmatrix}1&0\\0&-1\end{pmatrix} & -1\begin{pmatrix}1&0\\0&-1\end{pmatrix}\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix}$$

$\therefore$ L.H.S = R.H.S

4.16:

(a)

A B C	A' B' C'
0 0 0	0 0 0
0 0 1	0 0 1
0 1 0	0 1 1
0 1 1	0 1 0
1 0 0	1 0 1
1 0 1	1 0 0
1 1 0	1 1 1
1 1 1	1 1 1

A B C	A' B' C'
0 0 0	0 0 0
0 0 1	0 0 1
0 1 0	0 1 1
0 1 1	0 1 0
1 0 0	1 0 1
1 0 1	1 0 0
1 1 0	1 1 0
1 1 1	1 1 1

(b) Same

sat sun mon tue wed thu fri Date: / /

4.17:

anti controlled- CNOT gate : $\begin{pmatrix} 0&1&0&0\\ 1&0&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{pmatrix}$

~~Now, $(X\otimes X)$ CNOT $(X\otimes X)$ $= \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{pmatrix} \begin{pmatrix} 0\begin{pmatrix}0&1\\1&0\end{pmatrix} & 1\begin{pmatrix}0&1\\1&0\end{pmatrix}\\ 1\begin{pmatrix}0&1\\1&0\end{pmatrix} & 0\begin{pmatrix}0&1\\1&0\end{pmatrix} \end{pmatrix} = \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{pmatrix} \begin{pmatrix} 0&0&0&1\\ 0&0&1&0\\ 0&1&0&0\\ 1&0&0&0 \end{pmatrix}$~~

Now, $(X\otimes X)\, CNOT\, (X\otimes X) = \begin{pmatrix} 0&0&0&1\\ 0&0&1&0\\ 0&1&0&0\\ 1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{pmatrix} \begin{pmatrix} 0&0&0&1\\ 0&0&1&0\\ 0&1&0&0\\ 1&0&0&0 \end{pmatrix}$

$= \begin{pmatrix} 0&0&0&1\\ 0&0&1&0\\ 0&1&0&0\\ 1&0&0&0 \end{pmatrix} \begin{pmatrix} 0&0&0&1\\ 0&0&1&0\\ 1&0&0&0\\ 0&1&0&0 \end{pmatrix} = \begin{pmatrix} 0&1&0&0\\ 1&0&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{pmatrix}$

$\therefore$ L.H.S = R.H.S

~~4.18~~:

$CNOT|+\rangle|-\rangle = (H\otimes H)\, CNOT_{01}\, (H\otimes H)|+\rangle|-\rangle$

$= (H\otimes H)\, CNOT_{01}\, |0\rangle|1\rangle$

$= (H\otimes H)\, |1\rangle|0\rangle$

$= \frac{1}{2}\begin{pmatrix} 1&1&1&1\\ 1&-1&1&-1\\ 1&1&-1&-1\\ 1&-1&-1&1 \end{pmatrix}\begin{pmatrix} 0\\0\\1\\0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1\\1\\-1\\-1 \end{pmatrix}$

$= \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle - |11\rangle\right)$

$= \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle - \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$

Here, $\alpha_1\alpha_0 = \frac{1}{2}$ $\quad \alpha_1\beta_0 = \frac{1}{2}$ $\quad \alpha_0\beta_1 = -\frac{1}{2}$ $\quad \beta_0\beta_1 = -\frac{1}{2}$

$\alpha_1 = \frac{1}{2\alpha_0}$ $\quad \beta_0 = \alpha_0$ $\quad \beta_1 = -\frac{1}{2\alpha_0}$ $\quad \beta_1 = -\frac{1}{2\alpha_0}$

$\therefore f$

4.18:

$$|+-\rangle = \left\{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right\}\left\{\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right\} = \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle\right)$$

$$|-+\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle - |11\rangle\right)$$

$$|--\rangle = \frac{1}{2}\left(|00\rangle - |01\rangle - |10\rangle + |11\rangle\right)$$

