Transmit Beamforming and Iterative Water-Filling Based on SLNR for OFDMA Systems

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***Abstract*—This paper proposes a transmit beamforming and subcarrier power allocation method for orthogonal frequency division multiple access (OFDMA) systems based on "signal-to-leakage-plus-noise ratio" (SLNR). As the transmit beamforming vector control criterion, we employ the maximization of the SLNR at each base station, while the subcarrier power allocation is performed by iterative water-filling algorithm using the SLNR of each subcarrier. The SLNR based approach enables us to obtain closed form beamforming vector, and achieve power allocation without sending any signal to the mobile terminal. We also discuss the validity of the SLNR based beamforming vector in terms of Pareto optimality. Computer simulations show the promising performance of the proposed method and the validity of the analysis of the SLNR based beamforming vector.**

***Index Terms*—OFDMA, transmit beamforming, power allocation, SLNR, iterative water-filling, Pareto optimal**

I. Introduction

Much effort to apply block transmission schemes using cyclic prefix [1]–[3], such as orthogonal frequency division multiplexing (OFDM), to mobile communications systems has been made as typified by WiMAX system, where orthogonal frequency division multiple access (OFDMA) [4] is adopted for the physical layer / medium access control layer protocol. In cellular mobile communications systems, co-channel interference is one of the most serious problems, especially when frequency reuse factor is set to be 1 in order to achieve high spectral efficiency. Receive beamforming using multiple antenna elements with the combining vector based on maximum signal-to-interference-plus-noise ratio (SINR) criterion is effective for the interference suppression. However, employing multiple antenna elements and calculating the vector at the mobile terminal are undesirable from view points of cost and power consumption. Therefore, for downlink communications in the cellular systems, transmit beamforming is more attractive.

In this paper, we consider the downlink transmit beamforming for OFDMA systems. The determination of the transmit beamforming vector based on the SINR is difficult compared as the receive beamforming, because the received SINR of each user becomes a function of beamforming vectors of all the base stations. Moreover, SINRs at all the mobile terminals have to be taken into consideration simultaneously for the calculation of the beamforming vector. Instead of the simultaneous maximization of received SINRs, we utilize so-called *signal-to-leakage-plus-noise ratio (SLNR)* [5] as the optimization metric of beamforming vectors. The SLNR of the base station is defined as the ratio of the received signal power from the base station at the desired mobile terminal to the received signal power at mobile terminals in the other cells plus noise power. The same idea as the SLNR is used in [6], where the beamforming vector is determined by using the received SINR of *virtual uplink*. With the SLNR based criterion, each base station can obtain closed form beamforming vector based only on locally available information. We discuss the validity of the SLNR based beamforming vector in terms of the Pareto optimality. We also consider the power allocation problem over subcarriers combined with the transmit beamforming in order to further reduce the impact of the co-channel interference. As the power allocation strategy, we utilize *iterative water-filling* [7]–[9] for the SLNR instead of the SINR, which is commonly used in the iterative water-filling algorithm. The employment of the SLNR enables us to determine transmit power without sending any signal to mobile terminals. From the computer simulation results, we discuss achievable rate of the SLNR based beamforming vector comparing with the performance of maximum-ratio-combining (MRC) vector, zero-forcing (ZF) vector, and their linear combinations [10], [11]. Moreover, the sum-rate performance of the transmit beamforming and power allocation method is evaluated with highlighting the difference between the SLNR and the SINR based iterative water-filling power allocation algorithms.

