モバイルWiMAXのための通信路推定

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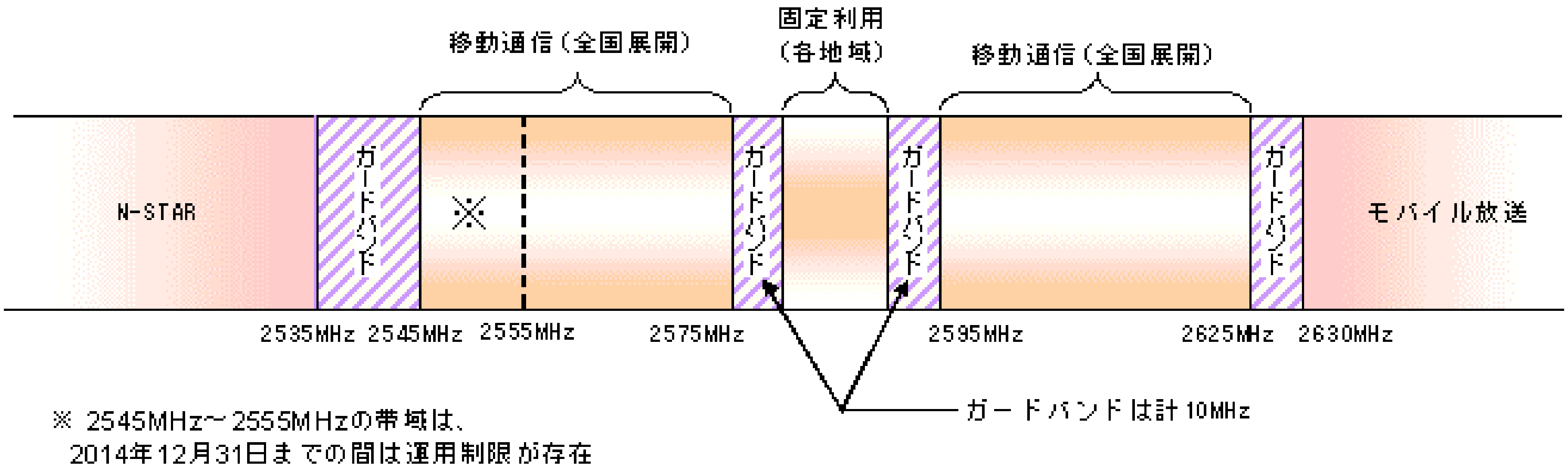
はじめに

-広帯域移動無線アクセスシステム@2.5GHz帯



-IEEE802.16e
-**モバイル**WiMAX (worldwide interoperability for microwave access)
-OFDMA (Orthogonal frequency division multiple access)

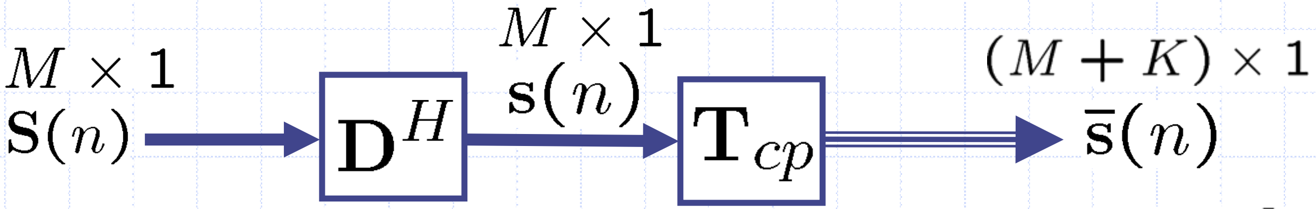
図:総務省報道資料より

AGENDA

- Block transmission using cyclic prefix
  (OFDM: orthogonal frequency division multiplexing)
- Orthogonal frequency division multiple access (OFDMA)
  (frame structure of IEEE 802.16 TDD and WiMAX)
- Uplink channel estimation scheme for OFDMA
- Downlink channel estimation scheme for OFDMA

Block transmission using cyclic prefix
(orthogonal frequency division multiplexing)

Transmitted signal block (vector)



$$\mathbf{S}(n) = \begin{bmatrix} S_0(n) \\ \vdots \\ S_{M-1}(n) \end{bmatrix}$$

IDFT

cyclic prefix (CP) insertion

$$\bar{\mathbf{s}}(n) = \begin{bmatrix} \bar{s}_0(n) \\ \vdots \\ \bar{s}_{M+K-1}(n) \end{bmatrix}$$

- transmitted signal block： $\bar{\mathbf{s}}(n) = \mathbf{T}_{cp}\mathbf{D}^H\mathbf{S}(n)$

- DFT matrix：

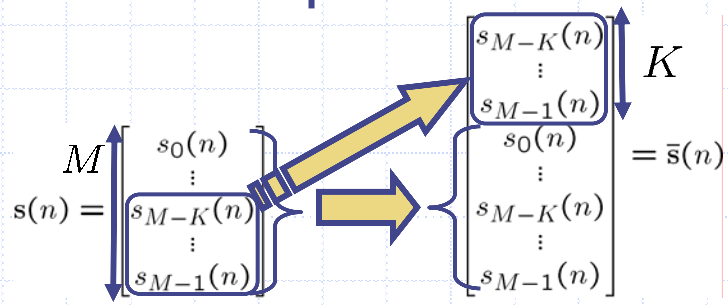
$$\mathbf{D} = \frac{1}{\sqrt{M}} \begin{bmatrix} e^{-j\frac{2\pi\times 0\times 0}{M}} & \dots & e^{-j\frac{2\pi\times 0\times (M-1)}{M}} \\ \vdots & & \vdots \\ e^{-j\frac{2\pi\times (M-1)\times 0}{M}} & \dots & e^{-j\frac{2\pi (M-1)\times (M-1)}{M}} \end{bmatrix}$$

$\mathbf{D}^{-1} = \mathbf{D}^H$

unitary

- CP insertion matrix：

$$\mathbf{T}_{cp} = \begin{bmatrix} \mathbf{0}_{K\times(M-K)} & \mathbf{I}_K \\ \multicolumn{2}{c}{\mathbf{I}_M} \end{bmatrix}$$



Received signal block @ time domain

$$\bar{\mathbf{r}}(n) = \begin{bmatrix} \bar{r}_0(n) \\ \vdots \\ \bar{r}_{M+K-1}(n) \end{bmatrix}$$

impulse response

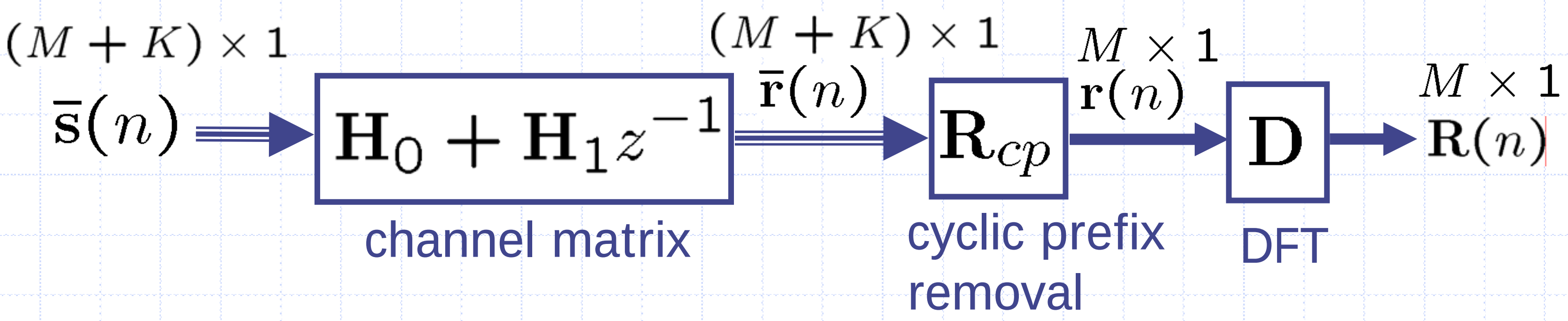
$$= \begin{bmatrix} 0 & \dots & h_L & \dots & h_0 & 0 & \dots & 0 \\ \vdots & & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & & \ddots & 0 \\ 0 & \dots & \dots & \dots & 0 & h_L & \dots & h_0 \end{bmatrix} \begin{bmatrix} \bar{s}_0(n-1) \\ \vdots \\ \bar{s}_{M+K-1}(n-1) \\ \bar{s}_0(n) \\ \vdots \\ \bar{s}_{M+K-1}(n) \end{bmatrix} \begin{matrix} \bar{\mathbf{s}}(n-1) \\ \\ \\ \bar{\mathbf{s}}(n) \end{matrix}$$

$$= \underbrace{\begin{bmatrix} 0 & \dots & h_L & \dots & h_1 \\ \vdots & & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & h_L \\ \vdots & & & & \vdots \\ 0 & \dots & \dots & \dots & 0 \end{bmatrix}}_{\mathbf{H}_1} \bar{\mathbf{s}}(n-1) + \underbrace{\begin{bmatrix} h_0 & & & \\ \vdots & \ddots & & \mathbf{0} \\ h_L & & \ddots & \\ & \ddots & & \ddots \\ \mathbf{0} & & h_L \quad \dots & h_0 \end{bmatrix}}_{\mathbf{H}_0} \bar{\mathbf{s}}(n)$$

$$= \mathbf{H}_1 \bar{\mathbf{s}}(n-1) + \mathbf{H}_0 \bar{\mathbf{s}}(n)$$

$$= \underline{(\mathbf{H}_0 + \mathbf{H}_1 z^{-1})}\bar{\mathbf{s}}(n)$$

Received signal block @ frequency domain



- Received signal block:

$$\mathbf{R}(n) = \mathbf{D}\mathbf{R}_{cp}\mathbf{H}_0\bar{\mathbf{s}}(n) + \mathbf{D}\boxed{\mathbf{R}_{cp}\mathbf{H}_1}\bar{\mathbf{s}}(n-1)$$

$$= \mathbf{D}\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}\mathbf{D}^H\mathbf{S}(n) \qquad \mathbf{0}_{M\times(M+K)}$$

- CP removal matrix:

$$\mathbf{R}_{cp} = \begin{bmatrix}\mathbf{0}_{M\times K} & \mathbf{I}_M\end{bmatrix}$$

$$\mathbf{R}_{cp}\mathbf{H}_1 = \begin{bmatrix}\mathbf{0}_{M\times K} & \mathbf{I}_M\end{bmatrix}\begin{bmatrix} 0 & \dots & h_L & \dots & h_1 \\ \vdots & & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & h_L \\ \vdots & & & & \vdots \\ 0 & \dots & \dots & \dots & 0 \end{bmatrix}$$

$$= \mathbf{0}_{M\times(M+K)}, \qquad (L \le K)$$

Equivalent channel model

-Channel including CP operations:

$$\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp} = \begin{bmatrix} h_0 & 0 & \cdots & 0 & h_L & \cdots & h_1 \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & h_L \\ h_L & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \cdots & 0 & h_L & \cdots & \cdots & h_0 \end{bmatrix}$$:circulant matrix

$$\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp} = \mathbf{D}^H\mathbf{\Lambda}\mathbf{D}$$

where $\mathbf{\Lambda} = \text{diag}[\lambda_0, \ldots, \lambda_{M-1}]$

frequency response: $\begin{bmatrix} \lambda_0 \\ \vdots \\ \lambda_{M-1} \end{bmatrix} = \mathbf{D} \begin{bmatrix} h_0 \\ \vdots \\ h_L \\ \mathbf{0}_{(M-L-1)\times 1} \end{bmatrix}$

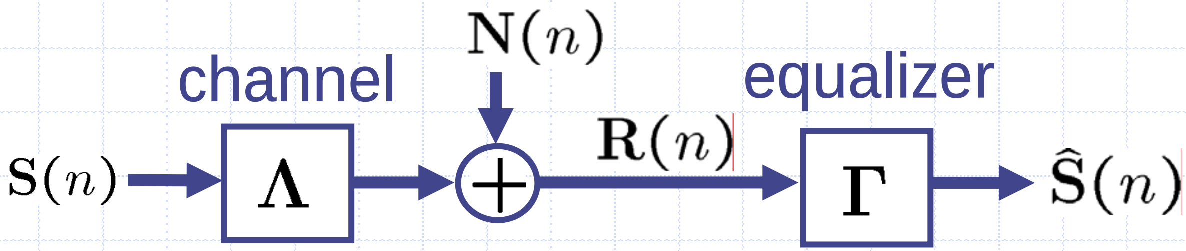
-Equivalent channel model:

$$\mathbf{S}(n) \to \boxed{\mathbf{D}^H} \to \boxed{\mathbf{D}} \to \boxed{\mathbf{\Lambda}} \to \boxed{\mathbf{D}^H} \to \boxed{\mathbf{D}} \to \mathbf{R}(n)$$

$$\mathbf{S}(n) \to \boxed{\mathbf{\Lambda}} \to \mathbf{R}(n)$$

$$\begin{aligned} \mathbf{R}(n) &= \mathbf{D}\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}\mathbf{D}^H\mathbf{S}(n) \\ &= \mathbf{\Lambda}\mathbf{S}(n) \end{aligned}$$