Now,

$$\begin{aligned}
CNOT\,|+\rangle|-\rangle &= (H\otimes H)\,CNOT_{01}\,(H\otimes H)\,|+\rangle|-\rangle \\
&= (H\otimes H)\,CNOT_{01}\,|0\rangle|1\rangle \\
&= (H\otimes H)\,|1\rangle|1\rangle \\
&= \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \\
&= \frac{1}{2}\left(|00\rangle - |01\rangle - |10\rangle + |11\rangle\right) \\
&= |-\rangle|-\rangle
\end{aligned}$$

$$\therefore CNOT\,|+\rangle|-\rangle = |-\rangle|-\rangle$$

$$\begin{aligned}
CNOT\,|-+\rangle &= (H\otimes H)\,CNOT_{01}\,(H\otimes H)\,|-\rangle|+\rangle \\
&= (H\otimes H)\,CNOT_{01}\,|1\rangle|0\rangle \\
&= (H\otimes H)\,|1\rangle|0\rangle \\
&= \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \\
&= \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right) \\
&= |-\rangle|+\rangle
\end{aligned}$$

$\therefore CNOT\,|-\rangle|+\rangle = |-\rangle|+\rangle$

$$CNOT\,|-\rangle|-\rangle = (H\otimes H)\,CNOT_{01}\,(H\otimes H)\,|-\rangle|-\rangle$$

$$= (H\otimes H)\,CNOT_{01}\,|1\rangle|1\rangle$$

$$= (H\otimes H)|0\rangle|1\rangle = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

$$= \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle\right)$$

$$= |+\rangle|-\rangle$$

$$\therefore\ CNOT|-\rangle|-\rangle = |+\rangle|-\rangle$$

4.19:

$$L.H.S = |\Phi^+\rangle\langle\Phi^+| + |\Phi^-\rangle\langle\Phi^-| + |\Psi^+\rangle\langle\Psi^+| + |\Psi^-\rangle\langle\Psi^-|$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}\frac{1}{\sqrt{2}}(1\,0\,0\,1) + \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}\frac{1}{\sqrt{2}}(1\,0\,0\,-1) + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}\frac{1}{\sqrt{2}}(0\,1\,1\,0)$$

$$+ \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}\frac{1}{\sqrt{2}}(0\,1\,-1\,0)$$

$$= \frac{1}{2}\begin{pmatrix} 1(1) & 1(0) & 1(0) & 1(1) \\ 0(1) & 0(0) & 0(0) & 0(1) \\ 0(1) & 0(0) & 0(0) & 0(1) \\ 1(1) & 1(0) & 1(0) & 1(1) \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1(1) & 1(0) & 1(0) & 1(-1) \\ 0(1) & 0(0) & 0(0) & 0(-1) \\ 0(1) & 0(0) & 0(0) & 0(-1) \\ -1(1) & -1(0) & -1(0) & -1(-1) \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0(0) & 0(1) & 0(1) & 0(0) \\ 1(0) & 1(1) & 1(1) & 1(0) \\ 1(0) & 1(1) & 1(1) & 1(0) \\ 0(0) & 0(1) & 0(1) & 0(0) \end{pmatrix}$$

$$+ \frac{1}{2}\begin{pmatrix} 0(0) & 0(1) & 0(-1) & 0(0) \\ 1(0) & 1(1) & 1(-1) & 1(0) \\ -1(0) & -1(1) & -1(-1) & -1(0) \\ 0(0) & 0(1) & 0(-1) & 0(0) \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = I$$

$$\therefore\ L.H.S = R.H.S$$

4.20:

Controlled Z gate, or CZ gate:

for any gate U, $CU = \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&a&c \\ 0&0&b&d \end{pmatrix}$

$\because \; Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \therefore \; CZ = \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&-1 \end{pmatrix}$.

