Contact wrench set

$w = \begin{bmatrix} f \\ p \times f \end{bmatrix}$

↓

linearized friction cone

$w \in K = \text{Convex Cone}\left(\begin{bmatrix} e_1 \\ p \times e_1 \end{bmatrix}, \dots, \begin{bmatrix} e_{ne} \\ p \times e_{ne} \end{bmatrix}\right)$

↓

Aggregated contact wrench set is in CWS,

Minkowski sum of each individual wrench set

$CWS = K_1 \oplus K_2 \dots$

- sum of sets: $A \oplus B = \{a + b \mid a \in A, b \in B\}$

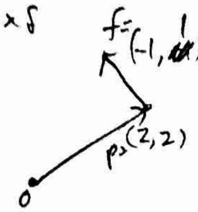
vs. union

ex. $A = \{1,2\}$, $B = \{3,4\}$

$A \oplus B = \{1+3, 1+4, 2+3, 2+4\}$

$A \cup B = \{1,2,3,4\}$

ex. $p \times f$




$\|f\| = \sqrt{2}$

$\|p\| = 2\sqrt{2}$

$\tau_0$ should be 4

$u \times v = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$ — vector

$(2,2) \times (-1, 1)$

$= 2(1) - 2(-1)$

$= 2 + 2 = 4$

$u \times v = \det(uv) = u_x v_y - u_y v_x$ — magnitude

$a \times b = \|a\| \|b\| \sin(\theta) n$

Decision variables

$r, \dot{r}, \ddot{r}, k_0, \dot{k}_0$

↑ com pos — angular momentum — minimize, humans do this — ZMP assumes zero

Feasible:

$a_i^T \begin{bmatrix} m\ddot{r} - mg \\ \dot{k}_0 - r \times mg \end{bmatrix} \leq 0, \; i = 0, \dots, n_a$

$\varepsilon_{max} = \min_{i=1,\dots,n_a} \bar{a}_i \begin{bmatrix} \dots \\ \dots \end{bmatrix}$

concave!

check / pointwise ~~min~~ among linear functions of decision variables

max this is convex

just have to make sure it can be minimized!

References for costs (ex. PD) can be arbitrary?

~~Are these constraints~~ or ~~velocities are~~

Costs have to be quadratic

Feasibility

$\dot{h} - w_g \overset{?}{\in} CWS$ — gravity wrench $w_g = \begin{bmatrix} mg \\ r \times mg \end{bmatrix}$

$h \in \mathbb{R}^6$ total robot momentum

$\dot{h} \approx \ddot{x}$

Once contact wrench $(f, \tau)$ is within CWS, we can compute necessary joint torques. → May not meet torque constraints

Atlas can ignore this

- Contact wrench set margin

Analogous to ε-ball in grasping

smallest magnitude of wrench disturbance that robot cannot resist, given contact locations and constraints

Goal: find motion s.t. CWS margin is maximized


How to check this?

Can we do a weighted cost between $\dot{h}$ and $B_\varepsilon$?

$(\dot{h}_{des} - \dot{h})$ ↑ min — $\varepsilon$ ↑ max

not linear — or $\varepsilon$ as constraint

Transform CWS convex hull ⟶ $\cap$ of halfspaces

use Double description method [44]

or

Q-hull [4] $\quad CWS = \{w \mid a_i^T w \leq 0, i = 1, \dots, n_a\}$

normal vectors

Centroidal momentum - stability criterion

$$f_G = M(g - \ddot{p}_G) \quad p_G = p_{COM}$$
$$\tau_G = p_G \times M(g - \ddot{p}_G) - \dot{L}$$

COG = COM when gravitational field is uniform across object

gravity + inertia forces

p: position wrt world frame $\Sigma_W$

where

$$M = \sum_{i=1}^{N} m_i$$

$$L = [L_x \; L_y \; L_z]^T = \sum_{i=1}^{N} \left[ m_i (p_{Li} - p_G) \times \dot{p}_{Li} + I_i \omega_i \right]$$

This term incorporate current momentum? Or coriolis forces from links ?

$$g = [0 \quad 0 \quad -g]^T$$

Make balance task by trying to move towards "center" of this convex cone?

