

# Abstraction in Reinforcement Learning

David Abel

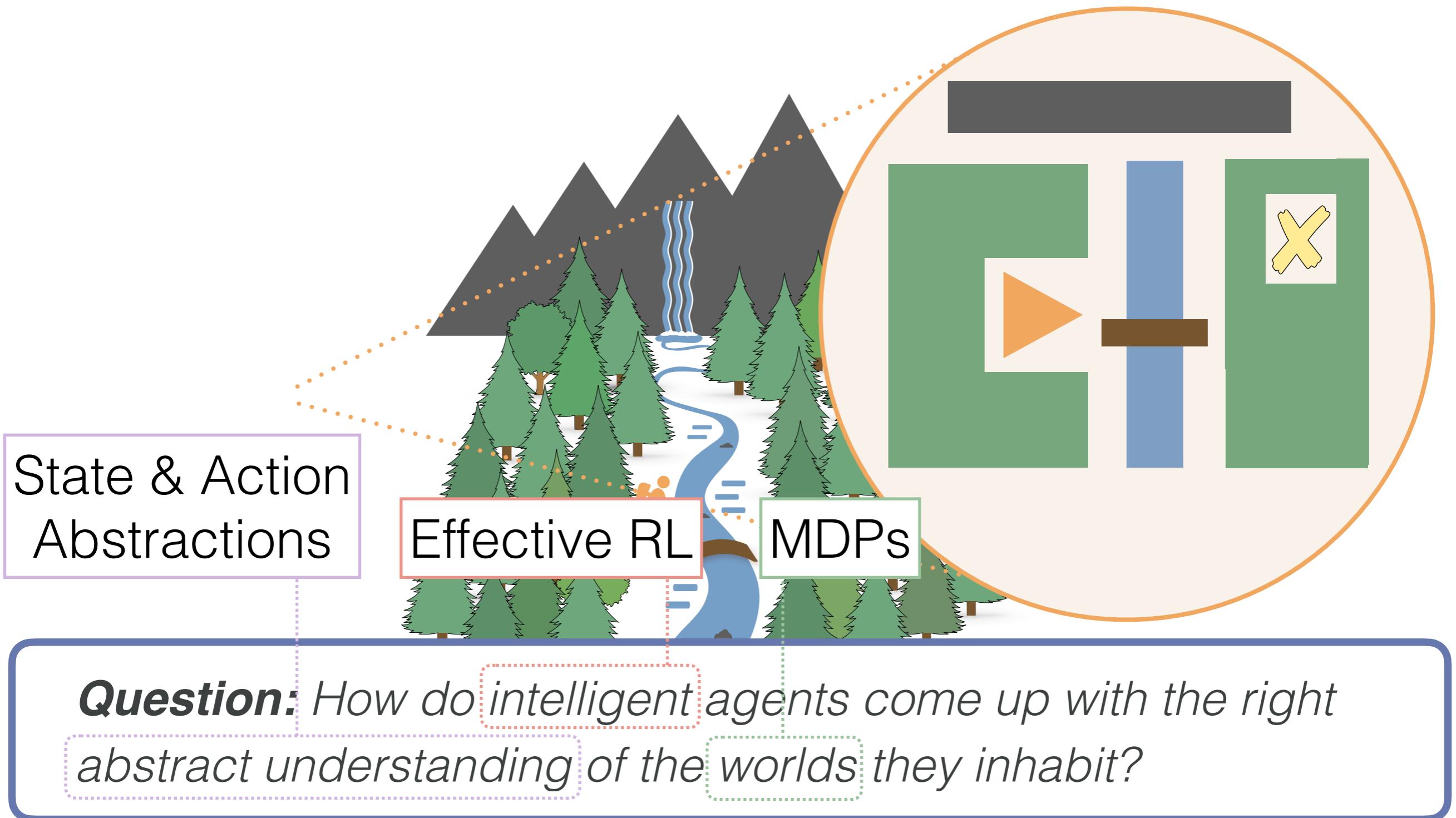
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*Purdue  
July 14, 2021*

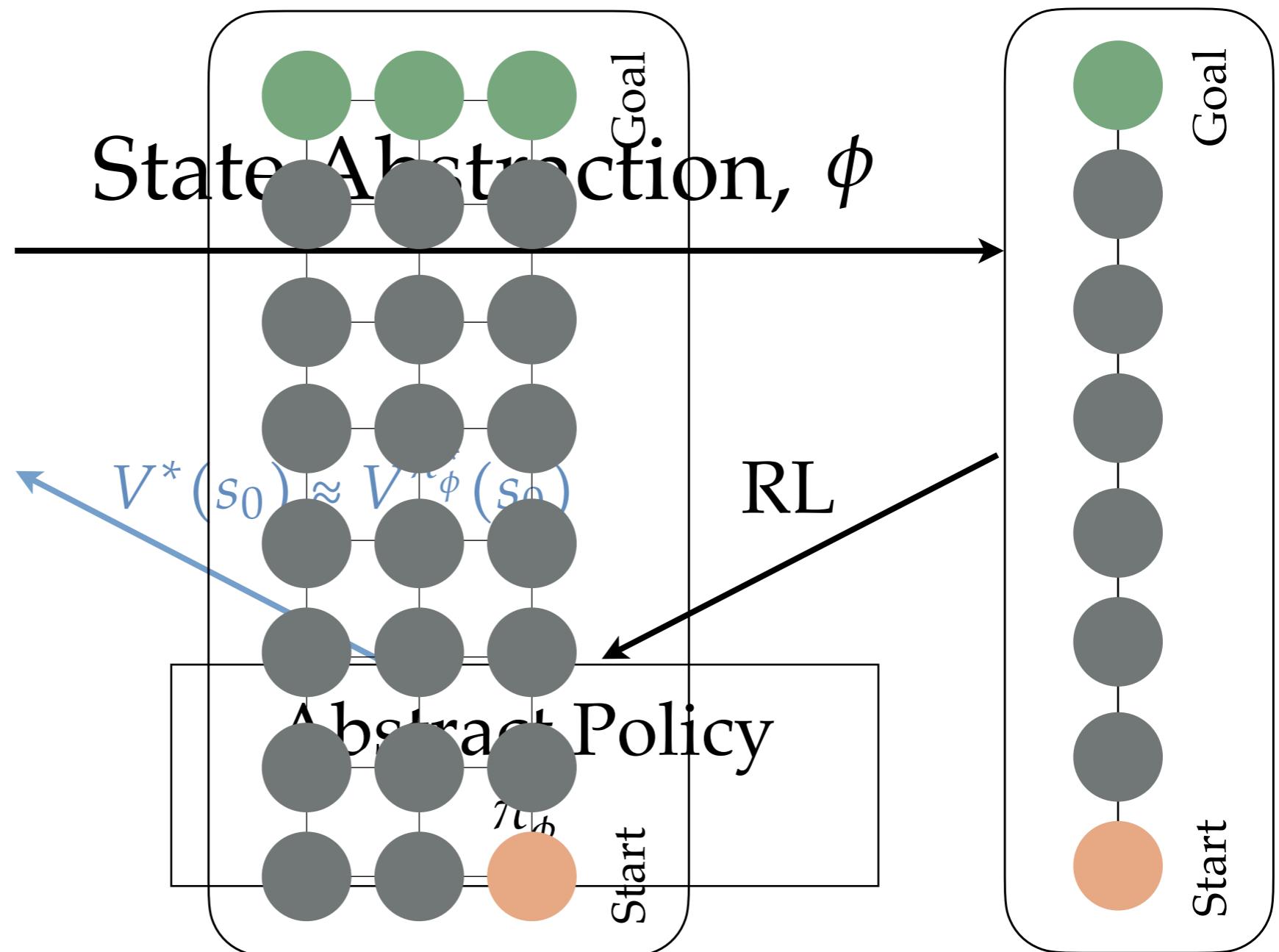
**Dissertation:** [david-abel.github.io/thesis.pdf](https://david-abel.github.io/thesis.pdf)

**Contact:** [dmabel@deepmind.com](mailto:dmabel@deepmind.com)

# Abstraction



# State Abstraction

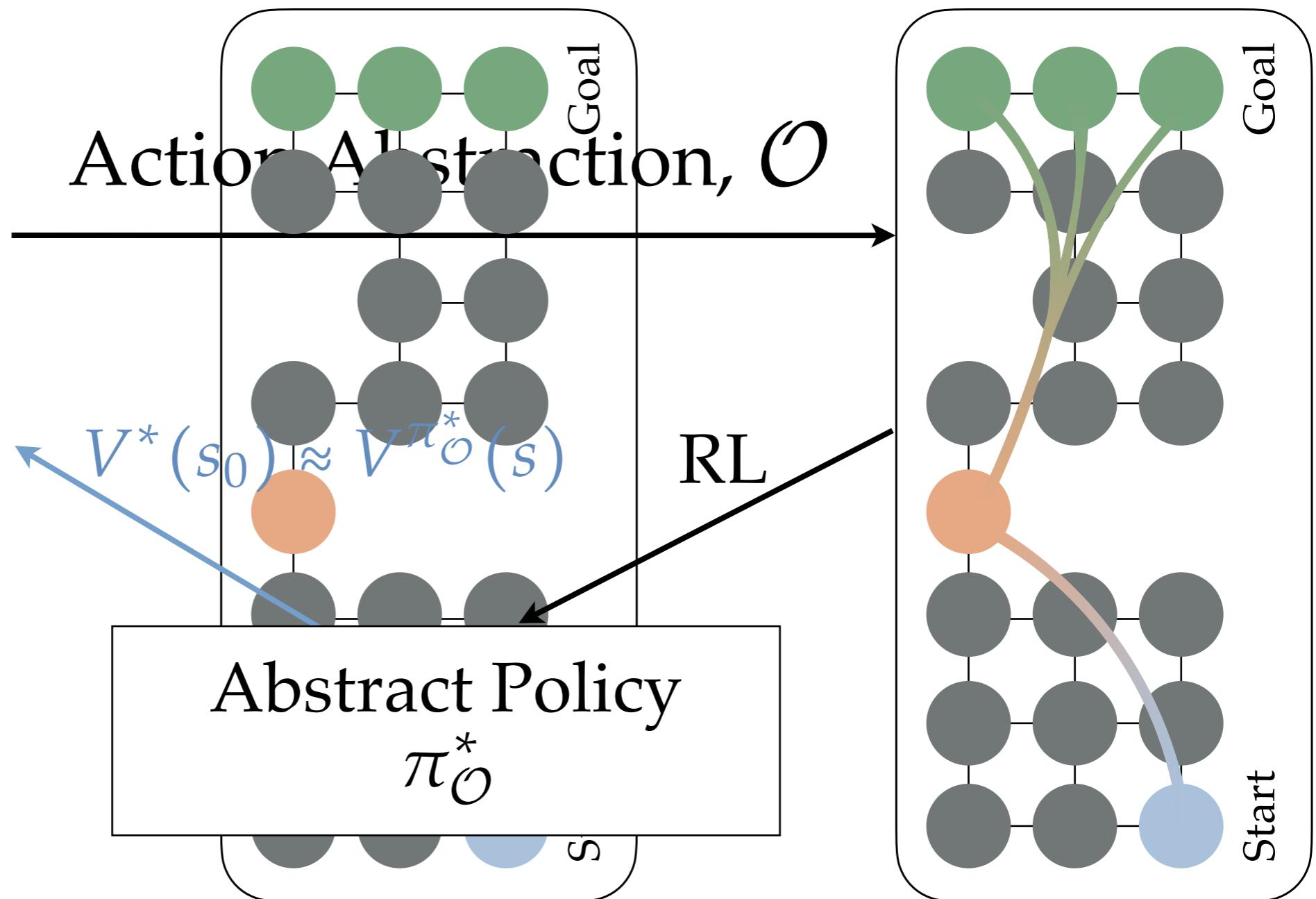


# State Abstraction

**Definition.** A state abstraction is a function  $\phi : \mathcal{S} \rightarrow \mathcal{S}_\phi$  that maps every ground state to an abstract state.

[Fox '73]	[Jong, Stone '05]	[Ortner et al. '07, '14, '19]
[Whitt '78]	[Ferns et al., '04, '06]	[Hutter '14, '16, '19]
[Singh et al. '95]	[Li et. al '06]	[Jiang et al., '14, 15]
[Dean, Givan '97]	[Whiteson et al.'07]	[Akrour et al., '18]
[Dieterich '00]	[Castro, Precup '09]	[Menashe, Stone '18]
[Andre, Russell '02]	[Mugan, Kuipers '12]	[Taïga et al. '18]
[Ravindran, Barto '03, '04]		[Hostetler et al. '14, '15, '17]

# Action Abstraction



# Action Abstraction

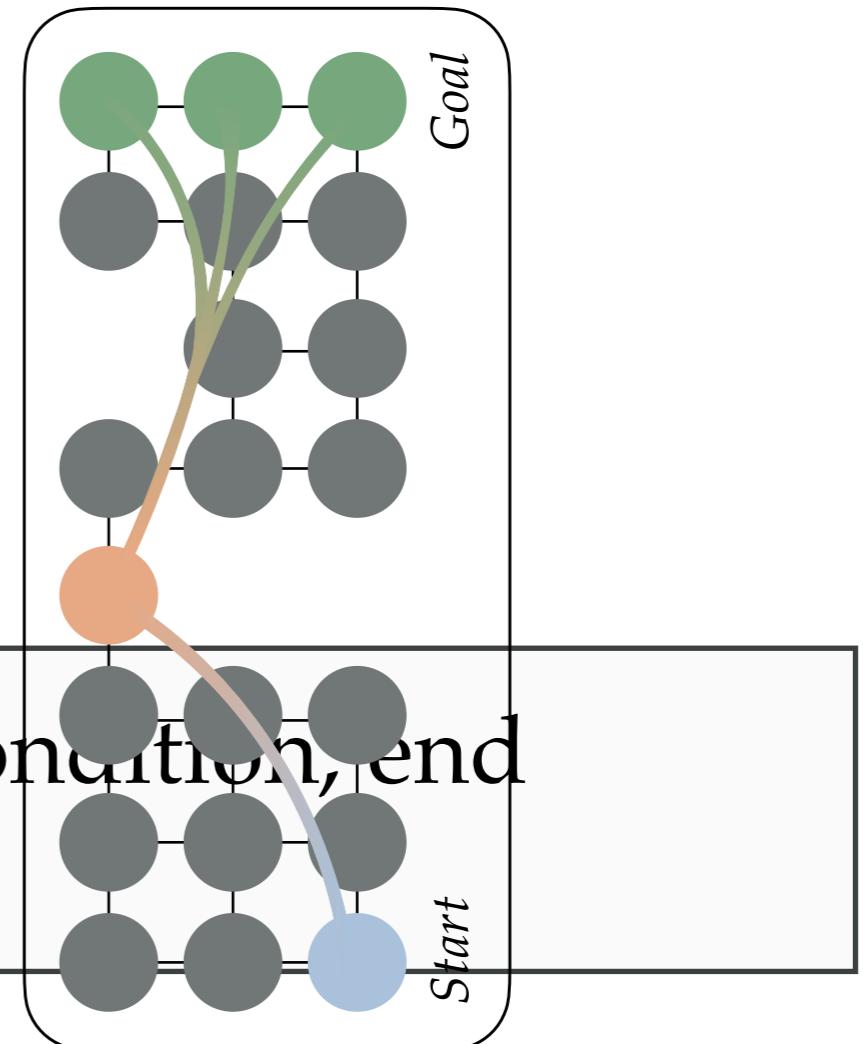
*Example:*

$$o_1 = (\text{blue circle}, \text{orange circle}, \pi_1)$$

[Sutton, Precup, Singh 1999]

$$o_2 = (\text{orange circle}, \text{green circle}, \pi_2)$$

**Definition (Option):** A start condition, end condition, and a policy.



# Action Abstraction

**Definition** (Action Abstraction): An action abstraction replaces the primitive actions with the option set  $\mathcal{O}$ .

[McGovern et. al. '97]

[Sutton, Precup, Singh '99]

[Simsek, Barto, '05, '08]

[Jong, Hester, Stone '08]

[Brunskill, Li '14]

[Ciosek, Silver '15]

[Konidaris et al. '06, '07, '09, '10, '18]

[Bacon et al. '17, '18]

[Fruit et al. '17, '17]

[Machado et al. '17]

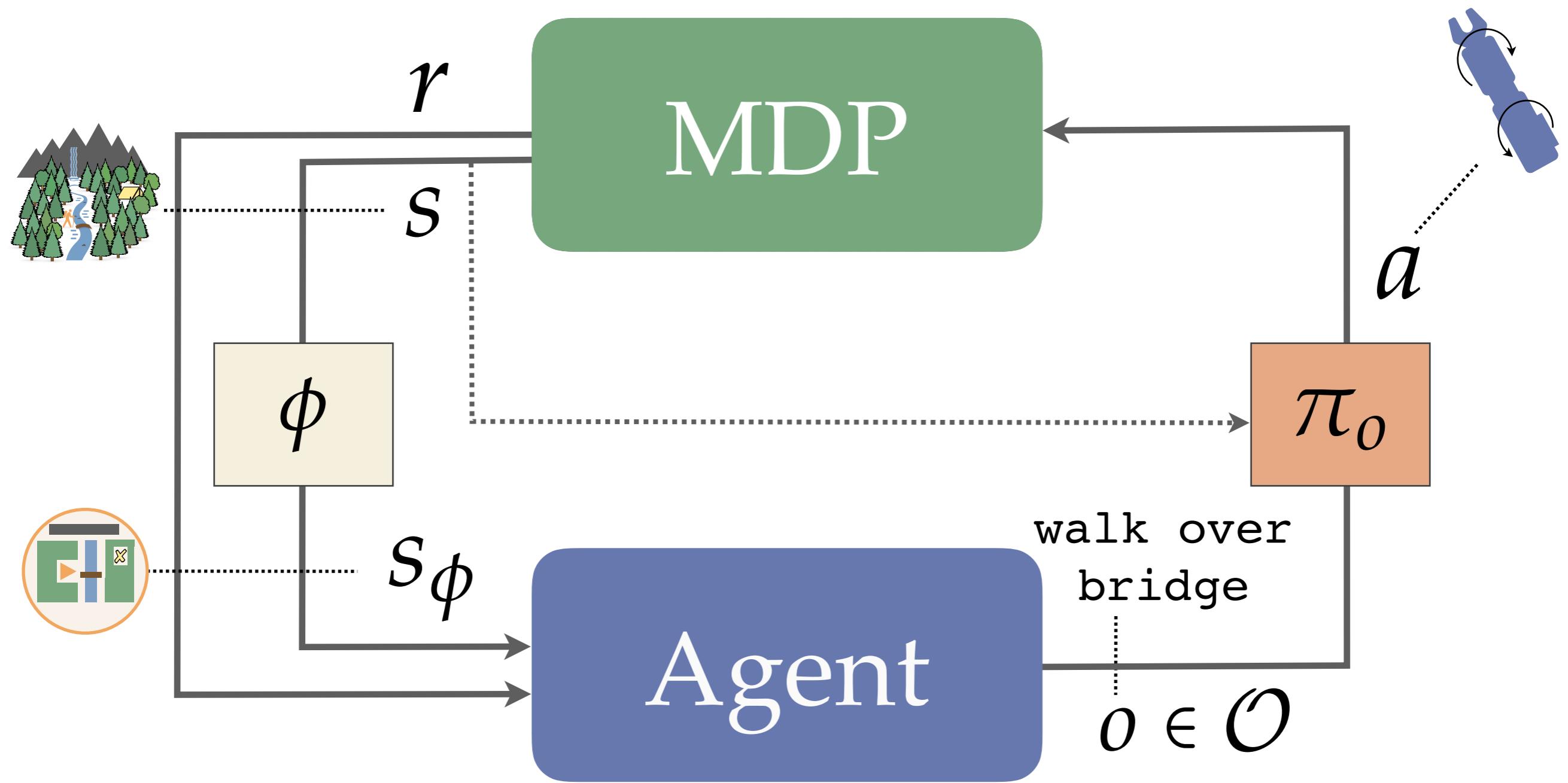
[Harutyunyan et al. '18]

[Eysenbach et al. '18]

[Majeed & Hutter '19]

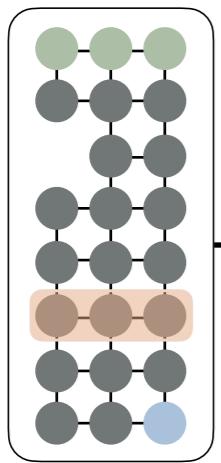
...and more!