Discrete frequency domain equalization



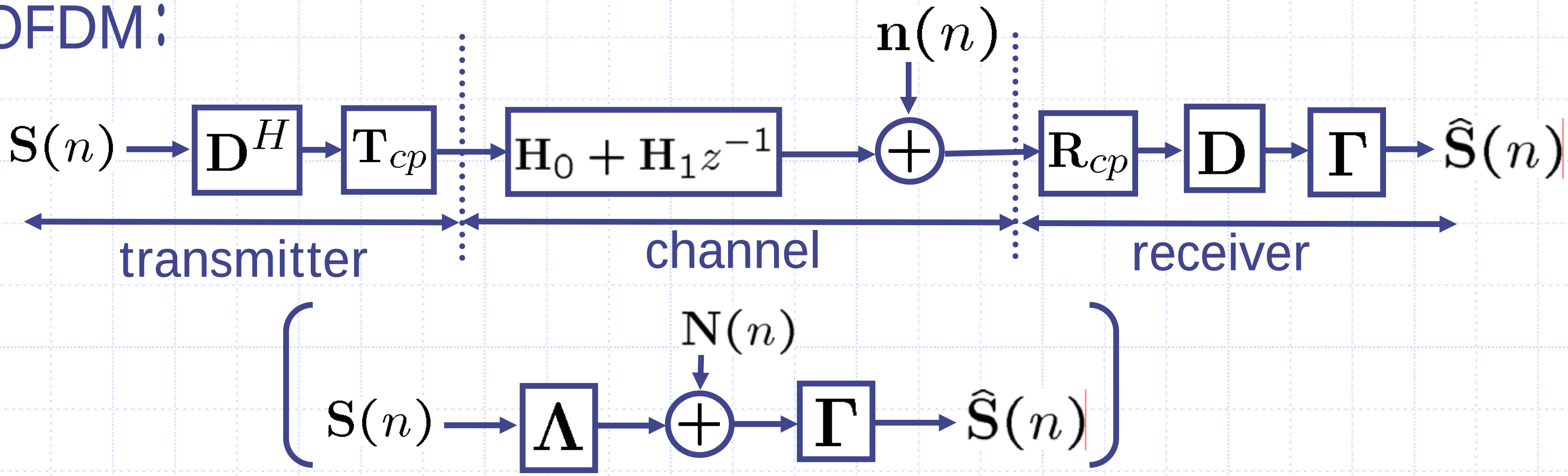
$\mathbf{\Lambda} = \mathrm{diag}[\lambda_0, \ldots, \lambda_{M-1}]$ $\quad$ $\mathbf{\Gamma} = \mathrm{diag}[\gamma_0, \ldots, \gamma_{M-1}]$

- zero-forcing (ZF) criterion: $\mathbf{\Gamma} = \mathrm{diag}\left[\frac{1}{\lambda_0}, \ldots, \frac{1}{\lambda_{M-1}}\right]$
- minimum mean-square-error (MMSE) criterion:

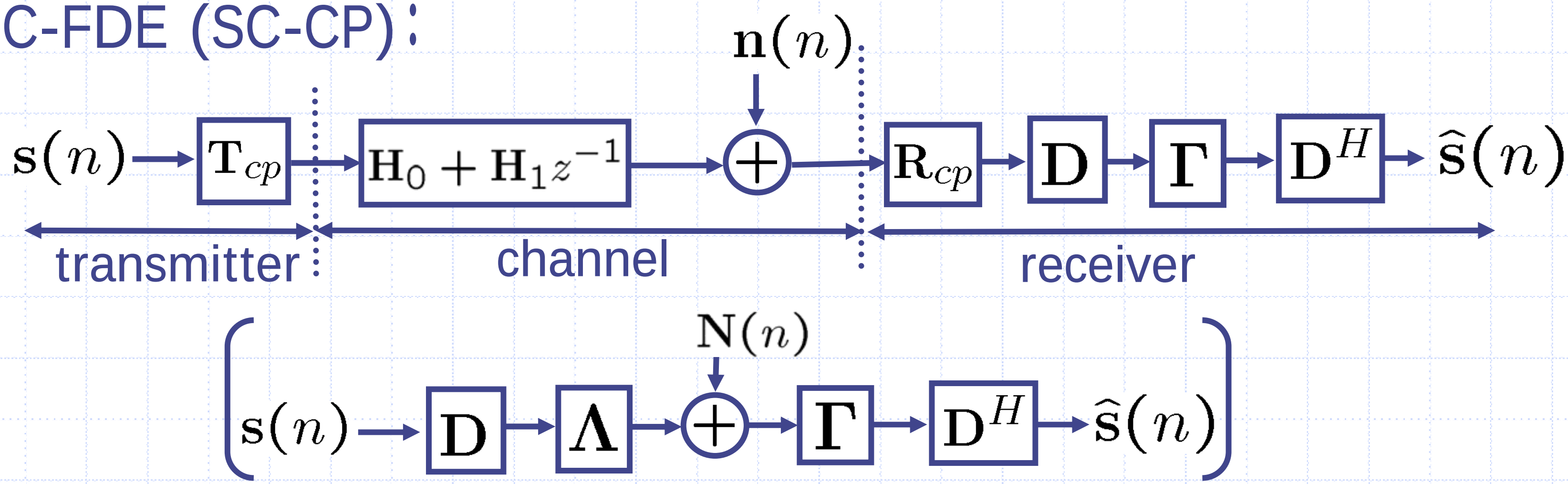
$$\mathbf{\Gamma} = \mathrm{diag}\left[\frac{\lambda_0^*}{|\lambda_0|^2 + \sigma_n^2/\sigma_s^2}, \ldots, \frac{\lambda_{M-1}^*}{|\lambda_{M-1}|^2 + \sigma_n^2/\sigma_s^2}\right]$$

OFDM and SC-FDE (SC-CP)

- OFDM：

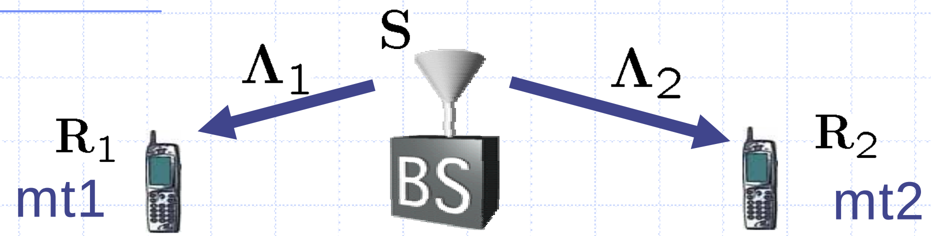


- SC-FDE (SC-CP)：



Orthogonal frequency division multiple access (OFDMA) (frame structure of IEEE 802.16 TDD)

OFDMA downlink transmission



$$\mathbf{\Lambda}_1 = \begin{bmatrix} \lambda_{0,1} & & \\ & \ddots & \\ & & \lambda_{M-1,1} \end{bmatrix}$$

$$\mathbf{\Lambda}_2 = \begin{bmatrix} \lambda_{0,2} & & \\ & \ddots & \\ & & \lambda_{M-1,2} \end{bmatrix}$$

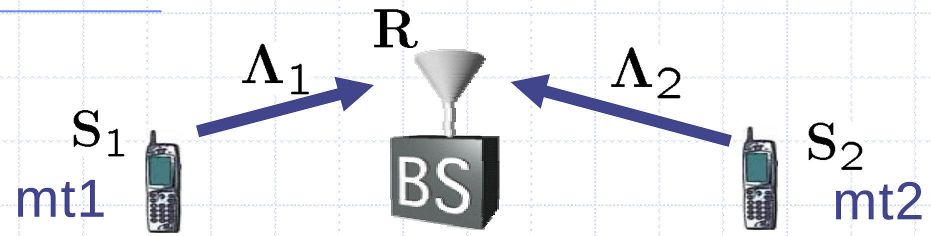
- transmitted signal @ BS:

$$\mathbf{S} = \begin{bmatrix} \boxed{\begin{matrix} s_0 \\ \vdots \\ s_{M/2-1} \end{matrix}} \\ \boxed{\begin{matrix} s_{M/2} \\ \vdots \\ s_{M-1} \end{matrix}} \end{bmatrix} \begin{matrix} \mathbf{S}_1 \\ \\ \mathbf{S}_2 \end{matrix}$$

- received signal @ mt1:

$$\begin{aligned} \mathbf{R}_1 &= \mathbf{\Lambda}_1 \mathbf{S} + \mathbf{N}_1 \\ &= \begin{bmatrix} \lambda_{0,1} & & \\ & \ddots & \\ & & \lambda_{M-1,1} \end{bmatrix} \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_2 \end{bmatrix} + \mathbf{N}_1 \end{aligned}$$

OFDMA uplink transmission



$$\mathbf{\Lambda}_1 = \begin{bmatrix} \lambda_{0,1} & & \\ & \ddots & \\ & & \lambda_{M-1,1} \end{bmatrix}$$

$$\mathbf{\Lambda}_2 = \begin{bmatrix} \lambda_{0,2} & & \\ & \ddots & \\ & & \lambda_{M-1,2} \end{bmatrix}$$

- transmitted signal @ mt1 and mt2:

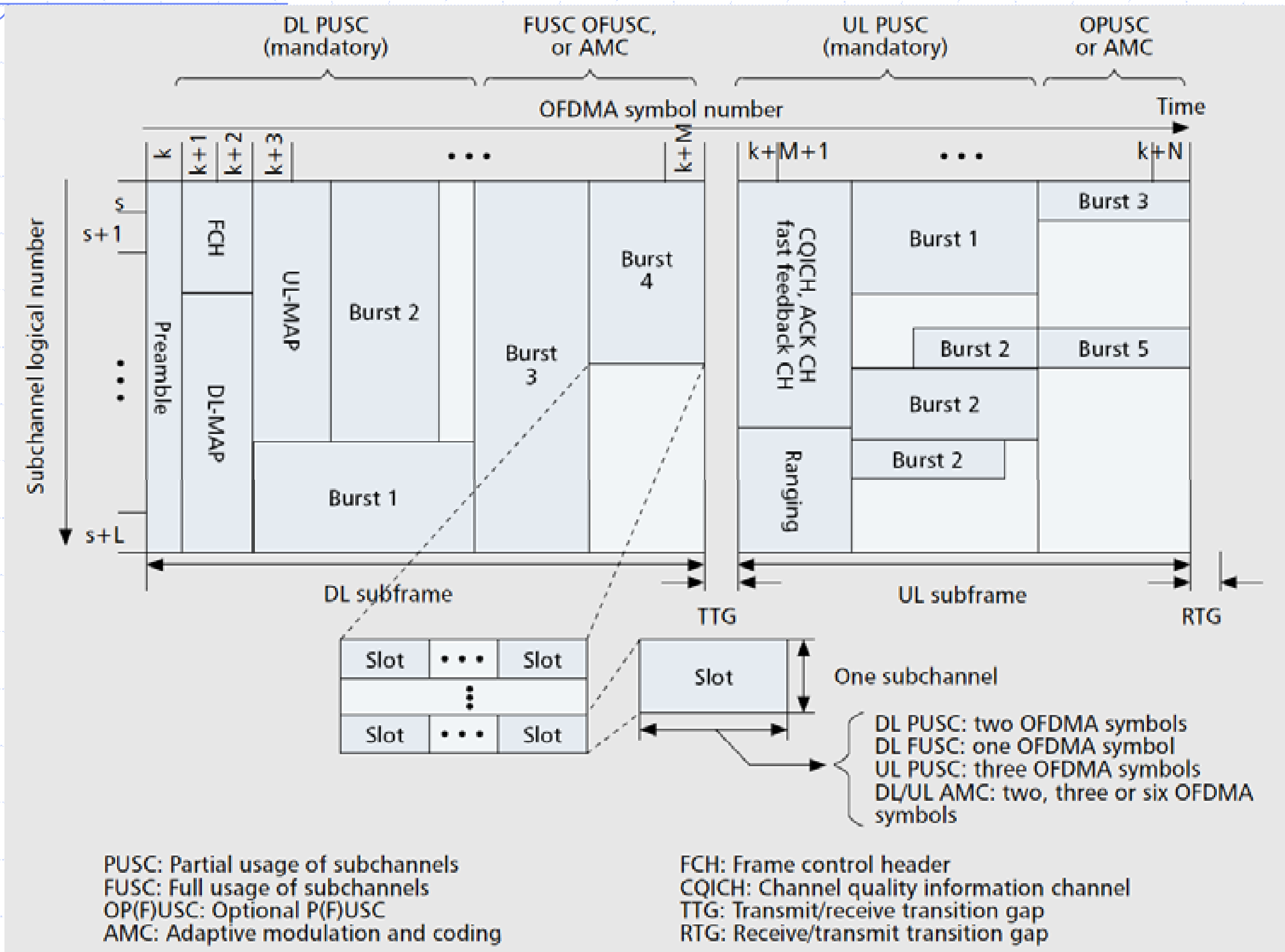
$$\mathbf{s}_1 = \begin{bmatrix} s_{0,1} \\ \vdots \\ s_{M/2-1,1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \mathbf{s}_2 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ s_{M/2,2} \\ \vdots \\ s_{M-1,2} \end{bmatrix}$$