II. Signal Model

Consider downlink of OFDMA systems with N pairs of base station and mobile terminal. For each base station, one mobile terminal out of the N mobile terminals is a desired terminal, while the signals from the rest of the $N-1$ base stations are considered as interference for the mobile terminal. Let $\mathbf{s}_i = [s_i^0, \cdots, s_i^{M-1}]^{\mathrm{T}}$ denote the frequency domain transmitted signal block from the i-th base station to the i-th mobile terminal, where M is the number of subcarriers and s_i^m is the symbol on the m-th subcarrier of the OFDMA

This work was supported in part by the KMRC R&D Grant for Mobile Wireless from Kinki Mobile Radio Center, Foundation, JAPAN, and by the Grant-in-Aid for Scientific Research, Grant no. 21760289, from the Ministry of Education, Science, Sports, and Culture of Japan.

signal. Moreover, P_i^m denotes the transmitted signal power of the i-th base station on the m-th subcarrier, and we define $\mathbf{w}_i^m = [w_i^{m,1}, \cdots, w_i^{m,Q}]^{\mathrm{T}}$ as the transmit beamforming weight vector on the m-th subcarrier of the transmitted signal from the i-th base station, where Q is the number of antenna elements at each base station and $\mathbf{w}_i^m$ is assumed to be unit norm, i.e., $||\mathbf{w}_i^m||^2 = 1$. Furthermore, assuming the length of the guard interval is greater than or equal to the order of the channel impulse response between the q-th antenna element of the i-th base station and the j-th mobile terminal $\mathbf{h}_{ji}^q = [h_{ji}^q(0), \cdots, h_{ji}^q(L-1)]^{\mathrm{T}}$, the frequency response between the q-th antenna element of the i-th base station and the j-th mobile terminal is given by

$$\begin{bmatrix} \lambda_{ji}^{0,q} \\ \vdots \\ \lambda_{ji}^{M-1,q} \end{bmatrix} = \mathbf{D} \begin{bmatrix} \mathbf{h}_{ji}^q \\ \mathbf{0}_{(M-L)\times 1} \end{bmatrix}, \quad (1)$$

where $\mathbf{D}$ is $M \times M$ a discrete Fourier transform (DFT) matrix, whose $\{m,n\}$ element is $\{\mathbf{D}\}_{m,n} = 1/\sqrt{M}\exp(-j\frac{2\pi mn}{M})$ and $\mathbf{0}_{(M-L)\times 1}$ denotes a zero vector of $(M-L) \times 1$. Defining the frequency response vector on the m-th subcarrier as $\boldsymbol{\lambda}_{ji}^m = [\lambda_{ji}^{m,1}, \cdots, \lambda_{ji}^{m,Q}]^{\mathrm{T}}$, the received signal at the j-th mobile terminal on the m-th subcarrier is written as

$$r_j^m = \sum_{i=1}^{N} \sqrt{P_i^m} (\mathbf{w}_i^m)^{\mathrm{H}} \boldsymbol{\lambda}_{ji}^m s_i^m + n_j^m, \quad (2)$$

where n_j^m denotes a zero mean additive white Gaussian noise (AWGN) at the j-th terminal on the m-th subcarrier with the variance of σ_n^2.

For the j-th mobile terminal, only the signal from the j-th base station is the desired signal, and the signals from the other base stations result in the interference. Therefore, the SINR at the j-th terminal on the m-th subcarrier is given by

$$\Gamma_j^m = \frac{P_j^m (\mathbf{w}_j^m)^{\mathrm{H}} \boldsymbol{\lambda}_{jj}^m (\boldsymbol{\lambda}_{jj}^m)^{\mathrm{H}} \mathbf{w}_j^m}{\sum_{\substack{i=1 \\ i\neq j}}^{N} P_i^m (\mathbf{w}_i^m)^{\mathrm{H}} \boldsymbol{\lambda}_{ji}^m (\boldsymbol{\lambda}_{ji}^m)^{\mathrm{H}} \mathbf{w}_i^m + \sigma_n^2}. \quad (3)$$

Unlike the case of the receive beamforming, (3) includes beamforming vectors of all the base stations $\mathbf{w}_1^m, \cdots, \mathbf{w}_N^m$ and channel frequency responses between the j-th mobile terminal and all the base stations $\boldsymbol{\lambda}_{j1}^m, \cdots, \boldsymbol{\lambda}_{jN}^m$, therefore, SINRs of all the users $\Gamma_1^m, \ldots, \Gamma_N^m$ have to be taken into the consideration simultaneously for the determination of the beamforming vectors and the power allocation. In order to avoid the complicated joint optimization problem and the huge overhead caused by exchanging the channel state information, we take a two-step suboptimal approach for the transmit beamforming and the power allocation based on SLNR rather than SINR as explained in the following sections.

