

MAE 171A Final Review

review slides

Error Analysis

Transducers: energy drawn from measured system $\leq \frac{1}{100}$ measuring energy

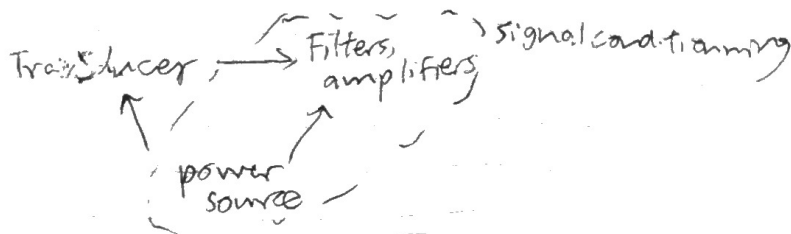
Key words: accuracy, precision, resolution, threshold, repeatability, reproducibility

Fahrenheit: 0 \approx coldest temp in Western Europe

100 \approx hottest \approx

32 freezing

212 boiling



Zero order hold

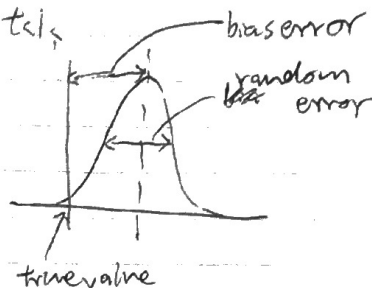


Multiplexer: N input channels \rightarrow one ADC

switching rate, cross talk

4 types of errors

- intrinsic
- application
- interface
- sampling and approximation



Integration

- Riemann sums
- Trapezoidal rule (linear, 1st order approx)
- Simpson's rule (parabolic, 2nd order approx)

$$\text{Variance } \sigma^2 = \frac{\sum (e_i - \mu)^2}{N}$$

$$\text{Gaussian } p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$$

10 20 30
0.68, 0.95, 0.997

$$z\text{-value} = \frac{e_i - \mu}{\sigma}$$

$$t\text{-distribution: } t = \frac{\bar{e}_i - \mu}{s/\sqrt{n}} \quad s: \text{sample SD, } s = \left[\frac{\sum (e_i - \bar{e})^2}{n-1} \right]^{1/2}$$

Larger n \rightarrow close to normal dist
 $n > 30$

Taylor Series

$$F = f(m_1, m_2, \dots, m_n)$$

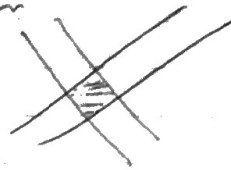
$$\delta F = \pm \left[\left(\frac{\partial F}{\partial m_1} \right)^2 \delta m_1^2 + \left(\frac{\partial F}{\partial m_2} \right)^2 \delta m_2^2 + \dots + \left(\frac{\partial F}{\partial m_n} \right)^2 \delta m_n^2 \right]^{1/2}$$

Water Tunnel

Laser (monochromatic beams cross) \rightarrow fringe pattern

(Gaussian intensity distributions of beams
 \rightarrow " of fringe

Faster particles \rightarrow higher f of intensity



$$p = \rho gh, \quad U_{\infty} = \left(\frac{2p_i \Delta h g}{\rho \omega} \right)^{1/2}$$

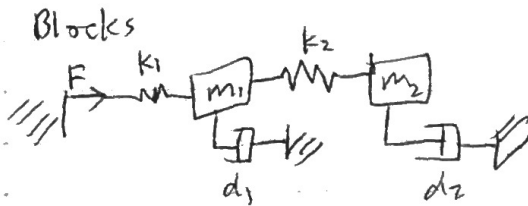
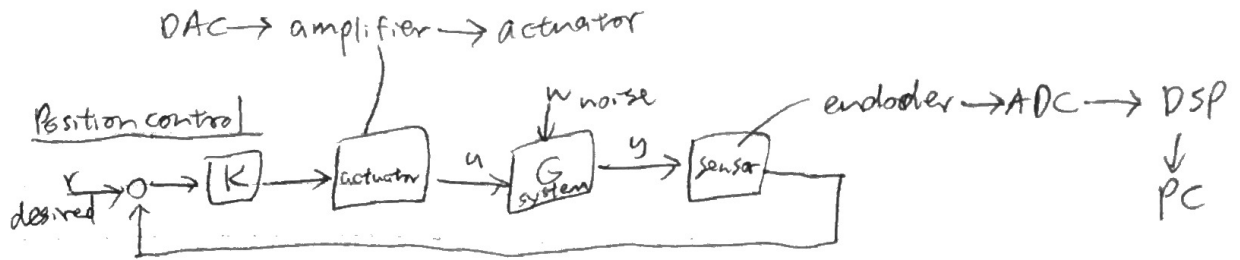
$$D_{\text{drag}} = \int p \times \cos \theta \times da$$