4.21:

(a) $SWAP\,|w_0\rangle = \cancel{(CNOT)(CNOT_{01})(CNOT)\,|w_0\rangle = (CNOT)(CNOT_{01})(CNOT)}$

$\cancel{\left(\frac{1}{2}|00\rangle - |01\rangle + |10\rangle + |11\rangle\right)}$

$\cancel{= (CNOT)(CNOT_{01})(CNOT_{10})\frac{1}{2}\left(|00\rangle - |01\rangle + |11\rangle + |10\rangle\right)}$

$= \cancel{(CN} \quad = SWAP\,\frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle + |11\rangle\right)$

$= \frac{1}{2}\left(|00\rangle - |10\rangle + |01\rangle + |11\rangle\right)$

$= \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right)$

$= |w_3\rangle$

$\therefore$ L.H.S = R.H.S

(b) $CNOT_{01}\,|w_2\rangle = CNOT_{01}\,\frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle - |11\rangle\right)$

$= \frac{1}{2}\left(|00\rangle + |11\rangle + |10\rangle - |01\rangle\right)$

$= \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle + |11\rangle\right)$

$= w_0$

$\therefore$ L.H.S = R.H.S

ⓑ $(X \otimes I)\,\frac{1}{2}\left(-|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$

$= \frac{1}{2}\left(-|10\rangle + |11\rangle + |00\rangle + |01\rangle\right)$

$= \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right) = \omega_3 \qquad \therefore L{\cdot}H{\cdot}S = R{\cdot}H{\cdot}S$

ⓓ $\text{CNOT}\,|\omega_3\rangle = \text{CNOT}\,\frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right)$

$= \frac{1}{2}\left(|00\rangle + |01\rangle - |11\rangle + |10\rangle\right)$

$= \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle - |11\rangle\right) = \omega_2$

$\therefore L{\cdot}H{\cdot}S = R{\cdot}H{\cdot}S$

4.22

ⓐ MS gate as a matrix :

$$\cancel{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & -i & i \\ 0 & 1 & 0 & 0 \\ 0 & -i & 1 & 0 \\ i & 0 & 0 & 1 \end{pmatrix}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & -i & 0 \\ 0 & -i & 1 & 0 \\ i & 0 & 0 & 1 \end{pmatrix}$$

ⓑ $MS^8 = \begin{pmatrix} 1&0&0&i \\ 0&1&-i&0 \\ 0&-i&1&0 \\ i&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&i \\ 0&1&-i&0 \\ 0&i&1&0 \\ i&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&i \\ 0&1&-i&0 \\ 0&-i&1&0 \\ i&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&i \\ 0&1&-i&0 \\ 0&-i&1&0 \\ -i&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&i \\ 0&1&-i&0 \\ 0&-i&1&0 \\ i&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&i \\ 0&1&-i&0 \\ 0&-i&1&0 \\ i&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&i \\ 0&1&-i&0 \\ 0&-i&1&0 \\ i&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&i \\ 0&1&-i&0 \\ 0&-i&1&0 \\ i&0&0&1 \end{pmatrix}$

$= \begin{pmatrix} 0&0&0&2i \\ 0&0&-2i&0 \\ 0&-2i&0&0 \\ 2i&0&0&0 \end{pmatrix}\begin{pmatrix} 0&0&0&2i \\ 0&0&-2i&0 \\ 0&-2i&0&0 \\ 2i&0&0&0 \end{pmatrix}\begin{pmatrix} 0&0&0&2i \\ 0&0&-2i&0 \\ 0&-2i&0&0 \\ 2i&0&0&0 \end{pmatrix}\begin{pmatrix} 0&0&0&2i \\ 0&0&-2i&0 \\ 0&-2i&0&0 \\ 2i&0&0&0 \end{pmatrix} \times \frac{1}{16}$