III. Beamforming Vector Control

We consider the maximization of SLNR [5] at each base station with respect to the beamforming vector. The SLNR of the j-th base station is defined as the ratio of the received signal power from the base station at the desired mobile terminal (the j-th terminal) to the received signal power at all the mobile terminals in the other cells plus noise power. This can be also considered as the ratio obtained from (3) by replacing the interference power observed at the j-th mobile terminal in the denominator with the interference power caused by the j-th base station. The SLNR of the m-th subcarrier at the j-th base station is written as

$$\tilde{\Gamma}_j^m = \frac{P_j^m (\mathbf{w}_j^m)^{\mathrm{H}} \boldsymbol{\lambda}_{jj}^m (\boldsymbol{\lambda}_{jj}^m)^{\mathrm{H}} \mathbf{w}_j^m}{\sum_{\substack{i=1 \\ i\neq j}}^{N} P_j^m (\mathbf{w}_j^m)^{\mathrm{H}} \boldsymbol{\lambda}_{ij}^m (\boldsymbol{\lambda}_{ij}^m)^{\mathrm{H}} \mathbf{w}_j^m + \sigma_n^2}. \quad (4)$$

Key issue here is that the SLNR is composed only by locally available information, while the SINR (3) includes some values, which are not directly observable for the j-th base station, like $\mathbf{w}_i^m$ or $\boldsymbol{\lambda}_{ji}$ for $i \neq j$.

Each transmit beamforming vector is controlled so that the SLNR of each base station is maximized. For given transmit power P_i^m, the beamforming vector, which maximizes (4) can be directly obtained as the eigenvector corresponding to the maximum eigenvalue of $[\sum_{\substack{i=1 \\ i\neq j}}^{N} P_j^m \boldsymbol{\lambda}_{ij}^m (\boldsymbol{\lambda}_{ij}^m)^{\mathrm{H}} + \sigma_n^2 \mathbf{I}_Q]^{-1} [P_j^m \boldsymbol{\lambda}_{jj}^m (\boldsymbol{\lambda}_{jj}^m)^{\mathrm{H}}]$,

$$(\mathbf{w}_j^m)^{\mathrm{SLNR}} = C_j^m \left(\sum_{i=1}^{N} P_i^m \boldsymbol{\lambda}_{ij}^m (\boldsymbol{\lambda}_{ij}^m)^{\mathrm{H}} + \sigma_n^2 \mathbf{I}_Q \right)^{-1} \sqrt{P_j^m} \boldsymbol{\lambda}_{jj}^m, \quad (5)$$

where C_j^m is a scaling factor, which keeps $(\mathbf{w}_j^m)^{\mathrm{SLNR}}$ unit norm, and $\mathbf{I}_Q$ denotes $Q \times Q$ identity matrix. In this way, by utilizing the SLNR, we can obtain closed form expression of the beamforming vector. The validity of the utilization of the SLNR based vector is discussed in Sec. V.

