1 Quadratic program

$$\min_x \tfrac{1}{2} x^T \boxed{Q} x + c^T x \quad (+c^T c)\ ?$$

s.t. $A_1 x = B_1$, $A_2 x \le B_2$

Q has to be real symmetric

x: { u - control (motor torques); $\ddot{q}$ - configuration accelerations; f - contact forces ← replace with λ (friction cone) }

$X = (\ddot{q}, \lambda, u)$

constraints:

- satisfy dynamics eqn of motion (1)
- non-sliding of contact points (7)
- contact forces inside (linearized) friction cone (6)
- actuation torques within limits (4) (angles?)

→ weighted tasks (soft constraints)

Robot: $q = (\underbrace{x_0, \theta_0}_{\text{base link}}, \hat{q})$ — $\hat{q}$: joint angles vector

Why ~~do we need~~? ~~Doesn't ensure this~~? → what makes sliding contacts hard?

Why QPs? Any other convex form that can be solved just as reliably and quickly?

$$M(q)\ddot{q} + N(q,\dot{q})\dot{q} = \overbrace{M(q)\begin{pmatrix} g \\ 0_4 \\ 0_r \end{pmatrix}}^{F_g} + \begin{pmatrix} 0_3 \\ 0_4 \\ u \end{pmatrix} + \sum_{k,i} J_{tk}(a_{k,i})^T f_{k_i} \quad (1)$$

$\forall k,i \quad J_{tk}(a_{k,i})\dot{q} = 0$

$\forall k,i \quad f_{k,i} \in K_{k,i}$

$\forall j \quad u_{j,min} \le u_j \le u_{j,max}$ (4)

linearize ↓

$$f_{k,i} = \sum_{\mu=1}^{V_{k,i}} \lambda_{k,i,\mu} V_{k,i,\mu}$$

vecs defining cone

$\forall k,i,u \quad \lambda_{k,i,u} \ge 0$ (6)

time differentiate ↓

$$J_{tk}(a_{k,i})\ddot{q} + \dot{J}_{tk}(a_{k,i})\dot{q} = 0 \quad (7)$$

How much the end affector moves with small perturbations in q and $\dot{q}$

k: index of each link in contact

f_k: force at link k

$K_{k,i}$: friction cone (Coulomb)

$a_{k,i}$: point of contact

$J_{tk}(p)$: $3 \times (3+4+j)$ translational Jacobian of link k ~~...~~.

μ: friction coefficient

i:

Abe et al ensure stable contacts by placing lower limit on force

(1), (4), (6), (7) as $A_1 X = B_1$, $A_2 X \le B_2$

$X = (\ddot{q}, \lambda, u)$

$\lfloor Y \rfloor = \begin{pmatrix} Y_1 \\ \vdots \\ Y_n \end{pmatrix}$ (ex. > 50N for 70kg character)

$\lceil Y \rceil = (Y_1 \dots Y_n)$

$$A_1 = \begin{pmatrix} M(q) & -\lceil J_{tk}(a_{k,i})^T V_{k,i,\mu} \rceil_{k,i,\mu} & -\begin{pmatrix} 0_{3\times r} \\ 0_{4\times r} \\ 1_{r\times r} \end{pmatrix} \\ \lfloor J_{tk}(a_{k,i}) \rfloor_{k,i} & 0_{3z\times\beta} & 0_{3z\times r} \end{pmatrix}$$

$(3+\cancel{4r})\times\beta$

should this be $\lfloor\cdot\rfloor$?

$\alpha = \dim(\ddot{q}) = 3+4+r$

$\beta = \dim(\lambda) = \#\text{contacts} \times \#\text{of force vecs}$

$\gamma = \dim(u) = r$ (4?)

$z = ?$

when would it not be 4? why does it have to be general

$$B_1 = \begin{pmatrix} -N(q,\dot{q})\dot{q} + M(q)\begin{pmatrix} g \\ 0_4 \\ 0_r \end{pmatrix} \\ \lfloor -\dot{J}_{tk}(a_{k,i})\dot{q} \rfloor_{k,i} \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0_{\beta\times\alpha} & -1_{\beta\times\beta} & 0_{\beta\times\gamma} \\ 0_{\gamma\times\alpha} & 0_{\gamma\times\beta} & -1_{\gamma\times\gamma} \\ 0_{\gamma\times\alpha} & 0_{\gamma\times\beta} & 1_{\gamma\times\gamma} \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0_\beta \\ -u_{min} \\ u_{max} \end{pmatrix}$$