- received signal @ BS:

$$\mathbf{R} = \mathbf{\Lambda}_1 \mathbf{S}_1 + \mathbf{\Lambda}_2 \mathbf{S}_2 + \mathbf{N}$$

$$= \begin{bmatrix} \lambda_{0,1} & & & & & \\ & \ddots & & & & \\ & & \lambda_{M/2-1,1} & & & \\ & & & \lambda_{M/2,2} & & \\ & & & & \ddots & \\ & & & & & \lambda_{M-1,2} \end{bmatrix} \begin{bmatrix} s_{0,1} \\ \vdots \\ s_{M/2-1,1} \\ s_{M/2,2} \\ \vdots \\ s_{M-1,2} \end{bmatrix} + \mathbf{N}$$

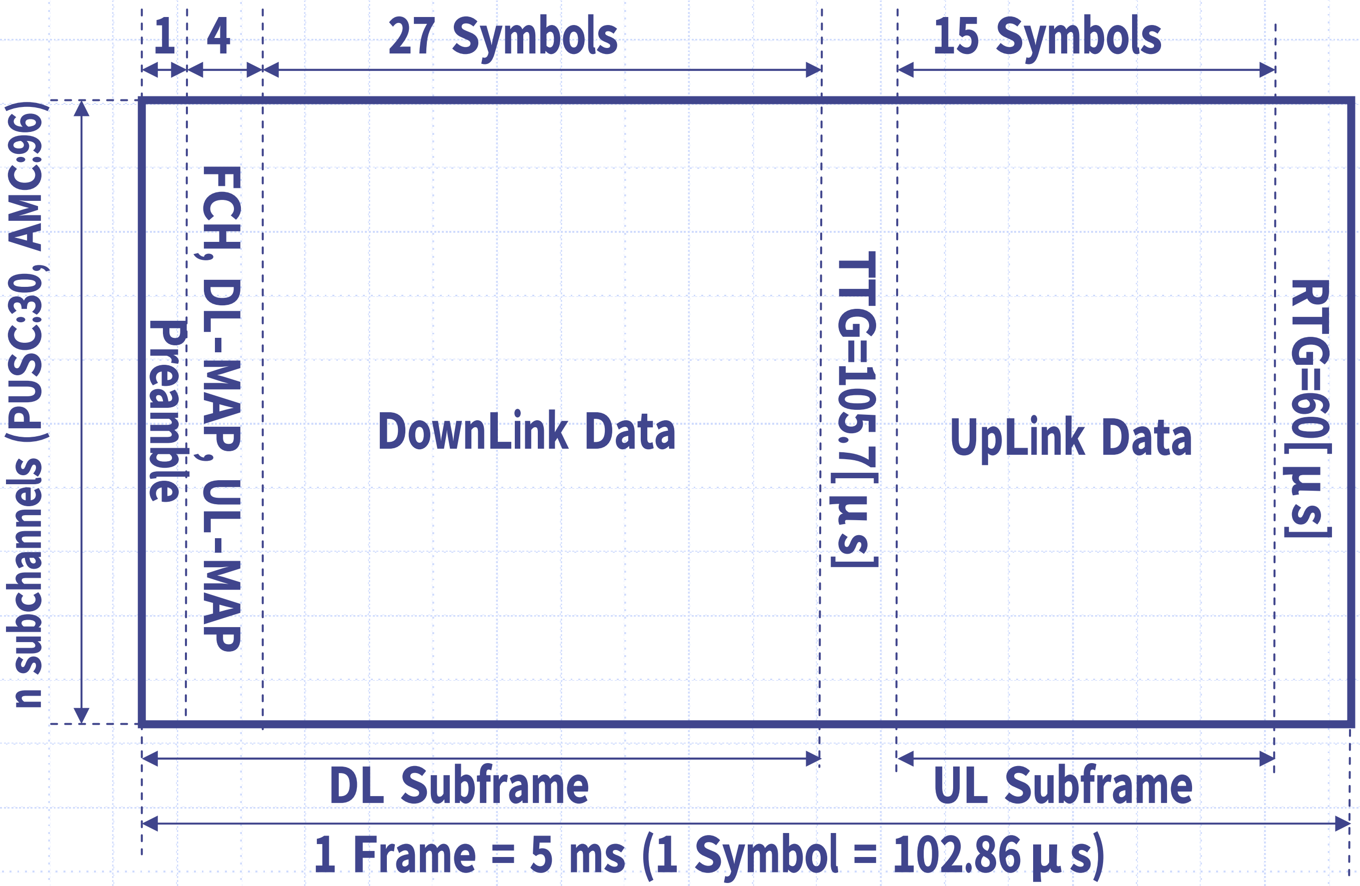
IEEE802.16TDD frame structure



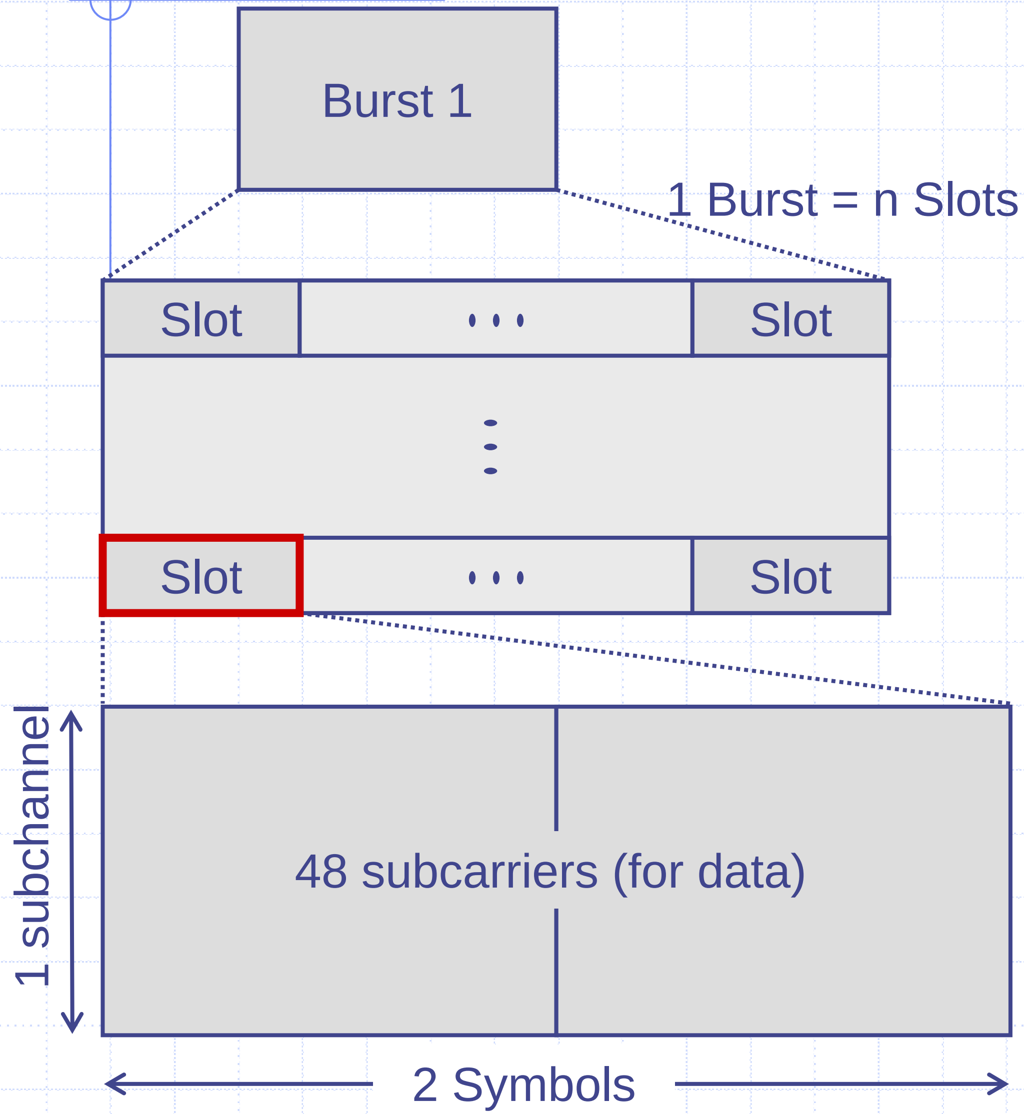
IEEE Commun. Mag. Dec. 2005

Example of WiMAX frame structure

WiMAX: worldwide interoperability for microwave access



Downlink PUSC (partial usage of subchannel)

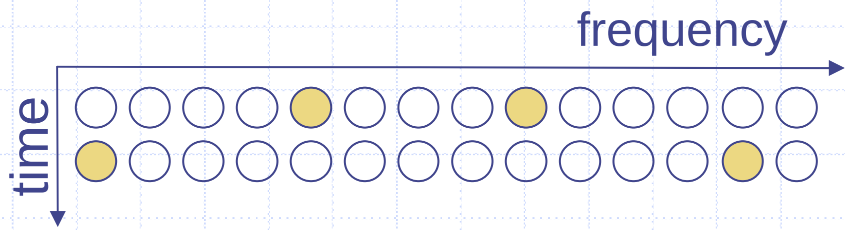


1 subchannel = 28 subcarriers
(24 for data, 4 for pilot)

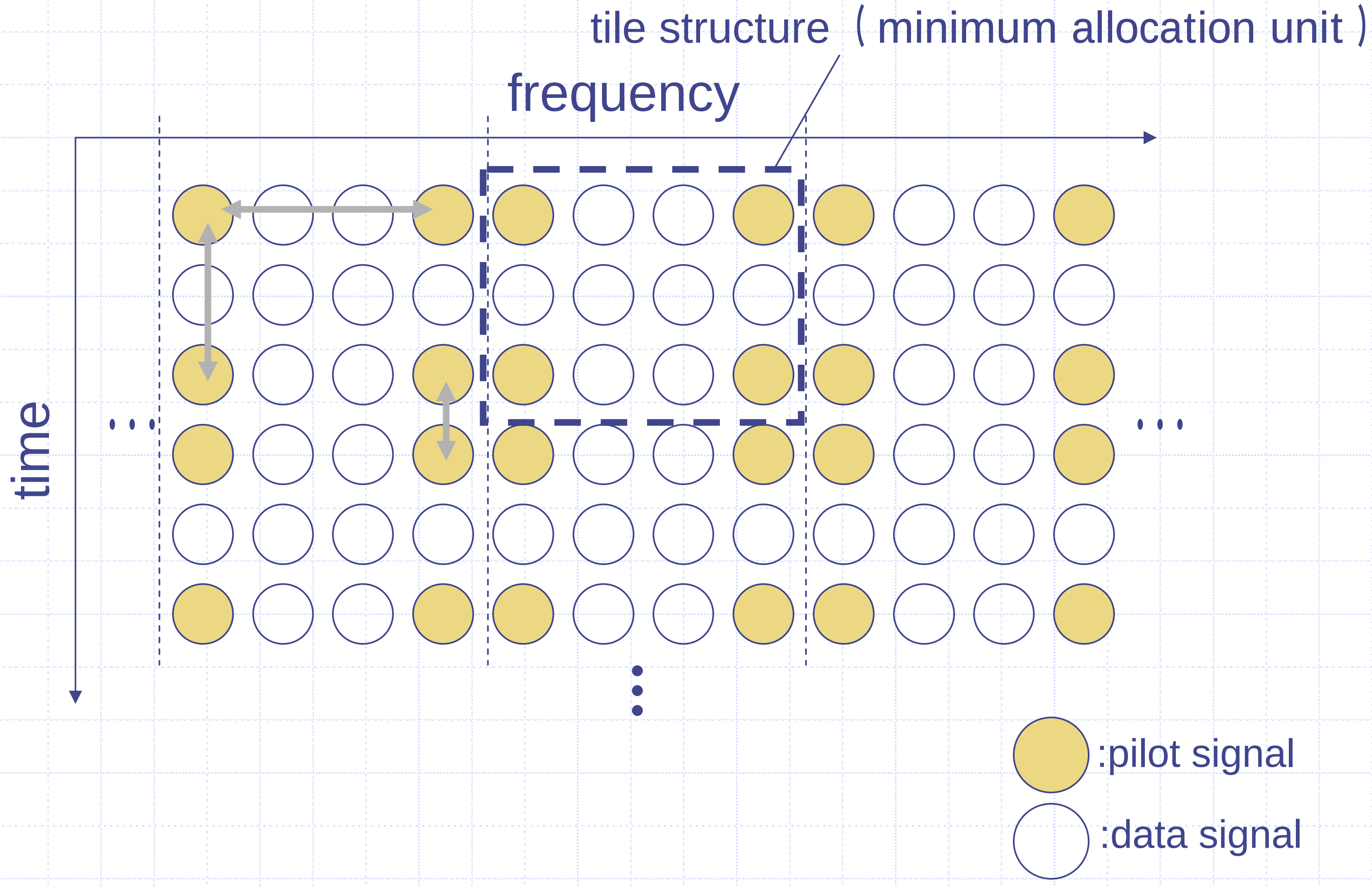
1 Symbol = 30 subchannels
= 840 subcarriers in total

