

# Drake QP Inverse Dynamics controller

$$\min_{\dot{v}, \beta} (J_{com} \dot{v} + J_{com} v - \ddot{p}_{com, des})^2 + (\dot{v} - \dot{v}_{des})^2 + \text{body acceleration costs} + \text{regularizations (including contacts)}$$

subject to:  $\lambda = \lambda + 10$   
 $A = [M_u - (J^T B)^T] \quad b = [h_u]$

$$X = \begin{bmatrix} \dot{v} \\ \beta \end{bmatrix} \begin{matrix} 36 \\ 32 \end{matrix} > 68$$

• Dynamics equality  $M_u \dot{v} + h_u = (J^T B)^T \beta$   $u = \text{only top } b \text{ for floating base rows.}$

•  $\dot{v}$  equality  $M_u \dot{v} - (J^T B)^T \beta = -h_u$  constrain  $\dot{v}$  values for indices that have desired values

• contact force basis inequality  $(\beta \geq 0)$   $A = [M_k \quad (J^T B)_k] \quad BL = T_{min} - h \quad BU = T_{max} - h$

• torque limit inequality in linear form  $T_{min} - h \leq M_k \dot{v} - (J^T B)_k \beta \leq T_{max} - h$

replace with centrodal moment matrix if  $\ddot{p} \rightarrow [p]$   
 $\| [L \dot{p}] - [\dot{p}]_{des} \|^2$   
 $J_{com} \dot{v} + J_{com} v - \ddot{p}_{des}$

- same as mc-rtc
- Dynamic equality
  - Contact force basis inequality

what to change for  $\lambda$ ?  $\beta$

## Costs

• Foot contact cost (soft no-slip constraint)  $36 \times 36$

$\ddot{p}_{contact} = J_c \ddot{q} + \dot{J}_c \dot{q} = 0 \rightarrow \min \| J_c \ddot{q} + \dot{J}_c \dot{q} - 0 \|^2$   
 $A = J_c \quad b = -\dot{J}_c \dot{q}$

## Quadratic cost!

$$\frac{1}{2} x^T Q x + b^T x + c$$

$Q = J_c^T J_c \quad c = -2(J_c^T (\dot{J}_c \dot{q}))^T$

$\|Ax - b\|_2^2 = x^T (A^T A) x - 2(A^T b)^T x$

• centrodal momentum change cost  $36 \times 36$

$\ddot{p}_c + \dot{p}_c = 0$

$J_c \ddot{q} + \dot{J}_c \dot{q} + k_d J_c \dot{q} = 0$   
 $A = J_c \quad b = -\dot{J}_c \dot{q}$

$\lambda' = \lambda + 10 > 0$   
 $\lambda > -10$

$x^T A^T A x + (A^T b - \dot{p}_{des})^T x$   
 $C \in \mathbb{R}^{6 \times N} \quad N = \text{Dof}$

centrodal momentum matrix: maps joint rates to centrodal momentum

$\begin{bmatrix} \dot{p} \\ \dot{L} \end{bmatrix} = C \dot{q} + \dot{c} \dot{q} \quad \begin{bmatrix} p \\ L \end{bmatrix} = C q$

$\min \| \begin{bmatrix} \dot{p} \\ \dot{L} \end{bmatrix} \|^2 \rightarrow \min \| C \dot{q} + \dot{c} \dot{q} \|^2$

$A = C \quad b = -\dot{C} \dot{q} - \begin{bmatrix} p \\ L \end{bmatrix}_{des}$

$\frac{1}{2} x^T (J_c^T J_c) x - (A^T b)^T x$   
 $J_c^T (J_c \ddot{q} + k_d J_c \dot{q})$

Note: no self-collision cost included here

$H = C \dot{q} V$   
 $\frac{\partial H}{\partial t} = \dot{C} \dot{q} + C \ddot{q}$

