博士課程学生 (Doctoral Course Students)

高橋　和音　(TAKAHASHI　Kazune)
(FMSP コース生)

A. 研究概要

1. **Homogeneous Hénon type elliptic equations with critical Sobolev growth [1] [2]**
   I worked on the following homogeneous Hénon type elliptic equations with critical Sobolev growth: $-\Delta u = \lambda \Psi u + |x|^{\alpha} u^{2^*-1}$, where Ψ is a given non-trivial bounded function which may vanish on the bounded boundary. I proved that there exists a positive solution if $\alpha > 0$ is sufficiently small for the dimension $N \geq 4$ by developing new families of test Talenti functions properly. It is known that there is another result for $N = 3$. Thus this seems to be the best result for this type of estimates.

2. **Kirchhoff-Hénon type equations with critical Sobolev growth [1] [8]**
   I applied these new families of functions above for the following Kirchhoff type equations with critical Sobolev growth: $-\left(a + b\|Du\|^{p-2}_{L^2(\Omega)}\right)\Delta u = \Psi u^{q-1} + |x|^{\alpha} u^{2^*-1}$. Kirchhoff-Hénon type equations had not been ever studied before my work. In addition, though the standard Kirchhoff-Hénon type equations are for the case $\alpha = 0$ and $p = 4$, my study covers the case $\alpha > 0$ and $2 < p < q < 2^*$ for the dimension $N \geq 3$. I introduced a new method to obtain estimates of the minimax energy where the fibering map cannot be explicitly solved, and proved the existence of a positive solution for small $\alpha \geq 0$.

3. **Stand wave solutions of nonlinear Schrödinger-Poisson systems [9]**
   This is a joint work with Hiroyuki Miyahara (UTokyo). We worked on stand wave solutions of the following nonhomogeneous nonlinear Schrödinger-Poisson systems: $-\Delta u - a\phi |u|^{q-1} u = \lambda u + b|u|^{p-1} u, -\Delta \phi = |u|^{q+1}$. There were many previous studies for the case the dimension $N = 3$, but our study covered the case $N \geq 3$. We proved existence and nonexistence theorem where each $p, q+1$ is critical or subcritical. Especially, for some specific case, we determined the range of λ where a non-trivial solution (u, ϕ) does exist or does not exist.

4. **Generalized Joseph-Lundgren exponent [4]**
   This is a joint work with Prof. Yasuhito Miyamoto (UTokyo). We worked on the following ordinary differential equation for $r \in (0, \infty)$: $r^{-(\gamma-1)}(r^{\alpha}|u'|^{\beta-1}u')' + |u|^{p-1}u = 0$. Here the left term represents a generalized radial differential operator that covers, for example, the N-dimensional usual Laplacian, m-Laplacian or k-Hessian. In previous research the generalized Joseph-Lundgren exponent for this operator was calculated, but there was a technical lowerbound for the exponent p. We removed that bound by transforming the equation and determined intersection numbers which role differently on p.

5. **Nonhomogeneous semilinear elliptic equations involving critical Sobolev exponent [5] [11]**
   I worked on the following nonhomogeneous semilinear elliptic equation involving the critical Sobolev exponent: $-\Delta u + au = bu^{2^*-1} + \lambda f$. I proved that provided b achieves its maximum at an inner point of the domain and a has a growth of the exponent q in some neighborhood of that point, then if the dimension of the domain is less than $6 + 2q$, there exist at least two positive solutions. It seems to be new that the coefficient of a linear term affects the dimension of the domain on which solutions exist.

6. **Zero-dimensional fold and cut [3] [6]**
   This is a joint work with Yasuhiko Asao (UTokyo), Prof. Erik D. Demaine (MIT), Prof. Martin L. Demaine (MIT), Hideaki Hosaka (Azabu high school), Prof. Akitoshi Kawamura (UTokyo) and Prof. Tomohiro Tachi (UTokyo). We showed how

to fold a piece of paper and punch one hole on given n points so as to produce any desired patterns of holds. There is 4 variants of problems; the paper is finite or infinite and we allow or forbid the crease on the points. In [6], we gave solutions for each case and the order of crease are bounded on the polynomial order of n and the paper ratio r. In the sequel paper [3], we also gave a definition of the complexity of folds, which will be useful for further studies that determine NP-hardness of complex folding problems.

7. **Application of SAT-solver for AI [10]**
It is known that n-SAT problems are NP-complete to solve for $n \geq 3$ but are solved quickly by SAT-solver in recent years. I applied it for AI in the international programming contest "SamurAI Coding 2016–17", which was held by Information Processing Society of Japan. I made an algorithm on SAT-solver to decide the all possible places of the hidden enemy logically by observing which places were conquered. It worked faster than a rudimentary algorithm by brute force.