# Abstraction in RL



## Part 1

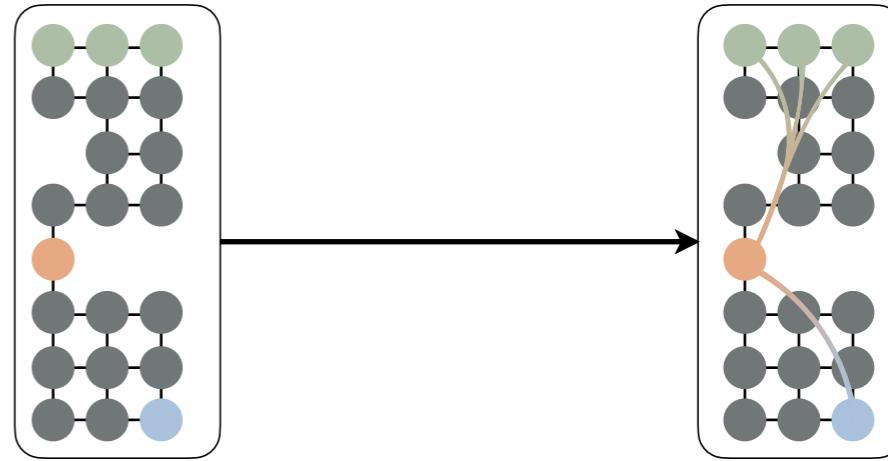
### STATE ABSTRACTION



1. Approximate State Abstraction  
*ICML 2016*
2. State Abstraction In Lifelong RL  
*ICML 2018*
3. State Abstraction As Compression  
*AAAI 2019*

## Part 2

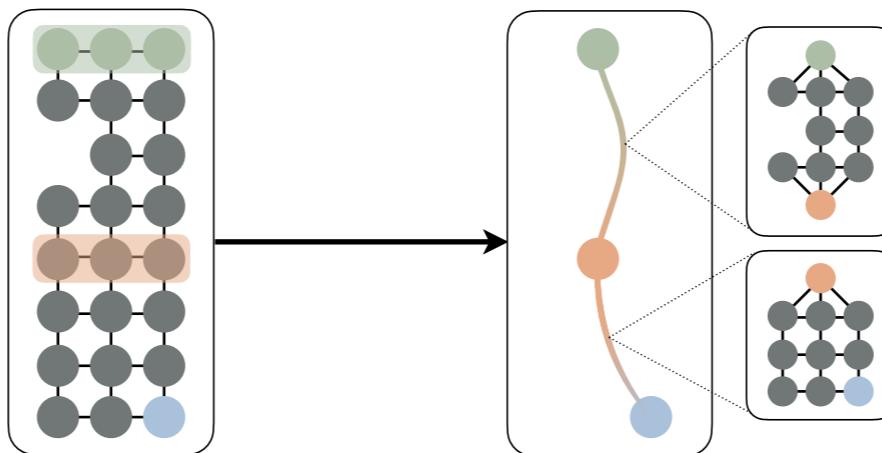
### ACTION ABSTRACTION



4. Options for Planning  
*ICML 2019*
5. Options for Exploration  
*ICML 2019*
6. A New Option Model  
*IJCAI 2019*

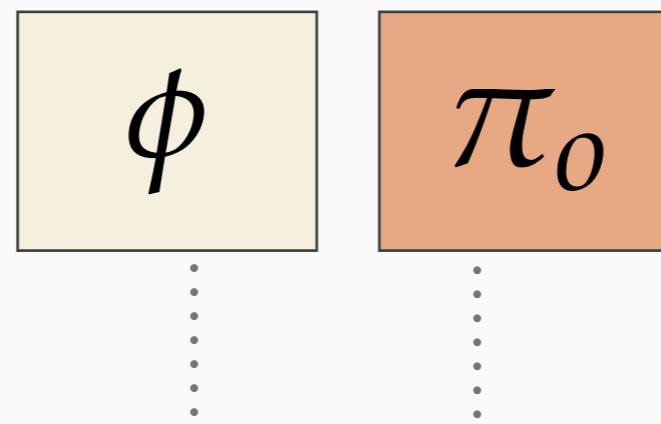
## Part 3

### STATE-ACTION ABSTRACTION



7. Value-Preserving Hierarchies  
*AISTATS 2020*

# Desirable Abstractions



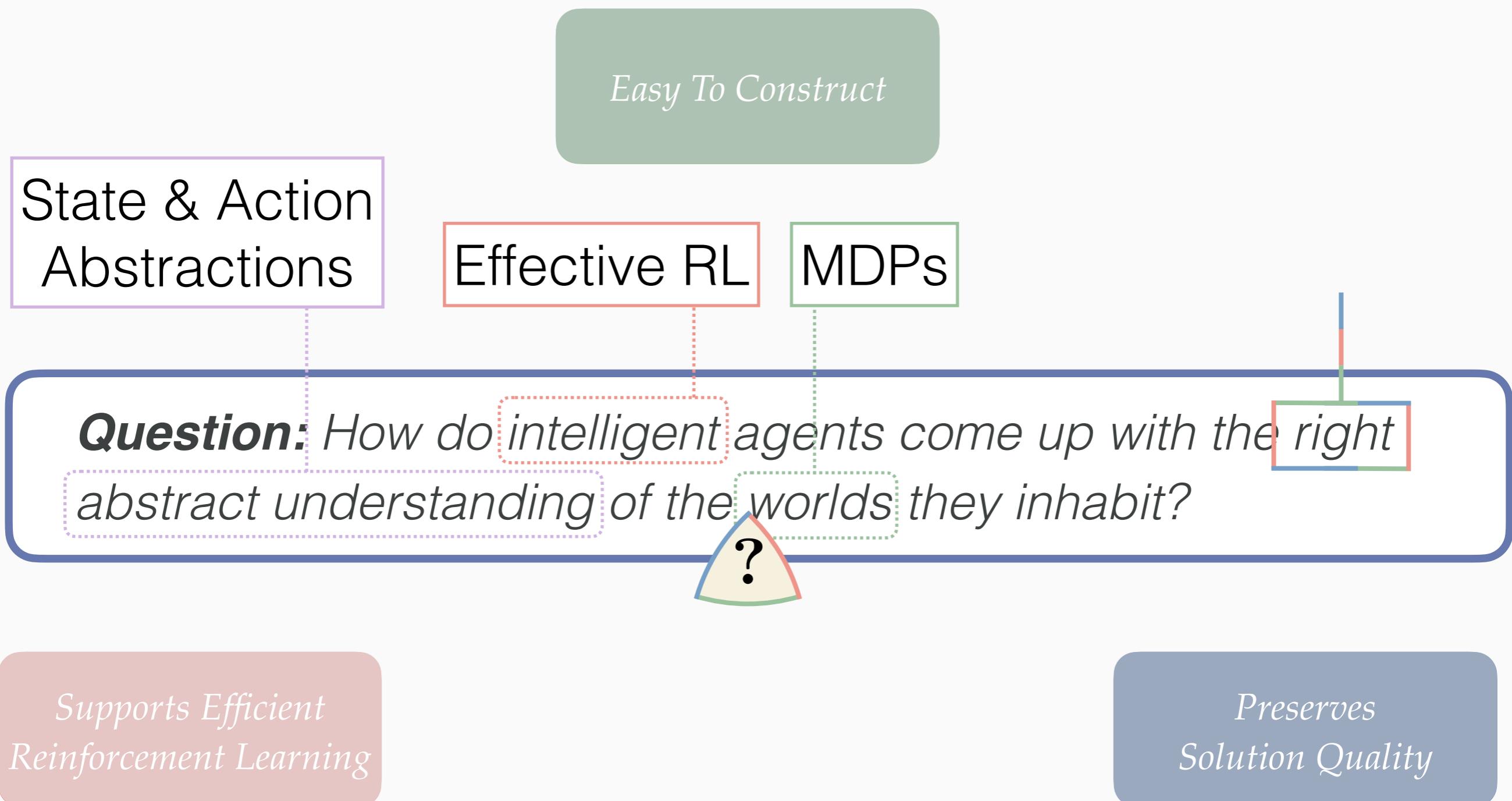
Q: Which kinds of abstractions are **desirable?**

Easy To Construct

Preserves  
Solution Quality

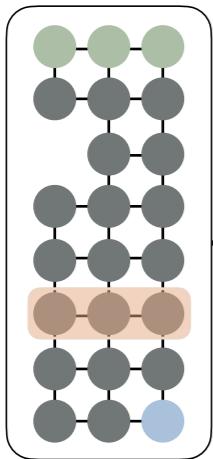
Supports Efficient  
Reinforcement  
Learning

# Abstraction Desiderata



## Part 1

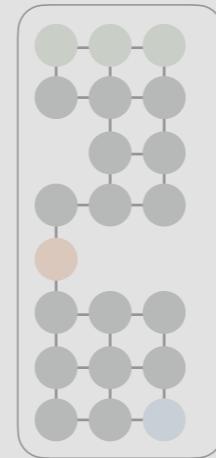
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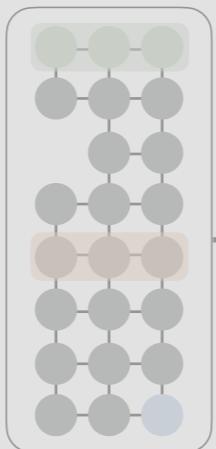
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## Part 3

### STATE-ACTION ABSTRACTION

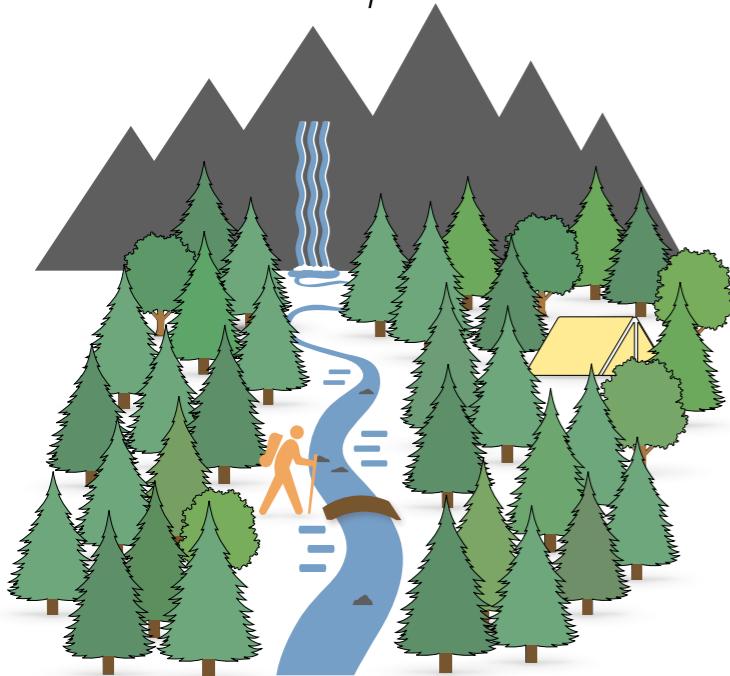


7. Value-Preserving Hierarchies  
*AISTATS 2020*

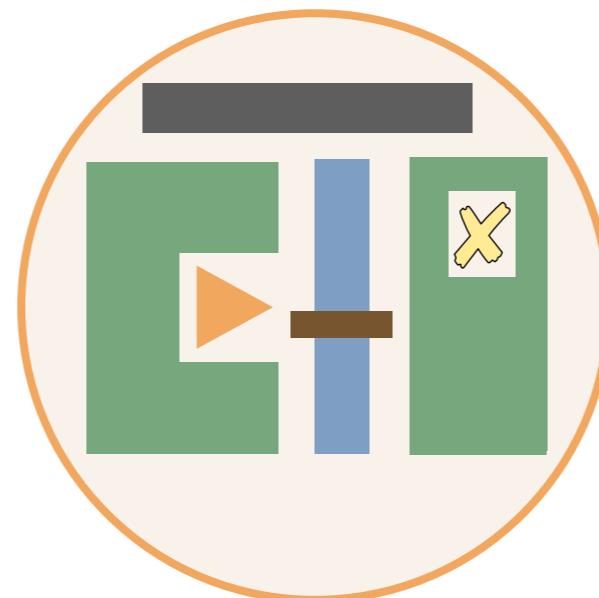
# State Abstraction as Compression

[AAJWL, AAAI 2019]

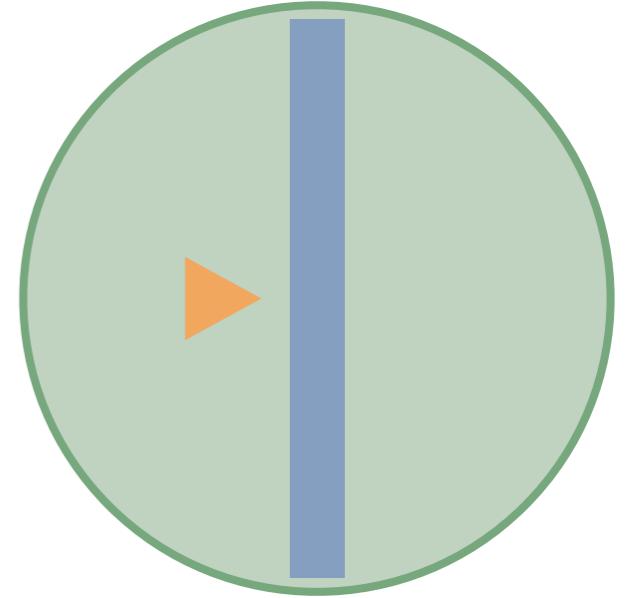
High Value  
No Compression



Some Value  
Some Compression



No Value  
High Compression



**Question:** How can we construct state abstractions that trade-off between compression and representational quality?

Dilip  
Arumugam

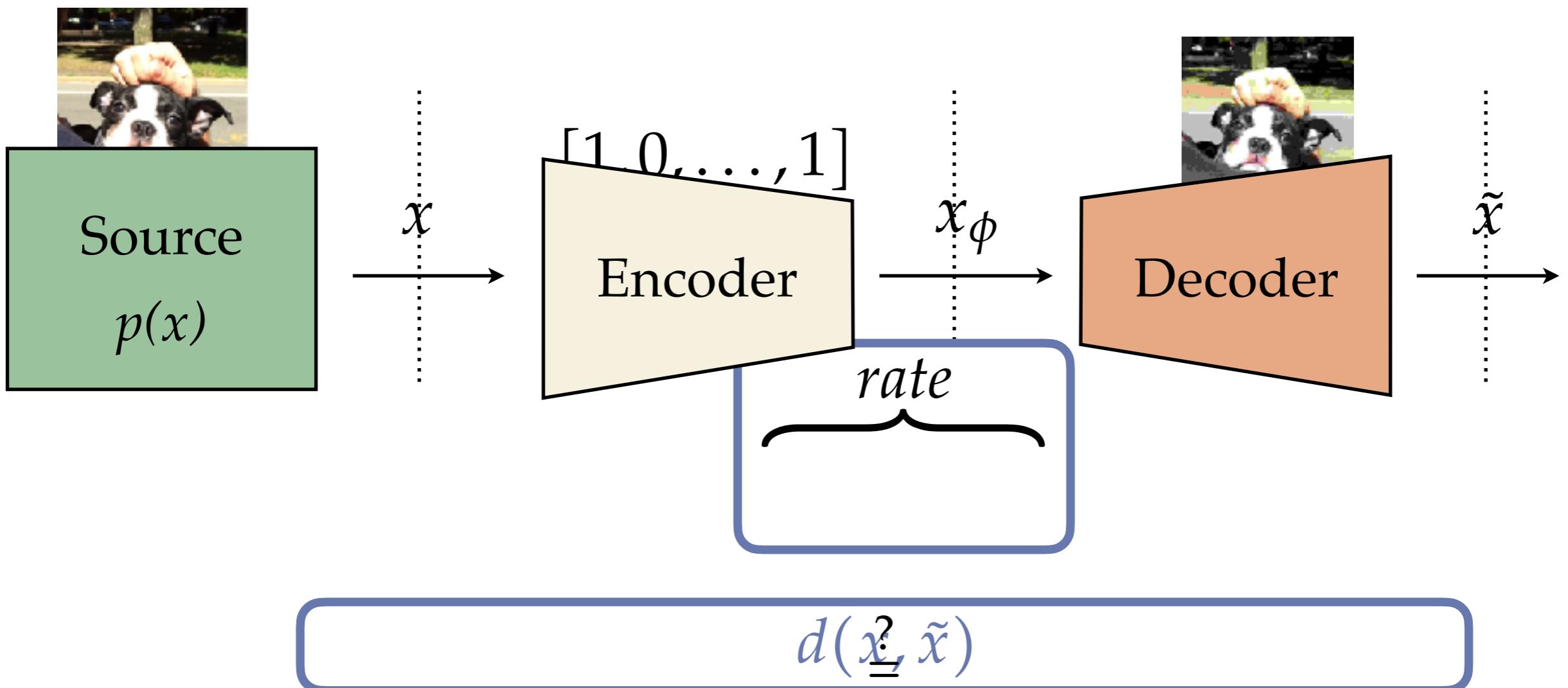
Kavosh  
Asadi

Yuu  
Jinnai

Lawson  
L.S. Wong

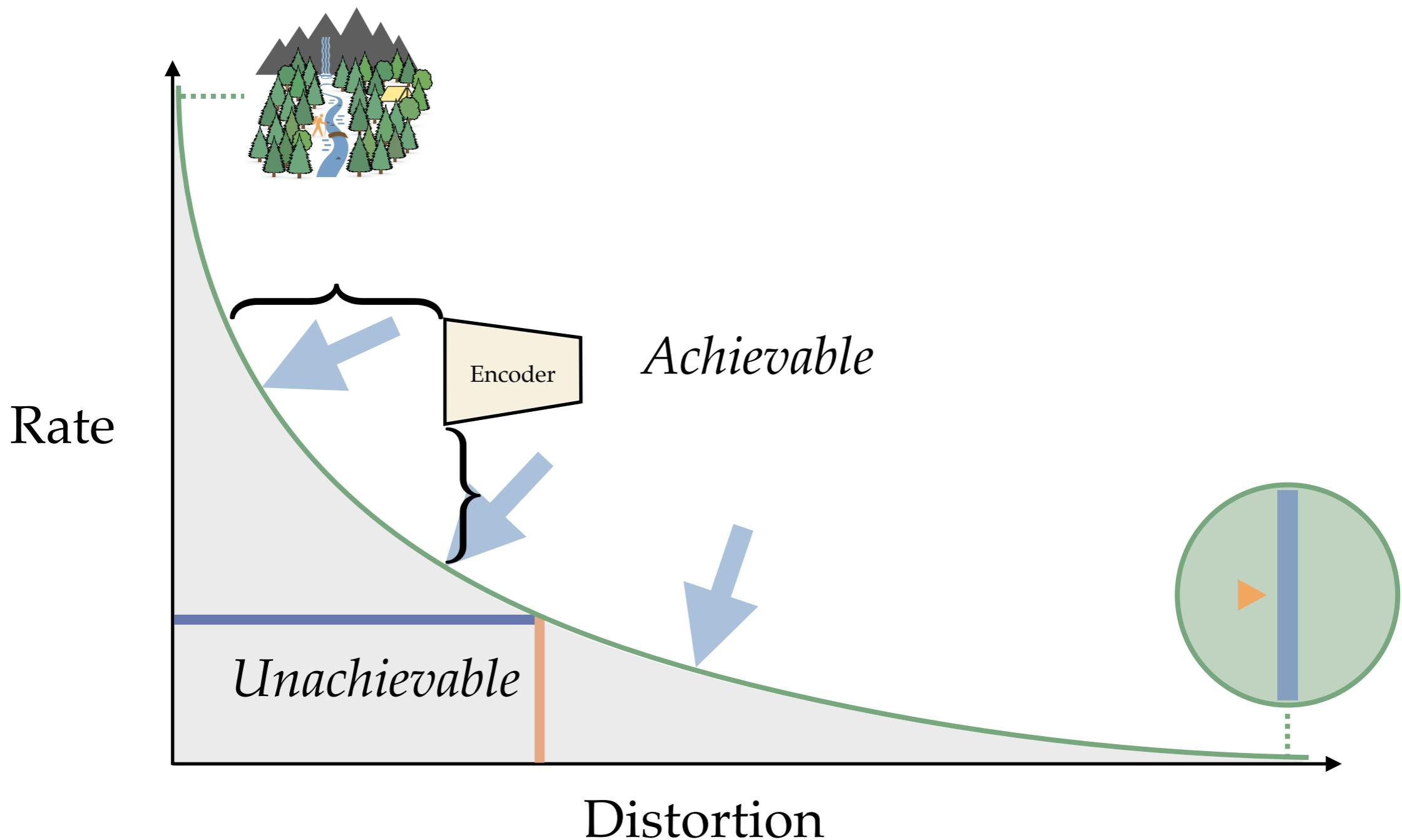
Michael L.  
Littman

# Rate-Distortion Theory

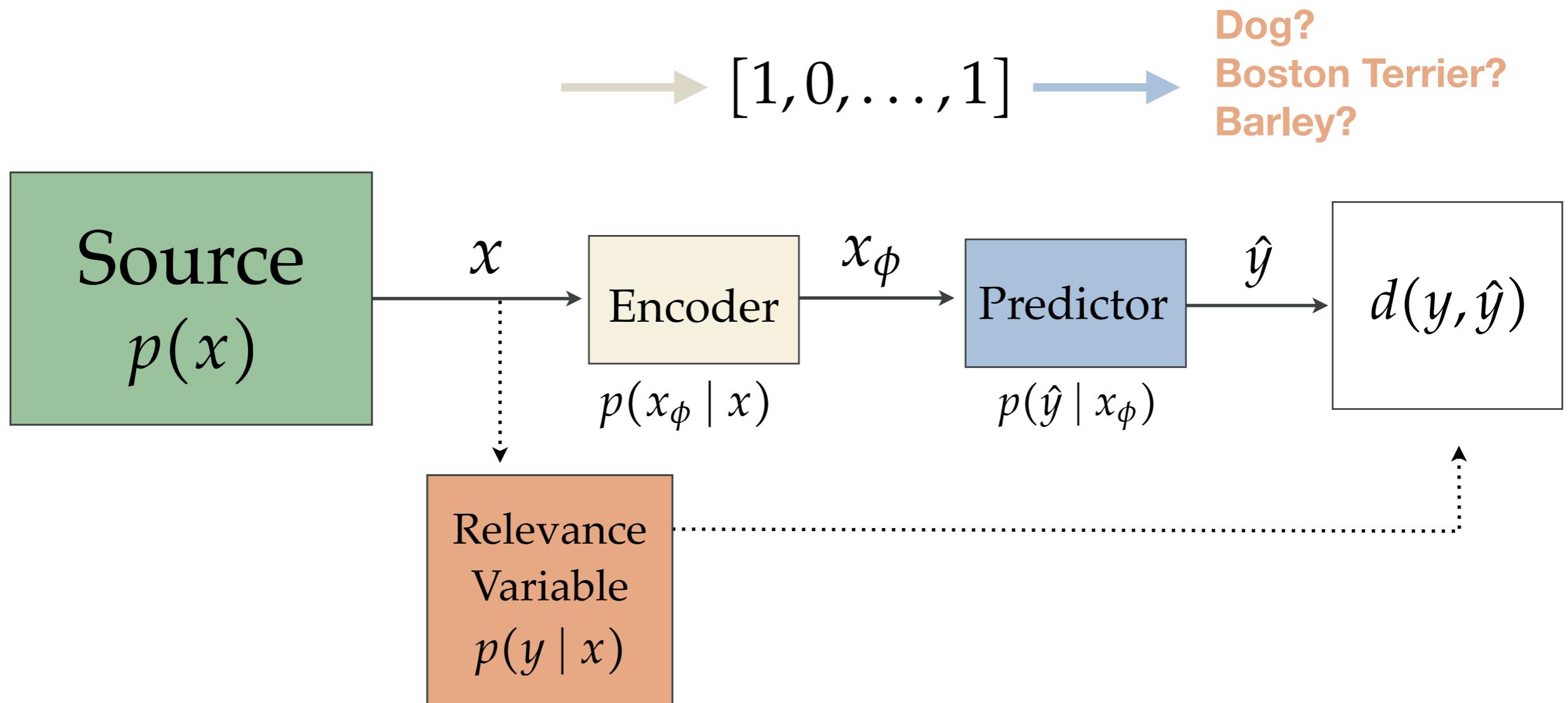


[Shannon '48, Berger '03]