Inequality constraint

$A_2 X \leq B_2$

?

$$\begin{array}{l} \beta \\ \gamma \\ \gamma \end{array}\begin{pmatrix} 0_{\beta\times\alpha} & -1_{\beta\times\beta} & 0_{\beta\times\gamma} \\ 0_{\gamma\times\alpha} & 0_{\gamma\times\beta} & -1_{\gamma\times\gamma} \\ 0_{\gamma\times\alpha} & 0_{\gamma\times\beta} & 1_{\gamma\times\gamma} \end{pmatrix}\begin{pmatrix} \ddot{q} \\ \lambda \\ u \end{pmatrix}\begin{array}{l} \alpha \\ \beta \\ \gamma \end{array} \leq \begin{pmatrix} 0_\beta \\ -u_{min} \\ u_{max} \end{pmatrix}$$

$$\begin{pmatrix} \ddot{q} \\ \lambda \\ u \end{pmatrix}\begin{array}{l}]\alpha \\ ]\beta \\ ]\gamma \end{array}$$

~~This is wrong?~~

~~Should A_2 be~~ ~~Is this wrong?~~

~~A_2 proposed $= A_2^T = (0.$~~

Abe's contact point tricks!

- Threshold (conservative) for contact force magnitude
- Conservative estimate for contact region, strict interior of actual

$$0\ddot{q} - 1_{\beta\times\beta}\lambda_\beta + 0u \leq 0_\beta \longrightarrow \lambda_\beta \geq 0 \quad (6)$$

$$0\ddot{q} + 0\lambda - 1_{\gamma\times\gamma}u \leq -u_{min} \longrightarrow u \geq u_{min}$$

$$0_{\gamma\times\alpha}\ddot{q} + 0\lambda + 1_{\gamma\times\gamma}u \leq u_{max} \longrightarrow u \leq u_{max} \quad (4)$$

✓

- Equality constraint should have (1), (7)

Figure out first:

(1) dynamics equation

Jacobian

(7) nonsliding constraint — Why do we need this? Doesn't friction cone constraint take care of this? ~~ie. If there is contact and~~

Assume $q, \dot{q}$ static at each timestep

↓

Dynamics linear in $\ddot{q}$

$$M(q)\ddot{q} - [J_{tk}(a_{ki})^T V_{k,i,\mu}]_{k,i,\mu}\lambda - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}u = -N(q,\dot{q})\dot{q} + M(q)\begin{pmatrix} g \\ 0 \\ 0 \end{pmatrix}$$

$$[J_{tk}(a_{k,i})]_{k,i}\ddot{q} + 0\lambda + 0u = [-\dot{J}_{tk}(a_{k,i})\dot{q}]_{k,i}$$

$$X = \begin{pmatrix} \ddot{q} \\ \lambda \\ u \end{pmatrix}\begin{array}{l} \alpha \\ \beta \\ \gamma \end{array}$$

$$M(q)\ddot{q} + \underbrace{N(q,\dot{q})}_{\text{coriolis}}\dot{q} = M(q)\begin{pmatrix} g \\ 0 \\ 0 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}}_{\text{motor torques}} + \underbrace{[J_{tk}(a_{ki})^T V_{k,i,\mu}]_{k,i,\mu}\lambda}$$

$$J_{tk}(a_{k,i})\ddot{q} + \dot{J}_{tk}(a_{k,i})\dot{q} = 0$$

$$\begin{bmatrix} (3+4+r)\times 1 \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}$$

J: $[3\times(3+4+r)]^T$

$\downarrow$ J^T, V

$[(3+4+r)\times 3]\,[3\times 1]$

$[(3+4+r)\times 1]\ \lambda\ 1\times 1$

$$= \sum_{k,i} J_{tk}(a_{k,i})^T f_{ki}$$

$$= \sum_{k,i} J_{tk}(a_{k,i})^T \sum_{\mu=1}^{4} \lambda_{k,i,\mu} V_{k,i,\mu}$$

2D particle

$\dot{g} = J_g \dot{q}$ $\quad Q = [J_g^T J_g]$ $\quad c = [-J_g^T [k_g (g_{ref} - g) - k_v J_g \dot{q} - \dot{J}_g \dot{q}]]$

$J_g = I$

Quadratic objectives — tasks/features

$$g: \mathbb{R}^{3+4+r} \rightarrow \mathbb{R}^d \quad \text{— dim(task)}$$

Should this be flipped?

$$\dot{g} = J_g \dot{q}$$

J_g – Jacobian of task $(3+4+r) \times d$

how small changes in feature affect q ?