Desired Centroidal Momentum

$$\begin{bmatrix} \text{angular} \\ \text{linear} \end{bmatrix} \begin{matrix} 3 \\ 3 \end{matrix}$$

$$\|H - H_{des}\|^2 \quad \text{constant}$$

$$\cancel{H^T H - 2H^T H_{des} + H_{des}^T H_{des}}$$

$$(C\dot{V} + \dot{C}V)^T (C\dot{V} + \dot{C}V) - 2(C\dot{V} + \dot{C}V)^T H_{des}$$

$$\cancel{H_{des}^T H_{des}}$$

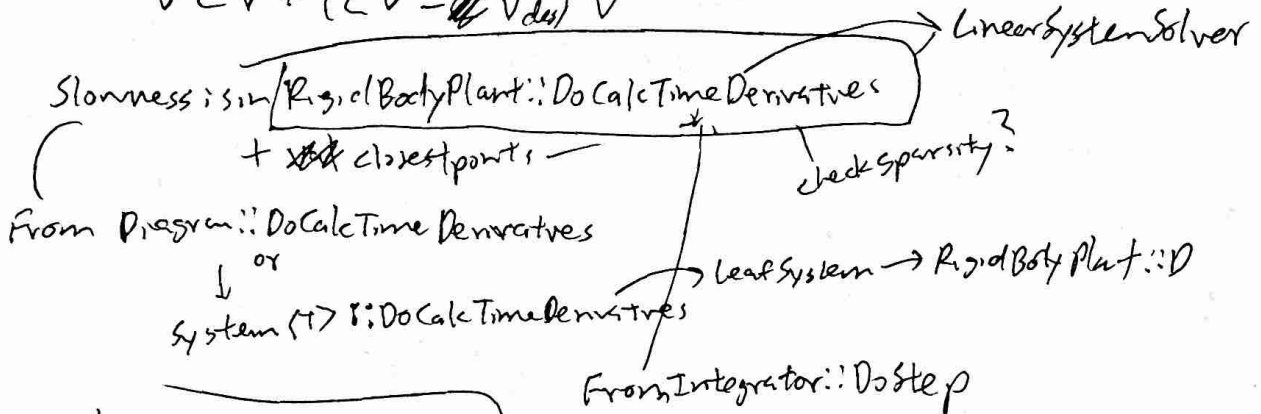
$$\min H^T H - 2H^T H_{des}$$

$$(C\dot{V} + \dot{C}V)^T (C\dot{V} + \dot{C}V) - 2(C\dot{V} + \dot{C}V)^T H_{des}$$

$$(C\dot{V})^T (C\dot{V}) + 2(C\dot{V})^T (\dot{C}V) + (\dot{C}V)^T (C\dot{V}) + (\dot{C}V)^T (\dot{C}V) - 2(C\dot{V} + \dot{C}V)^T H_{des}$$

constant

$$\dot{V}^T C \dot{V} + (\dot{C}V - \dot{V}_{des})^T \dot{V}$$



Look for IK solver

0.1

• New QP Plan type with separate body motions

• Try different integration scheme

50x50

colPivotQR 50000 ns  
Jacobi SBT 700000

$$x_{k+1} = \begin{bmatrix} 5/11 \\ -4/11 \\ 17/11 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

# Drake QP inverse dynamics controller

Dynamics:  $M(q)\ddot{v} + h(q, v) = S\tau + J^T\lambda$

acceleration / select a matrix

contact wrench in world frame

In actual implementation:

$M(q)\ddot{v} + h(q, v) = S\tau + J^T B \beta$

basis

replaced by set of point forces applied at contact points

Dynamics eq  
vel eq  
contact force basis  
torque limit eq  
LF, RF contact cost  
cen mom change cost  
pelvis, torso cost  
vel cost  
basis reg cost