IV. Power Allocation

For fixed beamforming vectors $\mathbf{w}_j^m$, we consider the transmit power allocation with total power constraint of

$$P = \sum_{m=0}^{M-1} P_j^m. \quad (6)$$

In order to maximize each user's achievable rate for given beamforming vectors, we can apply the idea of *iterative water-filling* [7]–[9], which is based on the well-known water-filling theorem [12] which says that more power should be allocated on the subcarrier with better SINR. The iterative water-filling can be implemented in a distributed manner, however, it requires actual signal transmission and feedback of observed SINR at the mobile terminal several times for the determination of the transmit power. The proposed method utilizes the same approach as the iterative water-filling but with the SLNR, where more power is allocated to the subcarrier if the frequency response of the desired channel $(\mathbf{w}_j^m)^{\mathrm{H}} \boldsymbol{\lambda}_{jj}^m$ is greater than that of the interference channel $(\mathbf{w}_j^m)^{\mathrm{H}} \boldsymbol{\lambda}_{ij}^m$, $i \neq j$. The utilization of the SLNR enables us to determine the transmit power without sending any signal to the mobile terminal, while the power allocation is performed by an iterative manner. Note that, although the system considered in the paper has multiple antennas, the iterative water-filling is performed not in spatial domain as in [9] but in frequency domain.

The transmit power of the j-th base station on the m-th subcarrier at the n-th iteration is described as $P_j^m(n)$ hereafter. Defining the sum of the leakage and noise power of the m-th subcarrier at the n-th iteration normalized by the channel frequency response including the effect of the beamforming vector as

$$X_j^m(n) = \frac{\sum_{\substack{i=1\\ i\neq j}}^{N} P_j^m(n)(\mathbf{w}_j^m)^{\mathrm{H}}\boldsymbol{\lambda}_{ij}^m(\boldsymbol{\lambda}_{ij}^m)^{\mathrm{H}}\mathbf{w}_j^m + \sigma_n^2}{(\mathbf{w}_j^m)^{\mathrm{H}}\boldsymbol{\lambda}_{jj}^m(\boldsymbol{\lambda}_{jj}^m)^{\mathrm{H}}\mathbf{w}_j^m}, \quad (7)$$

the proposed algorithm to update the transmit power is summarized as follows:

1) Initialize $K = 0,\ R(n) = \dfrac{P + \sum_m X_j^m(n)}{M}$.
2) If $R(n) > \max\limits_m X_j^m(n)$ then $\forall m,\ P_j^m(n+1) = R(n) - X_j^m(n)$ and exit, otherwise go to 3
3) Define a set $\mathcal{A} = \{m \mid R(n) \leq X_j^m(n)\}$ and $K = \mathrm{Card}(\mathcal{A})$, where Card denotes the number of elements in the set. Modify $R(n)$ as $R(n) = \dfrac{P + \sum_{m\notin\mathcal{A}} X_j^m(n)}{M-K}$, and if $R(n) > \max\limits_{m\notin\mathcal{A}} X_j^m(n)$ then go to 4, otherwise go to 3.
4) $P_j^m(n+1) = \begin{cases} R(n) - X_j^m(n) & (m \notin \mathcal{A}) \\ 0 & (m \in \mathcal{A}) \end{cases}$

V. Pareto Optimality of SLNR Based Transmit Beamforming Vector

We focus on the transmit beamforming on a certain subcarrier, so the superscript m is dropped and the transmit power P_j^m is set to be 1 in the sequel for simplicity. The achievable rate on the subcarrier at the j-th mobile terminal is given by

$$R_j(\mathbf{w}_1, \ldots, \mathbf{w}_N) = \log_2\left(1 + \frac{|\mathbf{w}_j^{\mathrm{H}}\boldsymbol{\lambda}_{jj}|^2}{\sum_{i\neq j}|\mathbf{w}_i^{\mathrm{H}}\boldsymbol{\lambda}_{ji}|^2 + \sigma_n^2}\right), \quad (8)$$

where the bandwidth is normalized to 1. The achievable rate region is defined by a set of all rate points $(R_1, \ldots, R_N)$ achieved by all the possible transmit beamforming vectors with unit norm. A rate point is Pareto optimal if it is impossible to increase one of the rate without decreasing at least one of the other rates [10].