■Slot : minimum allocation unit

1 Slot = 1 subchannel ✕ 2 Symbols
= 48 subcarriers (for data)
(ex. QPSK 1 Slot = 96bit)

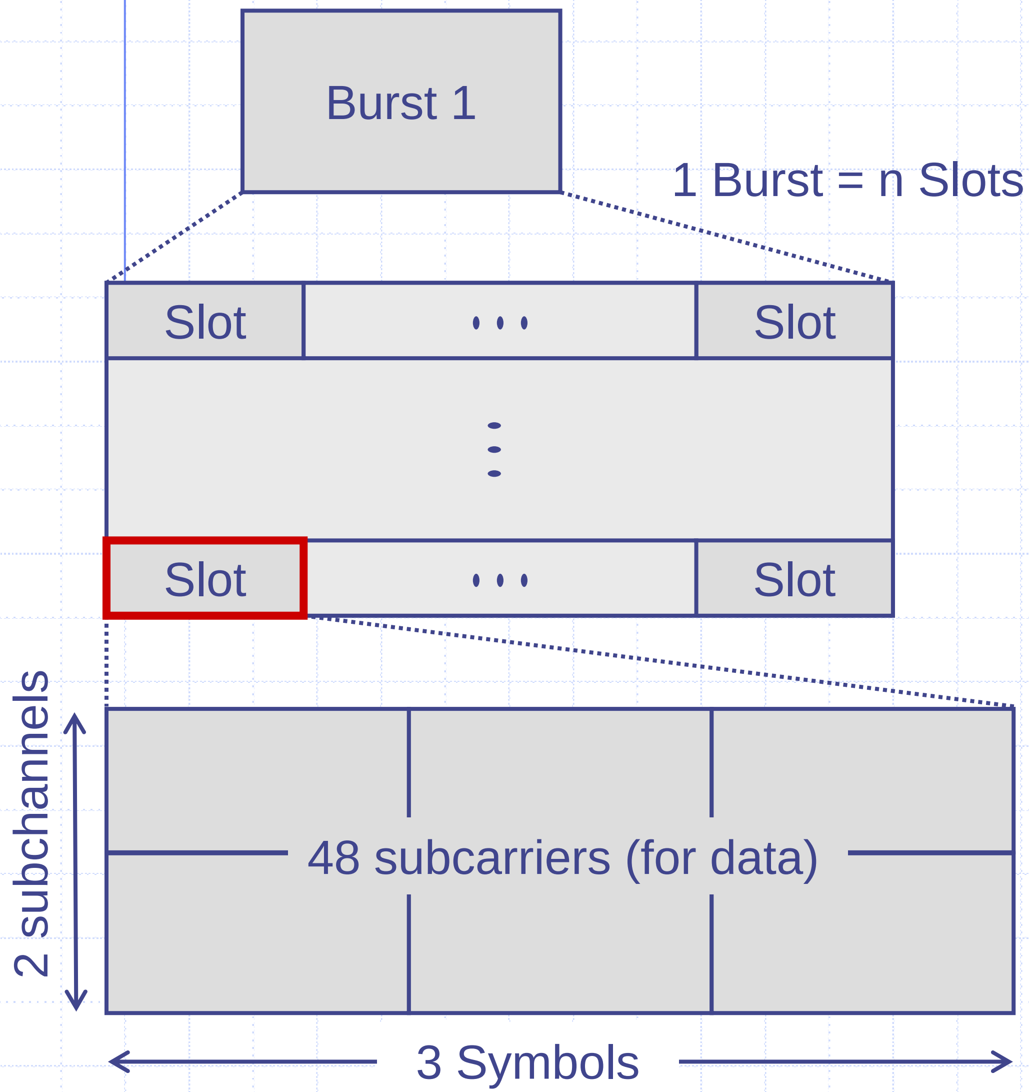


Uplink PUSC



tile structure（minimum allocation unit）
frequency
time
:pilot signal
:data signal

AMC (adaptive modulation and coding)

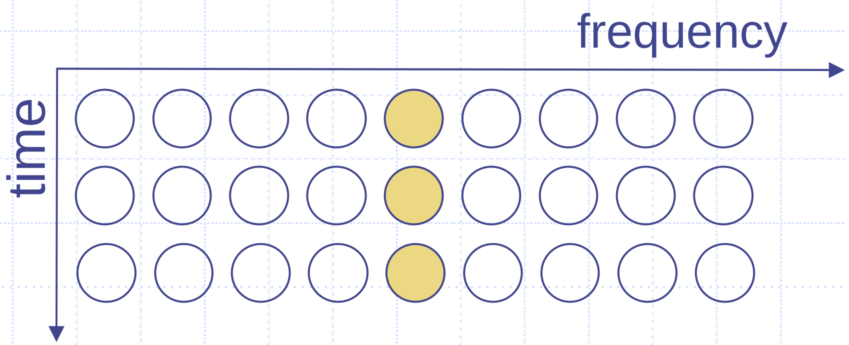


1 subchannel = 9 subcarriers
(8 for data, 1 for pilot)

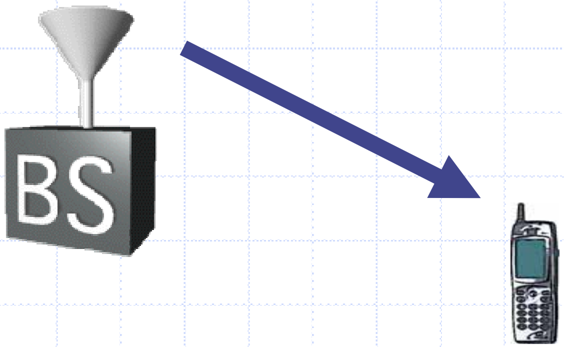
1 Symbol = 96 subchannels
= 864 subcarriers in total

■Slot : minimum allocation unit

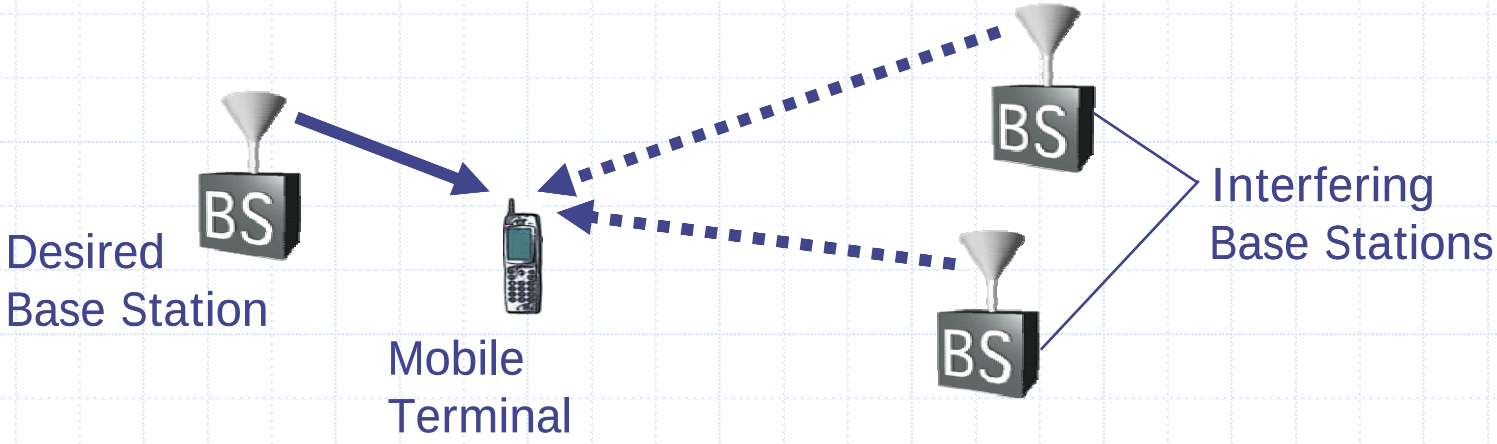
1 Slot = 2 subchannel ✕ 3 Symbols
= 48 subcarriers (for data)
(ex. QPSK 1 Slot = 96bit)



Downlink channel estimation scheme for OFDMA

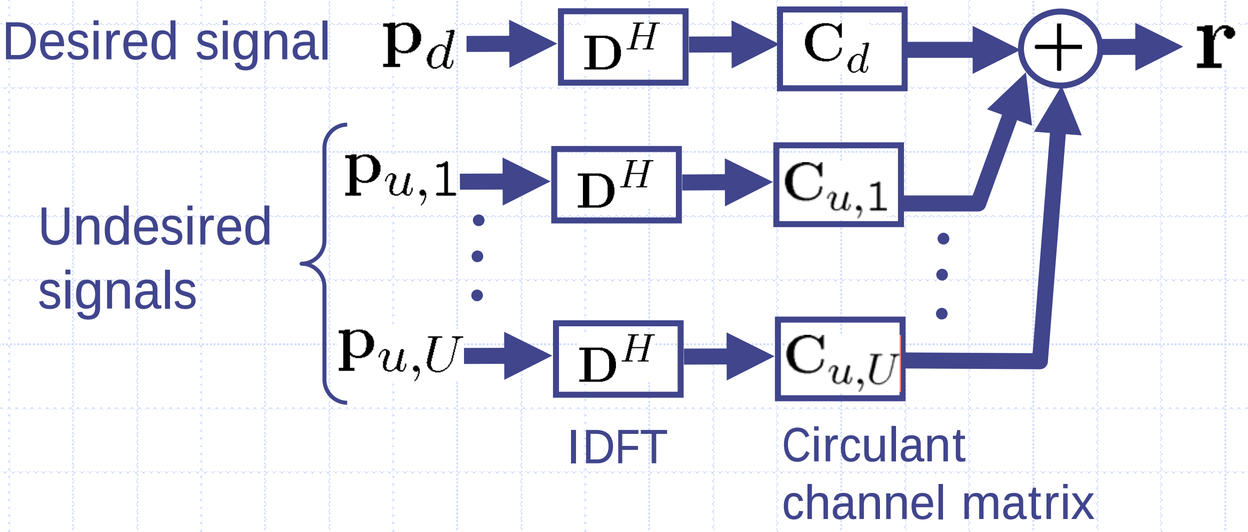


System model and assumptions



- There are one desired base station and U interfering base stations
- DFT window timings of all the base stations are synchronized
- All base stations use same carrier frequency
- The length of cyclic prefix K is greater than or equal to channel order L

Signal modeling



-Received Signal Block: $\mathbf{r} = \mathbf{C}_d \mathbf{D}^H \mathbf{p}_d + \sum_{i=1}^{U} \mathbf{C}_{u,i} \mathbf{D}^H \mathbf{p}_{u,i}$

$$\mathbf{C}_d = \begin{bmatrix} h_0^d & 0 & \dots & 0 & h_L^d & \dots & h_1^d \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & h_L^d \\ h_L^d & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \dots & 0 & h_L^d & \dots & \dots & h_0^d \end{bmatrix} \qquad \mathbf{C}_{u,i} = \begin{bmatrix} h_0^{u,i} & 0 & \dots & 0 & h_K^{u,i} & \dots & h_1^{u,i} \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & h_K^{u,i} \\ h_K^{u,i} & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \dots & 0 & h_K^{u,i} & \dots & \dots & h_0^{u,i} \end{bmatrix}$$

Signal modeling (cont'd)

$$\mathbf{r} = \mathbf{C}_d \mathbf{P}_d + \sum_{i=1}^{U} \mathbf{C}_{u,i} \mathbf{P}_{u,i}$$

$$\mathbf{P}_d = [P_0^d, \ldots, P_{M-1}^d]^T = \mathbf{D}^H \mathbf{p}_d,$$
$$\mathbf{P}_{u,i} = [P_0^{u,i}, \ldots, P_{M-1}^{u,i}]^T = \mathbf{D}^H \mathbf{p}_{u,i}, \quad (i = 1, \ldots, U)$$

-Property of circulant matrix:

$$\begin{bmatrix} a & c & b \\ b & a & c \\ c & b & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x & z & y \\ y & x & z \\ z & y & x \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\mathbf{r} = \mathbf{Q}_d^C \begin{bmatrix} \mathbf{h}_d \\ \mathbf{0}_{(M-K-1)\times 1} \end{bmatrix} + \sum_{i=1}^{U} \mathbf{Q}_{u,i}^C \begin{bmatrix} \mathbf{h}_{u,i} \\ \mathbf{0}_{(M-K-1)\times 1} \end{bmatrix}$$

$$\mathbf{Q}_d^C = \begin{bmatrix} P_0^d & P_{M-1}^d & \cdots & P_{M-2}^d \\ P_1^d & P_0^d & \ddots & \vdots \\ \vdots & \ddots & \ddots & P_{M-1}^d \\ P_{M-1}^d & \cdots & P_1^d & P_0^d \end{bmatrix}, \quad \mathbf{Q}_{u,i}^C = \begin{bmatrix} P_0^{u,i} & P_{M-1}^{u,i} & \cdots & P_1^{u,i} \\ P_1^{u,i} & P_0^{u,i} & \ddots & \vdots \\ \vdots & \ddots & \ddots & P_{M-1}^{u,i} \\ P_{M-1}^{u,i} & \cdots & P_1^{u,i} & P_0^{u,i} \end{bmatrix}$$

Channel estimation @ frequency domain

-Received Signal Block:

$$\mathbf{r} = \mathbf{C}_d \mathbf{D}^H \mathbf{p}_d + \sum_{i=1}^{U} \mathbf{C}_{u,i} \mathbf{D}^H \mathbf{p}_{u,i}$$