Desired motions:  $a_{des}$  in Cartesian

$a_{des} = J\dot{v} + \dot{J}v$

linear wrt  $\dot{v}$

$S = \begin{bmatrix} 0 \\ I \end{bmatrix}$  first 6 zeros

becomes  $\tau = (Torque_{linear})X + (Torque_{constant})$   
 $[M, -J^T B] \begin{bmatrix} \dot{v} \\ \beta \end{bmatrix} + [h]$

$\tau = M\ddot{v} + h - (J^T B)_\ell \beta$

where  $\ell$  means bottom  $N_\tau$  rows

Just solve for  $\dot{v}$  and  $\beta$

Constraints  $X = \begin{bmatrix} \dot{v} \\ \beta \end{bmatrix}$

equality:

- Dynamics
- contact constraints

can also be high weight cost terms

Inequality:

- Joint torque limit
- Limit on  $\beta$  (forces)

$\tau_{min} - h \leq M\ddot{v} - J^T B \beta \leq \tau_{max} - h$

$\min (J_{com}\dot{v} + \dot{J}_{com}v - \ddot{p}_{com})^2 + (\dot{v} - \dot{v}_{des})^2 + \text{body acceleration cost terms}$

36 DOF  $\dot{v}$   
32  $\beta$  basis weights  
68 total

① Dynamics eq  
 $A_{eq} 6 \times 68$  start 0

② vel eq  
 $15 \times 36$  0

Q had an infinity

$Ax = b$   
 $\begin{bmatrix} 15 \\ 68 \end{bmatrix} \begin{bmatrix} 36 \\ 68 \end{bmatrix} = \begin{bmatrix} 15 \\ 68 \end{bmatrix}$

~~contact force basis~~  $\lambda \geq 0$

$A_{ineq} 32 \times 32$  start 36

torque limit  $30 \times 68$  start 0

costs  
left foot contact

$36 \times 36$  start 0

right foot contact

$36 \times 36$  0

③ centroidal momentum change

$36 \times 36$  0

pelvis, torso, vel  $36 \times 36$  0

basis reg

$32 \times 32$  36

• make general methods for getting Q, c,  $A_{eq}$ ,  $A_{ineq}$ ,  $b_{eq}$ ,  $b_{ineq}$  from QP specifications in Math Program

see `mosek_solver.cc`

extract to somewhere else

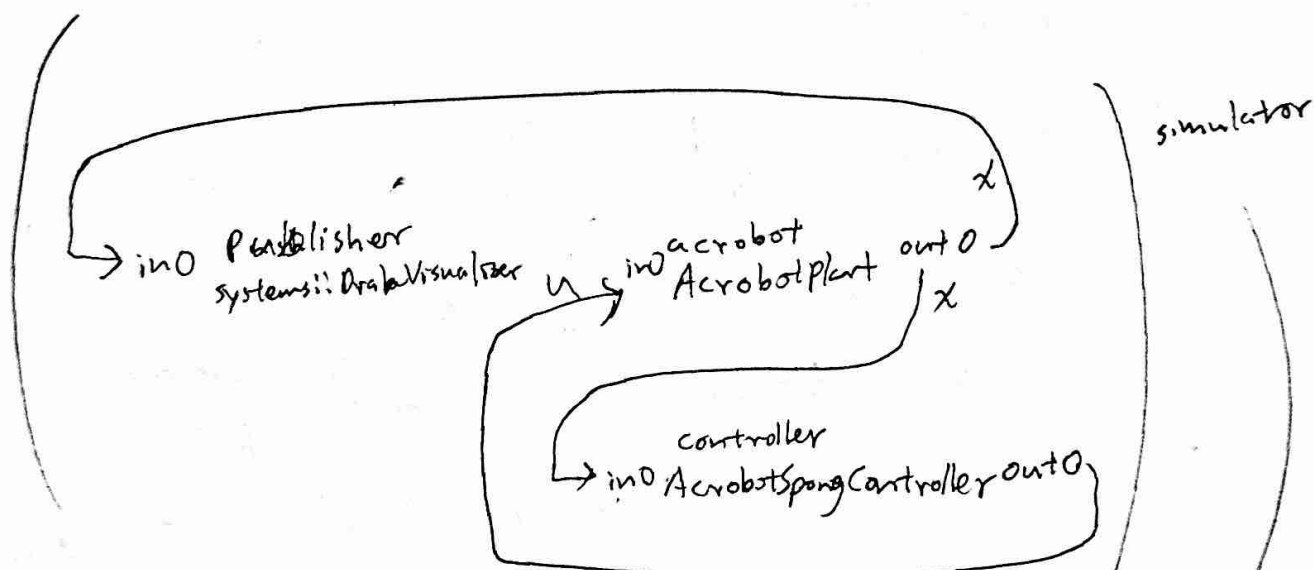
$X^T (J_c^T J_c) X + J_c^T (\dot{J}_c v)$

