

02-01

$$(1) \int x \sin 2x dx$$

$$\begin{aligned} &= \int x \left\{ -\frac{1}{2} \cos 2x \right\}' dx \\ &= -\frac{1}{2} x \cos 2x - \int -\frac{1}{2} \cos 2x dx \\ &= -\frac{1}{2} x \cos 2x - \frac{1}{4} \sin 2x + C \end{aligned}$$

$$(2) \int x \cos 3x dx$$

$$\begin{aligned} &= \int x \frac{1}{3} \{\sin 3x\}' dx \\ &= \frac{1}{3} x \sin 3x - \int \frac{1}{3} \sin 3x dx \\ &= \frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x + C \end{aligned}$$

$$(3) \int \frac{x}{\cos^2 x} dx$$

$$\begin{aligned} &= \int x \{\tan x\}' dx \\ &= x \tan x - \int \tan x dx \\ &= x \tan x - \int \frac{\sin x}{\cos x} dx \\ &= x \tan x - \int \frac{-\{\cos x\}'}{\cos x} dx \\ &= x \tan x - (-\log |\cos x|) + C \\ &= x \tan x + \log |\cos x| + C \end{aligned}$$

$$(4) \int x \log(x-2) dx$$

$$\begin{aligned} &= \int \frac{1}{2} (x^2 - 2^2)' \log(x-2) \\ &= \frac{1}{2} (x^2 - 2^2) \log(x-2) - \int \frac{1}{2} (x^2 - 2^2) \frac{1}{x-2} dx \\ &= \frac{1}{2} (x^2 - 2^2) \log(x-2) - \int \frac{1}{2} \frac{(x+2)(x-2)}{x-2} dx \\ &= \frac{1}{2} (x^2 - 2^2) \log(x-2) - \int \frac{1}{2} (x+2) dx \\ &= \frac{1}{2} (x^2 - 2^2) \log(x-2) - \left(\frac{1}{4} x^2 + x \right) + C \end{aligned}$$

$$(5) \int x^3 \log x dx$$

$$\begin{aligned} &= \int \frac{1}{4} (x^4)' \log x dx \\ &= \frac{1}{4} x^4 \log x - \int \frac{1}{4} x^4 \frac{1}{x} dx \\ &= \frac{1}{4} x^4 \log x - \int \frac{1}{4} x^3 dx \\ &= \frac{1}{4} x^4 \log x - \frac{1}{16} x^4 + C \end{aligned}$$

$$(6) \int (2x-1)e^x dx$$

$$\begin{aligned} &= \int (2x-1)(e^x)' dx \\ &= (2x-1)e^x - \int 2e^x dx \\ &= (2x-1)e^x - 2e^x + C \\ &= (2x-3)e^x + C \end{aligned}$$

$$(7) \int x e^{-3x} dx$$

$$\begin{aligned} &= \int -\frac{1}{3} x (e^{-3x})' dx \\ &= -\frac{1}{3} x e^{-3x} - \int -\frac{1}{3} e^{-3x} dx \\ &= -\frac{1}{3} x e^{-3x} - \frac{1}{9} e^{-3x} + C \\ &= e^{-3x} \left(-\frac{1}{3} x - \frac{1}{9} \right) + C \end{aligned}$$

$$(8) \int \log(x+2) dx$$

$$\begin{aligned} &= \int (x+2)' \log(x+2) dx \\ &= (x+2) \log(x+2) - \int (x+2) \frac{1}{x+2} dx \\ &= (x+2) \log(x+2) - \int 1 dx \\ &= (x+2) \log(x+2) - x + C \end{aligned}$$

$$(9) \int \log(1-x) dx$$

$$\begin{aligned} &= \int (x-1)' \log(1-x) dx \\ &= (x-1) \log(1-x) - \int (x-1) \frac{1}{1-x} dx \\ &= (x-1) \log(1-x) - \int -1 dx \\ &= (x-1) \log(1-x) + x + C \end{aligned}$$