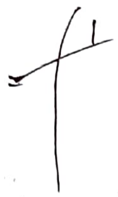
$= \frac{1}{16}\begin{pmatrix} -4&0&0&0 \\ 0&-4&0&0 \\ 0&0&-4&0 \\ 0&0&0&-4 \end{pmatrix}\begin{pmatrix} -4&0&0&0 \\ 0&-4&0&0 \\ 0&0&-4&0 \\ 0&0&0&-4 \end{pmatrix} = \frac{16}{16} \times \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix} = I$

(Proved)

1.23:

~~$U_{\text{Toffoli}} = (I \otimes S \otimes I) \cdot CNOT (I \otimes T^\dagger \otimes I) \cdot CNOT (I \otimes T \otimes T) \cdot CNOT (I \otimes I \otimes T^\dagger$~~

CNOT



$$U_{\text{Toffoli}} = (I \otimes I \otimes H) \cdot CNOT_{1\to2} \cdot (I \otimes I \otimes T^\dagger) \cdot CNOT_{0\to2} \cdot (I \otimes I \otimes T) \cdot$$

$$CNOT_{1\to2} \cdot (I \otimes I \otimes T^\dagger) \cdot CNOT_{0\to2} \cdot (I \otimes T \otimes T) \cdot (I \otimes I \otimes H) \cdot$$

$$CNOT_{0\to1} \cdot (I \otimes T^\dagger \otimes I) \cdot CNOT_{0\to1} (I \otimes S \otimes I)$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{-i\pi/4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-i\pi/4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-i\pi/4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{-i\pi/4} \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{i\pi/4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{i\pi/4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{i\pi/4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{i\pi/4} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \times$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{-i\pi/4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-i\pi/4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-i\pi/4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{-i\pi/4} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}(1+i) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}}(i+1) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}}(1+i) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}}(1+i) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\times \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}}(1-i) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}}(1-i) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}}(1-i) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}}(1-i) \end{pmatrix}$$

$$\times \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \end{pmatrix} \times \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&i&0&0&0&0&0 \\ 0&0&0&i&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&i&0 \\ 0&0&0&0&0&0&0&i \end{pmatrix}$$

Now, multiplying all matrices taking two at a time, with the help of computer, the result is:

$$\text{Toffoli} = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{pmatrix}$$

4.29:

(a) for $|000\rangle$ it converts into $|001\rangle$ and for $|001\rangle$, it turns into $|000\rangle$. Here bit flips if first two bits are zero, others remain the same.

(b)
$$\begin{pmatrix} 0&1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

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4.25:

ⓐ Yes, orthogonal

ⓑ Yes, orthogonal

ⓒ No, not orthogonal

4.26:

Circuit simulated via Quirk.

O/P is: $|s_4\rangle|s_3\rangle|a_3\rangle|c_3\rangle|s_2\rangle|a_2\rangle|c_2\rangle|s_1\rangle|a_1\rangle|c_1\rangle|s_0\rangle|a_0\rangle|c_0\rangle$ =

$|1110010110010\rangle$

4.27:

Modified the circuit by adding x gate before a's and after adding x gate to each bit of s.

O/P : $|0000100000000\rangle$

4.28:

ⓐ before CNOT gate, lowest 4 qubits are : c_3', a_3, $a_3 \oplus b_3$ and s_4. If we take CNOT of c_3' and $a_3 \oplus b_3$, then the o/p will be : $c_3' \oplus a_3 \oplus b_3$

$= s_3$

Thus, this simplification is correct.

~~4.29~~:

ⓑ for length n :

		Toffoli gate	no. of CNOT
C gates needed	: n	$\therefore$ $2n$ Toffoli gates	n
$C^{\dagger}$ gates needed	: $n-1$	$\therefore$ $2(n-1)$ Toffoli gates	$(n-1)$
S gates needed	: $n-1$	$\therefore$	$2(n-1)$
extra 1 CNOT gate	:		1
Total no. of gates	:	$2n+2(n-1)$ $= 4n-2$	$n+n-1+1$ $+2n-2$ $= 4n-2$

$\therefore$ Toffoli gate required : $4n-2$

$\therefore$ CNOT " " : $4n-2$

(Ans)