try on Drake

Message Naveen  
switched to Groeb for mosek

Look at PlanEval System

# Drake Acrobot swinging example



Basic procedure:

1. Set up diagram w/ nodes
  - publisher
  - robot
  - controller

2. ~~Build~~ Build diagram

3. Set up simulator w/ diagram

4. Initialize ~~diagram~~ simulator → start (stepTo(time(s)))

Diagram  
↓ goes into  
system  
↓ goes into  
simulator

just to set initial state

acrobot\_context  
via GetMutableSubsystemContext

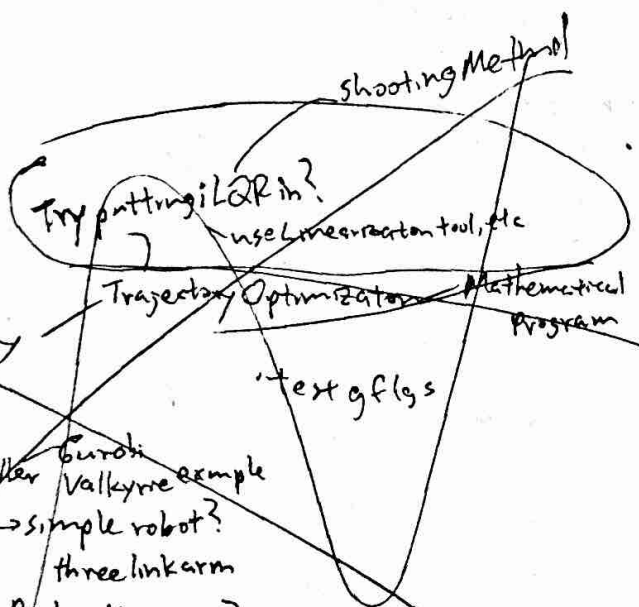
Controllers  
class with:

input port (state)

output port (plantInput,

control law { function to calculate controls })

Where are system dynamics (eg. for pendulum) calculated?



Look at:

- systems directory
- LCM basics
- Humanoid controller
- URDF parsing → simple robot? three link arm
- way to visualize Drake diagram?
- System::ToAutoDiffXd Support in System — how?

• see acrobot\_spong\_controller

for how to ~~use~~ linearize plant and use LQR

also note that acrobot example has state receiver and channel sender instead of just a plant, since it's meant to deal w/ a real robot