DFT(FFT)

$$\begin{aligned}
\mathbf{D}\mathbf{r} &= \mathbf{D}\mathbf{C}_d \mathbf{D}^H \mathbf{p}_d + \sum_{i=1}^{U} \mathbf{D}\mathbf{C}_{u,i} \mathbf{D}^H \mathbf{p}_{u,i} \\
&= \mathbf{D}\mathbf{D}^H \mathbf{\Lambda}_d \mathbf{D}\mathbf{D}^H \mathbf{p}_d + \sum_{i=1}^{U} \mathbf{D}\mathbf{D}^H \mathbf{\Lambda}_{u,i} \mathbf{D}\mathbf{D}^H \mathbf{p}_{u,i} \\
&= \mathbf{\Lambda}_d \mathbf{p}_d + \sum_{i=1}^{U} \mathbf{\Lambda}_{u,i} \mathbf{p}_{u,i}
\end{aligned}$$

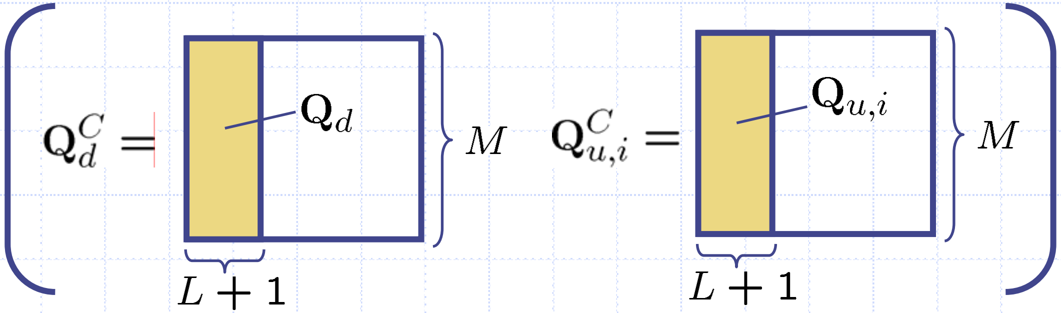
-Estimated Channel:

$$\hat{\lambda}_m^d = \lambda_m^d + \underbrace{\sum_{i=1}^{U} \frac{p_m^{u,i}}{p_m^d} \lambda_m^{u,i}}_{\text{Interference}}$$

Channel estimation @ time domain

-Received Signal Block:

$$
\begin{aligned}
\mathbf{r} &= \mathbf{Q}_d^C \begin{bmatrix} \mathbf{h}_d \\ \mathbf{0}_{(M-K-1)\times 1} \end{bmatrix} + \sum_{i=1}^{U} \mathbf{Q}_{u,i}^C \begin{bmatrix} \mathbf{h}_{u,i} \\ \mathbf{0}_{(M-K-1)\times 1} \end{bmatrix} \\
&= \mathbf{Q}_d \mathbf{h}_d + \sum_{i=1}^{U} \mathbf{Q}_{u,i} \mathbf{h}_{u,i}
\end{aligned}
$$



-Estimated Channel:

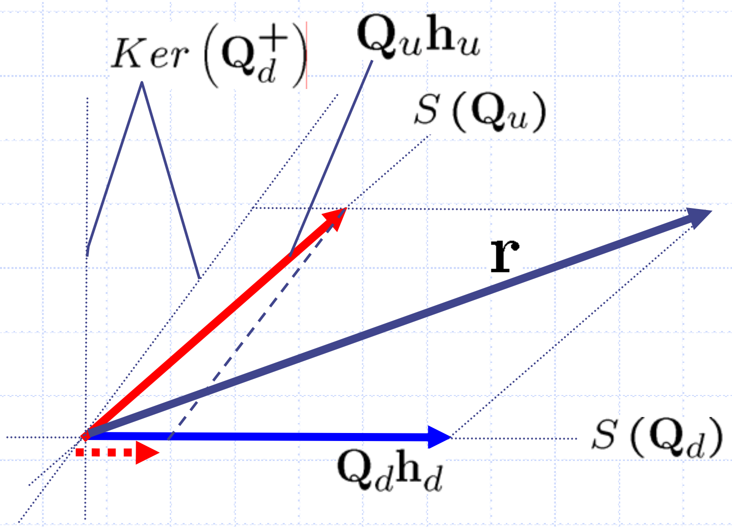
$$
\begin{aligned}
\hat{\mathbf{h}}_d &= \mathbf{Q}_d^{+} \mathbf{r} \\
&= \mathbf{h}_d + \underbrace{\sum_{i=1}^{U} \mathbf{Q}_d^{+} \mathbf{Q}_{u,i} \mathbf{h}_{u,i}}_{\text{Interference}}
\end{aligned}
$$

Pseudo-inverse of $\mathbf{Q}_d$:

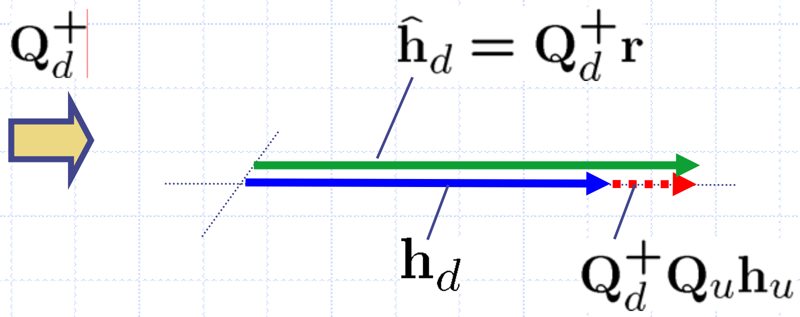
$$
\mathbf{Q}_d^{+} = (\mathbf{Q}_d^H \mathbf{Q}_d)^{-1} \mathbf{Q}_d^H
$$

Channel estimation @ time domain: geometric explanation

M-dimensional vector space



K+1-dimensional vector space



Channel estimation @ time domain with interference canceller

-Received Signal Block: $\mathbf{r} = \mathbf{Q}_d \mathbf{h}_d + \sum_{i=1}^{U} \mathbf{Q}_{u,i} \mathbf{h}_{u,i}$

-Estimated Channel (Interference):

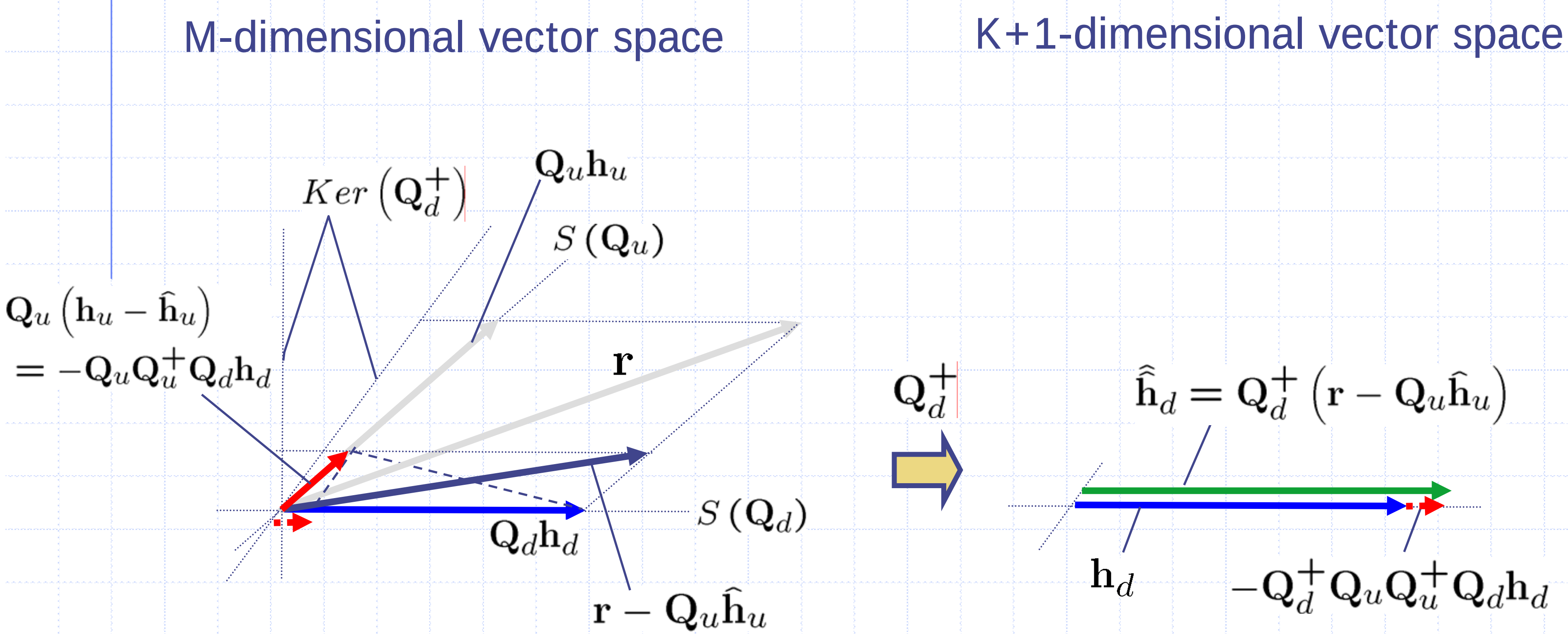
$$\begin{aligned} \hat{\mathbf{h}}_{u,i} &= \mathbf{Q}_{u,i}^{+} \mathbf{r} \\ &= \mathbf{h}_{u,i} + \mathbf{Q}_{u,i}^{+} \mathbf{Q}_d \mathbf{h}_d + \sum_{\substack{j=1 \\ j \neq i}}^{U} \mathbf{Q}_{u,i}^{+} \mathbf{Q}_{u,j} \mathbf{h}_{u,j} \end{aligned}$$

$$\mathbf{Q}_{u,i}^{+} = (\mathbf{Q}_{u,i}^{H} \mathbf{Q}_{u,i})^{-1} \mathbf{Q}_{u,i}^{H}$$

-Estimated Channel (Desired):

$$\begin{aligned} \hat{\hat{\mathbf{h}}}_d &= \mathbf{Q}_d^{+} \left(\mathbf{r} - \sum_{i=1}^{U} \mathbf{Q}_{u,i} \hat{\mathbf{h}}_{u,i} \right) \\ &= \mathbf{h}_d - \underbrace{\mathbf{Q}_d^{+} \sum_{i=1}^{U} \mathbf{Q}_{u,i} \mathbf{Q}_{u,i}^{+} \left(\mathbf{Q}_d \mathbf{h}_d + \sum_{\substack{j=1 \\ j \neq i}}^{U} \mathbf{Q}_{u,j} \mathbf{h}_{u,j} \right)}_{\text{Interference}} \end{aligned}$$

Channel estimation @ time domain with canceller: geometric explanation



Channel estimation @ time domain with iterative interference canceller

1) $k = 0$

2) Initial Estimation of $\mathbf{h}_d$: $\hat{\mathbf{h}}_d^k = \mathbf{Q}_d^+ \mathbf{r}$

3) Initial Estimation of $\mathbf{h}_{u,i}, \ (i = 1, \ldots, U)$: $\hat{\mathbf{h}}_d^k = \mathbf{Q}_d^+ \mathbf{r}$

4) $k = k + 1$

5) Estimated Channel (Desired):

