部分積分による定積分

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06-01

(1) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$
\begin{aligned}
\text{与式} &= \int_0^a f(x)dx \\
&\text{変数変換 } t = a - x \\
&dt = -dx \\
&= \int_{x=0}^{x=a} f(a-x)dx \\
&= \int_{t=a}^{t=0} f(t)(-dt) \\
&= \int_0^a f(t)dt \\
&= \int_0^a f(x)dx
\end{aligned}
$$

(2) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}dx$

$$
\begin{aligned}
\text{与式} &= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}dx \\
&\text{変数変換 } t = \frac{\pi}{2} - x \\
&dt = -dx \\
&= \int_{x=0}^{x=\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}dx \\
&= \int_{t=\frac{\pi}{2}}^{t=0} \frac{\cos(\frac{\pi}{2} - t)}{\cos(\frac{\pi}{2} - t) + \sin(\frac{\pi}{2} - t)}(-dt) \\
&= \int_0^{\frac{\pi}{2}} \frac{\sin t}{\sin t + \cos t}dt
\end{aligned}
$$

また　与式 $= \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x}dx$

よって　$2I = \int_0^{\frac{\pi}{2}} \left(\frac{\cos x + \sin x}{\cos x + \sin x}\right)dx$

$$
\begin{aligned}
&= \int_0^{\frac{\pi}{2}} 1dx \\
&= \frac{\pi}{2} \\
I &= \frac{\pi}{4}
\end{aligned}
$$

06-02

(1) $\int_0^{\frac{\pi}{2}} x\cos 3xdx$

$=$

(2) $\int_1^2 xe^{\frac{x}{2}}dx$

$=$

(3) $\int_1^{e^2} \log xdx$

$=$

(4) $\int_0^{\frac{\pi}{2}} (x-1)\sin xdx$

$=$

(5) $\int_{-} 4^3 \log(x+5)dx$

$=$

(6) $\int_1^e x^2 \log xdx$

$=$

06-03

(1) $\int_{-} 1^1 xe^{-3x^2}dx$

$=$

(2) $\int_{-\pi}^{\pi} x^2 \sin xdx$

$=$

06-04

(1) $\int_1^e \frac{\log x}{x^2} dx$

$=$

(2) $\int_- 1^1 x^2 e^{2x} dx$

$=$

(3) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \cos x dx$

$=$

06-05

(1)$I = \int_0^{\frac{\pi}{2}} e^x \sin x dx$

$=$

(2)$J = \int_0^{\frac{\pi}{2}} e - x \cos x dx$

$=$