MCDP formulation and implementation based on multiplicative decomposition of deformation gradient tensor

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1 MCDP

The plastic potential function is expressed as (Shunchuan et al., 2017)

Mohr-Coulomb:

$$f(\sigma)^{MC} = \sqrt{J_2^{\sigma}} + \frac{(B_I - 1)I_1^{\sigma} - 3f_c}{3(B_I + 1)\cos\theta^{\sigma} + \sqrt{3}(B_I - 1)\sin\theta^{\sigma}} \tag{1}$$

Drucker-Pragger:

$$f(\sigma^{DP}) = \sqrt{J_2^{\sigma}} + \frac{(B_I - 1)}{\sqrt{3}B_I + 2\sqrt{3}}I_1^{\sigma} - \frac{\sqrt{3}}{B_I + 2}f_c \tag{2}$$

$$B_I = \frac{1 + \sin\theta}{1 - \sin\theta}f_c = \frac{2c\cos\theta}{1 - \sin\theta} \tag{3}$$

The plastic potential should be described in terms of intermediate configuration. Therefore, above equations are written as below.

Mohr-Coulomb:

$$f(\bar{\mathbf{M}})^{MC} = \sqrt{J_2^{\bar{M}}} + \frac{(B_I - 1)I_1^{\bar{M}} - 3f_c}{3(B_I + 1)\cos\theta^{\bar{M}} + \sqrt{3}(B_I - 1)\sin\theta^{\bar{M}}} \tag{4}$$

Drucker-Pragger:

$$f(\bar{\mathbf{M}}^{DP}) = \sqrt{J_2^{\bar{M}}} + \frac{(B_I - 1)}{\sqrt{3}B_I + 2\sqrt{3}}I_1^{\bar{M}} - \frac{\sqrt{3}}{B_I + 2}f_c \tag{5}$$

$$B_I = \frac{1 + \sin\theta}{1 - \sin\theta}f_c = \frac{2c\cos\theta}{1 - \sin\theta} \tag{6}$$

Here, $\bar{M}$ is the Mandel stress tensor.

The invariant of the yield functions should also be shown as seen in

$$\bar{M}_{\bar{I}\bar{J}} = \bar{C}^e_{\bar{I}\bar{K}}\bar{S}_{\bar{K}\bar{J}} = F^e_{\bar{I}i}F^T_{i\bar{K}}F^{e-1}_{\bar{K}k}\sigma_{kj}F^{e-1}_{j\bar{J}} = JF^e_{\bar{I}i}\sigma_{ij}F^{e-1}_{j\bar{J}} \tag{7}$$

$$I_1^{\bar{M}} = \bar{M}_{\bar{I}\bar{I}} = JI_1^{\sigma} \tag{8}$$

$$J_2^{\bar{M}} = J^2 J_2^{\sigma} \tag{9}$$

$$J_3^{\bar{M}} = J^3 J_3^{\sigma} \tag{10}$$

$$\theta^{\bar{M}} = \theta^{\sigma} \tag{11}$$

$$\frac{\partial f(\bar{\mathbf{M}})^{DP}}{\partial \bar{M}}) = \frac{1}{2}\left(J_2^{\bar{M}}\right)^{-\frac{1}{2}}\left(\bar{M}^{\frac{I_1^{\bar{\mathbf{M}}}}{3}}\bar{\mathbf{G}}\right) + \frac{B_I - 1}{\sqrt{3}B_I + 2\sqrt{3}}\bar{\mathbf{G}} \tag{12}$$