For the special case of two pairs of the base station and the mobile terminal, we show the following proposition regarding the Pareto optimality of the SLNR based beamforming vector.

Proposition 1: For $N = 2$, transmit beamforming vectors, which maximize SLNRs, achieve a Pareto optimal rate point.

Proof: Let $\{\mathbf{w}_1^{\mathrm{SLNR}},\ \mathbf{w}_2^{\mathrm{SLNR}}\}$ be the beamforming vectors, which maximize SLNRs, and consider to change the SLNR solution to arbitrary beamforming vectors $\{\mathbf{w}_1^{'},\ \mathbf{w}_2^{'}\}$. With the change of vectors, we can write

$$|(\mathbf{w}_1^{'})^{\mathrm{H}}\boldsymbol{\lambda}_{11}|^2 = \alpha|(\mathbf{w}_1^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{11}|^2, \quad (9)$$
$$|(\mathbf{w}_1^{'})^{\mathrm{H}}\boldsymbol{\lambda}_{21}|^2 = \beta|(\mathbf{w}_1^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{21}|^2, \quad (10)$$
$$|(\mathbf{w}_2^{'})^{\mathrm{H}}\boldsymbol{\lambda}_{22}|^2 = \gamma|(\mathbf{w}_2^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{22}|^2, \quad (11)$$
$$|(\mathbf{w}_2^{'})^{\mathrm{H}}\boldsymbol{\lambda}_{12}|^2 = \delta|(\mathbf{w}_2^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{12}|^2, \quad (12)$$

where $\alpha, \beta, \gamma, \delta \in \mathbb{R}$, $\alpha > 0,\ \beta > 0,\ \gamma > 0$, and $\delta > 0$. Since $\{\mathbf{w}_1^{\mathrm{SLNR}},\ \mathbf{w}_2^{\mathrm{SLNR}}\}$ are the SLNR solution, SLNRs decrease with any change from the beamforming vectors as

$$\frac{|(\mathbf{w}_1^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{11}|^2}{|(\mathbf{w}_1^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{21}|^2 + \sigma_n^2} \geq \frac{\alpha|(\mathbf{w}_1^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{11}|^2}{\beta|(\mathbf{w}_1^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{21}|^2 + \sigma_n^2}, \quad (13)$$
$$\frac{|(\mathbf{w}_2^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{22}|^2}{|(\mathbf{w}_2^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{12}|^2 + \sigma_n^2} \geq \frac{\gamma|(\mathbf{w}_2^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{22}|^2}{\delta|(\mathbf{w}_2^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{12}|^2 + \sigma_n^2}. \quad (14)$$

The proposition is proved by contradiction. Suppose that the rate of user 1 increases with the change of beamforming vectors, namely, the SINR of user 1 increases with the change as

$$\frac{|(\mathbf{w}_1^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{11}|^2}{|(\mathbf{w}_2^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{12}|^2 + \sigma_n^2} < \frac{\alpha|(\mathbf{w}_1^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{11}|^2}{\delta|(\mathbf{w}_2^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{12}|^2 + \sigma_n^2}. \quad (15)$$

From (14) and (15), we have

$$\begin{aligned}\gamma(|(\mathbf{w}_2^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{12}|^2 + \sigma_n^2) &\leq \delta|(\mathbf{w}_2^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{12}|^2 + \sigma_n^2 \\ &< \alpha(|(\mathbf{w}_2^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{12}|^2 + \sigma_n^2),\end{aligned}$$

and this means

$$\gamma < \alpha. \quad (16)$$

Here, if we assume the rate of user 2 does not decrease with the change of vectors, then we have

$$\frac{|(\mathbf{w}_2^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{22}|^2}{|(\mathbf{w}_1^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{21}|^2 + \sigma_n^2} \leq \frac{\gamma|(\mathbf{w}_2^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{22}|^2}{\beta|(\mathbf{w}_1^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{21}|^2 + \sigma_n^2}. \quad (17)$$

