解析学 05 定積分

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5-1

(1)$f(x)=\int_1^{x-3}(x-3-p)\log p dp$

$$
\begin{aligned}
f(x) &= \int_1^{x-3}(x-3-p)\log p dp \\
&= \int_1^{x-3}(x-3)\log p - p\log p dp \\
&= (x-3)\int_1^{x-3}\log p dp - \int_1^{x-3} p\log p dp \\
\frac{df(x)}{dx} &= [p\log p - p]_1^{x-3} \\
&= (x-3)\log(x-3) - x + 4
\end{aligned}
$$

(2)$g(x)=-\int_0^{x^2}(x^2-s)e^s ds$

$$
\begin{aligned}
g(x) &= -x^2\int_0^{x^2} e^s ds + \int_0^{x^2} se^s ds \\
\frac{dg(x)}{dx} &= 2x(-\int_0^{x^2} e^s ds - x^2e^{x^2} + x^2e^{x^2}) \\
&= 2x(1-e^{x^2})
\end{aligned}
$$

5-2

(1) $\int_1^3 4x^3 dx$

$$
\begin{aligned}
&= [x^4]_1^3 \\
&= 3^4 - 1^4 \\
&= 80
\end{aligned}
$$

(2) $\int_1^4 \sqrt{x} dx$

$$
\begin{aligned}
&= [\frac{2}{3}x^{\frac{3}{2}}]_1^4 \\
&= \frac{2}{3}4\sqrt{4} - \frac{2}{3} \\
&= \frac{14}{3}
\end{aligned}
$$

(3) $\int_1^4 x^{-\frac{3}{2}} dx$

$$
\begin{aligned}
&= [-2x^{-\frac{1}{2}}]_1^4 \\
&= -2\frac{1}{\sqrt{4}} - 2 \\
&= -3
\end{aligned}
$$

(4) $\int_e^{e^2} \frac{dx}{x}$

$$
\begin{aligned}
&= [\log|x|]_e^{e+1} \\
&= 2 - 1 \\
&= 1
\end{aligned}
$$

(5) $\int_2^{e+1} \frac{dy}{1-y}$

$$
\begin{aligned}
&= [-\log|1-y|]_2^{e+1} \\
&= -1 + 0 \\
&= -1
\end{aligned}
$$

(6) $\int_0^{\pi} \sin\theta d\theta$

$$
\begin{aligned}
&= [-\cos\theta]_0^{\pi} \\
&= -0 - 1 \\
&= 1
\end{aligned}
$$

(7) $\int_{-}\frac{\pi}{2}^{\frac{\pi}{2}} \cos t dt$

$$= 2\int_0^1 \cos t dt$$
$$= 2[\sin t]_0^1$$
$$= 2$$

(8) $\int_0^{\log 2} e^{3x} dx$

$$= [\frac{1}{3}e^{3x}]_0^{\log 2}$$
$$= \frac{1}{3}e^{3\log 2} - \frac{1}{3}$$
$$= \frac{7}{3}$$

(9) $\int_1^2 2^x dx$

$$= [\frac{2^x}{\log 2}]_1^2$$
$$= \frac{2}{\log 2}$$

05-03

(1) $\int_0^{\frac{\pi}{2}} (\sin 2x + \cos 3x) dx$

$$= [-\frac{1}{2}\cos 2x + \frac{1}{3}\sin 3x]_0^{\frac{\pi}{2}}$$
$$= (\frac{1}{2} - \frac{1}{3}) - (-\frac{1}{2} + 0)$$
$$= \frac{2}{3}$$

(2) $\int_{\pi} t^3 \pi \cos(\frac{x}{4} - \frac{\pi}{4}) dx$

$$= [4\sin(\frac{x}{4} - \frac{\pi}{4})]_0^{\frac{\pi}{2}}$$
$$= 4 - 0$$
$$= 4$$

(3) $\int_1^2 \sin(\frac{2}{3}\pi t + \frac{\pi}{4}) dt$

$$= [-\frac{3}{2\pi}\cos(\frac{2}{3}\pi t + \frac{\pi}{4})]_1^2$$
$$= -\frac{3}{2\pi}(\cos\frac{19}{12}\pi - \cos\frac{11}{12}\pi)$$

(4) $\int_0^{\frac{\pi}{4}} \cos^2 x dx$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{2}(1 + \cos 2x) dx$$
$$= \frac{1}{2}[x + \frac{1}{2}\sin 2x]_0^{\frac{\pi}{4}}$$
$$= \frac{\pi + \sqrt{2}}{8}$$

(5) $\int_0^{\frac{\pi}{8}} \sin^2 2x dx$

$$= \int_0^{\frac{\pi}{8}} \frac{1}{2}(1 - \cos 4x) dx$$
$$= \frac{1}{2}[x - \frac{1}{4}\sin 4\pi]_0^{\frac{\pi}{8}}$$
$$= \frac{\pi - \sqrt{2}}{16}$$

(6) $\int_0^{\frac{\pi}{4}} \tan^2 x dx$

$$= \int_0^{\frac{\pi}{4}} (\frac{1}{\cos^2 x} - 1) dx$$
$$= [\tan x - x]_0^{\frac{\pi}{4}}$$
$$= 1 - \frac{\pi}{4}$$

05-04

(1) $\int_0^2 x(x^2+1)^3 dx$

$$t = x^2 + 1$$
$$dt = 2x dx$$
$$I = \frac{1}{2}\int_1^5 t^3 dt$$
$$= \frac{624}{8}$$

(2) $\int_1^2 \frac{x^2-2x}{x^3-3x^2+1} dx$

$$= [\frac{1}{3}\log|x^3 - 3x^2 + 1|]_1^2$$
$$= \frac{1}{3}\log 3$$