$$C_p = \frac{p - p_{\text{ref}}}{\frac{1}{2} \rho U^2} \rightarrow C_{Dp} = \frac{\int C_p \cos \theta da}{A} \rightarrow C_{Dp} = \int_0^{2\pi} C_p \cos \theta d\theta$$

relative pressure throughout a flow field

quantifies drag on object in environment

Why is $C_{Dp} \approx 0$ around cylinder?



$$m_1 \ddot{x}_1 = -k_1 x_1 - d_1 \dot{x}_1 - k_2 (x_1 - x_2) + F$$

$$m_2 \ddot{x}_2 = k_2 (x_1 - x_2) - d_2 \dot{x}_2$$

✓ Laplace

TR $Y(s) = T(s) U(s)$

← voltage to motor

← encoder position

$$\omega_n = \sqrt{F/k_m}$$

$$\omega_d = \omega_n \sqrt{1 - \beta^2}, \quad \beta = \frac{d}{2\sqrt{k_m}}$$

$$\hat{\omega}_d = 2\pi \frac{n}{t_n - t_0}$$

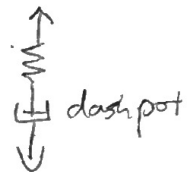
$$C(s) = k_p + \frac{k_0}{s} + k_d s \quad \rightarrow \quad K(s) = \frac{k_d s^2 + k_p s + k_i}{s}$$

root-locus (OLHP), Nyquist (not encircling -1)

Stress relaxation: Maxwell model

← plates fixed, stress decreases

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta}$$



Creep: Voight or four element model

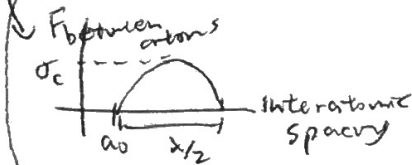
constant load



Fracture mechanics

Two ways to estimate strength

- theoretical cohesive strength: Force necessary to break atomic bonds
- work of fracture to create new surfaces: area under stress-strain curve



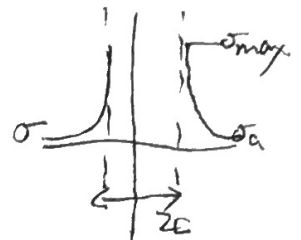
$$\sigma = E \epsilon$$

$$\lambda \sim 2a_0$$

$$\frac{E}{a_0} = \frac{2\pi\sigma_c}{\lambda} \rightarrow \sigma_c = \frac{E}{\pi}$$

$$\text{Work} = \int_0^{\lambda/2} \sigma_c \sin\left(\frac{2\pi x}{\lambda}\right) dx$$

$$\sigma_c = \sqrt{\frac{E\sigma_c}{a_0}} \quad \gamma_s = \text{surface energy}$$



$$\frac{\sigma_{max}}{\sigma_a} = 1 + \frac{2c}{b} \quad \text{Stress concentration factor}$$

$c/b = \text{larger for sharper cracks, } \frac{c}{b} = 1 \text{ for circle}$

$$\frac{\sigma_{max}}{\sigma_a} = K_t \quad \text{Stress concentration factor for sharp notches}$$

Griffith: Fibers are weakened by microscopic flaws on surface or interior of fiber

- Crack propagation \rightarrow release of elastic strain energy \rightarrow Decrease in energy
- crack extension \rightarrow new surfaces created \rightarrow increase of energy
- Plane strain condition

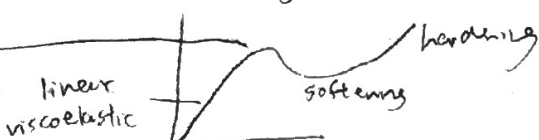
$$K_{Ic} = Y\sigma_c\sqrt{c} \quad \text{Fracture toughness [MPa}\sqrt{\text{m}}]$$

$Y = \text{geometrical constant}$

Intension: K_I : scale factor used to define magnitude of crack-tip stress field
 $K_I = K_{Ic}$ at onset of crack growth

Polymers: long chains of C, H, O molecules (ex. thermoplastics)

nonlinear viscoelastic



Glass transition temp

$T < T_g$ brittle, glassy

$T > T_g$ elastic & plastic deformation



Stress relaxation