From (13) and (17), we have

$$\begin{aligned}\alpha(|(\mathbf{w}_1^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{21}|^2 + \sigma_n^2) &\leq \beta|(\mathbf{w}_1^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{21}|^2 + \sigma_n^2 \\ &\leq \gamma(|(\mathbf{w}_1^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{21}|^2 + \sigma_n^2),\end{aligned}$$

and this means $\gamma \geq \alpha$, which is a contradiction. Thus, for any change of beamforming vectors, which increases user 1's rate, the rate of user 2 decreases and vice versa. ■

VI. Numerical Results

In this section, the performance of the transmit beamforming and power allocation method shown in Sec. III and IV, and the validity of the analysis in Sec. V are examined via computer simulations. Figs. 1 and 2 show examples of the achievable rate region (shaded area) of two base stations and mobile terminals with two antenna elements ($N = 2$ and $Q = 2$) for different channel realizations obtained by randomly generated beamforming vectors with norm less than 1. In the figures, SLNR denotes the rate point achieved by the beamforming vector of (5), while MRC and ZF stands for the rate point achieved by MRC vector and ZF vector, respectively. Since it has been proved that a point on the Pareto boundary is achieved by a linear combination of the MRC vector and the ZF vector [10], [11], we have evaluated the achievable rate of the linear combination by changing the weight of the two vectors (the dashed line connecting MRC and ZF). We can see that the SLNR based vector and the linear combination with a certain combination ratio can achieve rate points of Pareto boundary, while achieved rate points are different.

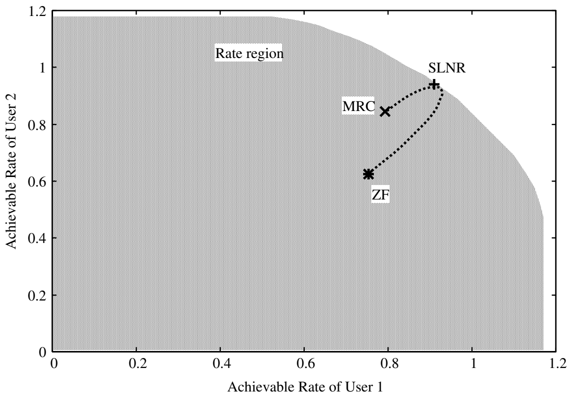

Fig. 1. Example of rate region 1 ($N = 2$ and $Q = 2$)

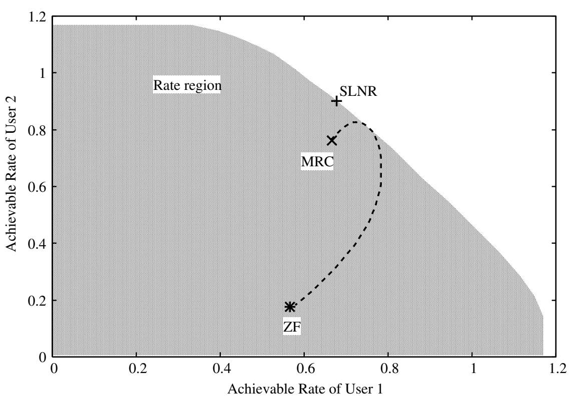

Fig. 2. Example of rate region 2 ($N = 2$ and $Q = 2$)

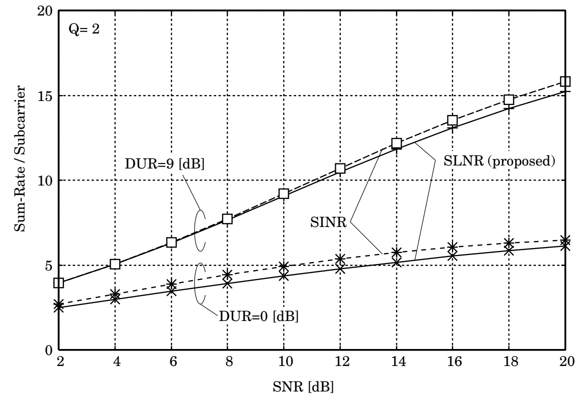

Fig. 3. Sum-rate per subcarrier ($Q = 2$)