# Rate-Distortion Theory

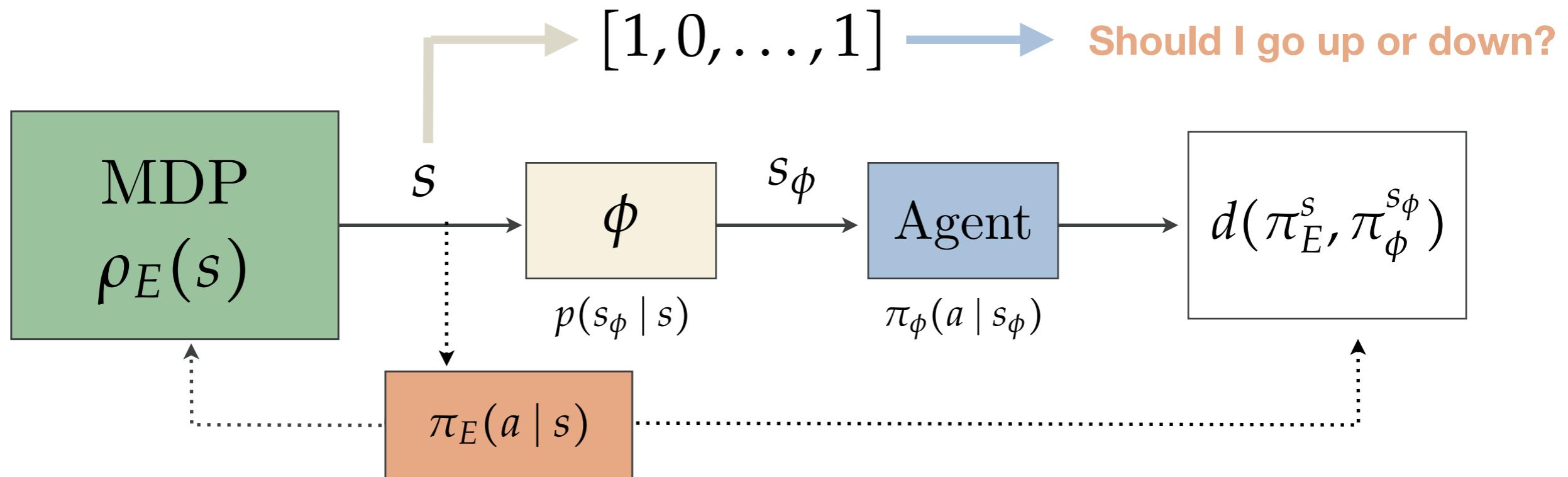


# Information Bottleneck



[Tishby, Pereira, Bialek '99]

# State Abstraction as Compression



$$\min_{\phi} \left( |\mathcal{S}_\phi| + \beta \mathbb{E}_{\rho_E(s)} \left[ V^{\pi_E}(s) - V^{\pi_\phi^*}(\phi(s)) \right] \right)$$

*Our Objective*

*Value Loss*

# State Abstraction as Compression

**Theorem.**

$$\min_{\phi} \left( |\mathcal{S}_\phi| + \beta \mathbb{E}_{\rho_E(s)} [V^{\pi_E}(s) - V^{\pi_\phi^*}(\phi(s))] \right)$$

*Our Objective*

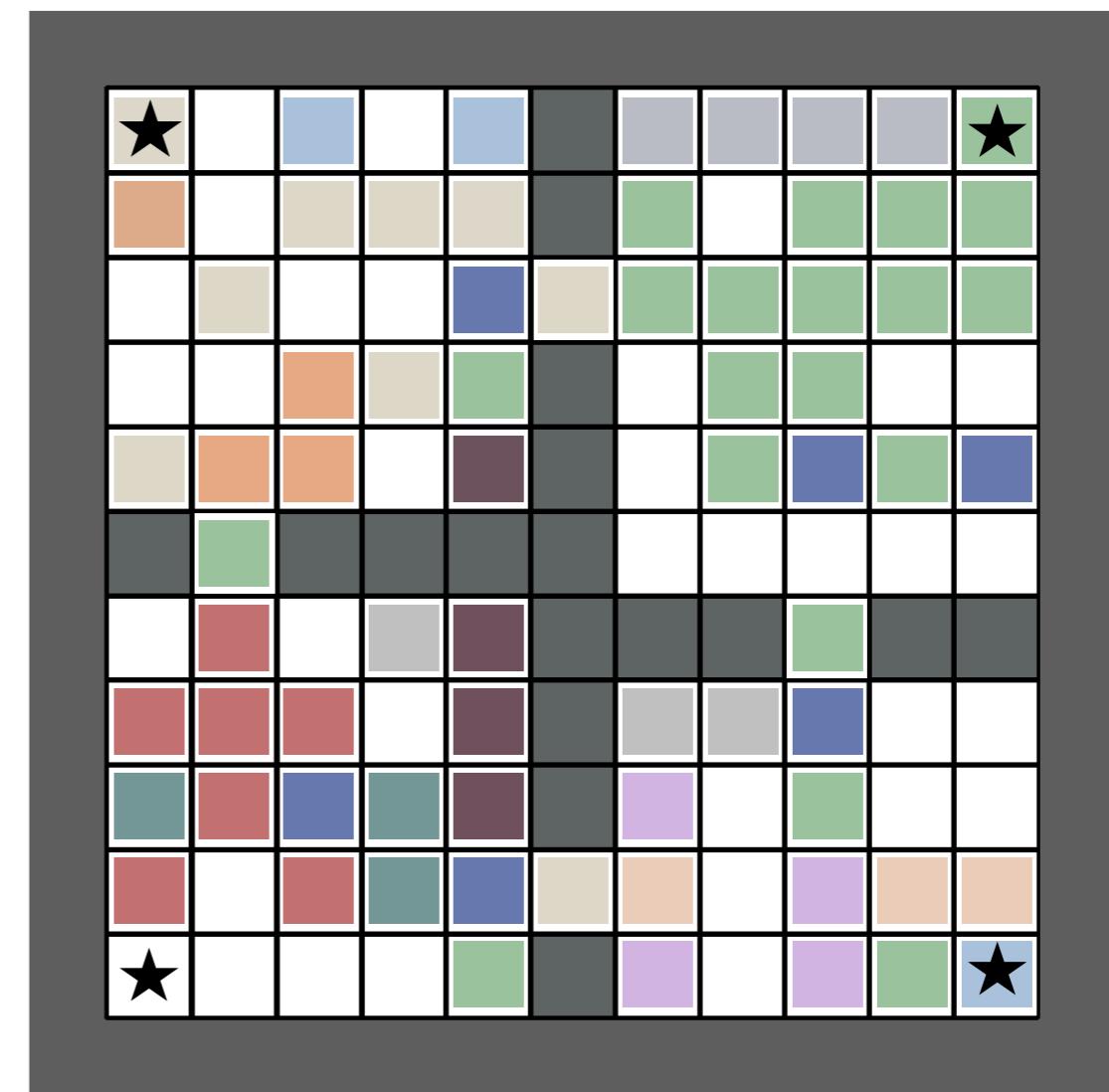
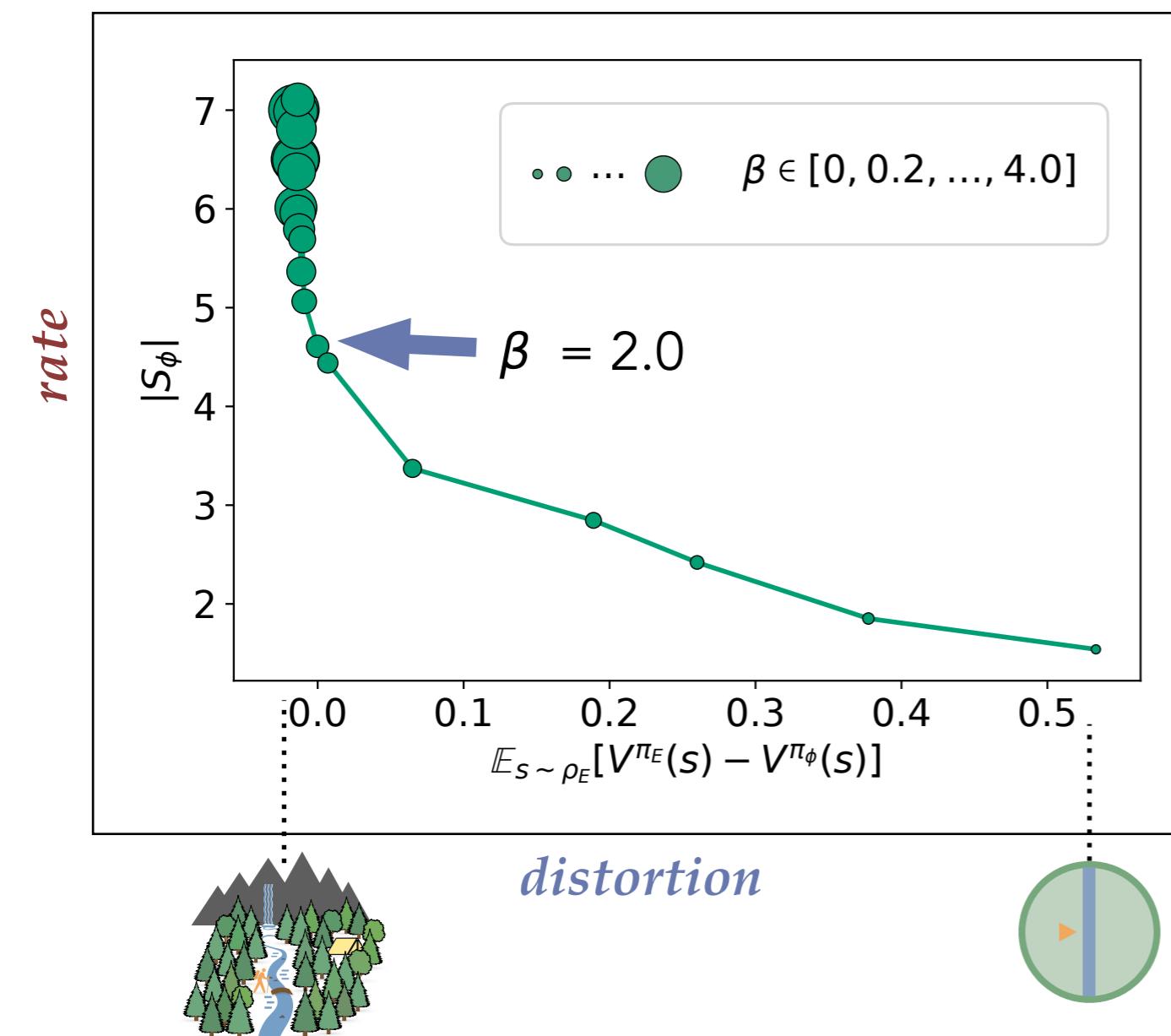
$\leq$

*DIB Objective*

[Strouse & Schwab, '17]

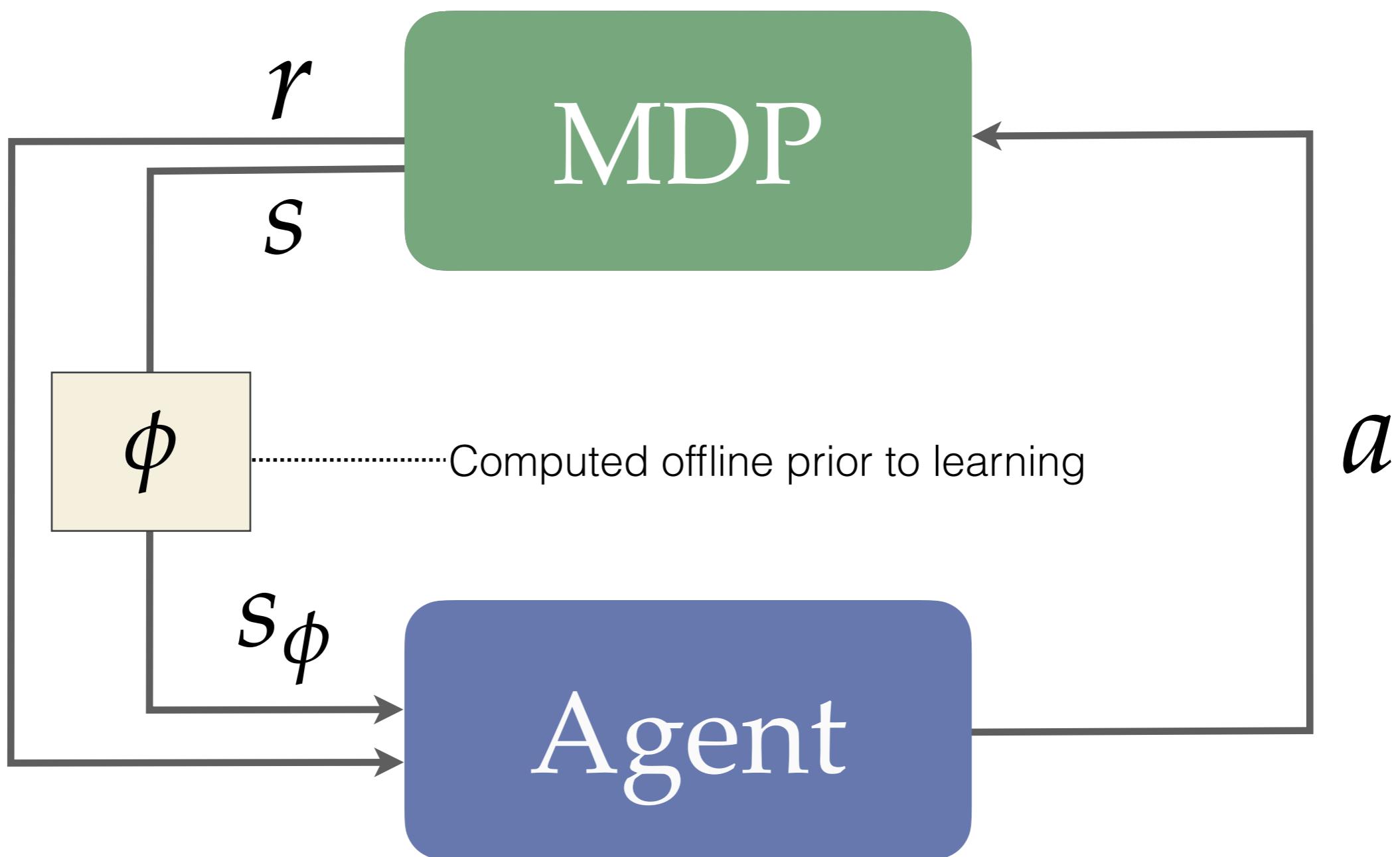
$$\min_{\phi} \left( \frac{H(\rho_\phi)}{\delta \log \frac{1}{\delta}} + 2\text{VMAX}\beta \mathbb{E}_{\rho_E(s)} [D_{\text{KL}}(\pi_E(s) \parallel \pi_\phi^*(\phi(s)))] \right)$$

# State Abstraction as Compression

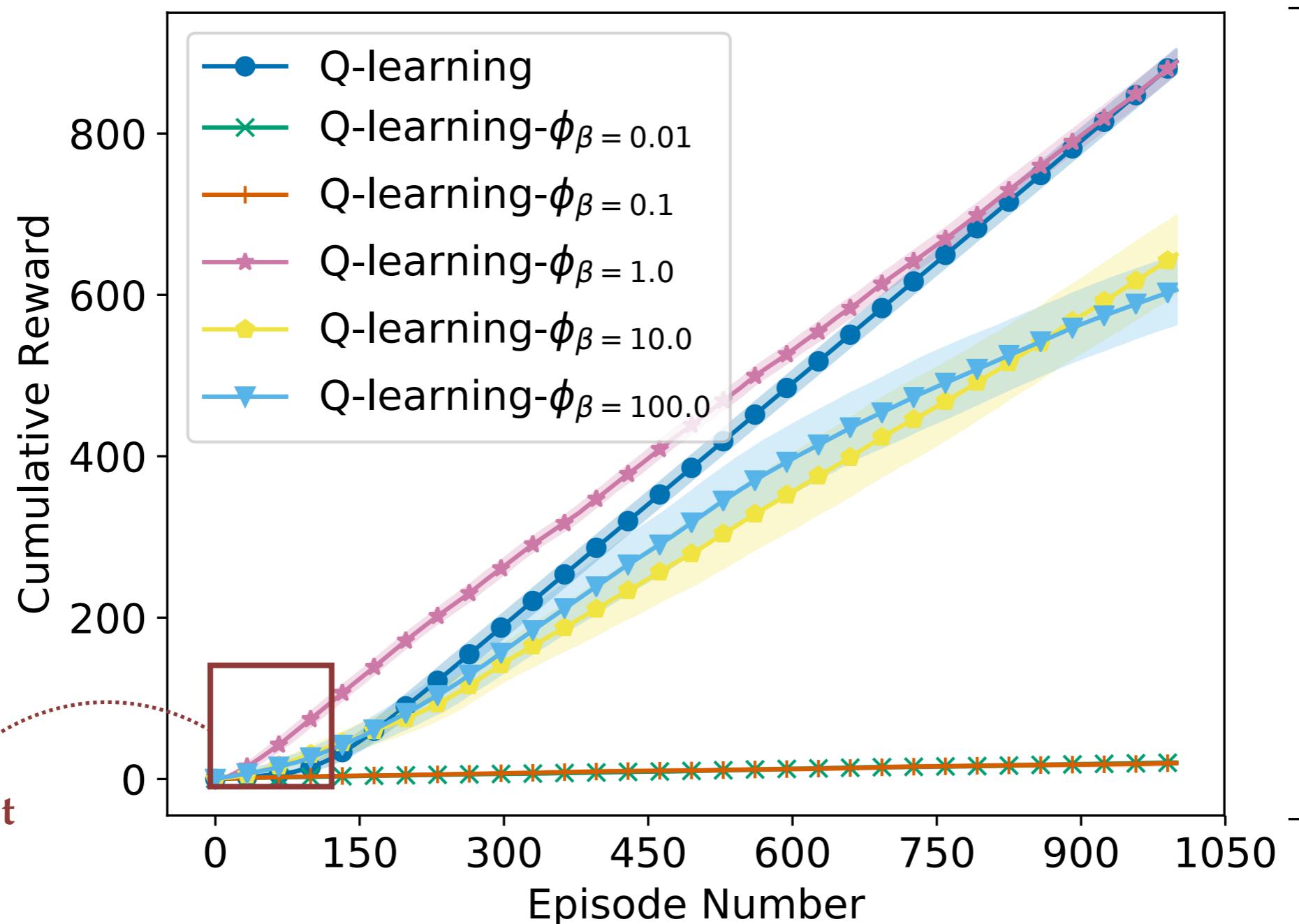


Multitask Abstraction

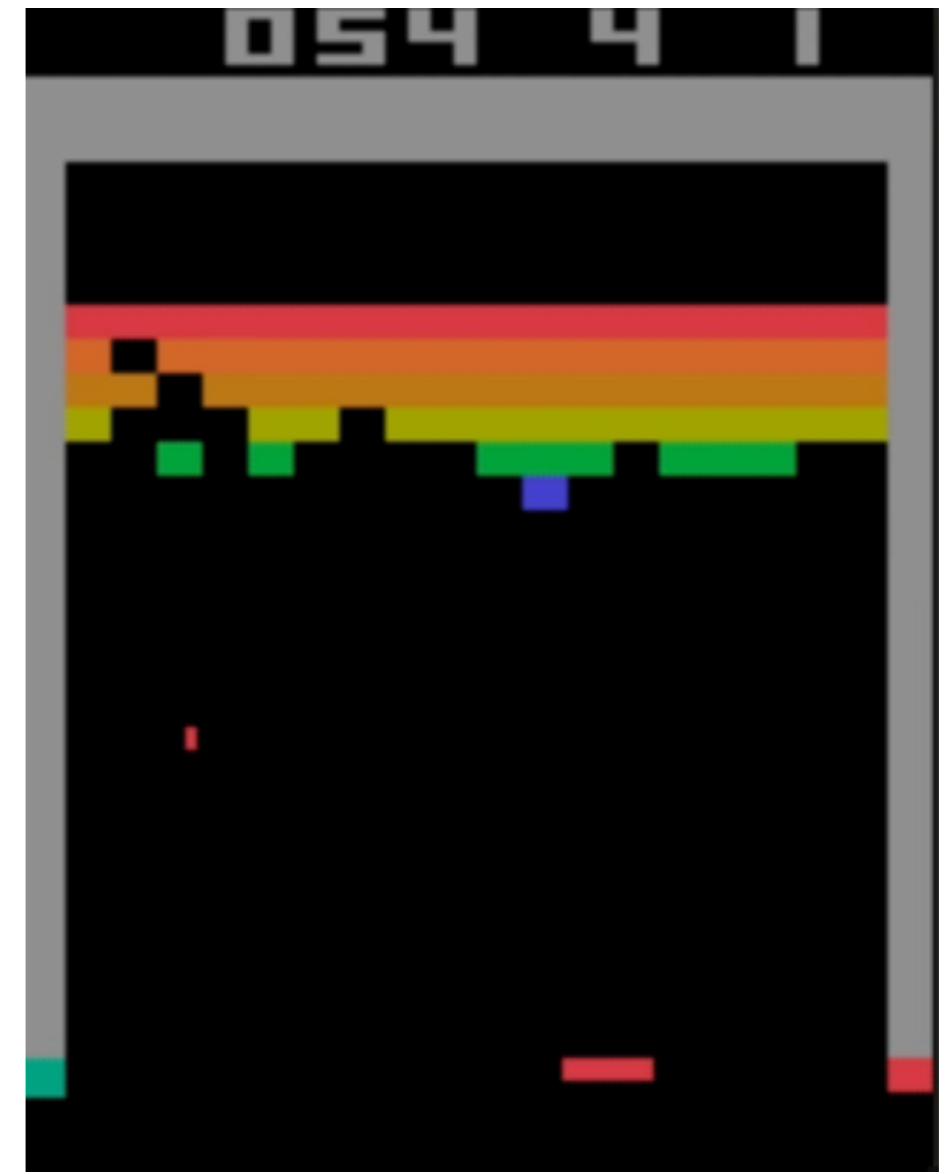
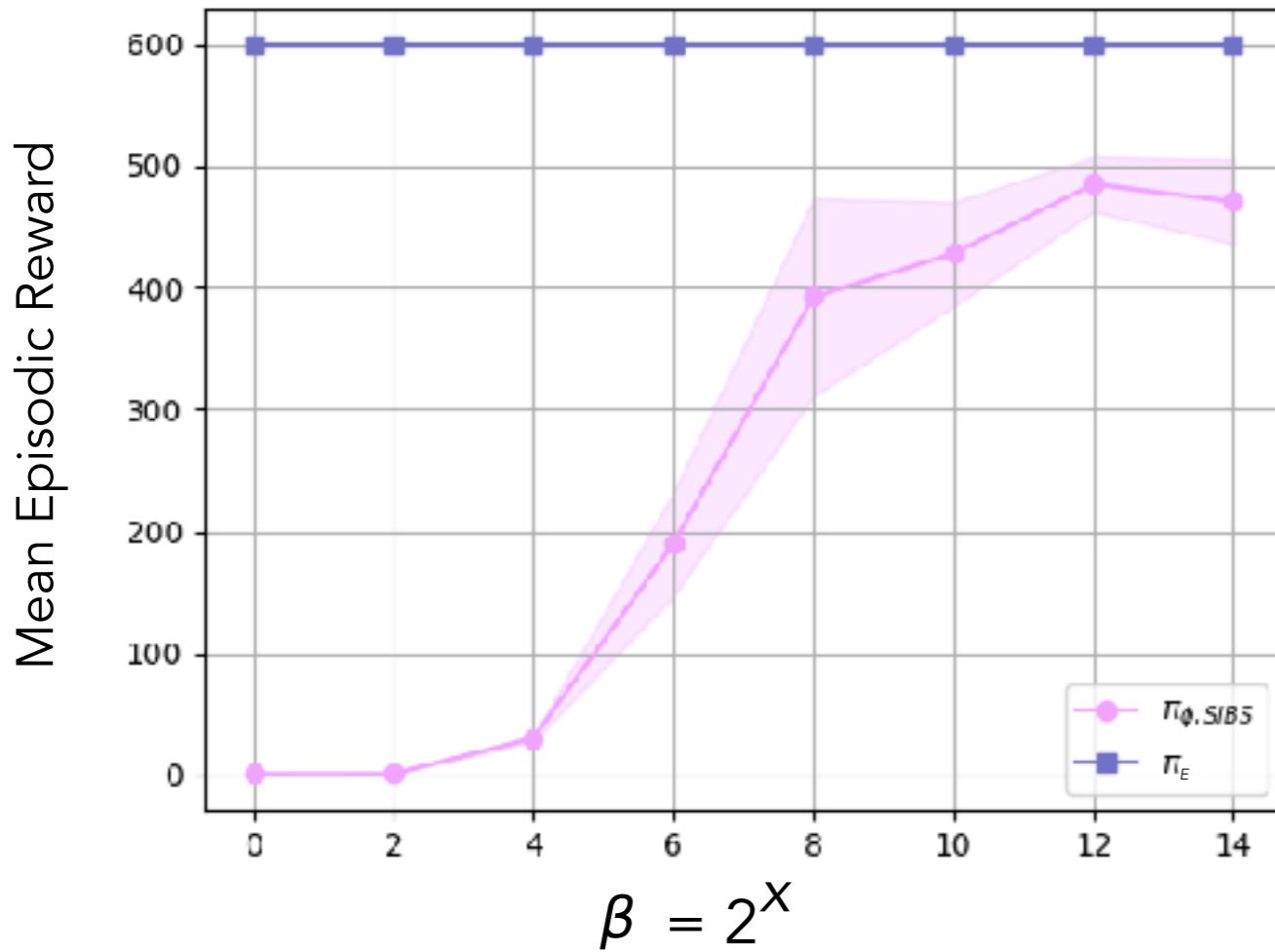
# Learning Experiments



