

解析学 02 課題

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02-01

$$(1) \int x \sin 2x dx$$

$$\begin{aligned} &= \int x \left\{ -\frac{1}{2} \cos 2x \right\}' dx \\ &= -\frac{1}{2} x \cos 2x - \int -\frac{1}{2} \cos 2x dx \\ &= -\frac{1}{2} x \cos 2x - \frac{1}{4} \sin 2x + C \end{aligned}$$

$$(2) \int x \cos 3x dx$$

$$\begin{aligned} &= \int x \frac{1}{3} \{\sin 3x\}' dx \\ &= \frac{1}{3} x \sin 3x - \int \frac{1}{3} \sin 3x dx \\ &= \frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x + C \end{aligned}$$

$$(3) \int \frac{x}{\cos^2 x} dx$$

$$\begin{aligned} &= \int x \{\tan x\}' dx \\ &= x \tan x - \int \tan x dx \\ &= x \tan x - \int \frac{\sin x}{\cos x} dx \\ &= x \tan x - \int \frac{-\{\cos x\}'}{\cos x} dx \\ &= x \tan x - (-\log |\cos x|) + C \\ &= x \tan x + \log |\cos x| + C \end{aligned}$$

$$(4) \int x \log(x-2) dx$$

$$\begin{aligned} &= \int \frac{1}{2} (x^2 - 2^2)' \log(x-2) \\ &= \frac{1}{2} (x^2 - 2^2) \log(x-2) - \int \frac{1}{2} (x^2 - 2^2) \frac{1}{x-2} dx \\ &= \frac{1}{2} (x^2 - 2^2) \log(x-2) - \int \frac{1}{2} \frac{(x+2)(x-2)}{x-2} dx \\ &= \frac{1}{2} (x^2 - 2^2) \log(x-2) - \int \frac{1}{2} (x+2) dx \\ &= \frac{1}{2} (x^2 - 2^2) \log(x-2) - \left(\frac{1}{4} x^2 + x \right) + C \end{aligned}$$

$$(5) \int x^3 \log x dx$$

$$\begin{aligned} &= \int \frac{1}{4} (x^4)' \log x dx \\ &= \frac{1}{4} x^4 \log x - \int \frac{1}{4} x^4 \frac{1}{x} dx \\ &= \frac{1}{4} x^4 \log x - \int \frac{1}{4} x^3 dx \\ &= \frac{1}{4} x^4 \log x - \frac{1}{16} x^4 + C \end{aligned}$$

$$(6) \int (2x - 1)e^x dx$$

$$\begin{aligned} &= \int (2x - 1)(e^x)' dx \\ &= (2x - 1)e^x - \int 2e^x dx \\ &= (2x - 1)e^x - 2e^x + C \\ &= (2x - 3)e^x + C \end{aligned}$$

$$(7) \int x e^{-3x} dx$$

$$\begin{aligned} &= \int -\frac{1}{3} x (e^{-3x})' dx \\ &= -\frac{1}{3} x e^{-3x} - \int -\frac{1}{3} e^{-3x} dx \\ &= -\frac{1}{3} x e^{-3x} - \frac{1}{9} e^{-3x} + C \end{aligned}$$