Set point objective

From Lasa heuristic

$$E_{\substack{\text{setpoint,}\\\text{goal}}}(X) = \frac{1}{2} \| \overbrace{k_p(g_{ref} - g) - k_v \dot{g}}^{\ddot{g}_{des}} - \ddot{g} \|^2$$

K_p, K_v hand tuned, $K_v = 2\sqrt{K_p}$

$$= \frac{1}{2} X^T Q X + C^T X + \cancel{\tfrac{1}{2} C^T C}$$

Does $\ddot{g}_{des}$ need to be PD controller?

$$Q = \begin{pmatrix} J_g^T J_g & O_{\alpha \times \beta} & O_{\alpha \times \gamma} \\ O_{\beta \times \alpha} & O_{\beta \times \beta} & O_{\beta \times \gamma} \\ O_{\gamma \times \alpha} & O_{\gamma \times \beta} & O_{\gamma \times \gamma} \end{pmatrix}, \quad C = \begin{pmatrix} -J_g^T (K_p(g_{ref} - g) - K_v J_g \dot{q} - \dot{J}_g \dot{q}) \\ O_\beta \\ O_\gamma \end{pmatrix}$$

~~Target~~ See Disney Research LA paper "Timing test" for PHRI

loose math

$$X = \begin{pmatrix} \ddot{q} \\ \lambda \\ u \end{pmatrix}$$

$$\cancel{\ddot{q}^T J_g^T J_g \ddot{q} - J_g^T (K_p(g_{ref} - g) - K_v J_g \dot{q} - \dot{J}_g \dot{q})}$$

minimize

$$\frac{1}{2} \ddot{q}^T J_g^T J_g \ddot{q} + (-J_g^T (k_p(g_{ref} - g) - k_v J_g \dot{q} - \dot{J}_g \dot{q})) \ddot{q} + \cancel{\tfrac{1}{2}(\cdots)^T(\cdots)}$$

~~Where does self collision constraint come in?~~

CoM task can be looser, to just keep CoM inside support polygon?

Target objective uses constant-jerk reference trajectory

Target objective

Check this

Also where is stiffness? in this task

$$\ddot{g}^i_{ref}(t) = \left(1 - \frac{t - t_0}{t_f - t_0}\right) \phi_{i,t_0} + \frac{t - t_0}{t_f - t_0} \psi_{i,t_0}, \quad t \in [t_0, t_f]$$

$$\begin{bmatrix} T^2/3 & T^2/6 \\ T/2 & T/2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} y_T - y_0 - \dot{y}_0 T \\ \dot{y}_T + \dot{y}_0 \end{bmatrix}$$

$$\frac{t_f - \cancel{t_0} - t + t_0}{t_f - t_0}$$

Found by integrating twice and writing boundary value conditions

solve with least-squares; pseudo-inverse

where $T = t_f - t_0$

$$\int_t^{t_f} \ddot{g}_{ref}\, dt = \int \left(1 - \frac{t - t_0}{t_f - t_0}\right) \phi_{i,t_0}\, dt + \int \frac{t - t_0}{t_f - t_0} \psi_{i,t_0}\, dt$$

$$\dot{g}_f - \dot{g} = \int \frac{(t_f - t)}{t_f - t_0} \phi_{i,t_0}\, dt + $$

$$E_{tgt,g}(X) = \sum_{i=1}^{d} \frac{1}{2} (\ddot{g}^i_{ref}(t_0) - \ddot{g}^i)^2$$

$$= \sum_{i=1}^{d} \frac{1}{2} (\phi_{i,t_0} - \ddot{g}^i)^2$$

First term ϕ dominates in beginning, then replaced gradually be second term ψ.

For large values $T >> 0$, equivalent to cubic Hermite interpolation

Does Karim's formulation use minimum torque objective?

If damping is not defined, is it just based on stiffness w/ heuristic?