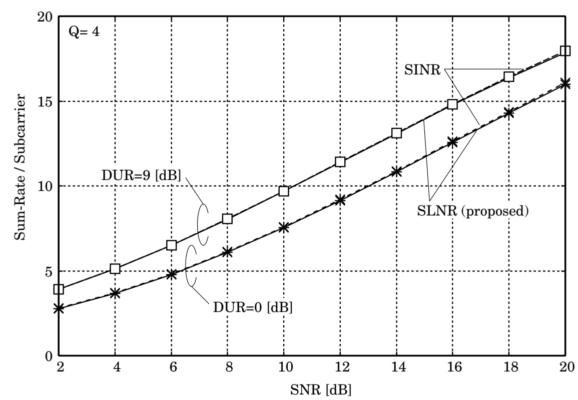

Fig. 4. Sum-rate per subcarrier ($Q = 4$)

Then, we have evaluated sum-rate per subcarrier by the proposed transmit beamforming and power allocation method. Here, the sum-rate per subcarrier is defined as

$$\frac{1}{N}\sum_{j=1}^{N}\sum_{m=0}^{M-1}\log_2(1+\hat{\Gamma}_j^m), \tag{18}$$

where $\hat{\Gamma}_j^m$ is the observed SINR of the j-th user on the m-th subcarrier in the simulation. Figs. 3 and 4 show the sum-rate performance with the number of antenna elements of $Q = 2$ and $Q = 4$, respectively. In the figures, the number of base stations (or mobile terminals) N is set to be 3, and DUR denotes the power ratio of the desired channel frequency response to the interference channel frequency response $E[|\lambda_{jj}^{m,q}|^2]/E[|\lambda_{ji}^{m,q}|^2]$. Note that the number of antenna elements is less than the number of incoming signals for the case of $Q = 2$. The performance of the proposed method using SINR in the iterative water-filling is also plotted in the same figures. From the figures, we can see that the degradation of the performance due to the employment of the SLNR in the iterative water-filling is rather small, especially for the case of $Q = 4$. In the performance evaluation, the overhead caused by the actual signal transmission and the feedback of the SINR required by the SINR based iterative water-filling is not taken into consideration, therefore, the proposed method may even outperform the SINR based approach if all the overheads are counted.

Finally, Figs. 5–8 show typical examples of the achieved power allocation of each user by the SLNR and the SINR based approaches for the number of antenna elements of $Q = 2$ and $Q = 4$. The same channel realization is used for the same number of antenna elements, and the SNR and the DUR are set to be 2 dB and 0 dB, respectively. From the figures, we can see that not only the SINR based but also the SLNR based iterative water-filling algorithms decrease the transmit power of a subcarrier, if the other users have large power on the corresponding subcarrier. This means that the automatic interference avoiding subcarrier allocation among the three users is achieved by the proposed SLNR based iterative water-filling algorithm. For $Q = 4$, the SLNR based iterative water-filling can achieve almost the same power allocation as that of the SINR based method. This is because the interference is almost completely suppressed for this case, and the SLNR coincides with the SINR in the absence of the interference. It should be also noted that, for $Q = 2$, the SLNR based approach results in completely different power allocation compared to the SINR based approach, while the degradation of the achievable sum-rate due to the employment