02-02

$$(1) \int x \cos 2x dx$$

$$\begin{aligned} &= \int \frac{1}{2} x (\sin 2x)' dx \\ &= \frac{1}{2} x \sin 2x - \int \frac{1}{2} \sin 2x dx \\ &= \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + C \end{aligned}$$

$$(2) \int x \sin 3x dx$$

$$\begin{aligned} &= \int -\frac{1}{3} x (\cos 3x)' dx \\ &= -\frac{1}{3} x \cos 3x - \int -\frac{1}{3} \cos 3x dx \\ &= -\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x + C \\ &= \cos 3x \left(-\frac{1}{3}x + \frac{1}{9}\right) + C \end{aligned}$$

$$(3) \int \frac{2x}{\cos^2 x} dx$$

$$\begin{aligned} &= \int 2x (\tan x)' dx \\ &= 2x \tan x - \int 2 \tan x dx \\ &= 2x \tan x + 2 \log |\cos x| + C \end{aligned}$$

02-03

$$(1) \int x e^x dx$$

$$\begin{aligned} &= \int x (e^x)' dx \\ &= x e^x - \int e^x dx \\ &= x e^x - e^x + C \\ &= e^x (x-1) + C \end{aligned}$$

$$(2) \int x e^{-2x} dx$$

$$\begin{aligned} &= \int -\frac{1}{2} x (e^{-2x})' dx \\ &= -\frac{1}{2} x e^{-2x} - \int -\frac{1}{2} e^{-2x} dx \\ &= -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C \\ &= e^{-2x} \left(-\frac{1}{2}x - \frac{1}{4}\right) + C \end{aligned}$$

$$(3) \int (2x-1) e^{2x} dx$$

$$\begin{aligned} &= \int \frac{1}{2} (2x-1) (e^{2x})' dx \\ &= \frac{1}{2} (2x-1) e^{2x} - \int e^{2x} dx \\ &= \frac{1}{2} (2x-1) e^{2x} - \frac{1}{2} e^{2x} + C \\ &= e^{2x} \left(x - \frac{5}{2}\right) + C \end{aligned}$$

$$(4) \int x^2 \log x dx$$

$$\begin{aligned} &= \int \frac{1}{3} (x^3)' \log x dx \\ &= \frac{1}{3} x^3 \log x - \int \frac{1}{3} x^3 \frac{1}{x} dx \\ &= \frac{1}{3} x^3 \log x - \frac{1}{9} x^3 + C \end{aligned}$$

$$(5) \int (x-1)^3 \log(x-1) dx$$

$$\begin{aligned} &= \int \frac{1}{4} \{(x-1)^4\}' \log(x-1) dx \\ &= \frac{1}{4} (x-1)^4 \log(x-1) - \int \frac{1}{4} (x-1)^4 \frac{1}{x-1} dx \\ &= \frac{1}{4} (x-1)^4 \log(x-1) - \frac{1}{16} (x-1)^4 + C \end{aligned}$$

$$(6) \int \log 2x dx$$

$$\begin{aligned} &= \int \frac{1}{2} (2x)' \log 2x dx \\ &= x \log 2x - \int \frac{1}{2} 2x \frac{1}{2x} dx \\ &= x \log 2x - \frac{1}{2} x + C \end{aligned}$$

$$(7) \int \log(3-x) dx$$

$$\begin{aligned} &= \int (x-3)' \log(3-x) dx \\ &= (x-3) \log(3-x) - \int (x-3) \frac{1}{3-x} dx \\ &= (x-3) \log(3-x) + x + C \end{aligned}$$

02-04

$$(1) \int x^2 \sin x dx$$

$$\begin{aligned} &= \int x^2 (-\cos x)' dx \\ &= -x^2 \cos x - \int -2x \cos x dx \\ &= -x^2 \cos x - \int -2x (\sin x)' dx \\ &= -x^2 \cos x - (-2x \sin x - \int -2 \sin x dx) \\ &= -x^2 \cos x + 2x \sin x - 2 \sin x + C \end{aligned}$$

$$(2) \int x^2 e^{-x} dx$$

$$\begin{aligned} &= \int -x^2 (e^{-x})' dx \\ &= -x^2 e^{-x} - \int -2x e^{-x} dx \\ &= -x^2 e^{-x} - \int 2x e^{-x} dx \\ &= -x^2 e^{-x} - (2x e^{-x} - \int 2e^{-x} dx) \\ &= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C \end{aligned}$$