Prioritized optimization (Lasa, Mordatch, Hertzmann)
(Lexicographic? Strictly prioritized?)

$$h_i = \min_x E_i(x)$$

s.t. $E_k(x) = h_k, \forall k < i$

$C(x) = 0$

$Dx + f \geq 0$

$h_1 = \min_x E_1(x)$
s.t. dynamics

$h_2 = \min_x E_2(x)$
s.t. dynamics
$+ h_1 = E_1(x)$

Key idea:
Optimal solutions to a quadratic must lie on a linear subspace

$E_k(x) = \|A_k x - b_k\|^2$

$A_k(x - x_k^*) = 0$

optimum of previous iteration

$A_k(x - x_k^*) = 0, \forall k < i$

BUT this has difficulties with rapidly changing constraints
Fails on active set methods

Reparametrization method — more flexibility

$E_1(x) = \frac{1}{2}\|A_1 x - b_1\|^2$

$\dot{E}_1(x) = A_1 x - b = 0$

$\frac{1}{2}(ax - b)^2$

$ax - b$

$X(w) = C_1 w + d_1$

null space basis
$C_1 = null(A_1)$
Find with SVD

any minimizer that satisfies dynamics constraints
Find with QP

plug into $E_2(x)$

$E_2(w) = \|A_2 X(w) - b_2\|^2$

$= \|A_2 C_1 w + A_2 d_1 - b_2\|^2$

plug into $Dx + f \geq 0$

$DC_1 w + Dd_1 + f \geq 0$

$d_2 = \arg\min_w E_2(w)$
s.t. $DC_1 w + Dd_1 + f \geq 0$

$x^* = C_1 d_2 + d_1$

Repeat recursively

Instead of equality constraints $C(x) = 0$

infeasible solutions due to numerical issues

Add top-priority objective $E(x) = \|C(x)\|^2$

minimized to zero

Combine objectives at same priority level with weights

Prioritized controllers allow control to be layered

Hierarchical control built into one optimization vs. hierarchy of simplified models

Disadvantages:

- Slower and more work to implement
  - QP + SVD at each priority level
  - Double precision arithmetic

$$Q_{sum} = \sum_{k=1}^{N} W_k Q_k \ , \ C_{sum} = \sum_{k=1}^{N} W_k C_k \longrightarrow \min_X \frac{1}{2} X^T Q_{sum} X + C_{sum}^T X$$

$$\text{s.t. } A_1 X = B_1,\ A_2 X \le B_2$$

- COM task is usually as setpoint objective, trying to keep the COM's horizontal position above the mid-point between two footprints.

Can ~~we generalize~~ this to other situations? — Yes

ex.

Place COM desired pos near midpoint of horizontal projection of all contact points.

This is a simple strategy, doesn't prevent falling on its own. Need to use in combination with standing posture task.

Recovery strategy when QP fails?

Concact change event

$$\eta = (e, s, t, \sigma) \in E \times S \times \mathbb{R} \times \{0, 1\}$$

link index / environment surface / timing / type 0: leaving contact, 1: entering contact

For short time after entering contact, do contact stabilization w/ QP (pause human tracking, new QP)

$\sigma = 0$: $E_1 \leftarrow E_1 \setminus \{e_j\}$, $E_2 \leftarrow E_2 \cup \{e_j\}$

$\sigma = 1$: $E_1 \leftarrow E_1 \cup \{e_j\}$, $E_2 \leftarrow E_2 \setminus \{e_j\}$

Track human marker ~~~~ while keeping contact end effector's tangential (~~surface~~ part) velocity zero, get contact orientation task to 1

What does this actually mean? Just flat contact?

Motion retargeting paper

$$\min_{\ddot{q}, u, f_c} \sum_{k=1}^{m} W_k \| \ddot{\tau}_k - \ddot{\tau}_k^d \|^2 \quad (1)$$

$$\text{s.t. } M\ddot{q} + N = Su + J_c^T \Lambda_c f_c \quad (2)$$

$$J_c \ddot{q} + \dot{J}_c \dot{q} = 0 \quad (3)$$

$$f_c \ge 0 \rightarrow f_c = \sum \lambda_i V_i \quad (4)$$

$$\lambda_i \ge 0$$

$$u_{min} \le u \le u_{max} \quad (5)$$

Dropped during initialization of posture, along w/ f_c and u from decision variables — Only Link position tasks + half-sitting posture task

$$\frac{q_{min} - q - \dot{q}\Delta t}{\frac{1}{2}\Delta t^2} \le \ddot{q} \le \frac{\dot{q}_{max} - \dot{q}}{\Delta t} \quad (6)$$

