

博士課程学生 (Doctoral Course Students)  
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 (FMSP コース生)

#### A. 研究概要

##### Differential equations

#### 1. **Stand wave solutions of nonlinear Schrödinger-Poisson systems** [5]

This is a joint work with Hiroyuki Miyahara (UTokyo). We worked on stand wave solutions of the following nonhomogeneous nonlinear Schrödinger-Poisson systems:  $-\Delta u - a\phi|u|^{q-1}u = \lambda u + b|u|^{p-1}u, -\Delta\phi = |u|^{q+1}$ . Though for the case the dimension  $N = 3$  there has been many previous studies, our study covers the case  $N \geq 3$ . We proved existence and nonexistence theorem where each  $a, b$  is critical or subcritical. Especially, for some specific case, we decided the range of  $\lambda$  where a non-trivial solution  $(u, \phi)$  does exist or does not exist.

#### 2. **Generalized Joseph-Lundgren exponent** [1]

This is a joint work with Prof. Yasuhito Miyamoto (UTokyo). We worked on the following ordinal differential equation for  $r \in (0, \infty)$ :  $r^{-(\gamma-1)}(r^\alpha|u'|^{\beta-1}u')' + |u|^{p-1}u = 0$ . Here the left term represents a generalized radial differential operator that covers, for example, the  $N$ -dimensional usual Laplacian,  $m$ -Laplacian or  $k$ -Hessian. In previous research the generalized Joseph-Lundgren exponent for this operator was calculated, but there was a technical underbound for the exponent  $p$ . We removed that bound by transforming the equation and determined intersection numbers which role differently on  $p$ .

#### 3. **Nonhomogeneous semilinear elliptic equations involving critical Sobolev exponent** [2] [7]

I worked on the following nonhomogeneous semilinear elliptic equation involving the critical Sobolev exponent:  $-\Delta u + au = bu^p + \lambda f$ . I proved that provided  $b$  achieves

its maximum at an inner point of the domain and  $a$  has a growth of the exponent  $q$  in some neighborhood of that point, then if the dimension of the domain is less than  $6 + 2q$ , there exist at least two positive solutions. It seems to be new that the coefficient of a linear term affects the dimension of the domain on which solutions exist.

##### Mathematical informatics

#### 4. **Zero-dimensional fold and cut** [3] [4]

This is a joint work with Yasuhiko Asao (UTokyo), Prof. Erik D. Demaine (MIT), Prof. Martin L. Demaine (MIT), Hideaki Hosaka (Azabu high school), Prof. Akitoshi Kawamura (UTokyo) and Prof. Tomohiro Tachi (UTokyo). We showed how to fold a piece of paper and punch one hole on given  $n$  points so as to produce any desired patterns of holds. There is 4 variants of problems; the paper is finite or infinite and we allow or forbid the crease on the points. In [4], we give solutions for each case and the order of crease are bounded on the polynomial order of  $n$  and the paper ratio  $r$ . In the sequel paper [3], we also give a definition of the complexity of folds, which will be useful for further studies that determine NP-hardness of complex folding problems.

#### 5. **Application of SAT-solver for AI** [6]

It is known that  $n$ -satisfiability problems are NP-hard to solve for  $n \geq 3$  but are solved quickly by SAT-solver in recent years. I applied it for AI in the international programming contest "SamuraiAI Coding 2016-17", which is held by Information Processing Society of Japan. I made an algorithm on SAT-solver to decide the place of the hidden enemy logically by observing which place is conquered. It worked faster than a rudimentary algorithm by brute force.

##### Social mathematics in FMSP

#### 6. **Control model for traffic lights**

This is a joint work with Xinchu HUANG

(UTokyo). We worked on discrete model of traffic lights which does not cause traffic jams. A observation data shows each number of cars for the pair of inlet and outlet of roads but there is ambiguity of route of each cars. We let the problem come to  $n$ -varieties transportation problem but it is known as NP-complete. We also suggest an algorithm that superimpose usual max flow problems.

## B. 発表論文

### Refereed Papers

1. Yasuhito Miyamoto and Kazune Takahashi: “Generalized Joseph-Lundgren exponent and intersection properties for supercritical quasilinear elliptic equations”, *Archiv der Mathematik* **108** (2017) 71–83.
2. Kazune Takahashi: “Semilinear elliptic equations with critical Sobolev exponent and non-homogeneous term”, Master Thesis, The University of Tokyo (2015).
3. Yasuhiko Asao, Erik Demaine, Martin Demaine, Hideaki Hosaka, Akitoshi Kawamura, Tomohiro Tachi and Kazune Takahashi: “Folding and Punching Paper”, to appear in *Journal of Information Processing*.

### Refereed Conference Abstracts

4. Yasuhiko Asao, Erik Demaine, Martin Demaine, Hideaki Hosaka, Akitoshi Kawamura, Tomohiro Tachi and Kazune Takahashi: “Folding and Punching Paper”, Abstracts from the 19th Japan Conference on Discrete and Computational Geometry, Graphs and Games (2016) 40–41.

### Preprints

5. Hiroyuki Miyahara and Kazune Takahashi: “Existence and Nonexistence of Standing Wave Solutions of Nonlinear Schrödinger-Poisson System”, preprint.

### Miscs

6. Kazune Takahashi: “Application of SAT-solver for AI on SamurAI Coding 2016–17”, (2017), <https://github.com/kazunetakahashi-thesis/SAT-solver-AI-project>.
7. Kazune Takahashi: “Semilinear elliptic equations with critical Sobolev exponent and non-homogeneous term”, to appear in *RIMS Kôkyûroku*.

## C. 口頭発表

### International Conferences

1. Semilinear elliptic equations with critical Sobolev exponent and non-homogeneous term, RIMS Workshop: Shapes and other properties of solutions of PDEs, RIMS, Kyoto University, Japan, Nov 2015. [Invited]
2. (With Yasuhiko Asao, Erik Demaine, Martin Demaine, Hideaki Hosaka, Akitoshi Kawamura and Tomohiro Tachi) Folding and Punching Paper, The 19th Japan Conference on Discrete and Computational Geometry, Graphs, and Games, Tokyo University of Science, Japan, Sep 2016.

### Domestic Conferences

3. Existence and Nonexistence of Standing Wave Solutions of Nonlinear Schrödinger-Poisson System, The 39th Differential Equation Seminar at Yokohama National University, Yokohama National University, Japan, Aug 2016. [Invited]

## D. 講義

### Teaching Assistant

1. Computational Mathematics I (Prof. Shingo Ichii): Through exercise students learn technique and knowledge in computers and network which is needed to study mathematical sciences. I made new teaching materials for the script language Ruby. I updated the original fast-moved attendance management system by Ruby

on Rails. (For third-year students in School of Sciences)

2. Computational Mathematics II (Prof. Shingo Ichii): Students work for their own purpose in computers and network. I helped a third-year student to learn Ruby and to develop an introductory network application. (For third-year students in School of Sciences)

#### G. 受賞

##### **International Programming Contests**

1. SamurAI Coding 2014–15, World Final: 6th place, 77th Information Processing Society of Japan National Convention, Kyoto University, Japan, Mar 2015.

##### **Domestic Programming Contests**

2. Code Runner 2015, Final Round: 1st place, Recruit Career, Tokyo, Dec 2015.
3. Code Runner 2014, Final Round: 7th place, Recruit Career, Tokyo, Nov 2014.
4. Code Festival 2014 AI Challenge, Final Round: 3rd place, Recruit Holdings, Tokyo, Nov 2014.