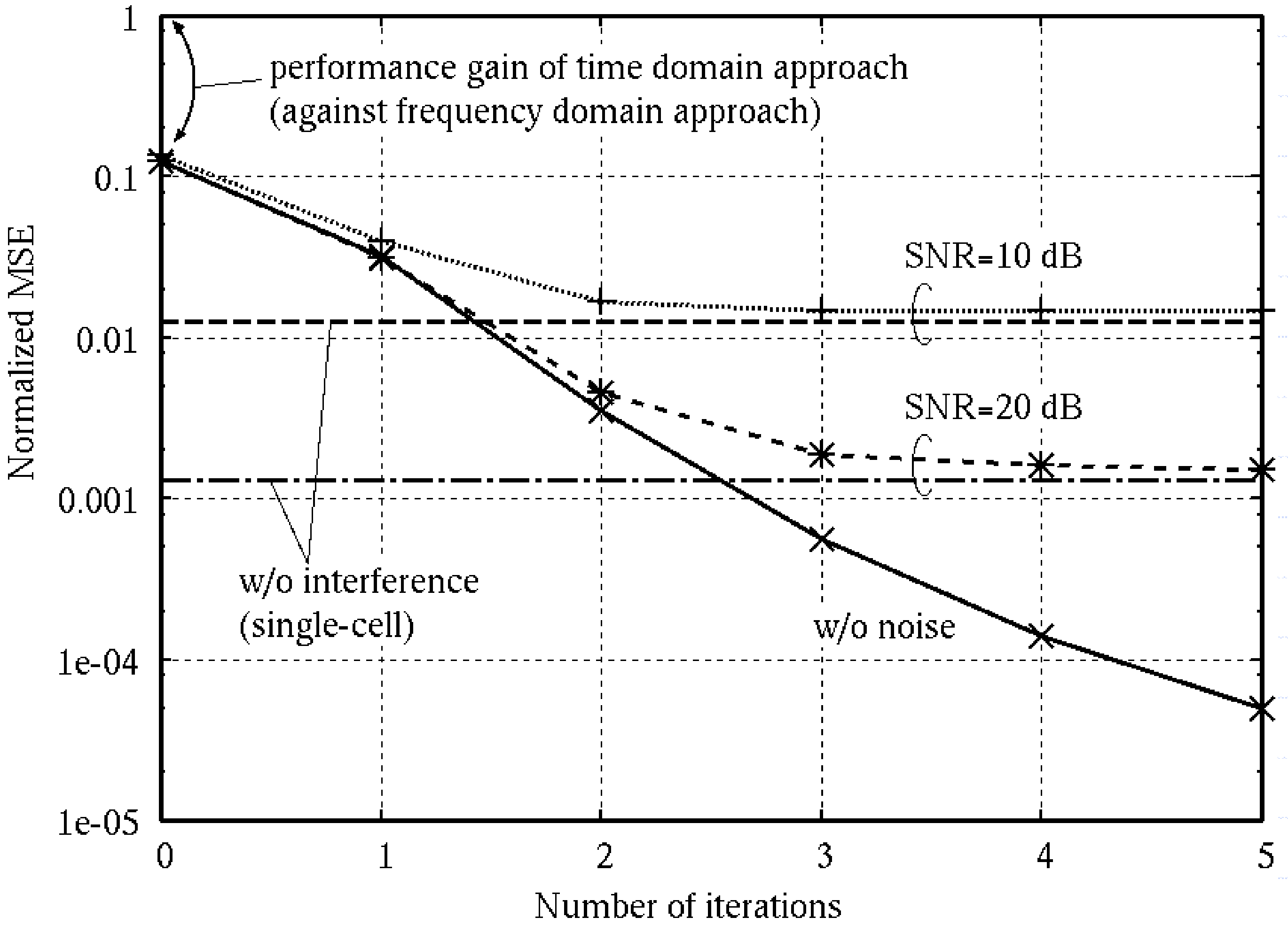
$$\hat{\mathbf{h}}_d^k = \mathbf{Q}_d^+ \left(\mathbf{r} - \sum_{i=1}^{U} \mathbf{Q}_{u,i} \hat{\mathbf{h}}_{u,i}^{k-1} \right)$$

6) Estimated Channel (Interference):

$$\hat{\mathbf{h}}_{u,i}^k = \mathbf{Q}_{u,i}^+ \left(\mathbf{r} - \mathbf{Q}_d \hat{\mathbf{h}}_d^{k-1} - \sum_{\substack{j=1 \\ j \neq i}}^{U} \mathbf{Q}_{u,j} \hat{\mathbf{h}}_{u,j}^{k-1} \right)$$

7) Go to step 4

Numerical results: OFDMA



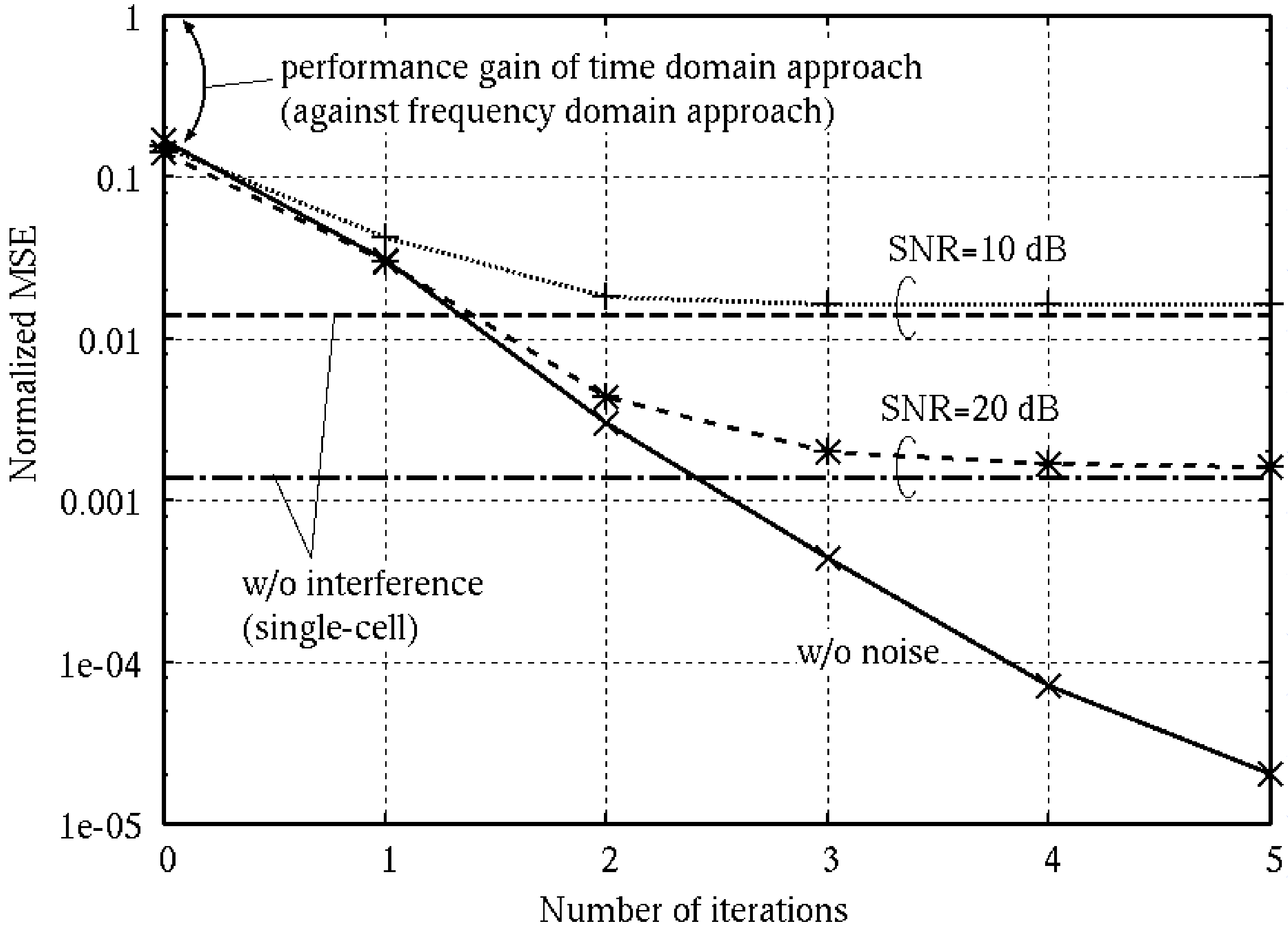
FFT Size: $M = 128$

Channel Order: $L = 16$

DUR: 0dB

$$\mathrm{NMSE} = E\left[\frac{\|\mathbf{h}_d - \hat{\mathbf{h}}_d\|^2}{\|\mathbf{h}_d\|^2}\right]$$

Numerical results: SC-CP



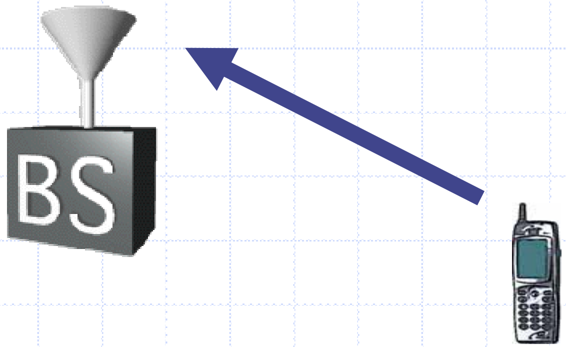
FFT Size: $M = 128$

Channel Order: $L = 16$

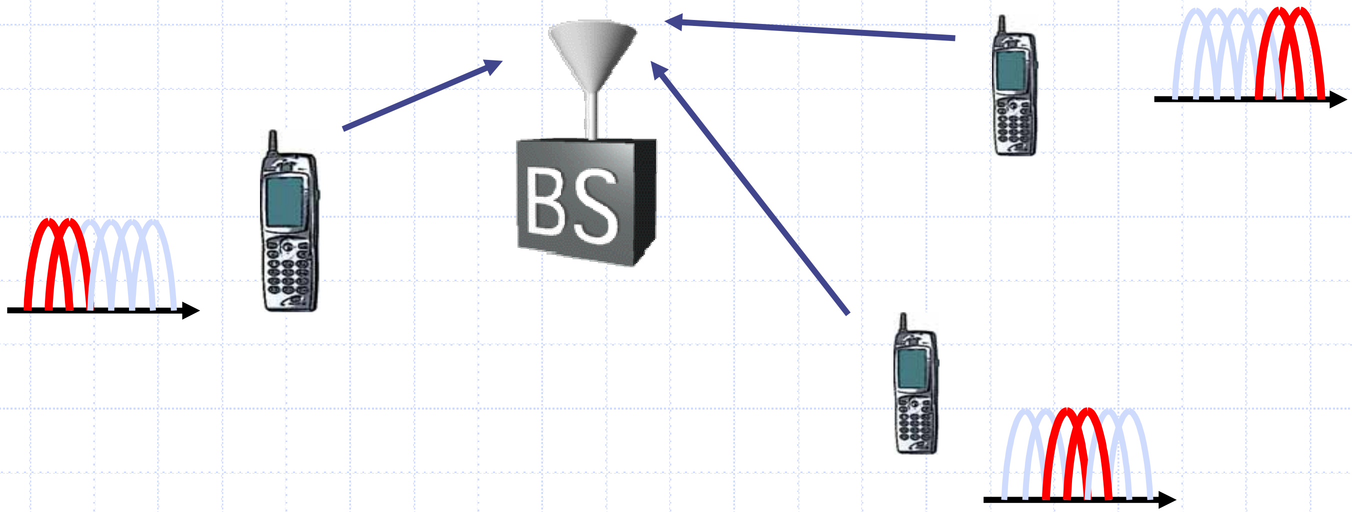
DUR: 0dB

$$\text{NMSE} = E\left[\frac{||\mathbf{h}_d - \hat{\mathbf{h}}_d||^2}{||\mathbf{h}_d||^2}\right]$$

Uplink channel estimation scheme for OFDMA

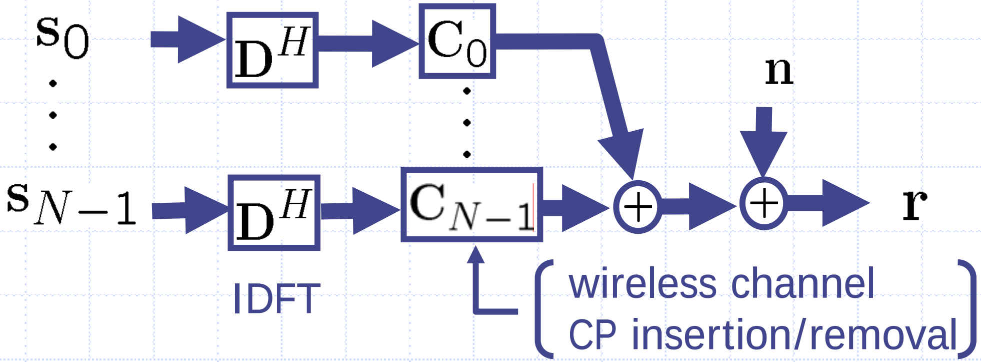


System model & assumptions



- consecutive subcarriers are assigned to a user
- no co-channel interference is considered
- no delayed signal beyond guard interval exists

Signal formulation



-Received Signal Block: $\mathbf{r} = \sum_{n=0}^{N-1} \mathbf{C}_n \mathbf{D}^H \mathbf{s}_n + \mathbf{n}$

$$\mathbf{s}_i = \begin{bmatrix} \mathbf{0}_{(\sum_{k=0}^{i-1} N_k)\times 1} \\ s_0^i \\ \vdots \\ s_{N_i-1}^i \\ \mathbf{0}_{(M-\sum_{k=0}^{i} N_k)\times 1} \end{bmatrix} \qquad M = \sum_{i=0}^{N-1} N_i \qquad \mathbf{C}_i = \begin{bmatrix} h_0^i & 0 & \dots & 0 & h_L^i & \dots & h_1^i \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & h_L^i \\ h_L^i & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \dots & 0 & h_L^i & \dots & \dots & h_0^i \end{bmatrix}$$

M: # of total subcarriers

N_k: # of subccarriers assigned to the k-th user

N: # of users

L: length of channle impulse response

Signal formulation (cont'd)

-Received Signal Block in Discrete Frequency Domain:

$$\begin{aligned}\mathbf{Dr} &= \sum_{n=0}^{N-1} \mathbf{D}\mathbf{C}_n\mathbf{D}^H\mathbf{s}_n + \mathbf{Dn} \\ &= \sum_{n=0}^{N-1} \boldsymbol{\Lambda}_n\mathbf{s}_n + \mathbf{N} \\ &= \sum_{n=0}^{N-1} \boldsymbol{\Lambda}_n\mathbf{P}_n\mathbf{s}_n + \mathbf{N} \\ &= \left(\sum_{n=0}^{N-1} \boldsymbol{\Lambda}_n\mathbf{P}_n\right)\left(\sum_{n=0}^{N-1} \mathbf{s}_n\right) \\ &= \boldsymbol{\Lambda}\mathbf{s} + \mathbf{N}\end{aligned}$$