Joint limit constraint — Look for details of this in ladder climbing paper

$$\ddot{d}(b_1, b_2) \ge \frac{1}{\Delta t}\left(-\zeta \frac{d(b_1, b_2) - \delta_s}{\delta_i - \delta_s} - \dot{d}(b_1, b_2)\right) \quad (8)$$

collision avoidance constraint b/w b_1 and b_2 of robot & env × pairs of possible collisions

$$\frac{\dot{q}_{min} - \dot{q}}{\Delta t} \le \ddot{q} \le \frac{\dot{q}_{max} - \dot{q}}{\Delta t} \quad (7)$$

Joint velocity limit constraint

ζ : damping coefficient

δ_s : security distance

δ_i : influence distance

Maintain two sets E_1, E_2

links in contact

Remove these ~~tasks~~ links' position and orientation tasks, since constraint 3 deals with that

Objective: tracking tasks (markers) + CoM (balance) + posture ~~~~

center of convex hull of ground projection of surfaces in E_1

Multirobot

why don't they acknowelege approaches of other DRC teams? or mordatch

What does a selection matrix look like?

Forces f: f_i^0, f_i^-, f_i^+ — forces applied by robot i on robots $j>i$

fixed env on robot — forces applied by robots $j<i$ on robot i

$\begin{bmatrix} F^0 \\ F^- \\ F^+ \end{bmatrix}$ stack of vectors

$F^- \in \mathbb{R}^{3k}$ — k: # of forces

Newton's third law

1. rest → remain at rest
2. F=ma
3. ~~opp~~ equal and opposite

$F^+ = \underbrace{(\Pi \otimes I_3)}_{\mathbb{R}^{3k \times 3k}} F^-$

$\begin{bmatrix} \Pi_{11} I_3 & \cdots & \Pi_{1k} I_3 \\ \vdots & \ddots & \vdots \\ \Pi_{k1} I_3 & \cdots & \Pi_{kk} I_3 \end{bmatrix}$

$\Pi \in \mathbb{R}^{k \times k}$ $\quad \otimes$ Knonecker product

$\Psi = \Pi \otimes I_3$

Decomposed into selection matrix blocks $\Psi_i \in \mathbb{R}^{3k_i \times 3k}$

$\Psi = \begin{pmatrix} \Psi_1 \\ \vdots \\ \Psi_n \end{pmatrix}$ — for each i, $f_i^+ = \Psi_i F^-$

~~this should be F^-?, pg 3~~

Typo in Multi-contact... pg 6 "monographs"

Dynamics:

$$M_i(q_i)\ddot{q}_i + N_i(q_i,\dot{q}_i) = J_{i,0}^T f_i^0 + J_{i,-}^T f_i^- - J_{i,+}^T \Psi_i F^- + S_i \tau_i$$

↓

$$M(q)\ddot{q} + N(q,\dot{q}) = J_0^T F^0 + (J_- - \Psi^T J_+)^T F^- + S\tau$$

coincidence of contact points: $(J_- - \Psi^T J_+)\dot{q} = 0$ $\quad$ Lagrange multipler?

$J_0 \dot{q} = 0$

To deal with model error and contact state mismatch, they use force controller.

Admittance controller (on any link equipped w/ F/T sensor)

↓

QP end-effector task $\quad V = K_I(f_{target} - f_{sensor} \cdot n)n \quad$ clamp $(V_{min}, V_{max}) = -0.05, 0.05$

gain $K_I = 5 \times 10^{-4}$ $\quad$ force along $\quad$ normal of surface

This lets manipulator "search" for a surface it is supposed to be in contact with

$\tilde{V}$

Low-pass filtered → QP task $(g^{ref}, \dot{g}^{ref}, \ddot{g}^{ref})$

Is "QP force task" same as end-effector task?

Derivation of setpoint task

$g = f(q)$ generalized state

↓

$\dot{g} = J\dot{q}$

↓ product rule

$\ddot{g} = J\ddot{q} + \dot{J}\dot{q}$

$\ddot{g}_{des} = k_p(g_{des} - g_{cur}) - k_d \dot{g}_{cur}$

$\|\ddot{g}_{des} - \ddot{g}\|^2$

↓ substituting

$\|k_p(g_{des} - g) - k_d\dot{g} - J\ddot{q} - \dot{J}\dot{q}\|^2$

Our decision variable is $\ddot{q}$

Rearranging...