8. **Control model for traffic lights [7]**
This is a joint work with Xinchi Huang (UTokyo). We worked on discrete model of traffic lights which would not cause traffic jams. An observation data showed each number of cars for the pair of inlet and outlet of roads but there was ambiguity of the route of each cars. We let the problem come to n-varieties transportation problem but it is known as NP-complete. Therefore we also suggested an algorithm that superimpose usual max flow problems.

B. 発表論文

Refereed Papers

1. Kazune Takahashi: "Hénon type elliptic equations with critical Sobolev growth", Doctor Dissertation, The University of Tokyo (2019). *Based on [2] and [8].*

2. Kazune Takahashi: "Positive solution for an Hénon type equation with critical Sobolev growth", Electronic Journal of Differential Equations **2018(194)** (2018) 1–17.

3. Yasuhiko Asao, Erik Demaine, Martin Demaine, Hideaki Hosaka, Akitoshi Kawamura, Tomohiro Tachi and Kazune Takahashi: "Folding and Punching Paper", Journal of Information Processing **25** (2017) 590–600.

4. Yasuhito Miyamoto and Kazune Takahashi: "Generalized Joseph-Lundgren exponent and intersection properties for supercritical quasilinear elliptic equations", Archiv der Mathematik **108(1)** (2017) 71–83.

5. Kazune Takahashi: "Semilinear elliptic equations with critical Sobolev exponent and non-homogeneous term", Master Thesis, The University of Tokyo (2015).

Refereed Conference Abstracts

6. Yasuhiko Asao, Erik Demaine, Martin Demaine, Hideaki Hosaka, Akitoshi Kawamura, Tomohiro Tachi and Kazune Takahashi: "Folding and Punching Paper", Abstracts from the 19th Japan Conference on Discrete and Computational Geometry, Graphs and Games (2016) 40–41.

Non-Refereed Papers

7. Xinchi Huang, Kazune Takahashi, Yasuhisa Kishi, Masahiko Kanai, and Takafumi Mase: "A modified model on a traffic network and signal optimization", Suurikagaku Jissenkenkyu Letter, LMSR **2018(4)** (2018) 1–5.

Preprints

8. Kazune Takahashi: "Positive solutions of Kirchhoff-Hénon type elliptic equations with critical Sobolev growth", submitted.

9. Hiroyuki Miyahara and Kazune Takahashi: "Existence and Nonexistence of Standing

Wave Solutions of Nonlinear Schrödinger-Poisson System", preprint.

Misc

10. Kazune Takahashi: "Application of SAT-solver for AI on SamurAI Coding 2016–17", Open report for Exercise of Mathematical Science IV, 2016 (2017). *In Japanese.*

11. Kazune Takahashi: "Semilinear elliptic equations with critical Sobolev exponent and non-homogeneous term",RIMS Kôkyûroku **2006** (2016) 1–11.

C. 口頭発表

International Conferences

1. (With Yasuhiko Asao, Erik Demaine, Martin Demaine, Hideaki Hosaka, Akitoshi Kawamura and Tomohiro Tachi) Folding and Punching Paper, The 19th Japan Conference on Discrete and Computational Geometry, Graphs, and Games, Tokyo University of Science, Japan, Sep 2016.

2. Semilinear elliptic equations with critical Sobolev exponent and non-homogeneous term, RIMS Workshop: Shapes and other properties of solutions of PDEs, RIMS, Kyoto University, Japan, Nov 2015. [Invited]

Domestic Conferences

3. Existence and Nonexistence of Standing Wave Solutions of Nonlinear Schrödinger-Poisson System, The 39th Differential Equation Seminar at Yokohama National University, Yokohama National University, Japan, Aug 2016. [Invited]

G. 受賞

Awards

1. (With Yasuhiko Asao, Erik Demaine, Martin Demaine, Hideaki Hosaka, Akitoshi Kawamura and Tomohiro Tachi) Folding and Punching Paper, Specially Selected Paper, Information Processing Society of Japan, Aug 2017.

International Programming Contests

2. SamurAI Coding 2014–15, World Final: 6th place, 77th Information Processing Society of Japan National Convention, Kyoto University, Japan, Mar 2015.

Domestic Programming Contests

3. Code Runner 2015, Final Round: 1st place, Recruit Career, Tokyo, Dec 2015.

4. Code Runner 2014, Final Round: 7th place, Recruit Career, Tokyo, Nov 2014.

5. Code Festival 2014 AI Challenge, Final Round: 3rd place, Recruit Holdings, Tokyo, Nov 2014.