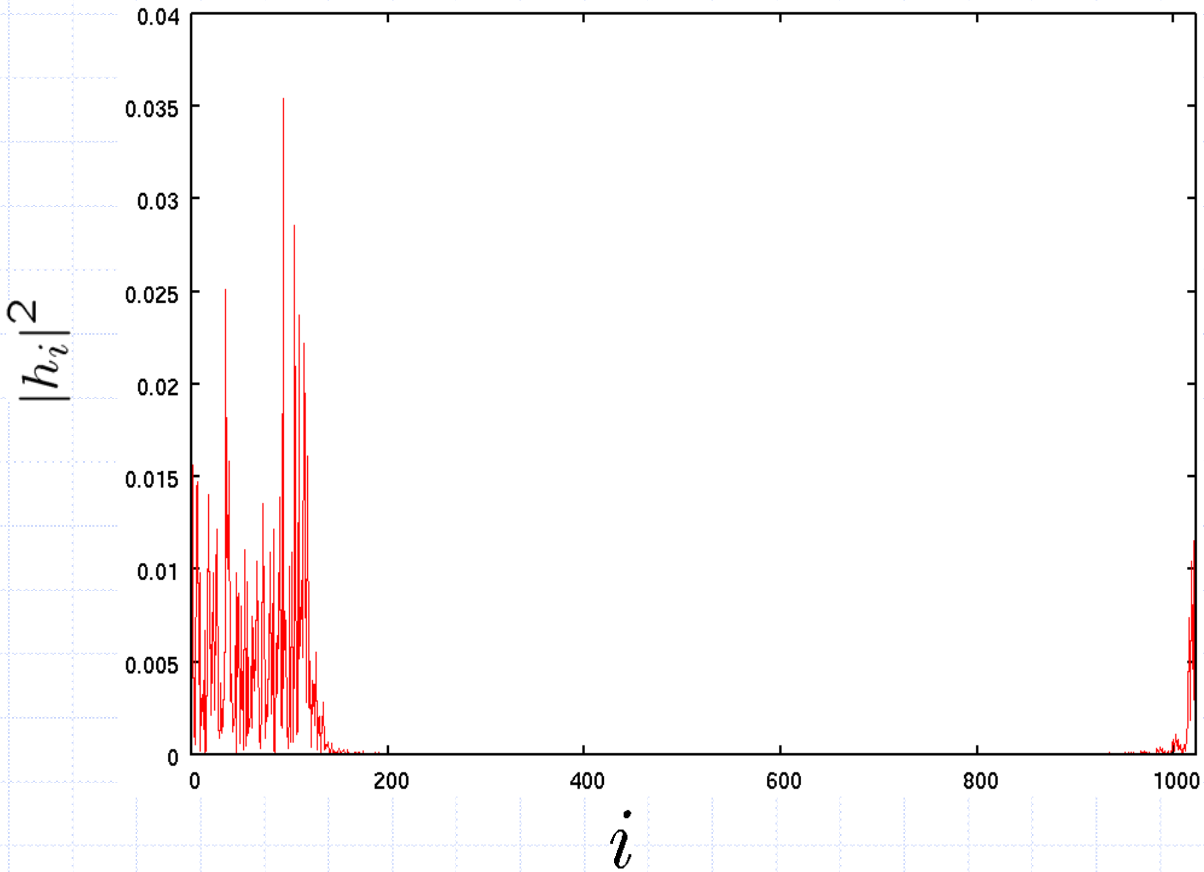
$$\boldsymbol{\Lambda}_n = \mathrm{diag}\left[\lambda_0^n \quad \cdots \quad \lambda_{M-1}^n\right]$$

$$\mathbf{P}_n = \mathrm{diag}\left[\underbrace{0\cdots0}_{\sum_{k=0}^{n-1} N_k} \quad \underbrace{1\cdots1}_{N_n} \quad 0\cdots0\right]$$

$$\mathbf{s} = \left[s_0^0\cdots s_{N_0-1}^0 \quad \cdots \quad s_0^{N-1}\cdots s_{N_{N-1}-1}^{N-1}\right]^T$$

$$\begin{aligned}\boldsymbol{\Lambda} &= \mathrm{diag}\left[\lambda_0 \quad \cdots \quad \lambda_{M-1}\right] \\ &= \mathrm{diag}\left[\lambda_0^0\cdots\lambda_{N_0-1}^0 \quad \cdots \quad \lambda_0^{N-1}\cdots\lambda_{N_{N-1}-1}^{N-1}\right]\end{aligned}$$

Example of overall impulse response



$M = 1024$

$N_k = 64$ for all k

$N = 16$

$L = 128$

$E[|h_l^k|^2] = \frac{1}{L+1}$

for all k and l

Overall frequency response

$$\begin{bmatrix} \lambda_0^n \\ \vdots \\ \lambda_{M-1}^n \end{bmatrix} = \mathbf{D} \begin{bmatrix} \overbrace{\begin{matrix} h_0^n \\ \vdots \\ h_L^n \end{matrix}}^{\mathbf{h}_n} \\ \mathbf{0}_{(M-L-1)\times 1} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_0\mathbf{h}_n \\ \mathbf{D}_1\mathbf{h}_n \\ \vdots \\ \mathbf{D}_{N-1}\mathbf{h}_n \end{bmatrix}$$

DFT Matrix:

$$\mathbf{D} = \begin{array}{c} \begin{bmatrix} \mathbf{D}_0 & \mathbf{D}_{0,c} \\ \mathbf{D}_1 & \mathbf{D}_{1,c} \\ \vdots & \vdots \\ \mathbf{D}_{N-1} & \mathbf{D}_{N-1,c} \end{bmatrix} \begin{array}{l} \}N_0 \\ \}N_1 \\ \\ \}N_{N-1} \end{array} \\ \underbrace{\quad\quad}_{L+1} \end{array}$$

$$\begin{bmatrix} \lambda_0 \\ \vdots \\ \lambda_{M-1} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_0\mathbf{h}_0 \\ \mathbf{D}_1\mathbf{h}_1 \\ \vdots \\ \mathbf{D}_{N-1}\mathbf{h}_{N-1} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{D}_0 & & & \\ & \mathbf{D}_1 & & \\ & & \ddots & \\ & & & \mathbf{D}_{N-1} \end{bmatrix} \begin{bmatrix} \mathbf{h}_0 \\ \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_{N-1} \end{bmatrix}$$

$$\begin{bmatrix} \lambda_0 \\ \vdots \\ \lambda_{M-1} \end{bmatrix} = \begin{bmatrix} \lambda_0^0 \\ \vdots \\ \lambda_{N_0-1}^0 \\ \lambda_{N_0}^1 \\ \vdots \\ \lambda_{N_0+N_1-1}^1 \\ \vdots \\ \lambda_{\sum_{k=0}^{N-2} N_k}^{N-1} \\ \vdots \\ \lambda_{\sum_{k=0}^{N-1} N_k-1}^{N-1} \end{bmatrix} \begin{array}{l} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} N_0 \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} N_1 \\ \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} N_{N-1} \end{array}$$

Overall impulse response

$$\begin{bmatrix} h_0 \\ \vdots \\ h_{M-1} \end{bmatrix} = \mathbf{D}^H \begin{bmatrix} \lambda_0 \\ \vdots \\ \lambda_{M-1} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{D}_0^H & \mathbf{D}_1^H & \cdots & \mathbf{D}_{N-1}^H \\ \mathbf{D}_{0,c}^H & \mathbf{D}_{1,c}^H & \cdots & \mathbf{D}_{N-1,c}^H \end{bmatrix} \begin{bmatrix} \mathbf{D}_0 & & & \\ & \mathbf{D}_1 & & \\ & & \ddots & \\ & & & \mathbf{D}_{N-1} \end{bmatrix} \begin{bmatrix} \mathbf{h}_0 \\ \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_{N-1} \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{n=0}^{N-1} \mathbf{D}_n^H \mathbf{D}_n \mathbf{h}_n \\ \sum_{n=0}^{N-1} \mathbf{D}_{n,c}^H \mathbf{D}_n \mathbf{h}_n \end{bmatrix}$$

Overall impulse response (cont'd)

$$h_i = \frac{1}{M}\sum_{n=0}^{N-1}\sum_{l=0}^{L} \frac{e^{j\frac{2\pi}{M}(i-l)\sum_{v=0}^{n-1} N_v}\left(1 - e^{j\frac{2\pi}{M}(i-l)N_n}\right)}{1 - e^{j\frac{2\pi}{M}(i-l)}}$$

$$\lim_{i\to l} \frac{1 - e^{j\frac{2\pi}{M}(i-l)N_n}}{1 - e^{j\frac{2\pi}{M}(i-l)}} = N_n$$

$$\Downarrow$$

$$E[|h_i|^2] = \frac{1}{M^2}\sum_{n=0}^{N-1}\sum_{l=0}^{L} \frac{\sigma_{l,n}^2\left(1 - \cos\frac{2\pi}{M}(i-l)N_n\right)}{1 - \cos\frac{2\pi}{M}(i-l)}$$

$$E[h_l^n h_{l'}^{n'}] = \begin{cases} \sigma_{l,n}^2, & l = l',\ n = n' \\ 0, & else \end{cases}$$

$$\Downarrow$$

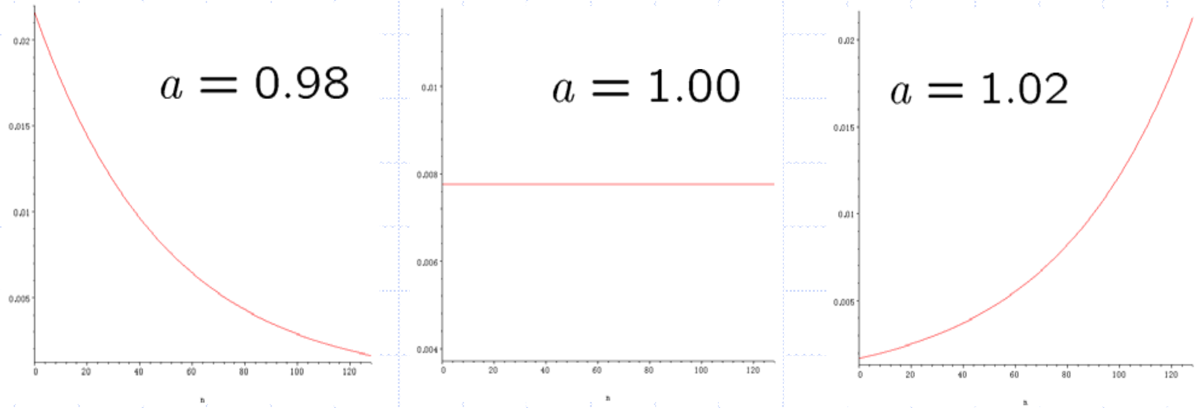
$$E[|h_i|^2] = \frac{1}{CM^2}\sum_{n=0}^{N-1}\sum_{l=0}^{L} \frac{a^l\left(1 - \cos\frac{2\pi}{M}(i-l)N_n\right)}{1 - \cos\frac{2\pi}{M}(i-l)}$$

$$\sigma_{l,k}^2 = \sigma_l^2, \quad \text{(assumption)}$$

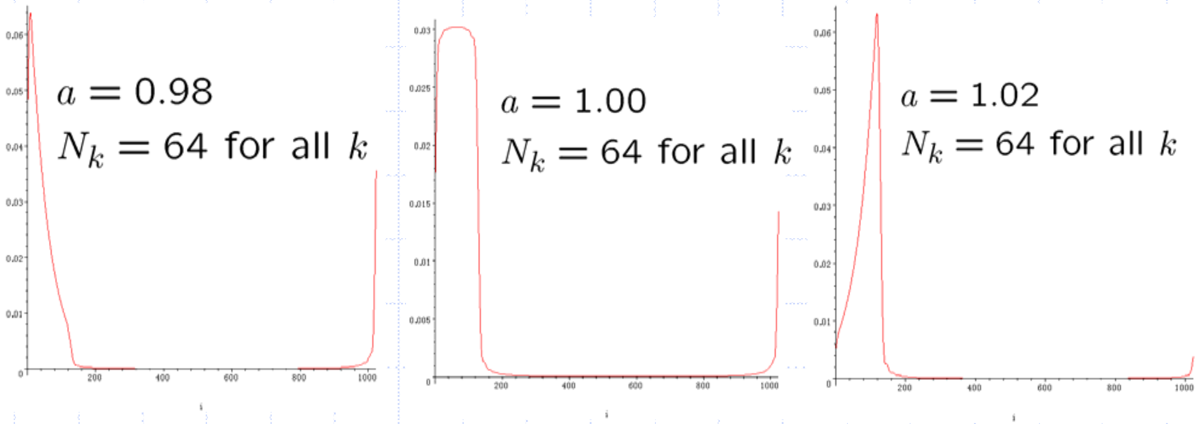
$$= \frac{1}{C}a^l, \quad C = \sum_{l=0}^{L} a^l$$