4.29:

Draper's adder is implemented and the final state is:

$$|s\rangle|b\rangle = |1010\rangle|0011\rangle$$

4.30:

ⓐ $4(4)-2 = 16-2 = 14$ Toffoli gates

$4(4) = 16$ CNOT gates

ⓑ $4(8)-2 = 30$ Toffoli gates

$4(8) = 32$ CNOT gates

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4.31:

for length 1 qbit, we need c_0, a_0, b_0 and s, or b_1

for " 2 qubit " " $c_0, c_1, a_0, a_1, b_0, b_1$ and b_2

$\therefore$ for bits of length 4, the number of qubits needed :

$3n + 1$

4.32:

ⓐ Tom Wong, author

ⓑ hands, quantum, some, not faster

4.33:

ⓐ missing complex amplitudes

ⓑ entanglement (no 2-qubit gates)

ⓒ entanglement (SWAP can't generate entanglement)

4.34:

1. $\{$Toffoli, H, S$\}$
2. $\{$CNOT, $R_{\pi/8}$, S$\}$
3. $\{$CNOT, H, T$\}$

4.35:

Here, $|\psi 00\rangle = \alpha|000\rangle + \beta|100\rangle$

$CNOT_{2,1}(|\psi 00\rangle) = \alpha|000\rangle + \beta|110\rangle \xrightarrow{CNOT_{2,0}} \alpha|000\rangle + \beta|111\rangle$

$\cancel{CNOT_{2,0}(|\psi 00\rangle)}$ $= \alpha|0_L\rangle + \beta|1_L\rangle$ = R.H.S (shown)

4.36:

(a) No error, nothing gates

(b) X gate to rightmost qubit

(c) X gate to leftmost qubit

(d) X gate to middle qubit

4.37:

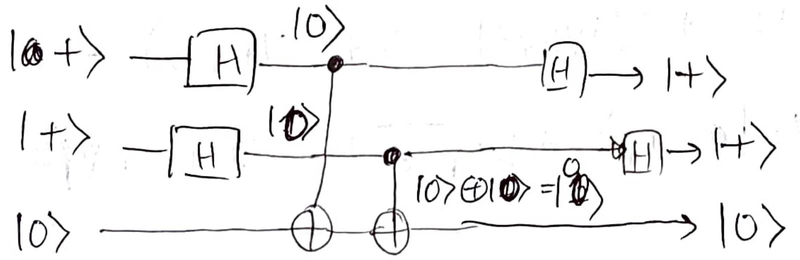
Here, $|\psi 00\rangle = \alpha|000\rangle + \beta|100\rangle \xrightarrow{CNOT_{2,1}} \alpha|000\rangle + \beta|110\rangle$

$\xrightarrow{CNOT_{2,0}} \alpha|000\rangle + \beta|111\rangle \xrightarrow{H\otimes H\otimes H} \alpha|+++\rangle + \beta|---\rangle$

(shown)

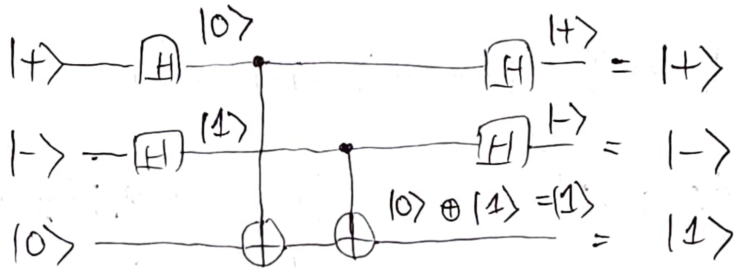
4.38:

(a) ~~$|+\rangle\otimes|+\rangle$~~ ~~Initial state : $|++0\rangle$~~



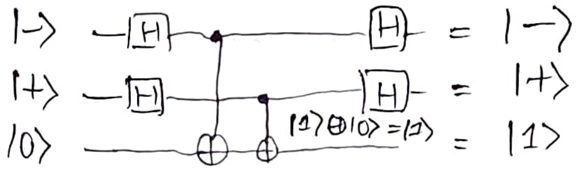
$|0++\rangle$

(b)