Contact forces and torques

$$f_c = \sum_{k=1}^{K} \sum_{l=1}^{L} \varepsilon_k^l (n_k + \mu_k t_k^l)$$

$$\tau_c = \sum_{k=1}^{K} \sum_{l=1}^{L} \varepsilon_k^l p_k \times (n_k + \mu_k t_k^l)$$

These form "polyhedral convex cone in the space of the contact force and torque"

"contact wrench"

~~weighting terms~~ nonnegative scalar

Friction cone approximation


$L = 4$ usually

Stable = sufficient friction exists at contact

· Arbitrary forces, independent of normal force

· Stability is wrt specific $(f, \tau)$

· Strongly stable if $(-f_G, -\tau_G)$ is inside polyhedral convex cone of contact wrench

Kind of like grasp map?

SVDs to compute ~~grasp maps~~ stability

non-zero singular values to check for force closure

Convex hull of vectors:

$$\mathrm{conv}(\{v_i\}) = \left\{ \sum k_i v_i \mid k_i \geq 0, \sum k_i = 1 \right\}$$

Set of vectors positively spans $\mathbb{R}^n$ iff origin is in interior of convex hull

$$\mathrm{pos}(\{v_i\}) = \mathbb{R}^n \longleftrightarrow 0 \in \mathrm{int}(\mathrm{conv}(\{v_i\}))$$

It takes at least $n+1$ vectors to positively span $\mathbb{R}^n$

vectors will be wrenches

$$w = \begin{bmatrix} f \\ p \times f \end{bmatrix}$$

from arbitrary reference frame

pHRI: Think of human as object to be manipulated?

Does this help ~~[illegible]~~ deal with contact changes?

~~Vai thesis~~

- maximize contact wrench set margin — convex function w/ linear constraints $0 \le \varepsilon \le \bar{a}_i[\cdots]$
- minimize upper bound of centroidal angular momentum — convex function
- smooth motion

→ QP! — when polytope — linear programming or second-order cone, depending on representation of CoM limits

or second-order cone program when ellipsoid

How do they set reference for the optimization? — desired motion — here?

or in contact positions

convex quadratic function of decision variables

slack variable for angular momentum

$$\min_{\substack{r,\dot{r},\ddot{r},s,\\ \varepsilon,k_0,\dot{k}_0}} \sum_i C_s\, s[i] - C_\varepsilon \varepsilon[i] + C_{\ddot{r}}\, \ddot{r}[i]^T \ddot{r}[i]$$

weighting terms

This is over trajectory, like most other works.

Can we make this reactive?

Because contact points are defined, in motion retargeting (real-time) but we don't know where they will be (?)

— What's the application of this?

Human-robot collaboration

Teleoperation?

~~If we know where they'll be~~

If we don't need real-time? We know where the contacts will be over entire trajectory and we ~~...~~ don't have to optimize locally.

In our QP: $X$: $u$ : motor torques; $\ddot{q}$ : joint accelerations; $f$ : contact forces (replaced by $\lambda$, friction cone vector weights)

Basically all tasks formulated in terms of task Jacobian $J_g$

How $\dot{q}$ affects task. — linearized approximation

Can we do that here?

$$\frac{1}{2} x^T Q x + C^T x + C^T C$$

$$CWS = \{ W \mid a_i^T W \le 0,\ i = 1, \ldots, n_a \}$$

$$a_i^T(\dot{h} - W_g) \le 0,\ i = 1 \ldots$$

$$\dot{h} = \begin{bmatrix} m\ddot{r} \\ \dot{k}_0 \end{bmatrix},\ W_g = \begin{bmatrix} mg \\ r \times mg \end{bmatrix}$$

Feasible if

$$a_i^T \begin{bmatrix} m\boxed{\ddot{r}} - mg \\ \dot{k}_0 - r \times mg \end{bmatrix} \le 0$$

This is what we control

→ Add robustness by maximizing $\varepsilon$-ball

6×6 matrix that transforms wrench at $p_w$ to origin at world frame

$$B_\varepsilon = \{ \dot{h} - W_g + T(p_w, I) W \mid W^T Q_W W \le \varepsilon^2 \} \subset CWS$$

$$\varepsilon_{max} = \min_{i=1,\ldots,n_a} \bar{a}_i \begin{bmatrix} m\ddot{r} - mg \\ \dot{k}_0 - r \times mg \end{bmatrix}$$

where

$$\bar{a}_i = -\left[ a_i^T T(p_w, I) Q_W^{-1} T^T(p_w, I) a_i \right]^{-\frac{1}{2}} a_i^T$$