Numerical results: overall impulse response

Delay Power Spectrum: $L = 128$

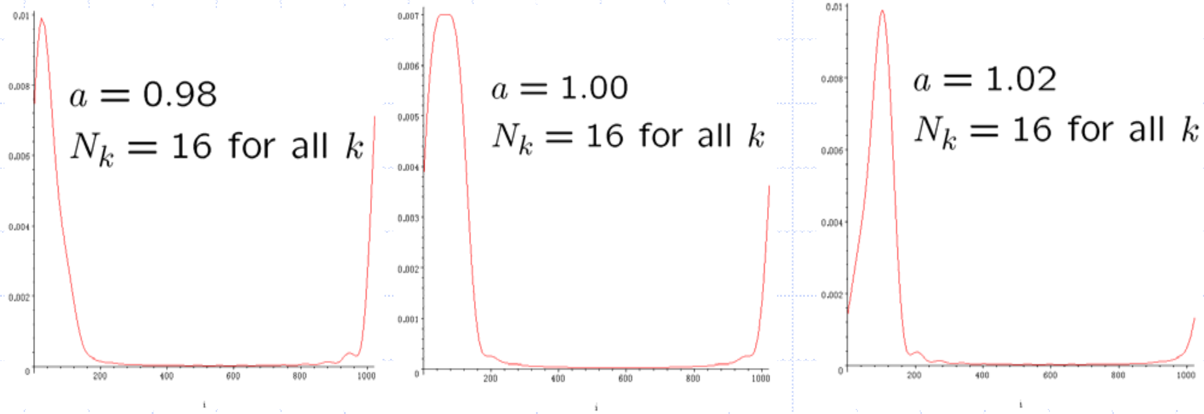


$E[|h_i|^2]: \; N = 16, \; L = 128, \; M = 1024$

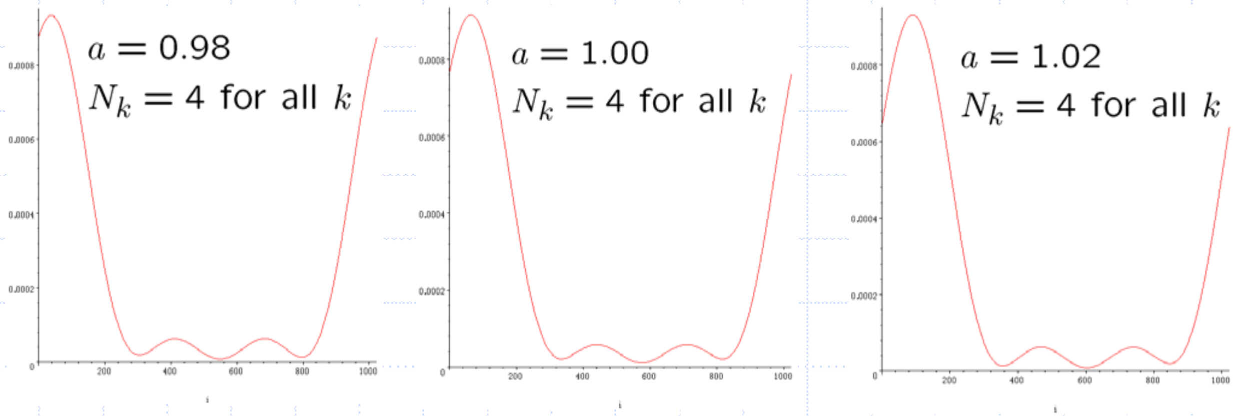


Numerical results: overall impulse response (cont'd)

$E[|h_i|^2]: \; N = 64, \; L = 128, \; M = 1024$

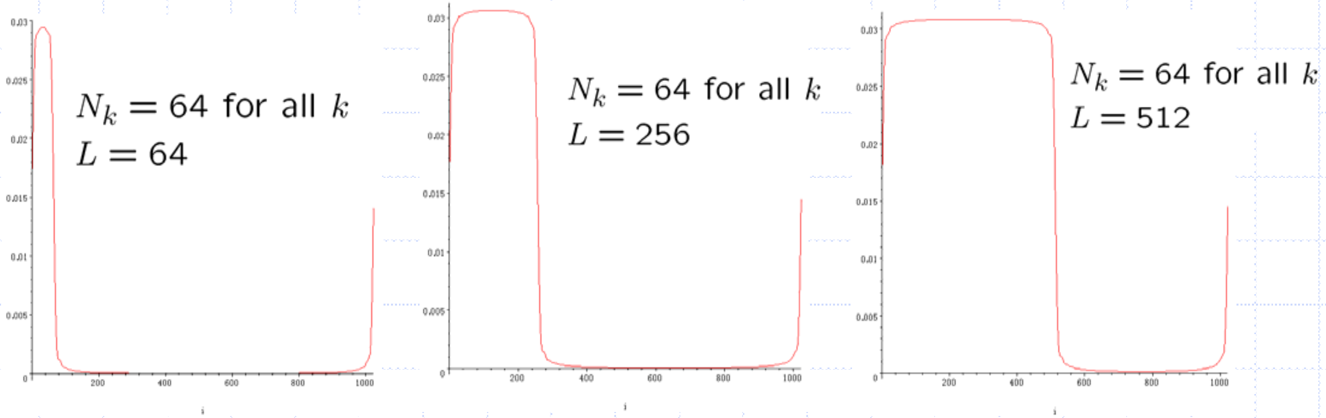


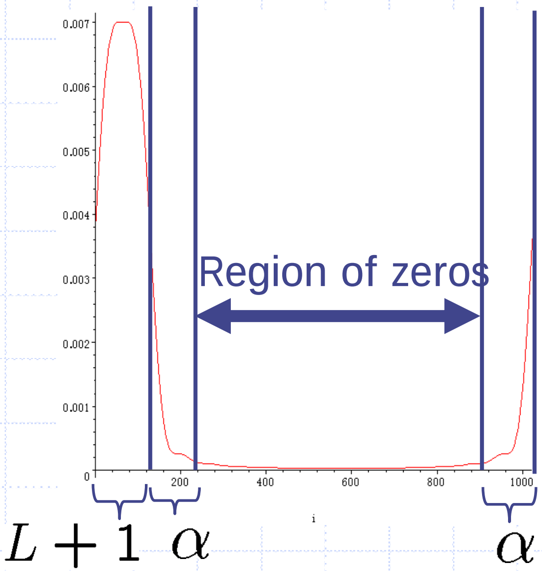
$E[|h_i|^2]: \; N = 256, \; L = 128, \; M = 1024$



Numerical results: overall impulse response (cont'd)

$E[|h_i|^2]: \; a = 1.00, \; N = 16, \; M = 1024$

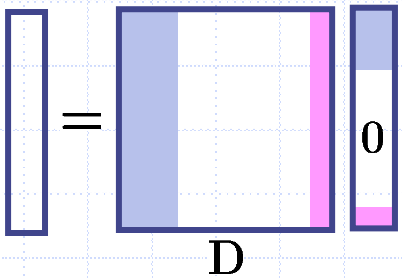


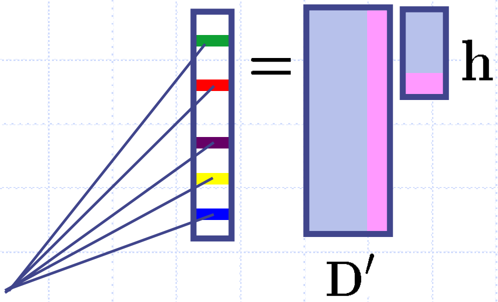


Only $L + 2\alpha + 1$ elements of overall impulse response have nonzero value

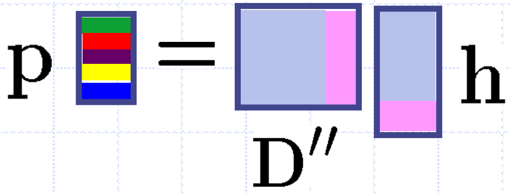
Least square channel estimation

$$\begin{bmatrix} \lambda_0 \\ \vdots \\ \lambda_{M-1} \end{bmatrix} = \mathbf{D} \begin{bmatrix} h_0 \\ \vdots \\ h_{M-1} \end{bmatrix}$$



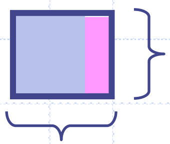


Pilot subcarriers



if $\mathbf{D}''$ is tall...

$$\begin{aligned} \hat{\mathbf{h}} &= \mathbf{D}''^{\dagger}\hat{\mathbf{p}} \\ &= (\mathbf{D}''^{H}\mathbf{D}'')^{-1}\mathbf{D}''^{H}\hat{\mathbf{p}} \end{aligned}$$

$\mathbf{D}'' =$ 

# of pilot subcarriers

# of nonzero elements in overall impulse response

Remedy for interpolation approach

$$\begin{bmatrix} \hat{\lambda}_0 \\ \vdots \\ \hat{\lambda}_{M-1} \end{bmatrix}$$: Estimated frequency response using interpolation

⇓ IDFT

$$\begin{bmatrix} \hat{h}_0 \\ \vdots \\ \hat{h}_{M-1} \end{bmatrix} = \mathbf{D}^H \begin{bmatrix} \hat{\lambda}_0 \\ \vdots \\ \hat{\lambda}_{M-1} \end{bmatrix}$$

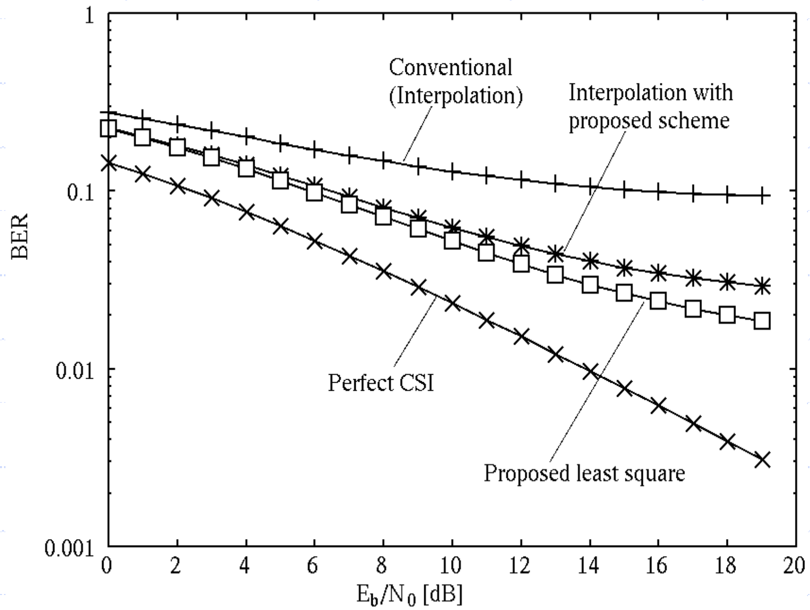
⇓ force to be zero at corresponding elements

$$\begin{bmatrix} \hat{h}'_0 \\ \vdots \\ \hat{h}'_{M-1} \end{bmatrix} = \boxed{0}$$

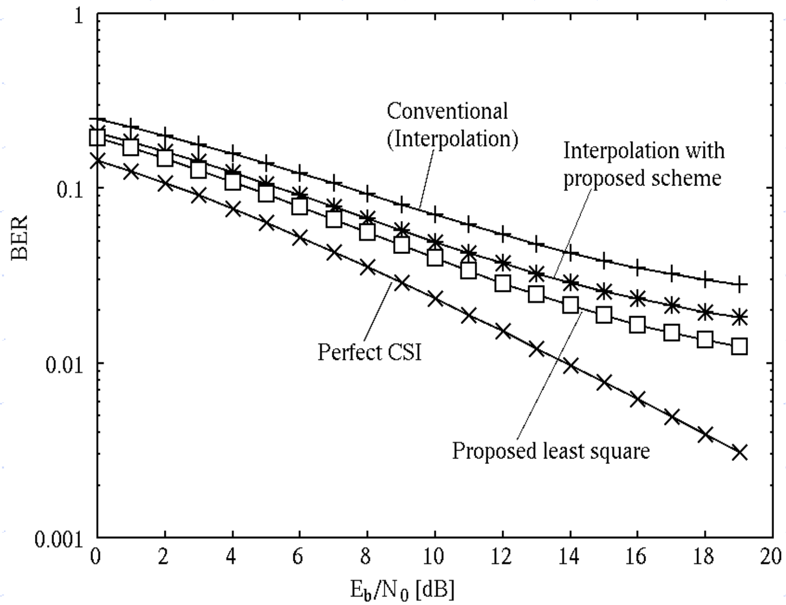
⇓ DFT

$$\begin{bmatrix} \hat{\lambda}_0 \\ \vdots \\ \hat{\lambda}_{M-1} \end{bmatrix}$$: Improved Estimation

Computer simulation results



$N_k = 64$ for all k
$N = 16$
$L = 128$
$M = 1024$
# of pilot signal: 256



$N_k = 64$ for all k
$N = 16$
$L = 128$
$M = 1024$
# of pilot signal: 512

Conclusion

- brief introduction of OFDM and OFDMA
- frame structure of mobile WiMAX (IEEE 802.16 TDD)
- uplink and downlink channel estimation schemes for OFDMA

- All the channel estimation methods considered in this talk are based on known (pilot or preamble) signal
- Utilization of FEC for channel estimation means channel estimation based not on known signal but on data signal
- Such an approach will be of great help especially for mobile WiMAX uplink