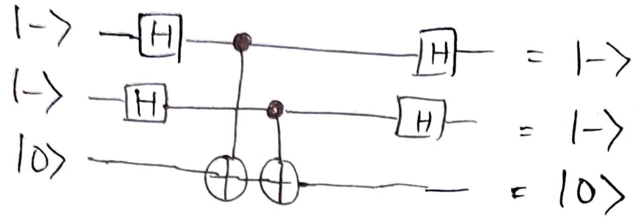
$|1-+\rangle$

(c)



$|1+-\rangle$

(d)



$\uparrow$ $|0--\rangle$

(e) O/P is zero for even no. of $|+\rangle$'s or $|-\rangle$, else one.

4.39:

row 1: move 2nd H gate at the end of the circuit

row 2: $H^2 = I$, $\therefore$ remove two H gates in the middle.

row 3: move 1st H gate at the begining of the circuit.

Thus, the circuit becomes equivalent.

4.40:

$\alpha(i\sqrt{1-\epsilon^2}|+++\rangle + \epsilon|-++\rangle) + \beta(i\sqrt{1-\epsilon^2}|---\rangle + \epsilon|+--\rangle) =$

$\alpha i\sqrt{1-\epsilon^2}|+++\rangle + \alpha\epsilon|-++\rangle + \beta i\sqrt{1-\epsilon^2}|---\rangle + \beta\epsilon|+--\rangle$

(a) $|\alpha i\sqrt{1-\epsilon^2}|^2 + |\beta i\sqrt{1-\epsilon^2}|^2 = (\alpha^2+\beta^2) + (1-\epsilon^2) = 1-\epsilon^2$

Resulting state: $\alpha|+++\rangle + \beta|---\rangle$, no error

(b) ϵ^2, Resulting state: $\alpha|-++\rangle + \beta|+--\rangle$, apply $(Z \otimes I \otimes I)$

then, $\alpha|+++\rangle - \beta|---\rangle$, then apply $(X \otimes I \otimes I)$ $\therefore$ $\alpha|+++\rangle + \beta|---\rangle$

(c) probability is zero

(d) probability is zero

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4.11:

(a) $|\psi 00000000\rangle = \alpha|000000000\rangle + \beta|100000000\rangle$

$$\xrightarrow{CNOT_{8:5,2}} \alpha|000000000\rangle + \beta|100100100\rangle$$

(shown)

(b) after (a):

$$\xrightarrow{H_8, H_5, H_2} \alpha|+00+00+00\rangle + \beta|-00-00-00\rangle$$

$$= \alpha\left(\tfrac{1}{\sqrt{2}}(|0\rangle+|1\rangle)|00\rangle\tfrac{1}{\sqrt{2}}(|0\rangle+|1\rangle)|00\rangle\tfrac{1}{\sqrt{2}}(|0\rangle+|1\rangle)|00\rangle\right) + \beta\left(\tfrac{1}{\sqrt{2}}(|100\rangle-|100\rangle)\tfrac{1}{\sqrt{2}}(|000\rangle-|100\rangle)\tfrac{1}{\sqrt{2}}(|000\rangle-|100\rangle)\right)$$

$$= \frac{\alpha}{\sqrt{2}}(|000\rangle+|100\rangle)(|000\rangle+|100\rangle)(|000\rangle+|100\rangle) +$$

$$\frac{\beta}{\sqrt{2}}(|000\rangle-|100\rangle)(|000\rangle-|100\rangle)(|000\rangle-|100\rangle)$$

(shown)

(c) after (b):

$$\xrightarrow{CNOT_{all}} \frac{\alpha}{\sqrt{2}}(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)(|000\rangle+|111\rangle) +$$

$$\frac{\beta}{\sqrt{2}}(|000\rangle-|100\rangle)(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)$$

$$= \alpha|0_L\rangle + \beta|1_L\rangle$$

(shown)

4.42:

$$\begin{array}{ccc} \cancel{0}\,1 & 1 & 1 \\ 0\,0\,\cancel{0}\,1 & \cancel{0}\,0\,0 & 0\,0\,0 \\ 8\ 7\ 6 & 5\ 4\ 3 & 2\ 1\ 0 \end{array}$$

Here, q_6, q_5 and q_1 are flipped.

we can correct them by applying X gate to each of them.

