

# 解析学 05 定積分

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2025/05/13

5-1

$$(1) f(x) = \int_1^{x-3} (x-3-p) \log p dp$$

$$\begin{aligned} f(x) &= \int_1^{x-3} (x-3-p) \log p dp \\ &= \int_1^{x-3} (x-3) \log p - p \log p dp \\ &= (x-3) \int_1^{x-3} \log p dp - \int_1^{x-3} p \log p dp \end{aligned}$$

$$\begin{aligned} \frac{df(x)}{dx} &= [p \log p - p]_1^{x-3} \\ &= (x-3) \log(x-3) - x + 4 \end{aligned}$$

$$(2) \int_1^4 \sqrt{x} dx$$

$$\begin{aligned} &= \left[ \frac{2}{3} x^{\frac{3}{2}} \right]_1^4 \\ &= \frac{2}{3} 4\sqrt{4} - \frac{2}{3} \\ &= \frac{14}{3} \end{aligned}$$

$$(3) \int_1^4 x^{-\frac{3}{2}} dx$$

$$\begin{aligned} &= [-2x^{-\frac{1}{2}}]_1^4 \\ &= -2 \frac{1}{\sqrt{4}} - 2 \\ &= -3 \end{aligned}$$

$$(2) g(x) = - \int_0^{x^2} (x^2 - s) e^s ds$$

$$\begin{aligned} g(x) &= -x^2 \int_0^{x^2} e^s ds + \int_0^{x^2} s e^s ds \\ \frac{dg(x)}{dx} &= 2x \left( - \int_0^{x^2} e^s ds - x^2 e^{x^2} + x^2 e^{x^2} \right) \\ &= 2x(1 - e^{x^2}) \end{aligned}$$

$$(4) \int_e^{e^2} \frac{dx}{x}$$

$$\begin{aligned} &= [\log |x|]_e^{e^2} \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

$$(5) \int_2^{e+1} \frac{dy}{1-y}$$

$$\begin{aligned} &= [-\log |1-y|]_2^{e+1} \\ &= -1 + 0 \\ &= -1 \end{aligned}$$

5-2

$$(1) \int_1^3 4x^3 dx$$

$$\begin{aligned} &= [x^4]_1^3 \\ &= 3^4 - 1^4 \\ &= 80 \end{aligned}$$

$$(6) \int_0^\pi \sin \theta d\theta$$

$$\begin{aligned} &= [-\cos \theta]_0^\pi \\ &= -0 - 1 \\ &= 1 \end{aligned}$$

$$(7) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t dt$$

$$\begin{aligned} &= 2 \int_0^1 \cos t dt \\ &= 2[\sin t]_0^1 \\ &= 2 \end{aligned}$$

$$(8) \int_0^{\log 2} e^{3x} dx$$

$$\begin{aligned} &= \left[\frac{1}{3}e^{3x}\right]_0^{\log 2} \\ &= \frac{1}{3}e^{3\log 2} - \frac{1}{3} \\ &= \frac{7}{3} \end{aligned}$$

$$(9) \int_1^2 2^x dx$$

$$\begin{aligned} &= \left[\frac{2^x}{\log 2}\right]_1^2 \\ &= \frac{2}{\log 2} \end{aligned}$$

05-03

$$(1) \int_0^{\frac{\pi}{2}} (\sin 2x + \cos 3x) dx$$

$$\begin{aligned} &= \left[-\frac{1}{2}\cos 2x + \frac{1}{3}\sin 3x\right]_0^{\frac{\pi}{2}} \\ &= \left(\frac{1}{2} - \frac{1}{3}\right) - \left(-\frac{1}{2} + 0\right) \\ &= \frac{2}{3} \end{aligned}$$

$$(2) \int_{\pi} t^3 \pi \cos\left(\frac{x}{4} - \frac{\pi}{4}\right) dx$$

$$\begin{aligned} &= \left[4\sin\left(\frac{x}{4} - \frac{\pi}{4}\right)\right]_0^{\frac{\pi}{2}} \\ &= 4 - 0 \\ &= 4 \end{aligned}$$

$$(3) \int_1^2 \sin\left(\frac{2}{3}\pi t + \frac{\pi}{4}\right) dt$$

$$\begin{aligned} &= \left[-\frac{3}{2\pi} \cos\left(\frac{2}{3}\pi t + \frac{\pi}{4}\right)\right]_1^2 \\ &= -\frac{3}{2\pi} \left(\cos \frac{19}{12}\pi - \cos \frac{11}{12}\pi\right) \end{aligned}$$

$$(4) \int_0^{\frac{\pi}{4}} \cos^2 x dx$$

$$\begin{aligned} &= \int_0^{\frac{\pi}{4}} \frac{1}{2}(1 + \cos 2x) dx \\ &= \frac{1}{2} \left[x + \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}} \\ &= \frac{\pi + \sqrt{2}}{8} \end{aligned}$$

$$(5) \int_0^{\frac{\pi}{8}} \sin^2 2x dx$$

$$\begin{aligned} &= \int_0^{\frac{\pi}{8}} \frac{1}{2}(1 - \cos 4x) dx \\ &= \frac{1}{2} \left[x - \frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{8}} \\ &= \frac{\pi - \sqrt{2}}{16} \end{aligned}$$

$$(6) \int_0^{\frac{\pi}{4}} \tan^2 x dx$$

$$\begin{aligned} &= \int_0^{\frac{\pi}{4}} \left(\frac{1}{\cos^2 x} - 1\right) dx \\ &= [\tan x - x]_0^{\frac{\pi}{4}} \\ &= 1 - \frac{\pi}{4} \end{aligned}$$

05-04

$$(1) \int_0^2 x(x^2 + 1)^3 dx$$

$$\begin{aligned} t &= x^2 + 1 \\ dt &= 2x dx \\ I &= \frac{1}{2} \int_1^5 t^3 dt \\ &= \frac{624}{8} \end{aligned}$$

$$(2) \int_1^2 \frac{x^2 - 2x}{x^3 - 3x^2 + 1} dx$$

$$\begin{aligned} &= \left[\frac{1}{3} \log |x^3 - 3x^2 + 1|\right]_1^2 \\ &= \frac{1}{3} \log 3 \end{aligned}$$