# Experiments: Four Rooms



# Experiments: Breakout



# Extension: Continuous State

**Theorem.** For any  $\delta \in (0, 1)$ ,  $n$  the size of the training data set,  $\Delta \in \mathbb{R}$  training loss, and  $\rho$  a fixed distribution on states used to train  $\tilde{\phi} \in \Phi$ , with probability at least  $1 - \delta$ :

$$\mathbb{E}_{s \sim \rho} \left[ \|(\pi^*(\cdot | s) - \pi_{\tilde{\phi}}(\cdot | s))\|_1 \right] \leq \frac{\Delta}{2} + 2\sqrt{2} \text{Rad}(\Phi) + \sqrt{\frac{2 \ln \frac{1}{\delta}}{n}}$$

[Barlett, Mendelson '02]

*training error*



*led  
project* →

Kavosh  
Asadi

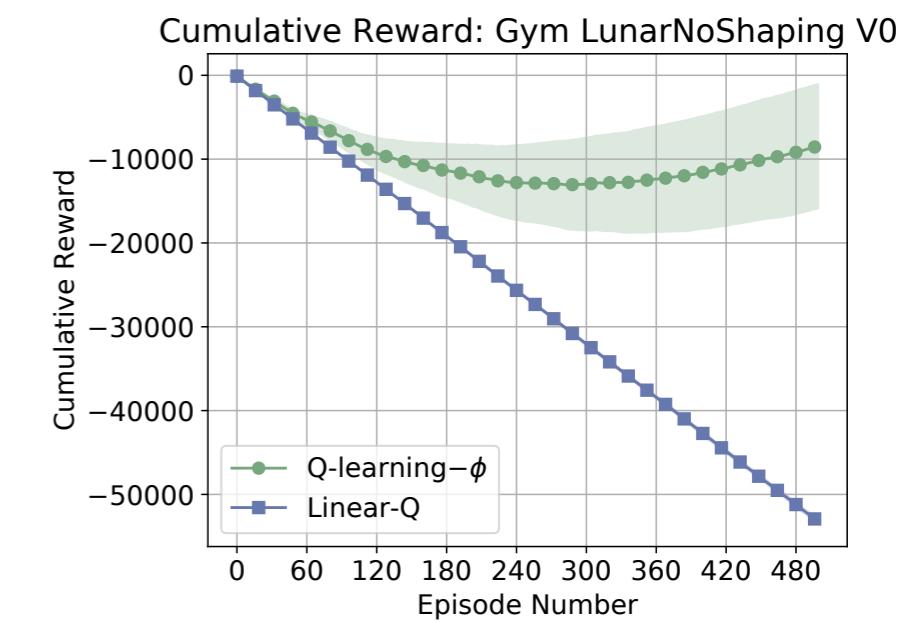
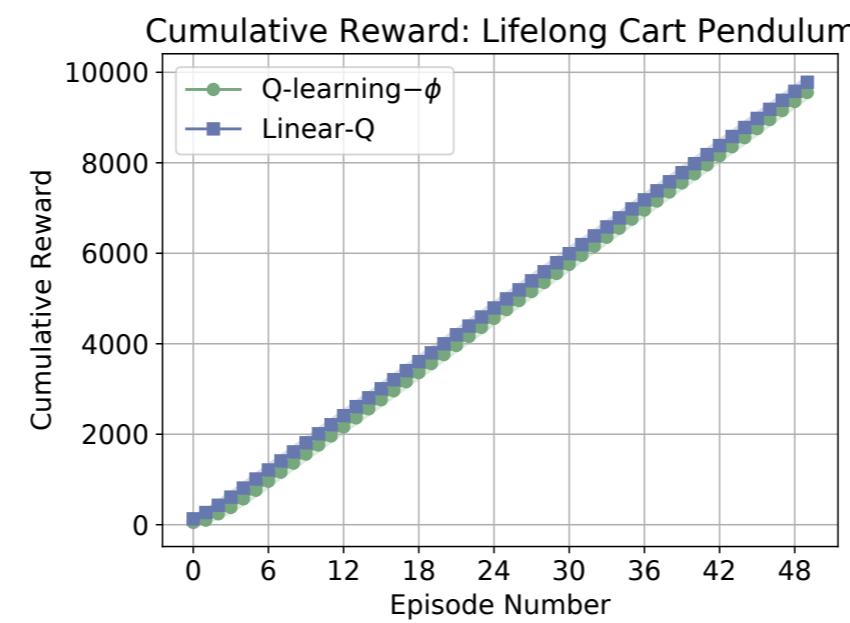
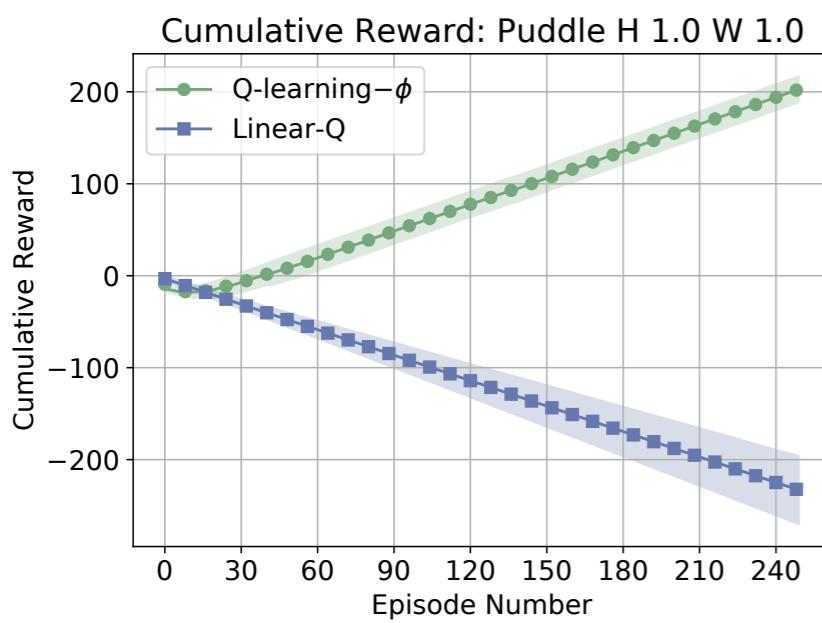
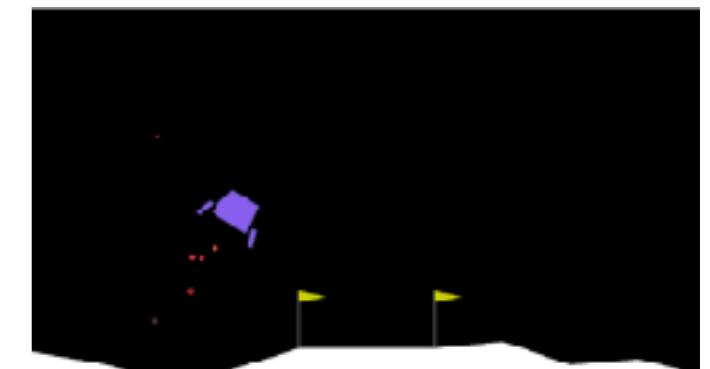
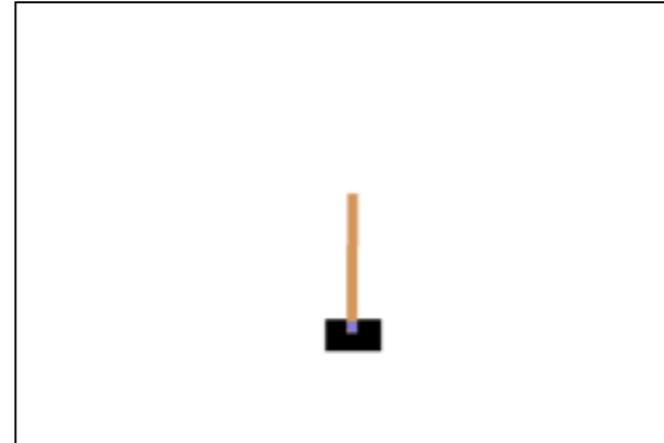
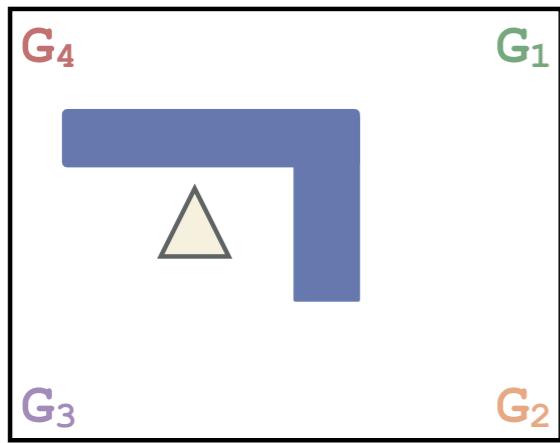
*hypothesis  
class richness*



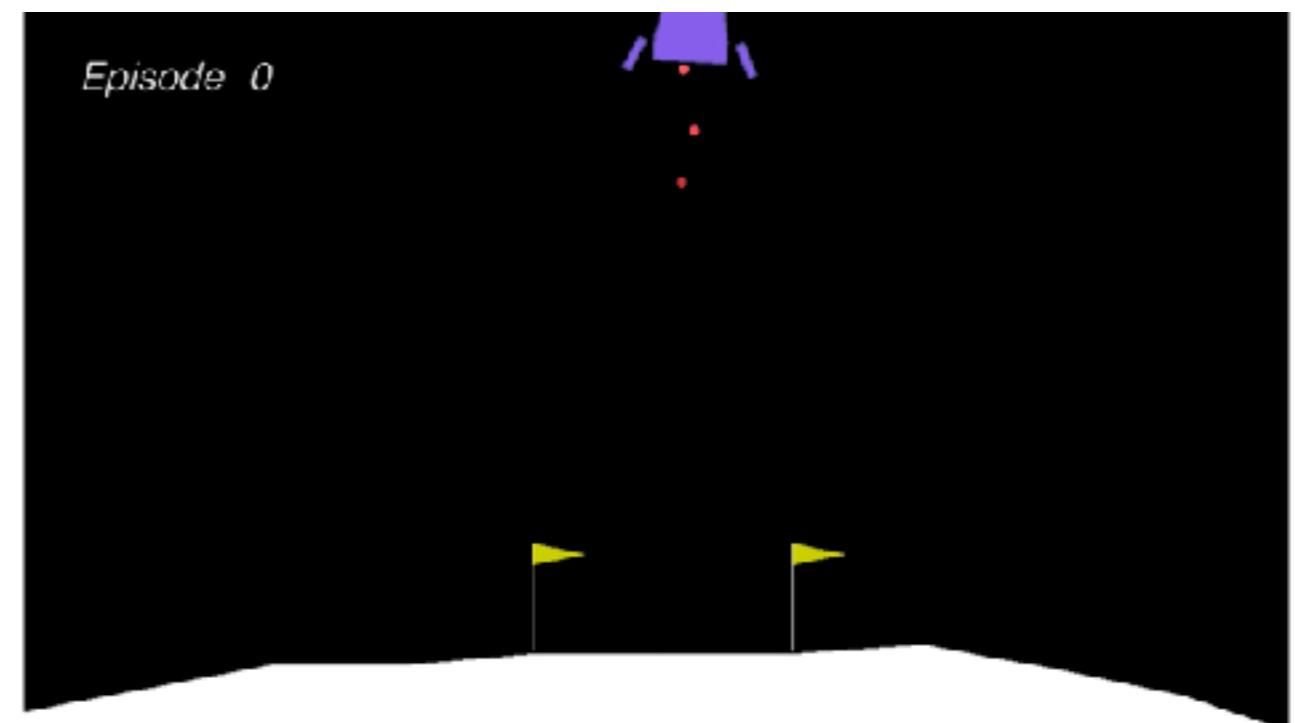
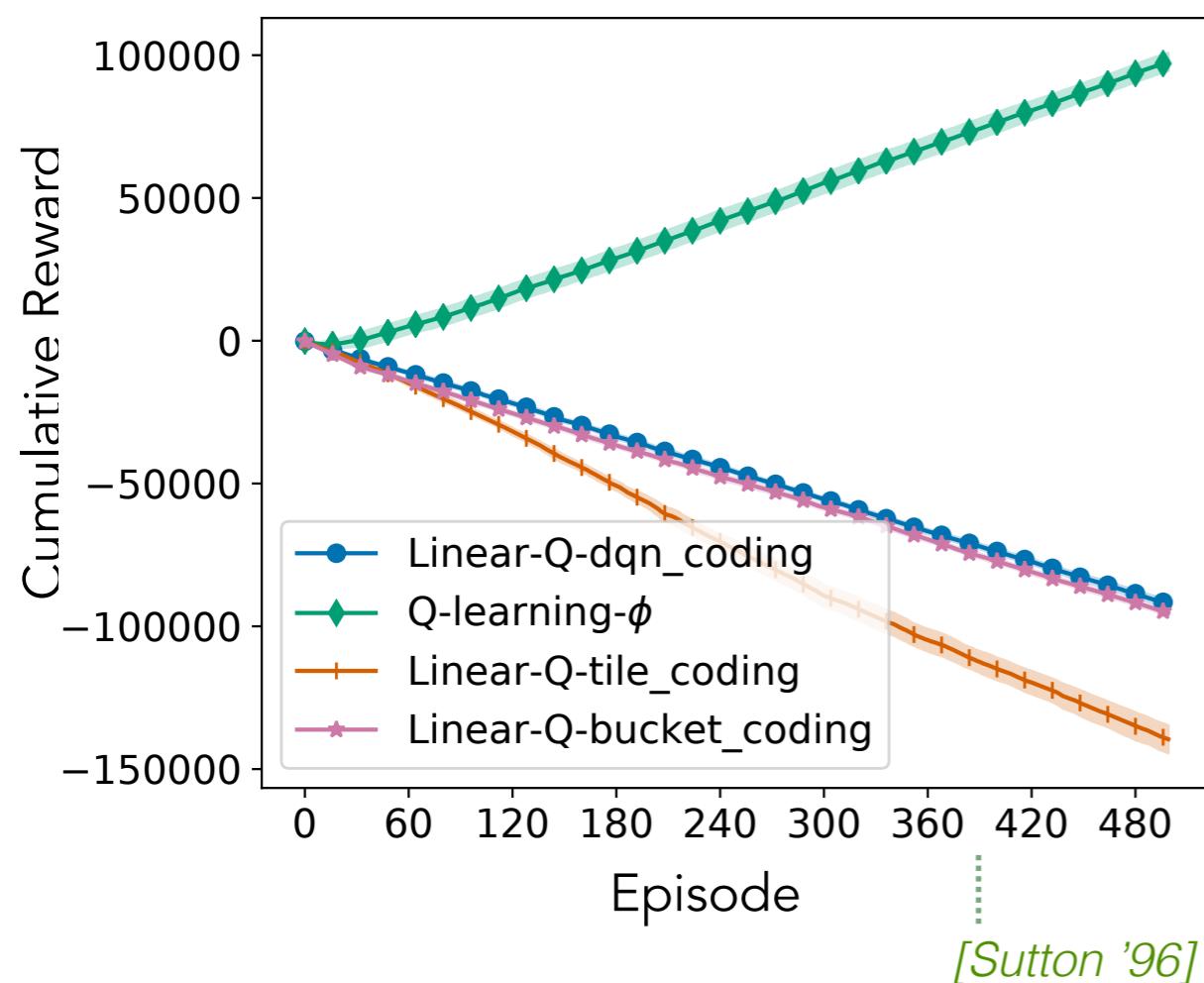
Michael L.  
Littman

*size of  
training data*

# Extension: Continuous State



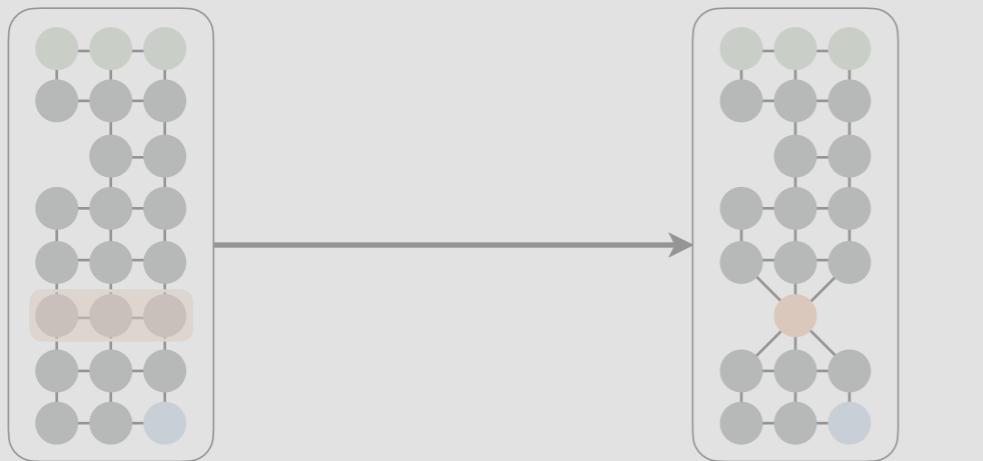
# Experiments: Lunar Lander



Tabular Q-Learning with  $\phi$

## Part 1

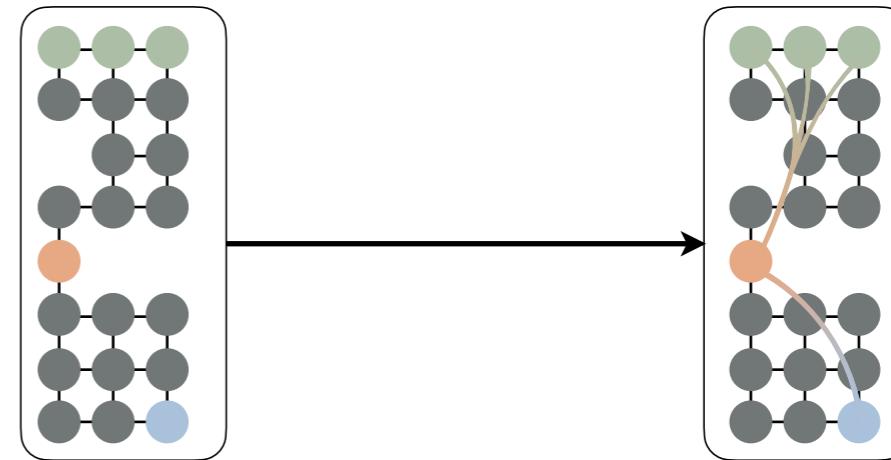
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1. Approximate State Abstraction  
*ICML 2016*
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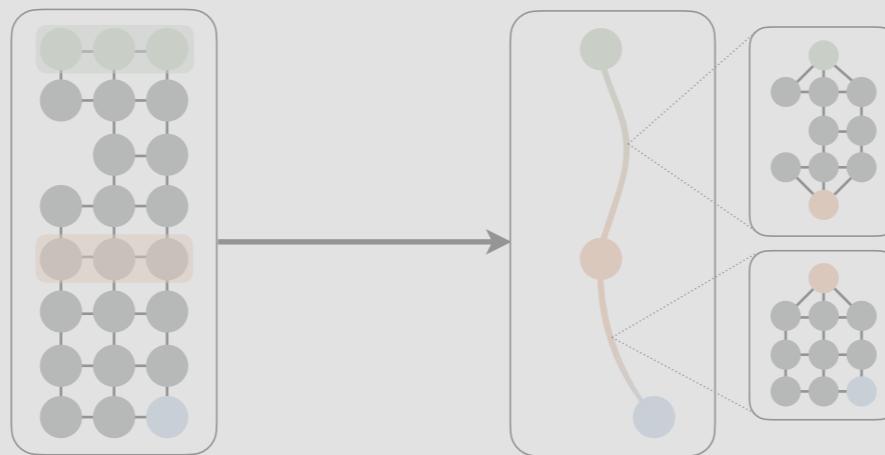
### ACTION ABSTRACTION