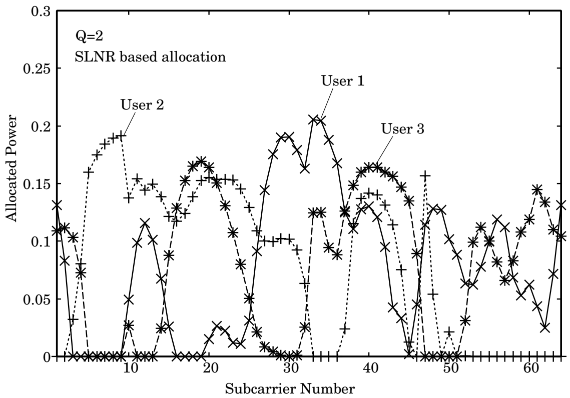

Fig. 5. Example of power allocationSLNR based ($Q = 2$)

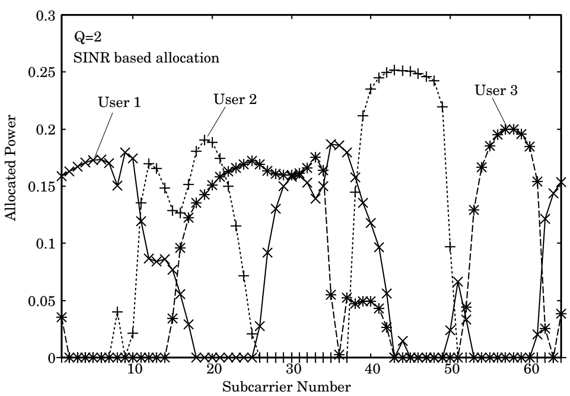

Fig. 6. Example of power allocationSINR based ($Q = 2$)

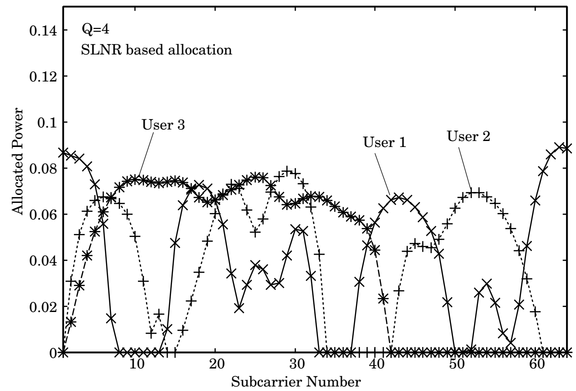

Fig. 7. Example of power allocationSLNR based ($Q = 4$)

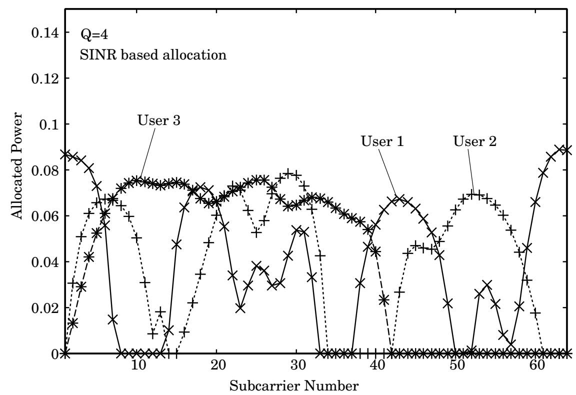

Fig. 8. Example of power allocationSINR based ($Q = 4$)

of the SLNR is rather small.

VII. Conclusion

We have considered transmit beamforming and power allocation method for downlink OFDMA systems. The beamforming vector is determined based on the maximization of the SLNR, while the transmit power is allocated by the iterative water-filling based on the SLNR. We have also discussed the Pareto optimality of the SLNR based beamforming vector and proved its optimality for $N = 2$. Computer simulation results show that the SLNR based beamforming vector and the linear combination of the MRC and ZF vectors with a certain combination ratio achieve different Pareto optimal points. Moreover, the degradation of the achievable sum-rate due to the employment of the SLNR based power allocation is rather small, and the difference is almost negligible for the case with a sufficient number of antenna elements.

Future work will include the analytical evaluation of the SLNR based beamforming vector for a larger number of users in terms of Pareto optimality.

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