$\|-J\ddot{q} + [k_p(g_{des} - g) - k_d\dot{g} - \dot{J}\dot{q}]\|^2$

recall: $\|Ax - b\|_2^2 = x^T(A^TA)x - 2(A^Tb)^Tx + b^Tb$

$A = -J \quad b = k_p(g_{des} - g) - k_d\dot{g} - \dot{J}\dot{q}$

$x = \ddot{q}$

$\ddot{q}^T(J^TJ)\ddot{q} + 2(J^Tb)^T\ddot{q} + b^Tb$

Ignore constants and stuff that doesn't depend on $\ddot{q}$

Wieber 2005

Jacobian: How much does output $\vec{y}$ change in $\vec{y} = f(\vec{x})$ if x_i ~~changes~~ increases by a little?

Linear approximation of f at x_0

$$J = \left[\frac{\partial f}{\partial x_1} \cdots \frac{\partial f}{\partial x_n}\right] = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \\ \vdots & \ddots & \\ \frac{\partial f_m}{\partial x_1} & & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

q - configuration vector

$\dot{q}, \ddot{q}$

why?

$$\dot{x}_k = J_{tk}(q)\dot{q} \quad , \quad \ddot{x}_k = J_{tk}(q)\ddot{q} + \dot{J}_{tk}(q,\dot{q})\dot{q}$$

$$\omega_k = J_{Rk}(q)\dot{q} \quad , \quad \dot{\omega} = J_{Rk}(q)\ddot{q} + \dot{J}_{Rk}(q,\dot{q})\dot{q}$$

virtual work $\rightarrow$ Gauss's principle

~~Objects with~~

"Acc of set of solids s.t. ~~some~~ constraints deviates the least possible from acc that it would have had without the constraints"

joints on robot

$$D = \sum_k \tfrac{1}{2}(\ddot{x}_k - \ddot{\underline{x}}_k)^T m_k (\ddot{x}_k - \ddot{\underline{x}}_k) + \tfrac{1}{2}(\dot{\omega}_k - \dot{\underline{\omega}}_k) I_k (\dot{\omega}_k - \dot{\underline{\omega}}_k)$$

deviation

Newton-Euler eqns

$$m_k \ddot{\underline{x}}_k = f_k$$

$$I_k \dot{\underline{\omega}}_k - \underbrace{(I_k \omega_k) \times \omega_k}_{\text{gyroscopic term, b.c. Euler eqn defined in frame attached to the solid}} = \tau_k$$

To minimize D, do some math

$$\frac{\partial D}{\partial \ddot{q}} = M(q)\ddot{q} + N(q,\dot{q})\dot{q} - F = 0$$

inertia matrix $M(q) = \sum_k J_{tk}^T m_k J_{tk} + J_{Rk}^T I_k J_{Rk}$

nonlinear dynamics $N(q,\dot{q}) = \sum_k J_{tk}^T m_k \dot{J}_{tk} + J_{Rk}^T I_k \dot{J}_{Rk} - J_{Rk}^T \underbrace{(I_k J_{Rk} \dot{q})}_{\text{skew symmetric matrix}} \times J_{Rk}$

forces $F = \sum_k J_{tk}^T f_k + J_{Rk}^T \tau_k$

$$\begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}$$

Note: different order from "Using a multi-objective ....." by Karim

$$q = \begin{array}{c} \begin{bmatrix} \hat{q} \\ x_0 \\ \theta_0 \end{bmatrix} \end{array} \begin{array}{l} \}\,r \\ 3 \\ 4 \end{array}$$

$$\dot{x}_k = R_0 \hat{J}_{Jk} \dot{\hat{q}} + \dot{x}_0 - (x_k - x_0) \times R_0 J_{R_0} \dot{\theta}_0$$

$$\omega_k = \hat{J}_{Rk} \dot{\hat{q}} + R_k^T R_0 J_{R_0} \dot{\theta}_0$$

$$J_{tk} = \left[R_0 \hat{J}_{Jk} \quad 1_{3\times 3} \quad -(x_k - x_0) \times R_0 J_{R_0}\right]$$

$$J_{Rk} = \left[\hat{J}_{Rk} \quad 0_{3\times 3} \quad R_k^T R_0 J_{R_0}\right]$$