4. Options for Planning  
*ICML 2019*
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*IJCAI 2019*

## Part 3

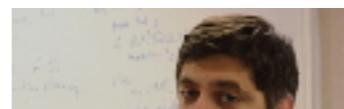
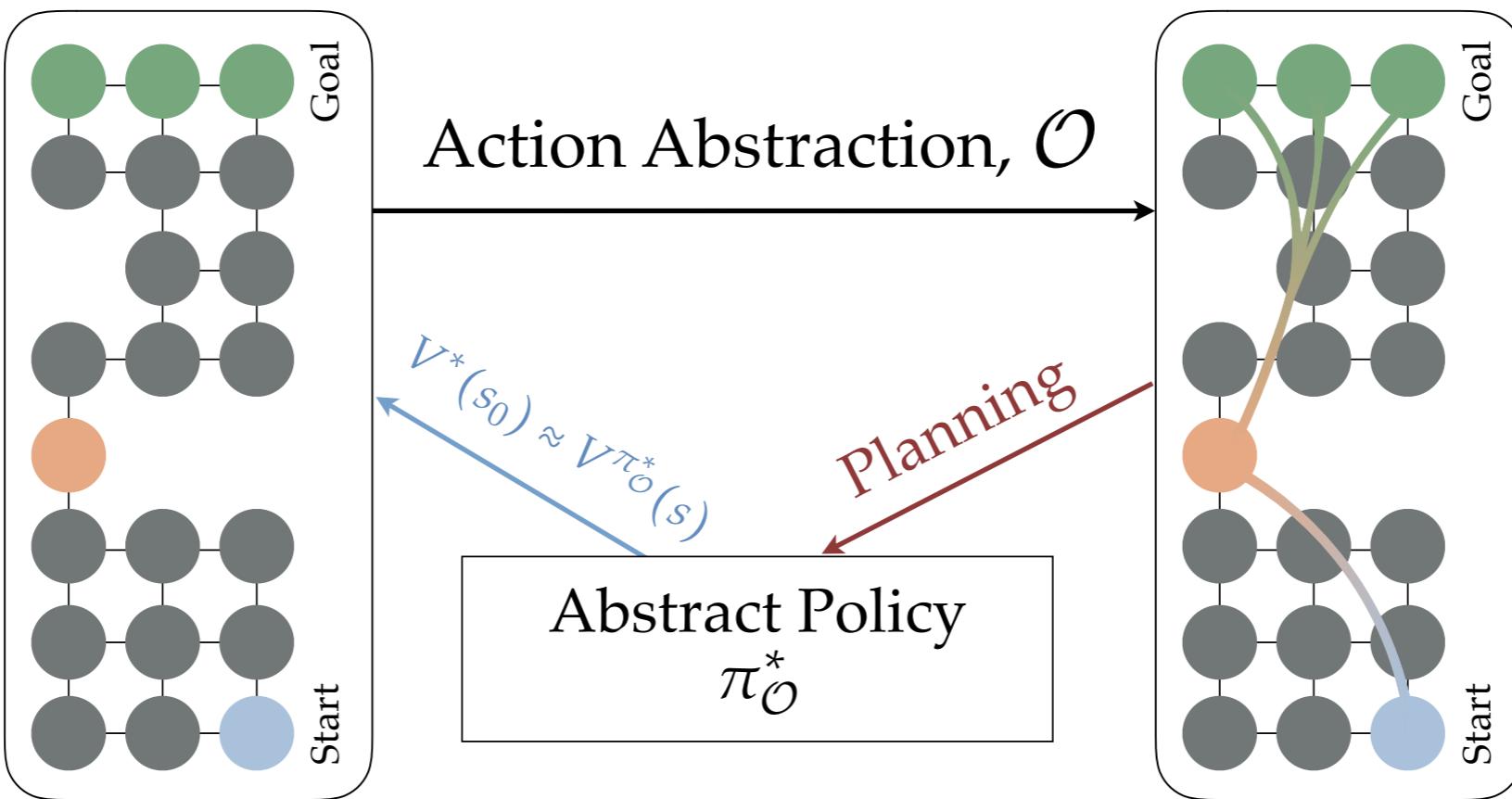
### STATE-ACTION ABSTRACTION



7. Value-Preserving Hierarchies  
*AISTATS 2020*

# Options for Planning

[JAHLK, ICML 2019]



**Question:** How can we find the set of options that make planning as fast as possible?

project → Jinnai

Hershkowitz

Littman

Konidaris

# Options for Planning

[JAHLK, ICML 2019]

**Theorem.** Finding the set of options that minimizes planning time is:

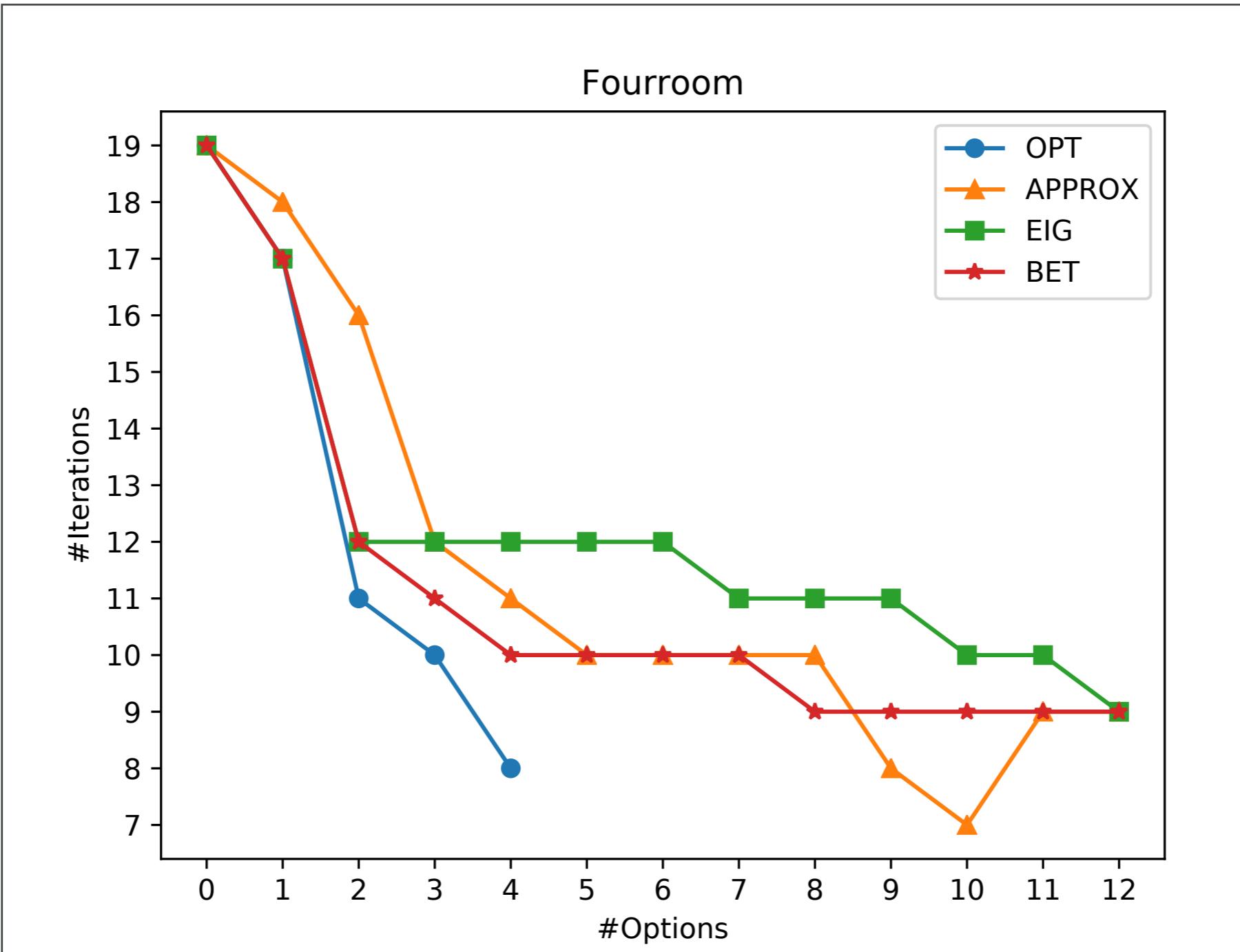
- 1) NP-hard in general.
- 2)  $2^{\log^{1-\varepsilon} n}$  -hard to approximate.<sup>1</sup>

<sup>1</sup>Unless  $\text{NP} \subseteq \text{DTIME}(n^{\text{poly log } n})$  [Dinitz et al. 2012]

**Question:** How can we find the set of options that make planning as fast as possible?

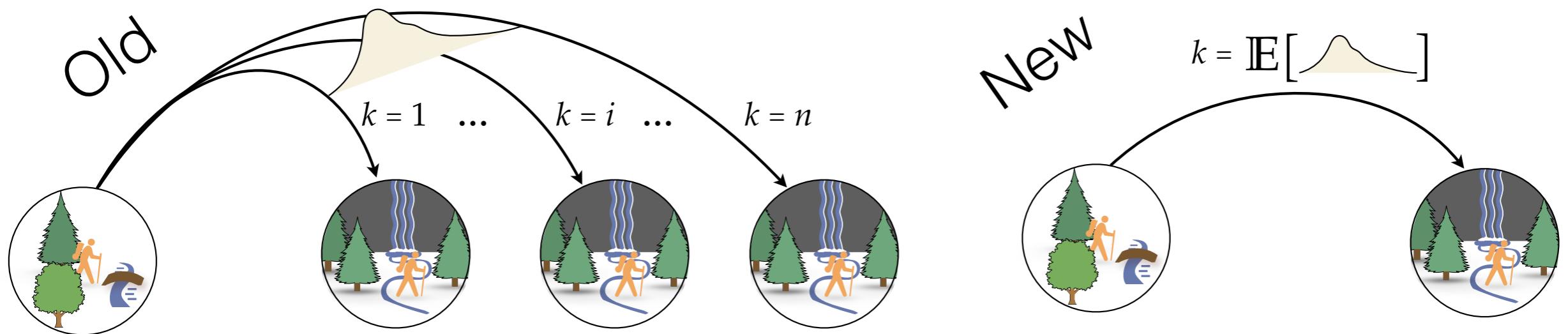
# Empirical Evaluation

[JAHLK, ICML 2019]



# A New Option Model

[AWdL, IJCAI 2019]



*jointly led  
project*



John  
Winder



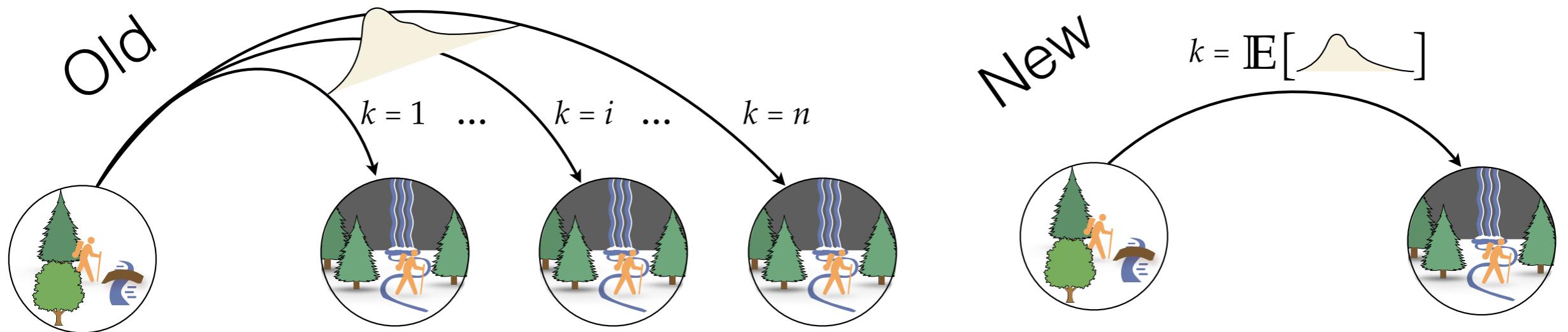
Marie  
desJardins



Michael L.  
Littman

# A New Option Model

[AWdL, IJCAI 2019]



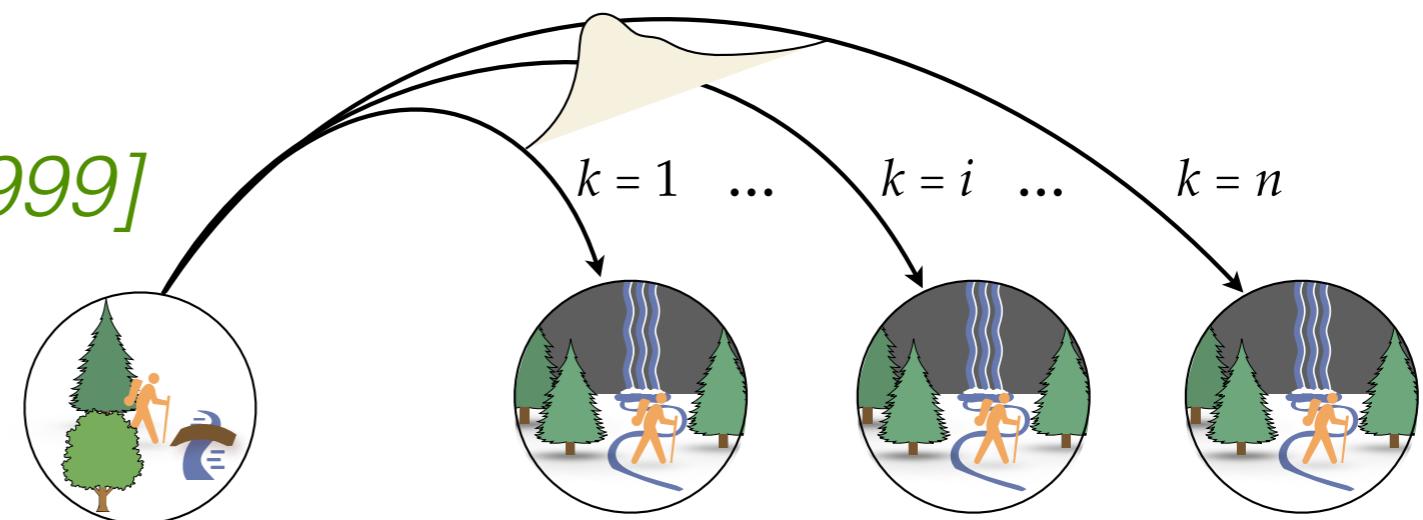
**Question:** How can we efficiently estimate the transition and reward models of options?

# A New Option Model

[AWdL, IJCAI 2019]

## **Multi-Time Model**

[Sutton, Precup, Singh 1999]



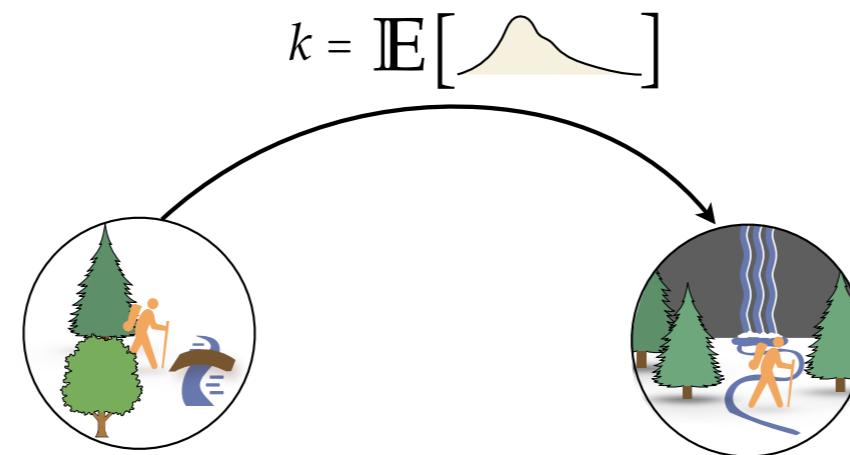
$$T_\gamma(s' | s, o) := \sum_{k=0}^{\infty} \gamma^k \beta(s_k) \mathbb{P}(s_k = s' | s, o)$$

$$R_\gamma(s, o) := \mathbb{E}_{k, s_1 \dots k} [r_1 + \gamma r_2 \dots + \gamma^{k-1} r_k | s, o]$$

# A New Option Model

[AWdL, IJCAI 2019]

## ***Expected Length Model***



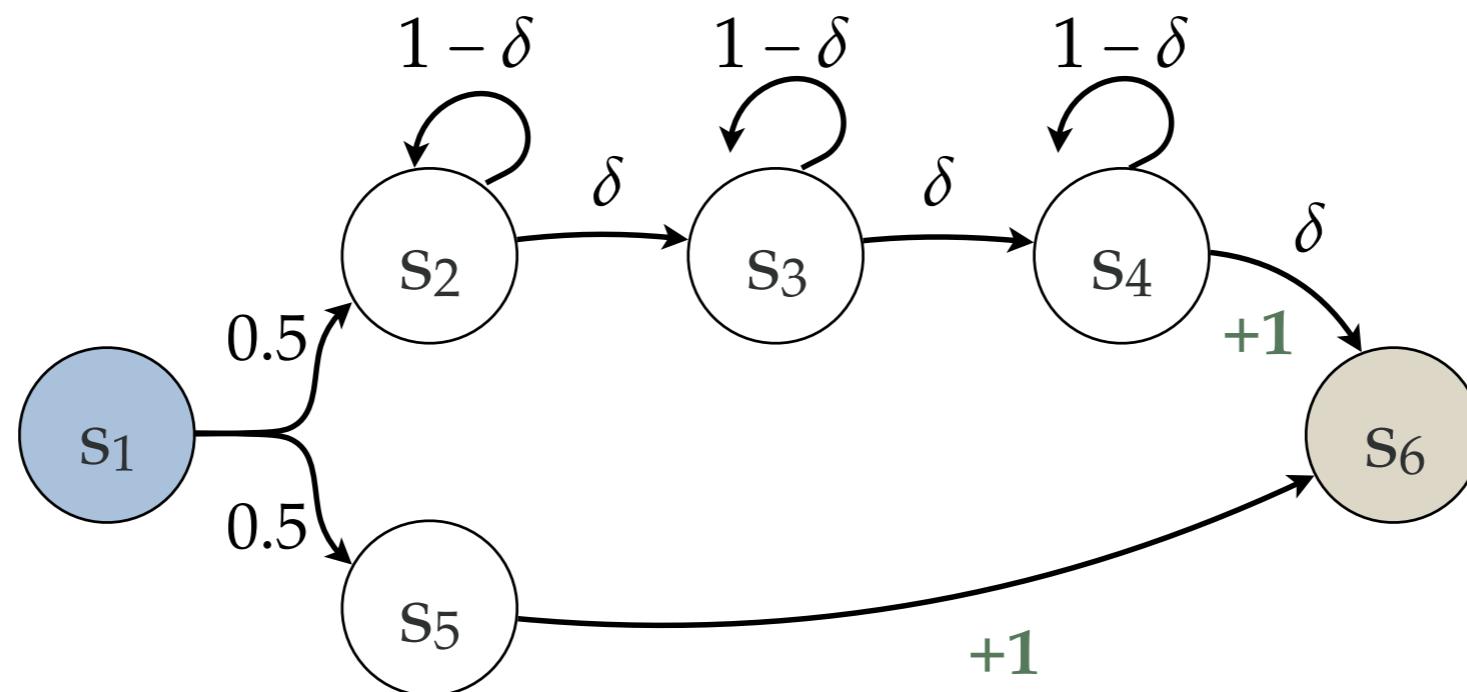
$$T_{\mu_k}(s' | s, o) := \gamma^{\mu_k} p(s' | s, o),$$

$$R_{\mu_k}(s, o, s') := \gamma^{\mu_k} \mathbb{E} [r_1 + r_2 \dots + r_{\mu_k} | s, o],$$

where  $\mu_k = \mathbb{E}[k | s, o]$ .

# A New Option Model

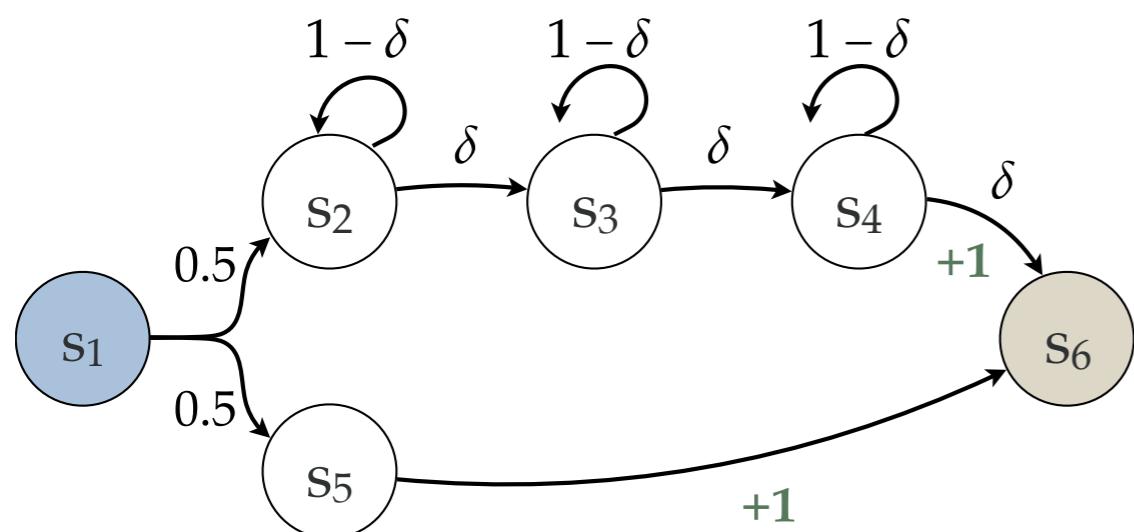
[AWdL, IJCAI 2019]



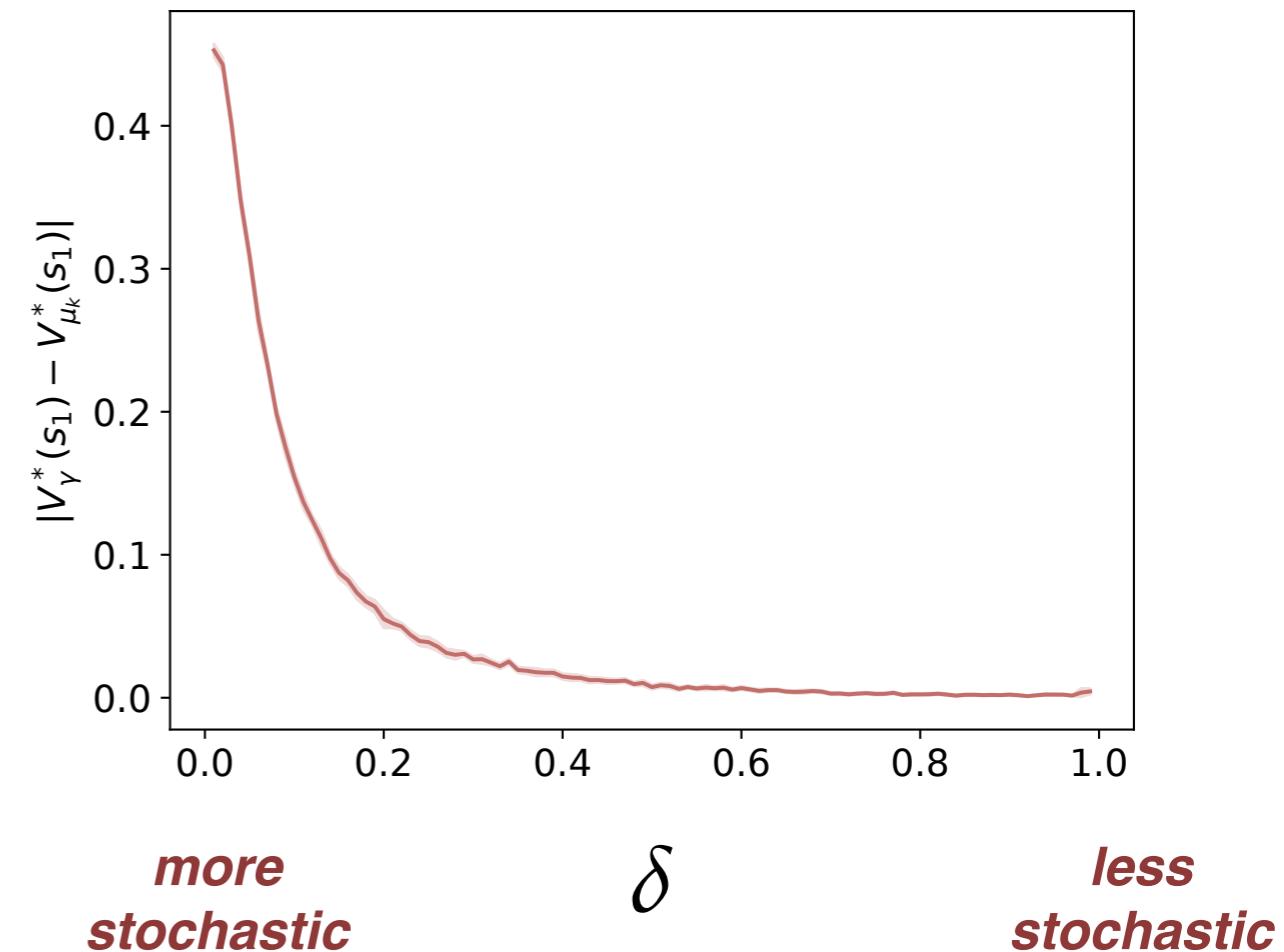
$$\mathbb{P}(s_6, k \mid s, o)$$

# A New Option Model

[AWdL, IJCAI 2019]



