The microscopic phonon properties we have seen in the previous sections are　located in specific paths or planes in the Brillouin zone. In order to more　rigorously inspect the LTCs, we examine phonon properties taken over the Brillouin zone. As such properties, in Fig.5 are shown　phonon densities of states (DOS), cumulative thermal conductivities and their　frequency derivatives, weighted DOS with the squares of the group velocity components (vλ,*x* and vλ,*z*), and finally,　frequency distributions of linewidths.

Firstly, we relate DOS (Fig.5-a) with the cumulative thermal conductivity ***κ****c* (Fig.5-b). The first DOS peaks indicated by arrows are related to the flattening of the acoustic branches at the Brillouin zone boundaries. As ***κ****c* increasing up to around 6, 12 and 10 THz for the α, β and γ phases, the phonons contributing to the LTCs are mainly located on the frequencies below the first peaks for the α and β phases, indicating that the main heat carriers are the phonons on the acoustic branches. In contrast, almost a half of the contributions to the LTCin β-Si3N4 are derived from the phonons above the first peak, indicating that the low frequency optical phonons should contribute to this component significantly. Therefore *κ*c in the γ phase resembles closely to *κxx*c in the β phase, but the respective phonon properties are much different. The weighted DOS in the γ phase exceeds those with vλ,*z*2 in the β phase, while the linewidths are, in the frequency range responsible for the LTC, twice larger.

In Figs.5-b and c, the directional differences in the derivatives of *κ*c*ii* in the α and β phases are qualitatively well consistent with the directional differences in the weighted DOS. The relatively larger intensities in the weighted DOS of vλ,*z*2 in β-Si3N4, critically causes the large anisotropy in its LTCs.

Fig.5-d shows significantly similar linewidth distributions between the α and β phases, which let the group velocities alone play the critical role on the different degrees of the anisotropy in the LTCs. Since it is curious that the linewidths are similar between these phases although their group velocities have marked differences, we investigate this similarity further. Recently Togo *et al*. have shown that the frequency profiles of the imaginary part of self-energy Γλ(ω), where　Γλ(ωλ)= Γλ, are characterized by the three phonon selection rules.[ phono3py]. More previously, Lindsay et al. have pointed out the significant correlation between LTC and phase space available for the three-phonon scattering according to the selection rule. In Eq.(1) of the linewidths, Φ-λλ’λ’’, the Fourier-transformed third order IFC, partly contains the selection rules (momentum conservation)[phono3py]. In the present study, as a rigorous inspection, we examine the impacts on the linewidths of Φ-λλ’λ’’ and the whole selection rules (momentum and energy conservation), one by one.