4.43:

Circuit constructed and simulated via Quirk.

4.44:

(a) $\frac{\alpha}{2^{3/2}}(|000\rangle + |111\rangle)(|000\rangle - |111\rangle)(|000\rangle + |111\rangle)$ H_8, H_5, H_2

$+ \frac{\beta}{2^{3/2}}(|000\rangle - |111\rangle)(|000\rangle + |111\rangle)(|000\rangle - |111\rangle)$

let, $|\psi\rangle = \frac{\alpha}{2^{3/2}} \cdot |\psi_0\rangle + \frac{\beta}{2^{3/2}}|\psi_1\rangle$

After applying CNOT and H gate:

$|000\rangle + |111\rangle \longrightarrow |000\rangle$

$|000\rangle - |111\rangle \longrightarrow |100\rangle$

$\therefore |\psi_0\rangle = |000\rangle|100\rangle|000\rangle = |000100000\rangle$

$|\psi_1\rangle = |100\rangle|000\rangle|100\rangle = |100000100\rangle$

$\therefore |\psi\rangle = \alpha|000100000\rangle + \beta|100000100\rangle$ (shown)

(b) After applying 4 CNOTs,

ancilla 1 stores : $q_2 \oplus q_5$

ancilla 2 stores : $q_5 \oplus q_8$

for qubits with coefficient α : $q_0 = 0$ $q_5 = 1$ $q_8 = 0$

$\therefore$ ancilla 1 : $0 \oplus 1 = 1$

ancilla 2 : $1 \oplus 0 = 1$

$\therefore$ α part : $\alpha(|000100000\rangle \otimes |11\rangle)$

for qubits with coefficient β : $q_2 = 1, q_5 = 0, q_8 = 1$

ancilla 1 : $q_2 \oplus q_5 = 1 \oplus 0 = 1$

ancilla 2 : $q_5 \oplus q_8 = 0 \oplus 1 = 1$

$\therefore$ β part : $\beta(|100000100\rangle \otimes |\cancel{00}11\rangle)$

final state : $\alpha(|000100000\rangle \otimes |11\rangle) + \beta(|\cancel{0000}\,100000100\rangle \otimes |11\rangle)$

$\therefore$ ancilla qubits correctly encode the phase parity of the adjacent triplets. for this circuits

(c) at the end of the circuits, H gate is applied again and CNOTs are integrated at the leftmost qubit of each triplet. $\therefore$

$$|000100000\rangle = (|000\rangle + |111\rangle)(|000\rangle - |111\rangle)(|000\rangle + |111\rangle)$$

$$|100000100\rangle = (|000\rangle - |111\rangle)(|000\rangle + |111\rangle)(|000\rangle - |111\rangle)$$

$\therefore$ The circuit is same again.

(d) ~~Applying Z gate~~ from the parities, we can say that middle triplet ~~was~~ had a phase flip.

By applying Z gate to the leftmost qubit in middle triplet, phase flip can be corrected.

4.45:

Quantum circuit is constructed in Quirk.

we apply $|0\rangle\langle 0|$ to simulate the ~~ze~~ conversion of anulla to $|0\rangle$

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4.46:

(a) 010 001 100 for α part
876 543 210

qubit $|q_7\rangle$, $|q_3\rangle$ and $|q_2\rangle$ were flipped

Can be fixed by applying X gate to each one.

(b) phase error at triplet_0. Can be fixed by applying Z gate to it.