**Value Difference**



$$\mathbb{P}(s_6, k \mid s, o)$$

# A New Option Model

[AWdL, IJCAI 2019]

**Lemma.** *There exists a  $\tau \geq 1$  such that*

$$|T_\gamma(s' | s, o) - T_{\mu_k}(s' | s, o)| \leq \gamma^{\mu_{k,o} - \tau} (2\tau + 1) e^{-\beta_{\min}}.$$

**Lemma.** *In stochastic shortest path MDPs,*

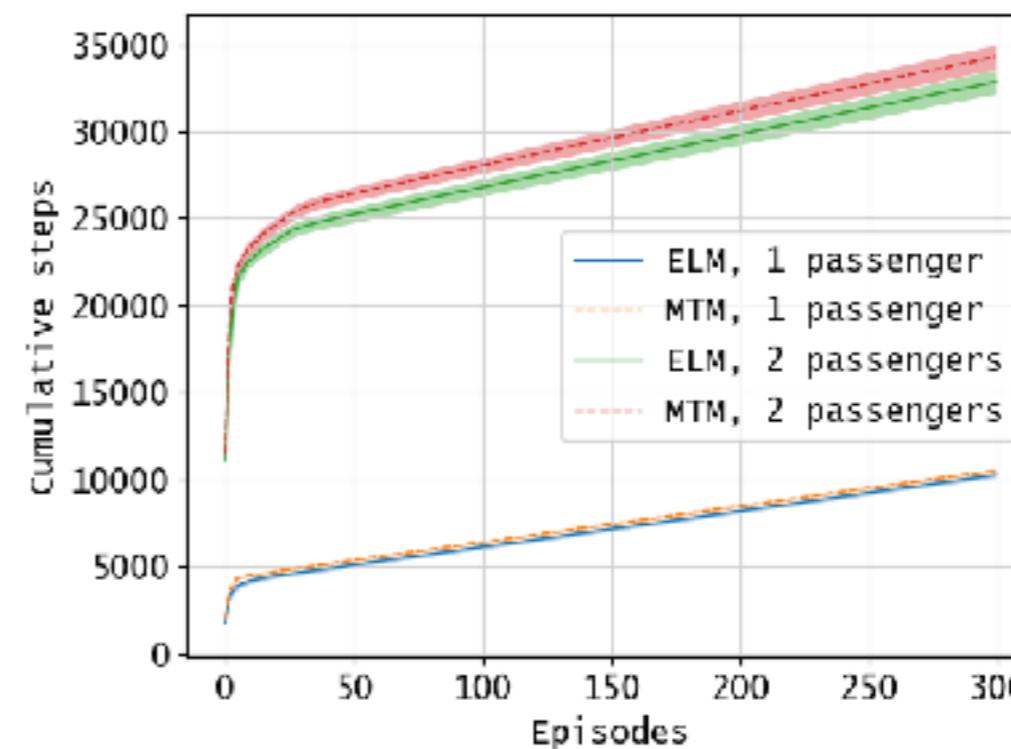
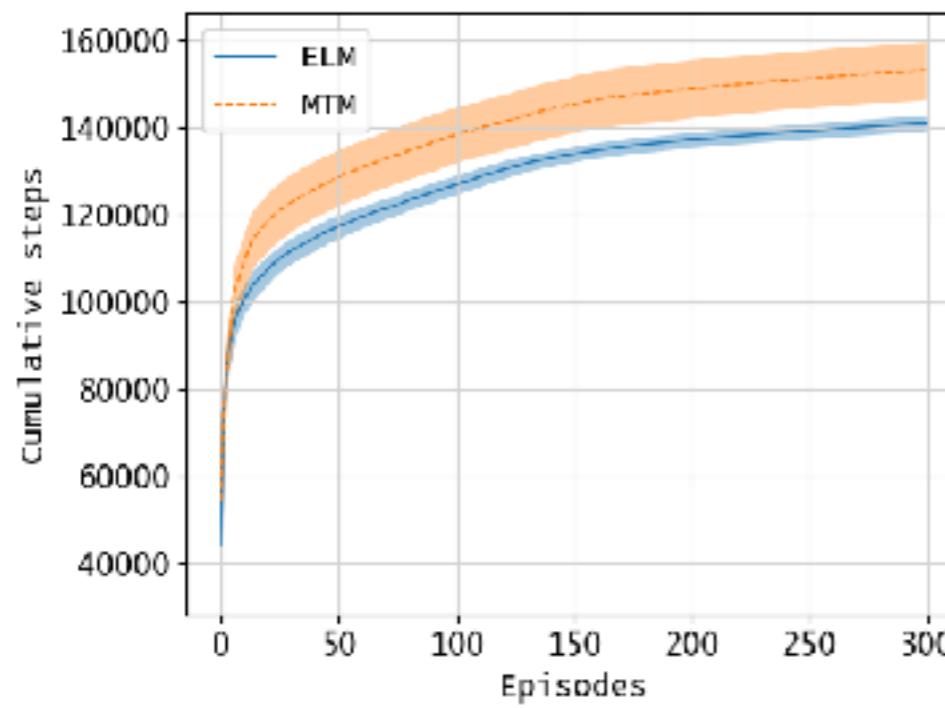
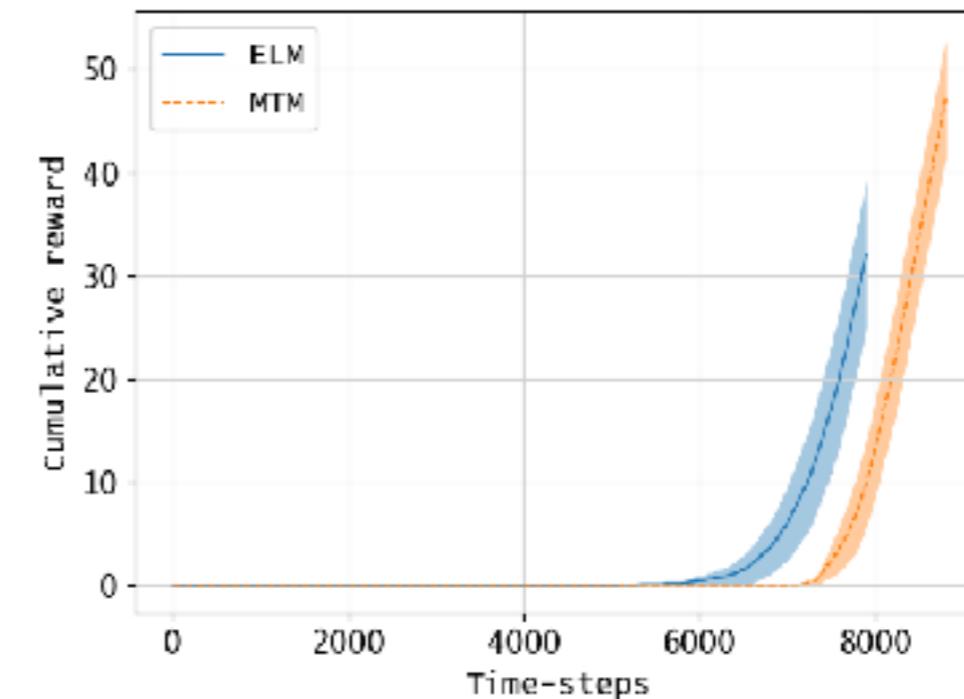
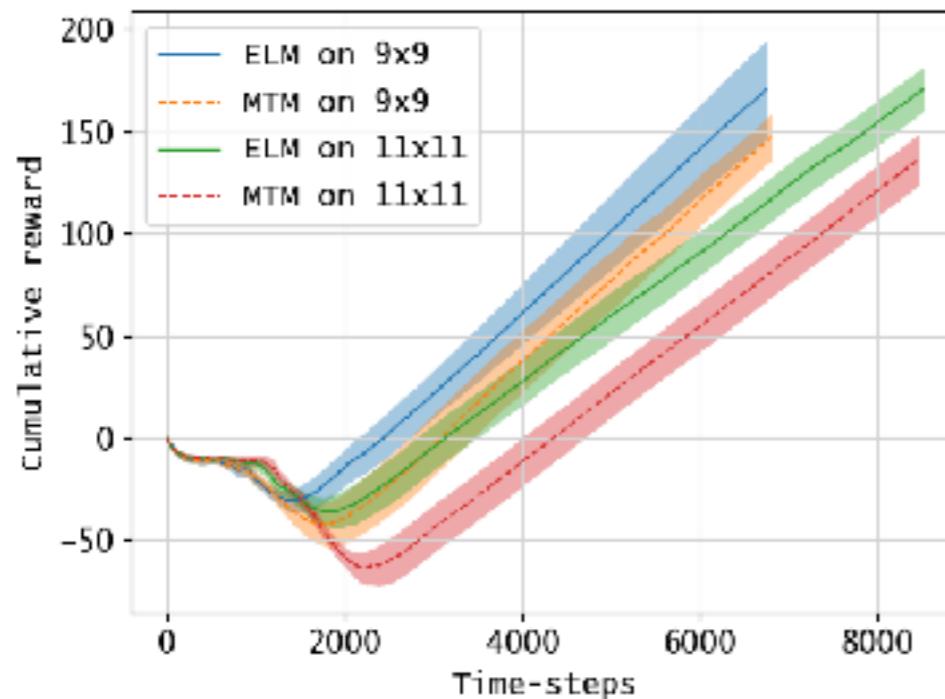
$$|R_\gamma(s, o) - R_{\mu_k}(s, o)| = |T_\gamma(s_g | s, o) - T_{\mu_k}(s_g | s, o)|.$$

**Theorem.** *In stochastic shortest path MDPs,*

$$|V_\gamma^{\pi_o}(s) - V_{\mu_k}^{\pi_o}(s)| \leq \frac{\varepsilon(1 - \gamma^{\mu_k}) + \gamma^{\mu_k} \frac{\varepsilon}{2} \text{RMAX}}{(1 - \gamma^{\mu_k})(1 - \gamma^{\mu_k} + \frac{\varepsilon}{2} \gamma^{\mu_k})}.$$

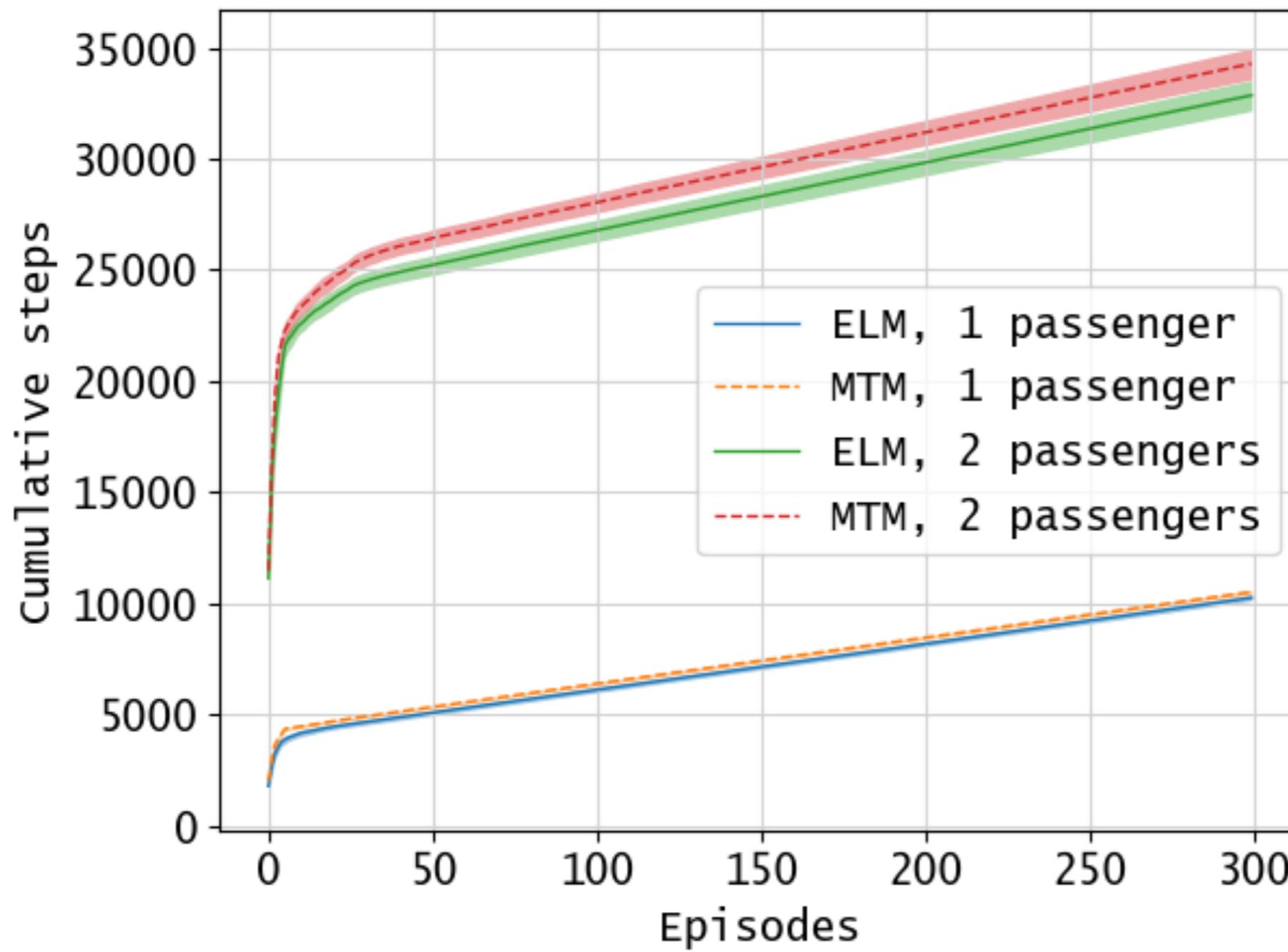
# A New Option Model

[AWdL, IJCAI 2019]



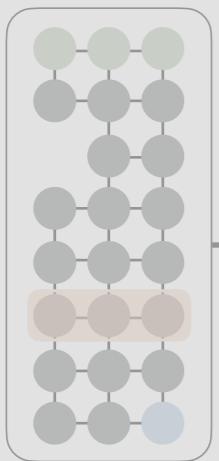
# A New Option Model

[AWdL, IJCAI 2019]



## Part 1

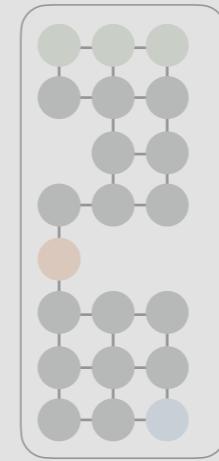
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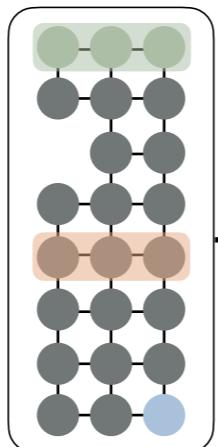
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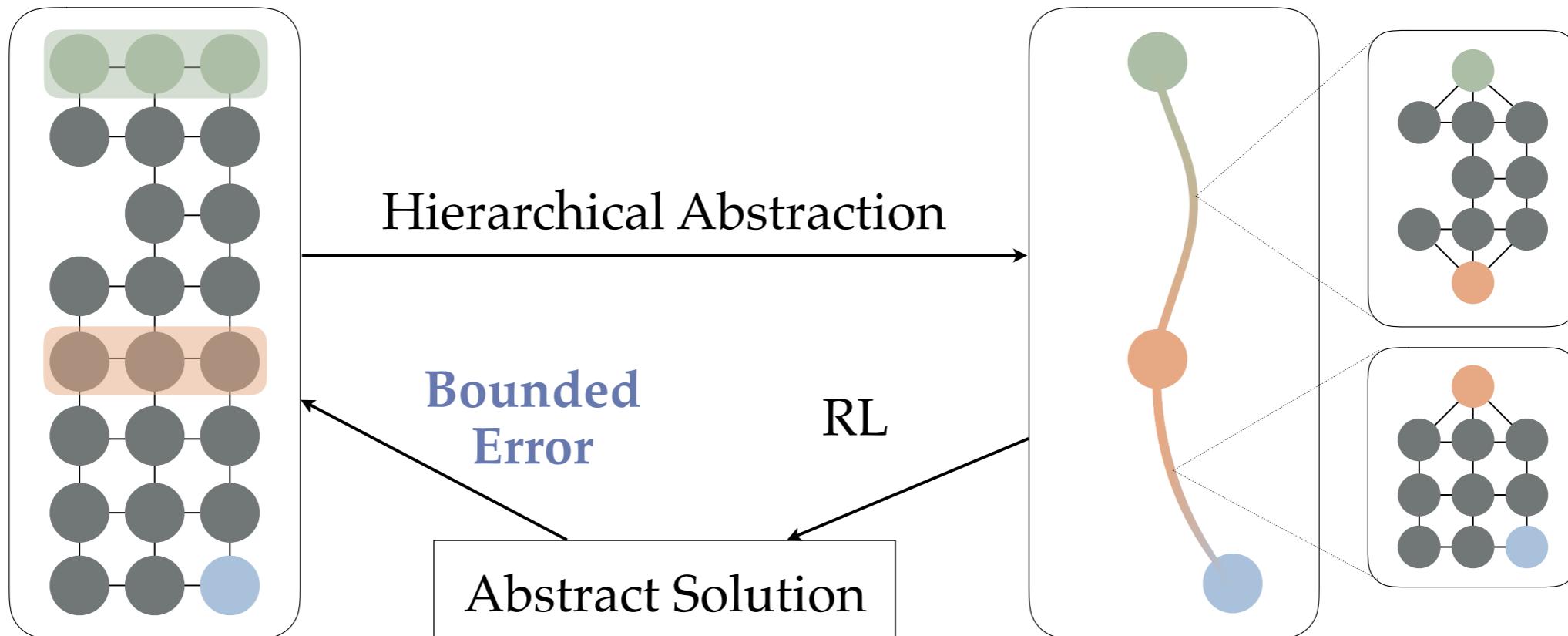
### STATE-ACTION ABSTRACTION



7. Value-Preserving Hierarchies  
*AISTATS 2020*

# Value-Preserving Hierarchies

[AUKAPL, AISTATS 2020]