plant inside controller just tracks real robot

$3 \times 10^{-4}$

0.0033

0.05

PlanEval ~~state~~

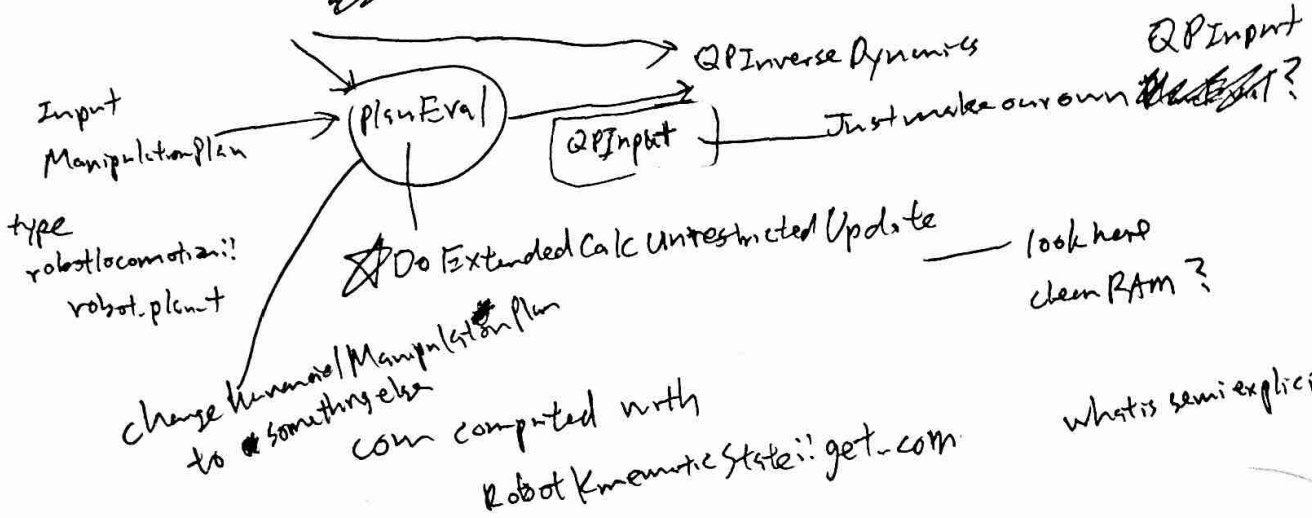
Try larger timesteps

inputs:

- state estimate
- ManipPlan

Outputs:

RobotStateMsg  
HumanoidStatus



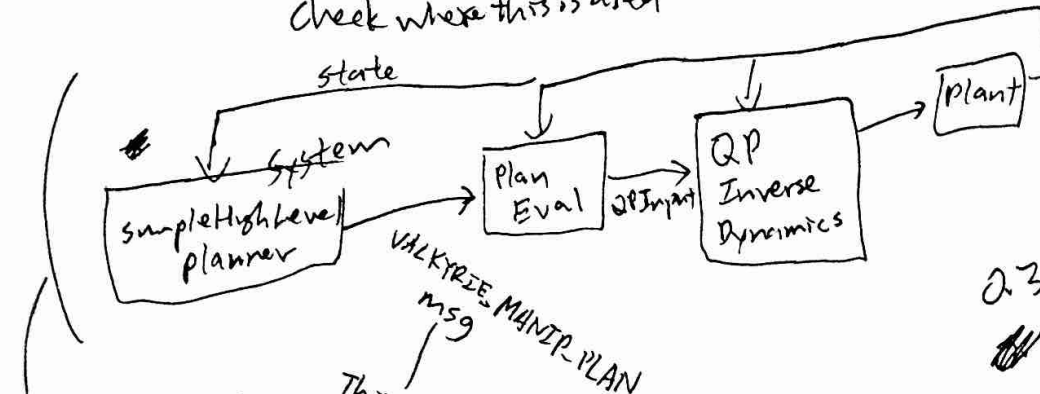
Try hardcoding desired centroidal momentum PD controller

Is control loop running at same rate as simulation?  
No, see dt in PlanEvalBaseSystem, QPInverseDynamicsSystem

student Teacher  
Person

Check where this is used

Person p = new Student



0 1 2 3

0.308

-0.39

0.93

make this

LCMDrivenLoop(Diagram) subscriber, null, lcm, timeconverts  
SimpleHighLevelPlanner(System)  
Diagram w/LCM subs, pubs

LCMDrivenLoop

Blocked by robot statesub

(see Valkyrie balancing demo, humanoid-controller.h)

This message type (class) contains the details on how to process desired behavior from high-level planner into QPInput. (desired accelerations and positions)