Nathan  
Umbohner



Khimya  
Khetarpal



Dilip  
Arumugam

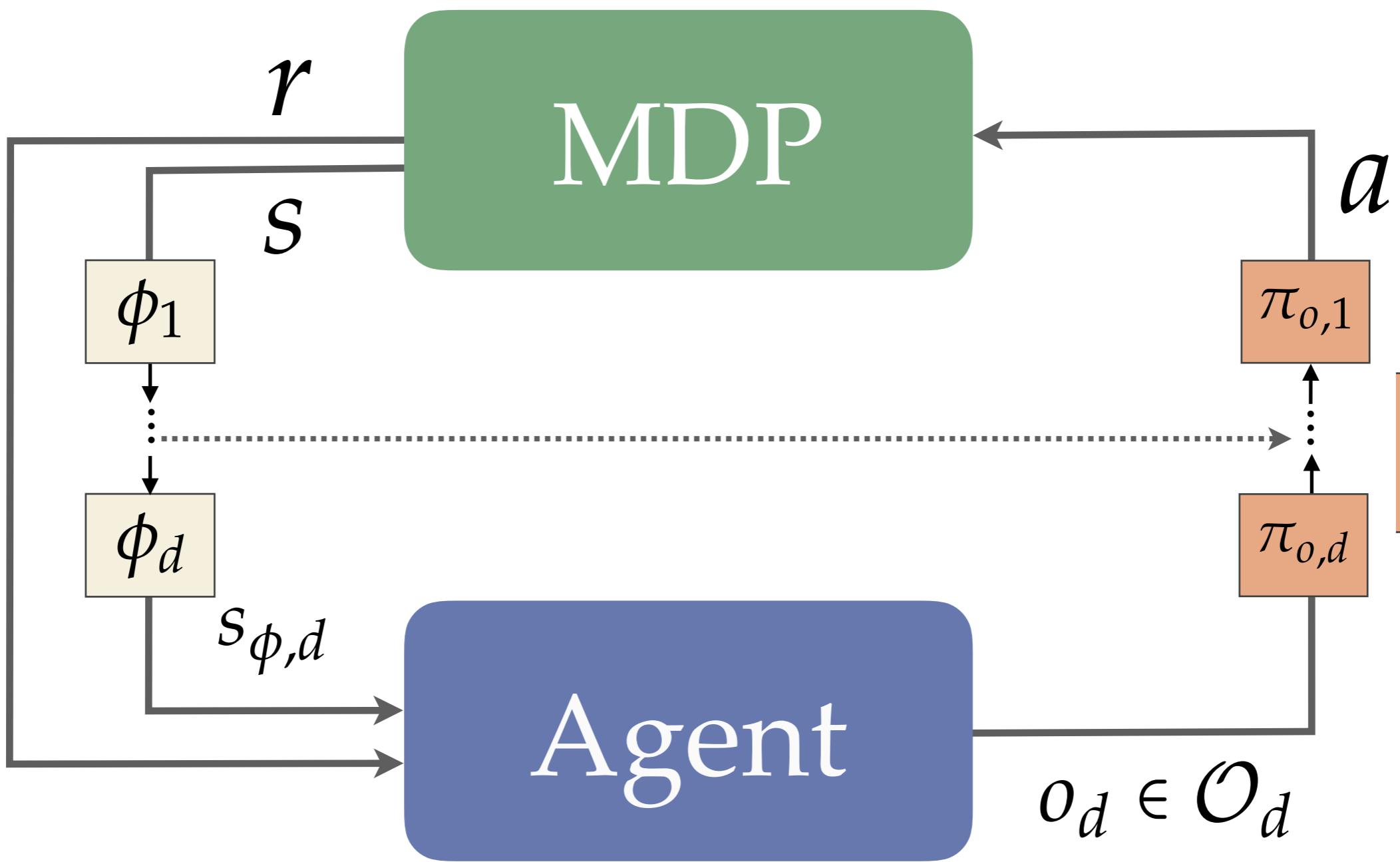


Doina  
Precup

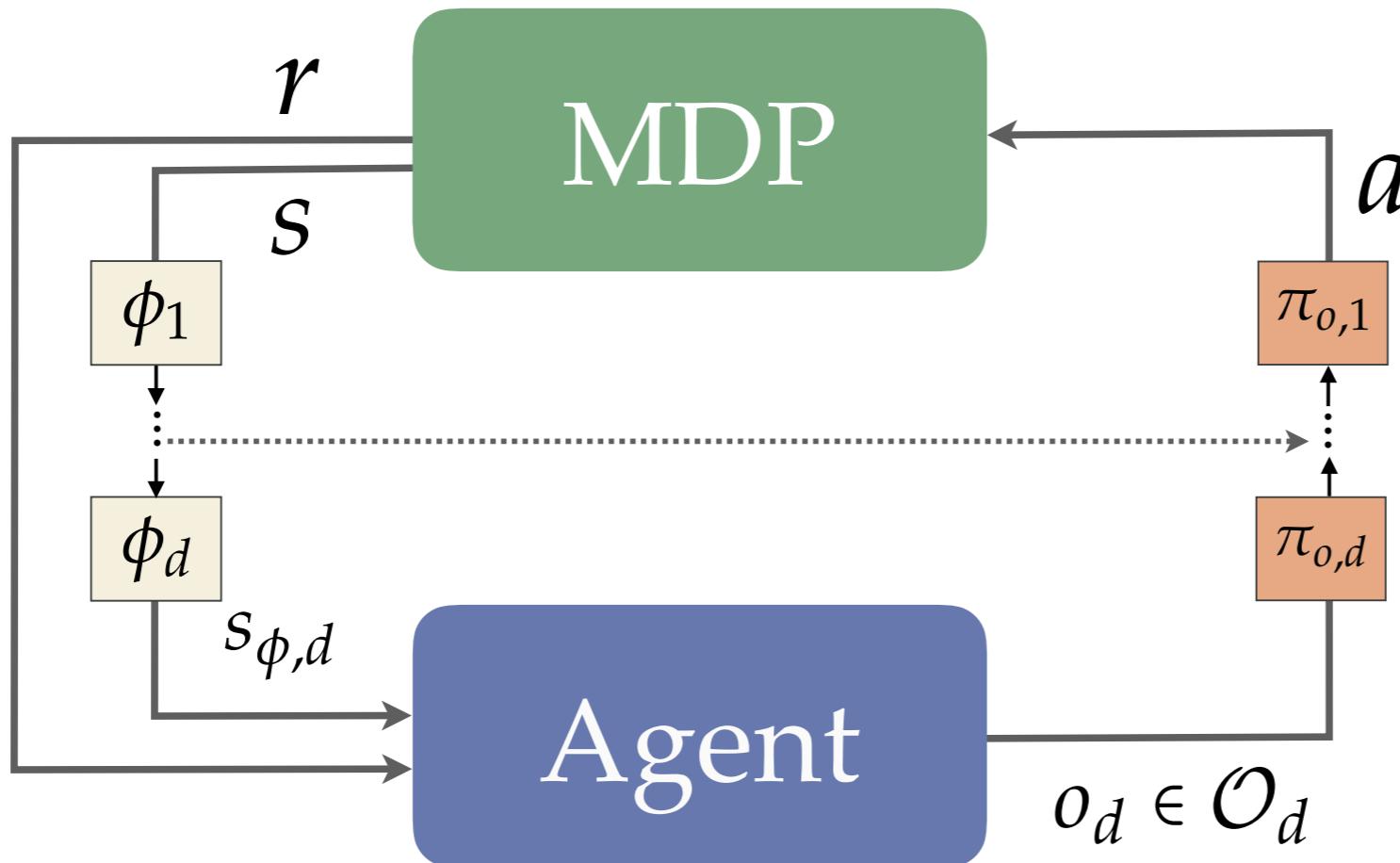


Michael L.  
Littman

# Hierarchical RL



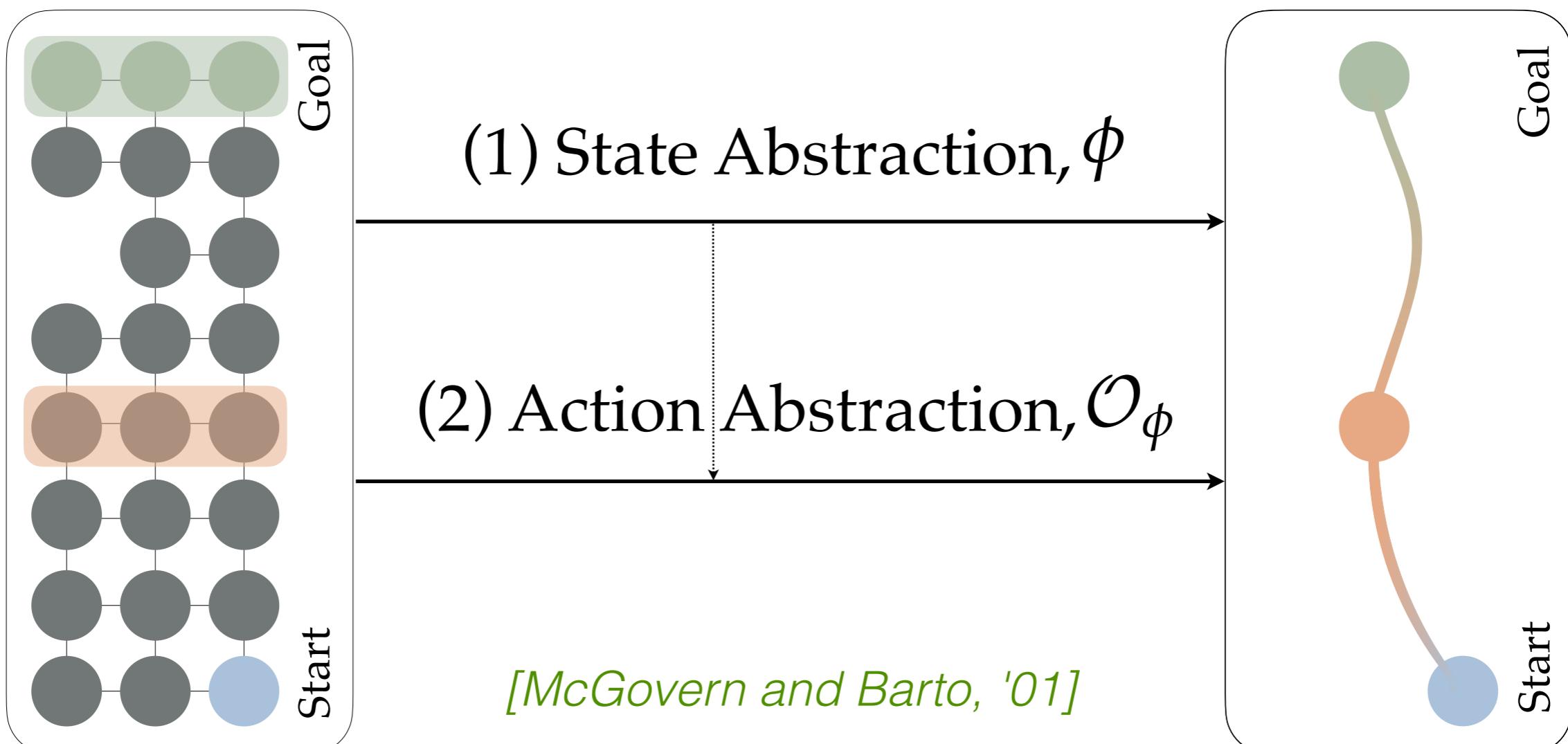
# Value-Preserving Hierarchical RL



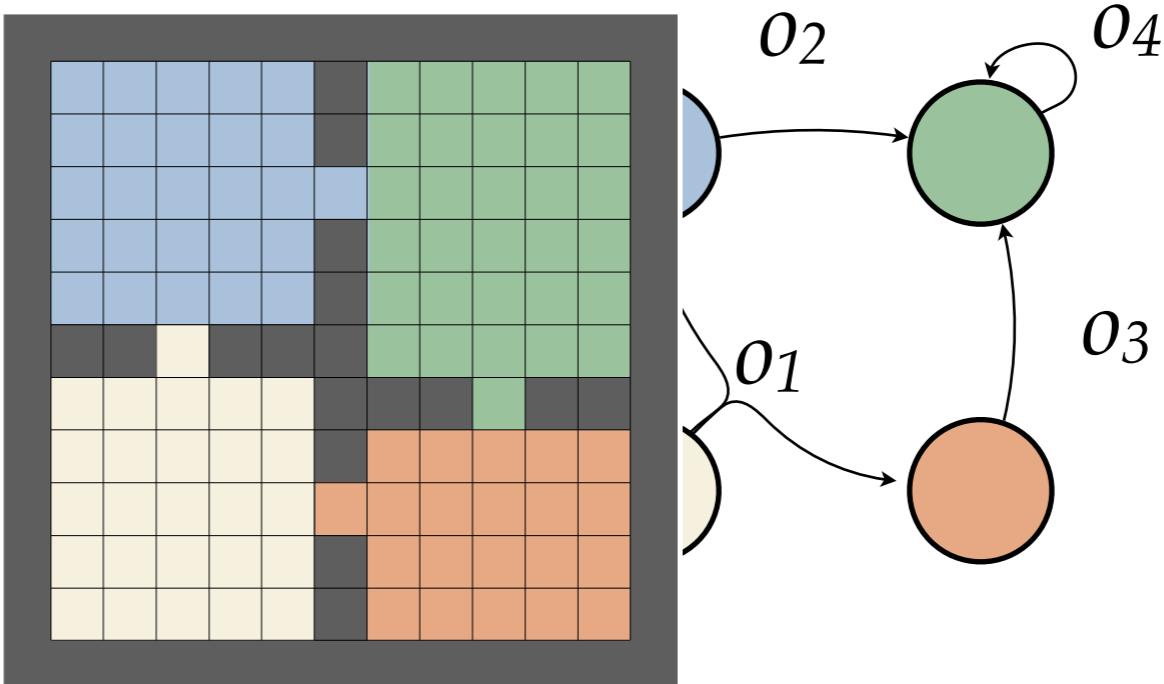
[Ravindran, Barto '03, '04]  
[Majeed & Hutter '19]

**Question:** Which combinations of state abstractions and options preserve representation of good behavior?

# $\phi$ -Relative Options



# $\phi$ -Relative Options



Given  $\phi$ ... Options must respect the abstract state boundaries.

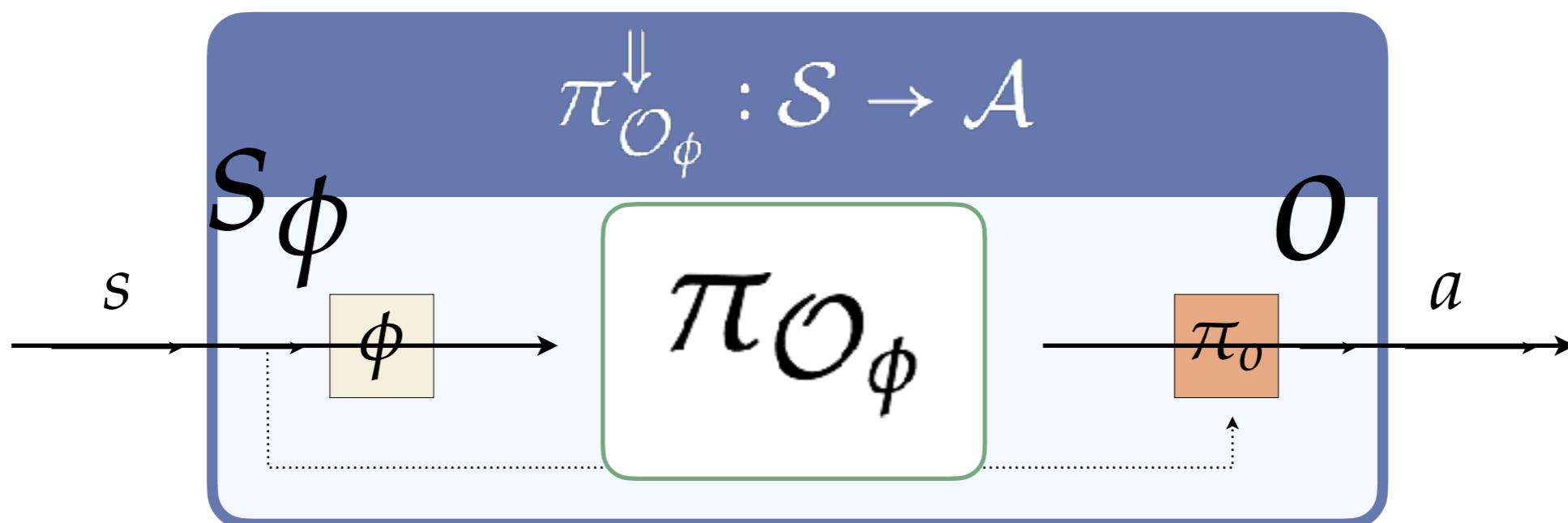
**Definition.** A set of options is said to be  $\phi$ -relative, denoted  $\mathcal{O}_\phi$ , if:

1. Each  $o \in \mathcal{O}_\phi$  initiates in some  $s_\phi$ , terminates when  $s \notin s_\phi$ .
2. For each abstract state, there is at least one  $o \in \mathcal{O}$  that initiates in that state.

# $\phi$ -Relative Options

(1) State Abstraction,  $\phi$

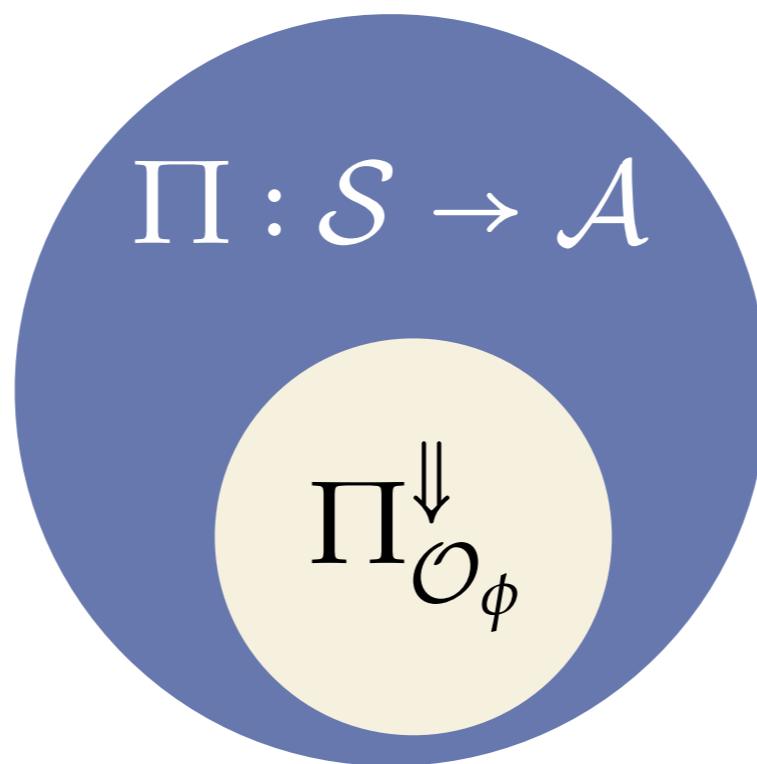
(2) Action Abstraction,  $\mathcal{O}_\phi$



# $\phi$ -Relative Options

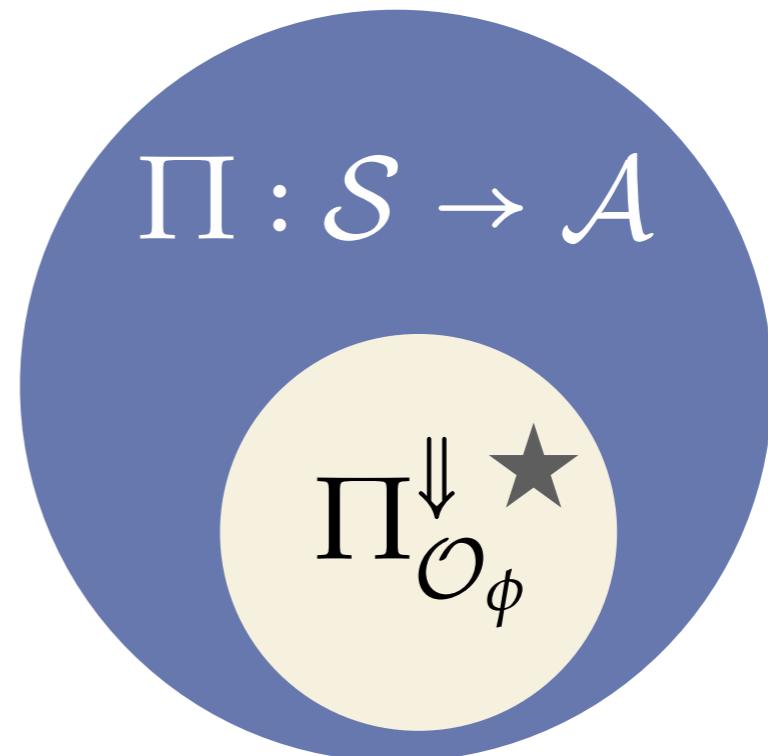
(1) State Abstraction,  $\phi$

(2) Action Abstraction,  $\mathcal{O}_\phi$



# $\phi$ -Relative Options

- (1) State Abstraction,  $\phi$
- (2) Action Abstraction,  $\mathcal{O}_\phi$



**Question:** Which  $\phi, \mathcal{O}_\phi$  pairs induce a policy class  $\Pi_{\mathcal{O}_\phi}^{\downarrow \star}$  such that the best abstract policy is still pretty good?

# Value-Preserving Abstractions

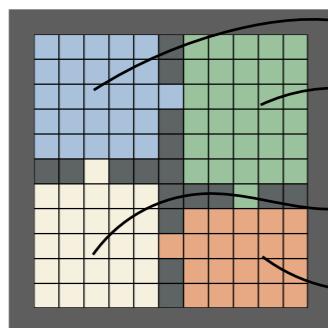
**Theorem.** There exist at least four classes of  $\phi, \mathcal{O}_\phi$  with bounded value loss:

$$\min_{\pi_{\mathcal{O}_\phi}^{\downarrow} \in \Pi_{\mathcal{O}_\phi}^{\downarrow}} \max_{s \in \mathcal{S}} \left( V^*(s) - V^{\pi_{\mathcal{O}_\phi}^{\downarrow}}(s) \right) \leq \eta_p,$$

where  $\eta_p$  varies depending on the class.

**Question:** Which  $\phi, \mathcal{O}_\phi$  pairs induce a policy class  $\Pi_{\mathcal{O}_\phi}^{\downarrow}$  such that the best abstract policy is still pretty good?

# $\phi$ -Relative Option Classes



$$\forall s_\phi \in \mathcal{S}_\phi \exists o \in \Omega(s_\phi) :$$

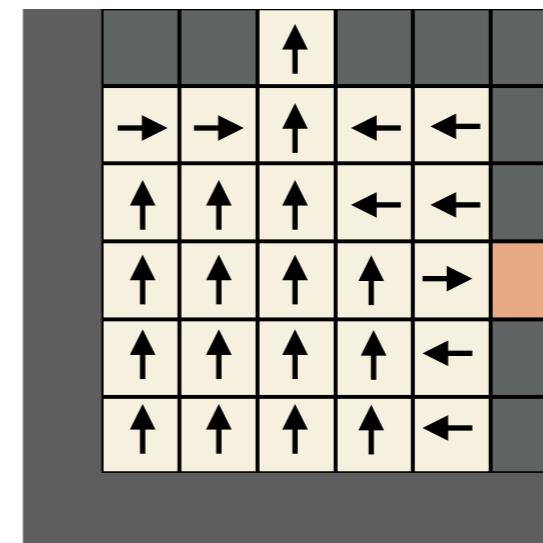
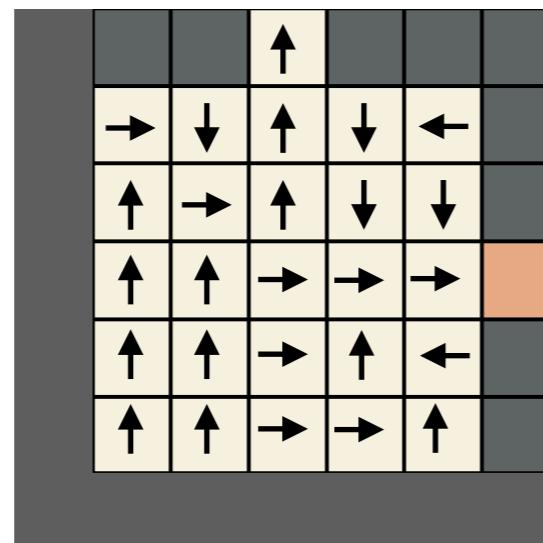
All options that initiate in  $s_\phi$

$o$

$\approx$

$o_{s_\phi}^*$

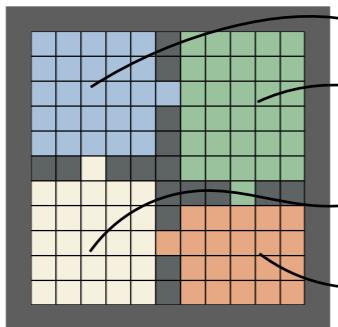
Determines option class



$$o_{s_\phi}^* = (s \in \mathcal{S}_\phi, s \notin \mathcal{S}_\phi, \pi^*)$$

*initiate*      *terminate*      *policy*

# $\phi$ -Relative Option Classes



$$\forall s_\phi \in \mathcal{S}_\phi \exists o \in \Omega(s_\phi) :$$

All options that initiate in  $s_\phi$

*Expressive  $Q^*$  Options*

$$|Q^*(s_\phi, o) - Q^*(s_\phi, o_{s_\phi}^*)| \leq \varepsilon_Q$$

*Expressive Model Options*

$$|R_\gamma(s_\phi, o^*) - R_\gamma(s_\phi, o)| \leq \varepsilon_R$$

and

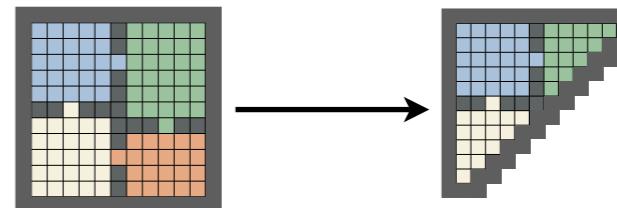
$$\|T_\gamma(\cdot | s_\phi, o^*) - T_\gamma(\cdot | s_\phi, o)\|_2 \leq \varepsilon_T$$

*Expressive  $k$ -Step Options*

$$\max_{s \in s_\phi, s' \in \mathcal{S}} |\mathbb{P}(s', k | s, o_{s_\phi}^*) - \mathbb{P}(s', k | s, o)| \leq \tau$$

[Nachum et al. 2019]

*Homomorphism Options*



[Ravindran and Barto '02, '03, '04]

# Value-Preserving Abstractions

**Theorem.**  $\min_{\pi_{\mathcal{O}_\phi}^\downarrow \in \Pi_{\mathcal{O}_\phi}^\downarrow} \max_{s \in \mathcal{S}} \left( V^*(s) - V^{\pi_{\mathcal{O}_\phi}^\downarrow}(s) \right) \leq \eta_p,$

*Expressive  $Q^*$  Options*

$$\frac{\varepsilon_Q}{1 - \gamma}$$

*Expressive Model Options*

$$\frac{\varepsilon_R + |\mathcal{S}| \varepsilon_T \text{RMAX}}{(1 - \gamma)^2}$$

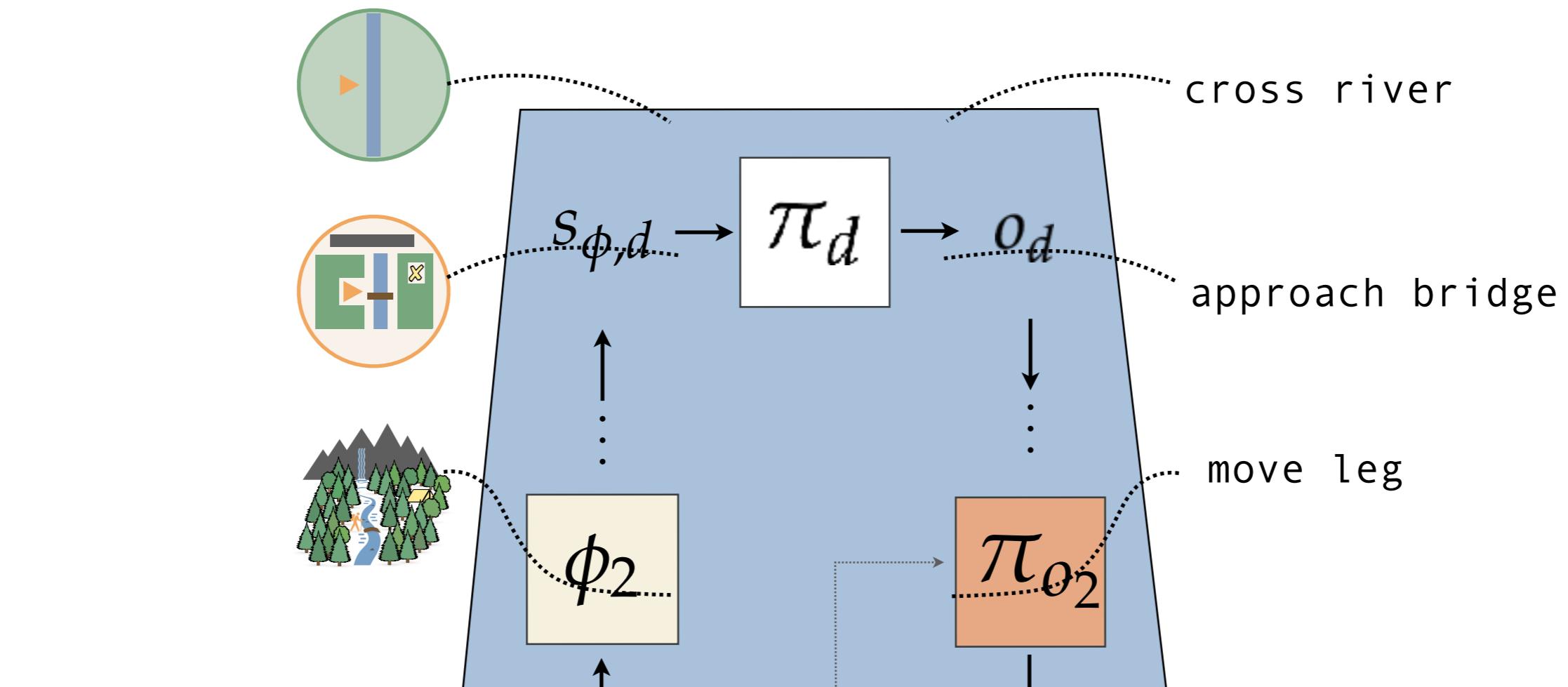
*Expressive  $k$ -Step Options*

$$\frac{\tau \gamma |\mathcal{S}|}{(1 - \gamma)^2}$$

*Homomorphism Options*

$$\frac{2}{1 - \gamma} \left( \varepsilon_r + \frac{\gamma \text{RMAX}}{1 - \gamma} \frac{\varepsilon_p}{2} \right)$$

# Value-Preserving Hierarchies

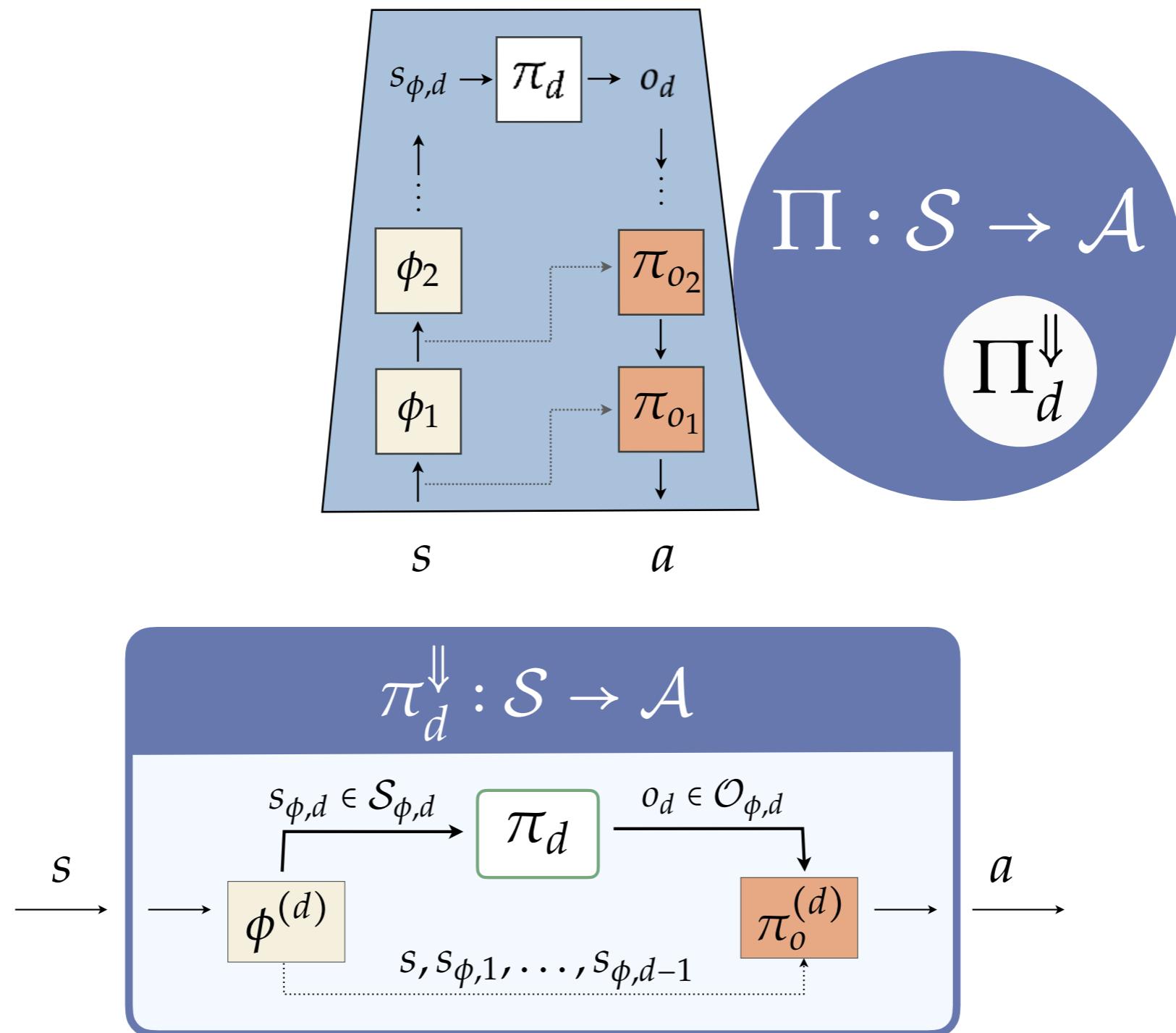


**Definition.** A depth  $d$  hierarchy  $H_d$  is defined by the pair

$$\phi^{(d)} = (\phi_1, \phi_2, \dots, \phi_d),$$

$$\mathcal{O}^{(d)} = (\mathcal{O}_1, \mathcal{O}_2, \dots, \mathcal{O}_d).$$

# Value-Preserving Hierarchies



# Value-Preserving Hierarchies

**Assumption 1.** *The value function is consistent throughout the hierarchy.*

- **value expressivity**

**Assumption 2.** *Subsequent levels of the hierarchy can represent policies similar in value to the best policy at the previous level.*

- **policy expressivity**

# Value-Preserving Hierarchies

value expressivity  
policy expressivity

**Assumption 1.** *The value function is consistent throughout the hierarchy.*

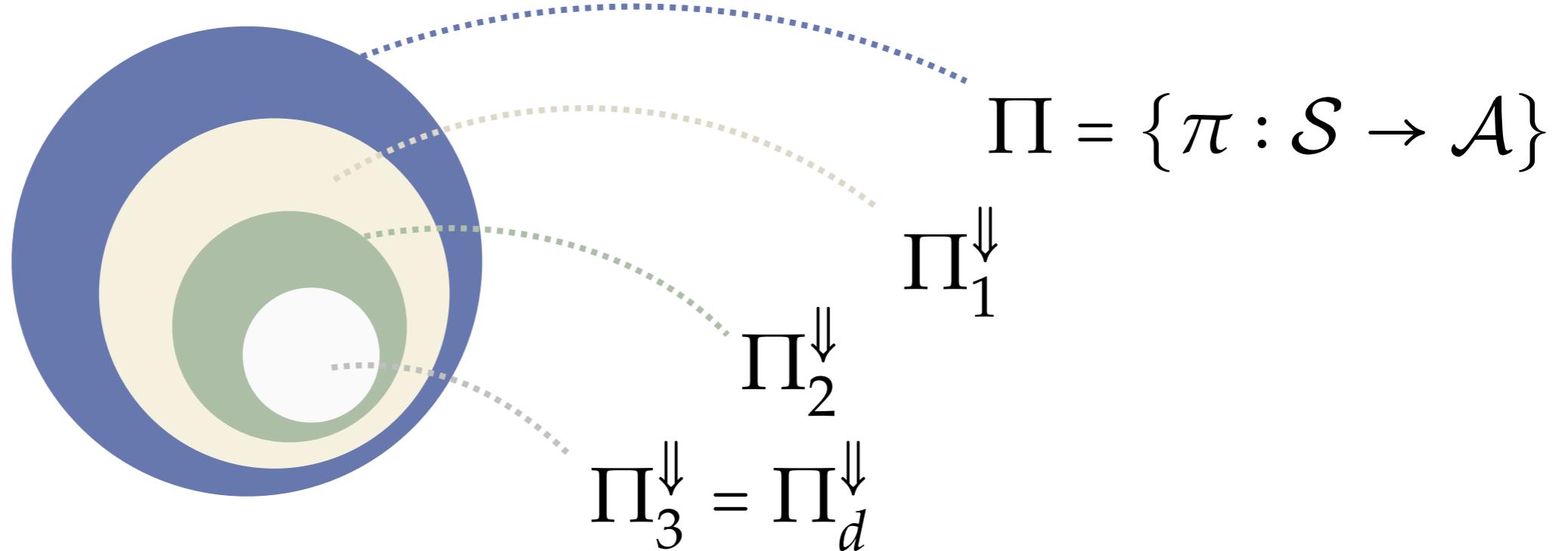
**Assumption 2.** *Subsequent levels of the hierarchy can represent policies similar in value to the best policy at the previous level.*

**Theorem.** Any hierarchy  $H_d$  that satisfies Assumptions 1 and 2 has bounded value loss:

$$\min_{\pi_d^\downarrow \in \Pi_d^\downarrow} \max_{s \in \mathcal{S}} \left( V^*(s) - V^{\pi_d^\downarrow}(s) \right) \leq d(\kappa + \ell)$$

value expressivity  
depth      policy expressivity

# Value-Preserving Hierarchies



**Theorem.** Any hierarchy  $H_d$  that satisfies Assumptions 1 and 2 has bounded value loss:

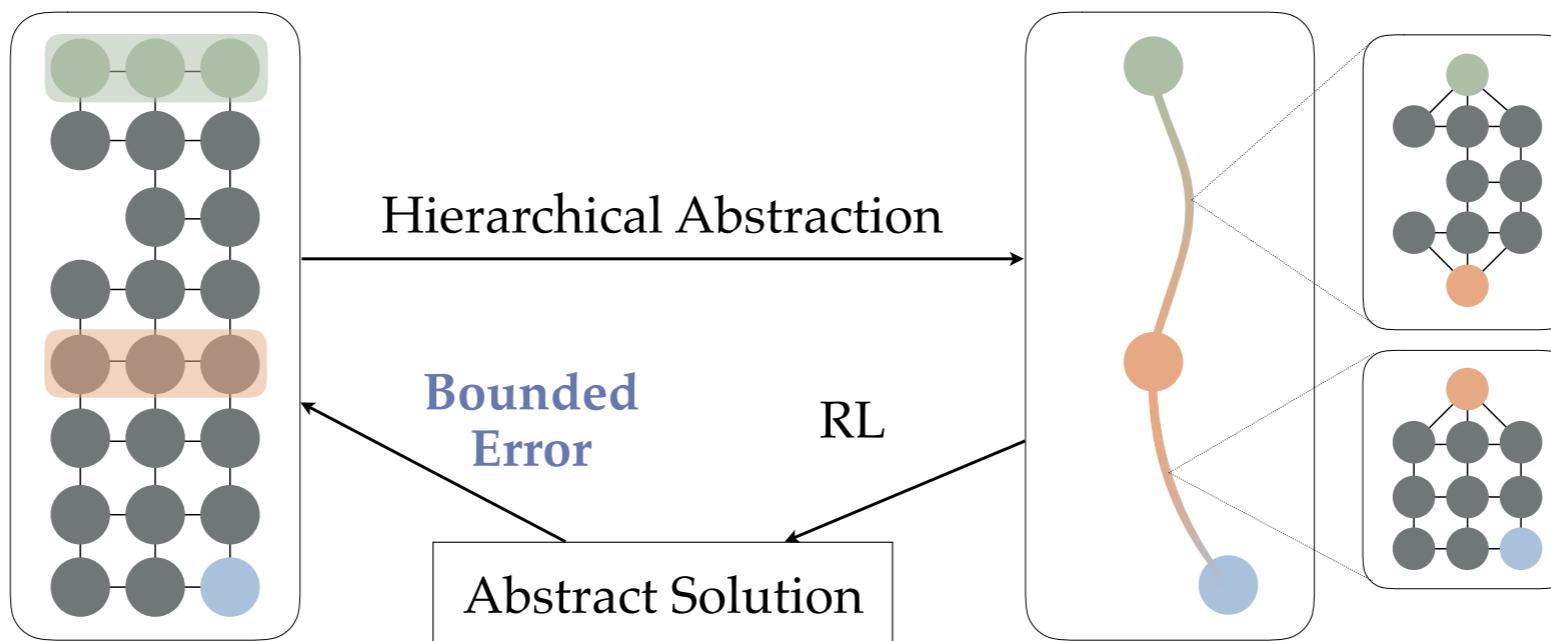
$$\min_{\pi_d^\downarrow \in \Pi_d^\downarrow} \max_{s \in \mathcal{S}} \left( V^*(s) - V^{\pi_d^\downarrow}(s) \right) \leq \underbrace{d}_{\text{depth}} (\underbrace{\kappa + \ell}_{\text{policy expressivity}})$$

value expressivity

depth

policy expressivity

# Value-Preserving Hierarchies



**Theorem.** Any hierarchy  $H_d$  that satisfies Assumptions 1 and 2 has bounded value loss:

$$\min_{\pi_d^\downarrow \in \Pi_d^\downarrow} \max_{s \in \mathcal{S}} \left( V^*(s) - V^{\pi_d^\downarrow}(s) \right) \leq \underbrace{d}_{\text{depth}} (\underbrace{\kappa + \ell}_{\text{policy expressivity}})$$

value expressivity

# Thanks to Mentors!

*Masters*



Joshua  
Schechter



Stefanie  
Tellex

*Ph.D*



Michael L.  
Littman

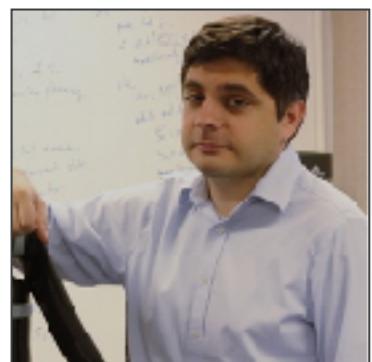


David  
Liben-Nowell



Ana  
Moltchanova

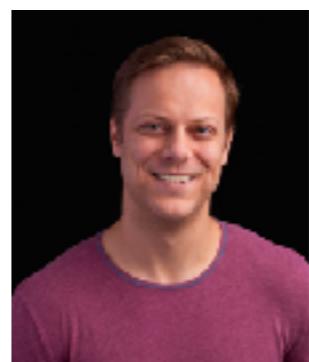
*Undergrad*



George  
Konidaris



Peter  
Stone



Will  
Dabney



Fernando  
Diaz



Owain  
Evans

*Committee*

*Internships*

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Cam  
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Kavosh  
Asadi



Gabriel  
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Stephen  
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Jonathon  
Cohen



Marie  
desJardins



Tom  
Griffiths



Yue  
Guo



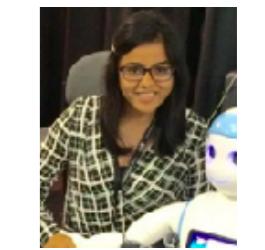
D. Ellis  
Hershkowitz



Mark  
Ho



Yuu  
Jinnai



Khimya  
Khetarpal



Akshay  
Krishnamurthy



Lucas  
Lehnert



James  
MacGlashan



Jee Won  
Park



Doina  
Precup



Emily  
Reif



Mark  
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John  
Salvatier



Robert  
Schapire



Andreas  
Stuhlmüller



Nathan  
Umbanhower



Edward  
Williams

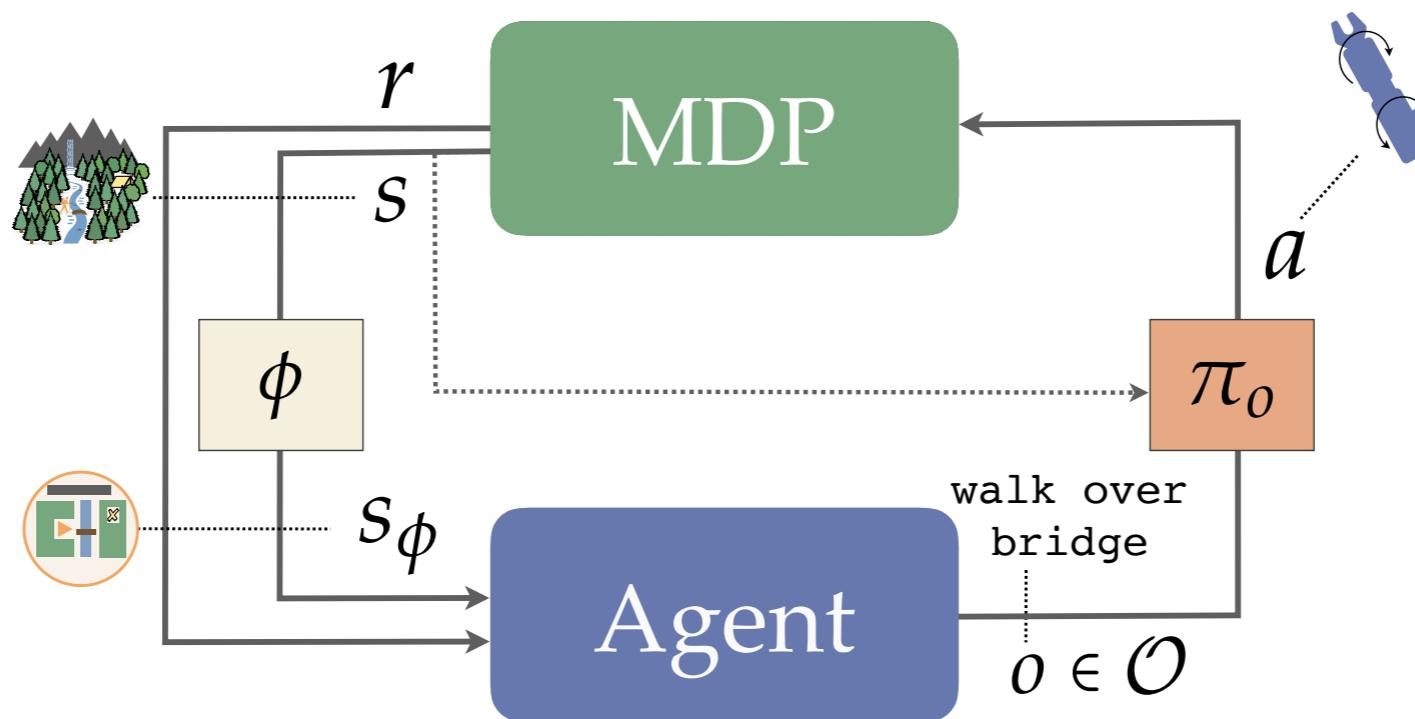


John  
Winder



Lawson  
Wong

# Summary



**Question:** How do effective RL **agents** come up with the right **state** and **action** abstractions of the **MDPs** they inhabit?

**Dissertation:** [david-abel.github.io/thesis.pdf](https://david-abel.github.io/thesis.pdf)

**Contact:** dmabel@deepmind.com